Computer algebra independent integration tests

4-Trig-functions/4.3-Tangent/4.3.7-d-trig-^m-a+b-c-tan-^n-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [499]. This is test number [106].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (499)	% 0. (0)
Mathematica	% 99.6 (497)	% 0.4 (2)
Maple	% 81.76 (408)	% 18.24 (91)
Maxima	% 37.88 (189)	% 62.12 (310)
Fricas	% 80.76 (403)	% 19.24 (96)
Sympy	% 16.23 (81)	% 83.77 (418)
Giac	% 56.71 (283)	% 43.29 (216)

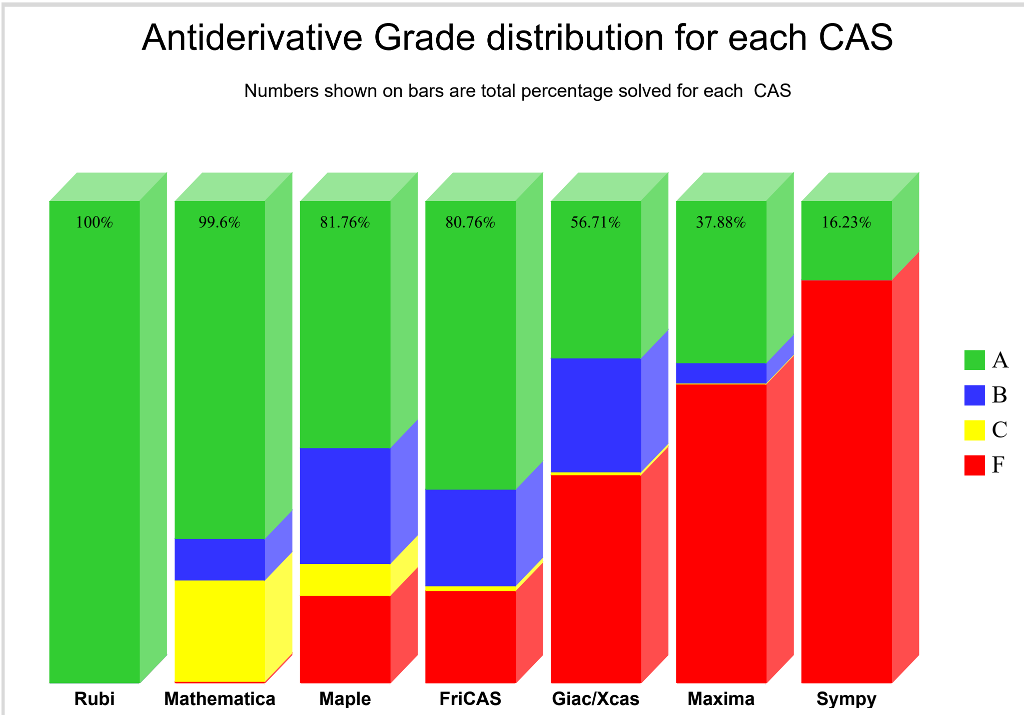
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

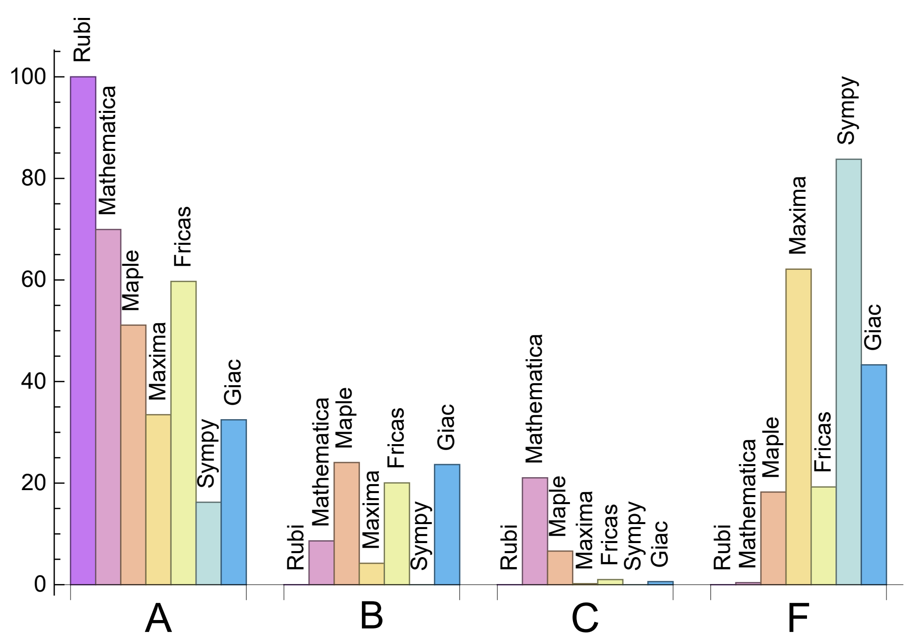
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	69.94	8.62	21.04	0.4
Maple	51.1	24.05	6.61	18.24
Maxima	33.47	4.21	0.2	62.12
Fricas	59.72	20.04	1.	19.24
Sympy	16.23	0.	0.	83.77
Giac	32.46	23.65	0.6	43.29

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	109.13	0.98	90.	1.
Mathematica	2.06	199.24	1.89	93.	1.
Maple	0.47	6074.87	35.98	153.	1.56
Maxima	1.32	148.97	2.13	89.	1.47
Fricas	3.67	945.59	7.41	510.	6.48
Sympy	18.88	489.25	5.55	117.	1.81
Giac	3.86	610.52	6.54	205.	1.94

1.4 list of integrals that has no closed form antiderivative

{174, 178, 420, 424, 486, 487, 488, 489, 490, 494, 495, 499}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {48, 84, 85, 97, 136, 145, 148, 152, 153, 154, 155, 157, 158, 159, 160, 164, 165, 170, 171, 172, 173, 176, 304, 305, 318, 330, 331, 339, 342, 343, 345, 354, 355, 356, 357, 360, 368, 369, 370, 371, 372, 422, 437, 438, 475, 481, 484, 485, 496, 497, 498}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

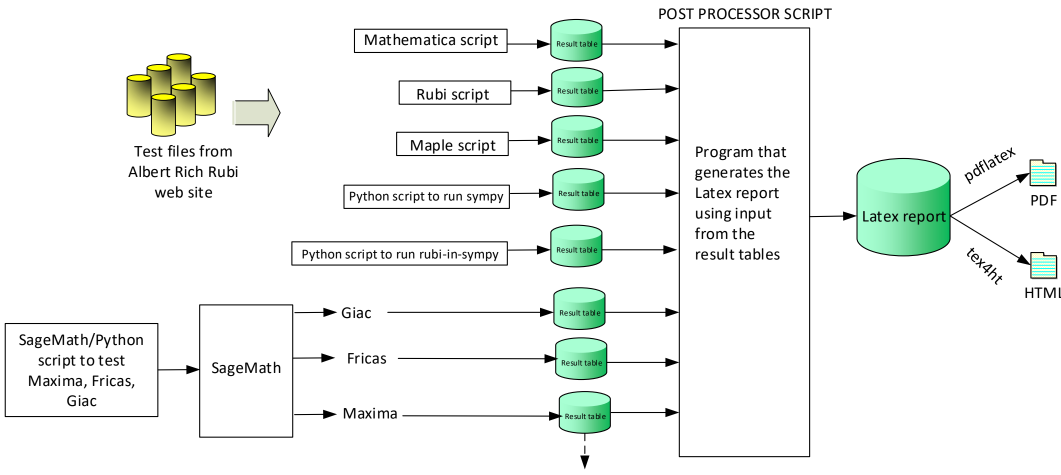
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 13, 14, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 104, 105, 106, 109, 116, 117, 118, 121, 124, 125, 126, 127, 128, 129, 130, 133, 137, 138, 139, 140, 141, 142, 149, 150, 151, 154, 155, 156, 161, 162, 163, 166, 167, 168, 169, 172, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 198, 199, 200, 201, 202, 203, 206, 207, 209, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 289, 292, 293, 294, 295, 296, 297, 298, 306, 307, 308, 309, 310, 311, 320, 321, 322, 323, 324, 325, 329, 334, 341, 353, 359, 360, 361, 362, 363, 364, 365, 366, 377, 381, 382, 383, 384, 385, 389, 390, 391, 393, 394, 395, 396, 397, 398, 400, 401, 403, 404, 406, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 476, 477, 478, 479, 480, 482, 483, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 499 }

B grade: { 33, 34, 35, 47, 48, 57, 58, 59, 60, 95, 96, 97, 107, 108, 119, 120, 131, 132, 143, 144, 145, 153, 157, 158, 160, 173, 176, 204, 205, 249, 287, 288, 290, 291, 368, 369, 370, 371, 372, 422, 450, 475, 497 }

C grade: { 8, 10, 11, 12, 16, 17, 18, 98, 99, 100, 101, 102, 103, 110, 111, 112, 113, 114, 115, 122, 123, 134, 135, 136, 146, 147, 148, 152, 159, 164, 165, 170, 171, 195, 196, 197, 208, 210, 270, 299, 300, 301, 302, 303, 304, 305, 312, 313, 314, 315, 316, 317, 318, 319, 326, 327, 328, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 354, 355, 356, 357, 358, 374, 375, 376, 378, 379, 380, 386, 387, 388, 392, 399, 402, 405, 407, 437, 438, 449, 481, 484, 485, 496, 498 }

F grade: { 367, 373}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 47, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 63, 64, 65, 66, 70, 71, 72, 75, 76, 77, 78, 79, 82, 89, 90, 91, 116, 117, 124, 125, 126, 127, 130, 136, 137, 138, 139, 140, 141, 142, 148, 149, 150, 151, 174, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 235, 236, 248, 249, 252, 253, 254, 255, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 320, 321, 322, 326, 327, 328, 329, 333, 334, 335, 339, 340, 341, 342, 346, 347, 348, 355, 374, 375, 376, 377, 378, 380, 382, 383, 384, 385, 386, 390, 396, 397, 420, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 438, 440, 441, 442, 443, 444, 445, 446, 448, 449, 452, 453, 454, 455, 457, 458, 459, 460, 464, 465, 466, 467, 469, 470, 471, 472, 486, 487, 488, 489, 490, 494, 495, 499 }

B grade: { 44, 45, 46, 50, 59, 60, 61, 62, 67, 68, 69, 73, 74, 80, 81, 83, 84, 85, 86, 87, 88, 92, 93, 94, 95, 96, 97, 100, 104, 105, 106, 107, 108, 109, 112, 118, 119, 120, 121, 128, 129, 131, 132, 133, 143, 144, 145, 146, 147, 198, 204, 205, 206, 216, 229, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 256, 279, 296, 297, 298, 299, 300, 301, 302, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 319, 323, 324, 325, 336, 337, 338, 349, 350, 351, 352, 353, 354, 379, 381, 389, 393, 394, 400, 401, 403, 404, 437, 439, 447, 450, 451, 456, 461, 462, 463, 468, 473 }

C grade: { 29, 98, 99, 101, 102, 103, 110, 111, 113, 114, 115, 122, 123, 134, 135, 169, 303, 304, 305, 316, 317, 318, 330, 331, 332, 343, 344, 345, 387, 388, 392, 399, 479 }

F grade: { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 170, 171, 172, 173, 175, 176, 177, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 391, 395, 398, 402, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 421, 422, 423, 474, 475, 476, 477, 478, 480, 481, 482, 483, 484, 485, 491, 492, 493, 496, 497, 498 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 116, 117, 118, 129, 130, 141, 142, 167, 168, 169, 174, 178, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 224, 225, 226, 227, 228, 229, 237, 240, 241, 250, 251, 252, 253, 262, 264, 273, 274, 276, 278, 279, 281, 283, 284, 285, 286, 287, 288, 289, 291, 374, 375, 376, 377, 380, 381, 382, 383, 384, 420, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 477, 478, 479, 486, 487, 488, 489, 490, 494, 495, 499

B grade: { 128, 140, 179, 180, 238, 239, 242, 257, 258, 259, 261, 263, 265, 266, 267, 269, 270, 271, 272, 277, 282 }

C grade: { 290 }

F grade: { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 119, 120, 121, 122, 123, 124, 125, 126, 127, 131, 132, 133, 134, 135, 136, 137, 138, 139, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 170, 171, 172, 173, 175, 176, 177, 217, 218, 219, 220, 221, 222, 223, 230, 231, 232, 233, 234, 235, 236, 243, 244, 245, 246, 247, 248, 249, 254, 255, 256, 260, 268, 275, 280, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 378, 379, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 421, 422, 423, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 480, 481, 482, 483, 484, 485, 491, 492, 493, 496, 497, 498 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 13, 14, 15, 16, 17, 18, 29, 30, 31, 32, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 68, 69, 70, 74, 75, 76, 92, 93, 94, 95, 96, 97, 100, 104, 105, 106, 107, 108, 109, 112, 114, 115, 116, 117, 118, 119, 120, 121, 124, 125, 126, 127, 128, 129, 130, 132, 136, 137, 138, 140, 141, 142, 149, 167, 168, 169, 174, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 230, 231, 233, 234, 235, 236, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 275, 277, 278, 282, 283, 284, 285, 286, 289, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 338, 339, 341, 342, 343, 344, 345, 353, 357, 358, 374, 375, 376, 377, 380, 381, 382, 383, 384, 389, 390, 391, 393, 394, 395, 396, 397, 398, 420, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 460, 461, 462, 463, 464, 465, 472, 473, 477, 478, 479, 486, 487, 488, 489, 490, 494, 495, 499 }

B grade: { 33, 34, 35, 47, 48, 60, 65, 66, 67, 71, 72, 73, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 98, 99, 101, 102, 103, 111, 113, 122, 123, 131, 133, 135, 143, 144, 145, 148, 224, 228, 229, 232, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 256, 274, 276, 279, 280, 281, 287, 288, 333, 334, 335, 336, 337, 340, 346, 347, 348, 349, 350, 351, 352, 354, 355, 356, 385, 386, 400, 401, 402, 403, 404, 405, 426, 458, 459, 466, 467, 468, 469, 470, 471 }

C grade: { 290, 291, 292, 378, 379 }

F grade: { 7, 8, 9, 10, 11, 12, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 110, 134, 139, 146, 147, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 170, 171, 172, 173, 175, 176, 177, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 387, 388, 392, 399, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 421, 422, 423, 474, 475, 476, 480, 481, 482, 483, 484, 485, 491, 492, 493, 496, 497, 498 }

2.1.6 Sympy

A grade: { 39, 51, 64, 76, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 224, 225, 226, 230, 231, 232, 233, 237, 238, 239, 250, 251, 252, 253, 254, 255, 260, 268, 275, 278, 280, 289, 335, 348, 374, 375, 376, 377, 380, 381, 382, 383, 384, 385, 433, 488, 494 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 203, 209, 210, 216, 223, 227, 228, 229, 234, 235, 236, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 276, 277, 279, 281, 282, 283, 284, 285, 286, 287, 288, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 378, 379, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 489, 490, 491, 492, 493, 495, 496, 497, 498, 499 }

2.1.7 Giac

A grade: { 1, 2, 3, 7, 8, 9, 10, 16, 17, 18, 31, 32, 40, 41, 42, 44, 45, 52, 53, 54, 57, 61, 62, 63, 65, 66, 67, 70, 74, 76, 77, 78, 79, 82, 87, 88, 89, 90, 91, 174, 178, 182, 183, 184, 188, 201, 208, 213, 214, 227, 230, 231, 232, 233, 234, 235, 236, 240, 243, 244, 245, 246, 247, 248, 249, 255, 256, 257, 258, 259, 260, 261, 263, 266, 267, 268, 269, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 289, 293, 294, 295, 306, 307, 308, 320, 321, 322, 333, 334, 335, 346, 347, 348, 378, 379, 380, 385, 386, 389, 390, 396, 397, 400, 401, 420, 424, 425, 426, 427, 428, 431, 432, 433, 437, 438, 439, 440, 444, 445, 446, 450, 451, 452, 453, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 473, 486, 487, 488, 489, 490, 494, 495, 499 }

B grade: { 4, 5, 6, 13, 14, 15, 33, 34, 35, 36, 37, 38, 39, 46, 47, 48, 50, 51, 55, 56, 58, 59, 60, 64, 68, 69, 71, 72, 73, 75, 80, 81, 83, 84, 85, 86, 93, 94, 118, 179, 180, 181, 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 202, 203, 204, 205, 206, 207, 209, 210, 211, 212, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 229, 237, 238, 239, 241, 242, 250, 251, 252, 253, 254, 262, 264, 265, 270, 271, 288, 374, 375, 376, 377, 381, 382, 383, 384, 403, 404, 429, 434, 435, 436, 447, 448, 449, 454, 455, 467, 472 }

C grade: { 290, 291, 292 }

F grade: { 11, 12, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 43, 49, 92, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 272, 283, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 387, 388, 391, 392, 393, 394, 395, 398, 399, 402, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 421, 422, 423, 430, 441, 442, 443, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 491, 492, 493, 496, 497, 498 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	56	58	63	180	0	80
normalized size	1	1.	0.57	0.59	0.64	1.84	0.	0.82
time (sec)	N/A	0.039	0.373	0.146	1.626	1.939	0.	1.42

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	47	48	46	135	0	59
normalized size	1	1.	0.77	0.79	0.75	2.21	0.	0.97
time (sec)	N/A	0.029	0.104	0.02	1.632	1.953	0.	1.354

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	37	26	100	0	38
normalized size	1	1.	1.	1.16	0.81	3.12	0.	1.19
time (sec)	N/A	0.017	0.039	0.024	1.635	1.959	0.	1.259

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	39	47	45	119	0	113
normalized size	1	1.	1.26	1.52	1.45	3.84	0.	3.65
time (sec)	N/A	0.023	0.082	0.043	1.637	1.851	0.	1.326

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	56	64	62	177	0	300
normalized size	1	1.	0.85	0.97	0.94	2.68	0.	4.55
time (sec)	N/A	0.037	0.392	0.02	1.647	1.895	0.	1.628

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	68	74	89	207	0	392
normalized size	1	1.	0.7	0.76	0.92	2.13	0.	4.04
time (sec)	N/A	0.039	0.246	0.023	1.69	2.052	0.	1.999

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	199	266	240	0	0	412
normalized size	1	1.	0.55	0.73	0.66	0.	0.	1.13
time (sec)	N/A	0.146	0.829	0.044	1.671	0.	0.	1.733

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	54	236	189	0	0	356
normalized size	1	1.	0.19	0.83	0.66	0.	0.	1.24
time (sec)	N/A	0.127	0.078	0.019	1.722	0.	0.	1.427

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	161	208	180	0	0	274
normalized size	1	1.	0.63	0.82	0.71	0.	0.	1.07
time (sec)	N/A	0.115	0.25	0.021	1.702	0.	0.	1.345

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	43	211	170	0	0	355
normalized size	1	1.	0.17	0.83	0.67	0.	0.	1.39
time (sec)	N/A	0.115	0.034	0.026	1.611	0.	0.	1.322

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	45	236	220	0	0	0
normalized size	1	1.	0.15	0.79	0.74	0.	0.	0.
time (sec)	N/A	0.128	0.074	0.023	1.687	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	45	272	232	0	0	0
normalized size	1	1.	0.12	0.75	0.64	0.	0.	0.
time (sec)	N/A	0.148	0.058	0.023	1.663	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	86	84	107	247	0	1381
normalized size	1	1.	0.47	0.46	0.59	1.36	0.	7.59
time (sec)	N/A	0.064	0.719	0.037	1.668	2.539	0.	14.408

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	66	64	72	163	0	1457
normalized size	1	1.	0.6	0.58	0.65	1.48	0.	13.25
time (sec)	N/A	0.042	0.735	0.016	1.574	2.453	0.	4.772

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	41	42	35	88	0	338
normalized size	1	1.	0.82	0.84	0.7	1.76	0.	6.76
time (sec)	N/A	0.021	0.091	0.021	1.638	2.075	0.	1.406

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	43	40	36	93	0	65
normalized size	1	1.	0.84	0.78	0.71	1.82	0.	1.27
time (sec)	N/A	0.021	0.05	0.026	1.537	2.018	0.	1.531

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	45	63	68	162	0	177
normalized size	1	1.	0.38	0.53	0.57	1.36	0.	1.49
time (sec)	N/A	0.045	0.049	0.02	1.637	1.932	0.	1.877

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	45	83	95	225	0	265
normalized size	1	1.	0.25	0.45	0.52	1.23	0.	1.45
time (sec)	N/A	0.065	0.033	0.021	1.653	1.923	0.	2.791

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	62	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.108	0.615	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	60	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.068	0.118	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.038	0.119	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	60	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.05	0.122	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	60	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.067	0.116	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	62	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.068	0.115	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	57	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.041	10.709	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	49	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.049	0.26	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.041	0.479	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	53	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.031	0.248	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	18076	0	59	0	0
normalized size	1	1.	1.	564.88	0.	1.84	0.	0.
time (sec)	N/A	0.019	0.024	3.234	0.	1.921	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	104	92	84	161	0	0
normalized size	1	1.	1.49	1.31	1.2	2.3	0.	0.
time (sec)	N/A	0.062	0.063	0.105	1.052	1.99	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	72	72	59	111	0	103
normalized size	1	1.	1.5	1.5	1.23	2.31	0.	2.15
time (sec)	N/A	0.047	0.049	0.063	1.065	1.933	0.	1.487

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	46	52	42	65	0	55
normalized size	1	1.	1.64	1.86	1.5	2.32	0.	1.96
time (sec)	N/A	0.026	0.044	0.033	0.95	1.794	0.	1.421

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	51	36	54	162	0	80
normalized size	1	1.	2.04	1.44	2.16	6.48	0.	3.2
time (sec)	N/A	0.028	0.029	0.039	0.989	1.966	0.	1.387

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	123	76	103	325	0	246
normalized size	1	1.	2.41	1.49	2.02	6.37	0.	4.82
time (sec)	N/A	0.053	0.045	0.08	1.039	1.967	0.	1.452

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	276	120	136	481	0	346
normalized size	1	1.	3.49	1.52	1.72	6.09	0.	4.38
time (sec)	N/A	0.07	6.056	0.074	1.056	2.087	0.	1.568

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	89	122	150	230	0	10661
normalized size	1	1.	0.87	1.2	1.47	2.25	0.	104.52
time (sec)	N/A	0.118	0.342	0.091	1.443	1.997	0.	6.028

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	58	102	111	176	0	5873
normalized size	1	1.	0.78	1.38	1.5	2.38	0.	79.36
time (sec)	N/A	0.073	0.344	0.062	1.58	1.959	0.	4.283

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	81	69	131	0	533
normalized size	1	1.	0.93	1.76	1.5	2.85	0.	11.59
time (sec)	N/A	0.048	0.217	0.033	1.545	1.862	0.	1.556

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	28	29	31	46	20	340
normalized size	1	1.	1.47	1.53	1.63	2.42	1.05	17.89
time (sec)	N/A	0.013	0.013	0.003	1.564	1.888	0.22	1.44

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	32	82	0	35
normalized size	1	1.	1.	0.96	1.33	3.42	0.	1.46
time (sec)	N/A	0.032	0.021	0.078	1.062	1.929	0.	1.562

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	60	54	54	163	0	72
normalized size	1	1.	1.43	1.29	1.29	3.88	0.	1.71
time (sec)	N/A	0.043	0.069	0.077	1.059	1.799	0.	1.511

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	106	83	80	236	0	107
normalized size	1	1.	1.66	1.3	1.25	3.69	0.	1.67
time (sec)	N/A	0.053	0.049	0.072	1.117	1.876	0.	1.472

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	97	185	140	258	0	0
normalized size	1	1.	0.91	1.73	1.31	2.41	0.	0.
time (sec)	N/A	0.107	0.719	0.084	0.984	2.059	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	72	155	108	193	0	194
normalized size	1	1.	0.9	1.94	1.35	2.41	0.	2.42
time (sec)	N/A	0.083	0.52	0.049	1.073	1.936	0.	1.747

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	48	125	96	136	0	127
normalized size	1	1.	0.89	2.31	1.78	2.52	0.	2.35
time (sec)	N/A	0.046	0.289	0.049	0.98	1.964	0.	1.762

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	66	124	92	228	0	197
normalized size	1	1.	1.27	2.38	1.77	4.38	0.	3.79
time (sec)	N/A	0.055	0.167	0.05	1.054	2.036	0.	1.708

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	376	100	150	414	0	339
normalized size	1	1.	4.59	1.22	1.83	5.05	0.	4.13
time (sec)	N/A	0.109	6.13	0.061	1.142	2.028	0.	1.644

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	447	183	220	699	0	564
normalized size	1	1.	3.63	1.49	1.79	5.68	0.	4.59
time (sec)	N/A	0.13	6.188	0.058	1.011	2.152	0.	1.683

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	96	199	176	293	0	0
normalized size	1	1.	0.79	1.63	1.44	2.4	0.	0.
time (sec)	N/A	0.131	1.476	0.047	1.494	2.069	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	71	168	117	224	0	1905
normalized size	1	1.	0.84	1.98	1.38	2.64	0.	22.41
time (sec)	N/A	0.107	0.686	0.045	1.589	1.956	0.	2.299

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	73	87	78	117	68	516
normalized size	1	1.	1.59	1.89	1.7	2.54	1.48	11.22
time (sec)	N/A	0.033	0.718	0.003	1.466	1.919	0.431	1.466

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	44	48	55	157	0	59
normalized size	1	1.	0.96	1.04	1.2	3.41	0.	1.28
time (sec)	N/A	0.054	0.468	0.046	1.023	1.874	0.	1.642

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	59	81	89	224	0	113
normalized size	1	1.	0.84	1.16	1.27	3.2	0.	1.61
time (sec)	N/A	0.072	0.442	0.082	0.979	1.955	0.	1.654

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	88	136	119	339	0	173
normalized size	1	1.	0.95	1.46	1.28	3.65	0.	1.86
time (sec)	N/A	0.09	0.774	0.059	0.966	2.038	0.	1.677

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	177	205	0	675	0	509
normalized size	1	1.	1.51	1.75	0.	5.77	0.	4.35
time (sec)	N/A	0.184	3.049	0.062	0.	2.387	0.	1.423

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	149	107	0	473	0	243
normalized size	1	1.	1.77	1.27	0.	5.63	0.	2.89
time (sec)	N/A	0.123	0.653	0.055	0.	2.47	0.	1.416

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	121	63	0	350	0	109
normalized size	1	1.	2.02	1.05	0.	5.83	0.	1.82
time (sec)	N/A	0.055	0.26	0.046	0.	1.782	0.	1.408

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	144	75	0	460	0	149
normalized size	1	1.	2.4	1.25	0.	7.67	0.	2.48
time (sec)	N/A	0.07	0.205	0.063	0.	2.091	0.	1.387

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	195	189	0	815	0	285
normalized size	1	1.	2.19	2.12	0.	9.16	0.	3.2
time (sec)	N/A	0.103	0.669	0.078	0.	2.281	0.	1.435

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	326	344	0	1523	0	505
normalized size	1	1.	2.51	2.65	0.	11.72	0.	3.88
time (sec)	N/A	0.176	6.25	0.081	0.	2.521	0.	1.477

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	140	545	0	1214	0	394
normalized size	1	1.	0.79	3.06	0.	6.82	0.	2.21
time (sec)	N/A	0.293	0.562	0.075	0.	2.278	0.	1.393

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	99	304	0	898	0	257
normalized size	1	1.	0.77	2.36	0.	6.96	0.	1.99
time (sec)	N/A	0.152	0.262	0.063	0.	2.077	0.	1.444

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	69	137	0	674	0	153
normalized size	1	1.	0.84	1.67	0.	8.22	0.	1.87
time (sec)	N/A	0.097	0.14	0.062	0.	1.882	0.	1.401

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	49	52	0	406	280	231
normalized size	1	1.	0.98	1.04	0.	8.12	5.6	4.62
time (sec)	N/A	0.075	0.066	0.019	0.	1.689	3.631	1.395

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	46	0	606	0	84
normalized size	1	1.	1.	0.96	0.	12.62	0.	1.75
time (sec)	N/A	0.062	0.118	0.063	0.	1.821	0.	1.409

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	73	107	0	884	0	131
normalized size	1	1.	0.96	1.41	0.	11.63	0.	1.72
time (sec)	N/A	0.09	0.292	0.075	0.	1.82	0.	1.443

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	103	191	0	1319	0	204
normalized size	1	1.	0.98	1.82	0.	12.56	0.	1.94
time (sec)	N/A	0.115	0.799	0.083	0.	1.966	0.	1.469

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	215	388	0	1330	0	757
normalized size	1	1.	1.05	1.9	0.	6.52	0.	3.71
time (sec)	N/A	0.309	3.717	0.082	0.	2.59	0.	1.431

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	182	269	0	1015	0	497
normalized size	1	1.	1.37	2.02	0.	7.63	0.	3.74
time (sec)	N/A	0.18	2.986	0.078	0.	2.208	0.	1.389

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	146	114	0	699	0	207
normalized size	1	1.	1.45	1.13	0.	6.92	0.	2.05
time (sec)	N/A	0.073	0.764	0.067	0.	1.945	0.	1.373

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	184	179	0	1102	0	360
normalized size	1	1.	1.67	1.63	0.	10.02	0.	3.27
time (sec)	N/A	0.128	0.818	0.084	0.	2.684	0.	1.404

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	218	229	0	1565	0	559
normalized size	1	1.	1.48	1.56	0.	10.65	0.	3.8
time (sec)	N/A	0.181	6.182	0.101	0.	2.526	0.	1.424

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	392	428	0	2477	0	740
normalized size	1	1.	1.87	2.04	0.	11.8	0.	3.52
time (sec)	N/A	0.275	6.335	0.109	0.	2.775	0.	1.439

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	136	411	0	1600	0	359
normalized size	1	1.	0.69	2.1	0.	8.16	0.	1.83
time (sec)	N/A	0.25	1.515	0.092	0.	2.479	0.	1.392

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	111	240	0	1300	0	892
normalized size	1	1.	0.8	1.74	0.	9.42	0.	6.46
time (sec)	N/A	0.156	1.588	0.077	0.	2.277	0.	1.568

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	88	160	0	865	2086	171
normalized size	1	1.	0.91	1.65	0.	8.92	21.51	1.76
time (sec)	N/A	0.083	1.025	0.023	0.	1.854	40.459	1.354

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	83	75	0	888	0	126
normalized size	1	1.	1.01	0.91	0.	10.83	0.	1.54
time (sec)	N/A	0.074	0.57	0.08	0.	1.974	0.	1.414

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	112	169	0	1374	0	192
normalized size	1	1.	0.97	1.46	0.	11.84	0.	1.66
time (sec)	N/A	0.146	0.763	0.095	0.	2.21	0.	1.38

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	151	281	0	2028	0	286
normalized size	1	1.	0.83	1.54	0.	11.14	0.	1.57
time (sec)	N/A	0.223	1.617	0.102	0.	2.235	0.	1.392

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	278	844	0	2313	0	1195
normalized size	1	1.	1.05	3.2	0.	8.76	0.	4.53
time (sec)	N/A	0.411	5.48	0.089	0.	3.158	0.	1.675

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	230	504	0	1723	0	760
normalized size	1	1.	1.28	2.8	0.	9.57	0.	4.22
time (sec)	N/A	0.246	5.439	0.082	0.	2.962	0.	1.604

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	170	221	0	1249	0	301
normalized size	1	1.	1.23	1.6	0.	9.05	0.	2.18
time (sec)	N/A	0.091	1.61	0.073	0.	2.724	0.	1.635

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	247	408	0	2356	0	717
normalized size	1	1.	1.49	2.46	0.	14.19	0.	4.32
time (sec)	N/A	0.217	3.002	0.09	0.	4.473	0.	1.646

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	205	205	286	435	0	3227	0	836
normalized size	1	1.	1.4	2.12	0.	15.74	0.	4.08
time (sec)	N/A	0.293	6.533	0.109	0.	3.791	0.	1.644

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	259	259	468	560	0	3969	0	1241
normalized size	1	1.	1.81	2.16	0.	15.32	0.	4.79
time (sec)	N/A	0.376	6.561	0.112	0.	3.854	0.	1.665

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	194	598	0	2664	0	1793
normalized size	1	1.	0.78	2.39	0.	10.66	0.	7.17
time (sec)	N/A	0.33	0.921	0.082	0.	3.269	0.	3.302

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	164	430	0	2402	0	381
normalized size	1	1.	0.85	2.23	0.	12.45	0.	1.97
time (sec)	N/A	0.246	2.595	0.081	0.	2.876	0.	1.611

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	138	350	0	1643	0	288
normalized size	1	1.	0.92	2.33	0.	10.95	0.	1.92
time (sec)	N/A	0.159	1.906	0.024	0.	1.986	0.	1.296

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	144	108	0	1300	0	147
normalized size	1	1.	1.29	0.96	0.	11.61	0.	1.31
time (sec)	N/A	0.088	0.827	0.086	0.	2.343	0.	1.621

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	146	235	0	1999	0	236
normalized size	1	1.	0.95	1.53	0.	12.98	0.	1.53
time (sec)	N/A	0.206	1.632	0.099	0.	2.552	0.	1.624

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	346	380	0	2931	0	355
normalized size	1	1.	1.5	1.65	0.	12.69	0.	1.54
time (sec)	N/A	0.303	1.674	0.102	0.	2.742	0.	1.623

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	208	7044	0	941	0	0
normalized size	1	1.	1.29	43.75	0.	5.84	0.	0.
time (sec)	N/A	0.165	3.252	0.752	0.	3.365	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	170	4296	0	674	0	410
normalized size	1	1.	1.5	38.02	0.	5.96	0.	3.63
time (sec)	N/A	0.105	1.054	0.243	0.	3.453	0.	1.656

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	140	144	0	510	0	176
normalized size	1	1.	1.94	2.	0.	7.08	0.	2.44
time (sec)	N/A	0.057	0.544	0.069	0.	3.392	0.	1.619

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	295	719	0	1319	0	0
normalized size	1	1.	3.51	8.56	0.	15.7	0.	0.
time (sec)	N/A	0.087	7.383	0.251	0.	2.863	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	460	2075	0	2132	0	0
normalized size	1	1.	3.62	16.34	0.	16.79	0.	0.
time (sec)	N/A	0.139	3.452	0.31	0.	6.426	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	1059	5378	0	3058	0	0
normalized size	1	1.	5.66	28.76	0.	16.35	0.	0.
time (sec)	N/A	0.232	6.53	0.296	0.	8.878	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	330	2498	0	4998	0	0
normalized size	1	1.	1.75	13.22	0.	26.44	0.	0.
time (sec)	N/A	0.237	3.662	0.413	0.	89.259	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	273	1343	0	4471	0	0
normalized size	1	1.	2.13	10.49	0.	34.93	0.	0.
time (sec)	N/A	0.135	3.676	0.193	0.	8.631	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	203	169	0	1046	0	0
normalized size	1	1.	2.39	1.99	0.	12.31	0.	0.
time (sec)	N/A	0.054	0.856	0.04	0.	2.127	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	156	1215	0	834	0	0
normalized size	1	1.	2.36	18.41	0.	12.64	0.	0.
time (sec)	N/A	0.079	2.059	0.428	0.	2.333	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	204	2441	0	1076	0	0
normalized size	1	1.	2.04	24.41	0.	10.76	0.	0.
time (sec)	N/A	0.095	4.375	0.349	0.	3.281	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	287	3769	0	1428	0	0
normalized size	1	1.	2.04	26.73	0.	10.13	0.	0.
time (sec)	N/A	0.128	3.285	0.336	0.	8.094	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	233	2399	0	1053	0	0
normalized size	1	1.	1.03	10.57	0.	4.64	0.	0.
time (sec)	N/A	0.216	5.523	0.453	0.	5.675	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	188	1104	0	768	0	0
normalized size	1	1.	1.01	5.94	0.	4.13	0.	0.
time (sec)	N/A	0.165	1.78	0.183	0.	4.518	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	170	359	0	667	0	0
normalized size	1	1.	1.5	3.18	0.	5.9	0.	0.
time (sec)	N/A	0.082	1.214	0.053	0.	3.903	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	418	1249	0	1905	0	0
normalized size	1	1.	3.29	9.83	0.	15.	0.	0.
time (sec)	N/A	0.141	4.816	0.164	0.	7.219	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	1022	2904	0	2515	0	0
normalized size	1	1.	6.12	17.39	0.	15.06	0.	0.
time (sec)	N/A	0.206	6.592	0.192	0.	7.955	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	409	6194	0	3437	0	0
normalized size	1	1.	1.83	27.78	0.	15.41	0.	0.
time (sec)	N/A	0.362	4.855	0.352	0.	10.117	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	278	2630	0	0	0	0
normalized size	1	1.	1.25	11.85	0.	0.	0.	0.
time (sec)	N/A	0.322	4.749	0.416	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	324	2261	0	4698	0	0
normalized size	1	1.	1.96	13.7	0.	28.47	0.	0.
time (sec)	N/A	0.202	5.177	0.235	0.	60.221	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	233	297	0	1382	0	0
normalized size	1	1.	1.86	2.38	0.	11.06	0.	0.
time (sec)	N/A	0.1	1.334	0.026	0.	5.671	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	220	1355	0	964	0	0
normalized size	1	1.	2.2	13.55	0.	9.64	0.	0.
time (sec)	N/A	0.099	2.55	0.203	0.	3.408	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	177	4594	0	1231	0	0
normalized size	1	1.	1.09	28.36	0.	7.6	0.	0.
time (sec)	N/A	0.137	1.74	0.586	0.	8.767	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	213	6988	0	1624	0	0
normalized size	1	1.	1.09	35.65	0.	8.29	0.	0.
time (sec)	N/A	0.171	2.047	0.868	0.	26.699	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	112	169	289	292	0	0
normalized size	1	1.	0.78	1.17	2.01	2.03	0.	0.
time (sec)	N/A	0.144	2.023	0.286	1.04	2.043	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	74	104	143	174	0	0
normalized size	1	1.	0.84	1.18	1.62	1.98	0.	0.
time (sec)	N/A	0.101	1.433	0.221	1.027	1.856	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	52	78	47	104	0	119
normalized size	1	1.	1.41	2.11	1.27	2.81	0.	3.22
time (sec)	N/A	0.047	0.603	0.069	0.984	1.761	0.	1.778

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	226	351	0	346	0	0
normalized size	1	1.	5.38	8.36	0.	8.24	0.	0.
time (sec)	N/A	0.068	2.349	0.21	0.	2.454	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	303	2801	0	699	0	0
normalized size	1	1.	3.33	30.78	0.	7.68	0.	0.
time (sec)	N/A	0.115	2.682	0.25	0.	2.542	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	278	6334	0	1071	0	0
normalized size	1	1.	1.94	44.29	0.	7.49	0.	0.
time (sec)	N/A	0.16	4.331	0.238	0.	2.699	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	314	1169	0	1864	0	0
normalized size	1	1.	2.15	8.01	0.	12.77	0.	0.
time (sec)	N/A	0.167	4.011	0.277	0.	23.758	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	270	795	0	1667	0	0
normalized size	1	1.	2.9	8.55	0.	17.92	0.	0.
time (sec)	N/A	0.107	3.005	0.206	0.	3.52	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	67	0	311	0	0
normalized size	1	1.	1.	1.46	0.	6.76	0.	0.
time (sec)	N/A	0.034	0.07	0.029	0.	2.072	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	49	57	0	113	0	0
normalized size	1	1.	1.63	1.9	0.	3.77	0.	0.
time (sec)	N/A	0.07	0.248	0.178	0.	2.089	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	68	86	0	211	0	0
normalized size	1	1.	0.92	1.16	0.	2.85	0.	0.
time (sec)	N/A	0.09	0.414	0.194	0.	3.122	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	90	148	0	339	0	0
normalized size	1	1.	0.73	1.2	0.	2.76	0.	0.
time (sec)	N/A	0.137	1.818	0.214	0.	6.822	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	186	67748	525	528	0	0
normalized size	1	1.	0.93	340.44	2.64	2.65	0.	0.
time (sec)	N/A	0.187	1.844	3.807	1.146	3.591	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	106	14991	292	358	0	0
normalized size	1	1.	0.81	114.44	2.23	2.73	0.	0.
time (sec)	N/A	0.134	1.14	1.089	1.098	2.743	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	72	103	112	236	0	0
normalized size	1	1.	0.95	1.36	1.47	3.11	0.	0.
time (sec)	N/A	0.064	1.554	0.053	1.02	2.459	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	333	3491	0	842	0	0
normalized size	1	1.	3.96	41.56	0.	10.02	0.	0.
time (sec)	N/A	0.097	4.854	0.211	0.	3.414	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	308	5633	0	1087	0	0
normalized size	1	1.	2.43	44.35	0.	8.56	0.	0.
time (sec)	N/A	0.158	4.097	0.261	0.	3.543	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	350	10582	0	1656	0	0
normalized size	1	1.	1.87	56.59	0.	8.86	0.	0.
time (sec)	N/A	0.236	4.757	0.228	0.	3.791	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	325	6025	0	0	0	0
normalized size	1	1.	1.74	32.22	0.	0.	0.	0.
time (sec)	N/A	0.223	3.316	1.602	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	282	1614	0	2133	0	0
normalized size	1	1.	2.1	12.04	0.	15.92	0.	0.
time (sec)	N/A	0.157	2.771	0.218	0.	51.087	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	214	104	0	722	0	0
normalized size	1	1.	2.52	1.22	0.	8.49	0.	0.
time (sec)	N/A	0.057	7.092	0.021	0.	2.21	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	74	109	0	211	0	0
normalized size	1	1.	1.19	1.76	0.	3.4	0.	0.
time (sec)	N/A	0.098	0.653	0.17	0.	4.866	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	119	170	0	359	0	0
normalized size	1	1.	1.04	1.49	0.	3.15	0.	0.
time (sec)	N/A	0.122	0.869	0.194	0.	27.599	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	135	264	0	0	0	0
normalized size	1	1.	0.79	1.54	0.	0.	0.	0.
time (sec)	N/A	0.179	1.287	0.239	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	294	391	721	829	0	0
normalized size	1	1.	1.19	1.58	2.91	3.34	0.	0.
time (sec)	N/A	0.231	2.282	1.559	1.075	6.251	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	205	262	416	598	0	0
normalized size	1	1.	1.22	1.56	2.48	3.56	0.	0.
time (sec)	N/A	0.159	1.45	0.614	1.067	4.429	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	124	147	182	454	0	0
normalized size	1	1.	1.05	1.25	1.54	3.85	0.	0.
time (sec)	N/A	0.078	1.213	0.058	1.035	2.968	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	305	27448	0	1561	0	0
normalized size	1	1.	2.24	201.82	0.	11.48	0.	0.
time (sec)	N/A	0.146	4.707	1.415	0.	3.212	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	385	38486	0	2014	0	0
normalized size	1	1.	2.18	217.44	0.	11.38	0.	0.
time (sec)	N/A	0.209	4.259	2.254	0.	3.421	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	237	237	1142	49917	0	2449	0	0
normalized size	1	1.	4.82	210.62	0.	10.33	0.	0.
time (sec)	N/A	0.324	6.814	4.138	0.	4.148	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	378	7943	0	0	0	0
normalized size	1	1.	1.54	32.29	0.	0.	0.	0.
time (sec)	N/A	0.333	5.652	1.49	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	309	2511	0	0	0	0
normalized size	1	1.	1.71	13.87	0.	0.	0.	0.
time (sec)	N/A	0.209	4.287	0.217	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	1331	176	0	1245	0	0
normalized size	1	1.	9.93	1.31	0.	9.29	0.	0.
time (sec)	N/A	0.113	9.42	0.021	0.	2.474	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	133	153	0	358	0	0
normalized size	1	1.	1.37	1.58	0.	3.69	0.	0.
time (sec)	N/A	0.106	0.955	0.232	0.	47.768	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	140	245	0	0	0	0
normalized size	1	1.	0.96	1.68	0.	0.	0.	0.
time (sec)	N/A	0.149	1.022	0.297	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	174	371	0	0	0	0
normalized size	1	1.	0.79	1.69	0.	0.	0.	0.
time (sec)	N/A	0.229	2.167	0.228	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	292	0	0	0	0	0
normalized size	1	1.	3.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.153	2.078	0.777	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	275	0	0	0	0	0
normalized size	1	1.	2.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.141	2.344	0.658	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	208	208	283	0	0	0	0	0
normalized size	1	1.	1.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.223	7.427	0.885	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	184	0	0	0	0	0
normalized size	1	1.	1.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	3.877	0.583	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	80	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.848	0.173	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	1215	0	0	0	0	0
normalized size	1	1.	13.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	15.071	0.214	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	252	0	0	0	0	0
normalized size	1	1.	2.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	2.179	0.228	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	3698	0	0	0	0	0
normalized size	1	1.	44.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	19.341	0.734	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	192	0	0	0	0	0
normalized size	1	1.	2.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.553	0.072	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.654	0.214	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	111	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	1.466	0.234	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	176	141	0	0	0	0	0
normalized size	1	0.98	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.201	1.293	0.258	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	295	0	0	0	0	0
normalized size	1	1.	3.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.181	2.02	0.414	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	517	0	0	0	0	0
normalized size	1	1.	8.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	2.485	7.694	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.053	2.855	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	31	0	50	126	0	0
normalized size	1	1.	0.94	0.	1.52	3.82	0.	0.
time (sec)	N/A	0.097	0.054	180.	1.165	1.112	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	59	0	99	250	0	0
normalized size	1	1.	0.86	0.	1.43	3.62	0.	0.
time (sec)	N/A	0.116	0.156	180.	1.245	1.087	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	89	171293	146	435	0	0
normalized size	1	1.	0.86	1647.05	1.4	4.18	0.	0.
time (sec)	N/A	0.128	0.287	9.345	1.252	1.134	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	506	0	0	0	0	0
normalized size	1	1.	5.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.142	2.855	5.38	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	284	0	0	0	0	0
normalized size	1	1.	3.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	1.306	5.46	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	77	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	0.202	3.082	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	217	0	0	0	0	0
normalized size	1	1.	2.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.146	7.307	3.131	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	2.831	2.056	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	81	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.137	0.5	0.706	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	2033	0	0	0	0	0
normalized size	1	1.	18.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.145	16.917	0.842	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	91	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.148	0.463	0.224	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	56	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	2.647	2.256	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	46	43	212	136	68	701
normalized size	1	1.	0.71	0.66	3.26	2.09	1.05	10.78
time (sec)	N/A	0.035	0.196	0.003	1.732	0.998	1.204	2.311

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	38	35	138	104	54	401
normalized size	1	1.	0.76	0.7	2.76	2.08	1.08	8.02
time (sec)	N/A	0.03	0.097	0.001	1.642	1.002	0.65	1.56

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	26	25	80	66	37	180
normalized size	1	1.	0.81	0.78	2.5	2.06	1.16	5.62
time (sec)	N/A	0.025	0.039	0.003	1.496	0.999	0.327	1.342

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	26	43	49	100	87	50
normalized size	1	1.	0.84	1.39	1.58	3.23	2.81	1.61
time (sec)	N/A	0.022	0.026	0.018	1.545	1.027	0.593	1.298

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	36	69	90	204	248	69
normalized size	1	1.	0.65	1.25	1.64	3.71	4.51	1.25
time (sec)	N/A	0.034	0.044	0.017	1.496	1.044	1.057	1.293

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	46	95	122	305	454	82
normalized size	1	1.	0.58	1.2	1.54	3.86	5.75	1.04
time (sec)	N/A	0.048	0.042	0.016	1.709	1.076	1.809	1.29

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	63	106	134	166	116	2321
normalized size	1	1.	0.85	1.43	1.81	2.24	1.57	31.36
time (sec)	N/A	0.049	0.225	0.006	1.144	1.103	0.885	7.185

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	65	78	95	126	88	1446
normalized size	1	1.	1.23	1.47	1.79	2.38	1.66	27.28
time (sec)	N/A	0.039	0.164	0.003	1.142	1.104	0.479	2.995

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	40	50	50	86	60	675
normalized size	1	1.	1.18	1.47	1.47	2.53	1.76	19.85
time (sec)	N/A	0.022	0.069	0.002	1.185	1.048	0.224	1.533

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	34	27	42	113	58	47
normalized size	1	1.	1.31	1.04	1.62	4.35	2.23	1.81
time (sec)	N/A	0.029	0.039	0.043	1.074	1.129	0.627	1.225

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	56	41	42	154	100	215
normalized size	1	1.	1.65	1.21	1.24	4.53	2.94	6.32
time (sec)	N/A	0.032	0.149	0.044	1.047	1.086	2.684	1.34

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	56	69	70	205	128	350
normalized size	1	1.	1.06	1.3	1.32	3.87	2.42	6.6
time (sec)	N/A	0.042	0.224	0.043	1.077	1.059	9.17	1.343

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	129	120	97	178	109	1467
normalized size	1	1.	1.61	1.5	1.21	2.22	1.36	18.34
time (sec)	N/A	0.052	0.047	0.004	1.69	1.079	1.185	6.264

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	97	92	77	134	82	855
normalized size	1	1.	1.62	1.53	1.28	2.23	1.37	14.25
time (sec)	N/A	0.043	0.037	0.004	1.689	1.057	0.664	2.64

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	65	64	55	90	54	390
normalized size	1	1.	1.62	1.6	1.38	2.25	1.35	9.75
time (sec)	N/A	0.034	0.024	0.003	1.64	1.028	0.344	1.562

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	28	29	31	46	20	340
normalized size	1	1.	1.47	1.53	1.63	2.42	1.05	17.89
time (sec)	N/A	0.012	0.006	0.	1.616	1.059	0.161	1.196

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	34	31	36	68	46	62
normalized size	1	1.	1.62	1.48	1.71	3.24	2.19	2.95
time (sec)	N/A	0.025	0.018	0.034	1.667	1.029	1.406	1.326

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	65	47	62	116	70	143
normalized size	1	1.	1.67	1.21	1.59	2.97	1.79	3.67
time (sec)	N/A	0.037	0.037	0.04	1.616	1.069	4.629	1.326

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	69	67	82	161	97	227
normalized size	1	1.	1.13	1.1	1.34	2.64	1.59	3.72
time (sec)	N/A	0.046	0.05	0.042	1.68	1.086	18.264	1.349

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	89	198	219	265	206	5148
normalized size	1	1.	0.85	1.89	2.09	2.52	1.96	49.03
time (sec)	N/A	0.109	0.339	0.006	1.111	1.195	1.683	17.98

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	72	151	171	209	160	3510
normalized size	1	1.	0.88	1.84	2.09	2.55	1.95	42.8
time (sec)	N/A	0.095	0.271	0.004	1.174	1.117	0.992	7.76

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	54	104	111	153	112	2039
normalized size	1	1.	0.87	1.68	1.79	2.47	1.81	32.89
time (sec)	N/A	0.06	0.213	0.004	1.128	1.21	0.527	3.389

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	65	60	80	161	97	124
normalized size	1	1.	1.27	1.18	1.57	3.16	1.9	2.43
time (sec)	N/A	0.064	0.12	0.052	1.087	1.17	2.487	1.58

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	51	62	69	234	131	223
normalized size	1	1.	0.91	1.11	1.23	4.18	2.34	3.98
time (sec)	N/A	0.082	0.243	0.056	1.111	1.124	9.123	1.64

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	61	91	82	238	0	420
normalized size	1	1.	0.8	1.2	1.08	3.13	0.	5.53
time (sec)	N/A	0.088	0.289	0.053	1.077	1.059	0.	1.688

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	243	226	159	293	212	3536
normalized size	1	1.	2.15	2.	1.41	2.59	1.88	31.29
time (sec)	N/A	0.088	0.083	0.004	1.606	1.126	2.397	13.362

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	190	179	131	238	165	2273
normalized size	1	1.	2.09	1.97	1.44	2.62	1.81	24.98
time (sec)	N/A	0.079	0.074	0.005	1.635	1.171	1.29	5.31

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	137	132	103	177	117	1265
normalized size	1	1.	1.99	1.91	1.49	2.57	1.7	18.33
time (sec)	N/A	0.074	0.056	0.004	1.617	1.094	0.732	2.823

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	73	87	78	117	68	516
normalized size	1	1.	1.59	1.89	1.7	2.54	1.48	11.22
time (sec)	N/A	0.031	0.576	0.003	1.644	1.064	0.382	1.47

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	66	53	62	115	73	66
normalized size	1	1.	1.74	1.39	1.63	3.03	1.92	1.74
time (sec)	N/A	0.065	0.103	0.04	1.645	1.04	5.506	1.669

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	71	60	77	143	0	165
normalized size	1	1.	1.61	1.36	1.75	3.25	0.	3.75
time (sec)	N/A	0.071	1.231	0.047	1.619	1.08	0.	1.749

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	104	91	105	204	0	300
normalized size	1	1.	1.53	1.34	1.54	3.	0.	4.41
time (sec)	N/A	0.077	0.102	0.053	1.624	1.095	0.	1.811

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	64	72	103	203	348	622
normalized size	1	1.	0.9	1.01	1.45	2.86	4.9	8.76
time (sec)	N/A	0.099	0.158	0.017	1.1	1.222	14.13	3.184

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	41	54	72	151	240	252
normalized size	1	1.	0.82	1.08	1.44	3.02	4.8	5.04
time (sec)	N/A	0.086	0.033	0.016	1.067	1.177	4.978	1.972

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	37	50	41	90	143	72
normalized size	1	1.	1.03	1.39	1.14	2.5	3.97	2.
time (sec)	N/A	0.053	0.028	0.014	1.055	1.089	3.348	1.487

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	57	76	66	169	398	80
normalized size	1	1.	0.89	1.19	1.03	2.64	6.22	1.25
time (sec)	N/A	0.082	0.05	0.067	1.051	1.2	16.75	1.446

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	63	150	92	297	743	551
normalized size	1	1.	0.71	1.69	1.03	3.34	8.35	6.19
time (sec)	N/A	0.113	0.25	0.08	1.112	1.227	68.366	1.489

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	83	264	130	367	0	755
normalized size	1	1.	0.72	2.3	1.13	3.19	0.	6.57
time (sec)	N/A	0.137	0.34	0.082	1.073	1.326	0.	1.534

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	92	102	0	618	685	521
normalized size	1	1.	1.08	1.2	0.	7.27	8.06	6.13
time (sec)	N/A	0.188	0.782	0.017	0.	1.217	20.898	4.154

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	70	70	0	495	493	405
normalized size	1	1.	1.11	1.11	0.	7.86	7.83	6.43
time (sec)	N/A	0.106	0.274	0.016	0.	1.194	9.167	2.278

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	49	52	0	409	292	184
normalized size	1	1.	0.98	1.04	0.	8.18	5.84	3.68
time (sec)	N/A	0.079	0.031	0.017	0.	1.141	3.607	1.564

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	49	52	0	406	280	231
normalized size	1	1.	0.98	1.04	0.	8.12	5.6	4.62
time (sec)	N/A	0.074	0.047	0.	0.	1.106	3.384	1.329

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	68	73	0	568	570	390
normalized size	1	1.	1.06	1.14	0.	8.88	8.91	6.09
time (sec)	N/A	0.111	0.237	0.067	0.	1.179	22.172	1.444

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	92	104	0	702	823	504
normalized size	1	1.	1.1	1.24	0.	8.36	9.8	6.
time (sec)	N/A	0.173	0.647	0.079	0.	1.192	89.714	1.61

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	121	158	0	811	0	602
normalized size	1	1.	1.07	1.4	0.	7.18	0.	5.33
time (sec)	N/A	0.241	1.868	0.082	0.	1.231	0.	1.731

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	73	149	173	423	1583	536
normalized size	1	1.	0.81	1.66	1.92	4.7	17.59	5.96
time (sec)	N/A	0.121	0.665	0.026	1.141	1.268	118.299	2.974

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	61	109	119	234	930	397
normalized size	1	1.	0.88	1.58	1.72	3.39	13.48	5.75
time (sec)	N/A	0.1	0.541	0.023	1.161	1.15	48.69	1.736

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	57	104	119	235	816	413
normalized size	1	1.	0.88	1.6	1.83	3.62	12.55	6.35
time (sec)	N/A	0.072	0.602	0.024	1.108	1.138	48.229	1.391

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	90	160	167	439	0	205
normalized size	1	1.	0.87	1.55	1.62	4.26	0.	1.99
time (sec)	N/A	0.121	1.986	0.092	1.12	1.277	0.	1.367

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	98	234	252	648	0	918
normalized size	1	1.	0.74	1.77	1.91	4.91	0.	6.95
time (sec)	N/A	0.156	0.826	0.101	1.164	1.39	0.	1.464

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	121	347	319	759	0	919
normalized size	1	1.	0.75	2.16	1.98	4.71	0.	5.71
time (sec)	N/A	0.179	1.011	0.109	1.153	1.474	0.	1.493

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	118	184	0	1065	2914	201
normalized size	1	1.	0.91	1.42	0.	8.19	22.42	1.55
time (sec)	N/A	0.197	1.228	0.023	0.	1.29	147.662	4.354

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	94	160	0	865	2179	171
normalized size	1	1.	0.99	1.68	0.	9.11	22.94	1.8
time (sec)	N/A	0.118	0.752	0.023	0.	1.258	52.049	2.202

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	87	151	0	892	2145	151
normalized size	1	1.	0.97	1.68	0.	9.91	23.83	1.68
time (sec)	N/A	0.104	0.508	0.022	0.	1.56	50.906	1.568

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	88	160	0	865	2086	171
normalized size	1	1.	0.91	1.65	0.	8.92	21.51	1.76
time (sec)	N/A	0.08	0.997	0.	0.	1.577	37.204	1.312

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	117	187	0	1142	0	231
normalized size	1	1.	0.91	1.46	0.	8.92	0.	1.8
time (sec)	N/A	0.192	2.799	0.087	0.	1.685	0.	1.412

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	137	218	0	1335	0	242
normalized size	1	1.	0.81	1.29	0.	7.9	0.	1.43
time (sec)	N/A	0.287	3.	0.098	0.	1.803	0.	1.44

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	165	272	0	1520	0	304
normalized size	1	1.	0.76	1.25	0.	6.97	0.	1.39
time (sec)	N/A	0.342	5.023	0.102	0.	1.928	0.	1.44

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	97	234	255	459	3225	632
normalized size	1	1.	0.9	2.17	2.36	4.25	29.86	5.85
time (sec)	N/A	0.152	1.069	0.026	1.128	1.563	173.463	3.334

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	87	193	262	473	2849	682
normalized size	1	1.	0.9	1.99	2.7	4.88	29.37	7.03
time (sec)	N/A	0.117	0.75	0.023	1.176	1.572	170.769	2.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	82	190	259	458	2876	876
normalized size	1	1.	0.88	2.04	2.78	4.92	30.92	9.42
time (sec)	N/A	0.088	0.657	0.026	1.101	1.583	171.352	1.688

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	126	289	338	895	0	360
normalized size	1	1.	0.85	1.95	2.28	6.05	0.	2.43
time (sec)	N/A	0.165	1.66	0.098	1.11	1.895	0.	1.631

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	144	362	466	1175	0	1218
normalized size	1	1.	0.8	2.	2.57	6.49	0.	6.73
time (sec)	N/A	0.214	1.923	0.105	1.132	2.053	0.	1.711

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	178	477	562	1324	0	2016
normalized size	1	1.	0.85	2.27	2.68	6.3	0.	9.6
time (sec)	N/A	0.238	2.47	0.109	1.183	2.33	0.	1.771

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	142	351	0	1646	0	290
normalized size	1	1.	0.93	2.29	0.	10.76	0.	1.9
time (sec)	N/A	0.229	2.171	0.026	0.	1.716	0.	4.324

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	136	338	0	1635	0	269
normalized size	1	1.	0.94	2.33	0.	11.28	0.	1.86
time (sec)	N/A	0.181	1.895	0.025	0.	1.717	0.	2.416

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	139	339	0	1638	0	270
normalized size	1	1.	0.97	2.35	0.	11.38	0.	1.88
time (sec)	N/A	0.154	1.977	0.023	0.	1.622	0.	1.753

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	138	350	0	1643	0	288
normalized size	1	1.	0.92	2.33	0.	10.95	0.	1.92
time (sec)	N/A	0.152	1.781	0.	0.	1.716	0.	1.268

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	174	379	0	1989	0	312
normalized size	1	1.	0.92	2.01	0.	10.52	0.	1.65
time (sec)	N/A	0.29	2.013	0.09	0.	1.937	0.	1.64

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	184	413	0	2261	0	352
normalized size	1	1.	0.77	1.72	0.	9.42	0.	1.47
time (sec)	N/A	0.365	4.259	0.101	0.	2.129	0.	1.639

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	949	466	0	2547	0	414
normalized size	1	1.	3.2	1.57	0.	8.58	0.	1.39
time (sec)	N/A	0.468	6.305	0.107	0.	2.458	0.	1.639

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	137	242	219	308	209	2982
normalized size	1	1.	1.19	2.1	1.9	2.68	1.82	25.93
time (sec)	N/A	0.073	1.885	0.006	1.699	1.41	1.542	4.993

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	102	154	140	204	126	1386
normalized size	1	1.	1.32	2.	1.82	2.65	1.64	18.
time (sec)	N/A	0.049	0.922	0.003	1.691	1.409	0.838	2.338

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	73	87	78	117	68	485
normalized size	1	1.	1.59	1.89	1.7	2.54	1.48	10.54
time (sec)	N/A	0.031	0.573	0.004	1.694	1.338	0.397	1.47

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	28	29	31	46	20	312
normalized size	1	1.	1.47	1.53	1.63	2.42	1.05	16.42
time (sec)	N/A	0.013	0.006	0.001	1.633	1.385	0.156	1.265

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	49	52	0	406	280	225
normalized size	1	1.	0.98	1.04	0.	8.12	5.6	4.5
time (sec)	N/A	0.074	0.07	0.02	0.	1.493	3.163	1.315

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	88	160	0	865	2086	165
normalized size	1	1.	0.91	1.65	0.	8.92	21.51	1.7
time (sec)	N/A	0.092	0.993	0.027	0.	1.523	36.543	1.321

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	138	350	0	1643	0	277
normalized size	1	1.	0.92	2.33	0.	10.95	0.	1.85
time (sec)	N/A	0.144	1.878	0.027	0.	1.687	0.	1.313

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	32	56	1161	171	0	65
normalized size	1	1.	0.59	1.04	21.5	3.17	0.	1.2
time (sec)	N/A	0.105	0.076	0.052	2.848	1.482	0.	1.141

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	20	29	373	55	0	39
normalized size	1	1.	0.67	0.97	12.43	1.83	0.	1.3
time (sec)	N/A	0.087	0.029	0.03	1.931	1.35	0.	1.16

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	24	39	398	147	0	54
normalized size	1	1.	0.67	1.08	11.06	4.08	0.	1.5
time (sec)	N/A	0.093	0.049	0.029	1.877	1.414	0.	1.175

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	0	30	10	14
normalized size	1	1.	1.	1.1	0.	3.	1.	1.4
time (sec)	N/A	0.043	0.009	0.023	0.	1.311	0.671	1.118

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	30	23	51	180	0	32
normalized size	1	1.	1.25	0.96	2.12	7.5	0.	1.33
time (sec)	N/A	0.073	0.018	0.096	1.867	1.396	0.	1.097

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	17	23	41	0	43
normalized size	1	1.	1.	1.21	1.64	2.93	0.	3.07
time (sec)	N/A	0.08	0.014	0.083	1.65	1.286	0.	1.113

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	38	51	409	167	0	68
normalized size	1	1.	0.84	1.13	9.09	3.71	0.	1.51
time (sec)	N/A	0.09	0.082	0.089	1.926	1.412	0.	1.107

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	22	25	39	70	0	80
normalized size	1	1.	0.65	0.74	1.15	2.06	0.	2.35
time (sec)	N/A	0.1	0.024	0.084	1.644	1.298	0.	1.097

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	31	34	88	234	0	89
normalized size	1	1.	0.86	0.94	2.44	6.5	0.	2.47
time (sec)	N/A	0.035	0.031	0.065	1.699	1.375	0.	1.435

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	22	29	755	82	0	42
normalized size	1	1.	0.69	0.91	23.59	2.56	0.	1.31
time (sec)	N/A	0.1	0.05	0.018	1.932	1.391	0.	1.133

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	34	54	1261	174	0	66
normalized size	1	1.	0.58	0.92	21.37	2.95	0.	1.12
time (sec)	N/A	0.12	0.063	0.016	2.862	1.378	0.	1.11

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	38	12	16
normalized size	1	1.	1.	0.93	0.	2.71	0.86	1.14
time (sec)	N/A	0.051	0.015	0.01	0.	1.337	3.302	1.053

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	32	181	140	0	50
normalized size	1	1.	0.92	0.86	4.89	3.78	0.	1.35
time (sec)	N/A	0.091	0.038	0.069	1.905	1.438	0.	1.082

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	27	54	181	159	0	84
normalized size	1	1.	0.82	1.64	5.48	4.82	0.	2.55
time (sec)	N/A	0.109	0.027	0.089	1.986	1.324	0.	1.104

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	43	62	751	193	0	2869
normalized size	1	1.	0.63	0.91	11.04	2.84	0.	42.19
time (sec)	N/A	0.039	0.068	0.02	1.908	1.299	0.	3.198

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	65	90	2388	236	0	0
normalized size	1	1.	0.66	0.92	24.37	2.41	0.	0.
time (sec)	N/A	0.049	0.216	0.021	2.875	1.425	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	17	26	50	50	0	36
normalized size	1	1.	0.68	1.04	2.	2.	0.	1.44
time (sec)	N/A	0.092	0.024	0.03	1.137	1.34	0.	1.102

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	49	38	57	190	0	54
normalized size	1	1.	1.58	1.23	1.84	6.13	0.	1.74
time (sec)	N/A	0.093	0.037	0.028	1.858	1.273	0.	1.106

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	34	14	16
normalized size	1	1.	1.	1.08	0.	2.83	1.17	1.33
time (sec)	N/A	0.046	0.013	0.023	0.	1.371	0.61	1.087

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	29	57	186	0	58
normalized size	1	1.	0.91	0.83	1.63	5.31	0.	1.66
time (sec)	N/A	0.08	0.033	0.079	1.923	1.402	0.	1.073

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	22	24	173	86	0	63
normalized size	1	1.	0.71	0.77	5.58	2.77	0.	2.03
time (sec)	N/A	0.092	0.027	0.086	1.88	1.402	0.	1.082

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	23	29	51	111	36	35
normalized size	1	1.	0.77	0.97	1.7	3.7	1.2	1.17
time (sec)	N/A	0.097	0.025	0.019	1.181	1.295	3.499	1.084

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	56	19	99	0	22
normalized size	1	1.	0.78	2.43	0.83	4.3	0.	0.96
time (sec)	N/A	0.101	0.018	0.016	1.925	1.342	0.	1.113

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	88	15	16
normalized size	1	1.	1.	0.93	0.	6.29	1.07	1.14
time (sec)	N/A	0.049	0.012	0.011	0.	1.351	2.252	1.081

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	47	38	65	258	0	76
normalized size	1	1.	0.89	0.72	1.23	4.87	0.	1.43
time (sec)	N/A	0.091	0.058	0.073	1.917	1.609	0.	1.091

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	31	31	304	139	0	74
normalized size	1	1.	0.52	0.52	5.07	2.32	0.	1.23
time (sec)	N/A	0.115	0.057	0.071	1.889	1.599	0.	1.108

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	25	18	92	0	0
normalized size	1	1.	1.	1.04	0.75	3.83	0.	0.
time (sec)	N/A	0.029	0.041	0.031	1.804	1.592	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	40	57	35	167	0	109
normalized size	1	1.	0.69	0.98	0.6	2.88	0.	1.88
time (sec)	N/A	0.035	0.061	0.019	1.955	1.541	0.	1.668

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	52	88	53	231	0	147
normalized size	1	1.	0.59	1.	0.6	2.62	0.	1.67
time (sec)	N/A	0.044	0.1	0.022	1.814	1.579	0.	1.879

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	62	119	68	293	0	185
normalized size	1	1.	0.53	1.01	0.58	2.48	0.	1.57
time (sec)	N/A	0.053	0.184	0.021	1.828	1.536	0.	2.112

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	52	19	24	228	0	39
normalized size	1	1.	2.36	0.86	1.09	10.36	0.	1.77
time (sec)	N/A	0.015	0.065	0.023	1.608	1.599	0.	1.071

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	44	4	4	185	0	22
normalized size	1	1.	14.67	1.33	1.33	61.67	0.	7.33
time (sec)	N/A	0.011	0.009	0.021	1.616	1.629	0.	1.132

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	36	12	15
normalized size	1	1.	1.	1.09	1.36	3.27	1.09	1.36
time (sec)	N/A	0.013	0.007	0.013	1.108	1.864	0.423	1.099

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	C	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	72	32	27	227	0	27
normalized size	1	1.	2.06	0.91	0.77	6.49	0.	0.77
time (sec)	N/A	0.022	0.067	0.02	1.594	1.854	0.	1.094

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	46	17	23	55	0	9
normalized size	1	1.	2.88	1.06	1.44	3.44	0.	0.56
time (sec)	N/A	0.017	0.007	0.029	1.845	1.872	0.	1.093

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	0	42	0	16
normalized size	1	1.	1.	1.08	0.	3.23	0.	1.23
time (sec)	N/A	0.02	0.007	0.029	0.	1.713	0.	1.089

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	109	166	0	770	0	193
normalized size	1	1.	0.93	1.42	0.	6.58	0.	1.65
time (sec)	N/A	0.146	1.36	0.046	0.	2.727	0.	1.206

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	82	114	0	621	0	131
normalized size	1	1.	0.93	1.3	0.	7.06	0.	1.49
time (sec)	N/A	0.114	0.352	0.027	0.	2.471	0.	1.177

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	59	91	0	518	0	81
normalized size	1	1.	0.95	1.47	0.	8.35	0.	1.31
time (sec)	N/A	0.071	0.05	0.032	0.	2.309	0.	1.155

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	72	615	0	953	0	0
normalized size	1	1.	0.97	8.31	0.	12.88	0.	0.
time (sec)	N/A	0.101	0.061	0.256	0.	1.722	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	115	2135	0	1467	0	0
normalized size	1	1.	1.	18.57	0.	12.76	0.	0.
time (sec)	N/A	0.148	0.36	0.319	0.	1.793	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	138	5676	0	1756	0	0
normalized size	1	1.	0.85	34.82	0.	10.77	0.	0.
time (sec)	N/A	0.212	1.314	0.31	0.	1.906	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	823	451	0	2030	0	0
normalized size	1	1.	3.71	2.03	0.	9.14	0.	0.
time (sec)	N/A	0.337	6.299	0.035	0.	14.409	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	767	323	0	1675	0	0
normalized size	1	1.	4.54	1.91	0.	9.91	0.	0.
time (sec)	N/A	0.21	6.207	0.03	0.	7.598	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	251	230	0	1388	0	0
normalized size	1	1.	2.04	1.87	0.	11.28	0.	0.
time (sec)	N/A	0.134	5.866	0.028	0.	3.454	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	203	169	0	1046	0	0
normalized size	1	1.	2.39	1.99	0.	12.31	0.	0.
time (sec)	N/A	0.053	0.798	0.	0.	2.164	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	64	2233	0	640	0	0
normalized size	1	1.	0.85	29.77	0.	8.53	0.	0.
time (sec)	N/A	0.096	0.252	0.349	0.	2.401	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	117	117	241	4518	0	751	0	0
normalized size	1	1.	2.06	38.62	0.	6.42	0.	0.
time (sec)	N/A	0.154	6.816	0.308	0.	2.312	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	167	167	339	6894	0	902	0	0
normalized size	1	1.	2.03	41.28	0.	5.4	0.	0.
time (sec)	N/A	0.231	14.219	0.401	0.	2.424	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	139	256	0	992	0	265
normalized size	1	1.	0.96	1.77	0.	6.84	0.	1.83
time (sec)	N/A	0.175	1.333	0.026	0.	2.299	0.	1.29

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	112	204	0	797	0	203
normalized size	1	1.	0.97	1.76	0.	6.87	0.	1.75
time (sec)	N/A	0.145	0.906	0.017	0.	2.269	0.	1.264

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	80	181	0	624	0	154
normalized size	1	1.	0.89	2.01	0.	6.93	0.	1.71
time (sec)	N/A	0.097	0.383	0.016	0.	2.24	0.	1.207

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	90	1765	0	1481	0	0
normalized size	1	1.	0.95	18.58	0.	15.59	0.	0.
time (sec)	N/A	0.131	0.251	0.186	0.	6.56	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	109	2011	0	1485	0	0
normalized size	1	1.	0.94	17.34	0.	12.8	0.	0.
time (sec)	N/A	0.174	0.329	0.221	0.	1.866	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	140	5224	0	1817	0	0
normalized size	1	1.	0.87	32.45	0.	11.29	0.	0.
time (sec)	N/A	0.222	1.362	0.197	0.	1.898	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	908	669	0	2572	0	0
normalized size	1	1.	3.09	2.28	0.	8.75	0.	0.
time (sec)	N/A	0.448	6.489	0.024	0.	36.296	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	833	510	0	2103	0	0
normalized size	1	1.	3.72	2.28	0.	9.39	0.	0.
time (sec)	N/A	0.355	6.356	0.017	0.	21.104	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	771	386	0	1739	0	0
normalized size	1	1.	4.48	2.24	0.	10.11	0.	0.
time (sec)	N/A	0.249	6.256	0.016	0.	7.995	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	233	297	0	1382	0	0
normalized size	1	1.	1.86	2.38	0.	11.06	0.	0.
time (sec)	N/A	0.095	1.271	0.	0.	4.834	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	256	3333	0	1829	0	0
normalized size	1	1.	2.25	29.24	0.	16.04	0.	0.
time (sec)	N/A	0.138	6.124	0.339	0.	6.952	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	78	6591	0	760	0	0
normalized size	1	1.	0.68	57.31	0.	6.61	0.	0.
time (sec)	N/A	0.173	0.29	0.319	0.	2.366	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	140	10026	0	927	0	0
normalized size	1	1.	0.85	60.76	0.	5.62	0.	0.
time (sec)	N/A	0.245	8.509	0.426	0.	2.475	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	259	461	0	1763	0	0
normalized size	1	1.	1.52	2.71	0.	10.37	0.	0.
time (sec)	N/A	0.179	1.25	0.059	0.	11.174	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	87	111	0	717	0	154
normalized size	1	1.	0.92	1.17	0.	7.55	0.	1.62
time (sec)	N/A	0.14	2.343	0.031	0.	2.166	0.	1.635

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	62	58	0	585	0	84
normalized size	1	1.	0.97	0.91	0.	9.14	0.	1.31
time (sec)	N/A	0.111	0.265	0.029	0.	2.105	0.	1.464

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	35	0	446	0	47
normalized size	1	1.	1.	0.85	0.	10.88	0.	1.15
time (sec)	N/A	0.064	0.027	0.03	0.	2.052	0.	1.4

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	72	496	0	1083	0	0
normalized size	1	1.	0.97	6.7	0.	14.64	0.	0.
time (sec)	N/A	0.108	0.079	0.24	0.	1.761	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	135	3601	0	1650	0	0
normalized size	1	1.	1.16	31.04	0.	14.22	0.	0.
time (sec)	N/A	0.163	0.733	0.276	0.	1.836	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	162	7641	0	1994	0	0
normalized size	1	1.	0.98	46.03	0.	12.01	0.	0.
time (sec)	N/A	0.214	1.873	0.266	0.	1.948	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	768	261	0	1918	0	0
normalized size	1	1.	4.34	1.47	0.	10.84	0.	0.
time (sec)	N/A	0.222	6.298	0.032	0.	15.203	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	713	165	0	1569	0	0
normalized size	1	1.	5.7	1.32	0.	12.55	0.	0.
time (sec)	N/A	0.139	6.23	0.03	0.	8.436	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	149	102	0	1179	0	0
normalized size	1	1.	1.73	1.19	0.	13.71	0.	0.
time (sec)	N/A	0.104	0.731	0.027	0.	2.8	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	67	0	311	0	0
normalized size	1	1.	1.	1.46	0.	6.76	0.	0.
time (sec)	N/A	0.031	0.068	0.	0.	1.977	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	212	1195	0	701	0	0
normalized size	1	1.	2.72	15.32	0.	8.99	0.	0.
time (sec)	N/A	0.119	9.343	0.307	0.	2.804	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	263	2433	0	844	0	0
normalized size	1	1.	2.19	20.27	0.	7.03	0.	0.
time (sec)	N/A	0.165	11.036	0.369	0.	2.83	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	794	3741	0	1022	0	0
normalized size	1	1.	4.67	22.01	0.	6.01	0.	0.
time (sec)	N/A	0.246	16.286	0.346	0.	2.857	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	84	141	0	987	0	134
normalized size	1	1.	0.86	1.44	0.	10.07	0.	1.37
time (sec)	N/A	0.169	0.365	0.026	0.	2.755	0.	1.402

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	75	92	0	829	0	103
normalized size	1	1.	1.03	1.26	0.	11.36	0.	1.41
time (sec)	N/A	0.133	0.362	0.017	0.	2.665	0.	1.374

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	56	68	0	790	56	93
normalized size	1	1.	0.81	0.99	0.	11.45	0.81	1.35
time (sec)	N/A	0.088	0.073	0.014	0.	2.748	13.658	1.289

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	91	32888	0	2129	0	0
normalized size	1	1.	0.86	310.26	0.	20.08	0.	0.
time (sec)	N/A	0.149	0.133	0.742	0.	2.25	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	115	54353	0	2851	0	0
normalized size	1	1.	0.73	346.2	0.	18.16	0.	0.
time (sec)	N/A	0.246	0.427	1.523	0.	2.354	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	142	79934	0	3421	0	0
normalized size	1	1.	0.66	371.79	0.	15.91	0.	0.
time (sec)	N/A	0.346	1.197	2.861	0.	2.695	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	182	182	787	286	0	2768	0	0
normalized size	1	1.	4.32	1.57	0.	15.21	0.	0.
time (sec)	N/A	0.251	6.393	0.023	0.	20.115	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	250	193	0	2291	0	0
normalized size	1	1.	2.03	1.57	0.	18.63	0.	0.
time (sec)	N/A	0.159	2.933	0.018	0.	10.804	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	154	131	0	686	0	0
normalized size	1	1.	1.9	1.62	0.	8.47	0.	0.
time (sec)	N/A	0.111	3.181	0.016	0.	2.131	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	214	104	0	722	0	0
normalized size	1	1.	2.52	1.22	0.	8.49	0.	0.
time (sec)	N/A	0.065	6.125	0.	0.	2.275	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	128	128	882	1305	0	1102	0	0
normalized size	1	1.	6.89	10.2	0.	8.61	0.	0.
time (sec)	N/A	0.184	13.503	0.304	0.	2.994	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	802	2577	0	1312	0	0
normalized size	1	1.	4.36	14.01	0.	7.13	0.	0.
time (sec)	N/A	0.259	16.383	0.266	0.	3.256	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	252	252	850	3925	0	1569	0	0
normalized size	1	1.	3.37	15.58	0.	6.23	0.	0.
time (sec)	N/A	0.368	16.484	0.355	0.	2.852	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	91	169	0	1338	0	185
normalized size	1	1.	0.79	1.47	0.	11.63	0.	1.61
time (sec)	N/A	0.204	0.453	0.027	0.	2.502	0.	1.492

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	84	118	0	1269	0	157
normalized size	1	1.	0.82	1.15	0.	12.32	0.	1.52
time (sec)	N/A	0.156	0.298	0.018	0.	2.506	0.	1.442

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	58	94	0	1230	83	142
normalized size	1	1.	0.59	0.95	0.	12.42	0.84	1.43
time (sec)	N/A	0.108	0.163	0.016	0.	2.511	19.734	1.341

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	94	331597	0	3623	0	0
normalized size	1	1.	0.64	2255.76	0.	24.65	0.	0.
time (sec)	N/A	0.214	0.37	15.647	0.	2.181	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	138	531573	0	4601	0	0
normalized size	1	1.	0.67	2580.45	0.	22.33	0.	0.
time (sec)	N/A	0.35	0.583	34.855	0.	2.663	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	165	790286	0	5387	0	0
normalized size	1	1.	0.61	2905.46	0.	19.81	0.	0.
time (sec)	N/A	0.443	1.992	58.722	0.	2.666	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	295	382	0	3800	0	0
normalized size	1	1.	1.73	2.23	0.	22.22	0.	0.
time (sec)	N/A	0.267	4.44	0.047	0.	18.962	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	260	291	0	1137	0	0
normalized size	1	1.	1.98	2.22	0.	8.68	0.	0.
time (sec)	N/A	0.159	6.038	0.046	0.	2.191	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	128	128	365	232	0	1176	0	0
normalized size	1	1.	2.85	1.81	0.	9.19	0.	0.
time (sec)	N/A	0.153	7.732	0.044	0.	2.148	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	1331	176	0	1245	0	0
normalized size	1	1.	9.93	1.31	0.	9.29	0.	0.
time (sec)	N/A	0.101	7.527	0.046	0.	2.132	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	186	186	1890	0	0	1693	0	0
normalized size	1	1.	10.16	0.	0.	9.1	0.	0.
time (sec)	N/A	0.279	15.914	0.219	0.	3.006	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	249	249	871	0	0	1937	0	0
normalized size	1	1.	3.5	0.	0.	7.78	0.	0.
time (sec)	N/A	0.378	16.506	0.23	0.	3.216	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	441	0	0	2286	0	0
normalized size	1	1.	1.35	0.	0.	6.99	0.	0.
time (sec)	N/A	0.496	16.319	0.255	0.	3.359	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	74	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	0.08	0.786	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	100	100	101	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	0.23	0.589	0.	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	106	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.171	0.735	0.398	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	73	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	0.126	0.316	0.	0.	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.081	0.294	0.	0.	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	98	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.165	0.314	0.	0.	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	142	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.168	0.645	0.296	0.	0.	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	172	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.253	2.542	0.301	0.	0.	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	2.876	0.418	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	2250	0	0	0	0	0
normalized size	1	1.	27.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	14.847	0.332	0.	0.	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	1992	0	0	0	0	0
normalized size	1	1.	24.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	14.549	0.283	0.	0.	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	192	0	0	0	0	0
normalized size	1	1.	2.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.378	0.001	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	1989	0	0	0	0	0
normalized size	1	1.	25.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	14.505	0.281	0.	0.	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	2468	0	0	0	0	0
normalized size	1	1.	29.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	14.556	0.283	0.	0.	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	3.614	0.316	0.	0.	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	224	321	351	559	301	8031
normalized size	1	1.	0.88	1.26	1.38	2.19	1.18	31.49
time (sec)	N/A	0.147	0.904	0.008	1.519	1.776	4.696	75.904

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	160	201	247	365	194	4219
normalized size	1	1.	0.95	1.2	1.47	2.17	1.15	25.11
time (sec)	N/A	0.098	0.429	0.007	1.523	1.672	1.908	17.739

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	107	108	112	213	94	1438
normalized size	1	1.	1.2	1.21	1.26	2.39	1.06	16.16
time (sec)	N/A	0.06	0.473	0.007	1.504	1.656	0.76	3.709

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	30	36	49	92	37	339
normalized size	1	1.	0.94	1.12	1.53	2.88	1.16	10.59
time (sec)	N/A	0.019	0.077	0.005	0.998	1.612	0.209	1.505

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	278	355	0	10283	0	451
normalized size	1	1.	1.09	1.39	0.	40.17	0.	1.76
time (sec)	N/A	0.38	0.575	0.027	0.	10.13	0.	1.587

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	558	558	575	1086	0	24260	0	811
normalized size	1	1.	1.03	1.95	0.	43.48	0.	1.45
time (sec)	N/A	0.728	6.279	0.036	0.	13.901	0.	1.655

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	57	34	45	149	34	46
normalized size	1	1.	1.54	0.92	1.22	4.03	0.92	1.24
time (sec)	N/A	0.061	0.022	0.026	1.507	1.384	0.201	1.099

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	196	412	358	564	386	10450
normalized size	1	1.	0.91	1.91	1.66	2.61	1.79	48.38
time (sec)	N/A	0.129	4.264	0.005	1.519	1.545	10.663	156.397

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	128	252	225	367	224	4724
normalized size	1	1.	0.89	1.75	1.56	2.55	1.56	32.81
time (sec)	N/A	0.082	0.974	0.006	1.534	1.374	4.196	34.499

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	75	134	123	208	116	1594
normalized size	1	1.	0.91	1.63	1.5	2.54	1.41	19.44
time (sec)	N/A	0.055	0.526	0.006	1.573	1.436	1.352	6.446

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	44	43	46	82	32	797
normalized size	1	1.	1.26	1.23	1.31	2.34	0.91	22.77
time (sec)	N/A	0.025	0.026	0.006	1.497	1.393	0.262	2.259

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	228	374	0	3252	2759	478
normalized size	1	1.	0.75	1.24	0.	10.77	9.14	1.58
time (sec)	N/A	0.321	0.541	0.031	0.	1.991	24.924	2.271

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	648	648	598	886	0	9750	0	698
normalized size	1	1.	0.92	1.37	0.	15.05	0.	1.08
time (sec)	N/A	0.661	6.274	0.037	0.	3.677	0.	2.397

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	650	650	219	531	0	0	0	0
normalized size	1	1.	0.34	0.82	0.	0.	0.	0.
time (sec)	N/A	0.56	0.725	0.12	0.	0.	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	106	123	0	0	0	0
normalized size	1	1.	0.3	0.35	0.	0.	0.	0.
time (sec)	N/A	0.224	0.36	0.069	0.	0.	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	145	181	0	1443	0	144
normalized size	1	1.	1.41	1.76	0.	14.01	0.	1.4
time (sec)	N/A	0.207	3.674	0.085	0.	2.674	0.	1.228

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	86	139	0	1285	0	117
normalized size	1	1.	0.96	1.54	0.	14.28	0.	1.3
time (sec)	N/A	0.118	0.041	0.048	0.	2.484	0.	1.181

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	98	0	0	2809	0	0
normalized size	1	1.	0.96	0.	0.	27.54	0.	0.
time (sec)	N/A	0.171	0.069	0.16	0.	3.583	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	643	643	550	537	0	0	0	0
normalized size	1	1.	0.86	0.84	0.	0.	0.	0.
time (sec)	N/A	0.496	16.836	0.056	0.	0.	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	189	374	0	1937	0	0
normalized size	1	1.	1.28	2.53	0.	13.09	0.	0.
time (sec)	N/A	0.31	6.044	0.058	0.	4.998	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	166	313	0	1609	0	0
normalized size	1	1.	1.32	2.48	0.	12.77	0.	0.
time (sec)	N/A	0.208	4.391	0.046	0.	4.23	0.	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	190	0	0	3501	0	0
normalized size	1	1.	1.23	0.	0.	22.59	0.	0.
time (sec)	N/A	0.267	2.999	0.138	0.	131.972	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	91	0	1251	0	101
normalized size	1	1.	1.	1.23	0.	16.91	0.	1.36
time (sec)	N/A	0.129	0.053	0.065	0.	2.974	0.	1.168

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	65	0	385	0	62
normalized size	1	1.	1.	1.59	0.	9.39	0.	1.51
time (sec)	N/A	0.068	0.015	0.053	0.	2.714	0.	1.13

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	0	0	1226	0	0
normalized size	1	1.	1.	0.	0.	17.51	0.	0.
time (sec)	N/A	0.158	0.048	0.148	0.	3.164	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	122	179	0	0	0	0
normalized size	1	1.	0.42	0.62	0.	0.	0.	0.
time (sec)	N/A	0.24	2.061	0.059	0.	0.	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	67	267	0	721	0	139
normalized size	1	1.	0.94	3.76	0.	10.15	0.	1.96
time (sec)	N/A	0.162	0.321	0.076	0.	3.272	0.	1.317

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	248	0	771	0	161
normalized size	1	1.	0.99	3.35	0.	10.42	0.	2.18
time (sec)	N/A	0.115	0.274	0.056	0.	3.269	0.	1.241

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	108	0	0	2276	0	0
normalized size	1	1.	0.89	0.	0.	18.81	0.	0.
time (sec)	N/A	0.214	0.561	0.125	0.	4.734	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	104	654	0	1273	0	806
normalized size	1	1.	0.95	6.	0.	11.68	0.	7.39
time (sec)	N/A	0.228	0.77	0.093	0.	4.049	0.	1.249

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	113	602	0	1355	0	834
normalized size	1	1.	0.97	5.15	0.	11.58	0.	7.13
time (sec)	N/A	0.187	0.765	0.065	0.	4.13	0.	1.37

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	149	0	0	3970	0	0
normalized size	1	1.	0.81	0.	0.	21.69	0.	0.
time (sec)	N/A	0.302	1.407	0.128	0.	7.161	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	151	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.713	1.283	0.257	0.	0.	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	304	0	0	0	0	0
normalized size	1	1.	2.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.331	0.603	0.198	0.	0.	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	460	460	385	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	1.283	6.279	0.201	0.	0.	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	617	617	381	0	0	0	0	0
normalized size	1	1.	0.62	0.	0.	0.	0.	0.
time (sec)	N/A	1.579	5.891	0.286	0.	0.	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	76	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.081	3.97	0.	0.	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.076	3.128	0.	0.	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.038	0.002	0.	0.	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	61	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	0.049	3.553	0.	0.	0.	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.076	3.549	0.	0.	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	0.081	3.42	0.	0.	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	61	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	0.063	0.505	0.	0.	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	61	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.057	5.706	0.	0.	0.	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	0.016	5.779	0.	0.	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	59	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	0.056	3.477	0.	0.	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	3.525	0.489	0.	0.	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	70	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.122	0.137	0.646	0.	0.	0.	0.

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	265	0	0	0	0	0
normalized size	1	1.	2.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.199	2.331	0.522	0.	0.	0.	0.

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	77	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.143	0.118	4.112	0.	0.	0.	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	56	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.139	9.022	0.536	0.	0.	0.	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	93	116	128	235	0	132
normalized size	1	1.	1.33	1.66	1.83	3.36	0.	1.89
time (sec)	N/A	0.051	0.067	0.088	1.106	1.562	0.	1.704

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	48	75	84	192	0	86
normalized size	1	1.	1.14	1.79	2.	4.57	0.	2.05
time (sec)	N/A	0.034	0.022	0.032	1.211	1.323	0.	1.668

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	47	44	62	115	0	65
normalized size	1	1.	1.68	1.57	2.21	4.11	0.	2.32
time (sec)	N/A	0.033	0.025	0.037	1.086	1.539	0.	1.681

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	44	36	39	74	0	49
normalized size	1	1.	1.38	1.12	1.22	2.31	0.	1.53
time (sec)	N/A	0.035	0.017	0.077	1.057	1.429	0.	1.558

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	52	72	63	117	0	2898
normalized size	1	1.	0.96	1.33	1.17	2.17	0.	53.67
time (sec)	N/A	0.05	0.186	0.082	1.019	1.485	0.	37.295

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	75	92	86	163	0	0
normalized size	1	1.	0.99	1.21	1.13	2.14	0.	0.
time (sec)	N/A	0.059	0.283	0.081	1.005	1.521	0.	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	75	94	76	180	0	95
normalized size	1	1.	1.1	1.38	1.12	2.65	0.	1.4
time (sec)	N/A	0.051	0.247	0.087	1.115	1.357	0.	1.468

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	53	66	53	135	0	65
normalized size	1	1.	1.15	1.43	1.15	2.93	0.	1.41
time (sec)	N/A	0.041	0.138	0.043	1.002	1.312	0.	1.531

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	33	34	92	36	34
normalized size	1	1.	1.	1.18	1.21	3.29	1.29	1.21
time (sec)	N/A	0.029	0.011	0.038	1.125	1.488	1.766	1.462

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	54	53	77	0	228
normalized size	1	1.	0.97	1.64	1.61	2.33	0.	6.91
time (sec)	N/A	0.039	0.056	0.036	1.64	1.722	0.	1.409

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	46	81	93	122	0	2714
normalized size	1	1.	0.75	1.33	1.52	2.	0.	44.49
time (sec)	N/A	0.046	0.128	0.082	1.649	1.685	0.	4.166

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	74	102	131	167	0	5072
normalized size	1	1.	0.85	1.17	1.51	1.92	0.	58.3
time (sec)	N/A	0.055	0.211	0.085	1.705	1.678	0.	5.932

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	128	128	875	248	211	343	0	225
normalized size	1	1.	6.84	1.94	1.65	2.68	0.	1.76
time (sec)	N/A	0.165	10.817	0.066	1.125	1.777	0.	1.863

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	347	178	161	284	0	162
normalized size	1	1.	3.61	1.85	1.68	2.96	0.	1.69
time (sec)	N/A	0.086	8.491	0.04	1.04	1.404	0.	1.827

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	66	125	142	266	0	140
normalized size	1	1.	1.06	2.02	2.29	4.29	0.	2.26
time (sec)	N/A	0.09	0.367	0.046	1.098	1.542	0.	1.906

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	71	104	97	197	0	130
normalized size	1	1.	1.27	1.86	1.73	3.52	0.	2.32
time (sec)	N/A	0.064	0.408	0.049	1.161	1.48	0.	1.941

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	52	89	76	169	0	0
normalized size	1	1.	0.91	1.56	1.33	2.96	0.	0.
time (sec)	N/A	0.06	0.159	0.052	1.015	1.477	0.	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	77	153	109	231	0	0
normalized size	1	1.	0.9	1.78	1.27	2.69	0.	0.
time (sec)	N/A	0.08	0.366	0.056	1.092	1.522	0.	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	116	183	140	290	0	0
normalized size	1	1.	1.02	1.61	1.23	2.54	0.	0.
time (sec)	N/A	0.099	0.608	0.093	1.129	1.513	0.	0.

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	106	157	115	284	0	159
normalized size	1	1.	1.1	1.64	1.2	2.96	0.	1.66
time (sec)	N/A	0.084	0.354	0.062	1.113	1.43	0.	2.412

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	83	111	89	231	0	108
normalized size	1	1.	1.12	1.5	1.2	3.12	0.	1.46
time (sec)	N/A	0.069	0.514	0.059	1.131	1.513	0.	2.264

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	57	57	167	0	57
normalized size	1	1.	1.	1.16	1.16	3.41	0.	1.16
time (sec)	N/A	0.054	0.14	0.053	1.029	1.446	0.	1.776

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	111	89	166	0	802
normalized size	1	1.	1.	2.02	1.62	3.02	0.	14.58
time (sec)	N/A	0.077	0.394	0.043	1.674	1.412	0.	1.955

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	65	122	131	176	0	5287
normalized size	1	1.	0.75	1.4	1.51	2.02	0.	60.77
time (sec)	N/A	0.085	0.324	0.044	1.584	1.383	0.	47.857

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	87	166	177	235	0	6057
normalized size	1	1.	0.71	1.36	1.45	1.93	0.	49.65
time (sec)	N/A	0.131	0.358	0.053	1.555	1.418	0.	61.645

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	207	224	0	724	0	177
normalized size	1	1.	2.3	2.49	0.	8.04	0.	1.97
time (sec)	N/A	0.143	1.328	0.087	0.	2.037	0.	1.76

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	53	111	0	413	0	119
normalized size	1	1.	0.9	1.88	0.	7.	0.	2.02
time (sec)	N/A	0.081	0.104	0.077	0.	1.693	0.	2.222

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	36	0	279	0	63
normalized size	1	1.	1.	0.9	0.	6.98	0.	1.58
time (sec)	N/A	0.046	0.047	0.056	0.	1.329	0.	1.729

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	61	0	414	0	99
normalized size	1	1.	1.	1.02	0.	6.9	0.	1.65
time (sec)	N/A	0.08	0.094	0.063	0.	1.535	0.	1.703

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	115	98	0	617	0	217
normalized size	1	1.	1.31	1.11	0.	7.01	0.	2.47
time (sec)	N/A	0.122	0.492	0.064	0.	1.572	0.	1.653

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	148	165	0	892	0	431
normalized size	1	1.	1.17	1.31	0.	7.08	0.	3.42
time (sec)	N/A	0.148	1.714	0.071	0.	1.765	0.	1.765

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	103	206	0	992	0	204
normalized size	1	1.	0.95	1.91	0.	9.19	0.	1.89
time (sec)	N/A	0.11	0.923	0.073	0.	1.783	0.	1.63

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	74	127	0	807	0	130
normalized size	1	1.	0.96	1.65	0.	10.48	0.	1.69
time (sec)	N/A	0.091	0.331	0.07	0.	1.535	0.	1.612

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	66	0	644	0	84
normalized size	1	1.	1.	1.27	0.	12.38	0.	1.62
time (sec)	N/A	0.067	0.144	0.063	0.	1.627	0.	1.65

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	24	0	487	0	54
normalized size	1	1.	1.	0.75	0.	15.22	0.	1.69
time (sec)	N/A	0.053	0.052	0.064	0.	1.498	0.	1.71

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	78	137	0	690	0	149
normalized size	1	1.	0.94	1.65	0.	8.31	0.	1.8
time (sec)	N/A	0.104	0.185	0.069	0.	1.688	0.	1.658

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	113	303	0	929	0	247
normalized size	1	1.	0.88	2.35	0.	7.2	0.	1.91
time (sec)	N/A	0.167	0.428	0.077	0.	1.75	0.	1.651

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	254	389	0	1447	0	331
normalized size	1	1.	1.52	2.33	0.	8.66	0.	1.98
time (sec)	N/A	0.267	4.104	0.109	0.	2.421	0.	1.808

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	191	236	0	929	0	207
normalized size	1	1.	1.75	2.17	0.	8.52	0.	1.9
time (sec)	N/A	0.141	0.807	0.099	0.	2.019	0.	1.819

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	75	80	0	598	0	123
normalized size	1	1.	0.95	1.01	0.	7.57	0.	1.56
time (sec)	N/A	0.08	0.238	0.09	0.	1.541	0.	1.739

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	92	102	0	738	0	151
normalized size	1	1.	0.98	1.09	0.	7.85	0.	1.61
time (sec)	N/A	0.083	0.237	0.074	0.	1.623	0.	1.785

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	119	118	0	973	0	205
normalized size	1	1.	1.04	1.04	0.	8.54	0.	1.8
time (sec)	N/A	0.181	0.522	0.085	0.	1.81	0.	1.724

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	147	164	0	1307	0	444
normalized size	1	1.	1.03	1.15	0.	9.14	0.	3.1
time (sec)	N/A	0.213	1.523	0.092	0.	2.081	0.	1.75

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	135	275	0	1349	0	243
normalized size	1	1.	1.06	2.17	0.	10.62	0.	1.91
time (sec)	N/A	0.142	0.732	0.093	0.	1.853	0.	1.705

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	104	181	0	1089	0	173
normalized size	1	1.	1.	1.74	0.	10.47	0.	1.66
time (sec)	N/A	0.136	0.62	0.085	0.	1.729	0.	1.731

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	83	112	0	844	0	124
normalized size	1	1.	1.08	1.45	0.	10.96	0.	1.61
time (sec)	N/A	0.076	0.29	0.088	0.	1.738	0.	1.709

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	63	57	0	771	0	95
normalized size	1	1.	0.95	0.86	0.	11.68	0.	1.44
time (sec)	N/A	0.062	0.266	0.083	0.	1.694	0.	1.737

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	116	248	0	1370	0	1083
normalized size	1	1.	0.78	1.68	0.	9.26	0.	7.32
time (sec)	N/A	0.186	1.148	0.084	0.	1.954	0.	147.814

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	148	413	0	1786	0	363
normalized size	1	1.	0.7	1.95	0.	8.42	0.	1.71
time (sec)	N/A	0.301	2.021	0.088	0.	2.147	0.	1.731

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	81	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.186	0.808	0.	0.	0.	0.

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	2033	0	0	0	0	0
normalized size	1	1.	18.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	16.163	0.923	0.	0.	0.	0.

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	89	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	0.157	0.267	0.	0.	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	122	0	143	266	0	0
normalized size	1	1.	1.23	0.	1.44	2.69	0.	0.
time (sec)	N/A	0.135	2.143	0.532	1.258	1.495	0.	0.

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	87	0	96	189	0	0
normalized size	1	1.	1.34	0.	1.48	2.91	0.	0.
time (sec)	N/A	0.111	2.094	0.514	1.117	1.416	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	25070	47	126	0	0
normalized size	1	1.	1.	808.71	1.52	4.06	0.	0.
time (sec)	N/A	0.093	0.023	2.749	1.031	1.422	0.	0.

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.005	0.003	0.	0.	0.	0.

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	61	61	1060	0	0	0	0	0
normalized size	1	1.	17.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	5.566	5.872	0.	0.	0.	0.

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	81	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.111	0.537	0.	0.	0.	0.

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	70	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.077	3.32	0.	0.	0.	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	482	0	0	0	0	0
normalized size	1	1.	6.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	3.509	7.773	0.	0.	0.	0.

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	82	82	1552	0	0	0	0	0
normalized size	1	1.	18.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	6.57	6.417	0.	0.	0.	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	2.686	2.257	0.	0.	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	3.623	0.381	0.	0.	0.	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	1.746	0.305	0.	0.	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	3.252	0.339	0.	0.	0.	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	10.538	0.75	0.	0.	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	165	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.191	2.193	0.428	0.	0.	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	122	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.13	1.619	0.378	0.	0.	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	0.11	0.333	0.	0.	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.769	0.297	0.	0.	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	8.139	0.501	0.	0.	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	299	0	0	0	0	0
normalized size	1	1.	3.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.189	1.923	0.924	0.	0.	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	292	0	0	0	0	0
normalized size	1	1.	2.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.183	3.504	0.773	0.	0.	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	104	319	0	0	0	0	0
normalized size	1	1.	3.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.214	2.071	0.292	0.	0.	0.	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	56	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	2.683	2.397	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [379] had the largest ratio of [0.9286]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.	14	0.214
2	A	3	3	1.	14	0.214
3	A	2	2	1.	14	0.143
4	A	2	2	1.	14	0.143
5	A	3	3	1.	14	0.214
6	A	4	3	1.	14	0.214
7	A	16	10	1.	14	0.714

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
8	A	14	10	1.	14	0.714
9	A	13	10	1.	14	0.714
10	A	13	10	1.	14	0.714
11	A	14	10	1.	14	0.714
12	A	16	10	1.	14	0.714
13	A	7	3	1.	14	0.214
14	A	5	3	1.	14	0.214
15	A	3	3	1.	14	0.214
16	A	3	3	1.	14	0.214
17	A	5	3	1.	14	0.214
18	A	7	3	1.	14	0.214
19	A	3	3	1.	14	0.214
20	A	3	3	1.	14	0.214
21	A	3	3	1.	14	0.214
22	A	3	3	1.	14	0.214
23	A	3	3	1.	14	0.214
24	A	3	3	1.	14	0.214
25	A	3	3	1.	12	0.25
26	A	3	3	1.	12	0.25
27	A	3	3	1.	12	0.25
28	A	3	3	1.	12	0.25
29	A	2	2	1.	14	0.143
30	A	3	2	1.	21	0.095
31	A	3	2	1.	21	0.095
32	A	3	2	1.	19	0.105
33	A	3	3	1.	19	0.158
34	A	4	4	1.	21	0.19
35	A	5	5	1.	21	0.238
36	A	6	6	1.	21	0.286
37	A	5	5	1.	21	0.238
38	A	4	4	1.	21	0.19
39	A	3	2	1.	12	0.167
40	A	3	2	1.	21	0.095
41	A	3	2	1.	21	0.095
42	A	3	2	1.	21	0.095
43	A	3	2	1.	23	0.087
44	A	3	2	1.	23	0.087
45	A	3	2	1.	21	0.095
46	A	4	3	1.	21	0.143
47	A	5	5	1.	23	0.217
48	A	6	5	1.	23	0.217
49	A	6	5	1.	23	0.217
50	A	5	5	1.	23	0.217
51	A	4	3	1.	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
52	A	3	2	1.	23	0.087
53	A	3	2	1.	23	0.087
54	A	3	2	1.	23	0.087
55	A	4	3	1.	23	0.13
56	A	4	4	1.	23	0.174
57	A	3	3	1.	21	0.143
58	A	4	4	1.	21	0.19
59	A	5	5	1.	23	0.217
60	A	6	6	1.	23	0.261
61	A	7	7	1.	23	0.304
62	A	6	6	1.	23	0.261
63	A	5	5	1.	23	0.217
64	A	3	3	1.	14	0.214
65	A	3	3	1.	23	0.13
66	A	4	4	1.	23	0.174
67	A	4	3	1.	23	0.13
68	A	6	5	1.	23	0.217
69	A	5	4	1.	23	0.174
70	A	4	4	1.	21	0.19
71	A	5	5	1.	21	0.238
72	A	6	6	1.	23	0.261
73	A	7	6	1.	23	0.261
74	A	7	6	1.	23	0.261
75	A	6	6	1.	23	0.261
76	A	5	5	1.	14	0.357
77	A	4	4	1.	23	0.174
78	A	5	4	1.	23	0.174
79	A	6	5	1.	23	0.217
80	A	7	6	1.	23	0.261
81	A	6	5	1.	23	0.217
82	A	5	4	1.	21	0.19
83	A	6	6	1.	21	0.286
84	A	7	6	1.	23	0.261
85	A	8	6	1.	23	0.261
86	A	8	6	1.	23	0.261
87	A	7	6	1.	23	0.261
88	A	6	6	1.	14	0.429
89	A	5	4	1.	23	0.174
90	A	6	5	1.	23	0.217
91	A	7	6	1.	23	0.261
92	A	6	6	1.	25	0.24
93	A	5	5	1.	25	0.2
94	A	4	4	1.	23	0.174
95	A	6	6	1.	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
96	A	7	7	1.	25	0.28
97	A	8	8	1.	25	0.32
98	A	8	8	1.	25	0.32
99	A	7	7	1.	25	0.28
100	A	6	6	1.	16	0.375
101	A	4	4	1.	25	0.16
102	A	5	5	1.	25	0.2
103	A	6	6	1.	25	0.24
104	A	7	7	1.	25	0.28
105	A	6	6	1.	25	0.24
106	A	5	5	1.	23	0.217
107	A	7	7	1.	23	0.304
108	A	8	8	1.	25	0.32
109	A	9	9	1.	25	0.36
110	A	9	9	1.	25	0.36
111	A	8	8	1.	25	0.32
112	A	7	7	1.	16	0.438
113	A	5	5	1.	25	0.2
114	A	6	6	1.	25	0.24
115	A	7	7	1.	25	0.28
116	A	4	4	1.	25	0.16
117	A	3	3	1.	25	0.12
118	A	2	2	1.	23	0.087
119	A	3	3	1.	23	0.13
120	A	5	5	1.	25	0.2
121	A	6	6	1.	25	0.24
122	A	6	6	1.	25	0.24
123	A	5	5	1.	25	0.2
124	A	3	3	1.	16	0.188
125	A	2	2	1.	25	0.08
126	A	3	3	1.	25	0.12
127	A	4	4	1.	25	0.16
128	A	5	5	1.	25	0.2
129	A	4	4	1.	25	0.16
130	A	3	3	1.	23	0.13
131	A	4	4	1.	23	0.174
132	A	6	6	1.	25	0.24
133	A	7	6	1.	25	0.24
134	A	7	6	1.	25	0.24
135	A	6	6	1.	25	0.24
136	A	4	4	1.	16	0.25
137	A	3	3	1.	25	0.12
138	A	4	4	1.	25	0.16
139	A	5	5	1.	25	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
140	A	6	6	1.	25	0.24
141	A	5	5	1.	25	0.2
142	A	4	4	1.	23	0.174
143	A	6	6	1.	23	0.261
144	A	7	6	1.	25	0.24
145	A	8	6	1.	25	0.24
146	A	8	6	1.	25	0.24
147	A	7	6	1.	25	0.24
148	A	6	6	1.	16	0.375
149	A	4	4	1.	25	0.16
150	A	5	5	1.	25	0.2
151	A	6	6	1.	25	0.24
152	A	3	3	1.	23	0.13
153	A	3	3	1.	25	0.12
154	A	5	5	1.	23	0.217
155	A	4	4	1.	23	0.174
156	A	3	3	1.	21	0.143
157	A	3	3	1.	21	0.143
158	A	3	3	1.	23	0.13
159	A	3	3	1.	23	0.13
160	A	3	3	1.	14	0.214
161	A	3	3	1.	23	0.13
162	A	4	4	1.	23	0.174
163	A	5	5	0.98	23	0.217
164	A	3	3	1.	25	0.12
165	A	3	3	1.	23	0.13
166	A	3	3	1.	14	0.214
167	A	3	3	1.	23	0.13
168	A	4	3	1.	23	0.13
169	A	4	3	1.	23	0.13
170	A	3	3	1.	23	0.13
171	A	3	3	1.	21	0.143
172	A	3	3	1.	21	0.143
173	A	3	3	1.	23	0.13
174	A	0	0	0.	0	0.
175	A	3	3	1.	23	0.13
176	A	4	4	1.	25	0.16
177	A	3	3	1.	25	0.12
178	A	0	0	0.	0	0.
179	A	4	3	1.	14	0.214
180	A	4	3	1.	14	0.214
181	A	4	3	1.	14	0.214
182	A	4	4	1.	14	0.286
183	A	5	4	1.	14	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	6	4	1.	14	0.286
185	A	4	3	1.	21	0.143
186	A	3	3	1.	21	0.143
187	A	2	2	1.	19	0.105
188	A	3	2	1.	19	0.105
189	A	3	3	1.	21	0.143
190	A	4	4	1.	21	0.19
191	A	5	3	1.	21	0.143
192	A	4	3	1.	21	0.143
193	A	3	3	1.	21	0.143
194	A	3	2	1.	12	0.167
195	A	2	2	1.	21	0.095
196	A	4	4	1.	21	0.19
197	A	5	4	1.	21	0.19
198	A	4	3	1.	23	0.13
199	A	4	3	1.	23	0.13
200	A	4	3	1.	21	0.143
201	A	4	3	1.	21	0.143
202	A	4	3	1.	23	0.13
203	A	4	3	1.	23	0.13
204	A	4	3	1.	23	0.13
205	A	4	3	1.	23	0.13
206	A	4	3	1.	23	0.13
207	A	4	3	1.	14	0.214
208	A	4	3	1.	23	0.13
209	A	4	3	1.	23	0.13
210	A	4	3	1.	23	0.13
211	A	4	3	1.	23	0.13
212	A	4	3	1.	23	0.13
213	A	5	4	1.	21	0.19
214	A	4	3	1.	21	0.143
215	A	4	3	1.	23	0.13
216	A	4	3	1.	23	0.13
217	A	6	6	1.	23	0.261
218	A	5	5	1.	23	0.217
219	A	4	4	1.	23	0.174
220	A	3	3	1.	14	0.214
221	A	5	5	1.	23	0.217
222	A	6	6	1.	23	0.261
223	A	7	6	1.	23	0.261
224	A	4	3	1.	23	0.13
225	A	4	3	1.	23	0.13
226	A	4	3	1.	21	0.143
227	A	4	3	1.	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
228	A	4	3	1.	23	0.13
229	A	4	3	1.	23	0.13
230	A	6	6	1.	23	0.261
231	A	5	5	1.	23	0.217
232	A	5	5	1.	23	0.217
233	A	5	5	1.	14	0.357
234	A	6	6	1.	23	0.261
235	A	7	6	1.	23	0.261
236	A	8	6	1.	23	0.261
237	A	4	3	1.	23	0.13
238	A	4	3	1.	23	0.13
239	A	4	3	1.	21	0.143
240	A	4	3	1.	21	0.143
241	A	4	3	1.	23	0.13
242	A	4	3	1.	23	0.13
243	A	6	6	1.	23	0.261
244	A	6	6	1.	23	0.261
245	A	6	6	1.	23	0.261
246	A	6	6	1.	14	0.429
247	A	7	7	1.	23	0.304
248	A	8	7	1.	23	0.304
249	A	9	7	1.	23	0.304
250	A	4	3	1.	14	0.214
251	A	4	3	1.	14	0.214
252	A	4	3	1.	14	0.214
253	A	3	2	1.	12	0.167
254	A	3	3	1.	14	0.214
255	A	5	5	1.	14	0.357
256	A	6	6	1.	14	0.429
257	A	5	4	1.	17	0.235
258	A	4	3	1.	17	0.176
259	A	4	4	1.	17	0.235
260	A	3	3	1.	15	0.2
261	A	4	4	1.	15	0.267
262	A	4	4	1.	17	0.235
263	A	5	5	1.	17	0.294
264	A	4	3	1.	17	0.176
265	A	4	4	1.	16	0.25
266	A	4	3	1.	17	0.176
267	A	5	5	1.	17	0.294
268	A	3	3	1.	15	0.2
269	A	5	5	1.	15	0.333
270	A	5	5	1.	17	0.294
271	A	5	5	1.	16	0.312

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
272	A	6	5	1.	16	0.312
273	A	4	3	1.	17	0.176
274	A	5	5	1.	17	0.294
275	A	3	3	1.	15	0.2
276	A	5	5	1.	15	0.333
277	A	5	4	1.	17	0.235
278	A	4	3	1.	17	0.176
279	A	4	4	1.	17	0.235
280	A	3	3	1.	15	0.2
281	A	6	5	1.	15	0.333
282	A	5	4	1.	17	0.235
283	A	3	3	1.	16	0.188
284	A	4	4	1.	16	0.25
285	A	5	4	1.	16	0.25
286	A	6	4	1.	16	0.25
287	A	4	4	1.	10	0.4
288	A	3	3	1.	10	0.3
289	A	3	3	1.	10	0.3
290	A	5	5	1.	12	0.417
291	A	4	4	1.	12	0.333
292	A	3	3	1.	12	0.25
293	A	7	6	1.	25	0.24
294	A	6	6	1.	25	0.24
295	A	5	5	1.	23	0.217
296	A	7	5	1.	23	0.217
297	A	8	6	1.	25	0.24
298	A	9	7	1.	25	0.28
299	A	9	8	1.	25	0.32
300	A	8	8	1.	25	0.32
301	A	7	7	1.	25	0.28
302	A	6	6	1.	16	0.375
303	A	5	5	1.	25	0.2
304	A	6	6	1.	25	0.24
305	A	7	6	1.	25	0.24
306	A	8	6	1.	25	0.24
307	A	7	6	1.	25	0.24
308	A	6	5	1.	23	0.217
309	A	8	6	1.	23	0.261
310	A	8	6	1.	25	0.24
311	A	9	7	1.	25	0.28
312	A	10	8	1.	25	0.32
313	A	9	8	1.	25	0.32
314	A	8	8	1.	25	0.32
315	A	7	7	1.	16	0.438

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
316	A	7	7	1.	25	0.28
317	A	6	6	1.	25	0.24
318	A	7	6	1.	25	0.24
319	A	8	8	1.	16	0.5
320	A	6	5	1.	25	0.2
321	A	5	5	1.	25	0.2
322	A	4	4	1.	23	0.174
323	A	7	5	1.	23	0.217
324	A	8	6	1.	25	0.24
325	A	9	7	1.	25	0.28
326	A	8	8	1.	25	0.32
327	A	7	7	1.	25	0.28
328	A	6	6	1.	25	0.24
329	A	3	3	1.	16	0.188
330	A	5	5	1.	25	0.2
331	A	6	6	1.	25	0.24
332	A	7	6	1.	25	0.24
333	A	6	5	1.	25	0.2
334	A	5	5	1.	25	0.2
335	A	5	5	1.	23	0.217
336	A	8	6	1.	23	0.261
337	A	9	7	1.	25	0.28
338	A	10	8	1.	25	0.32
339	A	8	8	1.	25	0.32
340	A	7	7	1.	25	0.28
341	A	4	4	1.	25	0.16
342	A	4	4	1.	16	0.25
343	A	6	6	1.	25	0.24
344	A	7	6	1.	25	0.24
345	A	8	6	1.	25	0.24
346	A	6	5	1.	25	0.2
347	A	6	6	1.	25	0.24
348	A	6	5	1.	23	0.217
349	A	9	7	1.	23	0.304
350	A	10	7	1.	25	0.28
351	A	11	8	1.	25	0.32
352	A	8	8	1.	25	0.32
353	A	6	6	1.	25	0.24
354	A	6	6	1.	25	0.24
355	A	6	6	1.	16	0.375
356	A	7	7	1.	25	0.28
357	A	8	7	1.	25	0.28
358	A	9	7	1.	25	0.28
359	A	4	4	1.	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
360	A	3	3	1.	25	0.12
361	A	5	4	1.	23	0.174
362	A	4	4	1.	23	0.174
363	A	3	3	1.	21	0.143
364	A	5	5	1.	21	0.238
365	A	6	6	1.	23	0.261
366	A	7	7	1.	23	0.304
367	A	3	3	1.	23	0.13
368	A	3	3	1.	23	0.13
369	A	3	3	1.	23	0.13
370	A	3	3	1.	14	0.214
371	A	3	3	1.	23	0.13
372	A	3	3	1.	23	0.13
373	A	3	3	1.	23	0.13
374	A	6	5	1.	14	0.357
375	A	6	5	1.	14	0.357
376	A	6	5	1.	14	0.357
377	A	3	2	1.	12	0.167
378	A	14	12	1.	14	0.857
379	A	21	13	1.	14	0.929
380	A	7	6	1.	8	0.75
381	A	4	3	1.	14	0.214
382	A	4	3	1.	14	0.214
383	A	4	3	1.	14	0.214
384	A	4	2	1.	12	0.167
385	A	13	9	1.	14	0.643
386	A	23	10	1.	14	0.714
387	A	8	7	1.	16	0.438
388	A	4	4	1.	16	0.25
389	A	8	7	1.	17	0.412
390	A	8	7	1.	15	0.467
391	A	11	10	1.	15	0.667
392	A	12	9	1.	17	0.529
393	A	9	7	1.	17	0.412
394	A	9	8	1.	15	0.533
395	A	13	12	1.	15	0.8
396	A	7	6	1.	17	0.353
397	A	4	4	1.	15	0.267
398	A	9	8	1.	15	0.533
399	A	4	4	1.	17	0.235
400	A	6	6	1.	17	0.353
401	A	6	6	1.	15	0.4
402	A	12	11	1.	15	0.733
403	A	7	6	1.	17	0.353

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
404	A	7	7	1.	15	0.467
405	A	14	12	1.	15	0.8
406	A	9	5	1.	29	0.172
407	A	7	4	1.	27	0.148
408	A	14	7	1.	29	0.241
409	A	15	7	1.	29	0.241
410	A	4	4	1.	25	0.16
411	A	4	4	1.	23	0.174
412	A	3	3	1.	14	0.214
413	A	4	4	1.	23	0.174
414	A	4	4	1.	23	0.174
415	A	4	4	1.	23	0.174
416	A	4	4	1.	23	0.174
417	A	4	4	1.	21	0.19
418	A	4	4	1.	21	0.19
419	A	4	4	1.	23	0.174
420	A	0	0	0.	0	0.
421	A	4	4	1.	23	0.174
422	A	4	4	1.	25	0.16
423	A	4	4	1.	25	0.16
424	A	0	0	0.	0	0.
425	A	4	4	1.	21	0.19
426	A	3	3	1.	19	0.158
427	A	3	3	1.	19	0.158
428	A	2	1	1.	21	0.048
429	A	3	2	1.	21	0.095
430	A	3	2	1.	21	0.095
431	A	3	2	1.	21	0.095
432	A	3	2	1.	21	0.095
433	A	2	1	1.	21	0.048
434	A	3	3	1.	21	0.143
435	A	4	4	1.	21	0.19
436	A	5	4	1.	21	0.19
437	A	5	5	1.	23	0.217
438	A	4	4	1.	21	0.19
439	A	5	4	1.	21	0.19
440	A	4	3	1.	23	0.13
441	A	3	2	1.	23	0.087
442	A	3	2	1.	23	0.087
443	A	3	2	1.	23	0.087
444	A	3	2	1.	23	0.087
445	A	3	2	1.	23	0.087
446	A	3	2	1.	23	0.087
447	A	5	4	1.	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
448	A	4	4	1.	23	0.174
449	A	5	5	1.	23	0.217
450	A	5	5	1.	23	0.217
451	A	4	4	1.	23	0.174
452	A	2	2	1.	21	0.095
453	A	3	3	1.	21	0.143
454	A	4	3	1.	23	0.13
455	A	4	3	1.	23	0.13
456	A	4	3	1.	23	0.13
457	A	4	3	1.	23	0.13
458	A	3	3	1.	23	0.13
459	A	2	2	1.	23	0.087
460	A	5	5	1.	23	0.217
461	A	6	6	1.	23	0.261
462	A	6	6	1.	23	0.261
463	A	5	5	1.	23	0.217
464	A	3	3	1.	23	0.13
465	A	3	3	1.	21	0.143
466	A	5	4	1.	21	0.19
467	A	5	4	1.	23	0.174
468	A	5	4	1.	23	0.174
469	A	5	4	1.	23	0.174
470	A	3	3	1.	23	0.13
471	A	3	3	1.	23	0.13
472	A	6	6	1.	23	0.261
473	A	7	6	1.	23	0.261
474	A	2	2	1.	23	0.087
475	A	3	3	1.	25	0.12
476	A	2	2	1.	25	0.08
477	A	4	3	1.	23	0.13
478	A	4	3	1.	23	0.13
479	A	3	3	1.	23	0.13
480	A	3	3	1.	14	0.214
481	A	3	3	1.	23	0.13
482	A	2	2	1.	23	0.087
483	A	2	2	1.	21	0.095
484	A	2	2	1.	21	0.095
485	A	2	2	1.	23	0.087
486	A	0	0	0.	0	0.
487	A	0	0	0.	0	0.
488	A	0	0	0.	0	0.
489	A	0	0	0.	0	0.
490	A	0	0	0.	0	0.
491	A	9	6	1.	25	0.24

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
492	A	7	6	1.	25	0.24
493	A	3	3	1.	25	0.12
494	A	0	0	0.	0	0.
495	A	0	0	0.	0	0.
496	A	4	4	1.	23	0.174
497	A	4	4	1.	25	0.16
498	A	4	4	1.	25	0.16
499	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int \left(b\tan^2(e+fx)\right)^{5/2}\,dx$

Optimal. Leaf size=98

$$\frac{b^2\tan^3(e+fx)\sqrt{b\tan^2(e+fx)}}{4f}-\frac{b^2\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f}-\frac{b^2\cot(e+fx)\sqrt{b\tan^2(e+fx)}\log(\cos(e+fx))}{f}$$

```
[Out] -((b^2*Cot[e + f*x]*Log[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]^2])/f) - (b^2*Tan
[e + f*x]*Sqrt[b*Tan[e + f*x]^2])/(2*f) + (b^2*Tan[e + f*x]^3*Sqrt[b*Tan[e
+ f*x]^2])/(4*f)
```

Rubi [A] time = 0.0390226, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 3475}

$$\frac{b^2\tan^3(e+fx)\sqrt{b\tan^2(e+fx)}}{4f}-\frac{b^2\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f}-\frac{b^2\cot(e+fx)\sqrt{b\tan^2(e+fx)}\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -((b^2*Cot[e + f*x]*Log[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]^2])/f) - (b^2*Tan
[e + f*x]*Sqrt[b*Tan[e + f*x]^2])/(2*f) + (b^2*Tan[e + f*x]^3*Sqrt[b*Tan[e
+ f*x]^2])/(4*f)
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(b\tan^2(e+fx)\right)^{5/2}\,dx = \left(b^2\cot(e+fx)\sqrt{b\tan^2(e+fx)}\right)\int \tan^5(e+fx)\,dx$$

$$= \frac{b^2\tan^3(e+fx)\sqrt{b\tan^2(e+fx)}}{4f} - \left(b^2\cot(e+fx)\sqrt{b\tan^2(e+fx)}\right)\int \tan^3(e+fx)\,dx$$

$$= -\frac{b^2\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f} + \frac{b^2\tan^3(e+fx)\sqrt{b\tan^2(e+fx)}}{4f} + \left(b^2\cot(e+fx)\sqrt{b\tan^2(e+fx)}\right)\ldots$$

$$= -\frac{b^2\cot(e+fx)\log(\cos(e+fx))\sqrt{b\tan^2(e+fx)}}{f} - \frac{b^2\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f} + \frac{b^2\tan\ldots}{\ldots}$$

Mathematica [A] time = 0.372675, size = 56, normalized size = 0.57

$$-\frac{\cot(e+fx)\left(b\tan^2(e+fx)\right)^{5/2}\left(2\cot^2(e+fx)+4\cot^4(e+fx)\log(\cos(e+fx))-1\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(Cot[e + f*x]*(-1 + 2*Cot[e + f*x]^2 + 4*Cot[e + f*x]^4*Log[Cos[e + f*x]])
*(b*Tan[e + f*x]^2)^(5/2))/(4*f)
```

Maple [A] time = 0.146, size = 58, normalized size = 0.6

$$\frac{\left(\tan\left(fx+e\right)\right)^4-2\left(\tan\left(fx+e\right)\right)^2+2\ln\left(1+\left(\tan\left(fx+e\right)\right)^2\right)}{4f\left(\tan\left(fx+e\right)\right)^5}\left(b\left(\tan\left(fx+e\right)\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/4/f*(b*tan(f*x+e)^2)^(5/2)*(tan(f*x+e)^4-2*tan(f*x+e)^2+2*ln(1+tan(f*x+e)
^2))/tan(f*x+e)^5
```

Maxima [A] time = 1.6261, size = 63, normalized size = 0.64

$$\frac{b^{\frac{5}{2}}\tan\left(fx+e\right)^4-2\,b^{\frac{5}{2}}\tan\left(fx+e\right)^2+2\,b^{\frac{5}{2}}\log\left(\tan\left(fx+e\right)^2+1\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b^(5/2)*tan(f*x + e)^4 - 2*b^(5/2)*tan(f*x + e)^2 + 2*b^(5/2)*log(tan(
f*x + e)^2 + 1))/f
```

Fricas [A] time = 1.93873, size = 180, normalized size = 1.84

$$\frac{\left(b^2\tan\left(fx+e\right)^4-2\,b^2\tan\left(fx+e\right)^2-2\,b^2\log\left(\frac{1}{\tan\left(fx+e\right)^2+1}\right)-3\,b^2\right)\sqrt{b\tan\left(fx+e\right)^2}}{4\,f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*tan(f*x + e)^4 - 2*b^2*tan(f*x + e)^2 - 2*b^2*log(1/(tan(f*x + e)^
2 + 1)) - 3*b^2)*sqrt(b*tan(f*x + e)^2)/(f*tan(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan^2\left(e+fx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**(5/2), x)
```

Giac [A] time = 1.4196, size = 80, normalized size = 0.82

$$\frac{1}{4}\,b^{\frac{5}{2}}\left(\frac{2\,\log\left(\tan\left(fx+e\right)^2+1\right)}{f}+\frac{f\tan\left(fx+e\right)^4-2\,f\tan\left(fx+e\right)^2}{f^2}\right)\mathrm{sgn}\left(\tan\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/4*b^(5/2)*(2*log(tan(f*x + e)^2 + 1)/f + (f*tan(f*x + e)^4 - 2*f*tan(f*x
+ e)^2)/f^2)*sgn(tan(f*x + e))
```

3.2 $\int \left(b\tan^2(e+fx)\right)^{3/2} dx$

Optimal. Leaf size=61

$$\frac{b\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f}+\frac{b\cot(e+fx)\sqrt{b\tan^2(e+fx)}\log(\cos(e+fx))}{f}$$

```
[Out] (b*Cot[e + f*x]*Log[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]^2])/f + (b*Tan[e + f*
x]*Sqrt[b*Tan[e + f*x]^2])/(2*f)
```

Rubi [A] time = 0.0292693, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 3475}

$$\frac{b\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f}+\frac{b\cot(e+fx)\sqrt{b\tan^2(e+fx)}\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (b*Cot[e + f*x]*Log[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]^2])/f + (b*Tan[e + f*
x]*Sqrt[b*Tan[e + f*x]^2])/(2*f)
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(b\tan^2(e+fx)\right)^{3/2} dx &= \left(b\cot(e+fx)\sqrt{b\tan^2(e+fx)}\right)\int \tan^3(e+fx)\,dx \\
&= \frac{b\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f}-\left(b\cot(e+fx)\sqrt{b\tan^2(e+fx)}\right)\int \tan(e+fx)\,dx \\
&= \frac{b\cot(e+fx)\log(\cos(e+fx))\sqrt{b\tan^2(e+fx)}}{f}+\frac{b\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f}
\end{aligned}$$

Mathematica [A] time = 0.104254, size = 47, normalized size = 0.77

$$\frac{\cot^3(e+fx)\left(b\tan^2(e+fx)\right)^{3/2}\left(\tan^2(e+fx)+2\log(\cos(e+fx))\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cot[e + f*x]^3*(b*Tan[e + f*x]^2)^(3/2)*(2*Log[Cos[e + f*x]] + Tan[e + f*x
]^2))/(2*f)
```

Maple [A] time = 0.02, size = 48, normalized size = 0.8

$$-\frac{-\left(\tan\left(fx+e\right)\right)^2+\ln\left(1+\left(\tan\left(fx+e\right)\right)^2\right)}{2f\left(\tan\left(fx+e\right)\right)^3}\left(b\left(\tan\left(fx+e\right)\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f*(b*tan(f*x+e)^2)^(3/2)*(-tan(f*x+e)^2+ln(1+tan(f*x+e)^2))/tan(f*x+e)
^3
```

Maxima [A] time = 1.63171, size = 46, normalized size = 0.75

$$\frac{b^{\frac{3}{2}}\tan\left(fx+e\right)^2-b^{\frac{3}{2}}\log\left(\tan\left(fx+e\right)^2+1\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b^(3/2)*tan(f*x + e)^2 - b^(3/2)*log(tan(f*x + e)^2 + 1))/f
```

Fricas [A] time = 1.95299, size = 135, normalized size = 2.21

$$\frac{\left(b\tan\left(fx+e\right)^2+b\log\left(\frac{1}{\tan\left(fx+e\right)^2+1}\right)+b\right)\sqrt{b\tan\left(fx+e\right)^2}}{2f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*tan(f*x + e)^2 + b*log(1/(tan(f*x + e)^2 + 1)) + b)*sqrt(b*tan(f*x +
 e)^2)/(f*tan(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**(3/2), x)
```

Giac [A] time = 1.35357, size = 59, normalized size = 0.97

$$\frac{1}{2}b^{\frac{3}{2}}\left(\frac{\tan\left(fx+e\right)^2}{f}-\frac{\log\left(\tan\left(fx+e\right)^2+1\right)}{f}\right)\mathrm{sgn}\left(\tan\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*b^(3/2)*(tan(f*x + e)^2/f - log(tan(f*x + e)^2 + 1)/f)*sgn(tan(f*x + e)
)
```

3.3 $\int \sqrt{b \tan^2(e+fx)}\,dx$

Optimal. Leaf size=32

$$-\frac{\cot(e+fx)\sqrt{b\tan^2(e+fx)}\log(\cos(e+fx))}{f}$$

```
[Out] -((Cot[e + f*x]*Log[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]^2])/f)
```

Rubi [A] time = 0.0172923, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3658, 3475}

$$-\frac{\cot(e+fx)\sqrt{b\tan^2(e+fx)}\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Log[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]^2])/f)
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\tan^2(e+fx)}\,dx &= \left(\cot(e+fx)\sqrt{b\tan^2(e+fx)}\right)\int \tan(e+fx)\,dx\\ &= -\frac{\cot(e+fx)\log(\cos(e+fx))\sqrt{b\tan^2(e+fx)}}{f}\end{aligned}$$

Mathematica [A] time = 0.0392099, size = 32, normalized size = 1.

$$-\frac{\cot(e+fx)\sqrt{b\tan^2(e+fx)}\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Log[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]^2])/f)
```

Maple [A] time = 0.024, size = 37, normalized size = 1.2

$$\frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^{2}\right)}{2\,f\tan\left(fx+e\right)}\sqrt{b\left(\tan\left(fx+e\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/f*(b*tan(f*x+e)^2)^(1/2)/tan(f*x+e)*ln(1+tan(f*x+e)^2)
```

Maxima [A] time = 1.63487, size = 26, normalized size = 0.81

$$\frac{\sqrt{b}\log\left(\tan\left(fx+e\right)^{2}+1\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b)*log(tan(f*x + e)^2 + 1)/f
```

Fricas [A] time = 1.9592, size = 100, normalized size = 3.12

$$-\frac{\sqrt{b\tan\left(fx+e\right)^{2}}\log\left(\frac{1}{\tan\left(fx+e\right)^{2}+1}\right)}{2\,f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(b*tan(f*x + e)^2)*log(1/(tan(f*x + e)^2 + 1))/(f*tan(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\tan^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(e + f*x)**2), x)
```

Giac [A] time = 1.25947, size = 38, normalized size = 1.19

$$\frac{\sqrt{b}\log\left(\tan\left(fx+e\right)^{2}+1\right)\operatorname{sgn}\left(\tan\left(fx+e\right)\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b)*log(tan(f*x + e)^2 + 1)*sgn(tan(f*x + e))/f
```

3.4 $\int \frac{1}{\sqrt{b \tan^2(e+fx)}} \, dx$

Optimal. Leaf size=31

$$\frac{\tan(e+fx)\log(\sin(e+fx))}{f\sqrt{b\tan^2(e+fx)}}$$

```
[Out] (Log[Sin[e + f*x]]*Tan[e + f*x])/(f*Sqrt[b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.0230007, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3658, 3475}

$$\frac{\tan(e+fx)\log(\sin(e+fx))}{f\sqrt{b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Tan[e + f*x]^2],x]
```

```
[Out] (Log[Sin[e + f*x]]*Tan[e + f*x])/(f*Sqrt[b*Tan[e + f*x]^2])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{b\tan^2(e+fx)}} \, dx &= \frac{\tan(e+fx)\int \cot(e+fx) \, dx}{\sqrt{b\tan^2(e+fx)}} \\
&= \frac{\log(\sin(e+fx))\tan(e+fx)}{f\sqrt{b\tan^2(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.0816531, size = 39, normalized size = 1.26

$$\frac{\tan(e+fx)(\log(\tan(e+fx))+\log(\cos(e+fx)))}{f\sqrt{b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Tan[e + f*x]^2],x]
```

```
[Out] ((Log[Cos[e + f*x]] + Log[Tan[e + f*x]])*Tan[e + f*x])/(f*Sqrt[b*Tan[e + f*
x]^2])
```

Maple [A] time = 0.043, size = 47, normalized size = 1.5

$$\frac{\tan\left(fx+e\right)\left(2\ln\left(\tan\left(fx+e\right)\right)-\ln\left(1+\left(\tan\left(fx+e\right)\right)^{2}\right)\right)}{2f}\frac{1}{\sqrt{b\left(\tan\left(fx+e\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/f*tan(f*x+e)*(2*ln(tan(f*x+e))-ln(1+tan(f*x+e)^2))/(b*tan(f*x+e)^2)^(1/
2)
```

Maxima [A] time = 1.63721, size = 45, normalized size = 1.45

$$-\frac{\frac{\log\left(\tan\left(fx+e\right)^{2}+1\right)}{\sqrt{b}}-\frac{2\log\left(\tan\left(fx+e\right)\right)}{\sqrt{b}}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(log(tan(f*x + e)^2 + 1)/sqrt(b) - 2*log(tan(f*x + e))/sqrt(b))/f
```

Fricas [A] time = 1.85147, size = 119, normalized size = 3.84

$$\frac{\sqrt{b\tan\left(fx+e\right)^{2}}\log\left(\frac{\tan\left(fx+e\right)^{2}}{\tan\left(fx+e\right)^{2}+1}\right)}{2bf\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(b*tan(f*x + e)^2)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))/(b*f*ta
n(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{b\tan^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*tan(e + f*x)**2), x)
```

Giac [B] time = 1.32646, size = 113, normalized size = 3.65

$$-\frac{\frac{2\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1\right)}{\sqrt{b}\operatorname{sgn}\left(\tan\left(fx+e\right)\right)}-\frac{\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)}{\sqrt{b}\operatorname{sgn}\left(\tan\left(fx+e\right)\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)/(sqrt(b)*sgn(tan(f*
x + e))) - log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/(sqrt(b)*sgn(tan(f*x
 + e))))/f
```

3.5 $\int \frac{1}{\left(b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{\cot(e+fx)}{2bf\sqrt{b\tan^2(e+fx)}}-\frac{\tan(e+fx)\log(\sin(e+fx))}{bf\sqrt{b\tan^2(e+fx)}}$$

```
[Out] -Cot[e + f*x]/(2*b*f*Sqrt[b*Tan[e + f*x]^2]) - (Log[Sin[e + f*x]]*Tan[e + f
*x])/(b*f*Sqrt[b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.0368908, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 3475}

$$-\frac{\cot(e+fx)}{2bf\sqrt{b\tan^2(e+fx)}}-\frac{\tan(e+fx)\log(\sin(e+fx))}{bf\sqrt{b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^2)^(-3/2),x]
```

```
[Out] -Cot[e + f*x]/(2*b*f*Sqrt[b*Tan[e + f*x]^2]) - (Log[Sin[e + f*x]]*Tan[e + f
*x])/(b*f*Sqrt[b*Tan[e + f*x]^2])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\tan(e+fx)\int \cot^3(e+fx)\,dx}{b\sqrt{b\tan^2(e+fx)}}$$
$$= -\frac{\cot(e+fx)}{2bf\sqrt{b\tan^2(e+fx)}} - \frac{\tan(e+fx)\int \cot(e+fx)\,dx}{b\sqrt{b\tan^2(e+fx)}}$$
$$= -\frac{\cot(e+fx)}{2bf\sqrt{b\tan^2(e+fx)}} - \frac{\log(\sin(e+fx))\tan(e+fx)}{bf\sqrt{b\tan^2(e+fx)}}$$

Mathematica [A] time = 0.391674, size = 56, normalized size = 0.85

$$-\frac{\tan^3(e+fx)\left(\cot^2(e+fx)+2\log(\tan(e+fx))+2\log(\cos(e+fx))\right)}{2f\left(b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^2)^(-3/2),x]
```

```
[Out] -((Cot[e + f*x]^2 + 2*Log[Cos[e + f*x]] + 2*Log[Tan[e + f*x]])*Tan[e + f*x]
^3)/(2*f*(b*Tan[e + f*x]^2)^(3/2))
```

Maple [A] time = 0.02, size = 64, normalized size = 1.

$$-\frac{\tan(fx+e)\left(2\ln\left(\tan(fx+e)\right)\left(\tan(fx+e)\right)^2-\ln\left(1+\left(\tan(fx+e)\right)^2\right)\left(\tan(fx+e)\right)^2+1\right)}{2f}\left(b\left(\tan(fx+e)\right)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f*tan(f*x+e)*(2*ln(tan(f*x+e))*tan(f*x+e)^2-ln(1+tan(f*x+e)^2)*tan(f*x
+e)^2+1)/(b*tan(f*x+e)^2)^(3/2)
```

Maxima [A] time = 1.64679, size = 62, normalized size = 0.94

$$\frac{\frac{\log\left(\tan(fx+e)^2+1\right)}{b^{\frac{3}{2}}} - \frac{2\log\left(\tan(fx+e)\right)}{b^{\frac{3}{2}}} - \frac{1}{b^{\frac{3}{2}}\tan(fx+e)^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(tan(f*x + e)^2 + 1)/b^(3/2) - 2*log(tan(f*x + e))/b^(3/2) - 1/(b^(
3/2)*tan(f*x + e)^2))/f
```

Fricas [A] time = 1.89531, size = 177, normalized size = 2.68

$$-\frac{\sqrt{b\tan\left(fx+e\right)^{2}}\left(\log\left(\frac{\tan\left(fx+e\right)^{2}}{\tan\left(fx+e\right)^{2}+1}\right)\tan\left(fx+e\right)^{2}+\tan\left(fx+e\right)^{2}+1\right)}{2\,b^{2}f\tan\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(b*tan(f*x + e)^2)*(log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f
*x + e)^2 + tan(f*x + e)^2 + 1)/(b^2*f*tan(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**(-3/2), x)
```

Giac [B] time = 1.62848, size = 300, normalized size = 4.55

$$-\frac{\frac{\mathrm{sgn}\left(-\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+1\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{8\log\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+1\right)\mathrm{sgn}\left(-\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+1\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{4\log\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\right)\mathrm{sgn}\left(-\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\right.}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{8\,b^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*(sgn(-tan(1/2*f*x + 1/2*e)^2 + 1)*tan(1/2*f*x + 1/2*e)^2/sgn(tan(1/2*f
*x + 1/2*e)) - 8*log(tan(1/2*f*x + 1/2*e)^2 + 1)*sgn(-tan(1/2*f*x + 1/2*e)^
2 + 1)/sgn(tan(1/2*f*x + 1/2*e)) + 4*log(tan(1/2*f*x + 1/2*e)^2)*sgn(-tan(1
/2*f*x + 1/2*e)^2 + 1)/sgn(tan(1/2*f*x + 1/2*e)) - (4*sgn(-tan(1/2*f*x + 1/
2*e)^2 + 1)*tan(1/2*f*x + 1/2*e)^2 - sgn(-tan(1/2*f*x + 1/2*e)^2 + 1))/(sgn
(tan(1/2*f*x + 1/2*e))*tan(1/2*f*x + 1/2*e)^2))/(b^(3/2)*f)
```

3.6 $\int \frac{1}{\left(b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{\cot^3(e+fx)}{4b^2f\sqrt{b\tan^2(e+fx)}}+\frac{\cot(e+fx)}{2b^2f\sqrt{b\tan^2(e+fx)}}+\frac{\tan(e+fx)\log(\sin(e+fx))}{b^2f\sqrt{b\tan^2(e+fx)}}$$

```
[Out] Cot[e + f*x]/(2*b^2*f*Sqrt[b*Tan[e + f*x]^2]) - Cot[e + f*x]^3/(4*b^2*f*Sqr
t[b*Tan[e + f*x]^2]) + (Log[Sin[e + f*x]]*Tan[e + f*x])/(b^2*f*Sqrt[b*Tan[e
 + f*x]^2])
```

Rubi [A] time = 0.0390056, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 3475}

$$-\frac{\cot^3(e+fx)}{4b^2f\sqrt{b\tan^2(e+fx)}}+\frac{\cot(e+fx)}{2b^2f\sqrt{b\tan^2(e+fx)}}+\frac{\tan(e+fx)\log(\sin(e+fx))}{b^2f\sqrt{b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^2)^(-5/2),x]
```

```
[Out] Cot[e + f*x]/(2*b^2*f*Sqrt[b*Tan[e + f*x]^2]) - Cot[e + f*x]^3/(4*b^2*f*Sqr
t[b*Tan[e + f*x]^2]) + (Log[Sin[e + f*x]]*Tan[e + f*x])/(b^2*f*Sqrt[b*Tan[e
 + f*x]^2])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\tan(e+fx)\int \cot^5(e+fx)\,dx}{b^2\sqrt{b\tan^2(e+fx)}}$$
$$= -\frac{\cot^3(e+fx)}{4b^2f\sqrt{b\tan^2(e+fx)}} - \frac{\tan(e+fx)\int \cot^3(e+fx)\,dx}{b^2\sqrt{b\tan^2(e+fx)}}$$
$$= \frac{\cot(e+fx)}{2b^2f\sqrt{b\tan^2(e+fx)}} - \frac{\cot^3(e+fx)}{4b^2f\sqrt{b\tan^2(e+fx)}} + \frac{\tan(e+fx)\int \cot(e+fx)\,dx}{b^2\sqrt{b\tan^2(e+fx)}}$$
$$= \frac{\cot(e+fx)}{2b^2f\sqrt{b\tan^2(e+fx)}} - \frac{\cot^3(e+fx)}{4b^2f\sqrt{b\tan^2(e+fx)}} + \frac{\log(\sin(e+fx))\tan(e+fx)}{b^2f\sqrt{b\tan^2(e+fx)}}$$

Mathematica [A] time = 0.246315, size = 68, normalized size = 0.7

$$\frac{\tan^5(e+fx)\left(-\cot^4(e+fx)+2\cot^2(e+fx)+4\log(\tan(e+fx))+4\log(\cos(e+fx))\right)}{4f\left(b\tan^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^2)^(-5/2),x]
```

```
[Out] ((2*Cot[e + f*x]^2 - Cot[e + f*x]^4 + 4*Log[Cos[e + f*x]] + 4*Log[Tan[e + f
*x]])*Tan[e + f*x]^5)/(4*f*(b*Tan[e + f*x]^2)^(5/2))
```

Maple [A] time = 0.023, size = 74, normalized size = 0.8

$$\frac{\tan\left(fx+e\right)\left(4\ln\left(\tan\left(fx+e\right)\right)\left(\tan\left(fx+e\right)\right)^4-2\ln\left(1+\left(\tan\left(fx+e\right)\right)^2\right)\left(\tan\left(fx+e\right)\right)^4+2\left(\tan\left(fx+e\right)\right)^2-\right.}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/4/f*tan(f*x+e)*(4*ln(tan(f*x+e))*tan(f*x+e)^4-2*ln(1+tan(f*x+e)^2)*tan(f*
x+e)^4+2*tan(f*x+e)^2-1)/(b*tan(f*x+e)^2)^(5/2)
```

Maxima [A] time = 1.68978, size = 89, normalized size = 0.92

$$-\frac{\frac{2\log\left(\tan\left(fx+e\right)^2+1\right)}{b^{\frac{5}{2}}}-\frac{4\log\left(\tan\left(fx+e\right)\right)}{b^{\frac{5}{2}}}-\frac{2\sqrt{b}\tan\left(fx+e\right)^2-\sqrt{b}}{b^3\tan\left(fx+e\right)^4}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*log(tan(f*x + e)^2 + 1)/b^(5/2) - 4*log(tan(f*x + e))/b^(5/2) - (2*
sqrt(b)*tan(f*x + e)^2 - sqrt(b))/(b^3*tan(f*x + e)^4))/f
```

Fricas [A] time = 2.05207, size = 207, normalized size = 2.13

$$\frac{\left(2\,\log\left(\frac{\tan\left(fx+e\right)^{2}}{\tan\left(fx+e\right)^{2}+1}\right)\tan\left(fx+e\right)^{4}+3\,\tan\left(fx+e\right)^{4}+2\,\tan\left(fx+e\right)^{2}-1\right)\sqrt{b\tan\left(fx+e\right)^{2}}}{4\,b^{3}f\tan\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 + 3*tan(f*x
+ e)^4 + 2*tan(f*x + e)^2 - 1)*sqrt(b*tan(f*x + e)^2)/(b^3*f*tan(f*x + e)^5
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**(-5/2), x)
```

Giac [B] time = 1.99926, size = 392, normalized size = 4.04

$$-\frac{\operatorname{sgn}\left(-\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-12\operatorname{sgn}\left(-\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/64*(sgn(-tan(1/2*f*x + 1/2*e)^2 + 1)*sgn(tan(1/2*f*x + 1/2*e))*tan(1/2*f
*x + 1/2*e)^4 - 12*sgn(-tan(1/2*f*x + 1/2*e)^2 + 1)*sgn(tan(1/2*f*x + 1/2*e
))*tan(1/2*f*x + 1/2*e)^2 + 64*log(tan(1/2*f*x + 1/2*e)^2 + 1)*sgn(-tan(1/2
*f*x + 1/2*e)^2 + 1)/sgn(tan(1/2*f*x + 1/2*e)) - 32*log(tan(1/2*f*x + 1/2*e
)^2)*sgn(-tan(1/2*f*x + 1/2*e)^2 + 1)/sgn(tan(1/2*f*x + 1/2*e)) + (48*sgn(-
tan(1/2*f*x + 1/2*e)^2 + 1)*tan(1/2*f*x + 1/2*e)^4 - 12*sgn(-tan(1/2*f*x +
1/2*e)^2 + 1)*tan(1/2*f*x + 1/2*e)^2 + sgn(-tan(1/2*f*x + 1/2*e)^2 + 1))/(s
gn(tan(1/2*f*x + 1/2*e))*tan(1/2*f*x + 1/2*e)^4))/(b^(5/2)*f)
```

3.7 $\int \left(b\tan^3(e+fx)\right)^{5/2}\,dx$

Optimal. Leaf size=364

$$\frac{2b^2\tan^5(e+fx)\sqrt{b\tan^3(e+fx)}}{13f}-\frac{2b^2\tan^3(e+fx)\sqrt{b\tan^3(e+fx)}}{9f}+\frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f}-\frac{b^2\tan^{-1}}{}$$

```
[Out] (-2*b^2*Cot[e + f*x]*Sqrt[b*Tan[e + f*x]^3])/f - (b^2*ArcTan[1 - Sqrt[2]*Sq
rt[Tan[e + f*x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2]*f*Tan[e + f*x]^(3/2)) +
(b^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2
]*f*Tan[e + f*x]^(3/2)) - (b^2*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e +
 f*x]]*Sqrt[b*Tan[e + f*x]^3])/(2*Sqrt[2]*f*Tan[e + f*x]^(3/2)) + (b^2*Log[
1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Sqrt[b*Tan[e + f*x]^3])/(2*S
qrt[2]*f*Tan[e + f*x]^(3/2)) + (2*b^2*Tan[e + f*x]*Sqrt[b*Tan[e + f*x]^3])/
(5*f) - (2*b^2*Tan[e + f*x]^3*Sqrt[b*Tan[e + f*x]^3])/(9*f) + (2*b^2*Tan[e
+ f*x]^5*Sqrt[b*Tan[e + f*x]^3])/(13*f)
```

Rubi [A] time = 0.145777, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3473, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{2b^2\tan^5(e+fx)\sqrt{b\tan^3(e+fx)}}{13f}-\frac{2b^2\tan^3(e+fx)\sqrt{b\tan^3(e+fx)}}{9f}+\frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f}-\frac{b^2\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^3)^(5/2),x]
```

```
[Out] (-2*b^2*Cot[e + f*x]*Sqrt[b*Tan[e + f*x]^3])/f - (b^2*ArcTan[1 - Sqrt[2]*Sq
rt[Tan[e + f*x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2]*f*Tan[e + f*x]^(3/2)) +
(b^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2
]*f*Tan[e + f*x]^(3/2)) - (b^2*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e +
 f*x]]*Sqrt[b*Tan[e + f*x]^3])/(2*Sqrt[2]*f*Tan[e + f*x]^(3/2)) + (b^2*Log[
1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Sqrt[b*Tan[e + f*x]^3])/(2*S
qrt[2]*f*Tan[e + f*x]^(3/2)) + (2*b^2*Tan[e + f*x]*Sqrt[b*Tan[e + f*x]^3])/
(5*f) - (2*b^2*Tan[e + f*x]^3*Sqrt[b*Tan[e + f*x]^3])/(9*f) + (2*b^2*Tan[e
+ f*x]^5*Sqrt[b*Tan[e + f*x]^3])/(13*f)
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(b\tan^3(e+fx)\right)^{5/2}\,dx = \frac{\left(b^2\sqrt{b\tan^3(e+fx)}\right)\int \tan^{\frac{15}{2}}(e+fx)\,dx}{\tan^{\frac{3}{2}}(e+fx)}$$

$$= \frac{2b^2\tan^5(e+fx)\sqrt{b\tan^3(e+fx)}}{13f} - \frac{\left(b^2\sqrt{b\tan^3(e+fx)}\right)\int \tan^{\frac{11}{2}}(e+fx)\,dx}{\tan^{\frac{3}{2}}(e+fx)}$$

$$= -\frac{2b^2\tan^3(e+fx)\sqrt{b\tan^3(e+fx)}}{9f} + \frac{2b^2\tan^5(e+fx)\sqrt{b\tan^3(e+fx)}}{13f} + \frac{\left(b^2\sqrt{b\tan^3(e\right.}}{\mathrm{t}}$$

$$= \frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f} - \frac{2b^2\tan^3(e+fx)\sqrt{b\tan^3(e+fx)}}{9f} + \frac{2b^2\tan^5(e+fx)}{13}$$

$$= -\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} + \frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f} - \frac{2b^2\tan^3(e+fx)}{9}$$

$$= -\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} + \frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f} - \frac{2b^2\tan^3(e+fx)}{9}$$

$$= -\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} + \frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f} - \frac{2b^2\tan^3(e+fx)}{9}$$

$$= -\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} + \frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f} - \frac{2b^2\tan^3(e+fx)}{9}$$

$$= -\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} + \frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f} - \frac{2b^2\tan^3(e+fx)}{9}$$

$$= -\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} - \frac{b^2\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\sqrt{b\tan^3}}{2\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}$$

$$= -\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} - \frac{b^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)} + \frac{b}{}$$

<u>Mathematica</u> [A] time = 0.829392, size = 199, normalized size = 0.55

$$\frac{b\left(b\tan^3(e+fx)\right)^{3/2}\left(360\tan^{\frac{13}{2}}(e+fx)-520\tan^{\frac{9}{2}}(e+fx)+936\tan^{\frac{5}{2}}(e+fx)-1170\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+f}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^3)^(5/2),x]
```

```
[Out] (b*(b*Tan[e + f*x]^3)^(3/2)*(-1170*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e +
f*x]]] + 1170*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] - 585*Sqrt[2]*
Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] + 585*Sqrt[2]*Log[1 + Sq
rt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 4680*Sqrt[Tan[e + f*x]] + 936*Ta
```

```
n[e + f*x]^(5/2) - 520*Tan[e + f*x]^(9/2) + 360*Tan[e + f*x]^(13/2)))/(2340
*f*Tan[e + f*x]^(9/2))
```

Maple [A] time = 0.044, size = 266, normalized size = 0.7

$$\frac{1}{2340 f \left(\tan \left(f x+e\right)\right)^{5} b^{4}}\left(b \left(\tan \left(f x+e\right)\right)^{3}\right)^{\frac{5}{2}}\left(360 \left(b \tan \left(f x+e\right)\right)^{13/2}-520 b^{2} \left(b \tan \left(f x+e\right)\right)^{9/2}+585 b^{6} \sqrt[4]{b^{2}}\sqrt{2}\ln\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^3)^(5/2),x)
```

```
[Out] 1/2340/f*(b*tan(f*x+e)^3)^(5/2)*(360*(b*tan(f*x+e))^(13/2)-520*b^2*(b*tan(f
*x+e))^(9/2)+585*b^6*(b^2)^(1/4)*2^(1/2)*ln(-(b*tan(f*x+e)+(b^2)^(1/4)*(b*t
an(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2))/((b^2)^(1/4)*(b*tan(f*x+e))^(1/2)*2^(
1/2)-b*tan(f*x+e)-(b^2)^(1/2)))+1170*b^6*(b^2)^(1/4)*2^(1/2)*arctan((2^(1/2
)*(b*tan(f*x+e))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+1170*b^6*(b^2)^(1/4)*2^(1/
2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4))+936*b^4*(
b*tan(f*x+e))^(5/2)-4680*b^6*(b*tan(f*x+e))^(1/2))/tan(f*x+e)^5/(b*tan(f*x+
e))^(5/2)/b^4
```

Maxima [A] time = 1.67078, size = 240, normalized size = 0.66

$$\frac{360 b^{\frac{5}{2}} \tan\left(f x+e\right)^{\frac{13}{2}}-520 b^{\frac{5}{2}} \tan\left(f x+e\right)^{\frac{9}{2}}+936 b^{\frac{5}{2}} \tan\left(f x+e\right)^{\frac{5}{2}}+585 \left(2 \sqrt{2}\sqrt{b}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(f x+\right.}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2340*(360*b^(5/2)*tan(f*x + e)^(13/2) - 520*b^(5/2)*tan(f*x + e)^(9/2) +
936*b^(5/2)*tan(f*x + e)^(5/2) + 585*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*
(sqrt(2) + 2*sqrt(tan(f*x + e)))) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(
sqrt(2) - 2*sqrt(tan(f*x + e)))) + sqrt(2)*sqrt(b)*log(sqrt(2)*sqrt(tan(f*x
 + e)) + tan(f*x + e) + 1) - sqrt(2)*sqrt(b)*log(-sqrt(2)*sqrt(tan(f*x + e)
) + tan(f*x + e) + 1))*b^2 - 4680*b^(5/2)*sqrt(tan(f*x + e)))/f
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan^3\left(e+fx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**3)**(5/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**3)**(5/2), x)
```

Giac [A] time = 1.73344, size = 412, normalized size = 1.13

$$\frac{1}{2340}\left(\frac{1170\sqrt{2}b\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan\left(fx+e\right)}\right)}{2\sqrt{|b|}}\right)}{f}+\frac{1170\sqrt{2}b\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan\left(fx+e\right)}\right)}{2\sqrt{|b|}}\right)}{f}+\frac{585\sqrt{2}b\cdot}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2340*(1170*sqrt(2)*b*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)
) + 2*sqrt(b*tan(f*x + e)))/sqrt(abs(b)))/f + 1170*sqrt(2)*b*sqrt(abs(b))*a
rctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(f*x + e)))/sqrt(abs
(b)))/f + 585*sqrt(2)*b*sqrt(abs(b))*log(b*tan(f*x + e) + sqrt(2)*sqrt(b*ta
n(f*x + e))*sqrt(abs(b)) + abs(b))/f - 585*sqrt(2)*b*sqrt(abs(b))*log(b*tan
(f*x + e) - sqrt(2)*sqrt(b*tan(f*x + e))*sqrt(abs(b)) + abs(b))/f + 8*(45*s
qrt(b*tan(f*x + e))*b^66*f^12*tan(f*x + e)^6 - 65*sqrt(b*tan(f*x + e))*b^66
*f^12*tan(f*x + e)^4 + 117*sqrt(b*tan(f*x + e))*b^66*f^12*tan(f*x + e)^2 -
585*sqrt(b*tan(f*x + e))*b^66*f^12)/(b^65*f^13))*b*sgn(tan(f*x + e))
```

3.8 $\int \left(b\tan^3(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=286

$$\frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f}-\frac{2b\sqrt{b\tan^3(e+fx)}}{3f}-\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}+\frac{b\tan^{-1}\left(\sqrt{2}\right.}{}$$

```
[Out] (-2*b*Sqrt[b*Tan[e + f*x]^3])/(3*f) - (b*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*
x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2]*f*Tan[e + f*x]^(3/2)) + (b*ArcTan[1 +
 Sqrt[2]*Sqrt[Tan[e + f*x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2]*f*Tan[e + f*x
]^(3/2)) + (b*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Sqrt[b*Tan
[e + f*x]^3])/(2*Sqrt[2]*f*Tan[e + f*x]^(3/2)) - (b*Log[1 + Sqrt[2]*Sqrt[Ta
n[e + f*x]] + Tan[e + f*x]]*Sqrt[b*Tan[e + f*x]^3])/(2*Sqrt[2]*f*Tan[e + f*
x]^(3/2)) + (2*b*Tan[e + f*x]^2*Sqrt[b*Tan[e + f*x]^3])/(7*f)
```

Rubi [A] time = 0.126522, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3473, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f}-\frac{2b\sqrt{b\tan^3(e+fx)}}{3f}-\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}+\frac{b\tan^{-1}\left(\sqrt{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^3)^(3/2),x]
```

```
[Out] (-2*b*Sqrt[b*Tan[e + f*x]^3])/(3*f) - (b*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*
x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2]*f*Tan[e + f*x]^(3/2)) + (b*ArcTan[1 +
 Sqrt[2]*Sqrt[Tan[e + f*x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2]*f*Tan[e + f*x
]^(3/2)) + (b*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Sqrt[b*Tan
[e + f*x]^3])/(2*Sqrt[2]*f*Tan[e + f*x]^(3/2)) - (b*Log[1 + Sqrt[2]*Sqrt[Ta
n[e + f*x]] + Tan[e + f*x]]*Sqrt[b*Tan[e + f*x]^3])/(2*Sqrt[2]*f*Tan[e + f*
x]^(3/2)) + (2*b*Tan[e + f*x]^2*Sqrt[b*Tan[e + f*x]^3])/(7*f)
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \left(b\tan^3(e+fx)\right)^{3/2}\,dx = \frac{\left(b\sqrt{b\tan^3(e+fx)}\right)\int \tan^{\frac{9}{2}}(e+fx)\,dx}{\tan^{\frac{3}{2}}(e+fx)}$$

$$= \frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f} - \frac{\left(b\sqrt{b\tan^3(e+fx)}\right)\int \tan^{\frac{5}{2}}(e+fx)\,dx}{\tan^{\frac{3}{2}}(e+fx)}$$

$$= -\frac{2b\sqrt{b\tan^3(e+fx)}}{3f} + \frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f} + \frac{\left(b\sqrt{b\tan^3(e+fx)}\right)\int \sqrt{\tan(e+\ldots}}{\tan^{\frac{3}{2}}(e+fx)}$$

$$= -\frac{2b\sqrt{b\tan^3(e+fx)}}{3f} + \frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f} + \frac{\left(b\sqrt{b\tan^3(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{\ldots}}{1+\ldots}\right.}{f\tan^{\frac{3}{2}}(e+f\ldots}$$

$$= -\frac{2b\sqrt{b\tan^3(e+fx)}}{3f} + \frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f} + \frac{\left(2b\sqrt{b\tan^3(e+fx)}\right)\text{Subst}\left(\int \frac{\ldots}{1\ldots}\right.}{f\tan^{\frac{3}{2}}(e+\ldots}$$

$$= -\frac{2b\sqrt{b\tan^3(e+fx)}}{3f} + \frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f} - \frac{\left(b\sqrt{b\tan^3(e+fx)}\right)\text{Subst}\left(\int \frac{1-\ldots}{1+\ldots}\right.}{f\tan^{\frac{3}{2}}(e+f\ldots}$$

$$= -\frac{2b\sqrt{b\tan^3(e+fx)}}{3f} + \frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f} + \frac{\left(b\sqrt{b\tan^3(e+fx)}\right)\text{Subst}\left(\int \frac{\ldots}{1-\ldots}\right.}{2f\tan^{\frac{3}{2}}(e\ldots}$$

$$= -\frac{2b\sqrt{b\tan^3(e+fx)}}{3f} + \frac{b\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\sqrt{b\tan^3(e+fx)}}{2\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)} - \frac{b\log\left(\ldots\right.}{\ldots}$$

$$= -\frac{2b\sqrt{b\tan^3(e+fx)}}{3f} - \frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)} + \frac{b\tan^{-1}\left(1+\sqrt{2}\sqrt{\ldots}\right.}{\sqrt{2}\ldots}$$

Mathematica [C] time = 0.0783766, size = 54, normalized size = 0.19

$$\frac{2b\sqrt{b\tan^3(e+fx)}\left(7\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\tan^2(e+fx)\right)+3\tan^2(e+fx)-7\right)}{21f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^3)^(3/2),x]
```

```
[Out] (2*b*Sqrt[b*Tan[e + f*x]^3]*(-7 + 7*Hypergeometric2F1[3/4, 1, 7/4, -Tan[e +
 f*x]^2] + 3*Tan[e + f*x]^2))/(21*f)
```

Maple [A] time = 0.019, size = 236, normalized size = 0.8

$$\frac{1}{84f\left(\tan\left(fx+e\right)\right)^3b^2}\left(b\left(\tan\left(fx+e\right)\right)^3\right)^{\frac{3}{2}}\left(24\left(b\tan\left(fx+e\right)\right)^{7/2}\sqrt[4]{b^2}-56b^2\left(b\tan\left(fx+e\right)\right)^{3/2}\sqrt[4]{b^2}+21b^4\sqrt{2}\ln\left(-\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^3)^(3/2),x)
```

```
[Out] 1/84/f*(b*tan(f*x+e)^3)^(3/2)*(24*(b*tan(f*x+e))^(7/2)*(b^2)^(1/4)-56*b^2*(
b*tan(f*x+e))^(3/2)*(b^2)^(1/4)+21*b^4*2^(1/2)*ln(-((b^2)^(1/4)*(b*tan(f*x+
e))^(1/2)*2^(1/2)-b*tan(f*x+e)-(b^2)^(1/2))/(b*tan(f*x+e)+(b^2)^(1/4)*(b*ta
n(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2)))+42*b^4*2^(1/2)*arctan((2^(1/2)*(b*tan
(f*x+e))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+42*b^4*2^(1/2)*arctan((2^(1/2)*(b*
tan(f*x+e))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4)))/tan(f*x+e)^3/(b*tan(f*x+e))^(3
/2)/b^2/(b^2)^(1/4)
```

Maxima [A] time = 1.7221, size = 189, normalized size = 0.66

$$24\, b^{\frac{3}{2}} \tan\left(fx+e\right)^{\frac{7}{2}} - 56\, b^{\frac{3}{2}} \tan\left(fx+e\right)^{\frac{3}{2}} + 21\, \Big(2\sqrt{2}\arctan\Big(\tfrac{1}{2}\sqrt{2}\Big(\sqrt{2}+2\sqrt{\tan\left(fx+e\right)}\Big)\Big) + 2\sqrt{2}\arctan\Big(-\tfrac{1}{2}\sqrt{2}\Big(\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/84*(24*b^(3/2)*tan(f*x + e)^(7/2) - 56*b^(3/2)*tan(f*x + e)^(3/2) + 21*(2
*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(f*x + e)))) + 2*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(f*x + e)))) - sqrt(2)*log(sqrt(2)*
sqrt(tan(f*x + e)) + tan(f*x + e) + 1) + sqrt(2)*log(-sqrt(2)*sqrt(tan(f*x
+ e)) + tan(f*x + e) + 1))*b^(3/2))/f
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan^{3}\left(e+fx\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**3)**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**3)**(3/2), x)
```

Giac [A] time = 1.42653, size = 356, normalized size = 1.24

$$\frac{1}{84}b\left(\frac{42\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan\left(fx+e\right)}\right)}{2\sqrt{|b|}}\right)}{bf}+\frac{42\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan\left(fx+e\right)}\right)}{2\sqrt{|b|}}\right)}{bf}-\frac{21\sqrt{2}|b|^{\frac{3}{2}}\log\left(b\tan\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/84*b*(42*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) +
2*sqrt(b*tan(f*x + e)))/sqrt(abs(b)))/(b*f) + 42*sqrt(2)*abs(b)^(3/2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(f*x + e)))/sqrt(abs(b))
)/(b*f) - 21*sqrt(2)*abs(b)^(3/2)*log(b*tan(f*x + e) + sqrt(2)*sqrt(b*tan(f
*x + e))*sqrt(abs(b)) + abs(b))/(b*f) + 21*sqrt(2)*abs(b)^(3/2)*log(b*tan(f
*x + e) - sqrt(2)*sqrt(b*tan(f*x + e))*sqrt(abs(b)) + abs(b))/(b*f) + 8*(3*
sqrt(b*tan(f*x + e))*b^21*f^6*tan(f*x + e)^3 - 7*sqrt(b*tan(f*x + e))*b^21*
f^6*tan(f*x + e))/(b^21*f^7))*sgn(tan(f*x + e))
```

3.9 $\int \sqrt{b \tan^3(e+fx)}\, dx$

Optimal. Leaf size=255

$$\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}+\frac{\sqrt{b\tan^3(e+fx)}\,\mathrm{lo}}{}$$

```
[Out] (2*Cot[e + f*x]*Sqrt[b*Tan[e + f*x]^3])/f + (ArcTan[1 - Sqrt[2]*Sqrt[Tan[e
+ f*x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2]*f*Tan[e + f*x]^(3/2)) - (ArcTan[1
 + Sqrt[2]*Sqrt[Tan[e + f*x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2]*f*Tan[e + f
*x]^(3/2)) + (Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Sqrt[b*Tan
[e + f*x]^3])/(2*Sqrt[2]*f*Tan[e + f*x]^(3/2)) - (Log[1 + Sqrt[2]*Sqrt[Tan[
e + f*x]] + Tan[e + f*x]]*Sqrt[b*Tan[e + f*x]^3])/(2*Sqrt[2]*f*Tan[e + f*x]
^(3/2))
```

Rubi [A] time = 0.115107, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3473, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}+\frac{\sqrt{b\tan^3(e+fx)}\,\mathrm{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]^3],x]
```


```
[Out] (2*Cot[e + f*x]*Sqrt[b*Tan[e + f*x]^3])/f + (ArcTan[1 - Sqrt[2]*Sqrt[Tan[e
+ f*x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2]*f*Tan[e + f*x]^(3/2)) - (ArcTan[1
 + Sqrt[2]*Sqrt[Tan[e + f*x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2]*f*Tan[e + f
*x]^(3/2)) + (Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Sqrt[b*Tan
[e + f*x]^3])/(2*Sqrt[2]*f*Tan[e + f*x]^(3/2)) - (Log[1 + Sqrt[2]*Sqrt[Tan[
e + f*x]] + Tan[e + f*x]]*Sqrt[b*Tan[e + f*x]^3])/(2*Sqrt[2]*f*Tan[e + f*x]
^(3/2))
```


Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{b\tan^3(e+fx)}\,dx = \frac{\sqrt{b\tan^3(e+fx)}\int \tan^{\frac{3}{2}}(e+fx)\,dx}{\tan^{\frac{3}{2}}(e+fx)}$$

$$= \frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} - \frac{\sqrt{b\tan^3(e+fx)}\int \frac{1}{\sqrt{\tan(e+fx)}}dx}{\tan^{\frac{3}{2}}(e+fx)}$$

$$= \frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} - \frac{\sqrt{b\tan^3(e+fx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}dx, x, \tan(e+fx)\right)}{f\tan^{\frac{3}{2}}(e+fx)}$$

$$= \frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} - \frac{\left(2\sqrt{b\tan^3(e+fx)}\right)\text{Subst}\left(\int \frac{1}{1+x^4}dx, x, \sqrt{\tan(e+fx)}\right)}{f\tan^{\frac{3}{2}}(e+fx)}$$

$$= \frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} - \frac{\sqrt{b\tan^3(e+fx)}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}dx, x, \sqrt{\tan(e+fx)}\right)}{f\tan^{\frac{3}{2}}(e+fx)} - \frac{\sqrt{b}}{}$$

$$= \frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} - \frac{\sqrt{b\tan^3(e+fx)}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}dx, x, \sqrt{\tan(e+fx)}\right)}{2f\tan^{\frac{3}{2}}(e+fx)} -$$

$$= \frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\sqrt{b\tan^3(e+fx)}}{2\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}$$

$$= \frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} + \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)} - \frac{\tan^{-1}(1}{}$$

Mathematica [A] time = 0.249944, size = 161, normalized size = 0.63

$$\frac{\sqrt{b\tan^3(e+fx)}\left(2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)-2\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)+8\sqrt{\tan(e+fx)}+\sqrt{2}\log(\right.}{4f\tan^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]^3],x]
```

```
[Out] ((2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 2*Sqrt[2]*ArcTan[1 + S
qrt[2]*Sqrt[Tan[e + f*x]]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + T
an[e + f*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] +
 8*Sqrt[Tan[e + f*x]])*Sqrt[b*Tan[e + f*x]^3])/(4*f*Tan[e + f*x]^(3/2))
```

Maple [A] time = 0.021, size = 208, normalized size = 0.8

$$-\frac{1}{4f\tan(fx+e)}\sqrt{b(\tan(fx+e))^3}\left(\sqrt[4]{b^2}\sqrt{2}\ln\left(-\left(b\tan(fx+e)+\sqrt[4]{b^2}\sqrt{b\tan(fx+e)}\sqrt{2}+\sqrt{b^2}\right)\left(\sqrt[4]{b^2}\sqrt{b\tan(f\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^3)^(1/2),x)
```

```
[Out] -1/4/f*(b*tan(f*x+e)^3)^(1/2)*((b^2)^(1/4)*2^(1/2)*ln(-(b*tan(f*x+e)+(b^2)^
(1/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2))/((b^2)^(1/4)*(b*tan(f*x+e))
^(1/2)*2^(1/2)-b*tan(f*x+e)-(b^2)^(1/2)))+2*(b^2)^(1/4)*2^(1/2)*arctan((2^(
1/2)*(b*tan(f*x+e))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+2*(b^2)^(1/4)*2^(1/2)*a
rctan((2^(1/2)*(b*tan(f*x+e))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4))-8*(b*tan(f*x+
e))^(1/2))/tan(f*x+e)/(b*tan(f*x+e))^(1/2)
```

Maxima [A] time = 1.70219, size = 180, normalized size = 0.71

$$-\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(fx+e\right)}\right)\right)+2\sqrt{2}\sqrt{b}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(fx+e\right)}\right)\right)+\sqrt{2}\sqrt{b}\log\left(\sqrt{2}\sqrt{\cdots}\right.}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(f*x + e)))
) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(f*x + e))))
 + sqrt(2)*sqrt(b)*log(sqrt(2)*sqrt(tan(f*x + e)) + tan(f*x + e) + 1) - sqr
t(2)*sqrt(b)*log(-sqrt(2)*sqrt(tan(f*x + e)) + tan(f*x + e) + 1) - 8*sqrt(b
)*sqrt(tan(f*x + e)))/f
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\tan^{3}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(e + f*x)**3), x)
```

Giac [A] time = 1.34484, size = 274, normalized size = 1.07

$$-\frac{1}{4}\left(\frac{2\sqrt{2}\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan\left(fx+e\right)}\right)}{2\sqrt{|b|}}\right)}{f}+\frac{2\sqrt{2}\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan\left(fx+e\right)}\right)}{2\sqrt{|b|}}\right)}{f}+\frac{\sqrt{2}\sqrt{|b|}\log\left(b\tan\left(fx+\cdots\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(2*sqrt(2)*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) + 2*s
qrt(b*tan(f*x + e)))/sqrt(abs(b)))/f + 2*sqrt(2)*sqrt(abs(b))*arctan(-1/2*s
qrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(f*x + e)))/sqrt(abs(b)))/f + sq
rt(2)*sqrt(abs(b))*log(b*tan(f*x + e) + sqrt(2)*sqrt(b*tan(f*x + e))*sqrt(a
bs(b)) + abs(b))/f - sqrt(2)*sqrt(abs(b))*log(b*tan(f*x + e) - sqrt(2)*sqrt
(b*tan(f*x + e))*sqrt(abs(b)) + abs(b))/f - 8*sqrt(b*tan(f*x + e))/f)*sgn(t
an(f*x + e))
```

3.10 $\int \frac{1}{\sqrt{b\tan^3(e+fx)}}\,dx$

Optimal. Leaf size=255

$$\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}f\sqrt{b\tan^3(e+fx)}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}f\sqrt{b\tan^3(e+fx)}}-\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}}-\frac{\tan^{\frac{3}{2}}(e}{}$$

```
[Out] (-2*Tan[e + f*x])/(f*Sqrt[b*Tan[e + f*x]^3]) + (ArcTan[1 - Sqrt[2]*Sqrt[Tan
[e + f*x]]]*Tan[e + f*x]^(3/2))/(Sqrt[2]*f*Sqrt[b*Tan[e + f*x]^3]) - (ArcTa
n[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]*Tan[e + f*x]^(3/2))/(Sqrt[2]*f*Sqrt[b*Tan
[e + f*x]^3]) - (Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Tan[e +
 f*x]^(3/2))/(2*Sqrt[2]*f*Sqrt[b*Tan[e + f*x]^3]) + (Log[1 + Sqrt[2]*Sqrt[T
an[e + f*x]] + Tan[e + f*x]]*Tan[e + f*x]^(3/2))/(2*Sqrt[2]*f*Sqrt[b*Tan[e
+ f*x]^3])
```

Rubi [A] time = 0.114648, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}f\sqrt{b\tan^3(e+fx)}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}f\sqrt{b\tan^3(e+fx)}}-\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}}-\frac{\tan^{\frac{3}{2}}(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Tan[e + f*x]^3],x]
```


```
[Out] (-2*Tan[e + f*x])/(f*Sqrt[b*Tan[e + f*x]^3]) + (ArcTan[1 - Sqrt[2]*Sqrt[Tan
[e + f*x]]]*Tan[e + f*x]^(3/2))/(Sqrt[2]*f*Sqrt[b*Tan[e + f*x]^3]) - (ArcTa
n[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]*Tan[e + f*x]^(3/2))/(Sqrt[2]*f*Sqrt[b*Tan
[e + f*x]^3]) - (Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Tan[e +
 f*x]^(3/2))/(2*Sqrt[2]*f*Sqrt[b*Tan[e + f*x]^3]) + (Log[1 + Sqrt[2]*Sqrt[T
an[e + f*x]] + Tan[e + f*x]]*Tan[e + f*x]^(3/2))/(2*Sqrt[2]*f*Sqrt[b*Tan[e
+ f*x]^3])
```


Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{b\tan^3(e+fx)}}\,dx = \frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\tan^{\frac{3}{2}}(e+fx)}\,dx}{\sqrt{b\tan^3(e+fx)}}$$

$$= -\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{\frac{3}{2}}(e+fx)\int \sqrt{\tan(e+fx)}\,dx}{\sqrt{b\tan^3(e+fx)}}$$

$$= -\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{\frac{3}{2}}(e+fx)\,\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f\sqrt{b\tan^3(e+fx)}}$$

$$= -\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}} - \frac{\left(2\tan^{\frac{3}{2}}(e+fx)\right)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(e+fx)}\right)}{f\sqrt{b\tan^3(e+fx)}}$$

$$= -\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}} + \frac{\tan^{\frac{3}{2}}(e+fx)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(e+fx)}\right)}{f\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{\frac{3}{2}}(e+fx)\,\text{Subst}}{f\sqrt{}}$$

$$= -\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{\frac{3}{2}}(e+fx)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(e+fx)}\right)}{2f\sqrt{b\tan^3(e+fx)}} - \tan^{\frac{3}{2}}(e+fx)\,\text{S}$$

$$= -\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}} - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\tan^{\frac{3}{2}}(e+fx)}{2\sqrt{2}f\sqrt{b\tan^3(e+fx)}} + \log\left(1+\sqrt{2}\sqrt{}\right.$$

$$= -\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}} + \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}f\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{\tan(e+f}\right.}{\sqrt{2}f\sqrt{b\tan^3(e}}$$

Mathematica [C] time = 0.0340779, size = 43, normalized size = 0.17

$$-\frac{2\tan(e+fx)\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\tan^2(e+fx)\right)}{f\sqrt{b\tan^3(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Tan[e + f*x]^3],x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[e + f*x]^2]*Tan[e + f*x])/(f*Sqrt[
b*Tan[e + f*x]^3])
```

Maple [A] time = 0.026, size = 211, normalized size = 0.8

$$-\frac{\tan(fx+e)}{4f}\left(\sqrt{2}\sqrt{b\tan(fx+e)}\ln\left(-\left(\sqrt[4]{b^2}\sqrt{b\tan(fx+e)}\sqrt{2}-b\tan(fx+e)-\sqrt{b^2}\right)\left(b\tan(fx+e)+\sqrt[4]{b^2}\sqrt{b\tan}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^3)^(1/2),x)
```

```
[Out] -1/4/f*tan(f*x+e)*(2^(1/2)*(b*tan(f*x+e))^(1/2)*ln(-((b^2)^(1/4)*(b*tan(f*x
+e))^(1/2)*2^(1/2)-b*tan(f*x+e)-(b^2)^(1/2))/(b*tan(f*x+e)+(b^2)^(1/4)*(b*t
an(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*2^(1/2)*(b*tan(f*x+e))^(1/2)*arcta
n((2^(1/2)*(b*tan(f*x+e))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+2*2^(1/2)*(b*tan(
f*x+e))^(1/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4)
)+8*(b^2)^(1/4))/(b*tan(f*x+e)^3)^(1/2)/(b^2)^(1/4)
```

Maxima [A] time = 1.6107, size = 170, normalized size = 0.67

$$-\frac{\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(fx+e\right)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(fx+e\right)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan\left(fx+e\right)}+\tan\left(fx+e\right)+1\right)+\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan\left(fx+e\right)}\right.}{\sqrt{b}}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(f*x + e)))) + 2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(f*x + e)))) - sqrt(2)*log(
sqrt(2)*sqrt(tan(f*x + e)) + tan(f*x + e) + 1) + sqrt(2)*log(-sqrt(2)*sqrt(
tan(f*x + e)) + tan(f*x + e) + 1))/sqrt(b) + 8/(sqrt(b)*sqrt(tan(f*x + e)))
)/f
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\tan^{3}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*tan(e + f*x)**3), x)
```

Giac [A] time = 1.32153, size = 355, normalized size = 1.39

$$-\frac{1}{4}b^{2}\left(\frac{2\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan\left(fx+e\right)}\right)}{2\sqrt{|b|}}\right)}{b^{4}f\operatorname{sgn}\left(\tan\left(fx+e\right)\right)}+\frac{2\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan\left(fx+e\right)}\right)}{2\sqrt{|b|}}\right)}{b^{4}f\operatorname{sgn}\left(\tan\left(fx+e\right)\right)}-\frac{\sqrt{2}|b|^{\frac{3}{2}}\log\left(b\tan\left(\right.\right.}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*b^2*(2*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) +
 2*sqrt(b*tan(f*x + e)))/sqrt(abs(b)))/(b^4*f*sgn(tan(f*x + e))) + 2*sqrt(2
)*abs(b)^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(f*x
 + e)))/sqrt(abs(b)))/(b^4*f*sgn(tan(f*x + e))) - sqrt(2)*abs(b)^(3/2)*log(
b*tan(f*x + e) + sqrt(2)*sqrt(b*tan(f*x + e))*sqrt(abs(b)) + abs(b))/(b^4*f
*sgn(tan(f*x + e))) + sqrt(2)*abs(b)^(3/2)*log(b*tan(f*x + e) - sqrt(2)*sqr
t(b*tan(f*x + e))*sqrt(abs(b)) + abs(b))/(b^4*f*sgn(tan(f*x + e))) + 8/(sqr
t(b*tan(f*x + e))*b^2*f*sgn(tan(f*x + e))))
```

3.11 $\int \frac{1}{\left(b\tan^3(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=298

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}bf\sqrt{b\tan^3(e+fx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}bf\sqrt{b\tan^3(e+fx)}}+\frac{2}{3bf\sqrt{b\tan^3(e+fx)}}-$$

```
[Out] 2/(3*b*f*Sqrt[b*Tan[e + f*x]^3]) - (2*Cot[e + f*x]^2)/(7*b*f*Sqrt[b*Tan[e +
 f*x]^3]) - (ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]]*Tan[e + f*x]^(3/2))/(Sq
rt[2]*b*f*Sqrt[b*Tan[e + f*x]^3]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]
*Tan[e + f*x]^(3/2))/(Sqrt[2]*b*f*Sqrt[b*Tan[e + f*x]^3]) - (Log[1 - Sqrt[2
]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Tan[e + f*x]^(3/2))/(2*Sqrt[2]*b*f*Sqr
t[b*Tan[e + f*x]^3]) + (Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*
Tan[e + f*x]^(3/2))/(2*Sqrt[2]*b*f*Sqrt[b*Tan[e + f*x]^3])
```

Rubi [A] time = 0.12807, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3474, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}bf\sqrt{b\tan^3(e+fx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}bf\sqrt{b\tan^3(e+fx)}}+\frac{2}{3bf\sqrt{b\tan^3(e+fx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^3)^(-3/2),x]
```

```
[Out] 2/(3*b*f*Sqrt[b*Tan[e + f*x]^3]) - (2*Cot[e + f*x]^2)/(7*b*f*Sqrt[b*Tan[e +
 f*x]^3]) - (ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]]*Tan[e + f*x]^(3/2))/(Sq
rt[2]*b*f*Sqrt[b*Tan[e + f*x]^3]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]
*Tan[e + f*x]^(3/2))/(Sqrt[2]*b*f*Sqrt[b*Tan[e + f*x]^3]) - (Log[1 - Sqrt[2
]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Tan[e + f*x]^(3/2))/(2*Sqrt[2]*b*f*Sqr
t[b*Tan[e + f*x]^3]) + (Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*
Tan[e + f*x]^(3/2))/(2*Sqrt[2]*b*f*Sqrt[b*Tan[e + f*x]^3])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

IntegerQ[n]

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(b\tan^3(e+fx)\right)^{3/2}}\,dx = \frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\tan^{\frac{9}{2}}(e+fx)}\,dx}{b\sqrt{b\tan^3(e+fx)}}$$

$$= -\frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\tan^{\frac{5}{2}}(e+fx)}\,dx}{b\sqrt{b\tan^3(e+fx)}}$$

$$= \frac{2}{3bf\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} + \frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\sqrt{\tan(e+fx)}}\,dx}{b\sqrt{b\tan^3(e+fx)}}$$

$$= \frac{2}{3bf\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} + \frac{\tan^{\frac{3}{2}}(e+fx)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx, x, \tan\right.}{bf\sqrt{b\tan^3(e+fx)}}$$

$$= \frac{2}{3bf\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} + \frac{\left(2\tan^{\frac{3}{2}}(e+fx)\right)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\text{ta}}\right.}{bf\sqrt{b\tan^3(e+fx)}}$$

$$= \frac{2}{3bf\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} + \frac{\tan^{\frac{3}{2}}(e+fx)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(e}\right.}{bf\sqrt{b\tan^3(e+fx)}}$$

$$= \frac{2}{3bf\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} + \frac{\tan^{\frac{3}{2}}(e+fx)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\text{ta}}\right.}{2bf\sqrt{b\tan^3(e+fx)}}$$

$$= \frac{2}{3bf\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\text{t}}{2\sqrt{2}bf\sqrt{b\tan^3(e+fx)}}$$

$$= \frac{2}{3bf\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx}{\sqrt{2}bf\sqrt{b\tan^3(e+fx)}}$$

Mathematica [C] time = 0.0736688, size = 45, normalized size = 0.15

$$-\frac{2\tan(e+fx)\text{Hypergeometric2F1}\left(-\frac{7}{4},1,-\frac{3}{4},-\tan^2(e+fx)\right)}{7f\left(b\tan^3(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^3)^(-3/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-7/4, 1, -3/4, -Tan[e + f*x]^2]*Tan[e + f*x])/(7*f*(b
*Tan[e + f*x]^3)^(3/2))
```

Maple [A] time = 0.023, size = 236, normalized size = 0.8

$$\frac{\tan(fx+e)}{84\,fb^4}\left(21\sqrt[4]{b^2}\sqrt{2}\left(b\tan(fx+e)\right)^{7/2}\ln\left(-\frac{b\tan(fx+e)+\sqrt[4]{b^2}\sqrt{b\tan(fx+e)}\sqrt{2}+\sqrt{b^2}}{\sqrt[4]{b^2}\sqrt{b\tan(fx+e)}\sqrt{2}-b\tan(fx+e)-\sqrt{b^2}}\right)+42\sqrt[4]{b^2}\sqrt{2}\left(b\tan\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^3)^(3/2),x)
```

```
[Out] 1/84/f*tan(f*x+e)/b^4*(21*(b^2)^(1/4)*2^(1/2)*(b*tan(f*x+e))^(7/2)*ln(-(b*t
an(f*x+e)+(b^2)^(1/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2))/((b^2)^(1/4
)*(b*tan(f*x+e))^(1/2)*2^(1/2)-b*tan(f*x+e)-(b^2)^(1/2)))+42*(b^2)^(1/4)*2^
(1/2)*(b*tan(f*x+e))^(7/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)+(b^2)^(1/4)
)/(b^2)^(1/4))+42*(b^2)^(1/4)*2^(1/2)*(b*tan(f*x+e))^(7/2)*arctan((2^(1/2)*
(b*tan(f*x+e))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4))+56*b^4*tan(f*x+e)^2-24*b^4)/
(b*tan(f*x+e)^3)^(3/2)
```

Maxima [A] time = 1.68656, size = 220, normalized size = 0.74

$$\frac{\frac{21\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(fx+e\right)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(fx+e\right)}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan\left(fx+e\right)}+\tan\left(fx+e\right)+1\right)-\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan\left(fx+e\right)}+t\right.}{b^{\frac{3}{2}}}}{84\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/84*(21*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(f*x + e)))) +
2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(f*x + e)))) + sqrt(2)*l
og(sqrt(2)*sqrt(tan(f*x + e)) + tan(f*x + e) + 1) - sqrt(2)*log(-sqrt(2)*sq
rt(tan(f*x + e)) + tan(f*x + e) + 1))/b^(3/2) + 8*(21*sqrt(tan(f*x + e)) +
7/tan(f*x + e)^(3/2) - 3/tan(f*x + e)^(7/2))/b^(3/2) - 168*sqrt(tan(f*x + e
))/b^(3/2))/f
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan^{3}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**3)**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**3)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan\left(fx+e\right)^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^3)^(-3/2), x)
```

3.12 $\int \frac{1}{\left(b\tan^3(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=364

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}b^2f\sqrt{b\tan^3(e+fx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}b^2f\sqrt{b\tan^3(e+fx)}}+\frac{2\tan(e+fx)}{b^2f\sqrt{b\tan^3(e+fx)}}+\frac{\tan}{}$$

```
[Out] (-2*Cot[e + f*x])/(5*b^2*f*Sqrt[b*Tan[e + f*x]^3]) + (2*Cot[e + f*x]^3)/(9*
b^2*f*Sqrt[b*Tan[e + f*x]^3]) - (2*Cot[e + f*x]^5)/(13*b^2*f*Sqrt[b*Tan[e +
 f*x]^3]) + (2*Tan[e + f*x])/(b^2*f*Sqrt[b*Tan[e + f*x]^3]) - (ArcTan[1 - S
qrt[2]*Sqrt[Tan[e + f*x]]]*Tan[e + f*x]^(3/2))/(Sqrt[2]*b^2*f*Sqrt[b*Tan[e
+ f*x]^3]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]*Tan[e + f*x]^(3/2))/(S
qrt[2]*b^2*f*Sqrt[b*Tan[e + f*x]^3]) + (Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]
+ Tan[e + f*x]]*Tan[e + f*x]^(3/2))/(2*Sqrt[2]*b^2*f*Sqrt[b*Tan[e + f*x]^3]
) - (Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Tan[e + f*x]^(3/2))
/(2*Sqrt[2]*b^2*f*Sqrt[b*Tan[e + f*x]^3])
```

Rubi [A] time = 0.147612, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}b^2f\sqrt{b\tan^3(e+fx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}b^2f\sqrt{b\tan^3(e+fx)}}+\frac{2\tan(e+fx)}{b^2f\sqrt{b\tan^3(e+fx)}}+\frac{\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^3)^(-5/2),x]
```


```
[Out] (-2*Cot[e + f*x])/(5*b^2*f*Sqrt[b*Tan[e + f*x]^3]) + (2*Cot[e + f*x]^3)/(9*
b^2*f*Sqrt[b*Tan[e + f*x]^3]) - (2*Cot[e + f*x]^5)/(13*b^2*f*Sqrt[b*Tan[e +
 f*x]^3]) + (2*Tan[e + f*x])/(b^2*f*Sqrt[b*Tan[e + f*x]^3]) - (ArcTan[1 - S
qrt[2]*Sqrt[Tan[e + f*x]]]*Tan[e + f*x]^(3/2))/(Sqrt[2]*b^2*f*Sqrt[b*Tan[e
+ f*x]^3]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]*Tan[e + f*x]^(3/2))/(S
qrt[2]*b^2*f*Sqrt[b*Tan[e + f*x]^3]) + (Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]
+ Tan[e + f*x]]*Tan[e + f*x]^(3/2))/(2*Sqrt[2]*b^2*f*Sqrt[b*Tan[e + f*x]^3]
) - (Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Tan[e + f*x]^(3/2))
/(2*Sqrt[2]*b^2*f*Sqrt[b*Tan[e + f*x]^3])
```


Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(b\tan^3(e+fx)\right)^{5/2}}\,dx = \frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\tan^{\frac{15}{2}}(e+fx)}\,dx}{b^2\sqrt{b\tan^3(e+fx)}}$$

$$= -\frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\tan^{\frac{11}{2}}(e+fx)}\,dx}{b^2\sqrt{b\tan^3(e+fx)}}$$

$$= \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} + \frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\tan^{\frac{7}{2}}(e+fx)}\,dx}{b^2\sqrt{b\tan^3(e+fx)}}$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{\frac{3}{2}}(e+fx)\int}{b^2\sqrt{b\tan^3(}}$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\tan(e+fx}{b^2f\sqrt{b\tan^3(e+}}$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\tan(e+fx}{b^2f\sqrt{b\tan^3(e+}}$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\tan(e+fx}{b^2f\sqrt{b\tan^3(e+}}$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\tan(e+fx}{b^2f\sqrt{b\tan^3(e+}}$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\tan(e+fx}{b^2f\sqrt{b\tan^3(e+}}$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\tan(e+fx}{b^2f\sqrt{b\tan^3(e+}}$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\tan(e+fx}{b^2f\sqrt{b\tan^3(e+}}$$

Mathematica [C] time = 0.0584859, size = 45, normalized size = 0.12

$$-\frac{2\tan(e+fx)\text{Hypergeometric2F1}\left(-\frac{13}{4},1,-\frac{9}{4},-\tan^2(e+fx)\right)}{13f\left(b\tan^3(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^3)^(-5/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-13/4, 1, -9/4, -Tan[e + f*x]^2]*Tan[e + f*x])/(13*f*
(b*Tan[e + f*x]^3)^(5/2))
```

Maple [A] time = 0.023, size = 272, normalized size = 0.8

$$\frac{\tan\left(fx+e\right)}{2340\,f b^{6}}\left(585\sqrt{2}\left(b\tan\left(fx+e\right)\right)^{13/2}\ln\left(-\frac{\sqrt[4]{b^{2}}\sqrt{b\tan\left(fx+e\right)}\sqrt{2}-b\tan\left(fx+e\right)-\sqrt{b^{2}}}{b\tan\left(fx+e\right)+\sqrt[4]{b^{2}}\sqrt{b\tan\left(fx+e\right)}\sqrt{2}+\sqrt{b^{2}}}\right)+1170\sqrt{2}\left(b\tan\left(fx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^3)^(5/2),x)
```

```
[Out] 1/2340/f*tan(f*x+e)/b^6*(585*2^(1/2)*(b*tan(f*x+e))^(13/2)*ln(-((b^2)^(1/4)
*(b*tan(f*x+e))^(1/2)*2^(1/2)-b*tan(f*x+e)-(b^2)^(1/2))/(b*tan(f*x+e)+(b^2)
^(1/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2)))+1170*2^(1/2)*(b*tan(f*x+e
))^(13/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+11
70*2^(1/2)*(b*tan(f*x+e))^(13/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)-(b^2)
^(1/4))/(b^2)^(1/4))+4680*(b^2)^(1/4)*b^6*tan(f*x+e)^6-936*b^6*(b^2)^(1/4)*
tan(f*x+e)^4+520*b^6*(b^2)^(1/4)*tan(f*x+e)^2-360*b^6*(b^2)^(1/4))/(b*tan(f
*x+e)^3)^(5/2)/(b^2)^(1/4)
```

Maxima [A] time = 1.66337, size = 232, normalized size = 0.64

$$\frac{\frac{585\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(fx+e\right)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(fx+e\right)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan\left(fx+e\right)}+\tan\left(fx+e\right)+1\right)+\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan\left(fx\right.\right.}\right.\right.}{b^{\frac{5}{2}}}}{2340\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2340*(585*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(f*x + e))))
 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(f*x + e)))) - sqrt(2
)*log(sqrt(2)*sqrt(tan(f*x + e)) + tan(f*x + e) + 1) + sqrt(2)*log(-sqrt(2)
*sqrt(tan(f*x + e)) + tan(f*x + e) + 1))/b^(5/2) + 8*(585*sqrt(b)/sqrt(tan(
f*x + e)) - 117*sqrt(b)/tan(f*x + e)^(5/2) + 65*sqrt(b)/tan(f*x + e)^(9/2)
- 45*sqrt(b)/tan(f*x + e)^(13/2))/b^3)/f
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan^{3}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**3)**(5/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**3)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan\left(fx+e\right)^{3}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^3)^(-5/2), x)
```

3.13 $\int \left(b\tan^4(e+fx)\right)^{5/2}\,dx$

Optimal. Leaf size=182

$$\frac{b^2\tan^7(e+fx)\sqrt{b\tan^4(e+fx)}}{9f}-\frac{b^2\tan^5(e+fx)\sqrt{b\tan^4(e+fx)}}{7f}+\frac{b^2\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f}-\frac{b^2\tan(e+$$

```
[Out] (b^2*Cot[e + f*x]*Sqrt[b*Tan[e + f*x]^4])/f - b^2*x*Cot[e + f*x]^2*Sqrt[b*T
an[e + f*x]^4] - (b^2*Tan[e + f*x]*Sqrt[b*Tan[e + f*x]^4])/(3*f) + (b^2*Tan
[e + f*x]^3*Sqrt[b*Tan[e + f*x]^4])/(5*f) - (b^2*Tan[e + f*x]^5*Sqrt[b*Tan[
e + f*x]^4])/(7*f) + (b^2*Tan[e + f*x]^7*Sqrt[b*Tan[e + f*x]^4])/(9*f)
```

Rubi [A] time = 0.0635726, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$\frac{b^2\tan^7(e+fx)\sqrt{b\tan^4(e+fx)}}{9f}-\frac{b^2\tan^5(e+fx)\sqrt{b\tan^4(e+fx)}}{7f}+\frac{b^2\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f}-\frac{b^2\tan(e+$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^4)^(5/2),x]
```


```
[Out] (b^2*Cot[e + f*x]*Sqrt[b*Tan[e + f*x]^4])/f - b^2*x*Cot[e + f*x]^2*Sqrt[b*T
an[e + f*x]^4] - (b^2*Tan[e + f*x]*Sqrt[b*Tan[e + f*x]^4])/(3*f) + (b^2*Tan
[e + f*x]^3*Sqrt[b*Tan[e + f*x]^4])/(5*f) - (b^2*Tan[e + f*x]^5*Sqrt[b*Tan[
e + f*x]^4])/(7*f) + (b^2*Tan[e + f*x]^7*Sqrt[b*Tan[e + f*x]^4])/(9*f)
```


Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(b\tan^4(e+fx)\right)^{5/2}dx = \left(b^2\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\right)\int \tan^{10}(e+fx)\,dx$$
$$= \frac{b^2\tan^7(e+fx)\sqrt{b\tan^4(e+fx)}}{9f} - \left(b^2\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\right)\int \tan^8(e+fx)\,dx$$
$$= -\frac{b^2\tan^5(e+fx)\sqrt{b\tan^4(e+fx)}}{7f} + \frac{b^2\tan^7(e+fx)\sqrt{b\tan^4(e+fx)}}{9f} + \left(b^2\cot^2(e+fx)\sqrt{b\ldots}\right.$$
$$= \frac{b^2\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f} - \frac{b^2\tan^5(e+fx)\sqrt{b\tan^4(e+fx)}}{7f} + \frac{b^2\tan^7(e+fx)\sqrt{b\ldots}}{9f}$$
$$= -\frac{b^2\tan(e+fx)\sqrt{b\tan^4(e+fx)}}{3f} + \frac{b^2\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f} - \frac{b^2\tan^5(e+fx)\sqrt{b\ldots}}{7f}$$
$$= \frac{b^2\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f} - \frac{b^2\tan(e+fx)\sqrt{b\tan^4(e+fx)}}{3f} + \frac{b^2\tan^3(e+fx)\sqrt{b\tan^4\ldots}}{5f}$$
$$= \frac{b^2\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f} - b^2x\cot^2(e+fx)\sqrt{b\tan^4(e+fx)} - \frac{b^2\tan(e+fx)\sqrt{b\tan^4\ldots}}{3f}$$

Mathematica [A] time = 0.718507, size = 86, normalized size = 0.47

$$\frac{\cot(e+fx)\left(b\tan^4(e+fx)\right)^{5/2}\left(315\cot^8(e+fx)-105\cot^6(e+fx)+63\cot^4(e+fx)-45\cot^2(e+fx)-315\tan^{-1}(\tan\ldots\right.}{315f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^4)^(5/2),x]
```

```
[Out] (Cot[e + f*x]*(35 - 45*Cot[e + f*x]^2 + 63*Cot[e + f*x]^4 - 105*Cot[e + f*x
]^6 + 315*Cot[e + f*x]^8 - 315*ArcTan[Tan[e + f*x]]*Cot[e + f*x]^9)*(b*Tan[
e + f*x]^4)^(5/2))/(315*f)
```

Maple [A] time = 0.037, size = 84, normalized size = 0.5

$$-\frac{-35\left(\tan(fx+e)\right)^9+45\left(\tan(fx+e)\right)^7-63\left(\tan(fx+e)\right)^5+105\left(\tan(fx+e)\right)^3+315\arctan\left(\tan(fx+e)\right)-\ldots}{315f\left(\tan(fx+e)\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^4)^(5/2),x)
```

```
[Out] -1/315/f*(b*tan(f*x+e)^4)^(5/2)*(-35*tan(f*x+e)^9+45*tan(f*x+e)^7-63*tan(f*
x+e)^5+105*tan(f*x+e)^3+315*arctan(tan(f*x+e))-315*tan(f*x+e))/tan(f*x+e)^1
0
```

Maxima [A] time = 1.66781, size = 107, normalized size = 0.59

$$\frac{35b^{\frac{5}{2}}\tan\left(fx+e\right)^9-45b^{\frac{5}{2}}\tan\left(fx+e\right)^7+63b^{\frac{5}{2}}\tan\left(fx+e\right)^5-105b^{\frac{5}{2}}\tan\left(fx+e\right)^3-315\left(fx+e\right)b^{\frac{5}{2}}+315b^{\frac{5}{2}}\tan\left(f\ldots\right.}{315f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/315*(35*b^(5/2)*tan(f*x + e)^9 - 45*b^(5/2)*tan(f*x + e)^7 + 63*b^(5/2)*t
an(f*x + e)^5 - 105*b^(5/2)*tan(f*x + e)^3 - 315*(f*x + e)*b^(5/2) + 315*b^
(5/2)*tan(f*x + e))/f
```

Fricas [A] time = 2.53901, size = 247, normalized size = 1.36

$$\frac{\left(35\,b^{2}\tan\left(fx+e\right)^{9}-45\,b^{2}\tan\left(fx+e\right)^{7}+63\,b^{2}\tan\left(fx+e\right)^{5}-105\,b^{2}\tan\left(fx+e\right)^{3}-315\,b^{2}fx+315\,b^{2}\tan\left(fx+\right.\right.}{315\,f\tan\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^2*tan(f*x + e)^9 - 45*b^2*tan(f*x + e)^7 + 63*b^2*tan(f*x + e)^
5 - 105*b^2*tan(f*x + e)^3 - 315*b^2*f*x + 315*b^2*tan(f*x + e))*sqrt(b*tan
(f*x + e)^4)/(f*tan(f*x + e)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan^{4}\left(e+fx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**4)**(5/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**4)**(5/2), x)
```

Giac [B] time = 14.4079, size = 1381, normalized size = 7.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -1/315*(315*b^2*f*x*tan(f*x)^9*tan(e)^9 - 2835*b^2*f*x*tan(f*x)^8*tan(e)^8
+ 315*b^2*tan(f*x)^9*tan(e)^8 + 315*b^2*tan(f*x)^8*tan(e)^9 + 11340*b^2*f*x
*tan(f*x)^7*tan(e)^7 - 105*b^2*tan(f*x)^9*tan(e)^6 - 2835*b^2*tan(f*x)^8*ta
n(e)^7 - 2835*b^2*tan(f*x)^7*tan(e)^8 - 105*b^2*tan(f*x)^6*tan(e)^9 - 26460
*b^2*f*x*tan(f*x)^6*tan(e)^6 + 63*b^2*tan(f*x)^9*tan(e)^4 + 945*b^2*tan(f*x
)^8*tan(e)^5 + 11340*b^2*tan(f*x)^7*tan(e)^6 + 11340*b^2*tan(f*x)^6*tan(e)^
7 + 945*b^2*tan(f*x)^5*tan(e)^8 + 63*b^2*tan(f*x)^4*tan(e)^9 + 39690*b^2*f*
x*tan(f*x)^5*tan(e)^5 - 45*b^2*tan(f*x)^9*tan(e)^2 - 567*b^2*tan(f*x)^8*tan
(e)^3 - 3780*b^2*tan(f*x)^7*tan(e)^4 - 26460*b^2*tan(f*x)^6*tan(e)^5 - 2646
0*b^2*tan(f*x)^5*tan(e)^6 - 3780*b^2*tan(f*x)^4*tan(e)^7 - 567*b^2*tan(f*x)
```

```
^3*tan(e)^8 - 45*b^2*tan(f*x)^2*tan(e)^9 - 39690*b^2*f*x*tan(f*x)^4*tan(e)^
4 + 35*b^2*tan(f*x)^9 + 405*b^2*tan(f*x)^8*tan(e) + 2268*b^2*tan(f*x)^7*tan
(e)^2 + 8820*b^2*tan(f*x)^6*tan(e)^3 + 39690*b^2*tan(f*x)^5*tan(e)^4 + 3969
0*b^2*tan(f*x)^4*tan(e)^5 + 8820*b^2*tan(f*x)^3*tan(e)^6 + 2268*b^2*tan(f*x
)^2*tan(e)^7 + 405*b^2*tan(f*x)*tan(e)^8 + 35*b^2*tan(e)^9 + 26460*b^2*f*x*
tan(f*x)^3*tan(e)^3 - 45*b^2*tan(f*x)^7 - 567*b^2*tan(f*x)^6*tan(e) - 3780*
b^2*tan(f*x)^5*tan(e)^2 - 26460*b^2*tan(f*x)^4*tan(e)^3 - 26460*b^2*tan(f*x
)^3*tan(e)^4 - 3780*b^2*tan(f*x)^2*tan(e)^5 - 567*b^2*tan(f*x)*tan(e)^6 - 4
5*b^2*tan(e)^7 - 11340*b^2*f*x*tan(f*x)^2*tan(e)^2 + 63*b^2*tan(f*x)^5 + 94
5*b^2*tan(f*x)^4*tan(e) + 11340*b^2*tan(f*x)^3*tan(e)^2 + 11340*b^2*tan(f*x
)^2*tan(e)^3 + 945*b^2*tan(f*x)*tan(e)^4 + 63*b^2*tan(e)^5 + 2835*b^2*f*x*t
an(f*x)*tan(e) - 105*b^2*tan(f*x)^3 - 2835*b^2*tan(f*x)^2*tan(e) - 2835*b^2
*tan(f*x)*tan(e)^2 - 105*b^2*tan(e)^3 - 315*b^2*f*x + 315*b^2*tan(f*x) + 31
5*b^2*tan(e))*sqrt(b)/(f*tan(f*x)^9*tan(e)^9 - 9*f*tan(f*x)^8*tan(e)^8 + 36
*f*tan(f*x)^7*tan(e)^7 - 84*f*tan(f*x)^6*tan(e)^6 + 126*f*tan(f*x)^5*tan(e)
^5 - 126*f*tan(f*x)^4*tan(e)^4 + 84*f*tan(f*x)^3*tan(e)^3 - 36*f*tan(f*x)^2
*tan(e)^2 + 9*f*tan(f*x)*tan(e) - f)
```

3.14 $\int \left(b \tan^4(e+fx)\right)^{3/2} dx$

Optimal. Leaf size=110

$$\frac{b\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f}-\frac{b\tan(e+fx)\sqrt{b\tan^4(e+fx)}}{3f}-bx\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}+\frac{b\cot(e+fx)\sqrt{}}{f}$$

```
[Out] (b*Cot[e + f*x]*Sqrt[b*Tan[e + f*x]^4])/f - b*x*Cot[e + f*x]^2*Sqrt[b*Tan[e
 + f*x]^4] - (b*Tan[e + f*x]*Sqrt[b*Tan[e + f*x]^4])/(3*f) + (b*Tan[e + f*x
]^3*Sqrt[b*Tan[e + f*x]^4])/(5*f)
```

Rubi [A] time = 0.0419221, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$\frac{b\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f}-\frac{b\tan(e+fx)\sqrt{b\tan^4(e+fx)}}{3f}-bx\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}+\frac{b\cot(e+fx)\sqrt{}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^4)^(3/2),x]
```

```
[Out] (b*Cot[e + f*x]*Sqrt[b*Tan[e + f*x]^4])/f - b*x*Cot[e + f*x]^2*Sqrt[b*Tan[e
 + f*x]^4] - (b*Tan[e + f*x]*Sqrt[b*Tan[e + f*x]^4])/(3*f) + (b*Tan[e + f*x
]^3*Sqrt[b*Tan[e + f*x]^4])/(5*f)
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \left(b\tan^4(e+fx)\right)^{3/2} dx &= \left(b\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\right)\int \tan^6(e+fx)\,dx \\
&= \frac{b\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f} - \left(b\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\right)\int \tan^4(e+fx)\,dx \\
&= -\frac{b\tan(e+fx)\sqrt{b\tan^4(e+fx)}}{3f} + \frac{b\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f} + \left(b\cot^2(e+fx)\sqrt{b\tan^4\right. \\
&= \frac{b\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f} - \frac{b\tan(e+fx)\sqrt{b\tan^4(e+fx)}}{3f} + \frac{b\tan^3(e+fx)\sqrt{b\tan^4(e+\right.}{5f} \\
&= \frac{b\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f} - bx\cot^2(e+fx)\sqrt{b\tan^4(e+fx)} - \frac{b\tan(e+fx)\sqrt{b\tan^4(e-\right.}{3f}
\end{aligned}
$$

Mathematica [A] time = 0.735057, size = 66, normalized size = 0.6

$$\frac{\cot(e+fx)\left(b\tan^4(e+fx)\right)^{3/2}\left(15\cot^4(e+fx)-5\cot^2(e+fx)-15\tan^{-1}(\tan(e+fx))\cot^5(e+fx)+3\right)}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^4)^(3/2),x]
```

```
[Out] (Cot[e + f*x]*(3 - 5*Cot[e + f*x]^2 + 15*Cot[e + f*x]^4 - 15*ArcTan[Tan[e +
 f*x]]*Cot[e + f*x]^5)*(b*Tan[e + f*x]^4)^(3/2))/(15*f)
```

Maple [A] time = 0.016, size = 64, normalized size = 0.6

$$-\frac{-3\left(\tan\left(fx+e\right)\right)^5+5\left(\tan\left(fx+e\right)\right)^3+15\arctan\left(\tan\left(fx+e\right)\right)-15\tan\left(fx+e\right)}{15f\left(\tan\left(fx+e\right)\right)^6}\left(b\left(\tan\left(fx+e\right)\right)^4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^4)^(3/2),x)
```

```
[Out] -1/15/f*(b*tan(f*x+e)^4)^(3/2)*(-3*tan(f*x+e)^5+5*tan(f*x+e)^3+15*arctan(ta
n(f*x+e))-15*tan(f*x+e))/tan(f*x+e)^6
```

Maxima [A] time = 1.57448, size = 72, normalized size = 0.65

$$\frac{3b^{\frac{3}{2}}\tan\left(fx+e\right)^5-5b^{\frac{3}{2}}\tan\left(fx+e\right)^3-15\left(fx+e\right)b^{\frac{3}{2}}+15b^{\frac{3}{2}}\tan\left(fx+e\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*b^(3/2)*tan(f*x + e)^5 - 5*b^(3/2)*tan(f*x + e)^3 - 15*(f*x + e)*b^
(3/2) + 15*b^(3/2)*tan(f*x + e))/f
```

Fricas [A] time = 2.45319, size = 163, normalized size = 1.48

$$\frac{\left(3\,b\tan\left(fx+e\right)^{5}-5\,b\tan\left(fx+e\right)^{3}-15\,bfx+15\,b\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^{4}}}{15\,f\tan\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 - 5*b*tan(f*x + e)^3 - 15*b*f*x + 15*b*tan(f*x + e
))*sqrt(b*tan(f*x + e)^4)/(f*tan(f*x + e)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan^{4}\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**4)**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**4)**(3/2), x)
```

Giac [B] time = 4.77166, size = 1457, normalized size = 13.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/60*(15*pi - 60*f*x*tan(f*x)^5*tan(e)^5 - 15*pi*sgn(2*tan(f*x)^2*tan(e) +
2*tan(f*x)*tan(e)^2 - 2*tan(f*x) - 2*tan(e))*tan(f*x)^5*tan(e)^5 - 15*pi*ta
n(f*x)^5*tan(e)^5 + 30*arctan((tan(f*x)*tan(e) - 1)/(tan(f*x) + tan(e)))*ta
n(f*x)^5*tan(e)^5 + 30*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*ta
n(f*x)^5*tan(e)^5 + 300*f*x*tan(f*x)^4*tan(e)^4 + 75*pi*sgn(2*tan(f*x)^2*ta
n(e) + 2*tan(f*x)*tan(e)^2 - 2*tan(f*x) - 2*tan(e))*tan(f*x)^4*tan(e)^4 + 7
5*pi*tan(f*x)^4*tan(e)^4 - 150*arctan((tan(f*x)*tan(e) - 1)/(tan(f*x) + tan
(e)))*tan(f*x)^4*tan(e)^4 - 150*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e)
 - 1))*tan(f*x)^4*tan(e)^4 - 60*tan(f*x)^5*tan(e)^4 - 60*tan(f*x)^4*tan(e)^
5 - 600*f*x*tan(f*x)^3*tan(e)^3 - 150*pi*sgn(2*tan(f*x)^2*tan(e) + 2*tan(f*
x)*tan(e)^2 - 2*tan(f*x) - 2*tan(e))*tan(f*x)^3*tan(e)^3 + 20*tan(f*x)^5*ta
n(e)^2 - 150*pi*tan(f*x)^3*tan(e)^3 + 300*arctan((tan(f*x)*tan(e) - 1)/(tan
(f*x) + tan(e)))*tan(f*x)^3*tan(e)^3 + 300*arctan((tan(f*x) + tan(e))/(tan(
f*x)*tan(e) - 1))*tan(f*x)^3*tan(e)^3 + 300*tan(f*x)^4*tan(e)^3 + 300*tan(f
*x)^3*tan(e)^4 + 20*tan(f*x)^2*tan(e)^5 + 600*f*x*tan(f*x)^2*tan(e)^2 + 150
*pi*sgn(2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 - 2*tan(f*x) - 2*tan(e))*
tan(f*x)^2*tan(e)^2 - 12*tan(f*x)^5 - 100*tan(f*x)^4*tan(e) + 150*pi*tan(f*
x)^2*tan(e)^2 - 300*arctan((tan(f*x)*tan(e) - 1)/(tan(f*x) + tan(e)))*tan(f
*x)^2*tan(e)^2 - 300*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(
f*x)^2*tan(e)^2 - 600*tan(f*x)^3*tan(e)^2 - 600*tan(f*x)^2*tan(e)^3 - 100*t
```

```
an(f*x)*tan(e)^4 - 12*tan(e)^5 - 300*f*x*tan(f*x)*tan(e) - 75*pi*sgn(2*tan(
f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 - 2*tan(f*x) - 2*tan(e))*tan(f*x)*tan(e
) + 20*tan(f*x)^3 - 75*pi*tan(f*x)*tan(e) + 150*arctan((tan(f*x)*tan(e) - 1
)/(tan(f*x) + tan(e)))*tan(f*x)*tan(e) + 150*arctan((tan(f*x) + tan(e))/(ta
n(f*x)*tan(e) - 1))*tan(f*x)*tan(e) + 300*tan(f*x)^2*tan(e) + 300*tan(f*x)*
tan(e)^2 + 20*tan(e)^3 + 60*f*x + 15*pi*sgn(2*tan(f*x)^2*tan(e) + 2*tan(f*x
)*tan(e)^2 - 2*tan(f*x) - 2*tan(e)) - 30*arctan((tan(f*x)*tan(e) - 1)/(tan(
f*x) + tan(e))) - 30*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1)) - 60
*tan(f*x) - 60*tan(e))*b^(3/2)/(f*tan(f*x)^5*tan(e)^5 - 5*f*tan(f*x)^4*tan(
e)^4 + 10*f*tan(f*x)^3*tan(e)^3 - 10*f*tan(f*x)^2*tan(e)^2 + 5*f*tan(f*x)*t
an(e) - f)
```

3.15 $\int \sqrt{b\tan^4(e+fx)}\,dx$

Optimal. Leaf size=50

$$\frac{\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f}-x\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}$$

```
[Out] (Cot[e + f*x]*Sqrt[b*Tan[e + f*x]^4])/f - x*Cot[e + f*x]^2*Sqrt[b*Tan[e + f
*x]^4]
```

Rubi [A] time = 0.0207209, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$\frac{\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f}-x\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]^4],x]
```

```
[Out] (Cot[e + f*x]*Sqrt[b*Tan[e + f*x]^4])/f - x*Cot[e + f*x]^2*Sqrt[b*Tan[e + f
*x]^4]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\tan^4(e+fx)}\,dx &= \left(\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\right)\int \tan^2(e+fx)\,dx\\ &= \frac{\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f}-\left(\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\right)\int 1\,dx\\ &= \frac{\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f}-x\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\end{aligned}$$

Mathematica [A] time = 0.0914212, size = 41, normalized size = 0.82

$$-\frac{\cot(e+fx)\sqrt{b\tan^4(e+fx)}\left(\tan^{-1}(\tan(e+fx))\cot(e+fx)-1\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]^4],x]
```

```
[Out] -((Cot[e + f*x]*(-1 + ArcTan[Tan[e + f*x]]*Cot[e + f*x])*Sqrt[b*Tan[e + f*x
]^4])/f)
```

Maple [A] time = 0.021, size = 42, normalized size = 0.8

$$-\frac{-\tan\left(fx+e\right)+\arctan\left(\tan\left(fx+e\right)\right)}{f\left(\tan\left(fx+e\right)\right)^{2}}\sqrt{b\left(\tan\left(fx+e\right)\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^4)^(1/2),x)
```

```
[Out] -1/f*(b*tan(f*x+e)^4)^(1/2)*(-tan(f*x+e)+arctan(tan(f*x+e)))/tan(f*x+e)^2
```

Maxima [A] time = 1.63842, size = 35, normalized size = 0.7

$$-\frac{\left(fx+e\right)\sqrt{b}-\sqrt{b}\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -((f*x + e)*sqrt(b) - sqrt(b)*tan(f*x + e))/f
```

Fricas [A] time = 2.07455, size = 88, normalized size = 1.76

$$-\frac{\sqrt{b\tan\left(fx+e\right)^{4}}\left(fx-\tan\left(fx+e\right)\right)}{f\tan\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*tan(f*x + e)^4)*(f*x - tan(f*x + e))/(f*tan(f*x + e)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan^4\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(e + f*x)**4), x)
```

Giac [B] time = 1.40617, size = 338, normalized size = 6.76

$$\frac{\left(\pi - 4\,fx\tan\left(fx\right)\tan\left(e\right) - \pi\mathrm{sgn}\left(2\,\tan\left(fx\right)^{2}\tan\left(e\right) + 2\,\tan\left(fx\right)\tan\left(e\right)^{2} - 2\,\tan\left(fx\right) - 2\,\tan\left(e\right)\right)\tan\left(fx\right)\tan\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(pi - 4*f*x*tan(f*x)*tan(e) - pi*sgn(2*tan(f*x)^2*tan(e) + 2*tan(f*x)*t
an(e)^2 - 2*tan(f*x) - 2*tan(e))*tan(f*x)*tan(e) - pi*tan(f*x)*tan(e) + 2*a
rctan((tan(f*x)*tan(e) - 1)/(tan(f*x) + tan(e)))*tan(f*x)*tan(e) + 2*arctan
((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)*tan(e) + 4*f*x + pi*sg
n(2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 - 2*tan(f*x) - 2*tan(e)) - 2*ar
ctan((tan(f*x)*tan(e) - 1)/(tan(f*x) + tan(e))) - 2*arctan((tan(f*x) + tan(
e))/(tan(f*x)*tan(e) - 1)) - 4*tan(f*x) - 4*tan(e))*sqrt(b)/(f*tan(f*x)*tan
(e) - f)
```

3.16 $\int \frac{1}{\sqrt{b\tan^4(e+fx)}}\,dx$

Optimal. Leaf size=51

$$-\frac{x\tan^2(e+fx)}{\sqrt{b\tan^4(e+fx)}}-\frac{\tan(e+fx)}{f\sqrt{b\tan^4(e+fx)}}$$

```
[Out] -(Tan[e + f*x]/(f*Sqrt[b*Tan[e + f*x]^4])) - (x*Tan[e + f*x]^2)/Sqrt[b*Tan[
e + f*x]^4]
```

Rubi [A] time = 0.0211575, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$-\frac{x\tan^2(e+fx)}{\sqrt{b\tan^4(e+fx)}}-\frac{\tan(e+fx)}{f\sqrt{b\tan^4(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Tan[e + f*x]^4],x]
```

```
[Out] -(Tan[e + f*x]/(f*Sqrt[b*Tan[e + f*x]^4])) - (x*Tan[e + f*x]^2)/Sqrt[b*Tan[
e + f*x]^4]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{b\tan^4(e+fx)}}\,dx = \frac{\tan^2(e+fx)\int \cot^2(e+fx)\,dx}{\sqrt{b\tan^4(e+fx)}}$$

$$= -\frac{\tan(e+fx)}{f\sqrt{b\tan^4(e+fx)}} - \frac{\tan^2(e+fx)\int 1\,dx}{\sqrt{b\tan^4(e+fx)}}$$

$$= -\frac{\tan(e+fx)}{f\sqrt{b\tan^4(e+fx)}} - \frac{x\tan^2(e+fx)}{\sqrt{b\tan^4(e+fx)}}$$

Mathematica [C] time = 0.0503223, size = 43, normalized size = 0.84

$$-\frac{\tan(e+fx)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(e+fx)\right)}{f\sqrt{b\tan^4(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Tan[e + f*x]^4],x]
```

```
[Out] -((Hypergeometric2F1[-1/2, 1, 1/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(f*Sqrt[b
*Tan[e + f*x]^4]))
```

Maple [A] time = 0.026, size = 40, normalized size = 0.8

$$-\frac{\tan\left(fx+e\right)\left(\arctan\left(\tan\left(fx+e\right)\right)\tan\left(fx+e\right)+1\right)}{f}\frac{1}{\sqrt{b\left(\tan\left(fx+e\right)\right)^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^4)^(1/2),x)
```

```
[Out] -1/f*tan(f*x+e)*(arctan(tan(f*x+e))*tan(f*x+e)+1)/(b*tan(f*x+e)^4)^(1/2)
```

Maxima [A] time = 1.53725, size = 36, normalized size = 0.71

$$-\frac{\frac{fx+e}{\sqrt{b}}+\frac{1}{\sqrt{b}\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -((f*x + e)/sqrt(b) + 1/(sqrt(b)*tan(f*x + e)))/f
```

Fricas [A] time = 2.01841, size = 93, normalized size = 1.82

$$-\frac{\sqrt{b\tan\left(fx+e\right)^4}\left(fx\tan\left(fx+e\right)+1\right)}{bf\tan\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*tan(f*x + e)^4)*(f*x*tan(f*x + e) + 1)/(b*f*tan(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\tan^4\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*tan(e + f*x)**4), x)
```

Giac [A] time = 1.53148, size = 65, normalized size = 1.27

$$-\frac{\frac{2\left(fx+e\right)}{\sqrt{b}}-\frac{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{b}}+\frac{1}{\sqrt{b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*(f*x + e)/sqrt(b) - tan(1/2*f*x + 1/2*e)/sqrt(b) + 1/(sqrt(b)*tan(1
/2*f*x + 1/2*e)))/f
```

3.17 $\int \frac{1}{\left(b\tan^4(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{x\tan^2(e+fx)}{b\sqrt{b\tan^4(e+fx)}}-\frac{\tan(e+fx)}{bf\sqrt{b\tan^4(e+fx)}}-\frac{\cot^3(e+fx)}{5bf\sqrt{b\tan^4(e+fx)}}+\frac{\cot(e+fx)}{3bf\sqrt{b\tan^4(e+fx)}}$$

```
[Out] Cot[e + f*x]/(3*b*f*Sqrt[b*Tan[e + f*x]^4]) - Cot[e + f*x]^3/(5*b*f*Sqrt[b*
Tan[e + f*x]^4]) - Tan[e + f*x]/(b*f*Sqrt[b*Tan[e + f*x]^4]) - (x*Tan[e + f
*x]^2)/(b*Sqrt[b*Tan[e + f*x]^4])
```

Rubi [A] time = 0.0446079, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$-\frac{x\tan^2(e+fx)}{b\sqrt{b\tan^4(e+fx)}}-\frac{\tan(e+fx)}{bf\sqrt{b\tan^4(e+fx)}}-\frac{\cot^3(e+fx)}{5bf\sqrt{b\tan^4(e+fx)}}+\frac{\cot(e+fx)}{3bf\sqrt{b\tan^4(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^4)^(-3/2),x]
```

```
[Out] Cot[e + f*x]/(3*b*f*Sqrt[b*Tan[e + f*x]^4]) - Cot[e + f*x]^3/(5*b*f*Sqrt[b*
Tan[e + f*x]^4]) - Tan[e + f*x]/(b*f*Sqrt[b*Tan[e + f*x]^4]) - (x*Tan[e + f
*x]^2)/(b*Sqrt[b*Tan[e + f*x]^4])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{\left(b\tan^4(e+fx)\right)^{3/2}}\,dx = \frac{\tan^2(e+fx)\int \cot^6(e+fx)\,dx}{b\sqrt{b\tan^4(e+fx)}}$$
$$= -\frac{\cot^3(e+fx)}{5bf\sqrt{b\tan^4(e+fx)}} - \frac{\tan^2(e+fx)\int \cot^4(e+fx)\,dx}{b\sqrt{b\tan^4(e+fx)}}$$
$$= \frac{\cot(e+fx)}{3bf\sqrt{b\tan^4(e+fx)}} - \frac{\cot^3(e+fx)}{5bf\sqrt{b\tan^4(e+fx)}} + \frac{\tan^2(e+fx)\int \cot^2(e+fx)\,dx}{b\sqrt{b\tan^4(e+fx)}}$$
$$= \frac{\cot(e+fx)}{3bf\sqrt{b\tan^4(e+fx)}} - \frac{\cot^3(e+fx)}{5bf\sqrt{b\tan^4(e+fx)}} - \frac{\tan(e+fx)}{bf\sqrt{b\tan^4(e+fx)}} - \frac{\tan^2(e+fx)\int 1\,dx}{b\sqrt{b\tan^4(e+fx)}}$$
$$= \frac{\cot(e+fx)}{3bf\sqrt{b\tan^4(e+fx)}} - \frac{\cot^3(e+fx)}{5bf\sqrt{b\tan^4(e+fx)}} - \frac{\tan(e+fx)}{bf\sqrt{b\tan^4(e+fx)}} - \frac{x\tan^2(e+fx)}{b\sqrt{b\tan^4(e+fx)}}$$

Mathematica [C] time = 0.0486975, size = 45, normalized size = 0.38

$$-\frac{\tan(e+fx)\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)}{5f\left(b\tan^4(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^4)^(-3/2),x]
```

```
[Out] -(Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(5*f*(b*T
an[e + f*x]^4)^(3/2))
```

Maple [A] time = 0.02, size = 63, normalized size = 0.5

$$-\frac{\tan\left(fx+e\right)\left(15\arctan\left(\tan\left(fx+e\right)\right)\left(\tan\left(fx+e\right)\right)^5+15\left(\tan\left(fx+e\right)\right)^4-5\left(\tan\left(fx+e\right)\right)^2+3\right)}{15f}\left(b\left(\tan\left(fx+e\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^4)^(3/2),x)
```

```
[Out] -1/15/f*tan(f*x+e)*(15*arctan(tan(f*x+e))*tan(f*x+e)^5+15*tan(f*x+e)^4-5*ta
n(f*x+e)^2+3)/(b*tan(f*x+e)^4)^(3/2)
```

Maxima [A] time = 1.63716, size = 68, normalized size = 0.57

$$-\frac{\frac{15\left(fx+e\right)}{b^{\frac{3}{2}}}+\frac{15\tan\left(fx+e\right)^4-5\tan\left(fx+e\right)^2+3}{b^{\frac{3}{2}}\tan\left(fx+e\right)^5}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(f*x + e)/b^(3/2) + (15*tan(f*x + e)^4 - 5*tan(f*x + e)^2 + 3)/(b
^(3/2)*tan(f*x + e)^5))/f
```

Fricas [A] time = 1.93182, size = 162, normalized size = 1.36

$$-\frac{\left(15\,fx\tan\left(fx+e\right)^{5}+15\,\tan\left(fx+e\right)^{4}-5\,\tan\left(fx+e\right)^{2}+3\right)\sqrt{b\tan\left(fx+e\right)^{4}}}{15\,b^{2}f\tan\left(fx+e\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(15*f*x*tan(f*x + e)^5 + 15*tan(f*x + e)^4 - 5*tan(f*x + e)^2 + 3)*sq
rt(b*tan(f*x + e)^4)/(b^2*f*tan(f*x + e)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(b\tan^{4}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**4)**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**4)**(-3/2), x)
```

Giac [A] time = 1.87698, size = 177, normalized size = 1.49

$$-\frac{\frac{480\left(fx+e\right)}{\sqrt{b}}-\frac{3\,b^{\frac{9}{2}}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-35\,b^{\frac{9}{2}}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+330\,b^{\frac{9}{2}}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{b^{5}}+\frac{330\,\sqrt{b}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-35\,\sqrt{b}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+3\,\sqrt{b}}{b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}}}{480\,bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -1/480*(480*(f*x + e)/sqrt(b) - (3*b^(9/2)*tan(1/2*f*x + 1/2*e)^5 - 35*b^(9
/2)*tan(1/2*f*x + 1/2*e)^3 + 330*b^(9/2)*tan(1/2*f*x + 1/2*e))/b^5 + (330*s
qrt(b)*tan(1/2*f*x + 1/2*e)^4 - 35*sqrt(b)*tan(1/2*f*x + 1/2*e)^2 + 3*sqrt(
b))/(b*tan(1/2*f*x + 1/2*e)^5))/(b*f)
```

3.18 $\int \frac{1}{\left(b\tan^4(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{x\tan^2(e+fx)}{b^2\sqrt{b\tan^4(e+fx)}}-\frac{\tan(e+fx)}{b^2f\sqrt{b\tan^4(e+fx)}}-\frac{\cot^7(e+fx)}{9b^2f\sqrt{b\tan^4(e+fx)}}+\frac{\cot^5(e+fx)}{7b^2f\sqrt{b\tan^4(e+fx)}}-\frac{\cot^3(e+fx)}{5b^2f\sqrt{b\tan^4(e+fx)}}+$$

```
[Out] Cot[e + f*x]/(3*b^2*f*Sqrt[b*Tan[e + f*x]^4]) - Cot[e + f*x]^3/(5*b^2*f*Sqr
t[b*Tan[e + f*x]^4]) + Cot[e + f*x]^5/(7*b^2*f*Sqrt[b*Tan[e + f*x]^4]) - Co
t[e + f*x]^7/(9*b^2*f*Sqrt[b*Tan[e + f*x]^4]) - Tan[e + f*x]/(b^2*f*Sqrt[b*
Tan[e + f*x]^4]) - (x*Tan[e + f*x]^2)/(b^2*Sqrt[b*Tan[e + f*x]^4])
```

Rubi [A] time = 0.0646674, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$-\frac{x\tan^2(e+fx)}{b^2\sqrt{b\tan^4(e+fx)}}-\frac{\tan(e+fx)}{b^2f\sqrt{b\tan^4(e+fx)}}-\frac{\cot^7(e+fx)}{9b^2f\sqrt{b\tan^4(e+fx)}}+\frac{\cot^5(e+fx)}{7b^2f\sqrt{b\tan^4(e+fx)}}-\frac{\cot^3(e+fx)}{5b^2f\sqrt{b\tan^4(e+fx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^4)^(-5/2),x]
```

```
[Out] Cot[e + f*x]/(3*b^2*f*Sqrt[b*Tan[e + f*x]^4]) - Cot[e + f*x]^3/(5*b^2*f*Sqr
t[b*Tan[e + f*x]^4]) + Cot[e + f*x]^5/(7*b^2*f*Sqrt[b*Tan[e + f*x]^4]) - Co
t[e + f*x]^7/(9*b^2*f*Sqrt[b*Tan[e + f*x]^4]) - Tan[e + f*x]/(b^2*f*Sqrt[b*
Tan[e + f*x]^4]) - (x*Tan[e + f*x]^2)/(b^2*Sqrt[b*Tan[e + f*x]^4])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{\left(b\tan^4(e+fx)\right)^{5/2}}\,dx = \frac{\tan^2(e+fx)\int \cot^{10}(e+fx)\,dx}{b^2\sqrt{b\tan^4(e+fx)}}$$

$$= -\frac{\cot^7(e+fx)}{9b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\tan^2(e+fx)\int \cot^8(e+fx)\,dx}{b^2\sqrt{b\tan^4(e+fx)}}$$

$$= \frac{\cot^5(e+fx)}{7b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\cot^7(e+fx)}{9b^2f\sqrt{b\tan^4(e+fx)}} + \frac{\tan^2(e+fx)\int \cot^6(e+fx)\,dx}{b^2\sqrt{b\tan^4(e+fx)}}$$

$$= -\frac{\cot^3(e+fx)}{5b^2f\sqrt{b\tan^4(e+fx)}} + \frac{\cot^5(e+fx)}{7b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\cot^7(e+fx)}{9b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\tan^2(e+fx)}{b^2\sqrt{b\,\mathrm{ta}}}$$

$$= \frac{\cot(e+fx)}{3b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\cot^3(e+fx)}{5b^2f\sqrt{b\tan^4(e+fx)}} + \frac{\cot^5(e+fx)}{7b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\cot^7(e+f}{9b^2f\sqrt{b\tan^4(e}}$$

$$= \frac{\cot(e+fx)}{3b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\cot^3(e+fx)}{5b^2f\sqrt{b\tan^4(e+fx)}} + \frac{\cot^5(e+fx)}{7b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\cot^7(e+f}{9b^2f\sqrt{b\tan^4(e}}$$

$$= \frac{\cot(e+fx)}{3b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\cot^3(e+fx)}{5b^2f\sqrt{b\tan^4(e+fx)}} + \frac{\cot^5(e+fx)}{7b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\cot^7(e+f}{9b^2f\sqrt{b\tan^4(e}}$$

Mathematica [C] time = 0.0326329, size = 45, normalized size = 0.25

$$-\frac{\tan(e+fx)\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},-\tan^2(e+fx)\right)}{9f\left(b\tan^4(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^4)^(-5/2),x]
```

```
[Out] -(Hypergeometric2F1[-9/2, 1, -7/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(9*f*(b*T
an[e + f*x]^4)^(5/2))
```

Maple [A] time = 0.021, size = 83, normalized size = 0.5

$$-\frac{\tan\left(fx+e\right)\left(315\,\arctan\left(\tan\left(fx+e\right)\right)\left(\tan\left(fx+e\right)\right)^{9}+315\,\left(\tan\left(fx+e\right)\right)^{8}-105\,\left(\tan\left(fx+e\right)\right)^{6}+63\,\left(\tan\left(f\right.\right.}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^4)^(5/2),x)
```

```
[Out] -1/315/f*tan(f*x+e)*(315*arctan(tan(f*x+e))*tan(f*x+e)^9+315*tan(f*x+e)^8-1
05*tan(f*x+e)^6+63*tan(f*x+e)^4-45*tan(f*x+e)^2+35)/(b*tan(f*x+e)^4)^(5/2)
```

Maxima [A] time = 1.6532, size = 95, normalized size = 0.52

$$-\frac{\frac{315\left(fx+e\right)}{b^{\frac{5}{2}}}+\frac{315\,\tan\left(fx+e\right)^{8}-105\,\tan\left(fx+e\right)^{6}+63\,\tan\left(fx+e\right)^{4}-45\,\tan\left(fx+e\right)^{2}+35}{b^{\frac{5}{2}}\,\tan\left(fx+e\right)^{9}}}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/315*(315*(f*x + e)/b^(5/2) + (315*tan(f*x + e)^8 - 105*tan(f*x + e)^6 +
63*tan(f*x + e)^4 - 45*tan(f*x + e)^2 + 35)/(b^(5/2)*tan(f*x + e)^9))/f
```

Fricas [A] time = 1.92304, size = 225, normalized size = 1.23

$$-\frac{\left(315\,fx\tan\left(fx+e\right)^{9}+315\,\tan\left(fx+e\right)^{8}-105\,\tan\left(fx+e\right)^{6}+63\,\tan\left(fx+e\right)^{4}-45\,\tan\left(fx+e\right)^{2}+35\right)\sqrt{b\tan\left(f\right.}}{315\,b^{3}f\tan\left(fx+e\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/315*(315*f*x*tan(f*x + e)^9 + 315*tan(f*x + e)^8 - 105*tan(f*x + e)^6 +
63*tan(f*x + e)^4 - 45*tan(f*x + e)^2 + 35)*sqrt(b*tan(f*x + e)^4)/(b^3*f*t
an(f*x + e)^11)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan^{4}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**4)**(5/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**4)**(-5/2), x)
```

Giac [A] time = 2.79148, size = 265, normalized size = 1.45

$$-\frac{\frac{161280\left(fx+e\right)}{b^{\frac{5}{2}}}+\frac{121590\sqrt{b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{8}-18480\sqrt{b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}+3528\sqrt{b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-495\sqrt{b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+35\sqrt{b}}{b^{3}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{9}}-\frac{35\,b^{\frac{49}{2}}\tan\left(\frac{1}{2}fx+\frac{1}{2}\right.}{}}{161280\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -1/161280*(161280*(f*x + e)/b^(5/2) + (121590*sqrt(b)*tan(1/2*f*x + 1/2*e)^
8 - 18480*sqrt(b)*tan(1/2*f*x + 1/2*e)^6 + 3528*sqrt(b)*tan(1/2*f*x + 1/2*e
)^4 - 495*sqrt(b)*tan(1/2*f*x + 1/2*e)^2 + 35*sqrt(b))/(b^3*tan(1/2*f*x + 1
/2*e)^9) - (35*b^(49/2)*tan(1/2*f*x + 1/2*e)^9 - 495*b^(49/2)*tan(1/2*f*x +
 1/2*e)^7 + 3528*b^(49/2)*tan(1/2*f*x + 1/2*e)^5 - 18480*b^(49/2)*tan(1/2*f
*x + 1/2*e)^3 + 121590*b^(49/2)*tan(1/2*f*x + 1/2*e))/b^27)/f
```

3.19 $\int \left(b\tan^n(e+fx)\right)^{5/2}\,dx$

Optimal. Leaf size=71

$$\frac{2b^2\tan^{2n+1}(e+fx)\sqrt{b\tan^n(e+fx)}\text{Hypergeometric2F1}\left(1,\frac{1}{4}(5n+2),\frac{1}{4}(5n+6),-\tan^2(e+fx)\right)}{f(5n+2)}$$

```
[Out] (2*b^2*Hypergeometric2F1[1, (2 + 5*n)/4, (6 + 5*n)/4, -Tan[e + f*x]^2]*Tan[
e + f*x]^(1 + 2*n)*Sqrt[b*Tan[e + f*x]^n])/(f*(2 + 5*n))
```

Rubi [A] time = 0.0439371, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{2b^2\tan^{2n+1}(e+fx)\sqrt{b\tan^n(e+fx)}\,{}_2F_1\left(1,\frac{1}{4}(5n+2);\frac{1}{4}(5n+6);-\tan^2(e+fx)\right)}{f(5n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^n)^(5/2),x]
```

```
[Out] (2*b^2*Hypergeometric2F1[1, (2 + 5*n)/4, (6 + 5*n)/4, -Tan[e + f*x]^2]*Tan[
e + f*x]^(1 + 2*n)*Sqrt[b*Tan[e + f*x]^n])/(f*(2 + 5*n))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(b\tan^n(e+fx)\right)^{5/2} dx = \left(b^2 \tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\right)\int \tan^{\frac{5n}{2}}(e+fx)\,dx$$
$$= \frac{\left(b^2 \tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\right)\text{Subst}\left(\int \frac{x^{5n/2}}{1+x^2}dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{2b^2\, {}_2F_1\left(1,\frac{1}{4}(2+5n);\frac{1}{4}(6+5n);-\tan^2(e+fx)\right)\tan^{1+2n}(e+fx)\sqrt{b\tan^n(e+fx)}}{f(2+5n)}$$

Mathematica [A] time = 0.108413, size = 62, normalized size = 0.87

$$\frac{2\tan(e+fx)\left(b\tan^n(e+fx)\right)^{5/2}\text{Hypergeometric2F1}\left(1,\frac{1}{4}(5n+2),\frac{1}{4}(5n+6),-\tan^2(e+fx)\right)}{f(5n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^n)^(5/2),x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 + 5*n)/4, (6 + 5*n)/4, -Tan[e + f*x]^2]*Tan[e +
f*x]*(b*Tan[e + f*x]^n)^(5/2))/(f*(2 + 5*n))
```

Maple [F] time = 0.615, size = 0, normalized size = 0.

$$\int \left(b\left(\tan\left(fx+e\right)\right)^n\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^n)^(5/2),x)
```

```
[Out] int((b*tan(f*x+e)^n)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^n\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^n)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**n)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^{n}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^n)^(5/2), x)
```

3.20 $\int \left(b \tan^n(e+fx)\right)^{3/2} dx$

Optimal. Leaf size=65

$$\frac{2b \tan^{n+1}(e+fx)\sqrt{b \tan^n(e+fx)}\text{Hypergeometric2F1}\left(1, \frac{1}{4}(3n+2), \frac{3(n+2)}{4}, -\tan^2(e+fx)\right)}{f(3n+2)}$$

```
[Out] (2*b*Hypergeometric2F1[1, (2 + 3*n)/4, (3*(2 + n))/4, -Tan[e + f*x]^2]*Tan[
e + f*x]^(1 + n)*Sqrt[b*Tan[e + f*x]^n])/(f*(2 + 3*n))
```

Rubi [A] time = 0.0447993, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{2b \tan^{n+1}(e+fx)\sqrt{b \tan^n(e+fx)}\,{}_2F_1\left(1, \frac{1}{4}(3n+2); \frac{3(n+2)}{4}; -\tan^2(e+fx)\right)}{f(3n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^n)^(3/2),x]
```

```
[Out] (2*b*Hypergeometric2F1[1, (2 + 3*n)/4, (3*(2 + n))/4, -Tan[e + f*x]^2]*Tan[
e + f*x]^(1 + n)*Sqrt[b*Tan[e + f*x]^n])/(f*(2 + 3*n))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(b\tan^n(e+fx)\right)^{3/2} dx = \left(b\tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\right)\int \tan^{\frac{3n}{2}}(e+fx)\,dx$$
$$= \frac{\left(b\tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\right)\text{Subst}\left(\int \frac{x^{3n/2}}{1+x^2}dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{2b\,{}_2F_1\left(1,\frac{1}{4}(2+3n);\frac{3(2+n)}{4};-\tan^2(e+fx)\right)\tan^{1+n}(e+fx)\sqrt{b\tan^n(e+fx)}}{f(2+3n)}$$

Mathematica [A] time = 0.0683063, size = 60, normalized size = 0.92

$$\frac{2\tan(e+fx)\left(b\tan^n(e+fx)\right)^{3/2}\text{Hypergeometric2F1}\left(1,\frac{1}{4}(3n+2),\frac{3(n+2)}{4},-\tan^2(e+fx)\right)}{f(3n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^n)^(3/2),x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 + 3*n)/4, (3*(2 + n))/4, -Tan[e + f*x]^2]*Tan[e
+ f*x]*(b*Tan[e + f*x]^n)^(3/2))/(f*(2 + 3*n))
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int \left(b\left(\tan\left(fx+e\right)\right)^n\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^n)^(3/2),x)
```

```
[Out] int((b*tan(f*x+e)^n)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^n\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^n)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan^{n}\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**n)**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**n)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^{n}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^n)^(3/2), x)
```

3.21 $\int \sqrt{b \tan^n(e+fx)}\,dx$

Optimal. Leaf size=56

$$\frac{2\tan(e+fx)\sqrt{b\tan^n(e+fx)}\text{Hypergeometric2F1}\left(1,\frac{n+2}{4},\frac{n+6}{4},-\tan^2(e+fx)\right)}{f(n+2)}$$

```
[Out] (2*Hypergeometric2F1[1, (2 + n)/4, (6 + n)/4, -Tan[e + f*x]^2]*Tan[e + f*x]
*Sqrt[b*Tan[e + f*x]^n])/(f*(2 + n))
```

Rubi [A] time = 0.0418333, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{2\tan(e+fx)\sqrt{b\tan^n(e+fx)}\,{}_2F_1\left(1,\frac{n+2}{4};\frac{n+6}{4};-\tan^2(e+fx)\right)}{f(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]^n],x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 + n)/4, (6 + n)/4, -Tan[e + f*x]^2]*Tan[e + f*x]
*Sqrt[b*Tan[e + f*x]^n])/(f*(2 + n))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \sqrt{b\tan^n(e+fx)}\,dx = \left(\tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\right)\int \tan^{\frac{n}{2}}(e+fx)\,dx$$
$$= \frac{\left(\tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\right)\text{Subst}\left(\int \frac{x^{n/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{2\,{}_2F_1\left(1,\frac{2+n}{4};\frac{6+n}{4};-\tan^2(e+fx)\right)\tan(e+fx)\sqrt{b\tan^n(e+fx)}}{f(2+n)}$$

Mathematica [A] time = 0.037718, size = 56, normalized size = 1.

$$\frac{2\tan(e+fx)\sqrt{b\tan^n(e+fx)}\text{Hypergeometric2F1}\left(1,\frac{n+2}{4},\frac{n+6}{4},-\tan^2(e+fx)\right)}{f(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]^n],x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 + n)/4, (6 + n)/4, -Tan[e + f*x]^2]*Tan[e + f*x]
*Sqrt[b*Tan[e + f*x]^n])/(f*(2 + n))
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int \sqrt{b\left(\tan\left(fx+e\right)\right)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^n)^(1/2),x)
```

```
[Out] int((b*tan(f*x+e)^n)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx+e\right)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^n), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \tan^{n}\left(e + f x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**n)**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(e + f*x)**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \tan\left(f x + e\right)^{n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^n), x)
```

3.22 $\int \frac{1}{\sqrt{b\tan^n(e+fx)}}\,dx$

Optimal. Leaf size=62

$$\frac{2\tan(e+fx)\text{Hypergeometric2F1}\left(1,\frac{2-n}{4},\frac{6-n}{4},-\tan^2(e+fx)\right)}{f(2-n)\sqrt{b\tan^n(e+fx)}}$$

```
[Out] (2*Hypergeometric2F1[1, (2 - n)/4, (6 - n)/4, -Tan[e + f*x]^2]*Tan[e + f*x]
)/(f*(2 - n)*Sqrt[b*Tan[e + f*x]^n])
```

Rubi [A] time = 0.0476349, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{2\tan(e+fx)\,{}_2F_1\left(1,\frac{2-n}{4};\frac{6-n}{4};-\tan^2(e+fx)\right)}{f(2-n)\sqrt{b\tan^n(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Tan[e + f*x]^n],x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 - n)/4, (6 - n)/4, -Tan[e + f*x]^2]*Tan[e + f*x]
)/(f*(2 - n)*Sqrt[b*Tan[e + f*x]^n])
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{b\tan^n(e+fx)}}\,dx = \frac{\tan^{\frac{n}{2}}(e+fx)\int \tan^{-\frac{n}{2}}(e+fx)\,dx}{\sqrt{b\tan^n(e+fx)}}$$
$$= \frac{\tan^{\frac{n}{2}}(e+fx)\,\mathrm{Subst}\left(\int \frac{x^{-n/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f\sqrt{b\tan^n(e+fx)}}$$
$$= \frac{2\,{}_2F_1\left(1, \frac{2-n}{4}; \frac{6-n}{4}; -\tan^2(e+fx)\right)\tan(e+fx)}{f(2-n)\sqrt{b\tan^n(e+fx)}}$$

Mathematica [A] time = 0.0498712, size = 60, normalized size = 0.97

$$-\frac{2\tan(e+fx)\mathrm{Hypergeometric2F1}\left(1, \frac{2-n}{4}, \frac{6-n}{4}, -\tan^2(e+fx)\right)}{f(n-2)\sqrt{b\tan^n(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Tan[e + f*x]^n],x]
```

```
[Out] (-2*Hypergeometric2F1[1, (2 - n)/4, (6 - n)/4, -Tan[e + f*x]^2]*Tan[e + f*x
])/(f*(-2 + n)*Sqrt[b*Tan[e + f*x]^n])
```

Maple [F] time = 0.122, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\left(\tan\left(fx+e\right)\right)^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^n)^(1/2),x)
```

```
[Out] int(1/(b*tan(f*x+e)^n)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\tan\left(fx+e\right)^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*tan(f*x + e)^n), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \tan^n\left(e + fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**n)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*tan(e + f*x)**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \tan\left(fx + e\right)^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*tan(f*x + e)^n), x)
```

3.23 $\int \frac{1}{\left(b\tan^n(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=71

$$\frac{2\tan^{1-n}(e+fx)\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-3n),\frac{3(2-n)}{4},-\tan^2(e+fx)\right)}{bf(2-3n)\sqrt{b\tan^n(e+fx)}}$$

```
[Out] (2*Hypergeometric2F1[1, (2 - 3*n)/4, (3*(2 - n))/4, -Tan[e + f*x]^2]*Tan[e
+ f*x]^(1 - n))/(b*f*(2 - 3*n)*Sqrt[b*Tan[e + f*x]^n])
```

Rubi [A] time = 0.0481613, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{2\tan^{1-n}(e+fx)\,{}_2F_1\left(1,\frac{1}{4}(2-3n);\frac{3(2-n)}{4};-\tan^2(e+fx)\right)}{bf(2-3n)\sqrt{b\tan^n(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^n)^(-3/2),x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 - 3*n)/4, (3*(2 - n))/4, -Tan[e + f*x]^2]*Tan[e
+ f*x]^(1 - n))/(b*f*(2 - 3*n)*Sqrt[b*Tan[e + f*x]^n])
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(b\tan^n(e+fx)\right)^{3/2}}\,dx = \frac{\tan^{\frac{n}{2}}(e+fx)\int \tan^{-\frac{3n}{2}}(e+fx)\,dx}{b\sqrt{b\tan^n(e+fx)}}$$
$$= \frac{\tan^{\frac{n}{2}}(e+fx)\,\text{Subst}\left(\int \frac{x^{-3n/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{bf\sqrt{b\tan^n(e+fx)}}$$
$$= \frac{2\,{}_2F_1\left(1,\frac{1}{4}(2-3n);\frac{3(2-n)}{4};-\tan^2(e+fx)\right)\tan^{1-n}(e+fx)}{bf(2-3n)\sqrt{b\tan^n(e+fx)}}$$

Mathematica [A] time = 0.0670313, size = 60, normalized size = 0.85

$$-\frac{2\tan(e+fx)\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-3n),-\frac{3}{4}(n-2),-\tan^2(e+fx)\right)}{f(3n-2)\left(b\tan^n(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^n)^(-3/2),x]
```

```
[Out] (-2*Hypergeometric2F1[1, (2 - 3*n)/4, (-3*(-2 + n))/4, -Tan[e + f*x]^2]*Tan
[e + f*x])/(f*(-2 + 3*n)*(b*Tan[e + f*x]^n)^(3/2))
```

Maple [F] time = 0.116, size = 0, normalized size = 0.

$$\int \left(b\left(\tan\left(fx+e\right)\right)^n\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^n)^(3/2),x)
```

```
[Out] int(1/(b*tan(f*x+e)^n)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan\left(fx+e\right)^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^n)^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan^n\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**n)**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**n)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan\left(fx+e\right)^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^n)^(-3/2), x)
```

3.24 $\int \frac{1}{\left(b\tan^n(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=71

$$\frac{2\tan^{1-2n}(e+fx)\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-5n),\frac{1}{4}(6-5n),-\tan^2(e+fx)\right)}{b^2f(2-5n)\sqrt{b\tan^n(e+fx)}}$$

```
[Out] (2*Hypergeometric2F1[1, (2 - 5*n)/4, (6 - 5*n)/4, -Tan[e + f*x]^2]*Tan[e +
f*x]^(1 - 2*n))/(b^2*f*(2 - 5*n)*Sqrt[b*Tan[e + f*x]^n])
```

Rubi [A] time = 0.0481893, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{2\tan^{1-2n}(e+fx)\,_2F_1\left(1,\frac{1}{4}(2-5n);\frac{1}{4}(6-5n);-\tan^2(e+fx)\right)}{b^2f(2-5n)\sqrt{b\tan^n(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^n)^(-5/2),x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 - 5*n)/4, (6 - 5*n)/4, -Tan[e + f*x]^2]*Tan[e +
f*x]^(1 - 2*n))/(b^2*f*(2 - 5*n)*Sqrt[b*Tan[e + f*x]^n])
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(b\tan^n(e+fx)\right)^{5/2}}\,dx = \frac{\tan^{\frac{n}{2}}(e+fx)\int \tan^{-\frac{5n}{2}}(e+fx)\,dx}{b^2\sqrt{b\tan^n(e+fx)}}$$
$$= \frac{\tan^{\frac{n}{2}}(e+fx)\,\text{Subst}\left(\int \frac{x^{-5n/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{b^2 f\sqrt{b\tan^n(e+fx)}}$$
$$= \frac{2\,{}_2F_1\left(1,\frac{1}{4}(2-5n);\frac{1}{4}(6-5n);-\tan^2(e+fx)\right)\tan^{1-2n}(e+fx)}{b^2 f(2-5n)\sqrt{b\tan^n(e+fx)}}$$

Mathematica [A] time = 0.0679305, size = 62, normalized size = 0.87

$$-\frac{2\tan(e+fx)\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-5n),\frac{1}{4}(6-5n),-\tan^2(e+fx)\right)}{f(5n-2)\left(b\tan^n(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^n)^(-5/2),x]

[Out] (-2*Hypergeometric2F1[1, (2 - 5*n)/4, (6 - 5*n)/4, -Tan[e + f*x]^2]*Tan[e +
 f*x])/(f*(-2 + 5*n)*(b*Tan[e + f*x]^n)^(5/2))
```

Maple [F] time = 0.115, size = 0, normalized size = 0.

$$\int \left(b\left(\tan\left(fx+e\right)\right)^{n}\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^n)^(5/2),x)

[Out] int(1/(b*tan(f*x+e)^n)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan\left(fx+e\right)^{n}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(5/2),x, algorithm="maxima")

[Out] integrate((b*tan(f*x + e)^n)^(-5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan^{n}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**n)**(5/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**n)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan\left(fx+e\right)^{n}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^n)^(-5/2), x)
```

3.25 $\int \left(b\tan^n(e+fx)\right)^p \, dx$

Optimal. Leaf size=59

$$\frac{\tan(e+fx)\left(b\tan^n(e+fx)\right)^p \text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+1),\frac{1}{2}(np+3),-\tan^2(e+fx)\right)}{f(np+1)}$$

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*Tan[e + f*x]^n)^p)/(f*(1 + n*p))
```

Rubi [A] time = 0.0400539, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3659, 3476, 364}

$$\frac{\tan(e+fx)\left(b\tan^n(e+fx)\right)^p {}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*Tan[e + f*x]^n)^p)/(f*(1 + n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(b\tan^n(e+fx)\right)^p\,dx = \left(\tan^{-np}(e+fx)\left(b\tan^n(e+fx)\right)^p\right)\int \tan^{np}(e+fx)\,dx$$
$$= \frac{\left(\tan^{-np}(e+fx)\left(b\tan^n(e+fx)\right)^p\right)\text{Subst}\left(\int \frac{x^{np}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^n(e+fx)\right)^p}{f(1+np)}$$

Mathematica [A] time = 0.0413329, size = 57, normalized size = 0.97

$$\frac{\tan(e+fx)\left(b\tan^n(e+fx)\right)^p \text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+1),\frac{1}{2}(np+3),-\tan^2(e+fx)\right)}{fnp+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*Tan[e + f*x]^n)^p)/(f + f*n*p)
```

Maple [F] time = 10.709, size = 0, normalized size = 0.

$$\int \left(b\left(\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^n)^p,x)
```

```
[Out] int((b*tan(f*x+e)^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^n)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^n\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^n)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan^{n}\left(e+fx\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**n)**p,x)
```

```
[Out] Integral((b*tan(e + f*x)**n)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^n)^p, x)
```

3.26 $\int \left(b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=59

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p \text{Hypergeometric2F1}\left(1,\frac{1}{2}(2p+1),\frac{1}{2}(2p+3),-\tan^2(e+fx)\right)}{f(2p+1)}$$

```
[Out] (Hypergeometric2F1[1, (1 + 2*p)/2, (3 + 2*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*Tan[e + f*x]^2)^p)/(f*(1 + 2*p))
```

Rubi [A] time = 0.0401488, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3658, 3476, 364}

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p {}_2F_1\left(1,\frac{1}{2}(2p+1);\frac{1}{2}(2p+3);-\tan^2(e+fx)\right)}{f(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + 2*p)/2, (3 + 2*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*Tan[e + f*x]^2)^p)/(f*(1 + 2*p))
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(b\tan^2(e+fx)\right)^p\,dx = \left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int \tan^{2p}(e+fx)\,dx$$
$$= \frac{\left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\text{Subst}\left(\int \frac{x^{2p}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{{}_2F_1\left(1,\frac{1}{2}(1+2p);\frac{1}{2}(3+2p);-\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^2(e+fx)\right)^p}{f(1+2p)}$$

Mathematica [A] time = 0.0485892, size = 49, normalized size = 0.83

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p \text{Hypergeometric2F1}\left(1,p+\frac{1}{2},p+\frac{3}{2},-\tan^2(e+fx)\right)}{2fp+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1, 1/2 + p, 3/2 + p, -Tan[e + f*x]^2]*Tan[e + f*x]*(b*Ta
n[e + f*x]^2)^p)/(f + 2*f*p)
```

Maple [F] time = 0.26, size = 0, normalized size = 0.

$$\int \left(b\left(\tan\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan^2\left(e+fx\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p, x)
```

3.27 $\int \left(b\tan^3(e+fx)\right)^p\,dx$

Optimal. Leaf size=57

$$\frac{\tan(e+fx)\left(b\tan^3(e+fx)\right)^p \text{Hypergeometric2F1}\left(1,\frac{1}{2}(3p+1),\frac{3(p+1)}{2},-\tan^2(e+fx)\right)}{f(3p+1)}$$

```
[Out] (Hypergeometric2F1[1, (1 + 3*p)/2, (3*(1 + p))/2, -Tan[e + f*x]^2]*Tan[e +
f*x]*(b*Tan[e + f*x]^3)^p)/(f*(1 + 3*p))
```

Rubi [A] time = 0.0387346, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3658, 3476, 364}

$$\frac{\tan(e+fx)\left(b\tan^3(e+fx)\right)^p {}_2F_1\left(1,\frac{1}{2}(3p+1);\frac{3(p+1)}{2};-\tan^2(e+fx)\right)}{f(3p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^3)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + 3*p)/2, (3*(1 + p))/2, -Tan[e + f*x]^2]*Tan[e +
f*x]*(b*Tan[e + f*x]^3)^p)/(f*(1 + 3*p))
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(b\tan^3(e+fx)\right)^p dx = \left(\tan^{-3p}(e+fx)\left(b\tan^3(e+fx)\right)^p\right)\int \tan^{3p}(e+fx)\,dx$$
$$= \frac{\left(\tan^{-3p}(e+fx)\left(b\tan^3(e+fx)\right)^p\right)\text{Subst}\left(\int \frac{x^{3p}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{{}_2F_1\left(1,\frac{1}{2}(1+3p);\frac{3(1+p)}{2};-\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^3(e+fx)\right)^p}{f(1+3p)}$$

Mathematica [A] time = 0.0407189, size = 55, normalized size = 0.96

$$\frac{\tan(e+fx)\left(b\tan^3(e+fx)\right)^p\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(3p+1),\frac{3(p+1)}{2},-\tan^2(e+fx)\right)}{3fp+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^3)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + 3*p)/2, (3*(1 + p))/2, -Tan[e + f*x]^2]*Tan[e +
f*x]*(b*Tan[e + f*x]^3)^p)/(f + 3*f*p)
```

Maple [F] time = 0.479, size = 0, normalized size = 0.

$$\int \left(b\left(\tan\left(fx+e\right)\right)^3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^3)^p,x)
```

```
[Out] int((b*tan(f*x+e)^3)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^3)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^3\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^3)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan^3\left(e+fx\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**3)**p,x)
```

```
[Out] Integral((b*tan(e + f*x)**3)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^3)^p, x)
```

3.28 $\int \left(b\tan^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=59

$$\frac{\tan(e+fx)\left(b\tan^4(e+fx)\right)^p \text{Hypergeometric2F1}\left(1,\frac{1}{2}(4p+1),\frac{1}{2}(4p+3),-\tan^2(e+fx)\right)}{f(4p+1)}$$

```
[Out] (Hypergeometric2F1[1, (1 + 4*p)/2, (3 + 4*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*Tan[e + f*x]^4)^p)/(f*(1 + 4*p))
```

Rubi [A] time = 0.0379048, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3658, 3476, 364}

$$\frac{\tan(e+fx)\left(b\tan^4(e+fx)\right)^p {}_2F_1\left(1,\frac{1}{2}(4p+1);\frac{1}{2}(4p+3);-\tan^2(e+fx)\right)}{f(4p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^4)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + 4*p)/2, (3 + 4*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*Tan[e + f*x]^4)^p)/(f*(1 + 4*p))
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(b\tan^4(e+fx)\right)^p\,dx = \left(\tan^{-4p}(e+fx)\left(b\tan^4(e+fx)\right)^p\right)\int \tan^{4p}(e+fx)\,dx$$
$$= \frac{\left(\tan^{-4p}(e+fx)\left(b\tan^4(e+fx)\right)^p\right)\text{Subst}\left(\int \frac{x^{4p}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{{}_2F_1\left(1, \frac{1}{2}(1+4p); \frac{1}{2}(3+4p); -\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^4(e+fx)\right)^p}{f(1+4p)}$$

Mathematica [A] time = 0.0311672, size = 53, normalized size = 0.9

$$\frac{\tan(e+fx)\left(b\tan^4(e+fx)\right)^p \text{Hypergeometric2F1}\left(1, 2p+\frac{1}{2}, 2p+\frac{3}{2}, -\tan^2(e+fx)\right)}{4fp+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^4)^p,x]
```

```
[Out] (Hypergeometric2F1[1, 1/2 + 2*p, 3/2 + 2*p, -Tan[e + f*x]^2]*Tan[e + f*x]*(
b*Tan[e + f*x]^4)^p)/(f + 4*f*p)
```

Maple [F] time = 0.248, size = 0, normalized size = 0.

$$\int \left(b\left(\tan\left(fx+e\right)\right)^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^4)^p,x)
```

```
[Out] int((b*tan(f*x+e)^4)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^4)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^4\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^4)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan^4\left(e+fx\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**4)**p,x)
```

```
[Out] Integral((b*tan(e + f*x)**4)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^4)^p, x)
```

3.29 $\int \left(b\tan^n(e+fx)\right)^{\frac{1}{n}}\,dx$

Optimal. Leaf size=32

$$-\frac{\cot(e+fx)\log(\cos(e+fx))\left(b\tan^n(e+fx)\right)^{\frac{1}{n}}}{f}$$

```
[Out] -((Cot[e + f*x]*Log[Cos[e + f*x]]*(b*Tan[e + f*x]^n)^n^(-1))/f)
```

Rubi [A] time = 0.0187617, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3659, 3475}

$$-\frac{\cot(e+fx)\log(\cos(e+fx))\left(b\tan^n(e+fx)\right)^{\frac{1}{n}}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^n)^n^(-1),x]
```

```
[Out] -((Cot[e + f*x]*Log[Cos[e + f*x]]*(b*Tan[e + f*x]^n)^n^(-1))/f)
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(b\tan^n(e+fx)\right)^{\frac{1}{n}}\,dx &= \left(\cot(e+fx)\left(b\tan^n(e+fx)\right)^{\frac{1}{n}}\right)\int \tan(e+fx)\,dx\\ &= -\frac{\cot(e+fx)\log(\cos(e+fx))\left(b\tan^n(e+fx)\right)^{\frac{1}{n}}}{f}\end{aligned}$$

Mathematica [A] time = 0.0237513, size = 32, normalized size = 1.

$$-\frac{\cot(e+fx)\log(\cos(e+fx))\left(b\tan^n(e+fx)\right)^{\frac{1}{n}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^n)^n^(-1),x]
```

```
[Out] -((Cot[e + f*x]*Log[Cos[e + f*x]]*(b*Tan[e + f*x]^n)^n^(-1))/f)
```

Maple [C] time = 3.234, size = 18076, normalized size = 564.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^n)^(1/n),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan\left(fx + e\right)^{n}\right)^{\left(\frac{1}{n}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^n)^(1/n), x)
```

Fricas [A] time = 1.9208, size = 59, normalized size = 1.84

$$-\frac{b^{\left(\frac{1}{n}\right)} \log\left(\frac{1}{\tan\left(fx+e\right)^{2}+1}\right)}{2\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(1/n),x, algorithm="fricas")
```

```
[Out] -1/2*b^(1/n)*log(1/(tan(f*x + e)^2 + 1))/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan^{n}\left(e + fx\right)\right)^{\frac{1}{n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**n)**(1/n),x)
```

```
[Out] Integral((b*tan(e + f*x)**n)**(1/n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^{n}\right)^{\left(\frac{1}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(1/n),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^n)^(1/n), x)
```

3.30 $\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)dx$

Optimal. Leaf size=70

$$-\frac{(a-b)\cos^5(e+fx)}{5f}+\frac{(2a-3b)\cos^3(e+fx)}{3f}-\frac{(a-3b)\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -(((a - 3*b)*Cos[e + f*x])/f) + ((2*a - 3*b)*Cos[e + f*x]^3)/(3*f) - ((a -
b)*Cos[e + f*x]^5)/(5*f) + (b*Sec[e + f*x])/f
```

Rubi [A] time = 0.0620982, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3664, 448}

$$-\frac{(a-b)\cos^5(e+fx)}{5f}+\frac{(2a-3b)\cos^3(e+fx)}{3f}-\frac{(a-3b)\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - 3*b)*Cos[e + f*x])/f) + ((2*a - 3*b)*Cos[e + f*x]^3)/(3*f) - ((a -
b)*Cos[e + f*x]^5)/(5*f) + (b*Sec[e + f*x])/f
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)dx &= \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\left(a-b+bx^2\right)}{x^6}\,dx,x,\sec(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(b+\frac{a-b}{x^6}+\frac{-2a+3b}{x^4}+\frac{a-3b}{x^2}\right)dx,x,\sec(e+fx)\right)}{f}\\ &= -\frac{(a-3b)\cos(e+fx)}{f}+\frac{(2a-3b)\cos^3(e+fx)}{3f}-\frac{(a-b)\cos^5(e+fx)}{5f}+\frac{b\sec(\ldots}{\ldots}\end{aligned}$$

Mathematica [A] time = 0.0625546, size = 104, normalized size = 1.49

$$-\frac{5a\cos(e+fx)}{8f}+\frac{5a\cos(3(e+fx))}{48f}-\frac{a\cos(5(e+fx))}{80f}+\frac{19b\cos(e+fx)}{8f}-\frac{3b\cos(3(e+fx))}{16f}+\frac{b\cos(5(e+fx))}{80f}+\frac{b\,s\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-5*a*Cos[e + f*x])/(8*f) + (19*b*Cos[e + f*x])/(8*f) + (5*a*Cos[3*(e + f*x
)])/(48*f) - (3*b*Cos[3*(e + f*x)])/(16*f) - (a*Cos[5*(e + f*x)])/(80*f) +
(b*Cos[5*(e + f*x)])/(80*f) + (b*Sec[e + f*x])/f
```

Maple [A] time = 0.105, size = 92, normalized size = 1.3

$$\frac{1}{f}\left(-\frac{\cos\left(fx+e\right)a}{5}\left(\frac{8}{3}+\left(\sin\left(fx+e\right)\right)^{4}+\frac{4\left(\sin\left(fx+e\right)\right)^{2}}{3}\right)+b\left(\frac{\left(\sin\left(fx+e\right)\right)^{8}}{\cos\left(fx+e\right)}+\left(\frac{16}{5}+\left(\sin\left(fx+e\right)\right)^{6}+\frac{6\left(\sin\right.}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*(-1/5*a*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+b*(sin(f*x+e)^8/
cos(f*x+e)+(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)
))
```

Maxima [A] time = 1.05209, size = 84, normalized size = 1.2

$$-\frac{3\,(a-b)\cos\left(fx+e\right)^{5}-5\,(2\,a-3\,b)\cos\left(fx+e\right)^{3}+15\,(a-3\,b)\cos\left(fx+e\right)-\frac{15\,b}{\cos\left(fx+e\right)}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(3*(a - b)*cos(f*x + e)^5 - 5*(2*a - 3*b)*cos(f*x + e)^3 + 15*(a - 3*
b)*cos(f*x + e) - 15*b/cos(f*x + e))/f
```

Fricas [A] time = 1.99045, size = 161, normalized size = 2.3

$$-\frac{3\,(a-b)\cos\left(fx+e\right)^{6}-5\,(2\,a-3\,b)\cos\left(fx+e\right)^{4}+15\,(a-3\,b)\cos\left(fx+e\right)^{2}-15\,b}{15\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*(a - b)*cos(f*x + e)^6 - 5*(2*a - 3*b)*cos(f*x + e)^4 + 15*(a - 3*
b)*cos(f*x + e)^2 - 15*b)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.31 $\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)dx$

Optimal. Leaf size=48

$$\frac{(a-b)\cos^3(e+fx)}{3f}-\frac{(a-2b)\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -(((a - 2*b)*Cos[e + f*x])/f) + ((a - b)*Cos[e + f*x]^3)/(3*f) + (b*Sec[e +
 f*x])/f
```

Rubi [A] time = 0.0470021, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3664, 448}

$$\frac{(a-b)\cos^3(e+fx)}{3f}-\frac{(a-2b)\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - 2*b)*Cos[e + f*x])/f) + ((a - b)*Cos[e + f*x]^3)/(3*f) + (b*Sec[e +
 f*x])/f
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)dx &= \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)\left(a-b+bx^2\right)}{x^4}\,dx, x, \sec(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(b+\frac{-a+b}{x^4}+\frac{a-2b}{x^2}\right)dx, x, \sec(e+fx)\right)}{f}\\ &= -\frac{(a-2b)\cos(e+fx)}{f}+\frac{(a-b)\cos^3(e+fx)}{3f}+\frac{b\sec(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.0494669, size = 72, normalized size = 1.5

$$-\frac{3a\cos(e+fx)}{4f}+\frac{a\cos(3(e+fx))}{12f}+\frac{7b\cos(e+fx)}{4f}-\frac{b\cos(3(e+fx))}{12f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-3*a*Cos[e + f*x])/(4*f) + (7*b*Cos[e + f*x])/(4*f) + (a*Cos[3*(e + f*x)])
/(12*f) - (b*Cos[3*(e + f*x)])/(12*f) + (b*Sec[e + f*x])/f
```

Maple [A] time = 0.063, size = 72, normalized size = 1.5

$$\frac{1}{f}\left(-\frac{a\left(2+\left(\sin\left(fx+e\right)\right)^{2}\right)\cos\left(fx+e\right)}{3}+b\left(\frac{\left(\sin\left(fx+e\right)\right)^{6}}{\cos\left(fx+e\right)}+\left(\frac{8}{3}+\left(\sin\left(fx+e\right)\right)^{4}+\frac{4\left(\sin\left(fx+e\right)\right)^{2}}{3}\right)\cos\left(fx+e\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*(-1/3*a*(2+sin(f*x+e)^2)*cos(f*x+e)+b*(sin(f*x+e)^6/cos(f*x+e)+(8/3+sin
(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)))
```

Maxima [A] time = 1.06492, size = 59, normalized size = 1.23

$$\frac{(a-b)\cos\left(fx+e\right)^{3}-3\,(a-2\,b)\cos\left(fx+e\right)+\frac{3\,b}{\cos\left(fx+e\right)}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*((a - b)*cos(f*x + e)^3 - 3*(a - 2*b)*cos(f*x + e) + 3*b/cos(f*x + e))/
f
```

Fricas [A] time = 1.93285, size = 111, normalized size = 2.31

$$\frac{(a-b)\cos\left(fx+e\right)^{4}-3\,(a-2\,b)\cos\left(fx+e\right)^{2}+3\,b}{3\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/3*((a - b)*cos(f*x + e)^4 - 3*(a - 2*b)*cos(f*x + e)^2 + 3*b)/(f*cos(f*x
+ e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan^{2}\left(e+fx\right)\right)\sin^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*sin(e + f*x)**3, x)
```

Giac [A] time = 1.48716, size = 103, normalized size = 2.15

$$\frac{b}{f\cos\left(fx+e\right)}+\frac{af^{5}\cos\left(fx+e\right)^{3}-bf^{5}\cos\left(fx+e\right)^{3}-3\,af^{5}\cos\left(fx+e\right)+6\,bf^{5}\cos\left(fx+e\right)}{3\,f^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] b/(f*cos(f*x + e)) + 1/3*(a*f^5*cos(f*x + e)^3 - b*f^5*cos(f*x + e)^3 - 3*a
*f^5*cos(f*x + e) + 6*b*f^5*cos(f*x + e))/f^6
```

3.32 $\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=28

$$\frac{b\sec(e+fx)}{f}-\frac{(a-b)\cos(e+fx)}{f}$$

```
[Out] -(((a - b)*Cos[e + f*x])/f) + (b*Sec[e + f*x])/f
```

Rubi [A] time = 0.0261331, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3664, 14}

$$\frac{b\sec(e+fx)}{f}-\frac{(a-b)\cos(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - b)*Cos[e + f*x])/f) + (b*Sec[e + f*x])/f
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{a-b+bx^2}{x^2}\,dx,x,\sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(b+\frac{a-b}{x^2}\right)dx,x,\sec(e+fx)\right)}{f}\\
&= -\frac{(a-b)\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}
\end{aligned}$$

Mathematica [A] time = 0.0435174, size = 46, normalized size = 1.64

$$\frac{a\sin(e)\sin(fx)}{f}-\frac{a\cos(e)\cos(fx)}{f}+\frac{b\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*Cos[e]*Cos[f*x])/f) + (b*Cos[e + f*x])/f + (b*Sec[e + f*x])/f + (a*Sin
[e]*Sin[f*x])/f
```

Maple [A] time = 0.033, size = 52, normalized size = 1.9

$$\frac{1}{f}\left(-\cos\left(fx+e\right)a+b\left(\frac{\left(\sin\left(fx+e\right)\right)^{4}}{\cos\left(fx+e\right)}+\left(2+\left(\sin\left(fx+e\right)\right)^{2}\right)\cos\left(fx+e\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*(-cos(f*x+e)*a+b*(sin(f*x+e)^4/cos(f*x+e)+(2+sin(f*x+e)^2)*cos(f*x+e)))
```

Maxima [A] time = 0.950032, size = 42, normalized size = 1.5

$$\frac{b\left(\frac{1}{\cos\left(fx+e\right)}+\cos\left(fx+e\right)\right)-a\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] (b*(1/cos(f*x + e) + cos(f*x + e)) - a*cos(f*x + e))/f
```

Fricas [A] time = 1.79394, size = 65, normalized size = 2.32

$$-\frac{(a-b)\cos\left(fx+e\right)^{2}-b}{f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -((a - b)*cos(f*x + e)^2 - b)/(f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan^{2}\left(e+fx\right)\right)\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*sin(e + f*x), x)
```

Giac [A] time = 1.42117, size = 55, normalized size = 1.96

$$b\left(\frac{\cos\left(fx+e\right)}{f}+\frac{1}{f\cos\left(fx+e\right)}\right)-\frac{a\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] b*(cos(f*x + e)/f + 1/(f*cos(f*x + e))) - a*cos(f*x + e)/f
```

3.33 $\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=25

$$\frac{b\sec(e+fx)}{f}-\frac{a\tanh^{-1}(\cos(e+fx))}{f}$$

```
[Out] -((a*ArcTanh[Cos[e + f*x]])/f) + (b*Sec[e + f*x])/f
```

Rubi [A] time = 0.0277562, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3664, 388, 207}

$$\frac{b\sec(e+fx)}{f}-\frac{a\tanh^{-1}(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*ArcTanh[Cos[e + f*x]])/f) + (b*Sec[e + f*x])/f
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{a-b+bx^2}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{f}\\ &= \frac{b\sec(e+fx)}{f}+\frac{a\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{f}\\ &= -\frac{a\tanh^{-1}(\cos(e+fx))}{f}+\frac{b\sec(e+fx)}{f}\end{aligned}$$

Mathematica [B] time = 0.0292559, size = 51, normalized size = 2.04

$$\frac{a\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}-\frac{a\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*Log[Cos[e/2 + (f*x)/2]])/f) + (a*Log[Sin[e/2 + (f*x)/2]])/f + (b*Sec[e
 + f*x])/f
```

Maple [A] time = 0.039, size = 36, normalized size = 1.4

$$\frac{b}{f\cos\left(fx+e\right)}+\frac{a\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*b/cos(f*x+e)+1/f*a*ln(csc(f*x+e)-cot(f*x+e))
```

Maxima [A] time = 0.988657, size = 54, normalized size = 2.16

$$-\frac{a\log\left(\cos\left(fx+e\right)+1\right)-a\log\left(\cos\left(fx+e\right)-1\right)-\frac{2b}{\cos\left(fx+e\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a*log(cos(f*x + e) + 1) - a*log(cos(f*x + e) - 1) - 2*b/cos(f*x + e))
/f
```

Fricas [B] time = 1.96554, size = 162, normalized size = 6.48

$$-\frac{a\cos\left(fx+e\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-a\cos\left(fx+e\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-2b}{2f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(a*cos(f*x + e)*log(1/2*cos(f*x + e) + 1/2) - a*cos(f*x + e)*log(-1/2*
cos(f*x + e) + 1/2) - 2*b)/(f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\tan^2\left(e+fx\right)\right)\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*csc(e + f*x), x)
```

Giac [B] time = 1.38698, size = 80, normalized size = 3.2

$$\frac{a\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)+\frac{4b}{\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*(a*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 4*b/((cos(f*x + e) - 1
)/(cos(f*x + e) + 1) + 1))/f
```

3.34 $\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=51

$$-\frac{(a+2b)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{a\cot(e+fx)\csc(e+fx)}{2f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -((a + 2*b)*ArcTanh[Cos[e + f*x]])/(2*f) - (a*Cot[e + f*x]*Csc[e + f*x])/(2
*f) + (b*Sec[e + f*x])/f
```

Rubi [A] time = 0.052528, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3664, 455, 388, 207}

$$-\frac{(a+2b)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{a\cot(e+fx)\csc(e+fx)}{2f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a + 2*b)*ArcTanh[Cos[e + f*x]])/(2*f) - (a*Cot[e + f*x]*Csc[e + f*x])/(2
*f) + (b*Sec[e + f*x])/f
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)dx = \frac{\text{Subst}\left(\int \frac{x^2(a-b+bx^2)}{(-1+x^2)^2}dx, x, \sec(e+fx)\right)}{f}$$
$$= -\frac{a\cot(e+fx)\csc(e+fx)}{2f} - \frac{\text{Subst}\left(\int \frac{-a-2bx^2}{-1+x^2}dx, x, \sec(e+fx)\right)}{2f}$$
$$= -\frac{a\cot(e+fx)\csc(e+fx)}{2f} + \frac{b\sec(e+fx)}{f} + \frac{(a+2b)\,\text{Subst}\left(\int \frac{1}{-1+x^2}dx, x\right.}{2f}$$
$$= -\frac{(a+2b)\tanh^{-1}(\cos(e+fx))}{2f} - \frac{a\cot(e+fx)\csc(e+fx)}{2f} + \frac{b\sec(e+fx)}{f}$$

Mathematica [B] time = 0.0448324, size = 123, normalized size = 2.41

$$-\frac{a\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f} + \frac{a\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f} + \frac{a\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f} - \frac{a\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f} + \frac{b\sec(e+fx)}{f} + \frac{b\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(a*Csc[(e + f*x)/2]^2)/(8*f) - (a*Log[Cos[(e + f*x)/2]])/(2*f) - (b*Log[Co
s[(e + f*x)/2]])/f + (a*Log[Sin[(e + f*x)/2]])/(2*f) + (b*Log[Sin[(e + f*x)
/2]])/f + (a*Sec[(e + f*x)/2]^2)/(8*f) + (b*Sec[e + f*x])/f
```

Maple [A] time = 0.08, size = 76, normalized size = 1.5

$$\frac{b}{f\cos\left(fx+e\right)} + \frac{b\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f} - \frac{\cot\left(fx+e\right)a\csc\left(fx+e\right)}{2f} + \frac{a\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*b/cos(f*x+e)+1/f*b*ln(csc(f*x+e)-cot(f*x+e))-1/2*a*cot(f*x+e)*csc(f*x+e
)/f+1/2/f*a*ln(csc(f*x+e)-cot(f*x+e))
```

Maxima [A] time = 1.03911, size = 103, normalized size = 2.02

$$-\frac{(a+2b)\log\left(\cos\left(fx+e\right)+1\right)-(a+2b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left((a+2b)\cos\left(fx+e\right)^2-2b\right)}{\cos\left(fx+e\right)^3-\cos\left(fx+e\right)}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/4*((a + 2*b)*log(cos(f*x + e) + 1) - (a + 2*b)*log(cos(f*x + e) - 1) - 2
*((a + 2*b)*cos(f*x + e)^2 - 2*b)/(cos(f*x + e)^3 - cos(f*x + e)))/f
```

Fricas [B] time = 1.96736, size = 325, normalized size = 6.37

$$\frac{2\,(a+2\,b)\cos\left(fx+e\right)^{2}-\left((a+2\,b)\cos\left(fx+e\right)^{3}-(a+2\,b)\cos\left(fx+e\right)\right)\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)+\left((a+2\,b)\cos\left(fx\right.\right.}{4\left(f\cos\left(fx+e\right)^{3}-f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a + 2*b)*cos(f*x + e)^2 - ((a + 2*b)*cos(f*x + e)^3 - (a + 2*b)*cos
(f*x + e))*log(1/2*cos(f*x + e) + 1/2) + ((a + 2*b)*cos(f*x + e)^3 - (a + 2
*b)*cos(f*x + e))*log(-1/2*cos(f*x + e) + 1/2) - 4*b)/(f*cos(f*x + e)^3 - f
*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan^{2}\left(e+fx\right)\right)\csc^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*csc(e + f*x)**3, x)
```

Giac [B] time = 1.45158, size = 246, normalized size = 4.82

$$\frac{2\,(a+2\,b)\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)-\frac{a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{a+\frac{14\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{a\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{2\,b\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}}{\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+\frac{\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/8*(2*(a + 2*b)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - a*(cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) + (a + 14*b*(cos(f*x + e) - 1)/(cos(f*x + e) +
1) - a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 2*b*(cos(f*x + e) - 1)^2
/(cos(f*x + e) + 1)^2)/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + (cos(f*x +
e) - 1)^2/(cos(f*x + e) + 1)^2))/f
```

3.35 $\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)dx$

Optimal. Leaf size=79

$$-\frac{3(a+4b)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{(5a+4b)\cot(e+fx)\csc(e+fx)}{8f}-\frac{a\cot^3(e+fx)\csc(e+fx)}{4f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] (-3*(a + 4*b)*ArcTanh[Cos[e + f*x]])/(8*f) - ((5*a + 4*b)*Cot[e + f*x]*Csc[
e + f*x])/(8*f) - (a*Cot[e + f*x]^3*Csc[e + f*x])/(4*f) + (b*Sec[e + f*x])/
f
```

Rubi [A] time = 0.0701404, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3664, 455, 1157, 388, 207}

$$-\frac{3(a+4b)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{(5a+4b)\cot(e+fx)\csc(e+fx)}{8f}-\frac{a\cot^3(e+fx)\csc(e+fx)}{4f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```


```
[Out] (-3*(a + 4*b)*ArcTanh[Cos[e + f*x]])/(8*f) - ((5*a + 4*b)*Cot[e + f*x]*Csc[
e + f*x])/(8*f) - (a*Cot[e + f*x]^3*Csc[e + f*x])/(4*f) + (b*Sec[e + f*x])/
f
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a-b+bx^2\right)}{\left(-1+x^2\right)^3}dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{a\cot^3(e+fx)\csc(e+fx)}{4f} - \frac{\text{Subst}\left(\int \frac{-a-4ax^2-4bx^4}{\left(-1+x^2\right)^2}dx, x, \sec(e+fx)\right)}{4f}$$

$$= -\frac{(5a+4b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{a\cot^3(e+fx)\csc(e+fx)}{4f} - \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{(5a+4b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{a\cot^3(e+fx)\csc(e+fx)}{4f} + \frac{b\sec(e+f}{f}$$

$$= -\frac{3(a+4b)\tanh^{-1}(\cos(e+fx))}{8f} - \frac{(5a+4b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{a\cot^3(}{}$$

Mathematica [B] time = 6.05595, size = 276, normalized size = 3.49

$$-\frac{a\csc^4\left(\frac{1}{2}(e+fx)\right)}{64f} - \frac{3a\csc^2\left(\frac{1}{2}(e+fx)\right)}{32f} + \frac{a\sec^4\left(\frac{1}{2}(e+fx)\right)}{64f} + \frac{3a\sec^2\left(\frac{1}{2}(e+fx)\right)}{32f} + \frac{3a\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{8f} - \frac{3a\,\mathrm{l}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-3*a*Csc[(e + f*x)/2]^2)/(32*f) - (b*Csc[(e + f*x)/2]^2)/(8*f) - (a*Csc[(e
 + f*x)/2]^4)/(64*f) - (3*a*Log[Cos[(e + f*x)/2]])/(8*f) - (3*b*Log[Cos[(e
+ f*x)/2]])/(2*f) + (3*a*Log[Sin[(e + f*x)/2]])/(8*f) + (3*b*Log[Sin[(e + f
*x)/2]])/(2*f) + (3*a*Sec[(e + f*x)/2]^2)/(32*f) + (b*Sec[(e + f*x)/2]^2)/(
8*f) + (a*Sec[(e + f*x)/2]^4)/(64*f) + (b*Sin[(e + f*x)/2])/(f*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])) - (b*Sin[(e + f*x)/2])/(f*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]))
```

Maple [A] time = 0.074, size = 120, normalized size = 1.5

$$-\frac{b}{2f\left(\sin\left(fx+e\right)\right)^2\cos\left(fx+e\right)} + \frac{3b}{2f\cos\left(fx+e\right)} + \frac{3b\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{2f} - \frac{\cot\left(fx+e\right)a\left(\csc\left(fx+e\right)\right.}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/2/f*b/sin(f*x+e)^2/cos(f*x+e)+3/2/f*b/cos(f*x+e)+3/2/f*b*ln(csc(f*x+e)-c
ot(f*x+e))-1/4/f*a*cot(f*x+e)*csc(f*x+e)^3-3/8*a*cot(f*x+e)*csc(f*x+e)/f+3/
8/f*a*ln(csc(f*x+e)-cot(f*x+e))
```

Maxima [A] time = 1.05649, size = 136, normalized size = 1.72

$$-\frac{3\,(a+4\,b)\log\left(\cos\left(fx+e\right)+1\right)-3\,(a+4\,b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\,(a+4\,b)\cos\left(fx+e\right)^{4}-5\,(a+4\,b)\cos\left(fx+e\right)^{2}+8\,b\right)}{\cos\left(fx+e\right)^{5}-2\,\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/16*(3*(a + 4*b)*log(cos(f*x + e) + 1) - 3*(a + 4*b)*log(cos(f*x + e) - 1
) - 2*(3*(a + 4*b)*cos(f*x + e)^4 - 5*(a + 4*b)*cos(f*x + e)^2 + 8*b)/(cos(
f*x + e)^5 - 2*cos(f*x + e)^3 + cos(f*x + e)))/f
```

Fricas [B] time = 2.08742, size = 481, normalized size = 6.09

$$\frac{6\,(a+4\,b)\cos\left(fx+e\right)^{4}-10\,(a+4\,b)\cos\left(fx+e\right)^{2}-3\left((a+4\,b)\cos\left(fx+e\right)^{5}-2\,(a+4\,b)\cos\left(fx+e\right)^{3}+(a+4\,b)\cdots\right.}{16\left(f\cos\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(6*(a + 4*b)*cos(f*x + e)^4 - 10*(a + 4*b)*cos(f*x + e)^2 - 3*((a + 4*
b)*cos(f*x + e)^5 - 2*(a + 4*b)*cos(f*x + e)^3 + (a + 4*b)*cos(f*x + e))*lo
g(1/2*cos(f*x + e) + 1/2) + 3*((a + 4*b)*cos(f*x + e)^5 - 2*(a + 4*b)*cos(f
*x + e)^3 + (a + 4*b)*cos(f*x + e))*log(-1/2*cos(f*x + e) + 1/2) + 16*b)/(f
*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.56772, size = 346, normalized size = 4.38

$$\frac{12\,(a+4\,b)\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)-\frac{\left(a-\frac{8\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{8\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{18\,a\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{72\,b\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}\right)\left(\cos\left(fx+e\right)+1\right)^2}{\left(\cos\left(fx+e\right)-1\right)^2}-\frac{8\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/64*(12*(a + 4*b)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - (a - 8*a*(
cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 8*b*(cos(f*x + e) - 1)/(cos(f*x + e)
 + 1) + 18*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 72*b*(cos(f*x + e)
 - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x + e) - 1)^2 - 8
*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 8*b*(cos(f*x + e) - 1)/(cos(f*x
+ e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 128*b/((cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) + 1))/f
```

3.36 $\int \sin^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=102

$$-\frac{(a-b)\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{(13a-19b)\sin(e+fx)\cos^3(e+fx)}{24f}-\frac{(11a-29b)\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}x$$

```
[Out] (5*(a - 7*b)*x)/16 - ((11*a - 29*b)*Cos[e + f*x]*Sin[e + f*x])/(16*f) + ((1
3*a - 19*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - ((a - b)*Cos[e + f*x]^5*S
in[e + f*x])/(6*f) + (b*Tan[e + f*x])/f
```

Rubi [A] time = 0.118024, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3663, 455, 1814, 1157, 388, 203}

$$-\frac{(a-b)\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{(13a-19b)\sin(e+fx)\cos^3(e+fx)}{24f}-\frac{(11a-29b)\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}x$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (5*(a - 7*b)*x)/16 - ((11*a - 29*b)*Cos[e + f*x]*Sin[e + f*x])/(16*f) + ((1
3*a - 19*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - ((a - b)*Cos[e + f*x]^5*S
in[e + f*x])/(6*f) + (b*Tan[e + f*x])/f
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
```

```
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sin^6(e+fx)\left(a+b\tan^2(e+fx)\right)dx = \frac{\text{Subst}\left(\int \frac{x^6\left(a+bx^2\right)}{\left(1+x^2\right)^4}dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\cos^5(e+fx)\sin(e+fx)}{6f} - \frac{\text{Subst}\left(\int \frac{-a+b+6(a-b)x^2-6(a-b)x^4-6bx^6}{\left(1+x^2\right)^3}dx, x, \text{t}\right.}{6f}$$

$$= \frac{(13a-19b)\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{(a-b)\cos^5(e+fx)\sin(e+fx)}{6f} + \frac{\text{Sub}}{}$$

$$= -\frac{(11a-29b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(13a-19b)\cos^3(e+fx)\sin(e+fx)}{24f} -$$

$$= -\frac{(11a-29b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(13a-19b)\cos^3(e+fx)\sin(e+fx)}{24f} -$$

$$= \frac{5}{16}(a-7b)x - \frac{(11a-29b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(13a-19b)\cos^3(e+fx)\text{s}}{24f}$$

Mathematica [A] time = 0.34246, size = 89, normalized size = 0.87

$$\frac{(141b-45a)\sin(2(e+fx))+3(3a-5b)\sin(4(e+fx))-a\sin(6(e+fx))+60ae+60afx+b\sin(6(e+fx))+192b\tan(}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (60*a*e - 420*b*e + 60*a*f*x - 420*b*f*x + (-45*a + 141*b)*Sin[2*(e + f*x)]
 + 3*(3*a - 5*b)*Sin[4*(e + f*x)] - a*Sin[6*(e + f*x)] + b*Sin[6*(e + f*x)]
 + 192*b*Tan[e + f*x])/(192*f)
```

Maple [A] time = 0.091, size = 122, normalized size = 1.2

$$\frac{1}{f}\left(a\left(-\frac{\cos\left(fx+e\right)}{6}\left(\left(\sin\left(fx+e\right)\right)^{5}+\frac{5\left(\sin\left(fx+e\right)\right)^{3}}{4}+\frac{15\sin\left(fx+e\right)}{8}\right)+\frac{5fx}{16}+\frac{5e}{16}\right)+b\left(\frac{\left(\sin\left(fx+e\right)\right)^{9}}{\cos\left(fx+e\right)}+\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*(a*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/1
6*f*x+5/16*e)+b*(sin(f*x+e)^9/cos(f*x+e)+(sin(f*x+e)^7+7/6*sin(f*x+e)^5+35/
24*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)-35/16*f*x-35/16*e))
```

Maxima [A] time = 1.44308, size = 150, normalized size = 1.47

$$\frac{15\left(fx+e\right)(a-7b)+48b\tan\left(fx+e\right)-\frac{3(11a-29b)\tan\left(fx+e\right)^{5}+8(5a-17b)\tan\left(fx+e\right)^{3}+3(5a-19b)\tan\left(fx+e\right)}{\tan\left(fx+e\right)^{6}+3\tan\left(fx+e\right)^{4}+3\tan\left(fx+e\right)^{2}+1}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/48*(15*(f*x + e)*(a - 7*b) + 48*b*tan(f*x + e) - (3*(11*a - 29*b)*tan(f*x
 + e)^5 + 8*(5*a - 17*b)*tan(f*x + e)^3 + 3*(5*a - 19*b)*tan(f*x + e))/(tan
(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

Fricas [A] time = 1.9969, size = 230, normalized size = 2.25

$$\frac{15(a-7b)fx\cos\left(fx+e\right)-\left(8(a-b)\cos\left(fx+e\right)^{6}-2(13a-19b)\cos\left(fx+e\right)^{4}+3(11a-29b)\cos\left(fx+e\right)^{2}-48\right.}{48f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/48*(15*(a - 7*b)*f*x*cos(f*x + e) - (8*(a - b)*cos(f*x + e)^6 - 2*(13*a -
 19*b)*cos(f*x + e)^4 + 3*(11*a - 29*b)*cos(f*x + e)^2 - 48*b)*sin(f*x + e)
)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.02839, size = 10661, normalized size = 104.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/192*(21*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*
tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^7*tan(e)^7 + 60*a*f*x*t
an(f*x)^7*tan(e)^7 - 420*b*f*x*tan(f*x)^7*tan(e)^7 + 21*pi*b*sgn(-2*tan(f*x
)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^7*tan(e)
^7 + 63*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*ta
n(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^7*tan(e)^5 - 21*pi*b*sgn(
2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 +
 2*tan(f*x) - 2*tan(e))*tan(f*x)^6*tan(e)^6 + 63*pi*b*sgn(2*tan(f*x)^2*tan(
e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*t
an(e))*tan(f*x)^5*tan(e)^7 + 42*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(
e) - 1))*tan(f*x)^7*tan(e)^7 - 42*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*t
an(e) + 1))*tan(f*x)^7*tan(e)^7 + 180*a*f*x*tan(f*x)^7*tan(e)^5 - 1260*b*f*
x*tan(f*x)^7*tan(e)^5 + 63*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e
)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^7*tan(e)^5 - 60*a*f*x*tan(f*x)^6*tan(
e)^6 + 420*b*f*x*tan(f*x)^6*tan(e)^6 - 21*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2
*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^6*tan(e)^6 + 180*a*f*x
*tan(f*x)^5*tan(e)^7 - 1260*b*f*x*tan(f*x)^5*tan(e)^7 + 63*pi*b*sgn(-2*tan(
f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^5*tan
(e)^7 + 63*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2
*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^7*tan(e)^3 - 63*pi*b*s
gn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^
2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^6*tan(e)^4 + 189*pi*b*sgn(2*tan(f*x)^2*
tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) -
 2*tan(e))*tan(f*x)^5*tan(e)^5 + 126*b*arctan((tan(f*x) + tan(e))/(tan(f*x)
*tan(e) - 1))*tan(f*x)^7*tan(e)^5 - 126*b*arctan(-(tan(f*x) - tan(e))/(tan(
f*x)*tan(e) + 1))*tan(f*x)^7*tan(e)^5 - 63*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 -
 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))
*tan(f*x)^4*tan(e)^6 - 42*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1
))*tan(f*x)^6*tan(e)^6 + 42*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e)
+ 1))*tan(f*x)^6*tan(e)^6 + 60*a*tan(f*x)^7*tan(e)^6 - 420*b*tan(f*x)^7*tan
(e)^6 + 63*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2
*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^3*tan(e)^7 + 126*b*arc
tan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^5*tan(e)^7 - 126*b*
arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)^5*tan(e)^7 + 60
*a*tan(f*x)^6*tan(e)^7 - 420*b*tan(f*x)^6*tan(e)^7 + 180*a*f*x*tan(f*x)^7*t
an(e)^3 - 1260*b*f*x*tan(f*x)^7*tan(e)^3 + 63*pi*b*sgn(-2*tan(f*x)^2*tan(e)
 + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^7*tan(e)^3 - 180*a
*f*x*tan(f*x)^6*tan(e)^4 + 1260*b*f*x*tan(f*x)^6*tan(e)^4 - 63*pi*b*sgn(-2*
tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^6
*tan(e)^4 + 540*a*f*x*tan(f*x)^5*tan(e)^5 - 3780*b*f*x*tan(f*x)^5*tan(e)^5
+ 189*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*
tan(e))*tan(f*x)^5*tan(e)^5 - 180*a*f*x*tan(f*x)^4*tan(e)^6 + 1260*b*f*x*ta
n(f*x)^4*tan(e)^6 - 63*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2
+ 2*tan(f*x) - 2*tan(e))*tan(f*x)^4*tan(e)^6 + 180*a*f*x*tan(f*x)^3*tan(e)^
7 - 1260*b*f*x*tan(f*x)^3*tan(e)^7 + 63*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*t
an(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^3*tan(e)^7 + 21*pi*b*sgn
(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2
+ 2*tan(f*x) - 2*tan(e))*tan(f*x)^7*tan(e) - 63*pi*b*sgn(2*tan(f*x)^2*tan(e
)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*ta
n(e))*tan(f*x)^6*tan(e)^2 + 189*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*
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tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^5
*tan(e)^3 + 126*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x
)^7*tan(e)^3 - 126*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan
(f*x)^7*tan(e)^3 - 189*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^
2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^4*tan(e)^4
 - 126*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^6*tan(e
)^4 + 126*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)^6*t
an(e)^4 + 160*a*tan(f*x)^7*tan(e)^4 - 1120*b*tan(f*x)^7*tan(e)^4 + 189*pi*b
*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e
)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^3*tan(e)^5 + 378*b*arctan((tan(f*x) +
 tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^5*tan(e)^5 - 378*b*arctan(-(tan(f*
x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)^5*tan(e)^5 + 120*a*tan(f*x)^6*
tan(e)^5 - 840*b*tan(f*x)^6*tan(e)^5 - 63*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 -
2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*
tan(f*x)^2*tan(e)^6 - 126*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1
))*tan(f*x)^4*tan(e)^6 + 126*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e)
 + 1))*tan(f*x)^4*tan(e)^6 + 120*a*tan(f*x)^5*tan(e)^6 - 840*b*tan(f*x)^5*t
an(e)^6 + 21*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) +
 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)*tan(e)^7 + 126*b*arc
tan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^3*tan(e)^7 - 126*b*
arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^7 + 16
0*a*tan(f*x)^4*tan(e)^7 - 1120*b*tan(f*x)^4*tan(e)^7 + 60*a*f*x*tan(f*x)^7*
tan(e) - 420*b*f*x*tan(f*x)^7*tan(e) + 21*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2
*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^7*tan(e) - 180*a*f*x*t
an(f*x)^6*tan(e)^2 + 1260*b*f*x*tan(f*x)^6*tan(e)^2 - 63*pi*b*sgn(-2*tan(f*
x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^6*tan(e
)^2 + 540*a*f*x*tan(f*x)^5*tan(e)^3 - 3780*b*f*x*tan(f*x)^5*tan(e)^3 + 189*
pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e)
)*tan(f*x)^5*tan(e)^3 - 540*a*f*x*tan(f*x)^4*tan(e)^4 + 3780*b*f*x*tan(f*x)
^4*tan(e)^4 - 189*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*t
an(f*x) - 2*tan(e))*tan(f*x)^4*tan(e)^4 + 540*a*f*x*tan(f*x)^3*tan(e)^5 - 3
780*b*f*x*tan(f*x)^3*tan(e)^5 + 189*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f
*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^3*tan(e)^5 - 180*a*f*x*tan(f
*x)^2*tan(e)^6 + 1260*b*f*x*tan(f*x)^2*tan(e)^6 - 63*pi*b*sgn(-2*tan(f*x)^2
*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^2*tan(e)^6
+ 60*a*f*x*tan(f*x)*tan(e)^7 - 420*b*f*x*tan(f*x)*tan(e)^7 + 21*pi*b*sgn(-2
*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)*
tan(e)^7 - 21*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e)
+ 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^6 + 63*pi*b*sgn(2*t
an(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*
tan(f*x) - 2*tan(e))*tan(f*x)^5*tan(e) + 42*b*arctan((tan(f*x) + tan(e))/(t
an(f*x)*tan(e) - 1))*tan(f*x)^7*tan(e) - 42*b*arctan(-(tan(f*x) - tan(e))/(
tan(f*x)*tan(e) + 1))*tan(f*x)^7*tan(e) - 189*pi*b*sgn(2*tan(f*x)^2*tan(e)^
2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(
e))*tan(f*x)^4*tan(e)^2 - 126*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e)
 - 1))*tan(f*x)^6*tan(e)^2 + 126*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*ta
n(e) + 1))*tan(f*x)^6*tan(e)^2 + 132*a*tan(f*x)^7*tan(e)^2 - 924*b*tan(f*x)
^7*tan(e)^2 + 189*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan
(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^3*tan(e)^3 + 37
8*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^5*tan(e)^3 -
 378*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)^5*tan(e)
^3 + 20*a*tan(f*x)^6*tan(e)^3 - 140*b*tan(f*x)^6*tan(e)^3 - 189*pi*b*sgn(2*
tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2
*tan(f*x) - 2*tan(e))*tan(f*x)^2*tan(e)^4 - 378*b*arctan((tan(f*x) + tan(e)
)/(tan(f*x)*tan(e) - 1))*tan(f*x)^4*tan(e)^4 + 378*b*arctan(-(tan(f*x) - ta
n(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(e)^4 + 300*a*tan(f*x)^5*tan(e)^
4 - 2100*b*tan(f*x)^5*tan(e)^4 + 63*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn
(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*
x)*tan(e)^5 + 378*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f
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*x)^3*tan(e)^5 - 378*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*t
an(f*x)^3*tan(e)^5 + 300*a*tan(f*x)^4*tan(e)^5 - 2100*b*tan(f*x)^4*tan(e)^5
 - 21*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(
f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(e)^6 - 126*b*arctan((tan(f*x) +
tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^2*tan(e)^6 + 126*b*arctan(-(tan(f*x
) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^6 + 20*a*tan(f*x)^3*ta
n(e)^6 - 140*b*tan(f*x)^3*tan(e)^6 + 42*b*arctan((tan(f*x) + tan(e))/(tan(f
*x)*tan(e) - 1))*tan(f*x)*tan(e)^7 - 42*b*arctan(-(tan(f*x) - tan(e))/(tan(
f*x)*tan(e) + 1))*tan(f*x)*tan(e)^7 + 132*a*tan(f*x)^2*tan(e)^7 - 924*b*tan
(f*x)^2*tan(e)^7 - 60*a*f*x*tan(f*x)^6 + 420*b*f*x*tan(f*x)^6 - 21*pi*b*sgn
(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*
x)^6 + 180*a*f*x*tan(f*x)^5*tan(e) - 1260*b*f*x*tan(f*x)^5*tan(e) + 63*pi*b
*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*ta
n(f*x)^5*tan(e) - 540*a*f*x*tan(f*x)^4*tan(e)^2 + 3780*b*f*x*tan(f*x)^4*tan
(e)^2 - 189*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x
) - 2*tan(e))*tan(f*x)^4*tan(e)^2 + 540*a*f*x*tan(f*x)^3*tan(e)^3 - 3780*b*
f*x*tan(f*x)^3*tan(e)^3 + 189*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*ta
n(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^3*tan(e)^3 - 540*a*f*x*tan(f*x)^2*
tan(e)^4 + 3780*b*f*x*tan(f*x)^2*tan(e)^4 - 189*pi*b*sgn(-2*tan(f*x)^2*tan(
e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^2*tan(e)^4 + 180
*a*f*x*tan(f*x)*tan(e)^5 - 1260*b*f*x*tan(f*x)*tan(e)^5 + 63*pi*b*sgn(-2*ta
n(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)*tan
(e)^5 - 60*a*f*x*tan(e)^6 + 420*b*f*x*tan(e)^6 - 21*pi*b*sgn(-2*tan(f*x)^2*
tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(e)^6 - 63*pi*b*sg
n(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2
 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^4 - 42*b*arctan((tan(f*x) + tan(e))/(tan
(f*x)*tan(e) - 1))*tan(f*x)^6 + 42*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*
tan(e) + 1))*tan(f*x)^6 - 192*b*tan(f*x)^7 + 63*pi*b*sgn(2*tan(f*x)^2*tan(e
)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*ta
n(e))*tan(f*x)^3*tan(e) + 126*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e)
 - 1))*tan(f*x)^5*tan(e) - 126*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(
e) + 1))*tan(f*x)^5*tan(e) - 264*a*tan(f*x)^6*tan(e) + 504*b*tan(f*x)^6*tan
(e) - 189*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*
tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^2*tan(e)^2 - 378*b*arct
an((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^4*tan(e)^2 + 378*b*a
rctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(e)^2 - 360
*a*tan(f*x)^5*tan(e)^2 - 1512*b*tan(f*x)^5*tan(e)^2 + 63*pi*b*sgn(2*tan(f*x
)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*
x) - 2*tan(e))*tan(f*x)*tan(e)^3 + 378*b*arctan((tan(f*x) + tan(e))/(tan(f*
x)*tan(e) - 1))*tan(f*x)^3*tan(e)^3 - 378*b*arctan(-(tan(f*x) - tan(e))/(ta
n(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 - 960*a*tan(f*x)^4*tan(e)^3 - 63*pi
*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan
(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(e)^4 - 378*b*arctan((tan(f*x) + tan(e))/
(tan(f*x)*tan(e) - 1))*tan(f*x)^2*tan(e)^4 + 378*b*arctan(-(tan(f*x) - tan(
e))/(tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^4 - 960*a*tan(f*x)^3*tan(e)^4
+ 126*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)*tan(e)^5
 - 126*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e)
^5 - 360*a*tan(f*x)^2*tan(e)^5 - 1512*b*tan(f*x)^2*tan(e)^5 - 42*b*arctan((
tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(e)^6 + 42*b*arctan(-(tan(f*x)
 - tan(e))/(tan(f*x)*tan(e) + 1))*tan(e)^6 - 264*a*tan(f*x)*tan(e)^6 + 504*
b*tan(f*x)*tan(e)^6 - 192*b*tan(e)^7 - 180*a*f*x*tan(f*x)^4 + 1260*b*f*x*ta
n(f*x)^4 - 63*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f
*x) - 2*tan(e))*tan(f*x)^4 + 180*a*f*x*tan(f*x)^3*tan(e) - 1260*b*f*x*tan(f
*x)^3*tan(e) + 63*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*t
an(f*x) - 2*tan(e))*tan(f*x)^3*tan(e) - 540*a*f*x*tan(f*x)^2*tan(e)^2 + 378
0*b*f*x*tan(f*x)^2*tan(e)^2 - 189*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x
)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^2*tan(e)^2 + 180*a*f*x*tan(f*x
)*tan(e)^3 - 1260*b*f*x*tan(f*x)*tan(e)^3 + 63*pi*b*sgn(-2*tan(f*x)^2*tan(e
) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)*tan(e)^3 - 180*a*
```

```
f*x*tan(e)^4 + 1260*b*f*x*tan(e)^4 - 63*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*t
an(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(e)^4 - 63*pi*b*sgn(2*tan(f*x)
^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x
) - 2*tan(e))*tan(f*x)^2 - 126*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e
) - 1))*tan(f*x)^4 + 126*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1
))*tan(f*x)^4 + 132*a*tan(f*x)^5 - 924*b*tan(f*x)^5 + 21*pi*b*sgn(2*tan(f*x
)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*
x) - 2*tan(e))*tan(f*x)*tan(e) + 126*b*arctan((tan(f*x) + tan(e))/(tan(f*x)
*tan(e) - 1))*tan(f*x)^3*tan(e) - 126*b*arctan(-(tan(f*x) - tan(e))/(tan(f*
x)*tan(e) + 1))*tan(f*x)^3*tan(e) + 20*a*tan(f*x)^4*tan(e) - 140*b*tan(f*x)
^4*tan(e) - 63*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e)
 + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(e)^2 - 378*b*arctan((ta
n(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^2*tan(e)^2 + 378*b*arctan(
-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 + 300*a*tan
(f*x)^3*tan(e)^2 - 2100*b*tan(f*x)^3*tan(e)^2 + 126*b*arctan((tan(f*x) + ta
n(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)*tan(e)^3 - 126*b*arctan(-(tan(f*x) -
tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e)^3 + 300*a*tan(f*x)^2*tan(e)^
3 - 2100*b*tan(f*x)^2*tan(e)^3 - 126*b*arctan((tan(f*x) + tan(e))/(tan(f*x)
*tan(e) - 1))*tan(e)^4 + 126*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e)
 + 1))*tan(e)^4 + 20*a*tan(f*x)*tan(e)^4 - 140*b*tan(f*x)*tan(e)^4 + 132*a*
tan(e)^5 - 924*b*tan(e)^5 - 180*a*f*x*tan(f*x)^2 + 1260*b*f*x*tan(f*x)^2 -
63*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan
(e))*tan(f*x)^2 + 60*a*f*x*tan(f*x)*tan(e) - 420*b*f*x*tan(f*x)*tan(e) + 21
*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e
))*tan(f*x)*tan(e) - 180*a*f*x*tan(e)^2 + 1260*b*f*x*tan(e)^2 - 63*pi*b*sgn
(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(e)
^2 - 21*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*ta
n(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e)) - 126*b*arctan((tan(f*x) + tan(e))
/(tan(f*x)*tan(e) - 1))*tan(f*x)^2 + 126*b*arctan(-(tan(f*x) - tan(e))/(tan
(f*x)*tan(e) + 1))*tan(f*x)^2 + 160*a*tan(f*x)^3 - 1120*b*tan(f*x)^3 + 42*b
*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)*tan(e) - 42*b*a
rctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) + 120*a*t
an(f*x)^2*tan(e) - 840*b*tan(f*x)^2*tan(e) - 126*b*arctan((tan(f*x) + tan(e
))/(tan(f*x)*tan(e) - 1))*tan(e)^2 + 126*b*arctan(-(tan(f*x) - tan(e))/(tan
(f*x)*tan(e) + 1))*tan(e)^2 + 120*a*tan(f*x)*tan(e)^2 - 840*b*tan(f*x)*tan(
e)^2 + 160*a*tan(e)^3 - 1120*b*tan(e)^3 - 60*a*f*x + 420*b*f*x - 21*pi*b*sg
n(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e)) - 42*
b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1)) + 42*b*arctan(-(tan(f*x
) - tan(e))/(tan(f*x)*tan(e) + 1)) + 60*a*tan(f*x) - 420*b*tan(f*x) + 60*a*
tan(e) - 420*b*tan(e))/(f*tan(f*x)^7*tan(e)^7 + 3*f*tan(f*x)^7*tan(e)^5 - f
*tan(f*x)^6*tan(e)^6 + 3*f*tan(f*x)^5*tan(e)^7 + 3*f*tan(f*x)^7*tan(e)^3 -
3*f*tan(f*x)^6*tan(e)^4 + 9*f*tan(f*x)^5*tan(e)^5 - 3*f*tan(f*x)^4*tan(e)^6
 + 3*f*tan(f*x)^3*tan(e)^7 + f*tan(f*x)^7*tan(e) - 3*f*tan(f*x)^6*tan(e)^2
+ 9*f*tan(f*x)^5*tan(e)^3 - 9*f*tan(f*x)^4*tan(e)^4 + 9*f*tan(f*x)^3*tan(e)
^5 - 3*f*tan(f*x)^2*tan(e)^6 + f*tan(f*x)*tan(e)^7 - f*tan(f*x)^6 + 3*f*tan
(f*x)^5*tan(e) - 9*f*tan(f*x)^4*tan(e)^2 + 9*f*tan(f*x)^3*tan(e)^3 - 9*f*ta
n(f*x)^2*tan(e)^4 + 3*f*tan(f*x)*tan(e)^5 - f*tan(e)^6 - 3*f*tan(f*x)^4 + 3
*f*tan(f*x)^3*tan(e) - 9*f*tan(f*x)^2*tan(e)^2 + 3*f*tan(f*x)*tan(e)^3 - 3*
f*tan(e)^4 - 3*f*tan(f*x)^2 + f*tan(f*x)*tan(e) - 3*f*tan(e)^2 - f)
```

3.37 $\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=74

$$\frac{(a-b)\sin(e+fx)\cos^3(e+fx)}{4f}-\frac{(5a-9b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}x(a-5b)+\frac{b\tan(e+fx)}{f}$$

```
[Out] (3*(a - 5*b)*x)/8 - ((5*a - 9*b)*Cos[e + f*x]*Sin[e + f*x])/(8*f) + ((a - b
)*Cos[e + f*x]^3*Sin[e + f*x])/(4*f) + (b*Tan[e + f*x])/f
```

Rubi [A] time = 0.0733339, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3663, 455, 1157, 388, 203}

$$\frac{(a-b)\sin(e+fx)\cos^3(e+fx)}{4f}-\frac{(5a-9b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}x(a-5b)+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```


```
[Out] (3*(a - 5*b)*x)/8 - ((5*a - 9*b)*Cos[e + f*x]*Sin[e + f*x])/(8*f) + ((a - b
)*Cos[e + f*x]^3*Sin[e + f*x])/(4*f) + (b*Tan[e + f*x])/f
```


Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)dx = \frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)}{(1+x^2)^3}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{(a-b)\cos^3(e+fx)\sin(e+fx)}{4f} - \frac{\text{Subst}\left(\int \frac{a-b-4(a-b)x^2-4bx^4}{(1+x^2)^2}dx, x, \tan(e+\right.}{4f}$$

$$= -\frac{(5a-9b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{(a-b)\cos^3(e+fx)\sin(e+fx)}{4f} + \frac{\text{Su}}{}$$

$$= -\frac{(5a-9b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{(a-b)\cos^3(e+fx)\sin(e+fx)}{4f} + \frac{b\,\text{t}}{}$$

$$= \frac{3}{8}(a-5b)x - \frac{(5a-9b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{(a-b)\cos^3(e+fx)\sin(e}{4f}$$

Mathematica [A] time = 0.344238, size = 58, normalized size = 0.78

$$\frac{12(a-5b)(e+fx) - 8(a-2b)\sin(2(e+fx)) + (a-b)\sin(4(e+fx)) + 32b\tan(e+fx)}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (12*(a - 5*b)*(e + f*x) - 8*(a - 2*b)*Sin[2*(e + f*x)] + (a - b)*Sin[4*(e +
 f*x)] + 32*b*Tan[e + f*x])/(32*f)
```

Maple [A] time = 0.062, size = 102, normalized size = 1.4

$$\frac{1}{f}\left(a\left(-\frac{\cos(fx+e)}{4}\left((\sin(fx+e))^3 + \frac{3\sin(fx+e)}{2}\right) + \frac{3fx}{8} + \frac{3e}{8}\right) + b\left(\frac{(\sin(fx+e))^7}{\cos(fx+e)} + \left((\sin(fx+e))^5 + \frac{5(\text{si}}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*(a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+b*(sin
(f*x+e)^7/cos(f*x+e)+(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*
x+e)-15/8*f*x-15/8*e))
```

Maxima [A] time = 1.58039, size = 111, normalized size = 1.5

$$\frac{3\left(fx+e\right)(a-5b)+8b\tan\left(fx+e\right)-\frac{(5a-9b)\tan\left(fx+e\right)^3+(3a-7b)\tan\left(fx+e\right)}{\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/8*(3*(f*x + e)*(a - 5*b) + 8*b*tan(f*x + e) - ((5*a - 9*b)*tan(f*x + e)^3
 + (3*a - 7*b)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

Fricas [A] time = 1.95868, size = 176, normalized size = 2.38

$$\frac{3(a-5b)fx\cos\left(fx+e\right)+\left(2(a-b)\cos\left(fx+e\right)^4-(5a-9b)\cos\left(fx+e\right)^2+8b\right)\sin\left(fx+e\right)}{8f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/8*(3*(a - 5*b)*f*x*cos(f*x + e) + (2*(a - b)*cos(f*x + e)^4 - (5*a - 9*b)
*cos(f*x + e)^2 + 8*b)*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan^2\left(e+fx\right)\right)\sin^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*sin(e + f*x)**4, x)
```

Giac [B] time = 4.28335, size = 5873, normalized size = 79.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/64*(3*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*ta
n(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^5*tan(e)^5 + 24*a*f*x*tan
(f*x)^5*tan(e)^5 - 120*b*f*x*tan(f*x)^5*tan(e)^5 + 3*pi*b*sgn(-2*tan(f*x)^2
*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^5*tan(e)^5
 + 6*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*
x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^5*tan(e)^3 - 3*pi*b*sgn(2*tan
(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*ta
```

```
n(f*x) - 2*tan(e))*tan(f*x)^4*tan(e)^4 + 6*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 -
 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))
*tan(f*x)^3*tan(e)^5 + 6*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1)
)*tan(f*x)^5*tan(e)^5 - 6*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) +
1))*tan(f*x)^5*tan(e)^5 + 48*a*f*x*tan(f*x)^5*tan(e)^3 - 240*b*f*x*tan(f*x)
^5*tan(e)^3 + 6*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan
(f*x) - 2*tan(e))*tan(f*x)^5*tan(e)^3 - 24*a*f*x*tan(f*x)^4*tan(e)^4 + 120*
b*f*x*tan(f*x)^4*tan(e)^4 - 3*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*ta
n(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^4*tan(e)^4 + 48*a*f*x*tan(f*x)^3*t
an(e)^5 - 240*b*f*x*tan(f*x)^3*tan(e)^5 + 6*pi*b*sgn(-2*tan(f*x)^2*tan(e) +
 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^3*tan(e)^5 + 3*pi*b*
sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)
^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^5*tan(e) - 6*pi*b*sgn(2*tan(f*x)^2*tan
(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*
tan(e))*tan(f*x)^4*tan(e)^2 + 12*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2
*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^
3*tan(e)^3 + 12*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x
)^5*tan(e)^3 - 12*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(
f*x)^5*tan(e)^3 - 6*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*t
an(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^2*tan(e)^4 -
6*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^4*tan(e)^4 +
 6*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(e)^4
 + 24*a*tan(f*x)^5*tan(e)^4 - 120*b*tan(f*x)^5*tan(e)^4 + 3*pi*b*sgn(2*tan(
f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan
(f*x) - 2*tan(e))*tan(f*x)*tan(e)^5 + 12*b*arctan((tan(f*x) + tan(e))/(tan(
f*x)*tan(e) - 1))*tan(f*x)^3*tan(e)^5 - 12*b*arctan(-(tan(f*x) - tan(e))/(t
an(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^5 + 24*a*tan(f*x)^4*tan(e)^5 - 120*b
*tan(f*x)^4*tan(e)^5 + 24*a*f*x*tan(f*x)^5*tan(e) - 120*b*f*x*tan(f*x)^5*ta
n(e) + 3*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) -
 2*tan(e))*tan(f*x)^5*tan(e) - 48*a*f*x*tan(f*x)^4*tan(e)^2 + 240*b*f*x*tan
(f*x)^4*tan(e)^2 - 6*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 +
2*tan(f*x) - 2*tan(e))*tan(f*x)^4*tan(e)^2 + 96*a*f*x*tan(f*x)^3*tan(e)^3 -
 480*b*f*x*tan(f*x)^3*tan(e)^3 + 12*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f
*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^3*tan(e)^3 - 48*a*f*x*tan(f*
x)^2*tan(e)^4 + 240*b*f*x*tan(f*x)^2*tan(e)^4 - 6*pi*b*sgn(-2*tan(f*x)^2*ta
n(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^2*tan(e)^4 + 2
4*a*f*x*tan(f*x)*tan(e)^5 - 120*b*f*x*tan(f*x)*tan(e)^5 + 3*pi*b*sgn(-2*tan
(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)*tan(
e)^5 - 3*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*t
an(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^4 + 6*pi*b*sgn(2*tan(f*x
)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*
x) - 2*tan(e))*tan(f*x)^3*tan(e) + 6*b*arctan((tan(f*x) + tan(e))/(tan(f*x)
*tan(e) - 1))*tan(f*x)^5*tan(e) - 6*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)
*tan(e) + 1))*tan(f*x)^5*tan(e) - 12*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sg
n(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f
*x)^2*tan(e)^2 - 12*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan
(f*x)^4*tan(e)^2 + 12*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*
tan(f*x)^4*tan(e)^2 + 40*a*tan(f*x)^5*tan(e)^2 - 200*b*tan(f*x)^5*tan(e)^2
+ 6*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*
x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)*tan(e)^3 + 24*b*arctan((tan(f
*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^3*tan(e)^3 - 24*b*arctan(-(ta
n(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 + 24*a*tan(f*x)
^4*tan(e)^3 - 120*b*tan(f*x)^4*tan(e)^3 - 3*pi*b*sgn(2*tan(f*x)^2*tan(e)^2
- 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e)
)*tan(e)^4 - 12*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x
)^2*tan(e)^4 + 12*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(
f*x)^2*tan(e)^4 + 24*a*tan(f*x)^3*tan(e)^4 - 120*b*tan(f*x)^3*tan(e)^4 + 6*
b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)*tan(e)^5 - 6*b
*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e)^5 + 40*
```

```
a*tan(f*x)^2*tan(e)^5 - 200*b*tan(f*x)^2*tan(e)^5 - 24*a*f*x*tan(f*x)^4 + 1
20*b*f*x*tan(f*x)^4 - 3*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2
 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^4 + 48*a*f*x*tan(f*x)^3*tan(e) - 240*b*f
*x*tan(f*x)^3*tan(e) + 6*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^
2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^3*tan(e) - 96*a*f*x*tan(f*x)^2*tan(e)^2
 + 480*b*f*x*tan(f*x)^2*tan(e)^2 - 12*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan
(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^2*tan(e)^2 + 48*a*f*x*tan(
f*x)*tan(e)^3 - 240*b*f*x*tan(f*x)*tan(e)^3 + 6*pi*b*sgn(-2*tan(f*x)^2*tan(
e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)*tan(e)^3 - 24*a*
f*x*tan(e)^4 + 120*b*f*x*tan(e)^4 - 3*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan
(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(e)^4 - 6*pi*b*sgn(2*tan(f*x)^2*
tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) -
 2*tan(e))*tan(f*x)^2 - 6*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1
))*tan(f*x)^4 + 6*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(
f*x)^4 - 64*b*tan(f*x)^5 + 3*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan
(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)*tan(
e) + 12*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^3*tan(
e) - 12*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan
(e) - 80*a*tan(f*x)^4*tan(e) + 80*b*tan(f*x)^4*tan(e) - 6*pi*b*sgn(2*tan(f*
x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f
*x) - 2*tan(e))*tan(e)^2 - 24*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e)
 - 1))*tan(f*x)^2*tan(e)^2 + 24*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan
(e) + 1))*tan(f*x)^2*tan(e)^2 - 96*a*tan(f*x)^3*tan(e)^2 - 160*b*tan(f*x)^3
*tan(e)^2 + 12*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)
*tan(e)^3 - 12*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x
)*tan(e)^3 - 96*a*tan(f*x)^2*tan(e)^3 - 160*b*tan(f*x)^2*tan(e)^3 - 6*b*arc
tan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(e)^4 + 6*b*arctan(-(tan(
f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(e)^4 - 80*a*tan(f*x)*tan(e)^4 + 8
0*b*tan(f*x)*tan(e)^4 - 64*b*tan(e)^5 - 48*a*f*x*tan(f*x)^2 + 240*b*f*x*tan
(f*x)^2 - 6*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x
) - 2*tan(e))*tan(f*x)^2 + 24*a*f*x*tan(f*x)*tan(e) - 120*b*f*x*tan(f*x)*ta
n(e) + 3*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) -
 2*tan(e))*tan(f*x)*tan(e) - 48*a*f*x*tan(e)^2 + 240*b*f*x*tan(e)^2 - 6*pi*
b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*t
an(e)^2 - 3*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) +
2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e)) - 12*b*arctan((tan(f*x) + tan(
e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^2 + 12*b*arctan(-(tan(f*x) - tan(e))/(t
an(f*x)*tan(e) + 1))*tan(f*x)^2 + 40*a*tan(f*x)^3 - 200*b*tan(f*x)^3 + 6*b*
arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)*tan(e) - 6*b*arc
tan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) + 24*a*tan(
f*x)^2*tan(e) - 120*b*tan(f*x)^2*tan(e) - 12*b*arctan((tan(f*x) + tan(e))/(
tan(f*x)*tan(e) - 1))*tan(e)^2 + 12*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)
*tan(e) + 1))*tan(e)^2 + 24*a*tan(f*x)*tan(e)^2 - 120*b*tan(f*x)*tan(e)^2 +
 40*a*tan(e)^3 - 200*b*tan(e)^3 - 24*a*f*x + 120*b*f*x - 3*pi*b*sgn(-2*tan(
f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e)) - 6*b*arctan((
tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1)) + 6*b*arctan(-(tan(f*x) - tan(e))
/(tan(f*x)*tan(e) + 1)) + 24*a*tan(f*x) - 120*b*tan(f*x) + 24*a*tan(e) - 12
0*b*tan(e))/(f*tan(f*x)^5*tan(e)^5 + 2*f*tan(f*x)^5*tan(e)^3 - f*tan(f*x)^4
*tan(e)^4 + 2*f*tan(f*x)^3*tan(e)^5 + f*tan(f*x)^5*tan(e) - 2*f*tan(f*x)^4*
tan(e)^2 + 4*f*tan(f*x)^3*tan(e)^3 - 2*f*tan(f*x)^2*tan(e)^4 + f*tan(f*x)*t
an(e)^5 - f*tan(f*x)^4 + 2*f*tan(f*x)^3*tan(e) - 4*f*tan(f*x)^2*tan(e)^2 +
2*f*tan(f*x)*tan(e)^3 - f*tan(e)^4 - 2*f*tan(f*x)^2 + f*tan(f*x)*tan(e) - 2
*f*tan(e)^2 - f)
```

3.38 $\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=46

$$-\frac{(a-b)\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}x(a-3b)+\frac{b\tan(e+fx)}{f}$$

```
[Out] ((a - 3*b)*x)/2 - ((a - b)*Cos[e + f*x]*Sin[e + f*x])/(2*f) + (b*Tan[e + f*
x])/f
```

Rubi [A] time = 0.047587, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3663, 455, 388, 203}

$$-\frac{(a-b)\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}x(a-3b)+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((a - 3*b)*x)/2 - ((a - b)*Cos[e + f*x]*Sin[e + f*x])/(2*f) + (b*Tan[e + f*
x])/f
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)}{(1+x^2)^2}dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{(a-b)\cos(e+fx)\sin(e+fx)}{2f} - \frac{\text{Subst}\left(\int \frac{-a+b-2bx^2}{1+x^2}dx, x, \tan(e+fx)\right)}{2f}$$
$$= -\frac{(a-b)\cos(e+fx)\sin(e+fx)}{2f} + \frac{b\tan(e+fx)}{f} + \frac{(a-3b)\,\text{Subst}\left(\int \frac{1}{1+x^2}dx,\right.}{2f}$$
$$= \frac{1}{2}(a-3b)x - \frac{(a-b)\cos(e+fx)\sin(e+fx)}{2f} + \frac{b\tan(e+fx)}{f}$$

Mathematica [A] time = 0.216948, size = 43, normalized size = 0.93

$$\frac{2(a-3b)(e+fx)+(b-a)\sin(2(e+fx))+4b\tan(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (2*(a - 3*b)*(e + f*x) + (-a + b)*Sin[2*(e + f*x)] + 4*b*Tan[e + f*x])/(4*f
)
```

Maple [A] time = 0.033, size = 81, normalized size = 1.8

$$\frac{1}{f}\left(a\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b\left(\frac{(\sin(fx+e))^5}{\cos(fx+e)}+\left((\sin(fx+e))^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3f}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*(a*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+b*(sin(f*x+e)^5/cos(f*x+e
)+(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/2*f*x-3/2*e))
```

Maxima [A] time = 1.54547, size = 69, normalized size = 1.5

$$\frac{(fx+e)(a-3b)+2b\tan(fx+e)-\frac{(a-b)\tan(fx+e)}{\tan(fx+e)^2+1}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((f*x + e)*(a - 3*b) + 2*b*tan(f*x + e) - (a - b)*tan(f*x + e)/(tan(f*x
 + e)^2 + 1))/f
```

Fricas [A] time = 1.86203, size = 131, normalized size = 2.85

$$\frac{(a-3b)fx\cos\left(fx+e\right)-\left((a-b)\cos\left(fx+e\right)^{2}-2b\right)\sin\left(fx+e\right)}{2f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*((a - 3*b)*f*x*cos(f*x + e) - ((a - b)*cos(f*x + e)^2 - 2*b)*sin(f*x +
e))/(f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan^{2}\left(e+fx\right)\right)\sin^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*sin(e + f*x)**2, x)
```

Giac [B] time = 1.55597, size = 533, normalized size = 11.59

$$\frac{afx\tan\left(fx\right)^{3}\tan\left(e\right)^{3}-3bfx\tan\left(fx\right)^{3}\tan\left(e\right)^{3}+afx\tan\left(fx\right)^{3}\tan\left(e\right)-3bfx\tan\left(fx\right)^{3}\tan\left(e\right)-afx\tan\left(fx\right)^{2}\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*(a*f*x*tan(f*x)^3*tan(e)^3 - 3*b*f*x*tan(f*x)^3*tan(e)^3 + a*f*x*tan(f*
x)^3*tan(e) - 3*b*f*x*tan(f*x)^3*tan(e) - a*f*x*tan(f*x)^2*tan(e)^2 + 3*b*f
*x*tan(f*x)^2*tan(e)^2 + a*f*x*tan(f*x)*tan(e)^3 - 3*b*f*x*tan(f*x)*tan(e)^
3 + a*tan(f*x)^3*tan(e)^2 - 3*b*tan(f*x)^3*tan(e)^2 + a*tan(f*x)^2*tan(e)^3
 - 3*b*tan(f*x)^2*tan(e)^3 - a*f*x*tan(f*x)^2 + 3*b*f*x*tan(f*x)^2 + a*f*x*
tan(f*x)*tan(e) - 3*b*f*x*tan(f*x)*tan(e) - a*f*x*tan(e)^2 + 3*b*f*x*tan(e)
^2 - 2*b*tan(f*x)^3 - 2*a*tan(f*x)^2*tan(e) - 2*a*tan(f*x)*tan(e)^2 - 2*b*t
an(e)^3 - a*f*x + 3*b*f*x + a*tan(f*x) - 3*b*tan(f*x) + a*tan(e) - 3*b*tan(
e))/(f*tan(f*x)^3*tan(e)^3 + f*tan(f*x)^3*tan(e) - f*tan(f*x)^2*tan(e)^2 +
f*tan(f*x)*tan(e)^3 - f*tan(f*x)^2 + f*tan(f*x)*tan(e) - f*tan(e)^2 - f)
```

3.39 $\int \left(a+b\tan^2(e+fx)\right) dx$

Optimal. Leaf size=19

$$ax+\frac{b\tan(e+fx)}{f}-bx$$

```
[Out] a*x - b*x + (b*Tan[e + f*x])/f
```

Rubi [A] time = 0.0129236, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3473, 8}

$$ax+\frac{b\tan(e+fx)}{f}-bx$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Tan[e + f*x]^2,x]
```

```
[Out] a*x - b*x + (b*Tan[e + f*x])/f
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tan^2(e+fx)\right) dx &= ax+b\int \tan^2(e+fx)\,dx\\ &= ax+\frac{b\tan(e+fx)}{f}-b\int 1\,dx\\ &= ax-bx+\frac{b\tan(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.0134426, size = 28, normalized size = 1.47

$$ax-\frac{b\tan^{-1}(\tan(e+fx))}{f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Tan[e + f*x]^2,x]
```

```
[Out] a*x - (b*ArcTan[Tan[e + f*x]])/f + (b*Tan[e + f*x])/f
```

Maple [A] time = 0.003, size = 29, normalized size = 1.5

$$ax + \frac{b\tan\left(fx+e\right)}{f} - \frac{b\arctan\left(\tan\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*tan(f*x+e)^2,x)
```

```
[Out] a*x+b*tan(f*x+e)/f-b/f*arctan(tan(f*x+e))
```

Maxima [A] time = 1.56394, size = 31, normalized size = 1.63

$$ax - \frac{\left(fx+e-\tan\left(fx+e\right)\right)b}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] a*x - (f*x + e - tan(f*x + e))*b/f
```

Fricas [A] time = 1.88808, size = 46, normalized size = 2.42

$$\frac{(a-b)fx + b\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] ((a - b)*f*x + b*tan(f*x + e))/f
```

Sympy [A] time = 0.219814, size = 20, normalized size = 1.05

$$ax + b\left(\begin{cases} -x + \frac{\tan\left(e+fx\right)}{f} & \text{for } f \neq 0 \\ x\tan^2\left(e\right) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)**2,x)
```

```
[Out] a*x + b*Piecewise((-x + tan(e + f*x)/f, Ne(f, 0)), (x*tan(e)**2, True))
```

Giac [B] time = 1.43987, size = 340, normalized size = 17.89

$$ax + \frac{\left(\pi - 4\,fx\tan\left(fx\right)\tan\left(e\right) - \pi\mathrm{sgn}\left(2\,\tan\left(fx\right)^2\tan\left(e\right) + 2\,\tan\left(fx\right)\tan\left(e\right)^2 - 2\,\tan\left(fx\right) - 2\,\tan\left(e\right)\right)\tan\left(fx\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] a*x + 1/4*(pi - 4*f*x*tan(f*x)*tan(e) - pi*sgn(2*tan(f*x)^2*tan(e) + 2*tan(
f*x)*tan(e)^2 - 2*tan(f*x) - 2*tan(e))*tan(f*x)*tan(e) - pi*tan(f*x)*tan(e)
 + 2*arctan((tan(f*x)*tan(e) - 1)/(tan(f*x) + tan(e)))*tan(f*x)*tan(e) + 2*
arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)*tan(e) + 4*f*x +
 pi*sgn(2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 - 2*tan(f*x) - 2*tan(e))
- 2*arctan((tan(f*x)*tan(e) - 1)/(tan(f*x) + tan(e))) - 2*arctan((tan(f*x)
+ tan(e))/(tan(f*x)*tan(e) - 1)) - 4*tan(f*x) - 4*tan(e))*b/(f*tan(f*x)*tan
(e) - f)
```

3.40 $\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)dx$

Optimal. Leaf size=24

$$\frac{b\tan(e+fx)}{f}-\frac{a\cot(e+fx)}{f}$$

```
[Out] -((a*Cot[e + f*x])/f) + (b*Tan[e + f*x])/f
```

Rubi [A] time = 0.0322951, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3663, 14}

$$\frac{b\tan(e+fx)}{f}-\frac{a\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*Cot[e + f*x])/f) + (b*Tan[e + f*x])/f
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)dx &= \frac{\text{Subst}\left(\int \frac{a+bx^2}{x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(b+\frac{a}{x^2}\right)dx,x,\tan(e+fx)\right)}{f}\\
&= -\frac{a\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}
\end{aligned}$$

Mathematica [A] time = 0.0207985, size = 24, normalized size = 1.

$$\frac{b\tan(e+fx)}{f}-\frac{a\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*Cot[e + f*x])/f) + (b*Tan[e + f*x])/f
```

Maple [A] time = 0.078, size = 23, normalized size = 1.

$$\frac{b\tan\left(fx+e\right)-\cot\left(fx+e\right)a}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*(b*tan(f*x+e)-cot(f*x+e)*a)
```

Maxima [A] time = 1.0621, size = 32, normalized size = 1.33

$$\frac{b\tan\left(fx+e\right)-\frac{a}{\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] (b*tan(f*x + e) - a/tan(f*x + e))/f
```

Fricas [A] time = 1.92869, size = 82, normalized size = 3.42

$$-\frac{(a+b)\cos\left(fx+e\right)^2-b}{f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -((a + b)*cos(f*x + e)^2 - b)/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\tan^2\left(e+fx\right)\right)\csc^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*csc(e + f*x)**2, x)
```

<u>Giac</u> [A] time = 1.56161, size = 35, normalized size = 1.46

$$\frac{b\tan\left(fx+e\right)-\frac{a}{\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] (b*tan(f*x + e) - a/tan(f*x + e))/f
```

3.41 $\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=42

$$-\frac{(a+b)\cot(e+fx)}{f}-\frac{a\cot^3(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] -(((a + b)*Cot[e + f*x])/f) - (a*Cot[e + f*x]^3)/(3*f) + (b*Tan[e + f*x])/f
```

Rubi [A] time = 0.0432361, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3663, 448}

$$-\frac{(a+b)\cot(e+fx)}{f}-\frac{a\cot^3(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a + b)*Cot[e + f*x])/f) - (a*Cot[e + f*x]^3)/(3*f) + (b*Tan[e + f*x])/f
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)(a+bx^2)}{x^4}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(b+\frac{a}{x^4}+\frac{a+b}{x^2}\right)dx, x, \tan(e+fx)\right)}{f}\\ &= -\frac{(a+b)\cot(e+fx)}{f}-\frac{a\cot^3(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.0685022, size = 60, normalized size = 1.43

$$-\frac{2a\cot(e+fx)}{3f}-\frac{a\cot(e+fx)\csc^2(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}-\frac{b\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-2*a*Cot[e + f*x])/(3*f) - (b*Cot[e + f*x])/f - (a*Cot[e + f*x]*Csc[e + f*
x]^2)/(3*f) + (b*Tan[e + f*x])/f
```

Maple [A] time = 0.077, size = 54, normalized size = 1.3

$$\frac{1}{f}\left(b\left(\frac{1}{\cos\left(fx+e\right)\sin\left(fx+e\right)}-2\cot\left(fx+e\right)\right)+a\left(-\frac{2}{3}-\frac{\left(\csc\left(fx+e\right)\right)^{2}}{3}\right)\cot\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*(b*(1/sin(f*x+e)/cos(f*x+e)-2*cot(f*x+e))+a*(-2/3-1/3*csc(f*x+e)^2)*cot
(f*x+e))
```

Maxima [A] time = 1.05946, size = 54, normalized size = 1.29

$$\frac{3\,b\tan\left(fx+e\right)-\frac{3\,(a+b)\tan\left(fx+e\right)^{2}+a}{\tan\left(fx+e\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*b*tan(f*x + e) - (3*(a + b)*tan(f*x + e)^2 + a)/tan(f*x + e)^3)/f
```

Fricas [A] time = 1.79929, size = 163, normalized size = 3.88

$$-\frac{2\,(a+3\,b)\cos\left(fx+e\right)^{4}-3\,(a+3\,b)\cos\left(fx+e\right)^{2}+3\,b}{3\left(f\cos\left(fx+e\right)^{3}-f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a + 3*b)*cos(f*x + e)^4 - 3*(a + 3*b)*cos(f*x + e)^2 + 3*b)/((f*co
s(f*x + e)^3 - f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan^{2}\left(e+fx\right)\right)\csc^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*csc(e + f*x)**4, x)
```

Giac [A] time = 1.51136, size = 72, normalized size = 1.71

$$\frac{3\,b\tan\left(fx+e\right)-\frac{3\,a\tan\left(fx+e\right)^2+3\,b\tan\left(fx+e\right)^2+a}{\tan\left(fx+e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*b*tan(f*x + e) - (3*a*tan(f*x + e)^2 + 3*b*tan(f*x + e)^2 + a)/tan(f
*x + e)^3)/f
```

3.42 $\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=64

$$-\frac{(2a+b)\cot^3(e+fx)}{3f}-\frac{(a+2b)\cot(e+fx)}{f}-\frac{a\cot^5(e+fx)}{5f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] -(((a + 2*b)*Cot[e + f*x])/f) - ((2*a + b)*Cot[e + f*x]^3)/(3*f) - (a*Cot[e
 + f*x]^5)/(5*f) + (b*Tan[e + f*x])/f
```

Rubi [A] time = 0.0528497, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3663, 448}

$$-\frac{(2a+b)\cot^3(e+fx)}{3f}-\frac{(a+2b)\cot(e+fx)}{f}-\frac{a\cot^5(e+fx)}{5f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a + 2*b)*Cot[e + f*x])/f) - ((2*a + b)*Cot[e + f*x]^3)/(3*f) - (a*Cot[e
 + f*x]^5)/(5*f) + (b*Tan[e + f*x])/f
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2\left(a+bx^2\right)}{x^6}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(b+\frac{a}{x^6}+\frac{2a+b}{x^4}+\frac{a+2b}{x^2}\right)dx,x,\tan(e+fx)\right)}{f}\\ &= -\frac{(a+2b)\cot(e+fx)}{f}-\frac{(2a+b)\cot^3(e+fx)}{3f}-\frac{a\cot^5(e+fx)}{5f}+\frac{b\tan(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.0491428, size = 106, normalized size = 1.66

$$-\frac{8a\cot(e+fx)}{15f}-\frac{a\cot(e+fx)\csc^4(e+fx)}{5f}-\frac{4a\cot(e+fx)\csc^2(e+fx)}{15f}+\frac{b\tan(e+fx)}{f}-\frac{5b\cot(e+fx)}{3f}-\frac{b\cot}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-8*a*Cot[e + f*x])/(15*f) - (5*b*Cot[e + f*x])/(3*f) - (4*a*Cot[e + f*x]*C
sc[e + f*x]^2)/(15*f) - (b*Cot[e + f*x]*Csc[e + f*x]^2)/(3*f) - (a*Cot[e +
f*x]*Csc[e + f*x]^4)/(5*f) + (b*Tan[e + f*x])/f
```

Maple [A] time = 0.072, size = 83, normalized size = 1.3

$$\frac{1}{f}\left(b\left(-\frac{1}{3\left(\sin\left(fx+e\right)\right)^{3}\cos\left(fx+e\right)}+\frac{4}{3\cos\left(fx+e\right)\sin\left(fx+e\right)}-\frac{8\cot\left(fx+e\right)}{3}\right)+a\left(-\frac{8}{15}-\frac{\left(\csc\left(fx+e\right)\right)^{4}}{5}-\frac{4\,(\ldots}{\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*(b*(-1/3/sin(f*x+e)^3/cos(f*x+e)+4/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*x+
e))+a*(-8/15-1/5*csc(f*x+e)^4-4/15*csc(f*x+e)^2)*cot(f*x+e))
```

Maxima [A] time = 1.11731, size = 80, normalized size = 1.25

$$\frac{15\,b\tan\left(fx+e\right)-\frac{15\,(a+2\,b)\tan\left(fx+e\right)^{4}+5\,(2\,a+b)\tan\left(fx+e\right)^{2}+3\,a}{\tan\left(fx+e\right)^{5}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(15*b*tan(f*x + e) - (15*(a + 2*b)*tan(f*x + e)^4 + 5*(2*a + b)*tan(f*
x + e)^2 + 3*a)/tan(f*x + e)^5)/f
```

Fricas [A] time = 1.87629, size = 236, normalized size = 3.69

$$-\frac{8\,(a+5\,b)\cos\left(fx+e\right)^{6}-20\,(a+5\,b)\cos\left(fx+e\right)^{4}+15\,(a+5\,b)\cos\left(fx+e\right)^{2}-15\,b}{15\left(f\cos\left(fx+e\right)^{5}-2\,f\cos\left(fx+e\right)^{3}+f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a + 5*b)*cos(f*x + e)^6 - 20*(a + 5*b)*cos(f*x + e)^4 + 15*(a + 5
*b)*cos(f*x + e)^2 - 15*b)/((f*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(
f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.47163, size = 107, normalized size = 1.67

$$\frac{15\, b \tan\left(fx + e\right) - \frac{15\, a \tan\left(fx+e\right)^{4} + 30\, b \tan\left(fx+e\right)^{4} + 10\, a \tan\left(fx+e\right)^{2} + 5\, b \tan\left(fx+e\right)^{2} + 3\, a}{\tan\left(fx+e\right)^{5}}}{15\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(15*b*tan(f*x + e) - (15*a*tan(f*x + e)^4 + 30*b*tan(f*x + e)^4 + 10*a
*tan(f*x + e)^2 + 5*b*tan(f*x + e)^2 + 3*a)/tan(f*x + e)^5)/f
```

3.43 $\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=107

$$-\frac{\left(a^2-6ab+6b^2\right)\cos(e+fx)}{f}-\frac{(a-b)^2\cos^5(e+fx)}{5f}+\frac{2(a-2b)(a-b)\cos^3(e+fx)}{3f}+\frac{2b(a-2b)\sec(e+fx)}{f}+\frac{b^2\sec}{}$$

```
[Out] -(((a^2 - 6*a*b + 6*b^2)*Cos[e + f*x])/f) + (2*(a - 2*b)*(a - b)*Cos[e + f*
x]^3)/(3*f) - ((a - b)^2*Cos[e + f*x]^5)/(5*f) + (2*(a - 2*b)*b*Sec[e + f*x
])/f + (b^2*Sec[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.107058, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3664, 448}

$$-\frac{\left(a^2-6ab+6b^2\right)\cos(e+fx)}{f}-\frac{(a-b)^2\cos^5(e+fx)}{5f}+\frac{2(a-2b)(a-b)\cos^3(e+fx)}{3f}+\frac{2b(a-2b)\sec(e+fx)}{f}+\frac{b^2\sec}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(((a^2 - 6*a*b + 6*b^2)*Cos[e + f*x])/f) + (2*(a - 2*b)*(a - b)*Cos[e + f*
x]^3)/(3*f) - ((a - b)^2*Cos[e + f*x]^5)/(5*f) + (2*(a - 2*b)*b*Sec[e + f*x
])/f + (b^2*Sec[e + f*x]^3)/(3*f)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\left(a-b+bx^2\right)^2}{x^6}\,dx,x,\sec(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(2(a-2b)b+\frac{(a-b)^2}{x^6}+\frac{2(a-2b)(-a+b)}{x^4}+\frac{a^2-6ab+6b^2}{x^2}+b^2x^2\right)dx,x,\sec(e\right.}{f}\\ &= -\frac{\left(a^2-6ab+6b^2\right)\cos(e+fx)}{f}+\frac{2(a-2b)(a-b)\cos^3(e+fx)}{3f}-\frac{(a-b)^2\cos^5}{5f}\end{aligned}$$

Mathematica [A] time = 0.719437, size = 97, normalized size = 0.91

$$\frac{-30\left(5a^2-38ab+41b^2\right)\cos(e+fx)+5(5a-13b)(a-b)\cos(3(e+fx))-3(a-b)^2\cos(5(e+fx))+480b(a-2b)\operatorname{se}}{240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-30*(5*a^2 - 38*a*b + 41*b^2)*Cos[e + f*x] + 5*(5*a - 13*b)*(a - b)*Cos[3*
(e + f*x)] - 3*(a - b)^2*Cos[5*(e + f*x)] + 480*(a - 2*b)*b*Sec[e + f*x] +
80*b^2*Sec[e + f*x]^3)/(240*f)
```

Maple [A] time = 0.084, size = 185, normalized size = 1.7

$$\frac{1}{f}\left(-\frac{a^2\cos\left(fx+e\right)}{5}\left(\frac{8}{3}+\left(\sin\left(fx+e\right)\right)^4+\frac{4\left(\sin\left(fx+e\right)\right)^2}{3}\right)+2ab\left(\frac{\left(\sin\left(fx+e\right)\right)^8}{\cos\left(fx+e\right)}+\left(\frac{16}{5}+\left(\sin\left(fx+e\right)\right)^6+6/5\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-1/5*a^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+2*a*b*(sin(f*x
+e)^8/cos(f*x+e)+(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(
f*x+e))+b^2*(1/3*sin(f*x+e)^10/cos(f*x+e)^3-7/3*sin(f*x+e)^10/cos(f*x+e)-7/
3*(128/35+sin(f*x+e)^8+8/7*sin(f*x+e)^6+48/35*sin(f*x+e)^4+64/35*sin(f*x+e)
^2)*cos(f*x+e)))
```

Maxima [A] time = 0.983669, size = 140, normalized size = 1.31

$$-\frac{3\left(a^2-2ab+b^2\right)\cos\left(fx+e\right)^5-10\left(a^2-3ab+2b^2\right)\cos\left(fx+e\right)^3+15\left(a^2-6ab+6b^2\right)\cos\left(fx+e\right)-\frac{5\left(6\left(ab-2b^2\right.\right.}{\cos}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/15*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 10*(a^2 - 3*a*b + 2*b^2)*cos(
f*x + e)^3 + 15*(a^2 - 6*a*b + 6*b^2)*cos(f*x + e) - 5*(6*(a*b - 2*b^2)*cos
(f*x + e)^2 + b^2)/cos(f*x + e)^3)/f
```

Fricas [A] time = 2.05938, size = 258, normalized size = 2.41

$$-\frac{3\left(a^2-2ab+b^2\right)\cos\left(fx+e\right)^8-10\left(a^2-3ab+2b^2\right)\cos\left(fx+e\right)^6+15\left(a^2-6ab+6b^2\right)\cos\left(fx+e\right)^4-30\left(ab-\right.}{15f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^8 - 10*(a^2 - 3*a*b + 2*b^2)*cos(
f*x + e)^6 + 15*(a^2 - 6*a*b + 6*b^2)*cos(f*x + e)^4 - 30*(a*b - 2*b^2)*cos
(f*x + e)^2 - 5*b^2)/(f*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.44 $\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=80

$$\frac{(a-b)^2\cos^3(e+fx)}{3f}-\frac{(a-3b)(a-b)\cos(e+fx)}{f}+\frac{b(2a-3b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -(((a - 3*b)*(a - b)*Cos[e + f*x])/f) + ((a - b)^2*Cos[e + f*x]^3)/(3*f) +
((2*a - 3*b)*b*Sec[e + f*x])/f + (b^2*Sec[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.083143, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3664, 448}

$$\frac{(a-b)^2\cos^3(e+fx)}{3f}-\frac{(a-3b)(a-b)\cos(e+fx)}{f}+\frac{b(2a-3b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(((a - 3*b)*(a - b)*Cos[e + f*x])/f) + ((a - b)^2*Cos[e + f*x]^3)/(3*f) +
((2*a - 3*b)*b*Sec[e + f*x])/f + (b^2*Sec[e + f*x]^3)/(3*f)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a-b+bx^2)^2}{x^4}\,dx, x, \sec(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left((2a-3b)b-\frac{(a-b)^2}{x^4}+\frac{(a-3b)(a-b)}{x^2}+b^2x^2\right)dx, x, \sec(e+fx)\right)}{f}\\ &= -\frac{(a-3b)(a-b)\cos(e+fx)}{f}+\frac{(a-b)^2\cos^3(e+fx)}{3f}+\frac{(2a-3b)b\sec(e+}{f}\end{aligned}$$

Mathematica [A] time = 0.519778, size = 72, normalized size = 0.9

$$\frac{\left(-9a^2+42ab-33b^2\right)\cos(e+fx)+(a-b)^2\cos(3(e+fx))+4b\sec(e+fx)\left(6a+b\sec^2(e+fx)-9b\right)}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((-9*a^2 + 42*a*b - 33*b^2)*Cos[e + f*x] + (a - b)^2*Cos[3*(e + f*x)] + 4*b
*Sec[e + f*x]*(6*a - 9*b + b*Sec[e + f*x]^2))/(12*f)
```

Maple [B] time = 0.049, size = 155, normalized size = 1.9

$$\frac{1}{f}\left(-\frac{a^2\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx+e\right)}{3}+2\,ab\left(\frac{\left(\sin\left(fx+e\right)\right)^6}{\cos\left(fx+e\right)}+\left(8/3+\left(\sin\left(fx+e\right)\right)^4+4/3\,\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(f\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-1/3*a^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a*b*(sin(f*x+e)^6/cos(f*x+e)+(8
/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e))+b^2*(1/3*sin(f*x+e)^8/cos(f*x
+e)^3-5/3*sin(f*x+e)^8/cos(f*x+e)-5/3*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8
/5*sin(f*x+e)^2)*cos(f*x+e)))
```

Maxima [A] time = 1.07297, size = 108, normalized size = 1.35

$$\frac{\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^3-3\left(a^2-4\,ab+3\,b^2\right)\cos\left(fx+e\right)+\frac{3\left(2\,ab-3\,b^2\right)\cos\left(fx+e\right)^2+b^2}{\cos\left(fx+e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - 3*(a^2 - 4*a*b + 3*b^2)*cos(f*x +
 e) + (3*(2*a*b - 3*b^2)*cos(f*x + e)^2 + b^2)/cos(f*x + e)^3)/f
```

Fricas [A] time = 1.93564, size = 193, normalized size = 2.41

$$\frac{\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^6-3\left(a^2-4\,ab+3\,b^2\right)\cos\left(fx+e\right)^4+3\left(2\,ab-3\,b^2\right)\cos\left(fx+e\right)^2+b^2}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((a^2 - 2*a*b + b^2)*cos(f*x + e)^6 - 3*(a^2 - 4*a*b + 3*b^2)*cos(f*x +
 e)^4 + 3*(2*a*b - 3*b^2)*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.74691, size = 194, normalized size = 2.42

$$\frac{6\,ab\cos\left(fx+e\right)^{2}-9\,b^{2}\cos\left(fx+e\right)^{2}+b^{2}}{3\,f\cos\left(fx+e\right)^{3}}+\frac{a^{2}f^{11}\cos\left(fx+e\right)^{3}-2\,abf^{11}\cos\left(fx+e\right)^{3}+b^{2}f^{11}\cos\left(fx+e\right)^{3}-3\,a^{2}}{3\,f^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(6*a*b*cos(f*x + e)^2 - 9*b^2*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
+ 1/3*(a^2*f^11*cos(f*x + e)^3 - 2*a*b*f^11*cos(f*x + e)^3 + b^2*f^11*cos(f
*x + e)^3 - 3*a^2*f^11*cos(f*x + e) + 12*a*b*f^11*cos(f*x + e) - 9*b^2*f^11
*cos(f*x + e))/f^12
```

3.45 $\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=54

$$-\frac{(a-b)^2\cos(e+fx)}{f}+\frac{2b(a-b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -(((a - b)^2*Cos[e + f*x])/f) + (2*(a - b)*b*Sec[e + f*x])/f + (b^2*Sec[e +
 f*x]^3)/(3*f)
```

Rubi [A] time = 0.045866, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3664, 270}

$$-\frac{(a-b)^2\cos(e+fx)}{f}+\frac{2b(a-b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(((a - b)^2*Cos[e + f*x])/f) + (2*(a - b)*b*Sec[e + f*x])/f + (b^2*Sec[e +
 f*x]^3)/(3*f)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a-b+bx^2\right)^2}{x^2}\,dx, x, \sec(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(2(a-b)b+\frac{(a-b)^2}{x^2}+b^2x^2\right)dx, x, \sec(e+fx)\right)}{f}\\ &= -\frac{(a-b)^2\cos(e+fx)}{f}+\frac{2(a-b)b\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A] time = 0.289056, size = 48, normalized size = 0.89

$$\frac{b\sec(e+fx)\left(6a+b\sec^2(e+fx)-6b\right)-3(a-b)^2\cos(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-3*(a - b)^2*Cos[e + f*x] + b*Sec[e + f*x]*(6*a - 6*b + b*Sec[e + f*x]^2))
/(3*f)
```

Maple [B] time = 0.049, size = 125, normalized size = 2.3

$$\frac{1}{f}\left(-a^2\cos\left(fx+e\right)+2\,ab\left(\frac{\left(\sin\left(fx+e\right)\right)^4}{\cos\left(fx+e\right)}+\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx+e\right)\right)+b^2\left(\frac{\left(\sin\left(fx+e\right)\right)^6}{3\left(\cos\left(fx+e\right)\right)^3}-\frac{\left(\sin\left(fx\right.\right.}{\cos\left(fx\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-a^2*cos(f*x+e)+2*a*b*(sin(f*x+e)^4/cos(f*x+e)+(2+sin(f*x+e)^2)*cos(f*
x+e))+b^2*(1/3*sin(f*x+e)^6/cos(f*x+e)^3-sin(f*x+e)^6/cos(f*x+e)-(8/3+sin(f
*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)))
```

Maxima [A] time = 0.979617, size = 96, normalized size = 1.78

$$\frac{6\,ab\left(\frac{1}{\cos\left(fx+e\right)}+\cos\left(fx+e\right)\right)-b^2\left(\frac{6\,\cos\left(fx+e\right)^2-1}{\cos\left(fx+e\right)^3}+3\,\cos\left(fx+e\right)\right)-3\,a^2\cos\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(6*a*b*(1/cos(f*x + e) + cos(f*x + e)) - b^2*((6*cos(f*x + e)^2 - 1)/co
s(f*x + e)^3 + 3*cos(f*x + e)) - 3*a^2*cos(f*x + e))/f
```

Fricas [A] time = 1.96447, size = 136, normalized size = 2.52

$$-\frac{3\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^4-6\left(ab-b^2\right)\cos\left(fx+e\right)^2-b^2}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^4 - 6*(a*b - b^2)*cos(f*x + e)^2 -
 b^2)/(f*cos(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan^2\left(e+fx\right)\right)^2\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**2*sin(e + f*x), x)
```

Giac [A] time = 1.76229, size = 127, normalized size = 2.35

$$-\frac{a^2f^3\cos\left(fx+e\right)-2\,abf^3\cos\left(fx+e\right)+b^2f^3\cos\left(fx+e\right)}{f^4}+\frac{6\,ab\cos\left(fx+e\right)^2-6\,b^2\cos\left(fx+e\right)^2+b^2}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -(a^2*f^3*cos(f*x + e) - 2*a*b*f^3*cos(f*x + e) + b^2*f^3*cos(f*x + e))/f^4
 + 1/3*(6*a*b*cos(f*x + e)^2 - 6*b^2*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^
3)
```

3.46 $\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=52

$$-\frac{a^2\tanh^{-1}(\cos(e+fx))}{f}+\frac{b(2a-b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -((a^2*ArcTanh[Cos[e + f*x]])/f) + ((2*a - b)*b*Sec[e + f*x])/f + (b^2*Sec[
e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.0549429, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3664, 390, 207}

$$-\frac{a^2\tanh^{-1}(\cos(e+fx))}{f}+\frac{b(2a-b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a^2*ArcTanh[Cos[e + f*x]])/f) + ((2*a - b)*b*Sec[e + f*x])/f + (b^2*Sec[
e + f*x]^3)/(3*f)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a-b+bx^2)^2}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left((2a-b)b+b^2x^2+\frac{a^2}{-1+x^2}\right)dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{(2a-b)b\sec(e+fx)}{f} + \frac{b^2\sec^3(e+fx)}{3f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= -\frac{a^2\tanh^{-1}(\cos(e+fx))}{f} + \frac{(2a-b)b\sec(e+fx)}{f} + \frac{b^2\sec^3(e+fx)}{3f}$$

Mathematica [A] time = 0.167076, size = 66, normalized size = 1.27

$$\frac{3a^2\left(\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)+3b(2a-b)\sec(e+fx)+b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (3*a^2*(-Log[Cos[(e + f*x)/2]] + Log[Sin[(e + f*x)/2]]) + 3*(2*a - b)*b*Sec
[e + f*x] + b^2*Sec[e + f*x]^3)/(3*f)
```

Maple [B] time = 0.05, size = 124, normalized size = 2.4

$$\frac{b^2\left(\sin\left(fx+e\right)\right)^4}{3f\left(\cos\left(fx+e\right)\right)^3} - \frac{b^2\left(\sin\left(fx+e\right)\right)^4}{3f\cos\left(fx+e\right)} - \frac{b^2\left(\sin\left(fx+e\right)\right)^2\cos\left(fx+e\right)}{3f} - \frac{2b^2\cos\left(fx+e\right)}{3f} + 2\frac{ab}{f\cos\left(fx+e\right)} + \frac{a^2\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/3/f*b^2*sin(f*x+e)^4/cos(f*x+e)^3-1/3/f*b^2*sin(f*x+e)^4/cos(f*x+e)-1/3/f
*b^2*sin(f*x+e)^2*cos(f*x+e)-2/3/f*b^2*cos(f*x+e)+2/f*a*b/cos(f*x+e)+1/f*a^
2*ln(csc(f*x+e)-cot(f*x+e))
```

Maxima [A] time = 1.05444, size = 92, normalized size = 1.77

$$-\frac{3a^2\log\left(\cos\left(fx+e\right)+1\right)-3a^2\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\left(2ab-b^2\right)\cos\left(fx+e\right)^2+b^2\right)}{\cos\left(fx+e\right)^3}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*a^2*log(cos(f*x + e) + 1) - 3*a^2*log(cos(f*x + e) - 1) - 2*(3*(2*a
*b - b^2)*cos(f*x + e)^2 + b^2)/cos(f*x + e)^3)/f
```

Fricas [A] time = 2.03636, size = 228, normalized size = 4.38

$$-\frac{3\,a^2\cos\left(fx+e\right)^3\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)-3\,a^2\cos\left(fx+e\right)^3\log\left(-\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)-6\left(2\,ab-b^2\right)\cos\left(fx+e\right.}{6\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*a^2*cos(f*x + e)^3*log(1/2*cos(f*x + e) + 1/2) - 3*a^2*cos(f*x + e)
^3*log(-1/2*cos(f*x + e) + 1/2) - 6*(2*a*b - b^2)*cos(f*x + e)^2 - 2*b^2)/(
f*cos(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan^2\left(e+fx\right)\right)^2\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**2*csc(e + f*x), x)
```

Giac [B] time = 1.70784, size = 197, normalized size = 3.79

$$\frac{3\,a^2\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)+\frac{8\left(3\,ab-b^2+\frac{6\,ab\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{3\,b^2\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{3\,ab\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}\right)}{\left(\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*a^2*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 8*(3*a*b - b^2 + 6
*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 3*b^2*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) + 3*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) + 1)^3)/f
```

3.47 $\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=82

$$-\frac{a^2\csc^2(e+fx)\sec(e+fx)}{2f}+\frac{a(a+4b)\sec(e+fx)}{2f}-\frac{a(a+4b)\tanh^{-1}(\cos(e+fx))}{2f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -(a*(a + 4*b)*ArcTanh[Cos[e + f*x]])/(2*f) + (a*(a + 4*b)*Sec[e + f*x])/(2*
f) - (a^2*Csc[e + f*x]^2*Sec[e + f*x])/(2*f) + (b^2*Sec[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.109035, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3664, 463, 459, 321, 207}

$$-\frac{a^2\csc^2(e+fx)\sec(e+fx)}{2f}+\frac{a(a+4b)\sec(e+fx)}{2f}-\frac{a(a+4b)\tanh^{-1}(\cos(e+fx))}{2f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(a*(a + 4*b)*ArcTanh[Cos[e + f*x]])/(2*f) + (a*(a + 4*b)*Sec[e + f*x])/(2*
f) - (a^2*Csc[e + f*x]^2*Sec[e + f*x])/(2*f) + (b^2*Sec[e + f*x]^3)/(3*f)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a-b+bx^2\right)^2}{\left(-1+x^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{a^2\csc^2(e+fx)\sec(e+fx)}{2f} + \frac{\text{Subst}\left(\int \frac{x^2\left(a^2+4ab-2b^2+2b^2x^2\right)}{-1+x^2}\,dx, x, \sec(e+\right.}{2f}$$

$$= -\frac{a^2\csc^2(e+fx)\sec(e+fx)}{2f} + \frac{b^2\sec^3(e+fx)}{3f} + \frac{(a(a+4b))\,\text{Subst}\left(\int \frac{x}{-1}\right.}{2f}$$

$$= \frac{a(a+4b)\sec(e+fx)}{2f} - \frac{a^2\csc^2(e+fx)\sec(e+fx)}{2f} + \frac{b^2\sec^3(e+fx)}{3f} + \frac{(}{}$$

$$= -\frac{a(a+4b)\tanh^{-1}(\cos(e+fx))}{2f} + \frac{a(a+4b)\sec(e+fx)}{2f} - \frac{a^2\csc^2(e+fx)}{2f}$$

Mathematica [B] time = 6.12979, size = 376, normalized size = 4.59

$$\frac{\left(a^2+4ab\right)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f} + \frac{\left(-a^2-4ab\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f} - \frac{a^2\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f} + \frac{a^2\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f} +$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(a^2*Csc[(e + f*x)/2]^2)/(8*f) + ((-a^2 - 4*a*b)*Log[Cos[(e + f*x)/2]])/(2
*f) + ((a^2 + 4*a*b)*Log[Sin[(e + f*x)/2]])/(2*f) + (a^2*Sec[(e + f*x)/2]^2
)/(8*f) + b^2/(12*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) + (b^2*Sin[(e
+ f*x)/2])/(6*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) - (b^2*Sin[(e + f*
x)/2])/(6*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3) + b^2/(12*f*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^2) + (-12*a*b*Sin[(e + f*x)/2] - b^2*Sin[(e + f
*x)/2])/(6*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) + (12*a*b*Sin[(e + f*x)
/2] + b^2*Sin[(e + f*x)/2])/(6*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A] time = 0.061, size = 100, normalized size = 1.2

$$\frac{b^2}{3f\left(\cos\left(fx+e\right)\right)^3} + 2\frac{ab}{f\cos\left(fx+e\right)} + 2\frac{ab\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f} - \frac{a^2\csc\left(fx+e\right)\cot\left(fx+e\right)}{2f} + \frac{a^2\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/3/f*b^2/cos(f*x+e)^3+2/f*a*b/cos(f*x+e)+2/f*a*b*ln(csc(f*x+e)-cot(f*x+e))
-1/2/f*a^2*csc(f*x+e)*cot(f*x+e)+1/2/f*a^2*ln(csc(f*x+e)-cot(f*x+e))
```

Maxima [A] time = 1.14151, size = 150, normalized size = 1.83

$$-\frac{3\left(a^{2}+4\,ab\right)\log\left(\cos\left(fx+e\right)+1\right)-3\left(a^{2}+4\,ab\right)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\left(a^{2}+4\,ab\right)\cos\left(fx+e\right)^{4}-2\left(6\,ab-b^{2}\right)\cos\left(fx+e\right)^{2}-2\,b^{2}\right)}{\cos\left(fx+e\right)^{5}-\cos\left(fx+e\right)^{3}}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(3*(a^2 + 4*a*b)*log(cos(f*x + e) + 1) - 3*(a^2 + 4*a*b)*log(cos(f*x
+ e) - 1) - 2*(3*(a^2 + 4*a*b)*cos(f*x + e)^4 - 2*(6*a*b - b^2)*cos(f*x + e
)^2 - 2*b^2)/(cos(f*x + e)^5 - cos(f*x + e)^3))/f
```

Fricas [B] time = 2.02787, size = 414, normalized size = 5.05

$$\frac{6\left(a^{2}+4\,ab\right)\cos\left(fx+e\right)^{4}-4\left(6\,ab-b^{2}\right)\cos\left(fx+e\right)^{2}-4\,b^{2}-3\left(\left(a^{2}+4\,ab\right)\cos\left(fx+e\right)^{5}-\left(a^{2}+4\,ab\right)\cos\left(fx+e\right)^{3}\right)\cdots}{12\left(f\cos\left(fx+e\right)^{5}-f\,c\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^2 + 4*a*b)*cos(f*x + e)^4 - 4*(6*a*b - b^2)*cos(f*x + e)^2 - 4*b
^2 - 3*((a^2 + 4*a*b)*cos(f*x + e)^5 - (a^2 + 4*a*b)*cos(f*x + e)^3)*log(1/
2*cos(f*x + e) + 1/2) + 3*((a^2 + 4*a*b)*cos(f*x + e)^5 - (a^2 + 4*a*b)*cos
(f*x + e)^3)*log(-1/2*cos(f*x + e) + 1/2))/(f*cos(f*x + e)^5 - f*cos(f*x +
e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.64372, size = 339, normalized size = 4.13

$$-\frac{\frac{3\,a^{2}\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-6\left(a^{2}+4\,ab\right)\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)-\frac{3\left(a^{2}-\frac{2\,a^{2}\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{8\,ab\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}\right)\left(\cos\left(fx+e\right)+1\right)}{\cos\left(fx+e\right)-1}-\frac{16\left(6\,ab+b^{2}+\frac{12\,ab\left(\cos\left(fx+e\right)\cdots\right.}{\cos\left(fx+e\right)+1\cdots}\cdots\right.}{\cdots}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/24*(3*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 6*(a^2 + 4*a*b)*log(-(
cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 3*(a^2 - 2*a^2*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) - 8*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(f*x +
 e) + 1)/(cos(f*x + e) - 1) - 16*(6*a*b + b^2 + 12*a*b*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) + 6*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 3*b^2
*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((cos(f*x + e) - 1)/(cos(f*x +
e) + 1) + 1)^3)/f
```

3.48 $\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=123

$$\frac{\left(a^2+8ab+4b^2\right)\sec(e+fx)}{4f}-\frac{\left(3a^2+24ab+8b^2\right)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{a^2\csc^4(e+fx)\sec(e+fx)}{4f}-\frac{a(a+8b)\cot(e}{}$$

```
[Out] -((3*a^2 + 24*a*b + 8*b^2)*ArcTanh[Cos[e + f*x]])/(8*f) - (a*(a + 8*b)*Cot[
e + f*x]*Csc[e + f*x])/(8*f) + ((a^2 + 8*a*b + 4*b^2)*Sec[e + f*x])/(4*f) -
 (a^2*Csc[e + f*x]^4*Sec[e + f*x])/(4*f) + (b^2*Sec[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.129721, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3664, 463, 455, 1153, 207}

$$\frac{\left(a^2+8ab+4b^2\right)\sec(e+fx)}{4f}-\frac{\left(3a^2+24ab+8b^2\right)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{a^2\csc^4(e+fx)\sec(e+fx)}{4f}-\frac{a(a+8b)\cot(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((3*a^2 + 24*a*b + 8*b^2)*ArcTanh[Cos[e + f*x]])/(8*f) - (a*(a + 8*b)*Cot[
e + f*x]*Csc[e + f*x])/(8*f) + ((a^2 + 8*a*b + 4*b^2)*Sec[e + f*x])/(4*f) -
 (a^2*Csc[e + f*x]^4*Sec[e + f*x])/(4*f) + (b^2*Sec[e + f*x]^3)/(3*f)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a-b+bx^2\right)^2}{\left(-1+x^2\right)^3}dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{a^2\csc^4(e+fx)\sec(e+fx)}{4f} + \frac{\text{Subst}\left(\int \frac{x^4\left(a^2+8ab-4b^2+4b^2x^2\right)}{\left(-1+x^2\right)^2}dx, x, \sec(e+\right.}{4f}$$

$$= -\frac{a(a+8b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{a^2\csc^4(e+fx)\sec(e+fx)}{4f} - \frac{\text{Subst}}{}$$

$$= -\frac{a(a+8b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{a^2\csc^4(e+fx)\sec(e+fx)}{4f} - \frac{\text{Subst}}{}$$

$$= -\frac{a(a+8b)\cot(e+fx)\csc(e+fx)}{8f} + \frac{\left(a^2+8ab+4b^2\right)\sec(e+fx)}{4f} - \frac{a^2 \text{cs}}{}$$

$$= -\frac{\left(3a^2+24ab+8b^2\right)\tanh^{-1}(\cos(e+fx))}{8f} - \frac{a(a+8b)\cot(e+fx)\csc(e+}{8f}$$

Mathematica [B] time = 6.18752, size = 447, normalized size = 3.63

$$\frac{\left(3a^2+24ab+8b^2\right)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{8f} + \frac{\left(-3a^2-24ab-8b^2\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{8f} + \frac{\left(-3a^2-8ab\right)\csc^2\left(\frac{1}{2}(e+fx\right.}{32f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((-3*a^2 - 8*a*b)*Csc[(e + f*x)/2]^2)/(32*f) - (a^2*Csc[(e + f*x)/2]^4)/(64
*f) + ((-3*a^2 - 24*a*b - 8*b^2)*Log[Cos[(e + f*x)/2]])/(8*f) + ((3*a^2 + 2
4*a*b + 8*b^2)*Log[Sin[(e + f*x)/2]])/(8*f) + ((3*a^2 + 8*a*b)*Sec[(e + f*x
)/2]^2)/(32*f) + (a^2*Sec[(e + f*x)/2]^4)/(64*f) + b^2/(12*f*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^2) + (b^2*Sin[(e + f*x)/2])/(6*f*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^3) - (b^2*Sin[(e + f*x)/2])/(6*f*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^3) + b^2/(12*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2) + (
-12*a*b*Sin[(e + f*x)/2] - 7*b^2*Sin[(e + f*x)/2])/(6*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])) + (12*a*b*Sin[(e + f*x)/2] + 7*b^2*Sin[(e + f*x)/2])/(6
*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A] time = 0.058, size = 183, normalized size = 1.5

$$\frac{b^2}{3f\left(\cos\left(fx+e\right)\right)^3}+\frac{b^2}{f\cos\left(fx+e\right)}+\frac{b^2\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f}-\frac{ab}{f\left(\sin\left(fx+e\right)\right)^2\cos\left(fx+e\right)}+3\frac{ab}{f\cos\left(fx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/3/f*b^2/cos(f*x+e)^3+1/f*b^2/cos(f*x+e)+1/f*b^2*ln(csc(f*x+e)-cot(f*x+e))
-1/f*a*b/sin(f*x+e)^2/cos(f*x+e)+3/f*a*b/cos(f*x+e)+3/f*a*b*ln(csc(f*x+e)-c
ot(f*x+e))-1/4/f*a^2*cot(f*x+e)*csc(f*x+e)^3-3/8/f*a^2*csc(f*x+e)*cot(f*x+e
)+3/8/f*a^2*ln(csc(f*x+e)-cot(f*x+e))
```

Maxima [A] time = 1.01137, size = 220, normalized size = 1.79

$$-\frac{3\left(3a^2+24ab+8b^2\right)\log\left(\cos\left(fx+e\right)+1\right)-3\left(3a^2+24ab+8b^2\right)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\left(3a^2+24ab+8b^2\right)\cos\left(fx+e\right.\right.}{\cos}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/48*(3*(3*a^2 + 24*a*b + 8*b^2)*log(cos(f*x + e) + 1) - 3*(3*a^2 + 24*a*b
 + 8*b^2)*log(cos(f*x + e) - 1) - 2*(3*(3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e
)^6 - 5*(3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(6*a*b + b^2)*cos(f*x +
 e)^2 + 8*b^2)/(cos(f*x + e)^7 - 2*cos(f*x + e)^5 + cos(f*x + e)^3))/f
```

Fricas [B] time = 2.15241, size = 699, normalized size = 5.68

$$\frac{6\left(3a^2+24ab+8b^2\right)\cos\left(fx+e\right)^6-10\left(3a^2+24ab+8b^2\right)\cos\left(fx+e\right)^4+16\left(6ab+b^2\right)\cos\left(fx+e\right)^2+16b^2-3\left(\left(3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(6*(3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)^6 - 10*(3*a^2 + 24*a*b + 8*b^
2)*cos(f*x + e)^4 + 16*(6*a*b + b^2)*cos(f*x + e)^2 + 16*b^2 - 3*((3*a^2 +
24*a*b + 8*b^2)*cos(f*x + e)^7 - 2*(3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)^5
+ (3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)^3)*log(1/2*cos(f*x + e) + 1/2) + 3*
((3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)^7 - 2*(3*a^2 + 24*a*b + 8*b^2)*cos(f
*x + e)^5 + (3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)^3)*log(-1/2*cos(f*x + e)
+ 1/2))/(f*cos(f*x + e)^7 - 2*f*cos(f*x + e)^5 + f*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.68271, size = 564, normalized size = 4.59

$$-\frac{\frac{24\,a^{2}\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{48\,ab\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{3\,a^{2}\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-12\left(3\,a^{2}+24\,ab+8\,b^{2}\right)\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)+\frac{3\left(a^{2}-\frac{8\,a^{2}\left(\cos\left(fx+e\right)-\right.}{\cos\left(fx+e\right)+1}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/192*(24*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 48*a*b*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) - 3*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
- 12*(3*a^2 + 24*a*b + 8*b^2)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1)) +
 3*(a^2 - 8*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*a*b*(cos(f*x + e
) - 1)/(cos(f*x + e) + 1) + 18*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^
2 + 144*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 48*b^2*(cos(f*x + e
) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x + e) - 1)^2 -
256*(3*a*b + 2*b^2 + 6*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 3*b^2*(c
os(f*x + e) - 1)/(cos(f*x + e) + 1) + 3*a*b*(cos(f*x + e) - 1)^2/(cos(f*x +
 e) + 1)^2 + 3*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((cos(f*x + e
) - 1)/(cos(f*x + e) + 1) + 1)^3)/f
```

3.49 $\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=122

$$-\frac{\left(a^2-10ab+13b^2\right)\tan(e+fx)}{4f}+\frac{1}{8}x\left(3a^2-30ab+35b^2\right)+\frac{(a-b)^2\sin^4(e+fx)\tan(e+fx)}{4f}-\frac{(a-9b)(a-b)\sin(e+$$

```
[Out] ((3*a^2 - 30*a*b + 35*b^2)*x)/8 - ((a - 9*b)*(a - b)*Cos[e + f*x]*Sin[e + f
*x])/(8*f) - ((a^2 - 10*a*b + 13*b^2)*Tan[e + f*x])/(4*f) + ((a - b)^2*Sin[
e + f*x]^4*Tan[e + f*x])/(4*f) + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.131028, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3663, 463, 455, 1153, 203}

$$-\frac{\left(a^2-10ab+13b^2\right)\tan(e+fx)}{4f}+\frac{1}{8}x\left(3a^2-30ab+35b^2\right)+\frac{(a-b)^2\sin^4(e+fx)\tan(e+fx)}{4f}-\frac{(a-9b)(a-b)\sin(e+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((3*a^2 - 30*a*b + 35*b^2)*x)/8 - ((a - 9*b)*(a - b)*Cos[e + f*x]*Sin[e + f
*x])/(8*f) - ((a^2 - 10*a*b + 13*b^2)*Tan[e + f*x])/(4*f) + ((a - b)^2*Sin[
e + f*x]^4*Tan[e + f*x])/(4*f) + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^2}{\left(1+x^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{(a-b)^2\sin^4(e+fx)\tan(e+fx)}{4f} - \frac{\text{Subst}\left(\int \frac{x^4\left(a^2-10ab+5b^2-4b^2x^2\right)}{\left(1+x^2\right)^2}\,dx, x, \tan\right.}{4f}$$

$$= -\frac{(a-9b)(a-b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{(a-b)^2\sin^4(e+fx)\tan(e+f}{4f}$$

$$= -\frac{(a-9b)(a-b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{(a-b)^2\sin^4(e+fx)\tan(e+f}{4f}$$

$$= -\frac{(a-9b)(a-b)\cos(e+fx)\sin(e+fx)}{8f} - \frac{\left(a^2-10ab+13b^2\right)\tan(e+fx)}{4f}$$

$$= \frac{1}{8}\left(3a^2-30ab+35b^2\right)x - \frac{(a-9b)(a-b)\cos(e+fx)\sin(e+fx)}{8f} - \frac{\left(a^2-\right.}{}$$

Mathematica [A] time = 1.4757, size = 96, normalized size = 0.79

$$\frac{12\left(3a^2-30ab+35b^2\right)(e+fx) - 24\left(a^2-4ab+3b^2\right)\sin(2(e+fx)) + 3(a-b)^2\sin(4(e+fx)) + 32b\tan(e+fx)\left(6a\right.}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (12*(3*a^2 - 30*a*b + 35*b^2)*(e + f*x) - 24*(a^2 - 4*a*b + 3*b^2)*Sin[2*(e
 + f*x)] + 3*(a - b)^2*Sin[4*(e + f*x)] + 32*b*(6*a - 10*b + b*Sec[e + f*x]
^2)*Tan[e + f*x])/(96*f)
```

Maple [A] time = 0.047, size = 199, normalized size = 1.6

$$\frac{1}{f}\left(a^2\left(-\frac{\cos\left(fx+e\right)}{4}\left(\left(\sin\left(fx+e\right)\right)^3+\frac{3\sin\left(fx+e\right)}{2}\right)+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(\frac{\left(\sin\left(fx+e\right)\right)^7}{\cos\left(fx+e\right)}+\left(\left(\sin\left(fx+e\right)\right)^5+5\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2*a*
b*(sin(f*x+e)^7/cos(f*x+e)+(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*
cos(f*x+e)-15/8*f*x-15/8*e)+b^2*(1/3*sin(f*x+e)^9/cos(f*x+e)^3-2*sin(f*x+e)
^9/cos(f*x+e)-2*(sin(f*x+e)^7+7/6*sin(f*x+e)^5+35/24*sin(f*x+e)^3+35/16*sin
(f*x+e))*cos(f*x+e)+35/8*f*x+35/8*e))
```

Maxima [A] time = 1.49387, size = 176, normalized size = 1.44

$$\frac{8\,b^2\tan\left(fx+e\right)^3+3\left(3\,a^2-30\,ab+35\,b^2\right)\left(fx+e\right)+24\left(2\,ab-3\,b^2\right)\tan\left(fx+e\right)-\frac{3\left(\left(5\,a^2-18\,ab+13\,b^2\right)\tan\left(fx+e\right)^3+\left(3\,a^2-14\,a\cdots\right.\right.}{\tan\left(fx+e\right)^4+2\,\tan\left(fx+e\right)^2\cdots}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(8*b^2*tan(f*x + e)^3 + 3*(3*a^2 - 30*a*b + 35*b^2)*(f*x + e) + 24*(2*
a*b - 3*b^2)*tan(f*x + e) - 3*((5*a^2 - 18*a*b + 13*b^2)*tan(f*x + e)^3 + (
3*a^2 - 14*a*b + 11*b^2)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 +
 1))/f
```

Fricas [A] time = 2.06931, size = 293, normalized size = 2.4

$$\frac{3\left(3\,a^2-30\,ab+35\,b^2\right)fx\cos\left(fx+e\right)^3+\left(6\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^6-3\left(5\,a^2-18\,ab+13\,b^2\right)\cos\left(fx+e\right)^4+16\left(\cdots\right.\right.}{24\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(3*a^2 - 30*a*b + 35*b^2)*f*x*cos(f*x + e)^3 + (6*(a^2 - 2*a*b + b^
2)*cos(f*x + e)^6 - 3*(5*a^2 - 18*a*b + 13*b^2)*cos(f*x + e)^4 + 16*(3*a*b
- 5*b^2)*cos(f*x + e)^2 + 8*b^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.50 $\int \sin^2(e+fx) \left(a+b\tan^2(e+fx)\right)^2 \, dx$

Optimal. Leaf size=85

$$-\frac{(a-5b)(a-b)\tan(e+fx)}{2f}+\frac{(a-b)^2\sin^2(e+fx)\tan(e+fx)}{2f}+\frac{1}{2}x(a-5b)(a-b)+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] ((a - 5*b)*(a - b)*x)/2 - ((a - 5*b)*(a - b)*Tan[e + f*x])/(2*f) + ((a - b)
^2*Sin[e + f*x]^2*Tan[e + f*x])/(2*f) + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.107452, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3663, 463, 459, 321, 203}

$$-\frac{(a-5b)(a-b)\tan(e+fx)}{2f}+\frac{(a-b)^2\sin^2(e+fx)\tan(e+fx)}{2f}+\frac{1}{2}x(a-5b)(a-b)+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((a - 5*b)*(a - b)*x)/2 - ((a - 5*b)*(a - b)*Tan[e + f*x])/(2*f) + ((a - b)
^2*Sin[e + f*x]^2*Tan[e + f*x])/(2*f) + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^2}{\left(1+x^2\right)^2}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{(a-b)^2\sin^2(e+fx)\tan(e+fx)}{2f} - \frac{\text{Subst}\left(\int \frac{x^2\left(a^2-6ab+3b^2-2b^2x^2\right)}{1+x^2}dx, x, \tan\right.}{2f}$$

$$= \frac{(a-b)^2\sin^2(e+fx)\tan(e+fx)}{2f} + \frac{b^2\tan^3(e+fx)}{3f} - \frac{((a-5b)(a-b))\,\text{Su}}{}$$

$$= -\frac{(a-5b)(a-b)\tan(e+fx)}{2f} + \frac{(a-b)^2\sin^2(e+fx)\tan(e+fx)}{2f} + \frac{b^2\tan}{}$$

$$= \frac{1}{2}(a-5b)(a-b)x - \frac{(a-5b)(a-b)\tan(e+fx)}{2f} + \frac{(a-b)^2\sin^2(e+fx)\tan}{2f}$$

Mathematica [A] time = 0.686095, size = 71, normalized size = 0.84

$$\frac{6\left(a^2-6ab+5b^2\right)(e+fx)-3(a-b)^2\sin(2(e+fx))+4b\tan(e+fx)\left(6a+b\sec^2(e+fx)-7b\right)}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (6*(a^2 - 6*a*b + 5*b^2)*(e + f*x) - 3*(a - b)^2*Sin[2*(e + f*x)] + 4*b*(6*
a - 7*b + b*Sec[e + f*x]^2)*Tan[e + f*x])/(12*f)
```

Maple [B] time = 0.045, size = 168, normalized size = 2.

$$\frac{1}{f}\left(a^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2ab\left(\frac{(\sin(fx+e))^5}{\cos(fx+e)}+\left((\sin(fx+e))^3+3/2\sin(fx+e)\right)\cos(fx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+2*a*b*(sin(f*x+e)^5/cos
(f*x+e)+(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/2*f*x-3/2*e)+b^2*(1/3*si
n(f*x+e)^7/cos(f*x+e)^3-4/3*sin(f*x+e)^7/cos(f*x+e)-4/3*(sin(f*x+e)^5+5/4*s
in(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/2*f*x+5/2*e))
```

Maxima [A] time = 1.58921, size = 117, normalized size = 1.38

$$\frac{2\,b^2\tan\left(fx+e\right)^3+3\left(a^2-6\,ab+5\,b^2\right)\left(fx+e\right)+12\left(ab-b^2\right)\tan\left(fx+e\right)-\frac{3\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)}{\tan\left(fx+e\right)^2+1}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^2*tan(f*x + e)^3 + 3*(a^2 - 6*a*b + 5*b^2)*(f*x + e) + 12*(a*b - b
^2)*tan(f*x + e) - 3*(a^2 - 2*a*b + b^2)*tan(f*x + e)/(tan(f*x + e)^2 + 1))
/f
```

Fricas [A] time = 1.9561, size = 224, normalized size = 2.64

$$\frac{3\left(a^2-6\,ab+5\,b^2\right)fx\cos\left(fx+e\right)^3-\left(3\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^4-2\left(6\,ab-7\,b^2\right)\cos\left(fx+e\right)^2-2\,b^2\right)\sin\left(fx+e\right)}{6\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(a^2 - 6*a*b + 5*b^2)*f*x*cos(f*x + e)^3 - (3*(a^2 - 2*a*b + b^2)*co
s(f*x + e)^4 - 2*(6*a*b - 7*b^2)*cos(f*x + e)^2 - 2*b^2)*sin(f*x + e))/(f*c
os(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan^2\left(e+fx\right)\right)^2\sin^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**2*sin(e + f*x)**2, x)
```

Giac [B] time = 2.2994, size = 1905, normalized size = 22.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*a^2*f*x*tan(f*x)^5*tan(e)^5 - 18*a*b*f*x*tan(f*x)^5*tan(e)^5 + 15*b^
2*f*x*tan(f*x)^5*tan(e)^5 + 3*a^2*f*x*tan(f*x)^5*tan(e)^3 - 18*a*b*f*x*tan(
f*x)^5*tan(e)^3 + 15*b^2*f*x*tan(f*x)^5*tan(e)^3 - 9*a^2*f*x*tan(f*x)^4*tan
```

```
(e)^4 + 54*a*b*f*x*tan(f*x)^4*tan(e)^4 - 45*b^2*f*x*tan(f*x)^4*tan(e)^4 + 3
*a^2*f*x*tan(f*x)^3*tan(e)^5 - 18*a*b*f*x*tan(f*x)^3*tan(e)^5 + 15*b^2*f*x*
tan(f*x)^3*tan(e)^5 + 3*a^2*tan(f*x)^5*tan(e)^4 - 18*a*b*tan(f*x)^5*tan(e)^
4 + 15*b^2*tan(f*x)^5*tan(e)^4 + 3*a^2*tan(f*x)^4*tan(e)^5 - 18*a*b*tan(f*x
)^4*tan(e)^5 + 15*b^2*tan(f*x)^4*tan(e)^5 - 9*a^2*f*x*tan(f*x)^4*tan(e)^2 +
 54*a*b*f*x*tan(f*x)^4*tan(e)^2 - 45*b^2*f*x*tan(f*x)^4*tan(e)^2 + 12*a^2*f
*x*tan(f*x)^3*tan(e)^3 - 72*a*b*f*x*tan(f*x)^3*tan(e)^3 + 60*b^2*f*x*tan(f*
x)^3*tan(e)^3 - 9*a^2*f*x*tan(f*x)^2*tan(e)^4 + 54*a*b*f*x*tan(f*x)^2*tan(e
)^4 - 45*b^2*f*x*tan(f*x)^2*tan(e)^4 - 12*a*b*tan(f*x)^5*tan(e)^2 + 10*b^2*
tan(f*x)^5*tan(e)^2 - 12*a^2*tan(f*x)^4*tan(e)^3 + 36*a*b*tan(f*x)^4*tan(e)
^3 - 30*b^2*tan(f*x)^4*tan(e)^3 - 12*a^2*tan(f*x)^3*tan(e)^4 + 36*a*b*tan(f
*x)^3*tan(e)^4 - 30*b^2*tan(f*x)^3*tan(e)^4 - 12*a*b*tan(f*x)^2*tan(e)^5 +
10*b^2*tan(f*x)^2*tan(e)^5 + 9*a^2*f*x*tan(f*x)^3*tan(e) - 54*a*b*f*x*tan(f
*x)^3*tan(e) + 45*b^2*f*x*tan(f*x)^3*tan(e) - 12*a^2*f*x*tan(f*x)^2*tan(e)^
2 + 72*a*b*f*x*tan(f*x)^2*tan(e)^2 - 60*b^2*f*x*tan(f*x)^2*tan(e)^2 + 9*a^2
*f*x*tan(f*x)*tan(e)^3 - 54*a*b*f*x*tan(f*x)*tan(e)^3 + 45*b^2*f*x*tan(f*x)
*tan(e)^3 - 2*b^2*tan(f*x)^5 + 24*a*b*tan(f*x)^4*tan(e) - 30*b^2*tan(f*x)^4
*tan(e) + 18*a^2*tan(f*x)^3*tan(e)^2 - 36*a*b*tan(f*x)^3*tan(e)^2 + 10*b^2*
tan(f*x)^3*tan(e)^2 + 18*a^2*tan(f*x)^2*tan(e)^3 - 36*a*b*tan(f*x)^2*tan(e)
^3 + 10*b^2*tan(f*x)^2*tan(e)^3 + 24*a*b*tan(f*x)*tan(e)^4 - 30*b^2*tan(f*x
)*tan(e)^4 - 2*b^2*tan(e)^5 - 3*a^2*f*x*tan(f*x)^2 + 18*a*b*f*x*tan(f*x)^2
- 15*b^2*f*x*tan(f*x)^2 + 9*a^2*f*x*tan(f*x)*tan(e) - 54*a*b*f*x*tan(f*x)*t
an(e) + 45*b^2*f*x*tan(f*x)*tan(e) - 3*a^2*f*x*tan(e)^2 + 18*a*b*f*x*tan(e)
^2 - 15*b^2*f*x*tan(e)^2 - 12*a*b*tan(f*x)^3 + 10*b^2*tan(f*x)^3 - 12*a^2*t
an(f*x)^2*tan(e) + 36*a*b*tan(f*x)^2*tan(e) - 30*b^2*tan(f*x)^2*tan(e) - 12
*a^2*tan(f*x)*tan(e)^2 + 36*a*b*tan(f*x)*tan(e)^2 - 30*b^2*tan(f*x)*tan(e)^
2 - 12*a*b*tan(e)^3 + 10*b^2*tan(e)^3 - 3*a^2*f*x + 18*a*b*f*x - 15*b^2*f*x
 + 3*a^2*tan(f*x) - 18*a*b*tan(f*x) + 15*b^2*tan(f*x) + 3*a^2*tan(e) - 18*a
*b*tan(e) + 15*b^2*tan(e))/(f*tan(f*x)^5*tan(e)^5 + f*tan(f*x)^5*tan(e)^3 -
 3*f*tan(f*x)^4*tan(e)^4 + f*tan(f*x)^3*tan(e)^5 - 3*f*tan(f*x)^4*tan(e)^2
+ 4*f*tan(f*x)^3*tan(e)^3 - 3*f*tan(f*x)^2*tan(e)^4 + 3*f*tan(f*x)^3*tan(e)
 - 4*f*tan(f*x)^2*tan(e)^2 + 3*f*tan(f*x)*tan(e)^3 - f*tan(f*x)^2 + 3*f*tan
(f*x)*tan(e) - f*tan(e)^2 - f)
```

3.51 $\int \left(a+b\tan^2(e+fx)\right)^2 dx$

Optimal. Leaf size=46

$$\frac{b(2a-b)\tan(e+fx)}{f}+x(a-b)^2+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] (a - b)^2*x + ((2*a - b)*b*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.0331447, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3661, 390, 203}

$$\frac{b(2a-b)\tan(e+fx)}{f}+x(a-b)^2+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (a - b)^2*x + ((2*a - b)*b*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\tan^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{1+x^2} dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left((2a-b)b+b^2x^2+\frac{(a-b)^2}{1+x^2}\right) dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{(2a-b)b\tan(e+fx)}{f} + \frac{b^2\tan^3(e+fx)}{3f} + \frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \tan(e+fx)\right)}{f}$$
$$= (a-b)^2x + \frac{(2a-b)b\tan(e+fx)}{f} + \frac{b^2\tan^3(e+fx)}{3f}$$

Mathematica [A] time = 0.717783, size = 73, normalized size = 1.59

$$\frac{\tan(e+fx)\left(b\left(6a-b\left(3-\tan^2(e+fx)\right)\right)+\frac{3(a-b)^2\tanh^{-1}\left(\sqrt{-\tan^2(e+fx)}\right)}{\sqrt{-\tan^2(e+fx)}}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (Tan[e + f*x]*((3*(a - b)^2*ArcTanh[Sqrt[-Tan[e + f*x]^2]])/Sqrt[-Tan[e + f
*x]^2] + b*(6*a - b*(3 - Tan[e + f*x]^2))))/(3*f)
```

Maple [A] time = 0.003, size = 87, normalized size = 1.9

$$\frac{b^2\left(\tan\left(fx+e\right)\right)^3}{3f} + 2\frac{ab\tan\left(fx+e\right)}{f} - \frac{b^2\tan\left(fx+e\right)}{f} + \frac{\arctan\left(\tan\left(fx+e\right)\right)a^2}{f} - 2\frac{\arctan\left(\tan\left(fx+e\right)\right)ab}{f} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/3*b^2*tan(f*x+e)^3/f+2*a*b*tan(f*x+e)/f-b^2*tan(f*x+e)/f+1/f*arctan(tan(f
*x+e))*a^2-2/f*arctan(tan(f*x+e))*a*b+1/f*arctan(tan(f*x+e))*b^2
```

Maxima [A] time = 1.46565, size = 78, normalized size = 1.7

$$a^2x - \frac{2\left(fx+e-\tan\left(fx+e\right)\right)ab}{f} + \frac{\left(\tan\left(fx+e\right)^3+3fx+3e-3\tan\left(fx+e\right)\right)b^2}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x - 2*(f*x + e - tan(f*x + e))*a*b/f + 1/3*(tan(f*x + e)^3 + 3*f*x + 3*
e - 3*tan(f*x + e))*b^2/f
```

Fricas [A] time = 1.91938, size = 117, normalized size = 2.54

$$\frac{b^2\tan\left(fx+e\right)^3+3\left(a^2-2\,ab+b^2\right)fx+3\left(2\,ab-b^2\right)\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)*f*x + 3*(2*a*b - b^2)*tan(f
*x + e))/f
```

Sympy [A] time = 0.430802, size = 68, normalized size = 1.48

$$\begin{cases} a^2x-2abx+\frac{2ab\tan\left(e+fx\right)}{f}+b^2x+\frac{b^2\tan^3\left(e+fx\right)}{3f}-\frac{b^2\tan\left(e+fx\right)}{f} & \text{for } f\neq 0\\ x\left(a+b\tan^2\left(e\right)\right)^2 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((a**2*x - 2*a*b*x + 2*a*b*tan(e + f*x)/f + b**2*x + b**2*tan(e +
f*x)**3/(3*f) - b**2*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e)**2)**2, Tr
ue))
```

Giac [B] time = 1.46614, size = 516, normalized size = 11.22

$$\frac{3\,a^2fx\tan\left(fx\right)^3\tan\left(e\right)^3-6\,abfx\tan\left(fx\right)^3\tan\left(e\right)^3+3\,b^2fx\tan\left(fx\right)^3\tan\left(e\right)^3-9\,a^2fx\tan\left(fx\right)^2\tan\left(e\right)^2+18\,abfx\,t}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*f*x*tan(f*x)^3*tan(e)^3 - 6*a*b*f*x*tan(f*x)^3*tan(e)^3 + 3*b^2*
f*x*tan(f*x)^3*tan(e)^3 - 9*a^2*f*x*tan(f*x)^2*tan(e)^2 + 18*a*b*f*x*tan(f*
x)^2*tan(e)^2 - 9*b^2*f*x*tan(f*x)^2*tan(e)^2 - 6*a*b*tan(f*x)^3*tan(e)^2 +
 3*b^2*tan(f*x)^3*tan(e)^2 - 6*a*b*tan(f*x)^2*tan(e)^3 + 3*b^2*tan(f*x)^2*t
an(e)^3 + 9*a^2*f*x*tan(f*x)*tan(e) - 18*a*b*f*x*tan(f*x)*tan(e) + 9*b^2*f*
x*tan(f*x)*tan(e) - b^2*tan(f*x)^3 + 12*a*b*tan(f*x)^2*tan(e) - 9*b^2*tan(f
*x)^2*tan(e) + 12*a*b*tan(f*x)*tan(e)^2 - 9*b^2*tan(f*x)*tan(e)^2 - b^2*tan
(e)^3 - 3*a^2*f*x + 6*a*b*f*x - 3*b^2*f*x - 6*a*b*tan(f*x) + 3*b^2*tan(f*x)
 - 6*a*b*tan(e) + 3*b^2*tan(e))/(f*tan(f*x)^3*tan(e)^3 - 3*f*tan(f*x)^2*tan
(e)^2 + 3*f*tan(f*x)*tan(e) - f)
```

3.52 $\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=46

$$-\frac{a^2\cot(e+fx)}{f}+\frac{2ab\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] -((a^2*Cot[e + f*x])/f) + (2*a*b*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*
f)
```

Rubi [A] time = 0.0544183, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3663, 270}

$$-\frac{a^2\cot(e+fx)}{f}+\frac{2ab\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a^2*Cot[e + f*x])/f) + (2*a*b*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*
f)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(2ab+\frac{a^2}{x^2}+b^2x^2\right)dx,x,\tan(e+fx)\right)}{f}\\
&= -\frac{a^2\cot(e+fx)}{f}+\frac{2ab\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [A] time = 0.468013, size = 44, normalized size = 0.96

$$\frac{b\tan(e+fx)\left(6a+b\sec^2(e+fx)-b\right)-3a^2\cot(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-3*a^2*Cot[e + f*x] + b*(6*a - b + b*Sec[e + f*x]^2)*Tan[e + f*x])/(3*f)
```

Maple [A] time = 0.046, size = 48, normalized size = 1.

$$\frac{1}{f}\left(\frac{b^2(\sin(fx+e))^3}{3(\cos(fx+e))^3}+2\tan(fx+e)ab-a^2\cot(fx+e)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(1/3*b^2*sin(f*x+e)^3/cos(f*x+e)^3+2*tan(f*x+e)*a*b-a^2*cot(f*x+e))
```

Maxima [A] time = 1.02274, size = 55, normalized size = 1.2

$$\frac{b^2\tan(fx+e)^3+6ab\tan(fx+e)-\frac{3a^2}{\tan(fx+e)}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) - 3*a^2/tan(f*x + e))/f
```

Fricas [A] time = 1.87401, size = 157, normalized size = 3.41

$$-\frac{(3a^2+6ab-b^2)\cos(fx+e)^4-2(3ab-b^2)\cos(fx+e)^2-b^2}{3f\cos(fx+e)^3\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*((3*a^2 + 6*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a*b - b^2)*cos(f*x + e)^2
 - b^2)/(f*cos(f*x + e)^3*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan^2(e+fx)\right)^2\csc^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**2*csc(e + f*x)**2, x)
```

Giac [A] time = 1.64249, size = 59, normalized size = 1.28

$$\frac{b^2 \tan\left(fx+e\right)^3 + 6\,ab \tan\left(fx+e\right) - \frac{3\,a^2}{\tan\left(fx+e\right)}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) - 3*a^2/tan(f*x + e))/f
```

3.53 $\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=70

$$-\frac{a^2\cot^3(e+fx)}{3f}+\frac{b(2a+b)\tan(e+fx)}{f}-\frac{a(a+2b)\cot(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] -((a*(a + 2*b)*Cot[e + f*x])/f) - (a^2*Cot[e + f*x]^3)/(3*f) + (b*(2*a + b)
*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.0724582, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3663, 448}

$$-\frac{a^2\cot^3(e+fx)}{3f}+\frac{b(2a+b)\tan(e+fx)}{f}-\frac{a(a+2b)\cot(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a*(a + 2*b)*Cot[e + f*x])/f) - (a^2*Cot[e + f*x]^3)/(3*f) + (b*(2*a + b)
*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)\left(a+bx^2\right)^2}{x^4}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(b(2a+b)+\frac{a^2}{x^4}+\frac{a(a+2b)}{x^2}+b^2x^2\right)dx,x,\tan(e+fx)\right)}{f}\\ &= -\frac{a(a+2b)\cot(e+fx)}{f}-\frac{a^2\cot^3(e+fx)}{3f}+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A] time = 0.441543, size = 59, normalized size = 0.84

$$\frac{b\tan(e+fx)\left(6a+b\sec^2(e+fx)+2b\right)-a\cot(e+fx)\left(a\csc^2(e+fx)+2a+6b\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-(a*Cot[e + f*x]*(2*a + 6*b + a*Csc[e + f*x]^2)) + b*(6*a + 2*b + b*Sec[e
+ f*x]^2)*Tan[e + f*x])/(3*f)
```

Maple [A] time = 0.082, size = 81, normalized size = 1.2

$$\frac{1}{f}\left(-b^2\left(-\frac{2}{3}-\frac{\left(\sec\left(fx+e\right)\right)^2}{3}\right)\tan\left(fx+e\right)+2\,ab\left(\frac{1}{\cos\left(fx+e\right)\sin\left(fx+e\right)}-2\,\cot\left(fx+e\right)\right)+a^2\left(-\frac{2}{3}-\frac{\left(\csc\left(fx\right.\right.}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-b^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+2*a*b*(1/sin(f*x+e)/cos(f*x+e)
-2*cot(f*x+e))+a^2*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e))
```

Maxima [A] time = 0.979427, size = 89, normalized size = 1.27

$$\frac{b^2\tan\left(fx+e\right)^3+3\left(2\,ab+b^2\right)\tan\left(fx+e\right)-\frac{3\left(a^2+2\,ab\right)\tan\left(fx+e\right)^2+a^2}{\tan\left(fx+e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(2*a*b + b^2)*tan(f*x + e) - (3*(a^2 + 2*a*b)*t
an(f*x + e)^2 + a^2)/tan(f*x + e)^3)/f
```

Fricas [A] time = 1.95461, size = 224, normalized size = 3.2

$$-\frac{2\left(a^2+6\,ab+b^2\right)\cos\left(fx+e\right)^6-3\left(a^2+6\,ab+b^2\right)\cos\left(fx+e\right)^4+6\,ab\cos\left(fx+e\right)^2+b^2}{3\left(f\cos\left(fx+e\right)^5-f\cos\left(fx+e\right)^3\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^2 + 6*a*b + b^2)*cos(f*x + e)^6 - 3*(a^2 + 6*a*b + b^2)*cos(f*x
+ e)^4 + 6*a*b*cos(f*x + e)^2 + b^2)/((f*cos(f*x + e)^5 - f*cos(f*x + e)^3)
*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.65381, size = 113, normalized size = 1.61

$$\frac{b^2\tan\left(fx+e\right)^3+6\,ab\tan\left(fx+e\right)+3\,b^2\tan\left(fx+e\right)-\frac{3\,a^2\tan\left(fx+e\right)^2+6\,ab\tan\left(fx+e\right)^2+a^2}{\tan\left(fx+e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) + 3*b^2*tan(f*x + e) - (3*a^2*
tan(f*x + e)^2 + 6*a*b*tan(f*x + e)^2 + a^2)/tan(f*x + e)^3)/f
```

3.54 $\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=93

$$-\frac{(a^2+4ab+b^2)\cot(e+fx)}{f}-\frac{a^2\cot^5(e+fx)}{5f}+\frac{2b(a+b)\tan(e+fx)}{f}-\frac{2a(a+b)\cot^3(e+fx)}{3f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] -(((a^2 + 4*a*b + b^2)*Cot[e + f*x])/f) - (2*a*(a + b)*Cot[e + f*x]^3)/(3*f
) - (a^2*Cot[e + f*x]^5)/(5*f) + (2*b*(a + b)*Tan[e + f*x])/f + (b^2*Tan[e
+ f*x]^3)/(3*f)
```

Rubi [A] time = 0.0898972, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3663, 448}

$$-\frac{(a^2+4ab+b^2)\cot(e+fx)}{f}-\frac{a^2\cot^5(e+fx)}{5f}+\frac{2b(a+b)\tan(e+fx)}{f}-\frac{2a(a+b)\cot^3(e+fx)}{3f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(((a^2 + 4*a*b + b^2)*Cot[e + f*x])/f) - (2*a*(a + b)*Cot[e + f*x]^3)/(3*f
) - (a^2*Cot[e + f*x]^5)/(5*f) + (2*b*(a + b)*Tan[e + f*x])/f + (b^2*Tan[e
+ f*x]^3)/(3*f)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2(a+bx^2)^2}{x^6}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(2b(a+b)+\frac{a^2}{x^6}+\frac{2a(a+b)}{x^4}+\frac{a^2+4ab+b^2}{x^2}+b^2x^2\right)dx,x,\tan(e+fx)\right)}{f}\\
&= -\frac{(a^2+4ab+b^2)\cot(e+fx)}{f}-\frac{2a(a+b)\cot^3(e+fx)}{3f}-\frac{a^2\cot^5(e+fx)}{5f}
\end{aligned}$$

Mathematica [A] time = 0.774006, size = 88, normalized size = 0.95

$$\frac{5b\tan(e+fx)\left(6a+b\sec^2(e+fx)+5b\right)-\cot(e+fx)\left(3a^2\csc^4(e+fx)+8a^2+2a(2a+5b)\csc^2(e+fx)+50ab+15b\right.}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-(Cot[e + f*x]*(8*a^2 + 50*a*b + 15*b^2 + 2*a*(2*a + 5*b)*Csc[e + f*x]^2 +
 3*a^2*Csc[e + f*x]^4)) + 5*b*(6*a + 5*b + b*Sec[e + f*x]^2)*Tan[e + f*x])/
(15*f)
```

Maple [A] time = 0.059, size = 136, normalized size = 1.5

$$\frac{1}{f}\left(b^2\left(\frac{1}{3\sin(fx+e)(\cos(fx+e))^3}+\frac{4}{3\cos(fx+e)\sin(fx+e)}-\frac{8\cot(fx+e)}{3}\right)+2ab\left(-1/3\frac{1}{(\sin(fx+e))^3\cos(f\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(b^2*(1/3/sin(f*x+e)/cos(f*x+e)^3+4/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*x
+e))+2*a*b*(-1/3/sin(f*x+e)^3/cos(f*x+e)+4/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(
f*x+e))+a^2*(-8/15-1/5*csc(f*x+e)^4-4/15*csc(f*x+e)^2)*cot(f*x+e))
```

Maxima [A] time = 0.966183, size = 119, normalized size = 1.28

$$\frac{5\,b^2\tan\left(fx+e\right)^3+30\left(ab+b^2\right)\tan\left(fx+e\right)-\frac{15\left(a^2+4\,ab+b^2\right)\tan\left(fx+e\right)^4+10\left(a^2+ab\right)\tan\left(fx+e\right)^2+3\,a^2}{\tan\left(fx+e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(5*b^2*tan(f*x + e)^3 + 30*(a*b + b^2)*tan(f*x + e) - (15*(a^2 + 4*a*b
 + b^2)*tan(f*x + e)^4 + 10*(a^2 + a*b)*tan(f*x + e)^2 + 3*a^2)/tan(f*x + e
)^5)/f
```

Fricas [A] time = 2.0376, size = 339, normalized size = 3.65

$$-\frac{8\left(a^2+10\,ab+5\,b^2\right)\cos\left(fx+e\right)^8-20\left(a^2+10\,ab+5\,b^2\right)\cos\left(fx+e\right)^6+15\left(a^2+10\,ab+5\,b^2\right)\cos\left(fx+e\right)^4-10\left(3\,a\right.}{15\left(f\cos\left(fx+e\right)^7-2\,f\cos\left(fx+e\right)^5+f\cos\left(fx+e\right)^3\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a^2 + 10*a*b + 5*b^2)*cos(f*x + e)^8 - 20*(a^2 + 10*a*b + 5*b^2)*
cos(f*x + e)^6 + 15*(a^2 + 10*a*b + 5*b^2)*cos(f*x + e)^4 - 10*(3*a*b + b^2
)*cos(f*x + e)^2 - 5*b^2)/((f*cos(f*x + e)^7 - 2*f*cos(f*x + e)^5 + f*cos(f
*x + e)^3)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.67698, size = 173, normalized size = 1.86

$$\frac{5\,b^2\tan\left(fx+e\right)^3+30\,ab\tan\left(fx+e\right)+30\,b^2\tan\left(fx+e\right)-\frac{15\,a^2\tan\left(fx+e\right)^4+60\,ab\tan\left(fx+e\right)^4+15\,b^2\tan\left(fx+e\right)^4+10\,a^2\tan\left(fx+e\right.}{\tan\left(fx+e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(5*b^2*tan(f*x + e)^3 + 30*a*b*tan(f*x + e) + 30*b^2*tan(f*x + e) - (1
5*a^2*tan(f*x + e)^4 + 60*a*b*tan(f*x + e)^4 + 15*b^2*tan(f*x + e)^4 + 10*a
^2*tan(f*x + e)^2 + 10*a*b*tan(f*x + e)^2 + 3*a^2)/tan(f*x + e)^5)/f
```

3.55 $\int \frac{\sin^5(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=117

$$-\frac{a^2\cos(e+fx)}{f(a-b)^3}-\frac{a^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{f(a-b)^{7/2}}-\frac{\cos^5(e+fx)}{5f(a-b)}+\frac{(2a-b)\cos^3(e+fx)}{3f(a-b)^2}$$

```
[Out] -((a^2*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/((a - b)^(7/2)*f
)) - (a^2*Cos[e + f*x])/((a - b)^3*f) + ((2*a - b)*Cos[e + f*x]^3)/(3*(a -
b)^2*f) - Cos[e + f*x]^5/(5*(a - b)*f)
```

Rubi [A] time = 0.184031, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3664, 461, 205}

$$-\frac{a^2\cos(e+fx)}{f(a-b)^3}-\frac{a^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{f(a-b)^{7/2}}-\frac{\cos^5(e+fx)}{5f(a-b)}+\frac{(2a-b)\cos^3(e+fx)}{3f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```


```
[Out] -((a^2*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/((a - b)^(7/2)*f
)) - (a^2*Cos[e + f*x])/((a - b)^3*f) + ((2*a - b)*Cos[e + f*x]^3)/(3*(a -
b)^2*f) - Cos[e + f*x]^5/(5*(a - b)*f)
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^6\left(a-b+bx^2\right)}\,dx,x,\sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a-b)x^6}+\frac{-2a+b}{(a-b)^2x^4}+\frac{a^2}{(a-b)^3x^2}-\frac{a^2b}{(a-b)^3\left(a-b+bx^2\right)}\right)dx,x,\sec(e+fx)\right)}{f}$$

$$= -\frac{a^2\cos(e+fx)}{(a-b)^3f}+\frac{(2a-b)\cos^3(e+fx)}{3(a-b)^2f}-\frac{\cos^5(e+fx)}{5(a-b)f}-\frac{\left(a^2b\right)\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx,x,\text{se}\right.}{(a-b)^3f}$$

$$= -\frac{a^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{(a-b)^{7/2}f}-\frac{a^2\cos(e+fx)}{(a-b)^3f}+\frac{(2a-b)\cos^3(e+fx)}{3(a-b)^2f}-\frac{\cos^5(e+fx)}{5(a-b)f}$$

Mathematica [A] time = 3.04949, size = 177, normalized size = 1.51

$$\frac{\sqrt{a-b}\cos(e+fx)\left(4\left(7a^2-9ab+2b^2\right)\cos(2(e+fx))-89a^2-3(a-b)^2\cos(4(e+fx))-42ab+11b^2\right)+120a^2\sqrt{b}\,\text{t}}{120f(a-b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (120*a^2*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]] +
 120*a^2*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]] +
 Sqrt[a - b]*Cos[e + f*x]*(-89*a^2 - 42*a*b + 11*b^2 + 4*(7*a^2 - 9*a*b + 2
*b^2)*Cos[2*(e + f*x)] - 3*(a - b)^2*Cos[4*(e + f*x)]))/(120*(a - b)^(7/2)*
f)
```

Maple [A] time = 0.062, size = 205, normalized size = 1.8

$$-\frac{\left(\cos\left(fx+e\right)\right)^5a^2}{5f\left(a-b\right)^3}+\frac{2\left(\cos\left(fx+e\right)\right)^5ab}{5f\left(a-b\right)^3}-\frac{\left(\cos\left(fx+e\right)\right)^5b^2}{5f\left(a-b\right)^3}+\frac{2\left(\cos\left(fx+e\right)\right)^3a^2}{3f\left(a-b\right)^3}-\frac{\left(\cos\left(fx+e\right)\right)^3ab}{f\left(a-b\right)^3}+\frac{b^2\left(\text{co}\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/5/f/(a-b)^3*cos(f*x+e)^5*a^2+2/5/f/(a-b)^3*cos(f*x+e)^5*a*b-1/5/f/(a-b)^
3*cos(f*x+e)^5*b^2+2/3/f/(a-b)^3*cos(f*x+e)^3*a^2-1/f/(a-b)^3*cos(f*x+e)^3*
a*b+1/3/f/(a-b)^3*b^2*cos(f*x+e)^3-a^2*cos(f*x+e)/(a-b)^3/f+1/f*a^2*b/(a-b)
^3/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.38719, size = 675, normalized size = 5.77

$$\left[-\frac{6\left(a^2-2ab+b^2\right)\cos\left(fx+e\right)^5-10\left(2a^2-3ab+b^2\right)\cos\left(fx+e\right)^3+15a^2\sqrt{-\frac{b}{a-b}}\log\left(-\frac{(a-b)\cos\left(fx+e\right)^2-2(a-b)\sqrt{-\frac{b}{a-b}}\cos(\cdots}{(a-b)\cos\left(fx+e\right)^2+b}\cdots\right.}{30\left(a^3-3a^2b+3ab^2-b^3\right)f}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(6*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 10*(2*a^2 - 3*a*b + b^2)*cos
(f*x + e)^3 + 15*a^2*sqrt(-b/(a - b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a -
 b)*sqrt(-b/(a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 30*a
^2*cos(f*x + e))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f), -1/15*(3*(a^2 - 2*a*b
 + b^2)*cos(f*x + e)^5 - 5*(2*a^2 - 3*a*b + b^2)*cos(f*x + e)^3 + 15*a^2*sq
rt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + 15*a^2*cos(
f*x + e))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42267, size = 509, normalized size = 4.35

$$-\frac{\frac{15a^2b\arctan\left(-\frac{a\cos\left(fx+e\right)-b\cos\left(fx+e\right)-b}{\sqrt{ab-b^2}\cos\left(fx+e\right)+\sqrt{ab-b^2}}\right)}{\left(a^3-3a^2b+3ab^2-b^3\right)\sqrt{ab-b^2}}-\frac{2\left(8a^2+9ab-2b^2-\frac{40a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{30ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{10b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{80a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{10b^2(\cos(fx+e\cdots}{(\cos(fx+e)+\cdots}\cdots\right.}{\left(a^3-3a^2b+3ab^2-b^3\right)\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right)^5}\cdots}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*a^2*b*arctan(-(a*cos(f*x + e) - b*cos(f*x + e) - b)/(sqrt(a*b - b
^2)*cos(f*x + e) + sqrt(a*b - b^2)))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(
a*b - b^2)) - 2*(8*a^2 + 9*a*b - 2*b^2 - 40*a^2*(cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) - 30*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 10*b^2*(cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) + 80*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2 + 10*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 90*a*b*(cos(f*x
+ e) - 1)^3/(cos(f*x + e) + 1)^3 + 30*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e
) + 1)^3 + 15*a*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((a^3 - 3*a^2*
b + 3*a*b^2 - b^3)*((cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)^5))/f
```

3.56 $\int \frac{\sin^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=84

$$\frac{\cos^3(e+fx)}{3f(a-b)}-\frac{a\cos(e+fx)}{f(a-b)^2}-\frac{a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

```
[Out] -((a*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/((a - b)^(5/2)*f))
 - (a*Cos[e + f*x])/((a - b)^2*f) + Cos[e + f*x]^3/(3*(a - b)*f)
```

Rubi [A] time = 0.122553, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3664, 453, 325, 205}

$$\frac{\cos^3(e+fx)}{3f(a-b)}-\frac{a\cos(e+fx)}{f(a-b)^2}-\frac{a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/((a - b)^(5/2)*f))
 - (a*Cos[e + f*x])/((a - b)^2*f) + Cos[e + f*x]^3/(3*(a - b)*f)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4\left(a-b+bx^2\right)}\,dx,x,\sec(e+fx)\right)}{f}$$
$$= \frac{\cos^3(e+fx)}{3(a-b)f} + \frac{a\,\text{Subst}\left(\int \frac{1}{x^2\left(a-b+bx^2\right)}\,dx,x,\sec(e+fx)\right)}{(a-b)f}$$
$$= -\frac{a\cos(e+fx)}{(a-b)^2f} + \frac{\cos^3(e+fx)}{3(a-b)f} - \frac{(ab)\,\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx,x,\sec(e+fx)\right)}{(a-b)^2f}$$
$$= -\frac{a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} - \frac{a\cos(e+fx)}{(a-b)^2f} + \frac{\cos^3(e+fx)}{3(a-b)f}$$

Mathematica [A] time = 0.653202, size = 149, normalized size = 1.77

$$\frac{(a-b)\cos(e+fx)((a-b)\cos(2(e+fx))-5a-b)+6a\sqrt{b}\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)+6a\sqrt{b}\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\ldots}{\ldots}\right.}{6f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (6*a*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sq
rt[b]] + 6*a*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x
)/2])/Sqrt[b]] + (a - b)*Cos[e + f*x]*(-5*a - b + (a - b)*Cos[2*(e + f*x)])
)/(6*(a - b)^3*f)
```

Maple [A] time = 0.055, size = 107, normalized size = 1.3

$$\frac{a\left(\cos\left(fx+e\right)\right)^3}{3f(a-b)^2} - \frac{b\left(\cos\left(fx+e\right)\right)^3}{3f(a-b)^2} - \frac{\cos\left(fx+e\right)a}{f(a-b)^2} + \frac{ab}{f(a-b)^2}\arctan\left((a-b)\cos\left(fx+e\right)\frac{1}{\sqrt{b(a-b)}}\right)\frac{1}{\sqrt{b(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/3/f/(a-b)^2*a*cos(f*x+e)^3-1/3/f/(a-b)^2*b*cos(f*x+e)^3-a*cos(f*x+e)/(a-b
)^2/f+1/f*a*b/(a-b)^2/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/
2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.47034, size = 473, normalized size = 5.63

$$\left[\frac{2\,(a-b)\cos\left(fx+e\right)^{3}+3\,a\sqrt{-\frac{b}{a-b}}\log\left(\frac{(a-b)\cos\left(fx+e\right)^{2}+2\,(a-b)\sqrt{-\frac{b}{a-b}}\cos\left(fx+e\right)-b}{(a-b)\cos\left(fx+e\right)^{2}+b}\right)-6\,a\cos\left(fx+e\right)}{6\left(a^{2}-2\,ab+b^{2}\right)f}, \frac{(a-b)\cos\left(fx+e\right)^{3}\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(a - b)*cos(f*x + e)^3 + 3*a*sqrt(-b/(a - b))*log(((a - b)*cos(f*x
+ e)^2 + 2*(a - b)*sqrt(-b/(a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)
^2 + b)) - 6*a*cos(f*x + e))/((a^2 - 2*a*b + b^2)*f), 1/3*((a - b)*cos(f*x
+ e)^3 - 3*a*sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b
) - 3*a*cos(f*x + e))/((a^2 - 2*a*b + b^2)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.4162, size = 243, normalized size = 2.89

$$\frac{ab\arctan\left(\frac{a\cos\left(fx+e\right)-b\cos\left(fx+e\right)}{\sqrt{ab-b^{2}}}\right)}{\left(a^{2}-2\,ab+b^{2}\right)\sqrt{ab-b^{2}}f}+\frac{a^{2}f^{5}\cos\left(fx+e\right)^{3}-2\,abf^{5}\cos\left(fx+e\right)^{3}+b^{2}f^{5}\cos\left(fx+e\right)^{3}-3\,a^{2}f^{5}\cos\left(fx+\ldots\right.}{3\left(a^{3}f^{6}-3\,a^{2}bf^{6}+3\,ab^{2}f^{6}-b^{3}f^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] a*b*arctan((a*cos(f*x + e) - b*cos(f*x + e))/sqrt(a*b - b^2))/((a^2 - 2*a*b
 + b^2)*sqrt(a*b - b^2)*f) + 1/3*(a^2*f^5*cos(f*x + e)^3 - 2*a*b*f^5*cos(f*
x + e)^3 + b^2*f^5*cos(f*x + e)^3 - 3*a^2*f^5*cos(f*x + e) + 3*a*b*f^5*cos(
f*x + e))/(a^3*f^6 - 3*a^2*b*f^6 + 3*a*b^2*f^6 - b^3*f^6)
```

3.57 $\int \frac{\sin(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=60

$$-\frac{\cos(e+fx)}{f(a-b)}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/((a - b)^(3/2)*f)) -
 Cos[e + f*x]/((a - b)*f)
```

Rubi [A] time = 0.0550307, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3664, 325, 205}

$$-\frac{\cos(e+fx)}{f(a-b)}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```


```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/((a - b)^(3/2)*f)) -
 Cos[e + f*x]/((a - b)*f)
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a-b+bx^2)}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)}{(a-b)f} - \frac{b\,\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \sec(e+fx)\right)}{(a-b)f}$$

$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} - \frac{\cos(e+fx)}{(a-b)f}$$

Mathematica [B] time = 0.260307, size = 121, normalized size = 2.02

$$\frac{(b-a)\cos(e+fx)+\sqrt{b}\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)+\sqrt{b}\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b
]] + Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sq
rt[b]] + (-a + b)*Cos[e + f*x])/((a - b)^2*f)
```

Maple [A] time = 0.046, size = 63, normalized size = 1.1

$$-\frac{\cos(fx+e)}{(a-b)f}+\frac{b}{(a-b)f}\arctan\left((a-b)\cos(fx+e)\frac{1}{\sqrt{b(a-b)}}\right)\frac{1}{\sqrt{b(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -cos(f*x+e)/(a-b)/f+1/f*b/(a-b)/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*
(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.78239, size = 350, normalized size = 5.83

$$\left[-\frac{\sqrt{-\frac{b}{a-b}}\log\left(-\frac{(a-b)\cos\left(fx+e\right)^2-2\,(a-b)\sqrt{-\frac{b}{a-b}}\cos\left(fx+e\right)-b}{(a-b)\cos\left(fx+e\right)^2+b}\right)+2\,\cos\left(fx+e\right)}{2\,(a-b)f},\,-\frac{\sqrt{\frac{b}{a-b}}\arctan\left(-\frac{(a-b)\sqrt{\frac{b}{a-b}}\cos\left(fx+e\right)}{b}\right)+\cos\left(fx+e\right)}{(a-b)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-b/(a - b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a - b)*sqrt(-b/(a
 - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 2*cos(f*x + e))/((
a - b)*f), -(sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b
) + cos(f*x + e))/((a - b)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(e+fx\right)}{a+b\tan^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral(sin(e + f*x)/(a + b*tan(e + f*x)**2), x)
```

Giac [A] time = 1.40835, size = 109, normalized size = 1.82

$$-\frac{f\cos\left(fx+e\right)}{af^2-bf^2}+\frac{b\arctan\left(\frac{a\cos\left(fx+e\right)-b\cos\left(fx+e\right)}{\sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2}(a-b)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -f*cos(f*x + e)/(a*f^2 - b*f^2) + b*arctan((a*cos(f*x + e) - b*cos(f*x + e)
)/sqrt(a*b - b^2))/(sqrt(a*b - b^2)*(a - b)*f)
```

3.58 $\int \frac{\csc(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=60

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{af\sqrt{a-b}}-\frac{\tanh^{-1}(\cos(e+fx))}{af}$$

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(a*Sqrt[a - b]*f)) -
 ArcTanh[Cos[e + f*x]]/(a*f)
```

Rubi [A] time = 0.0702532, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3664, 391, 207, 205}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{af\sqrt{a-b}}-\frac{\tanh^{-1}(\cos(e+fx))}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(a*Sqrt[a - b]*f)) -
 ArcTanh[Cos[e + f*x]]/(a*f)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)(a-b+bx^2)}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{af} - \frac{b\,\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \sec(e+fx)\right)}{af}$$
$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}f} - \frac{\tanh^{-1}(\cos(e+fx))}{af}$$

Mathematica [B] time = 0.205482, size = 144, normalized size = 2.4

$$\frac{\sqrt{b}\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right) + \sqrt{b}\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right) - (a-b)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right) - \log\left(\sin\right.\right.}{af(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b
]] + Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sq
rt[b]] - (a - b)*(Log[Cos[(e + f*x)/2]] - Log[Sin[(e + f*x)/2]]))/(a*(a - b
)*f)
```

Maple [A] time = 0.063, size = 75, normalized size = 1.3

$$-\frac{\ln\left(\cos\left(fx+e\right)+1\right)}{2fa} + \frac{b}{fa}\arctan\left((a-b)\cos\left(fx+e\right)\frac{1}{\sqrt{b(a-b)}}\right)\frac{1}{\sqrt{b(a-b)}} + \frac{\ln\left(\cos\left(fx+e\right)-1\right)}{2fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/2/f/a*ln(cos(f*x+e)+1)+1/f*b/a/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(
b*(a-b))^(1/2))+1/2/f/a*ln(cos(f*x+e)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.09132, size = 460, normalized size = 7.67

$$\left[\frac{\sqrt{-\frac{b}{a-b}}\log\left(\frac{(a-b)\cos\left(fx+e\right)^2+2\,(a-b)\sqrt{-\frac{b}{a-b}}\cos\left(fx+e\right)-b}{(a-b)\cos\left(fx+e\right)^2+b}\right)-\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)}{2\,af},-\frac{2\sqrt{\frac{b}{a-b}}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/(a - b))*log(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/(a -
 b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) - log(1/2*cos(f*x + e)
 + 1/2) + log(-1/2*cos(f*x + e) + 1/2))/(a*f), -1/2*(2*sqrt(b/(a - b))*arct
an(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + log(1/2*cos(f*x + e) + 1/2) -
 log(-1/2*cos(f*x + e) + 1/2))/(a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(e+fx\right)}{a+b\tan^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)/(a + b*tan(e + f*x)**2), x)
```

Giac [B] time = 1.38654, size = 149, normalized size = 2.48

$$-\frac{\frac{2\,b\arctan\left(-\frac{a\cos\left(fx+e\right)-b\cos\left(fx+e\right)-b}{\sqrt{ab-b^2}\cos\left(fx+e\right)+\sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2}\,a}-\frac{\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)}{a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*arctan(-(a*cos(f*x + e) - b*cos(f*x + e) - b)/(sqrt(a*b - b^2)*co
s(f*x + e) + sqrt(a*b - b^2)))/(sqrt(a*b - b^2)*a) - log(-(cos(f*x + e) - 1
)/(cos(f*x + e) + 1))/a)/f
```

3.59 $\int \frac{\csc^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=89

$$-\frac{(a-2b)\tanh^{-1}(\cos(e+fx))}{2a^2f}-\frac{\sqrt{b}\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a^2f}-\frac{\cot(e+fx)\csc(e+fx)}{2af}$$

```
[Out] -((Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(a^2*f))
 - ((a - 2*b)*ArcTanh[Cos[e + f*x]])/(2*a^2*f) - (Cot[e + f*x]*Csc[e + f*x]
)/(2*a*f)
```

Rubi [A] time = 0.102602, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3664, 471, 522, 207, 205}

$$-\frac{(a-2b)\tanh^{-1}(\cos(e+fx))}{2a^2f}-\frac{\sqrt{b}\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a^2f}-\frac{\cot(e+fx)\csc(e+fx)}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(a^2*f))
 - ((a - 2*b)*ArcTanh[Cos[e + f*x]])/(2*a^2*f) - (Cot[e + f*x]*Csc[e + f*x]
)/(2*a*f)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2\left(a-b+bx^2\right)}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af} + \frac{\text{Subst}\left(\int \frac{a-b-bx^2}{\left(-1+x^2\right)\left(a-b+bx^2\right)}\,dx, x, \sec(e+fx)\right)}{2af}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af} + \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{2a^2f} - \frac{((a-b)b)\,\text{Subs}}{}$$

$$= -\frac{\sqrt{a-b}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a^2f} - \frac{(a-2b)\tanh^{-1}(\cos(e+fx))}{2a^2f} - \frac{\cot(e+fx)\csc(e+fx)}{2af}$$

Mathematica [B] time = 0.66921, size = 195, normalized size = 2.19

$$\frac{8\sqrt{b}\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right) + 8\sqrt{b}\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right) - a\csc^2\left(\frac{1}{2}(e+fx)\right) + a\sec^2\left(\frac{1}{2}(e+\right.}{8a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (8*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt
[b]] + 8*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2]
)/Sqrt[b]] - a*Csc[(e + f*x)/2]^2 - 4*a*Log[Cos[(e + f*x)/2]] + 8*b*Log[Cos
[(e + f*x)/2]] + 4*a*Log[Sin[(e + f*x)/2]] - 8*b*Log[Sin[(e + f*x)/2]] + a*
Sec[(e + f*x)/2]^2)/(8*a^2*f)
```

Maple [B] time = 0.078, size = 189, normalized size = 2.1

$$\frac{1}{4fa\left(\cos\left(fx+e\right)+1\right)} - \frac{\ln\left(\cos\left(fx+e\right)+1\right)}{4fa} + \frac{\ln\left(\cos\left(fx+e\right)+1\right)b}{2fa^2} + \frac{b}{fa}\arctan\left((a-b)\cos\left(fx+e\right)\frac{1}{\sqrt{b(a-b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/4/f/a/(cos(f*x+e)+1)-1/4/f/a*ln(cos(f*x+e)+1)+1/2/f/a^2*ln(cos(f*x+e)+1)*
b+1/f*b/a/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))-1/f*b^2/
a^2/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))+1/4/f/a/(cos(f
*x+e)-1)+1/4/f/a*ln(cos(f*x+e)-1)-1/2/f/a^2*ln(cos(f*x+e)-1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.28141, size = 815, normalized size = 9.16

$$\left[\frac{2\sqrt{-ab+b^2}\left(\cos\left(fx+e\right)^2-1\right)\log\left(-\frac{(a-b)\cos\left(fx+e\right)^2+2\sqrt{-ab+b^2}\cos\left(fx+e\right)-b}{(a-b)\cos\left(fx+e\right)^2+b}\right)+2\,a\cos\left(fx+e\right)-\left((a-2\,b)\cos\left(fx+e\right)^2-\right.}{4\left(a^2f\cos\left(fx+e\right)^2-a^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-a*b + b^2)*(cos(f*x + e)^2 - 1)*log(-((a - b)*cos(f*x + e)^2
+ 2*sqrt(-a*b + b^2)*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 2*a*
cos(f*x + e) - ((a - 2*b)*cos(f*x + e)^2 - a + 2*b)*log(1/2*cos(f*x + e) +
1/2) + ((a - 2*b)*cos(f*x + e)^2 - a + 2*b)*log(-1/2*cos(f*x + e) + 1/2))/(
a^2*f*cos(f*x + e)^2 - a^2*f), 1/4*(4*sqrt(a*b - b^2)*(cos(f*x + e)^2 - 1)*
arctan(sqrt(a*b - b^2)*cos(f*x + e)/b) + 2*a*cos(f*x + e) - ((a - 2*b)*cos(
f*x + e)^2 - a + 2*b)*log(1/2*cos(f*x + e) + 1/2) + ((a - 2*b)*cos(f*x + e)
^2 - a + 2*b)*log(-1/2*cos(f*x + e) + 1/2))/(a^2*f*cos(f*x + e)^2 - a^2*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3\left(e+fx\right)}{a+b\tan^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(a + b*tan(e + f*x)**2), x)
```

Giac [B] time = 1.43536, size = 285, normalized size = 3.2

$$\frac{\frac{2\,(a-2\,b)\log\left(-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)}{a^2}-\frac{8\sqrt{ab-b^2}\arctan\left(-\frac{a\cos(fx+e)-b\cos(fx+e)-b}{\sqrt{ab-b^2}\cos(fx+e)+\sqrt{ab-b^2}}\right)}{a^2}+\frac{\left(a-\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)\left(\cos(fx+e)+1\right)}{a^2\left(\cos(fx+e)-1\right)}-\frac{\cos(fx+e)-1}{a\left(\cos(fx+e)+1\right)}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/8*(2*(a - 2*b)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/a^2 - 8*sqrt(a
*b - b^2)*arctan(-(a*cos(f*x + e) - b*cos(f*x + e) - b)/(sqrt(a*b - b^2)*co
s(f*x + e) + sqrt(a*b - b^2)))/a^2 + (a - 2*a*(cos(f*x + e) - 1)/(cos(f*x +
 e) + 1) + 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(f*x + e) + 1)/(a
^2*(cos(f*x + e) - 1)) - (cos(f*x + e) - 1)/(a*(cos(f*x + e) + 1)))/f
```

3.60 $\int \frac{\csc^5(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=130

$$-\frac{(3a^2-12ab+8b^2)\tanh^{-1}(\cos(e+fx))}{8a^3f}-\frac{(5a-4b)\cot(e+fx)\csc(e+fx)}{8a^2f}-\frac{\sqrt{b}(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a^3f}-\frac{\cot^3(}{}$$

```
[Out] -(((a - b)^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(a^3*f
)) - ((3*a^2 - 12*a*b + 8*b^2)*ArcTanh[Cos[e + f*x]])/(8*a^3*f) - ((5*a - 4
*b)*Cot[e + f*x]*Csc[e + f*x])/(8*a^2*f) - (Cot[e + f*x]^3*Csc[e + f*x])/(4
*a*f)
```

Rubi [A] time = 0.175632, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3664, 470, 527, 522, 207, 205}

$$-\frac{(3a^2-12ab+8b^2)\tanh^{-1}(\cos(e+fx))}{8a^3f}-\frac{(5a-4b)\cot(e+fx)\csc(e+fx)}{8a^2f}-\frac{\sqrt{b}(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a^3f}-\frac{\cot^3(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```


```
[Out] -(((a - b)^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(a^3*f
)) - ((3*a^2 - 12*a*b + 8*b^2)*ArcTanh[Cos[e + f*x]])/(8*a^3*f) - ((5*a - 4
*b)*Cot[e + f*x]*Csc[e + f*x])/(8*a^2*f) - (Cot[e + f*x]^3*Csc[e + f*x])/(4
*a*f)
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^3\left(a-b+bx^2\right)}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4af} - \frac{\text{Subst}\left(\int \frac{-a+b+(-4a+3b)x^2}{\left(-1+x^2\right)^2\left(a-b+bx^2\right)}\,dx, x, \sec(e+fx)\right)}{4af}$$

$$= -\frac{(5a-4b)\cot(e+fx)\csc(e+fx)}{8a^2f} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af} - \frac{\text{Subst}\left(\int \frac{-(3a-4b)(a-b)+(5a}{\left(-1+x^2\right)\left(a-b+\right.}\right.}{8a}$$

$$= -\frac{(5a-4b)\cot(e+fx)\csc(e+fx)}{8a^2f} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af} - \frac{\left((a-b)^2b\right)\text{Subst}\left(\int \frac{}{a-b}\right.}{a^3}$$

$$= -\frac{(a-b)^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a^3f} - \frac{\left(3a^2-12ab+8b^2\right)\tanh^{-1}(\cos(e+fx))}{8a^3f} - \frac{(5a-4b)}{}$$

Mathematica [B] time = 6.24972, size = 326, normalized size = 2.51

$$\frac{\left(3a^2-12ab+8b^2\right)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{8a^3f} + \frac{\left(-3a^2+12ab-8b^2\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{8a^3f} + \frac{(4b-3a)\csc^2\left(\frac{1}{2}(e+fx)\right)}{32a^2f} +$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((a - b)^(3/2)*Sqrt[b]*ArcTan[(Sec[(e + f*x)/2]*(Sqrt[a - b]*Cos[(e + f*x)/
2] - Sqrt[a]*Sin[(e + f*x)/2]))/Sqrt[b]])/(a^3*f) + ((a - b)^(3/2)*Sqrt[b]*
ArcTan[(Sec[(e + f*x)/2]*(Sqrt[a - b]*Cos[(e + f*x)/2] + Sqrt[a]*Sin[(e + f
*x)/2]))/Sqrt[b]])/(a^3*f) + ((-3*a + 4*b)*Csc[(e + f*x)/2]^2)/(32*a^2*f) -
 Csc[(e + f*x)/2]^4/(64*a*f) + ((-3*a^2 + 12*a*b - 8*b^2)*Log[Cos[(e + f*x)
/2]])/(8*a^3*f) + ((3*a^2 - 12*a*b + 8*b^2)*Log[Sin[(e + f*x)/2]])/(8*a^3*f
```

```
) + ((3*a - 4*b)*Sec[(e + f*x)/2]^2)/(32*a^2*f) + Sec[(e + f*x)/2]^4/(64*a*
f)
```

Maple [B] time = 0.081, size = 344, normalized size = 2.7

$$\frac{1}{16fa\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{3}{16fa\left(\cos\left(fx+e\right)+1\right)}-\frac{b}{4fa^{2}\left(\cos\left(fx+e\right)+1\right)}-\frac{3\ln\left(\cos\left(fx+e\right)+1\right)}{16fa}+\frac{3\ln\left(\cos\left(f\right.\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/16/f/a/(cos(f*x+e)+1)^2+3/16/f/a/(cos(f*x+e)+1)-1/4/f/a^2/(cos(f*x+e)+1)*
b-3/16/f/a*ln(cos(f*x+e)+1)+3/4/f/a^2*ln(cos(f*x+e)+1)*b-1/2/f/a^3*ln(cos(f
*x+e)+1)*b^2+1/f*b/a/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2
))-2/f*b^2/a^2/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))+1/f
*b^3/a^3/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))-1/16/f/a/
(cos(f*x+e)-1)^2+3/16/f/a/(cos(f*x+e)-1)-1/4/f/a^2/(cos(f*x+e)-1)*b+3/16/f/
a*ln(cos(f*x+e)-1)-3/4/f/a^2*ln(cos(f*x+e)-1)*b+1/2/f/a^3*ln(cos(f*x+e)-1)*
b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.52071, size = 1523, normalized size = 11.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*a^2 - 4*a*b)*cos(f*x + e)^3 - 8*((a - b)*cos(f*x + e)^4 - 2*(a
- b)*cos(f*x + e)^2 + a - b)*sqrt(-a*b + b^2)*log(((a - b)*cos(f*x + e)^2 -
 2*sqrt(-a*b + b^2)*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) - 2*(5*
a^2 - 4*a*b)*cos(f*x + e) - ((3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3
*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 - 12*a*b + 8*b^2)*log(1/2*cos
(f*x + e) + 1/2) + ((3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a^2 - 12
*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 - 12*a*b + 8*b^2)*log(-1/2*cos(f*x + e
) + 1/2))/(a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f), 1/16*(2*
(3*a^2 - 4*a*b)*cos(f*x + e)^3 + 16*((a - b)*cos(f*x + e)^4 - 2*(a - b)*cos
(f*x + e)^2 + a - b)*sqrt(a*b - b^2)*arctan(sqrt(a*b - b^2)*cos(f*x + e)/b)
 - 2*(5*a^2 - 4*a*b)*cos(f*x + e) - ((3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^
4 - 2*(3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 - 12*a*b + 8*b^2)*log
```

```
(1/2*cos(f*x + e) + 1/2) + ((3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*
a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 - 12*a*b + 8*b^2)*log(-1/2*cos
(f*x + e) + 1/2))/(a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.47657, size = 505, normalized size = 3.88

$$-\frac{\frac{\frac{8\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{8\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{a^2}-\frac{4\left(3\,a^2-12\,ab+8\,b^2\right)\log\left(-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)}{a^3}+\frac{64\left(a^2b-2\,ab^2+b^3\right)\arctan\left(-\frac{a\cos(fx+e)-b\cos(fx+e)-b}{\sqrt{ab-b^2}\cos(fx+e)+\sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2}a^3}\cdots}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/64*((8*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 8*b*(cos(f*x + e) - 1)/
(cos(f*x + e) + 1) - a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^2 - 4*(
3*a^2 - 12*a*b + 8*b^2)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/a^3 + 6
4*(a^2*b - 2*a*b^2 + b^3)*arctan(-(a*cos(f*x + e) - b*cos(f*x + e) - b)/(sq
rt(a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/(sqrt(a*b - b^2)*a^3) + (a^2
 - 8*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 8*a*b*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) + 18*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 72*a
*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 48*b^2*(cos(f*x + e) - 1)^2/
(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(a^3*(cos(f*x + e) - 1)^2))/f
```

3.61 $\int \frac{\sin^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=178

$$-\frac{(11a^2-4ab+b^2)\sin(e+fx)\cos(e+fx)}{16f(a-b)^3}+\frac{x(15a^2b+5a^3-5ab^2+b^3)}{16(a-b)^4}-\frac{a^{5/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{f(a-b)^4}+\frac{\sin^3(e+fx)\,c}{6f(a}$$

```
[Out] ((5*a^3 + 15*a^2*b - 5*a*b^2 + b^3)*x)/(16*(a - b)^4) - (a^(5/2)*Sqrt[b]*Ar
cTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/((a - b)^4*f) - ((11*a^2 - 4*a*b + b^
2)*Cos[e + f*x]*Sin[e + f*x])/(16*(a - b)^3*f) + ((3*a - b)*Cos[e + f*x]^3*
Sin[e + f*x])/(8*(a - b)^2*f) + (Cos[e + f*x]^3*Sin[e + f*x]^3)/(6*(a - b)*
f)
```

Rubi [A] time = 0.292728, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3663, 470, 578, 527, 522, 203, 205}

$$-\frac{(11a^2-4ab+b^2)\sin(e+fx)\cos(e+fx)}{16f(a-b)^3}+\frac{x(15a^2b+5a^3-5ab^2+b^3)}{16(a-b)^4}-\frac{a^{5/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{f(a-b)^4}+\frac{\sin^3(e+fx)\,c}{6f(a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((5*a^3 + 15*a^2*b - 5*a*b^2 + b^3)*x)/(16*(a - b)^4) - (a^(5/2)*Sqrt[b]*Ar
cTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/((a - b)^4*f) - ((11*a^2 - 4*a*b + b^
2)*Cos[e + f*x]*Sin[e + f*x])/(16*(a - b)^3*f) + ((3*a - b)*Cos[e + f*x]^3*
Sin[e + f*x])/(8*(a - b)^2*f) + (Cos[e + f*x]^3*Sin[e + f*x]^3)/(6*(a - b)*
f)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
```

```
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^4(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)}{6(a-b)f} - \frac{\text{Subst}\left(\int \frac{x^2(3a-3(2a-b)x^2)}{(1+x^2)^3(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{6(a-b)f}$$

$$= \frac{(3a-b)\cos^3(e+fx)\sin(e+fx)}{8(a-b)^2f} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6(a-b)f} - \frac{\text{Subst}\left(\int \frac{3a(3a-b)-3(8a^2-3a}{(1+x^2)^2(a+bx}\right.}{24(a}$$

$$= -\frac{(11a^2-4ab+b^2)\cos(e+fx)\sin(e+fx)}{16(a-b)^3f} + \frac{(3a-b)\cos^3(e+fx)\sin(e+fx)}{8(a-b)^2f} + \cos^3(e$$

$$= -\frac{(11a^2-4ab+b^2)\cos(e+fx)\sin(e+fx)}{16(a-b)^3f} + \frac{(3a-b)\cos^3(e+fx)\sin(e+fx)}{8(a-b)^2f} + \cos^3(e$$

$$= \frac{(5a^3+15a^2b-5ab^2+b^3)x}{16(a-b)^4} - \frac{a^{5/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)^4f} - \frac{(11a^2-4ab+b^2)\cos(e+fx}{16(a-b)^3f}$$

Mathematica [A] time = 0.562333, size = 140, normalized size = 0.79

$$-\frac{-12\left(15a^2b+5a^3-5ab^2+b^3\right)(e+fx)+192a^{5/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)+(a-b)^3\sin(6(e+fx))-3(3a-b)(a-b)^2\sin}{192f(a-b)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(-12*(5*a^3 + 15*a^2*b - 5*a*b^2 + b^3)*(e + f*x) + 192*a^(5/2)*Sqrt[b]*Ar
cTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + 3*(a - b)*(5*a - b)*(3*a + b)*Sin[2*
(e + f*x)] - 3*(a - b)^2*(3*a - b)*Sin[4*(e + f*x)] + (a - b)^3*Sin[6*(e +
f*x)])/(192*(a - b)^4*f)
```

Maple [B] time = 0.075, size = 545, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/f*b/(a-b)^4*a^3/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-11/16/f/(a-
b)^4/(1+tan(f*x+e)^2)^3*tan(f*x+e)^5*a^3+15/16/f/(a-b)^4/(1+tan(f*x+e)^2)^3
*tan(f*x+e)^5*a^2*b-5/16/f/(a-b)^4/(1+tan(f*x+e)^2)^3*tan(f*x+e)^5*a*b^2+1/
16/f/(a-b)^4/(1+tan(f*x+e)^2)^3*tan(f*x+e)^5*b^3-5/6/f/(a-b)^4/(1+tan(f*x+e
)^2)^3*tan(f*x+e)^3*a^3+1/2/f/(a-b)^4/(1+tan(f*x+e)^2)^3*tan(f*x+e)^3*a^2*b
+1/2/f/(a-b)^4/(1+tan(f*x+e)^2)^3*tan(f*x+e)^3*a*b^2-1/6/f/(a-b)^4/(1+tan(f
*x+e)^2)^3*tan(f*x+e)^3*b^3-5/16/f/(a-b)^4/(1+tan(f*x+e)^2)^3*tan(f*x+e)*a^
3+1/16/f/(a-b)^4/(1+tan(f*x+e)^2)^3*tan(f*x+e)*a^2*b+5/16/f/(a-b)^4/(1+tan(
f*x+e)^2)^3*tan(f*x+e)*a*b^2-1/16/f/(a-b)^4/(1+tan(f*x+e)^2)^3*tan(f*x+e)*b
^3+5/16/f/(a-b)^4*arctan(tan(f*x+e))*a^3+15/16/f/(a-b)^4*arctan(tan(f*x+e))
*a^2*b-5/16/f/(a-b)^4*arctan(tan(f*x+e))*a*b^2+1/16/f/(a-b)^4*arctan(tan(f*
x+e))*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.27819, size = 1214, normalized size = 6.82

$$\left[\frac{12\sqrt{-ab}a^2\log\left(\frac{\left(a^2+6ab+b^2\right)\cos\left(fx+e\right)^4-2\left(3ab+b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a+b\right)\cos\left(fx+e\right)^3-b\cos\left(fx+e\right)\right)\sqrt{-ab}\sin\left(fx+e\right)+b^2}{\left(a^2-2ab+b^2\right)\cos\left(fx+e\right)^4+2\left(ab-b^2\right)\cos\left(fx+e\right)^2+b^2}\right)+3\left(5a^3+15a^2b-5\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/48*(12*sqrt(-a*b)*a^2*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b
 + b^2)*cos(f*x + e)^2 + 4*((a + b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-
a*b)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2
)*cos(f*x + e)^2 + b^2)) + 3*(5*a^3 + 15*a^2*b - 5*a*b^2 + b^3)*f*x - (8*(a
^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^5 - 2*(13*a^3 - 33*a^2*b + 27*a*
b^2 - 7*b^3)*cos(f*x + e)^3 + 3*(11*a^3 - 15*a^2*b + 5*a*b^2 - b^3)*cos(f*x
 + e))*sin(f*x + e))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*f), 1/48*
(24*sqrt(a*b)*a^2*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(a*b)/(a*b*co
s(f*x + e)*sin(f*x + e))) + 3*(5*a^3 + 15*a^2*b - 5*a*b^2 + b^3)*f*x - (8*(
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^5 - 2*(13*a^3 - 33*a^2*b + 27*a
*b^2 - 7*b^3)*cos(f*x + e)^3 + 3*(11*a^3 - 15*a^2*b + 5*a*b^2 - b^3)*cos(f*
x + e))*sin(f*x + e))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39315, size = 394, normalized size = 2.21

$$-\frac{\frac{48\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)a^3b}{\left(a^4-4a^3b+6a^2b^2-4ab^3+b^4\right)\sqrt{ab}}-\frac{3\left(5a^3+15a^2b-5ab^2+b^3\right)(fx+e)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}+\frac{33a^2\tan(fx+e)^5-12ab\tan(fx+e)^5+3b^2\tan(fx+e)^5+40a^2\tan\cdots}{(\cdots}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/48*(48*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt
(a*b)))*a^3*b/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*sqrt(a*b)) - 3*(
5*a^3 + 15*a^2*b - 5*a*b^2 + b^3)*(f*x + e)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*
a*b^3 + b^4) + (33*a^2*tan(f*x + e)^5 - 12*a*b*tan(f*x + e)^5 + 3*b^2*tan(f
*x + e)^5 + 40*a^2*tan(f*x + e)^3 + 16*a*b*tan(f*x + e)^3 - 8*b^2*tan(f*x +
 e)^3 + 15*a^2*tan(f*x + e) + 12*a*b*tan(f*x + e) - 3*b^2*tan(f*x + e))/((a
^3 - 3*a^2*b + 3*a*b^2 - b^3)*(tan(f*x + e)^2 + 1)^3))/f
```

3.62 $\int \frac{\sin^4(e+fx)}{a+b\tan^2(e+fx)} \, dx$

Optimal. Leaf size=129

$$\frac{x\left(3a^2+6ab-b^2\right)}{8(a-b)^3}-\frac{a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{f(a-b)^3}+\frac{\sin(e+fx)\cos^3(e+fx)}{4f(a-b)}-\frac{(5a-b)\sin(e+fx)\cos(e+fx)}{8f(a-b)^2}$$

```
[Out] ((3*a^2 + 6*a*b - b^2)*x)/(8*(a - b)^3) - (a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*
Tan[e + f*x])/Sqrt[a]])/((a - b)^3*f) - ((5*a - b)*Cos[e + f*x]*Sin[e + f*x
])/(8*(a - b)^2*f) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*(a - b)*f)
```

Rubi [A] time = 0.151545, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3663, 470, 527, 522, 203, 205}

$$\frac{x\left(3a^2+6ab-b^2\right)}{8(a-b)^3}-\frac{a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{f(a-b)^3}+\frac{\sin(e+fx)\cos^3(e+fx)}{4f(a-b)}-\frac{(5a-b)\sin(e+fx)\cos(e+fx)}{8f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((3*a^2 + 6*a*b - b^2)*x)/(8*(a - b)^3) - (a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*
Tan[e + f*x])/Sqrt[a]])/((a - b)^3*f) - ((5*a - b)*Cos[e + f*x]*Sin[e + f*x
])/(8*(a - b)^2*f) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*(a - b)*f)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f} - \frac{\text{Subst}\left(\int \frac{a+(-4a+b)x^2}{(1+x^2)^2(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{4(a-b)f}$$

$$= -\frac{(5a-b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f} + \frac{\text{Subst}\left(\int \frac{a(3a+b)-(5a-b)bx^2}{(1+x^2)(a+bx^2)}\right.}{8(a-b)^2}$$

$$= -\frac{(5a-b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f} - \frac{(a^2b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,\right.}{(a-b)^3f}$$

$$= \frac{(3a^2+6ab-b^2)x}{8(a-b)^3} - \frac{a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)^3f} - \frac{(5a-b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f} + \cos$$

Mathematica [A] time = 0.261784, size = 99, normalized size = 0.77

$$\frac{4\left(3a^2+6ab-b^2\right)(e+fx) - 32a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right) + (a-b)^2\sin(4(e+fx)) - 8a(a-b)\sin(2(e+fx))}{32f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (4*(3*a^2 + 6*a*b - b^2)*(e + f*x) - 32*a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan
[e + f*x])/Sqrt[a]] - 8*a*(a - b)*Sin[2*(e + f*x)] + (a - b)^2*Sin[4*(e + f
*x)])/(32*(a - b)^3*f)
```

Maple [B] time = 0.063, size = 304, normalized size = 2.4

$$-\frac{a^2b}{f(a-b)^3}\arctan\left(b\tan(fx+e)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{5(\tan(fx+e))^3a^2}{8f(a-b)^3\left(1+(\tan(fx+e))^2\right)^2} + \frac{3(\tan(fx+e))^3ab}{4f(a-b)^3\left(1+(\tan(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/f*a^2*b/(a-b)^3/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-5/8/f/(a-b)
^3/(1+tan(f*x+e)^2)^2*tan(f*x+e)^3*a^2+3/4/f/(a-b)^3/(1+tan(f*x+e)^2)^2*tan
(f*x+e)^3*a*b-1/8/f/(a-b)^3/(1+tan(f*x+e)^2)^2*tan(f*x+e)^3*b^2-3/8/f/(a-b)
^3/(1+tan(f*x+e)^2)^2*tan(f*x+e)*a^2+1/4/f/(a-b)^3/(1+tan(f*x+e)^2)^2*tan(f
*x+e)*a*b+1/8/f/(a-b)^3/(1+tan(f*x+e)^2)^2*tan(f*x+e)*b^2+3/8/f/(a-b)^3*arc
tan(tan(f*x+e))*a^2+3/4/f/(a-b)^3*arctan(tan(f*x+e))*a*b-1/8/f/(a-b)^3*arct
an(tan(f*x+e))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07707, size = 898, normalized size = 6.96

$$\left[\frac{\left(3\,a^2+6\,ab-b^2\right)fx-2\,\sqrt{-ab}a\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+b^2\right)\cos\left(fx+e\right)^2-4\left(\left(a+b\right)\cos\left(fx+e\right)^3-b\cos\left(fx+e\right)\right)\sqrt{-ab}\sin\left(fx+e\right)+b^2}{\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^4+2\left(ab-b^2\right)\cos\left(fx+e\right)^2+b^2}\right)}{8\left(a^3-3\,a^2b+3\,ab^2-b^3\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/8*((3*a^2 + 6*a*b - b^2)*f*x - 2*sqrt(-a*b)*a*log(((a^2 + 6*a*b + b^2)*c
os(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 - 4*((a + b)*cos(f*x + e)^3
- b*cos(f*x + e))*sqrt(-a*b)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f
*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2)) + (2*(a^2 - 2*a*b + b^2)*c
os(f*x + e)^3 - (5*a^2 - 6*a*b + b^2)*cos(f*x + e))*sin(f*x + e))/((a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*f), 1/8*((3*a^2 + 6*a*b - b^2)*f*x + 4*sqrt(a*b)*a*
arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(a*b)/(a*b*cos(f*x + e)*sin(f*x
 + e))) + (2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - (5*a^2 - 6*a*b + b^2)*cos
(f*x + e))*sin(f*x + e))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.44372, size = 257, normalized size = 1.99

$$-\frac{\frac{8\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)a^2b}{\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\sqrt{ab}}-\frac{\left(3\,a^2+6\,ab-b^2\right)\left(fx+e\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{5\,a\tan\left(fx+e\right)^3-b\tan\left(fx+e\right)^3+3\,a\tan\left(fx+e\right)+b\tan\left(fx+e\right)}{\left(a^2-2\,ab+b^2\right)\left(\tan\left(fx+e\right)^2+1\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/8*(8*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b)))*a^2*b/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a*b)) - (3*a^2 + 6*a*b -
b^2)*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (5*a*tan(f*x + e)^3 - b*ta
n(f*x + e)^3 + 3*a*tan(f*x + e) + b*tan(f*x + e))/((a^2 - 2*a*b + b^2)*(tan
(f*x + e)^2 + 1)^2))/f
```

3.63 $\int \frac{\sin^2(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=82

$$-\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{f(a-b)^2}-\frac{\sin(e+fx)\cos(e+fx)}{2f(a-b)}+\frac{x(a+b)}{2(a-b)^2}$$

```
[Out] ((a + b)*x)/(2*(a - b)^2) - (Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a]])/((a - b)^2*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*(a - b)*f)
```

Rubi [A] time = 0.0968373, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3663, 471, 522, 203, 205}

$$-\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{f(a-b)^2}-\frac{\sin(e+fx)\cos(e+fx)}{2f(a-b)}+\frac{x(a+b)}{2(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((a + b)*x)/(2*(a - b)^2) - (Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a]])/((a - b)^2*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*(a - b)*f)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2\left(a+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f} + \frac{\text{Subst}\left(\int \frac{a-bx^2}{\left(1+x^2\right)\left(a+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{2(a-b)f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f} - \frac{(ab)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(e+fx)\right)}{(a-b)^2f} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1+\cdots}\right.}{2(a\cdots}$$

$$= \frac{(a+b)x}{2(a-b)^2} - \frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)^2f} - \frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f}$$

Mathematica [A] time = 0.140003, size = 69, normalized size = 0.84

$$\frac{2(a+b)(e+fx)+(b-a)\sin(2(e+fx))-4\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{4f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (2*(a + b)*(e + f*x) - 4*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt
[a]] + (-a + b)*Sin[2*(e + f*x)])/(4*(a - b)^2*f)
```

Maple [A] time = 0.062, size = 137, normalized size = 1.7

$$-\frac{ab}{f(a-b)^2}\arctan\left(b\tan(fx+e)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{\tan(fx+e)a}{2f(a-b)^2\left(1+(\tan(fx+e))^2\right)} + \frac{b\tan(fx+e)}{2f(a-b)^2\left(1+(\tan(fx+e))^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/f*b/(a-b)^2*a/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-1/2/f/(a-b)^2
*tan(f*x+e)/(1+tan(f*x+e)^2)*a+1/2/f/(a-b)^2*tan(f*x+e)/(1+tan(f*x+e)^2)*b+
1/2/f/(a-b)^2*arctan(tan(f*x+e))*a+1/2/f/(a-b)^2*arctan(tan(f*x+e))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88243, size = 674, normalized size = 8.22

$$\left[\frac{2\,(a+b)fx-2\,(a-b)\cos\left(fx+e\right)\sin\left(fx+e\right)+\sqrt{-ab}\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a+b\right)\cos\left(fx+e\right)^3-b\cos\cdots\right.}{\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^4+2\left(ab-b^2\right)\cos\left(fx+e\right)^2+\cdots}\right.}{4\left(a^2-2\,ab+b^2\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a + b)*f*x - 2*(a - b)*cos(f*x + e)*sin(f*x + e) + sqrt(-a*b)*log(
((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 + 4*((
a + b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b)*sin(f*x + e) + b^2)/((a^
2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2)))/((a
^2 - 2*a*b + b^2)*f), 1/2*((a + b)*f*x - (a - b)*cos(f*x + e)*sin(f*x + e)
+ sqrt(a*b)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(a*b)/(a*b*cos(f*x
+ e)*sin(f*x + e))))/((a^2 - 2*a*b + b^2)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40066, size = 153, normalized size = 1.87

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)ab}{\left(a^2-2\,ab+b^2\right)\sqrt{ab}}-\frac{\left(fx+e\right)(a+b)}{a^2-2\,ab+b^2}+\frac{\tan\left(fx+e\right)}{\left(\tan\left(fx+e\right)^2+1\right)(a-b)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b)))*a*b/((a^2 - 2*a*b + b^2)*sqrt(a*b)) - (f*x + e)*(a + b)/(a^2 - 2*a*b
+ b^2) + tan(f*x + e)/((tan(f*x + e)^2 + 1)*(a - b)))/f
```

3.64 $\int \frac{1}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=50

$$\frac{x}{a-b}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}f(a-b)}$$

```
[Out] x/(a - b) - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*(a -
b)*f)
```

Rubi [A] time = 0.0747476, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3660, 3675, 205}

$$\frac{x}{a-b}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-1),x]
```

```
[Out] x/(a - b) - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*(a -
b)*f)
```

Rule 3660

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/(a -
 b), x] - Dist[b/(a - b), Int[Sec[e + f*x]^2/(a + b*Tan[e + f*x]^2), x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a, b]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+b\tan^2(e+fx)}\,dx = \frac{x}{a-b} - \frac{b\int \frac{\sec^2(e+fx)}{a+b\tan^2(e+fx)}\,dx}{a-b}$$
$$= \frac{x}{a-b} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(e+fx)\right)}{(a-b)f}$$
$$= \frac{x}{a-b} - \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)f}$$

Mathematica [A] time = 0.065608, size = 49, normalized size = 0.98

$$\frac{\tan^{-1}(\tan(e+fx)) - \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}}}{af-bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-1),x]
```

```
[Out] (ArcTan[Tan[e + f*x]] - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/Sq
rt[a])/(a*f - b*f)
```

Maple [A] time = 0.019, size = 52, normalized size = 1.

$$-\frac{b}{f(a-b)}\arctan\left(b\tan(fx+e)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{\arctan(\tan(fx+e))}{f(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/f/(a-b)*b/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))+1/f/(a-b)*arctan(
tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68936, size = 406, normalized size = 8.12

$$\left[\frac{4fx - \sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(fx+e)^4 - 6ab\tan(fx+e)^2 + a^2 + 4\left(ab\tan(fx+e)^3 - a^2\tan(fx+e)\right)\sqrt{-\frac{b}{a}}}{b^2\tan(fx+e)^4 + 2ab\tan(fx+e)^2 + a^2}\right)}{4(a-b)f}, \frac{2fx - \sqrt{\frac{b}{a}}\arctan\left(\frac{\left(b\tan(fx+e)^2 - a\right)\sqrt{\frac{b}{a}}}{2b\tan(fx+e)}\right)}{2(a-b)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*f*x - sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a
^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e
)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/((a - b)*f), 1/2*(2*f*x - sqrt(b/a)*arc
tan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/((a - b)*f)]
```

Sympy [A] time = 3.63093, size = 280, normalized size = 5.6

$$\begin{cases} \frac{\tilde{\infty}x}{\tan^2(e)} & \text{for } a = 0 \wedge b = 0 \wedge f = 0 \\ \frac{x}{a} & \text{for } b = 0 \\ \frac{-x - \frac{1}{f\tan(e+fx)}}{b} & \text{for } a = 0 \\ \frac{fx\tan^2(e+fx)}{2bf\tan^2(e+fx)+2bf} + \frac{fx}{2bf\tan^2(e+fx)+2bf} + \frac{\tan(e+fx)}{2bf\tan^2(e+fx)+2bf} & \text{for } a = b \\ \frac{x}{a+b\tan^2(e)} & \text{for } f = 0 \\ \frac{2i\sqrt{a}fx\sqrt{\frac{1}{b}}}{2ia^{\frac{3}{2}}f\sqrt{\frac{1}{b}} - 2i\sqrt{a}bf\sqrt{\frac{1}{b}}} - \frac{\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}} + \tan(e+fx)\right)}{2ia^{\frac{3}{2}}f\sqrt{\frac{1}{b}} - 2i\sqrt{a}bf\sqrt{\frac{1}{b}}} + \frac{\log\left(i\sqrt{a}\sqrt{\frac{1}{b}} + \tan(e+fx)\right)}{2ia^{\frac{3}{2}}f\sqrt{\frac{1}{b}} - 2i\sqrt{a}bf\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x/tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x/a, Eq(b, 0)
), ((-x - 1/(f*tan(e + f*x)))/b, Eq(a, 0)), (f*x*tan(e + f*x)**2/(2*b*f*tan
(e + f*x)**2 + 2*b*f) + f*x/(2*b*f*tan(e + f*x)**2 + 2*b*f) + tan(e + f*x)/
(2*b*f*tan(e + f*x)**2 + 2*b*f), Eq(a, b)), (x/(a + b*tan(e)**2), Eq(f, 0))
, (2*I*sqrt(a)*f*x*sqrt(1/b)/(2*I*a**(3/2)*f*sqrt(1/b) - 2*I*sqrt(a)*b*f*sq
rt(1/b)) - log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(2*I*a**(3/2)*f*sqrt(1/
b) - 2*I*sqrt(a)*b*f*sqrt(1/b)) + log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(
2*I*a**(3/2)*f*sqrt(1/b) - 2*I*sqrt(a)*b*f*sqrt(1/b)), True))
```

Giac [B] time = 1.39468, size = 231, normalized size = 4.62

$$-\frac{2\left(\frac{\sqrt{ab}\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan(fx+e)}{\sqrt{\frac{a+b-\sqrt{(a+b)^2-4ab}}{b}}}\right)\right)|b|}{(a-b)^2b-(ab+b^2)|-a+b|}+\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan(fx+e)}{\sqrt{\frac{a+b+\sqrt{(a+b)^2-4ab}}{b}}}\right)\right)b}{(a-b)^2+a|-a+b|+b|-a+b|}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -2*(sqrt(a*b)*(pi*floor((f*x + e)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(f*x +
e)/sqrt((a + b - sqrt((a + b)^2 - 4*a*b))/b)))*abs(b)/((a - b)^2*b - (a*b +
 b^2)*abs(-a + b)) + (pi*floor((f*x + e)/pi + 1/2) + arctan(2*sqrt(1/2)*tan
(f*x + e)/sqrt((a + b + sqrt((a + b)^2 - 4*a*b))/b)))*b/((a - b)^2 + a*abs(
```

```
-a + b) + b*abs(-a + b)))/f
```

3.65 $\int \frac{\csc^2(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=48

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{\cot(e+fx)}{af}$$

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(3/2)*f)) - Cot[e + f
*x]/(a*f)
```

Rubi [A] time = 0.0616512, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3663, 325, 205}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{\cot(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(3/2)*f)) - Cot[e + f
*x]/(a*f)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\cot(e+fx)}{af} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(e+fx)\right)}{af}$$
$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{\cot(e+fx)}{af}$$

Mathematica [A] time = 0.117831, size = 48, normalized size = 1.

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{\cot(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(3/2)*f)) - Cot[e + f
*x]/(a*f)
```

Maple [A] time = 0.063, size = 46, normalized size = 1.

$$-\frac{1}{fa\tan\left(fx+e\right)} - \frac{b}{fa}\arctan\left(b\tan\left(fx+e\right)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/f/a/tan(f*x+e)-1/f*b/a/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.82102, size = 606, normalized size = 12.62

$$\left[\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a^2+ab\right)\cos\left(fx+e\right)^3-ab\cos\left(fx+e\right)\right)\sqrt{-\frac{b}{a}}\sin\left(fx+e\right)+b^2}{\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^4+2\left(ab-b^2\right)\cos\left(fx+e\right)^2+b^2}\right)\sin\left(fx+e\right)-4\cos\left(fx\right.}{4\,af\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*
cos(f*x + e)^2 + 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/
a)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*
cos(f*x + e)^2 + b^2))*sin(f*x + e) - 4*cos(f*x + e))/(a*f*sin(f*x + e)), 1
/2*(sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos(f*x
+ e)*sin(f*x + e)))*sin(f*x + e) - 2*cos(f*x + e))/(a*f*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2\left(e+fx\right)}{a+b\tan^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*tan(e + f*x)**2), x)
```

Giac [A] time = 1.4093, size = 84, normalized size = 1.75

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)b}{\sqrt{ab}a}+\frac{1}{a\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))*
b/(sqrt(a*b)*a) + 1/(a*tan(f*x + e)))/f
```

3.66 $\int \frac{\csc^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{b}(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{(a-b)\cot(e+fx)}{a^2f}-\frac{\cot^3(e+fx)}{3af}$$

```
[Out] -(((a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(5/2)*f)) - (
(a - b)*Cot[e + f*x])/(a^2*f) - Cot[e + f*x]^3/(3*a*f)
```

Rubi [A] time = 0.0902362, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3663, 453, 325, 205}

$$-\frac{\sqrt{b}(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{(a-b)\cot(e+fx)}{a^2f}-\frac{\cot^3(e+fx)}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(5/2)*f)) - (
(a - b)*Cot[e + f*x])/(a^2*f) - Cot[e + f*x]^3/(3*a*f)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\cot^3(e+fx)}{3af} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{af}$$
$$= -\frac{(a-b)\cot(e+fx)}{a^2f} - \frac{\cot^3(e+fx)}{3af} - \frac{((a-b)b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(e+fx)\right)}{a^2f}$$
$$= -\frac{(a-b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{(a-b)\cot(e+fx)}{a^2f} - \frac{\cot^3(e+fx)}{3af}$$

Mathematica [A] time = 0.291959, size = 73, normalized size = 0.96

$$\frac{3\sqrt{b}(b-a)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right) - \sqrt{a}\cot(e+fx)\left(a\csc^2(e+fx)+2a-3b\right)}{3a^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (3*Sqrt[b]*(-a + b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] - Sqrt[a]*Cot[e
+ f*x]*(2*a - 3*b + a*Csc[e + f*x]^2))/(3*a^(5/2)*f)
```

Maple [A] time = 0.075, size = 107, normalized size = 1.4

$$-\frac{1}{3fa\left(\tan\left(fx+e\right)\right)^3} - \frac{1}{fa\tan\left(fx+e\right)} + \frac{b}{fa^2\tan\left(fx+e\right)} - \frac{b}{fa}\arctan\left(b\tan\left(fx+e\right)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{b^2}{fa^2}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/3/f/a/tan(f*x+e)^3-1/f/a/tan(f*x+e)+1/f/a^2/tan(f*x+e)*b-1/f*b/a/(a*b)^(
1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))+1/f*b^2/a^2/(a*b)^(1/2)*arctan(b*tan(
f*x+e)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.82012, size = 884, normalized size = 11.63

$$\left[-\frac{4\,(2\,a-3\,b)\cos\left(fx+e\right)^{3}+3\left((a-b)\cos\left(fx+e\right)^{2}-a+b\right)\sqrt{-\frac{b}{a}}\log\left(\frac{\left(a^{2}+6\,ab+b^{2}\right)\cos\left(fx+e\right)^{4}-2\left(3\,ab+b^{2}\right)\cos\left(fx+e\right)^{2}-4\left(\left(a^{2}+ab\right)\right.}{\left(a^{2}-2\,ab+b^{2}\right)\cos\left(fx+e\right)^{4}+2\left(ab-\right.}\right.}{12\left(a^{2}f\cos\left(fx+e\right)^{2}-a^{2}f\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*(2*a - 3*b)*cos(f*x + e)^3 + 3*((a - b)*cos(f*x + e)^2 - a + b)*s
qrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x
 + e)^2 - 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(
f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x
 + e)^2 + b^2))*sin(f*x + e) - 12*(a - b)*cos(f*x + e))/((a^2*f*cos(f*x + e
)^2 - a^2*f)*sin(f*x + e)), -1/6*(2*(2*a - 3*b)*cos(f*x + e)^3 - 3*((a - b)
*cos(f*x + e)^2 - a + b)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*
sqrt(b/a)/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(a - b)*cos(f*x +
 e))/((a^2*f*cos(f*x + e)^2 - a^2*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.44278, size = 131, normalized size = 1.72

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)\left(ab-b^{2}\right)}{\sqrt{ab}a^{2}}+\frac{3\,a\tan\left(fx+e\right)^{2}-3\,b\tan\left(fx+e\right)^{2}+a}{a^{2}\tan\left(fx+e\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b)))*(a*b - b^2)/(sqrt(a*b)*a^2) + (3*a*tan(f*x + e)^2 - 3*b*tan(f*x + e)^
2 + a)/(a^2*tan(f*x + e)^3))/f
```

3.67 $\int \frac{\csc^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=105

$$-\frac{\sqrt{b}(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}f}-\frac{(2a-b)\cot^3(e+fx)}{3a^2f}-\frac{(a-b)^2\cot(e+fx)}{a^3f}-\frac{\cot^5(e+fx)}{5af}$$

```
[Out] -(((a - b)^2*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(7/2)*f)) -
 ((a - b)^2*Cot[e + f*x])/(a^3*f) - ((2*a - b)*Cot[e + f*x]^3)/(3*a^2*f) -
Cot[e + f*x]^5/(5*a*f)
```

Rubi [A] time = 0.115417, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3663, 461, 205}

$$-\frac{\sqrt{b}(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}f}-\frac{(2a-b)\cot^3(e+fx)}{3a^2f}-\frac{(a-b)^2\cot(e+fx)}{a^3f}-\frac{\cot^5(e+fx)}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - b)^2*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(7/2)*f)) -
 ((a - b)^2*Cot[e + f*x])/(a^3*f) - ((2*a - b)*Cot[e + f*x]^3)/(3*a^2*f) -
Cot[e + f*x]^5/(5*a*f)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^6(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^6}+\frac{2a-b}{a^2x^4}+\frac{(a-b)^2}{a^3x^2}-\frac{(a-b)^2b}{a^3(a+bx^2)}\right) dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a-b)^2\cot(e+fx)}{a^3f}-\frac{(2a-b)\cot^3(e+fx)}{3a^2f}-\frac{\cot^5(e+fx)}{5af}-\frac{\left((a-b)^2b\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx\right.}{a^3f}$$

$$= -\frac{(a-b)^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}f}-\frac{(a-b)^2\cot(e+fx)}{a^3f}-\frac{(2a-b)\cot^3(e+fx)}{3a^2f}-\frac{\cot^5(e+f}{5af}$$

Mathematica [A] time = 0.79873, size = 103, normalized size = 0.98

$$\frac{-\sqrt{a}\cot(e+fx)\left(3a^2\csc^4(e+fx)+8a^2+a(4a-5b)\csc^2(e+fx)-25ab+15b^2\right)-15\sqrt{b}(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{15a^{7/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-15*(a - b)^2*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] - Sqrt[a]*Cot
[e + f*x]*(8*a^2 - 25*a*b + 15*b^2 + a*(4*a - 5*b)*Csc[e + f*x]^2 + 3*a^2*C
sc[e + f*x]^4))/(15*a^(7/2)*f)
```

Maple [B] time = 0.083, size = 191, normalized size = 1.8

$$-\frac{1}{5fa\left(\tan\left(fx+e\right)\right)^5}-\frac{2}{3fa\left(\tan\left(fx+e\right)\right)^3}+\frac{b}{3fa^2\left(\tan\left(fx+e\right)\right)^3}-\frac{1}{fa\tan\left(fx+e\right)}+2\frac{b}{fa^2\tan\left(fx+e\right)}-\frac{}{fa^3\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/5/f/a/tan(f*x+e)^5-2/3/f/a/tan(f*x+e)^3+1/3/f/a^2/tan(f*x+e)^3*b-1/f/a/t
an(f*x+e)+2/f/a^2/tan(f*x+e)*b-1/f/a^3/tan(f*x+e)*b^2-1/f*b/a/(a*b)^(1/2)*a
rctan(b*tan(f*x+e)/(a*b)^(1/2))+2/f*b^2/a^2/(a*b)^(1/2)*arctan(b*tan(f*x+e)
/(a*b)^(1/2))-1/f*b^3/a^3/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [B] time = 1.9664, size = 1319, normalized size = 12.56

$$\left[-\frac{4\left(8a^2-25ab+15b^2\right)\cos\left(fx+e\right)^5-20\left(4a^2-11ab+6b^2\right)\cos\left(fx+e\right)^3-15\left(\left(a^2-2ab+b^2\right)\cos\left(fx+e\right)^4-\dots\right.}{\dots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(4*(8*a^2 - 25*a*b + 15*b^2)*cos(f*x + e)^5 - 20*(4*a^2 - 11*a*b + 6
*b^2)*cos(f*x + e)^3 - 15*((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 - 2*
a*b + b^2)*cos(f*x + e)^2 + a^2 - 2*a*b + b^2)*sqrt(-b/a)*log(((a^2 + 6*a*b
 + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 + 4*((a^2 + a*b)*co
s(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*x + e) + b^2)/((a^2 - 2*a
*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2))*sin(f*x + e
) + 60*(a^2 - 2*a*b + b^2)*cos(f*x + e))/((a^3*f*cos(f*x + e)^4 - 2*a^3*f*c
os(f*x + e)^2 + a^3*f)*sin(f*x + e)), -1/30*(2*(8*a^2 - 25*a*b + 15*b^2)*co
s(f*x + e)^5 - 10*(4*a^2 - 11*a*b + 6*b^2)*cos(f*x + e)^3 - 15*((a^2 - 2*a*
b + b^2)*cos(f*x + e)^4 - 2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^2 + a^2 - 2*a*
b + b^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos
(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 30*(a^2 - 2*a*b + b^2)*cos(f*x + e)
)/((a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.46882, size = 204, normalized size = 1.94

$$-\frac{\frac{15\left(a^2b-2ab^2+b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)}{\sqrt{ab}a^3}+\frac{15a^2\tan\left(fx+e\right)^4-30ab\tan\left(fx+e\right)^4+15b^2\tan\left(fx+e\right)^4+10a^2\tan\left(fx+e\right)^2-5ab\tan\left(fx\dots\right.}{a^3\tan\left(fx+e\right)^5}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(a^2*b - 2*a*b^2 + b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + ar
ctan(b*tan(f*x + e)/sqrt(a*b)))/(sqrt(a*b)*a^3) + (15*a^2*tan(f*x + e)^4 -
30*a*b*tan(f*x + e)^4 + 15*b^2*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 - 5*a
*b*tan(f*x + e)^2 + 3*a^2)/(a^3*tan(f*x + e)^5))/f
```

3.68 $\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=204

$$-\frac{\left(5a^2+10ab-b^2\right)\cos(e+fx)}{5f(a-b)^4}-\frac{b\left(5a^2+2b^2\right)\sec(e+fx)}{10f(a-b)^4\left(a+b\sec^2(e+fx)-b\right)}+\frac{(10a-3b)\cos^3(e+fx)}{15f(a-b)^3}-\frac{\cos^5(e+f}{5f(a-b)\left(a+b\sec^2\right.}$$

```
[Out] -(a*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(2*(a -
 b)^(9/2)*f) - ((5*a^2 + 10*a*b - b^2)*Cos[e + f*x])/(5*(a - b)^4*f) + ((10
*a - 3*b)*Cos[e + f*x]^3)/(15*(a - b)^3*f) - Cos[e + f*x]^5/(5*(a - b)*f*(a
 - b + b*Sec[e + f*x]^2)) - (b*(5*a^2 + 2*b^2)*Sec[e + f*x])/(10*(a - b)^4*
f*(a - b + b*Sec[e + f*x]^2))
```

Rubi [A] time = 0.309352, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3664, 462, 456, 1261, 205}

$$-\frac{\left(5a^2+10ab-b^2\right)\cos(e+fx)}{5f(a-b)^4}-\frac{b\left(5a^2+2b^2\right)\sec(e+fx)}{10f(a-b)^4\left(a+b\sec^2(e+fx)-b\right)}+\frac{(10a-3b)\cos^3(e+fx)}{15f(a-b)^3}-\frac{\cos^5(e+f}{5f(a-b)\left(a+b\sec^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```


```
[Out] -(a*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(2*(a -
 b)^(9/2)*f) - ((5*a^2 + 10*a*b - b^2)*Cos[e + f*x])/(5*(a - b)^4*f) + ((10
*a - 3*b)*Cos[e + f*x]^3)/(15*(a - b)^3*f) - Cos[e + f*x]^5/(5*(a - b)*f*(a
 - b + b*Sec[e + f*x]^2)) - (b*(5*a^2 + 2*b^2)*Sec[e + f*x])/(10*(a - b)^4*
f*(a - b + b*Sec[e + f*x]^2))
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
```

```
a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^6\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{-10a+3b+5(a-b)x^2}{x^4\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+fx)\right)}{5(a-b)f}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(e+fx)\right)} - \frac{b\left(5a^2+2b^2\right)\sec(e+fx)}{10(a-b)^4f\left(a-b+b\sec^2(e+fx)\right)} - \frac{b\,\text{Subst}\left(\right.}{}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(e+fx)\right)} - \frac{b\left(5a^2+2b^2\right)\sec(e+fx)}{10(a-b)^4f\left(a-b+b\sec^2(e+fx)\right)} - \frac{b\,\text{Subst}\left(\right.}{}$$

$$= -\frac{\left(5a^2+10ab-b^2\right)\cos(e+fx)}{5(a-b)^4f} + \frac{(10a-3b)\cos^3(e+fx)}{15(a-b)^3f} - \frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(\right.}$$

$$= -\frac{a\sqrt{b}(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2(a-b)^{9/2}f} - \frac{\left(5a^2+10ab-b^2\right)\cos(e+fx)}{5(a-b)^4f} + \frac{(10a-3b)\cos^3(}{15(a-b)^3}$$

Mathematica [A] time = 3.71694, size = 215, normalized size = 1.05

$$\frac{\frac{(a-b)(5(5a+3b)\cos(3(e+fx))+3(b-a)\cos(5(e+fx)))-30\cos(e+fx)\left(a^2\left(\frac{8b}{(a-b)\cos(2(e+fx))+a+b}+5\right)+18ab+b^2\right)}{(a-b)^4} + \frac{120a\sqrt{b}(3a+4b)\tan^{-1}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{9/2}}}{240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((120*a*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])
/Sqrt[b]])/(a - b)^(9/2) + (120*a*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[a - b] +
 Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a - b)^(9/2) + (-30*Cos[e + f*x]*(18*
a*b + b^2 + a^2*(5 + (8*b)/(a + b + (a - b)*Cos[2*(e + f*x)]))) + (a - b)*(
```

```
5*(5*a + 3*b)*Cos[3*(e + f*x)] + 3*(-a + b)*Cos[5*(e + f*x)]))/(a - b)^4)/(
240*f)
```

Maple [B] time = 0.082, size = 388, normalized size = 1.9

$$-\frac{\left(\cos\left(fx+e\right)\right)^{5}a^{2}}{5f\left(a^{2}-2ab+b^{2}\right)\left(a-b\right)^{2}}+\frac{2\left(\cos\left(fx+e\right)\right)^{5}ab}{5f\left(a^{2}-2ab+b^{2}\right)\left(a-b\right)^{2}}-\frac{\left(\cos\left(fx+e\right)\right)^{5}b^{2}}{5f\left(a^{2}-2ab+b^{2}\right)\left(a-b\right)^{2}}+\frac{2\left(\cos\left(fx+e\right)\right)^{3}a^{2}}{3f\left(a^{2}-2ab+b^{2}\right)\left(a-b\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/5/f/(a^2-2*a*b+b^2)/(a-b)^2*cos(f*x+e)^5*a^2+2/5/f/(a^2-2*a*b+b^2)/(a-b)
^2*cos(f*x+e)^5*a*b-1/5/f/(a^2-2*a*b+b^2)/(a-b)^2*cos(f*x+e)^5*b^2+2/3/f/(a
^2-2*a*b+b^2)/(a-b)^2*cos(f*x+e)^3*a^2-2/3/f/(a^2-2*a*b+b^2)/(a-b)^2*cos(f*
x+e)^3*a*b-1/f/(a^2-2*a*b+b^2)/(a-b)^2*cos(f*x+e)*a^2-2/f/(a^2-2*a*b+b^2)/(
a-b)^2*cos(f*x+e)*a*b-1/2/f*a^2*b/(a-b)^4*cos(f*x+e)/(a*cos(f*x+e)^2-cos(f*
x+e)^2*b+b)+3/2/f*a^2*b/(a-b)^4/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*
(a-b))^(1/2))+2/f*a*b^2/(a-b)^4/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*
(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.59003, size = 1330, normalized size = 6.52

$$\left[-\frac{12\left(a^{3}-3a^{2}b+3ab^{2}-b^{3}\right)\cos\left(fx+e\right)^{7}-4\left(10a^{3}-23a^{2}b+16ab^{2}-3b^{3}\right)\cos\left(fx+e\right)^{5}+20\left(3a^{3}+a^{2}b-4ab^{2}\right)\cos}{60\left(\left(a^{5}-5a^{4}b+10a^{3}b^{2}-10a^{2}b^{3}+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(12*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 4*(10*a^3 - 23*
a^2*b + 16*a*b^2 - 3*b^3)*cos(f*x + e)^5 + 20*(3*a^3 + a^2*b - 4*a*b^2)*cos
(f*x + e)^3 - 15*(3*a^2*b + 4*a*b^2 + (3*a^3 + a^2*b - 4*a*b^2)*cos(f*x + e
)^2)*sqrt(-b/(a - b))*log(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/(a -
b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 30*(3*a^2*b + 4*a*b^2
)*cos(f*x + e))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*
f*cos(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f), -1/3
0*(6*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 2*(10*a^3 - 23*a^2*b
```

```
+ 16*a*b^2 - 3*b^3)*cos(f*x + e)^5 + 10*(3*a^3 + a^2*b - 4*a*b^2)*cos(f*x +
 e)^3 + 15*(3*a^2*b + 4*a*b^2 + (3*a^3 + a^2*b - 4*a*b^2)*cos(f*x + e)^2)*s
qrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + 15*(3*a^2*
b + 4*a*b^2)*cos(f*x + e))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*
b^4 - b^5)*f*cos(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^
5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2)**2,x)

[Out] Timed out
```

Giac [B] time = 1.43139, size = 757, normalized size = 3.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")

[Out] -1/30*(15*(3*a^2*b + 4*a*b^2)*arctan(-(a*cos(f*x + e) - b*cos(f*x + e) - b)
/(sqrt(a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/((a^4 - 4*a^3*b + 6*a^2*
b^2 - 4*a*b^3 + b^4)*sqrt(a*b - b^2)) + 30*(a^2*b + a^2*b*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) - 2*a*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/((a^4
 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(a + 2*a*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2)) - 4*(8*a^2 + 34*a*b + 3*b^2 - 40*a^2*(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) - 140*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1
) + 80*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 160*a*b*(cos(f*x + e
) - 1)^2/(cos(f*x + e) + 1)^2 + 30*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2 - 180*a*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 30*a*b*(cos(f*x
 + e) - 1)^4/(cos(f*x + e) + 1)^4 + 15*b^2*(cos(f*x + e) - 1)^4/(cos(f*x +
e) + 1)^4)/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*((cos(f*x + e) - 1)
/(cos(f*x + e) + 1) - 1)^5))/f
```

3.69 $\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=133

$$\frac{\cos^3(e+fx)}{3f(a-b)^2}-\frac{(a+b)\cos(e+fx)}{f(a-b)^3}-\frac{ab\sec(e+fx)}{2f(a-b)^3\left(a+b\sec^2(e+fx)-b\right)}-\frac{\sqrt{b}(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2f(a-b)^{7/2}}$$

```
[Out] -(Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(2*(a - b
)^(7/2)*f) - ((a + b)*Cos[e + f*x])/((a - b)^3*f) + Cos[e + f*x]^3/(3*(a -
b)^2*f) - (a*b*Sec[e + f*x])/(2*(a - b)^3*f*(a - b + b*Sec[e + f*x]^2))
```

Rubi [A] time = 0.180449, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3664, 456, 1261, 205}

$$\frac{\cos^3(e+fx)}{3f(a-b)^2}-\frac{(a+b)\cos(e+fx)}{f(a-b)^3}-\frac{ab\sec(e+fx)}{2f(a-b)^3\left(a+b\sec^2(e+fx)-b\right)}-\frac{\sqrt{b}(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2f(a-b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```


```
[Out] -(Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(2*(a - b
)^(7/2)*f) - ((a + b)*Cos[e + f*x])/((a - b)^3*f) + Cos[e + f*x]^3/(3*(a -
b)^2*f) - (a*b*Sec[e + f*x])/(2*(a - b)^3*f*(a - b + b*Sec[e + f*x]^2))
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{ab\sec(e+fx)}{2(a-b)^3f\left(a-b+b\sec^2(e+fx)\right)} - \frac{b\,\text{Subst}\left(\int \frac{\frac{2}{(a-b)b}-\frac{2ax^2}{(a-b)^2b}+\frac{ax^4}{(a-b)^3}}{x^4\left(a-b+bx^2\right)}\,dx, x, \sec(e+fx)\right)}{2f}$$

$$= -\frac{ab\sec(e+fx)}{2(a-b)^3f\left(a-b+b\sec^2(e+fx)\right)} - \frac{b\,\text{Subst}\left(\int \left(\frac{2}{(a-b)^2bx^4}+\frac{2(a+b)}{b(-a+b)^3x^2}+\frac{3a+2b}{(a-b)^3\left(a-b+bx^2\right.}\right.\right.}{2f}$$

$$= -\frac{(a+b)\cos(e+fx)}{(a-b)^3f} + \frac{\cos^3(e+fx)}{3(a-b)^2f} - \frac{ab\sec(e+fx)}{2(a-b)^3f\left(a-b+b\sec^2(e+fx)\right)} - \frac{(b(3a+2\ldots}{}$$

$$= -\frac{\sqrt{b}(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2(a-b)^{7/2}f} - \frac{(a+b)\cos(e+fx)}{(a-b)^3f} + \frac{\cos^3(e+fx)}{3(a-b)^2f} - \frac{ab\ldots}{2(a-b)^3f(\ldots}$$

Mathematica [A] time = 2.98622, size = 182, normalized size = 1.37

$$\frac{-\frac{\cos(e+fx)\left(\frac{12ab}{(a-b)\cos(2(e+fx))+a+b}+9a+15b\right)+(b-a)\cos(3(e+fx))}{(a-b)^3}+\frac{6\sqrt{b}(3a+2b)\tan^{-1}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{7/2}}+\frac{6\sqrt{b}(3a+2b)\tan^{-1}\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx\ldots\right.}{\sqrt{b}}\right)}{(a-b)^{7/2}}}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((6*Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqr
t[b]])/(a - b)^(7/2) + (6*Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[a - b] + Sqrt[a]
*Tan[(e + f*x)/2])/Sqrt[b]])/(a - b)^(7/2) - (Cos[e + f*x]*(9*a + 15*b + (1
2*a*b)/(a + b + (a - b)*Cos[2*(e + f*x)])) + (-a + b)*Cos[3*(e + f*x)])/(a
- b)^3)/(12*f)
```

Maple [B] time = 0.078, size = 269, normalized size = 2.

$$\frac{a\left(\cos\left(fx+e\right)\right)^3}{3f\left(a^2-2ab+b^2\right)(a-b)} - \frac{b\left(\cos\left(fx+e\right)\right)^3}{3f\left(a^2-2ab+b^2\right)(a-b)} - \frac{\cos\left(fx+e\right)a}{f\left(a^2-2ab+b^2\right)(a-b)} - \frac{\cos\left(fx+e\right)b}{f\left(a^2-2ab+b^2\right)(a-b)} - \frac{\ldots}{2f\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/3/f/(a^2-2*a*b+b^2)/(a-b)*a*cos(f*x+e)^3-1/3/f/(a^2-2*a*b+b^2)/(a-b)*b*co
s(f*x+e)^3-1/f/(a^2-2*a*b+b^2)/(a-b)*a*cos(f*x+e)-1/f/(a^2-2*a*b+b^2)/(a-b)
```

```
*cos(f*x+e)*b-1/2/f*b/(a-b)^3*a*cos(f*x+e)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b
)+3/2/f*b/(a-b)^3/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))*
a+1/f*b^2/(a-b)^3/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.20797, size = 1015, normalized size = 7.63

$$\left[\frac{4\left(a^2-2ab+b^2\right)\cos\left(fx+e\right)^5-4\left(3a^2-ab-2b^2\right)\cos\left(fx+e\right)^3-3\left(\left(3a^2-ab-2b^2\right)\cos\left(fx+e\right)^2+3ab+2b^2\right)\sqrt{}}{12\left(\left(a^4-4a^3b+6a^2b^2-4ab^3+b^4\right)f\cos\left(fx+e\right)^2+\left(a^3b-3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 4*(3*a^2 - a*b - 2*b^2)*cos(f
*x + e)^3 - 3*((3*a^2 - a*b - 2*b^2)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt(-
b/(a - b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a - b)*sqrt(-b/(a - b))*cos(f*
x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) - 6*(3*a*b + 2*b^2)*cos(f*x + e))
/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^3*b - 3
*a^2*b^2 + 3*a*b^3 - b^4)*f), 1/6*(2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 2
*(3*a^2 - a*b - 2*b^2)*cos(f*x + e)^3 - 3*((3*a^2 - a*b - 2*b^2)*cos(f*x +
e)^2 + 3*a*b + 2*b^2)*sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f
*x + e)/b) - 3*(3*a*b + 2*b^2)*cos(f*x + e))/((a^4 - 4*a^3*b + 6*a^2*b^2 -
4*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38939, size = 497, normalized size = 3.74

$$\frac{a^4f^{11}\cos\left(fx+e\right)^3-4a^3bf^{11}\cos\left(fx+e\right)^3+6a^2b^2f^{11}\cos\left(fx+e\right)^3-4ab^3f^{11}\cos\left(fx+e\right)^3+b^4f^{11}\cos\left(fx+e\right)^3-3}{3\left(a^6f^{12}-6a^5bf^{12}+15a^4b^2f^{12}-20a^3b^3f^{12}+15a^2b^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(a^4*f^11*cos(f*x + e)^3 - 4*a^3*b*f^11*cos(f*x + e)^3 + 6*a^2*b^2*f^11
*cos(f*x + e)^3 - 4*a*b^3*f^11*cos(f*x + e)^3 + b^4*f^11*cos(f*x + e)^3 - 3
*a^4*f^11*cos(f*x + e) + 6*a^3*b*f^11*cos(f*x + e) - 6*a*b^3*f^11*cos(f*x +
 e) + 3*b^4*f^11*cos(f*x + e))/(a^6*f^12 - 6*a^5*b*f^12 + 15*a^4*b^2*f^12 -
 20*a^3*b^3*f^12 + 15*a^2*b^4*f^12 - 6*a*b^5*f^12 + b^6*f^12) - 1/2*a*b*cos
(f*x + e)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(a*cos(f*x + e)^2 - b*cos(f*x +
e)^2 + b)*f) + 1/2*(3*a*b + 2*b^2)*arctan((a*cos(f*x + e) - b*cos(f*x + e))
/sqrt(a*b - b^2))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a*b - b^2)*f)
```

3.70 $\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2} dx$

Optimal. Leaf size=101

$$-\frac{3\cos(e+fx)}{2f(a-b)^2}+\frac{\cos(e+fx)}{2f(a-b)\left(a+b\sec^2(e+fx)-b\right)}-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2f(a-b)^{5/2}}$$

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(2*(a - b)^(5/2)*f)
 - (3*Cos[e + f*x])/(2*(a - b)^2*f) + Cos[e + f*x]/(2*(a - b)*f*(a - b + b*
Sec[e + f*x]^2))
```

Rubi [A] time = 0.0731347, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3664, 290, 325, 205}

$$-\frac{3\cos(e+fx)}{2f(a-b)^2}+\frac{\cos(e+fx)}{2f(a-b)\left(a+b\sec^2(e+fx)-b\right)}-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(2*(a - b)^(5/2)*f)
 - (3*Cos[e + f*x])/(2*(a - b)^2*f) + Cos[e + f*x]/(2*(a - b)*f*(a - b + b*
Sec[e + f*x]^2))
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\cos(e+fx)}{2(a-b)f\left(a-b+b\sec^2(e+fx)\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{x^2\left(a-b+bx^2\right)}\,dx, x, \sec(e+fx)\right)}{2(a-b)f}$$
$$= -\frac{3\cos(e+fx)}{2(a-b)^2f} + \frac{\cos(e+fx)}{2(a-b)f\left(a-b+b\sec^2(e+fx)\right)} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \sec(\right.}{2(a-b)^2f}$$
$$= -\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}f} - \frac{3\cos(e+fx)}{2(a-b)^2f} + \frac{\cos(e+fx)}{2(a-b)f\left(a-b+b\sec^2(e+fx)\right)}$$

Mathematica [A] time = 0.764116, size = 146, normalized size = 1.45

$$\frac{\frac{2\cos(e+fx)\left(-\frac{b}{(a-b)\cos(2(e+fx))+a+b}-1\right)}{(a-b)^2} + \frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{5/2}} + \frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{5/2}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((3*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a -
b)^(5/2) + (3*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[
b]])/(a - b)^(5/2) + (2*Cos[e + f*x]*(-1 - b/(a + b + (a - b)*Cos[2*(e + f*
x)])))/(a - b)^2)/(2*f)
```

Maple [A] time = 0.067, size = 114, normalized size = 1.1

$$-\frac{\cos\left(fx+e\right)}{f\left(a^2-2ab+b^2\right)} - \frac{b\cos\left(fx+e\right)}{2f(a-b)^2\left(a\left(\cos\left(fx+e\right)\right)^2-\left(\cos\left(fx+e\right)\right)^2b+b\right)} + \frac{3b}{2f(a-b)^2}\arctan\left((a-b)\cos\left(fx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/f/(a^2-2*a*b+b^2)*cos(f*x+e)-1/2/f*b/(a-b)^2*cos(f*x+e)/(a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)+3/2/f*b/(a-b)^2/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(
b*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94515, size = 699, normalized size = 6.92

$$\left[-\frac{4\,(a-b)\cos\left(fx+e\right)^{3}-3\left((a-b)\cos\left(fx+e\right)^{2}+b\right)\sqrt{-\frac{b}{a-b}}\log\left(\frac{(a-b)\cos\left(fx+e\right)^{2}+2\,(a-b)\sqrt{-\frac{b}{a-b}}\cos\left(fx+e\right)-b}{(a-b)\cos\left(fx+e\right)^{2}+b}\right)+6\,b\cos\left(fx+\right.}{4\left(\left(a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}\right)f\cos\left(fx+e\right)^{2}+\left(a^{2}b-2\,ab^{2}+b^{3}\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a - b)*cos(f*x + e)^3 - 3*((a - b)*cos(f*x + e)^2 + b)*sqrt(-b/(a
 - b))*log(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/(a - b))*cos(f*x + e
) - b)/((a - b)*cos(f*x + e)^2 + b)) + 6*b*cos(f*x + e))/((a^3 - 3*a^2*b +
3*a*b^2 - b^3)*f*cos(f*x + e)^2 + (a^2*b - 2*a*b^2 + b^3)*f), -1/2*(2*(a -
b)*cos(f*x + e)^3 + 3*((a - b)*cos(f*x + e)^2 + b)*sqrt(b/(a - b))*arctan(-
(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + 3*b*cos(f*x + e))/((a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*f*cos(f*x + e)^2 + (a^2*b - 2*a*b^2 + b^3)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37288, size = 207, normalized size = 2.05

$$-\frac{f^{3}\cos\left(fx+e\right)}{a^{2}f^{4}-2\,abf^{4}+b^{2}f^{4}}+\frac{3\,b\arctan\left(\frac{a\cos\left(fx+e\right)-b\cos\left(fx+e\right)}{\sqrt{ab-b^{2}}}\right)}{2\left(a^{2}-2\,ab+b^{2}\right)\sqrt{ab-b^{2}}f}-\frac{b\cos\left(fx+e\right)}{2\left(a\cos\left(fx+e\right)^{2}-b\cos\left(fx+e\right)^{2}+b\right)\left(a^{2}-2\,ab+b^{2}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -f^3*cos(f*x + e)/(a^2*f^4 - 2*a*b*f^4 + b^2*f^4) + 3/2*b*arctan((a*cos(f*x
 + e) - b*cos(f*x + e))/sqrt(a*b - b^2))/((a^2 - 2*a*b + b^2)*sqrt(a*b - b^
2)*f) - 1/2*b*cos(f*x + e)/((a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)*(a^2
- 2*a*b + b^2)*f)
```

3.71 $\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{\sqrt{b}(3a-2b)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^2f(a-b)^{3/2}}-\frac{\tanh^{-1}(\cos(e+fx))}{a^2f}-\frac{b\sec(e+fx)}{2af(a-b)\left(a+b\sec^2(e+fx)-b\right)}$$

```
[Out] -((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(2*a^2*(a
 - b)^(3/2)*f) - ArcTanh[Cos[e + f*x]]/(a^2*f) - (b*Sec[e + f*x])/(2*a*(a -
 b)*f*(a - b + b*Sec[e + f*x]^2))
```

Rubi [A] time = 0.127508, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3664, 414, 522, 207, 205}

$$-\frac{\sqrt{b}(3a-2b)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^2f(a-b)^{3/2}}-\frac{\tanh^{-1}(\cos(e+fx))}{a^2f}-\frac{b\sec(e+fx)}{2af(a-b)\left(a+b\sec^2(e+fx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```


```
[Out] -((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(2*a^2*(a
 - b)^(3/2)*f) - ArcTanh[Cos[e + f*x]]/(a^2*f) - (b*Sec[e + f*x])/(2*a*(a -
 b)*f*(a - b + b*Sec[e + f*x]^2))
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{b\sec(e+fx)}{2a(a-b)f\left(a-b+b\sec^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-b-bx^2}{(-1+x^2)(a-b+bx^2)}\,dx, x, \sec(e+fx)\right)}{2a(a-b)f}$$

$$= -\frac{b\sec(e+fx)}{2a(a-b)f\left(a-b+b\sec^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{a^2 f} - \frac{((3a-2b)b)\,\text{S}}{}$$

$$= -\frac{(3a-2b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^2(a-b)^{3/2}f} - \frac{\tanh^{-1}(\cos(e+fx))}{a^2 f} - \frac{b\sec(e+fx)}{2a(a-b)f\left(a-b+b\sec^2(e+\right.}$$

Mathematica [A] time = 0.818277, size = 184, normalized size = 1.67

$$\frac{-\frac{2ab\cos(e+fx)}{(a-b)((a-b)\cos(2(e+fx))+a+b)} + \frac{\sqrt{b}(3a-2b)\tan^{-1}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{3/2}} + \frac{\sqrt{b}(3a-2b)\tan^{-1}\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{3/2}} + 2\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2a^2 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[
b]])/(a - b)^(3/2) + ((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan
[(e + f*x)/2])/Sqrt[b]])/(a - b)^(3/2) - (2*a*b*Cos[e + f*x])/((a - b)*(a +
 b + (a - b)*Cos[2*(e + f*x)])) - 2*Log[Cos[(e + f*x)/2]] + 2*Log[Sin[(e +
f*x)/2]])/(2*a^2*f)
```

Maple [A] time = 0.084, size = 179, normalized size = 1.6

$$-\frac{\ln\left(\cos\left(fx+e\right)+1\right)}{2fa^2} - \frac{b\cos\left(fx+e\right)}{2fa\left(a-b\right)\left(a\left(\cos\left(fx+e\right)\right)^2-\left(\cos\left(fx+e\right)\right)^2 b+b\right)} + \frac{3b}{2fa\left(a-b\right)}\arctan\left((a-b)\cos\left(fx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f/a^2*ln(cos(f*x+e)+1)-1/2/f*b/a/(a-b)*cos(f*x+e)/(a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)+3/2/f*b/a/(a-b)/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a
-b))^(1/2))-1/f*b^2/a^2/(a-b)/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a
-b))^(1/2))+1/2/f/a^2*ln(cos(f*x+e)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.68441, size = 1102, normalized size = 10.02

$$\left[-\frac{2\,ab\cos\left(fx+e\right)-\left(\left(3\,a^{2}-5\,ab+2\,b^{2}\right)\cos\left(fx+e\right)^{2}+3\,ab-2\,b^{2}\right)\sqrt{-\frac{b}{a-b}}\log\left(\frac{(a-b)\cos\left(fx+e\right)^{2}+2\,(a-b)\sqrt{-\frac{b}{a-b}}\cos\left(fx+e\right)-\cdots}{(a-b)\cos\left(fx+e\right)^{2}+b}\cdots\right.}{4\left(\left(a^{4}-2\,a^{3}\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*cos(f*x + e) - ((3*a^2 - 5*a*b + 2*b^2)*cos(f*x + e)^2 + 3*a*b
 - 2*b^2)*sqrt(-b/(a - b))*log(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/
(a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 2*((a^2 - 2*a*b
+ b^2)*cos(f*x + e)^2 + a*b - b^2)*log(1/2*cos(f*x + e) + 1/2) - 2*((a^2 -
2*a*b + b^2)*cos(f*x + e)^2 + a*b - b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^
4 - 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b - a^2*b^2)*f), -1/2*(a*b*c
os(f*x + e) + ((3*a^2 - 5*a*b + 2*b^2)*cos(f*x + e)^2 + 3*a*b - 2*b^2)*sqrt
(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + ((a^2 - 2*a*b
 + b^2)*cos(f*x + e)^2 + a*b - b^2)*log(1/2*cos(f*x + e) + 1/2) - ((a^2 - 2
*a*b + b^2)*cos(f*x + e)^2 + a*b - b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^4
 - 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b - a^2*b^2)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.4036, size = 360, normalized size = 3.27

$$-\frac{\frac{\left(3\,ab-2\,b^2\right)\arctan\left(-\frac{a\cos\left(fx+e\right)-b\cos\left(fx+e\right)-b}{\sqrt{ab-b^2}\cos\left(fx+e\right)+\sqrt{ab-b^2}}\right)}{\left(a^3-a^2b\right)\sqrt{ab-b^2}}+\frac{2\left(ab+\frac{ab\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{2\,b^2\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}\right)}{\left(a^3-a^2b\right)\left(a+\frac{2\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{4\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{a\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}\right)}-\frac{\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((3*a*b - 2*b^2)*arctan(-(a*cos(f*x + e) - b*cos(f*x + e) - b)/(sqrt(a
*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/((a^3 - a^2*b)*sqrt(a*b - b^2))
+ 2*(a*b + a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b^2*(cos(f*x + e)
- 1)/(cos(f*x + e) + 1))/((a^3 - a^2*b)*(a + 2*a*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2)) - log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/
a^2)/f
```

3.72 $\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=147

$$-\frac{b\sec(e+fx)}{a^2f\left(a+b\sec^2(e+fx)-b\right)}-\frac{(a-4b)\tanh^{-1}(\cos(e+fx))}{2a^3f}-\frac{\sqrt{b}(3a-4b)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^3f\sqrt{a-b}}-\frac{\cot(e+fx)\csc(}{2af\left(a+b\sec^2(e+\right.}$$

```
[Out] -((3*a - 4*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(2*a^3*Sq
rt[a - b]*f) - ((a - 4*b)*ArcTanh[Cos[e + f*x]])/(2*a^3*f) - (Cot[e + f*x]*
Csc[e + f*x])/(2*a*f*(a - b + b*Sec[e + f*x]^2)) - (b*Sec[e + f*x])/(a^2*f*
(a - b + b*Sec[e + f*x]^2))
```

Rubi [A] time = 0.181454, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3664, 471, 527, 522, 207, 205}

$$-\frac{b\sec(e+fx)}{a^2f\left(a+b\sec^2(e+fx)-b\right)}-\frac{(a-4b)\tanh^{-1}(\cos(e+fx))}{2a^3f}-\frac{\sqrt{b}(3a-4b)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^3f\sqrt{a-b}}-\frac{\cot(e+fx)\csc(}{2af\left(a+b\sec^2(e+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((3*a - 4*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(2*a^3*Sq
rt[a - b]*f) - ((a - 4*b)*ArcTanh[Cos[e + f*x]])/(2*a^3*f) - (Cot[e + f*x]*
Csc[e + f*x])/(2*a*f*(a - b + b*Sec[e + f*x]^2)) - (b*Sec[e + f*x])/(a^2*f*
(a - b + b*Sec[e + f*x]^2))
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{a-b-3bx^2}{\left(-1+x^2\right)\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+fx)\right)}{2af}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)} - \frac{b\sec(e+fx)}{a^2f\left(a-b+b\sec^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2(a-2b)(a-b)-4(a-b)}{\left(-1+x^2\right)\left(a-b+bx^2\right.}\right.}{4a^2(a-}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)} - \frac{b\sec(e+fx)}{a^2f\left(a-b+b\sec^2(e+fx)\right)} + \frac{(a-4b)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx\right.}{2a^3f}$$

$$= -\frac{(3a-4b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^3\sqrt{a-b}f} - \frac{(a-4b)\tanh^{-1}(\cos(e+fx))}{2a^3f} - \frac{\cot(e+fx)\csc(e+}{2af\left(a-b+b\sec^2(e\right.}$$

Mathematica [A] time = 6.18172, size = 218, normalized size = 1.48

$$\frac{-\frac{8ab\cos(e+fx)}{(a-b)\cos(2(e+fx))+a+b} + \frac{4\sqrt{b}(3a-4b)\tan^{-1}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{\sqrt{a-b}} + \frac{4\sqrt{b}(3a-4b)\tan^{-1}\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{\sqrt{a-b}} + 4(a-4b)\log\left(\sin\left(\frac{1}{2}(e+f\right.\right.}{8a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((4*(3*a - 4*b)*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqr
t[b]])/Sqrt[a - b] + (4*(3*a - 4*b)*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*T
an[(e + f*x)/2])/Sqrt[b]])/Sqrt[a - b] - (8*a*b*Cos[e + f*x])/(a + b + (a -
 b)*Cos[2*(e + f*x)]) - a*Csc[(e + f*x)/2]^2 - 4*(a - 4*b)*Log[Cos[(e + f*x
```

```
)/2]] + 4*(a - 4*b)*Log[Sin[(e + f*x)/2]] + a*Sec[(e + f*x)/2]^2)/(8*a^3*f)
```

Maple [A] time = 0.101, size = 229, normalized size = 1.6

$$\frac{1}{4fa^2\left(\cos\left(fx+e\right)+1\right)}-\frac{\ln\left(\cos\left(fx+e\right)+1\right)}{4fa^2}+\frac{\ln\left(\cos\left(fx+e\right)+1\right)b}{fa^3}-\frac{b\cos\left(fx+e\right)}{2fa^2\left(a\left(\cos\left(fx+e\right)\right)^2-\left(\cos\left(fx+e\right)\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/4/f/a^2/(cos(f*x+e)+1)-1/4/f/a^2*ln(cos(f*x+e)+1)+1/f/a^3*ln(cos(f*x+e)+1
)*b-1/2/f*b/a^2*cos(f*x+e)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+3/2/f*b/a^2/(b
*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))-2/f*b^2/a^3/(b*(a-b)
)^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))+1/4/f/a^2/(cos(f*x+e)-1)+1
/4/f/a^2*ln(cos(f*x+e)-1)-1/f/a^3*ln(cos(f*x+e)-1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.52578, size = 1565, normalized size = 10.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2 - 2*a*b)*cos(f*x + e)^3 + 4*a*b*cos(f*x + e) - ((3*a^2 - 7*a*b
 + 4*b^2)*cos(f*x + e)^4 - (3*a^2 - 10*a*b + 8*b^2)*cos(f*x + e)^2 - 3*a*b
+ 4*b^2)*sqrt(-b/(a - b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a - b)*sqrt(-b/
(a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) - ((a^2 - 5*a*b +
4*b^2)*cos(f*x + e)^4 - (a^2 - 6*a*b + 8*b^2)*cos(f*x + e)^2 - a*b + 4*b^2)
*log(1/2*cos(f*x + e) + 1/2) + ((a^2 - 5*a*b + 4*b^2)*cos(f*x + e)^4 - (a^2
 - 6*a*b + 8*b^2)*cos(f*x + e)^2 - a*b + 4*b^2)*log(-1/2*cos(f*x + e) + 1/2
))/((a^4 - a^3*b)*f*cos(f*x + e)^4 - a^3*b*f - (a^4 - 2*a^3*b)*f*cos(f*x +
e)^2), 1/4*(2*(a^2 - 2*a*b)*cos(f*x + e)^3 + 4*a*b*cos(f*x + e) - 2*((3*a^2
 - 7*a*b + 4*b^2)*cos(f*x + e)^4 - (3*a^2 - 10*a*b + 8*b^2)*cos(f*x + e)^2
- 3*a*b + 4*b^2)*sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x +
e)/b) - ((a^2 - 5*a*b + 4*b^2)*cos(f*x + e)^4 - (a^2 - 6*a*b + 8*b^2)*cos(f
*x + e)^2 - a*b + 4*b^2)*log(1/2*cos(f*x + e) + 1/2) + ((a^2 - 5*a*b + 4*b^
2)*cos(f*x + e)^4 - (a^2 - 6*a*b + 8*b^2)*cos(f*x + e)^2 - a*b + 4*b^2)*log
(-1/2*cos(f*x + e) + 1/2))/((a^4 - a^3*b)*f*cos(f*x + e)^4 - a^3*b*f - (a^4
```

```
 - 2*a^3*b)*f*cos(f*x + e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42445, size = 559, normalized size = 3.8

$$\frac{\frac{6\,(a-4\,b)\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)}{a^3}-\frac{12\left(3\,ab-4\,b^2\right)\arctan\left(-\frac{a\cos\left(fx+e\right)-b\cos\left(fx+e\right)-b}{\sqrt{ab-b^2}\cos\left(fx+e\right)+\sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2}\,a^3}-\frac{3\left(\cos\left(fx+e\right)-1\right)}{a^2\left(\cos\left(fx+e\right)+1\right)}+\frac{3\,a^2+\frac{4\,a^2\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{28\,ab\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{a^2\left(\cos\left(f\cdots\right.\right.}{\left(\cos\left(fx\cdots\right.\right.}}{a^3\left(\frac{a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{2\,a\left(\cos\left(f\cdots\right.\right.}{\left(\cos\left(fx\cdots\right.\right.}\right.}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(6*(a - 4*b)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/a^3 - 12*(3*a
*b - 4*b^2)*arctan(-(a*cos(f*x + e) - b*cos(f*x + e) - b)/(sqrt(a*b - b^2)*
cos(f*x + e) + sqrt(a*b - b^2)))/(sqrt(a*b - b^2)*a^3) - 3*(cos(f*x + e) -
1)/(a^2*(cos(f*x + e) + 1)) + (3*a^2 + 4*a^2*(cos(f*x + e) - 1)/(cos(f*x +
e) + 1) - 28*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - a^2*(cos(f*x + e)
- 1)^2/(cos(f*x + e) + 1)^2 + 16*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2 - 2*a^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 8*a*b*(cos(f*x + e)
 - 1)^3/(cos(f*x + e) + 1)^3)/(a^3*(a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
 + 2*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 4*b*(cos(f*x + e) - 1)^2
/(cos(f*x + e) + 1)^2 + a*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)))/f
```

3.73 $\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=210

$$-\frac{3\left(a^2-8ab+8b^2\right)\tanh^{-1}(\cos(e+fx))}{8a^4f}-\frac{3b(3a-4b)\sec(e+fx)}{8a^3f\left(a+b\sec^2(e+fx)-b\right)}-\frac{3\sqrt{b}(a-2b)\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^4f}-$$

```
[Out] (-3*(a - 2*b)*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]
])/(2*a^4*f) - (3*(a^2 - 8*a*b + 8*b^2)*ArcTanh[Cos[e + f*x]])/(8*a^4*f) -
((5*a - 6*b)*Cot[e + f*x]*Csc[e + f*x])/(8*a^2*f*(a - b + b*Sec[e + f*x]^2)
) - (Cot[e + f*x]^3*Csc[e + f*x])/(4*a*f*(a - b + b*Sec[e + f*x]^2)) - (3*(
3*a - 4*b)*b*Sec[e + f*x])/(8*a^3*f*(a - b + b*Sec[e + f*x]^2))
```

Rubi [A] time = 0.275453, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3664, 470, 527, 522, 207, 205}

$$-\frac{3\left(a^2-8ab+8b^2\right)\tanh^{-1}(\cos(e+fx))}{8a^4f}-\frac{3b(3a-4b)\sec(e+fx)}{8a^3f\left(a+b\sec^2(e+fx)-b\right)}-\frac{3\sqrt{b}(a-2b)\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^4f}-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```


```
[Out] (-3*(a - 2*b)*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]
])/(2*a^4*f) - (3*(a^2 - 8*a*b + 8*b^2)*ArcTanh[Cos[e + f*x]])/(8*a^4*f) -
((5*a - 6*b)*Cot[e + f*x]*Csc[e + f*x])/(8*a^2*f*(a - b + b*Sec[e + f*x]^2)
) - (Cot[e + f*x]^3*Csc[e + f*x])/(4*a*f*(a - b + b*Sec[e + f*x]^2)) - (3*(
3*a - 4*b)*b*Sec[e + f*x])/(8*a^3*f*(a - b + b*Sec[e + f*x]^2))
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^3\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-a+b+(-4a+5b)x^2}{\left(-1+x^2\right)^2\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+fx)\right)}{4af}$$

$$= -\frac{(5a-6b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-3(a-2b)(a-b)}{(-1+x^2)(}\right.}{}$$

$$= -\frac{(5a-6b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)} - \frac{3(3a-4b)b\sec(e}{8a^3f\left(a-b+b\sec^2(\right.}$$

$$= -\frac{(5a-6b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)} - \frac{3(3a-4b)b\sec(e}{8a^3f\left(a-b+b\sec^2(\right.}$$

$$= -\frac{3(a-2b)\sqrt{a-b}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^4f} - \frac{3\left(a^2-8ab+8b^2\right)\tanh^{-1}(\cos(e+fx))}{8a^4f} - \frac{(5a}{8a}$$

Mathematica [A] time = 6.33467, size = 392, normalized size = 1.87

$$\frac{b^2\cos(e+fx)-ab\cos(e+fx)}{a^3f(a\cos(2(e+fx))+a-b\cos(2(e+fx))+b)} + \frac{3\left(a^2-8ab+8b^2\right)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{8a^4f} - \frac{3\left(a^2-8ab+8b^2\right)\log\left(\cos\right.}{8a^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (3*(a - 2*b)*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sec[(e + f*x)/2]*(Sqrt[a - b]*Cos[
(e + f*x)/2] - Sqrt[a]*Sin[(e + f*x)/2]))/Sqrt[b]])/(2*a^4*f) + (3*(a - 2*b
)*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sec[(e + f*x)/2]*(Sqrt[a - b]*Cos[(e + f*x)/2
] + Sqrt[a]*Sin[(e + f*x)/2]))/Sqrt[b]])/(2*a^4*f) + (-(a*b*Cos[e + f*x]) +
 b^2*Cos[e + f*x])/(a^3*f*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])
) + ((-3*a + 8*b)*Csc[(e + f*x)/2]^2)/(32*a^3*f) - Csc[(e + f*x)/2]^4/(64*a
^2*f) - (3*(a^2 - 8*a*b + 8*b^2)*Log[Cos[(e + f*x)/2]])/(8*a^4*f) + (3*(a^2
 - 8*a*b + 8*b^2)*Log[Sin[(e + f*x)/2]])/(8*a^4*f) + ((3*a - 8*b)*Sec[(e +
f*x)/2]^2)/(32*a^3*f) + Sec[(e + f*x)/2]^4/(64*a^2*f)
```

Maple [B] time = 0.109, size = 428, normalized size = 2.

$$\frac{1}{16 f a^2 \left(\cos \left(f x+e\right)+1\right)^2}+\frac{3}{16 f a^2 \left(\cos \left(f x+e\right)+1\right)}-\frac{b}{2 f a^3 \left(\cos \left(f x+e\right)+1\right)}-\frac{3 \ln \left(\cos \left(f x+e\right)+1\right)}{16 f a^2}+\frac{3 \ln (\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/16/f/a^2/(cos(f*x+e)+1)^2+3/16/f/a^2/(cos(f*x+e)+1)-1/2/f/a^3/(cos(f*x+e)
+1)*b-3/16/f/a^2*ln(cos(f*x+e)+1)+3/2/f/a^3*ln(cos(f*x+e)+1)*b-3/2/f/a^4*ln
(cos(f*x+e)+1)*b^2-1/2/f*b/a^2*cos(f*x+e)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)
+1/2/f*b^2/a^3*cos(f*x+e)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+3/2/f*b/a^2/(b*
(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))-9/2/f*b^2/a^3/(b*(a-b
))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))+3/f*b^3/a^4/(b*(a-b))^(1/
2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))-1/16/f/a^2/(cos(f*x+e)-1)^2+3/1
6/f/a^2/(cos(f*x+e)-1)-1/2/f/a^3/(cos(f*x+e)-1)*b+3/16/f/a^2*ln(cos(f*x+e)-
1)-3/2/f/a^3*ln(cos(f*x+e)-1)*b+3/2/f/a^4*ln(cos(f*x+e)-1)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.77455, size = 2477, normalized size = 11.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(6*(a^3 - 5*a^2*b + 4*a*b^2)*cos(f*x + e)^5 - 2*(5*a^3 - 24*a^2*b + 2
4*a*b^2)*cos(f*x + e)^3 - 12*((a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^6 - (2*a^2
 - 7*a*b + 6*b^2)*cos(f*x + e)^4 + (a^2 - 5*a*b + 6*b^2)*cos(f*x + e)^2 + a
*b - 2*b^2)*sqrt(-a*b + b^2)*log(((a - b)*cos(f*x + e)^2 - 2*sqrt(-a*b + b^
```

```
2)*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) - 6*(3*a^2*b - 4*a*b^2)*
cos(f*x + e) - 3*((a^3 - 9*a^2*b + 16*a*b^2 - 8*b^3)*cos(f*x + e)^6 - (2*a^
3 - 19*a^2*b + 40*a*b^2 - 24*b^3)*cos(f*x + e)^4 + a^2*b - 8*a*b^2 + 8*b^3
+ (a^3 - 11*a^2*b + 32*a*b^2 - 24*b^3)*cos(f*x + e)^2)*log(1/2*cos(f*x + e)
 + 1/2) + 3*((a^3 - 9*a^2*b + 16*a*b^2 - 8*b^3)*cos(f*x + e)^6 - (2*a^3 - 1
9*a^2*b + 40*a*b^2 - 24*b^3)*cos(f*x + e)^4 + a^2*b - 8*a*b^2 + 8*b^3 + (a^
3 - 11*a^2*b + 32*a*b^2 - 24*b^3)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1
/2))/((a^5 - a^4*b)*f*cos(f*x + e)^6 + a^4*b*f - (2*a^5 - 3*a^4*b)*f*cos(f*
x + e)^4 + (a^5 - 3*a^4*b)*f*cos(f*x + e)^2), 1/16*(6*(a^3 - 5*a^2*b + 4*a*
b^2)*cos(f*x + e)^5 - 2*(5*a^3 - 24*a^2*b + 24*a*b^2)*cos(f*x + e)^3 + 24*(
(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^6 - (2*a^2 - 7*a*b + 6*b^2)*cos(f*x + e)
^4 + (a^2 - 5*a*b + 6*b^2)*cos(f*x + e)^2 + a*b - 2*b^2)*sqrt(a*b - b^2)*ar
ctan(sqrt(a*b - b^2)*cos(f*x + e)/b) - 6*(3*a^2*b - 4*a*b^2)*cos(f*x + e) -
 3*((a^3 - 9*a^2*b + 16*a*b^2 - 8*b^3)*cos(f*x + e)^6 - (2*a^3 - 19*a^2*b +
 40*a*b^2 - 24*b^3)*cos(f*x + e)^4 + a^2*b - 8*a*b^2 + 8*b^3 + (a^3 - 11*a^
2*b + 32*a*b^2 - 24*b^3)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((
a^3 - 9*a^2*b + 16*a*b^2 - 8*b^3)*cos(f*x + e)^6 - (2*a^3 - 19*a^2*b + 40*a
*b^2 - 24*b^3)*cos(f*x + e)^4 + a^2*b - 8*a*b^2 + 8*b^3 + (a^3 - 11*a^2*b +
 32*a*b^2 - 24*b^3)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^5 - a
^4*b)*f*cos(f*x + e)^6 + a^4*b*f - (2*a^5 - 3*a^4*b)*f*cos(f*x + e)^4 + (a^
5 - 3*a^4*b)*f*cos(f*x + e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.43885, size = 740, normalized size = 3.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(12*(a^2 - 8*a*b + 8*b^2)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/
a^4 - 96*(a^2*b - 3*a*b^2 + 2*b^3)*arctan(-(a*cos(f*x + e) - b*cos(f*x + e)
 - b)/(sqrt(a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/(sqrt(a*b - b^2)*a^
4) - (8*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*a*b*(cos(f*x + e) -
1)/(cos(f*x + e) + 1) - a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^4
- (a^2 - 8*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 16*a*b*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) + 18*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
 - 144*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 144*b^2*(cos(f*x + e
) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(a^4*(cos(f*x + e) - 1)
^2) - 64*(a^2*b - a*b^2 + a^2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 3*a
*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2*b^3*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1))/((a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f
*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)
^2)*a^4))/f
```

3.74 $\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=196

$$\frac{3x\left(a^2+6ab+b^2\right)}{8(a-b)^4}-\frac{3\sqrt{a}\sqrt{b}(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2f(a-b)^4}-\frac{3b(3a+b)\tan(e+fx)}{8f(a-b)^3\left(a+b\tan^2(e+fx)\right)}+\frac{\sin(e+fx)\cos^3(e+}{4f(a-b)\left(a+b\tan^2(e\right.}$$

```
[Out] (3*(a^2 + 6*a*b + b^2)*x)/(8*(a - b)^4) - (3*Sqrt[a]*Sqrt[b]*(a + b)*ArcTan
[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2*(a - b)^4*f) - ((5*a + b)*Cos[e + f*x]
*Sin[e + f*x])/(8*(a - b)^2*f*(a + b*Tan[e + f*x]^2)) + (Cos[e + f*x]^3*Sin
[e + f*x])/(4*(a - b)*f*(a + b*Tan[e + f*x]^2)) - (3*b*(3*a + b)*Tan[e + f*
x])/(8*(a - b)^3*f*(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.249952, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3663, 470, 527, 522, 203, 205}

$$\frac{3x\left(a^2+6ab+b^2\right)}{8(a-b)^4}-\frac{3\sqrt{a}\sqrt{b}(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2f(a-b)^4}-\frac{3b(3a+b)\tan(e+fx)}{8f(a-b)^3\left(a+b\tan^2(e+fx)\right)}+\frac{\sin(e+fx)\cos^3(e+}{4f(a-b)\left(a+b\tan^2(e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```


```
[Out] (3*(a^2 + 6*a*b + b^2)*x)/(8*(a - b)^4) - (3*Sqrt[a]*Sqrt[b]*(a + b)*ArcTan
[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2*(a - b)^4*f) - ((5*a + b)*Cos[e + f*x]
*Sin[e + f*x])/(8*(a - b)^2*f*(a + b*Tan[e + f*x]^2)) + (Cos[e + f*x]^3*Sin
[e + f*x])/(4*(a - b)*f*(a + b*Tan[e + f*x]^2)) - (3*b*(3*a + b)*Tan[e + f*
x])/(8*(a - b)^3*f*(a + b*Tan[e + f*x]^2))
```


Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^3\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{a+(-4a-b)x^2}{\left(1+x^2\right)^2\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4(a-b)f}$$

$$= -\frac{(5a+b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{3a(a+b)-3b(\cdots}{(1+x^2)(a+\cdots}\right.}{8(\cdots}$$

$$= -\frac{(5a+b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{3b(3a+b)\tan(\cdots}{8(a-b)^3f\left(a+b\tan\cdots\right.}$$

$$= -\frac{(5a+b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{3b(3a+b)\tan(\cdots}{8(a-b)^3f\left(a+b\tan\cdots\right.}$$

$$= \frac{3\left(a^2+6ab+b^2\right)x}{8(a-b)^4} - \frac{3\sqrt{a}\sqrt{b}(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2(a-b)^4f} - \frac{(5a+b)\cos(e+fx)\sin(e+fx\cdots}{8(a-b)^2f\left(a+b\tan^2(e+fx)\cdots\right.}$$

Mathematica [A] time = 1.51482, size = 136, normalized size = 0.69

$$\frac{12\left(a^2+6ab+b^2\right)(e+fx)+(a-b)^2\sin(4(e+fx))-8(a+b)(a-b)\sin(2(e+fx))-48\sqrt{a}\sqrt{b}(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\cdots\right.}{32f(a-b)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (12*(a^2 + 6*a*b + b^2)*(e + f*x) - 48*Sqrt[a]*Sqrt[b]*(a + b)*ArcTan[(Sqrt
[b]*Tan[e + f*x])/Sqrt[a]] - 8*(a - b)*(a + b)*Sin[2*(e + f*x)] - (16*a*(a
- b)*b*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)]) + (a - b)^2*Sin
[4*(e + f*x)])/(32*(a - b)^4*f)
```

Maple [B] time = 0.092, size = 411, normalized size = 2.1

$$-\frac{a^2 b \tan\left(fx+e\right)}{2f(a-b)^4\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)}+\frac{ab^2\tan\left(fx+e\right)}{2f(a-b)^4\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)}-\frac{3a^2b}{2f(a-b)^4}\arctan\left(b\tan\left(fx+e\right)\frac{1}{\sqrt{ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*a^2*b/(a-b)^4*tan(f*x+e)/(a+b*tan(f*x+e)^2)+1/2/f*a*b^2/(a-b)^4*tan(
f*x+e)/(a+b*tan(f*x+e)^2)-3/2/f*a^2*b/(a-b)^4/(a*b)^(1/2)*arctan(b*tan(f*x+
e)/(a*b)^(1/2))-3/2/f*a*b^2/(a-b)^4/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(
1/2))-5/8/f/(a-b)^4/(1+tan(f*x+e)^2)^2*tan(f*x+e)^3*a^2+1/4/f/(a-b)^4/(1+ta
n(f*x+e)^2)^2*tan(f*x+e)^3*a*b+3/8/f/(a-b)^4/(1+tan(f*x+e)^2)^2*tan(f*x+e)^
3*b^2-3/8/f/(a-b)^4/(1+tan(f*x+e)^2)^2*tan(f*x+e)*a^2+5/8/f/(a-b)^4/(1+tan(
f*x+e)^2)^2*tan(f*x+e)*b^2-1/4/f/(a-b)^4/(1+tan(f*x+e)^2)^2*tan(f*x+e)*a*b+
9/4/f/(a-b)^4*arctan(tan(f*x+e))*a*b+3/8/f/(a-b)^4*arctan(tan(f*x+e))*b^2+3
/8/f/(a-b)^4*arctan(tan(f*x+e))*a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.47919, size = 1600, normalized size = 8.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a^3 + 5*a^2*b - 5*a*b^2 - b^3)*f*x*cos(f*x + e)^2 + 3*(a^2*b + 6*a
*b^2 + b^3)*f*x + 3*((a^2 - b^2)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-a*b)*log
(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 + 4*(
(a + b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b)*sin(f*x + e) + b^2)/((a
^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2)) + (
2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^5 - (5*a^3 - 9*a^2*b + 3*a*b
^2 + b^3)*cos(f*x + e)^3 - 3*(3*a^2*b - 2*a*b^2 - b^3)*cos(f*x + e))*sin(f*
x + e))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*f*cos(f*
x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f), 1/8*(3*(a^3
```

```
+ 5*a^2*b - 5*a*b^2 - b^3)*f*x*cos(f*x + e)^2 + 3*(a^2*b + 6*a*b^2 + b^3)*f
*x + 6*((a^2 - b^2)*cos(f*x + e)^2 + a*b + b^2)*sqrt(a*b)*arctan(1/2*((a +
b)*cos(f*x + e)^2 - b)*sqrt(a*b)/(a*b*cos(f*x + e)*sin(f*x + e))) + (2*(a^3
 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^5 - (5*a^3 - 9*a^2*b + 3*a*b^2 + b
^3)*cos(f*x + e)^3 - 3*(3*a^2*b - 2*a*b^2 - b^3)*cos(f*x + e))*sin(f*x + e)
)/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*f*cos(f*x + e)
^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3924, size = 359, normalized size = 1.83

$$\frac{\frac{3\left(a^2+6ab+b^2\right)(fx+e)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}-\frac{4ab\tan(fx+e)}{\left(a^3-3a^2b+3ab^2-b^3\right)\left(b\tan(fx+e)^2+a\right)}-\frac{12\left(a^2b+ab^2\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{\left(a^4-4a^3b+6a^2b^2-4ab^3+b^4\right)\sqrt{ab}}-\frac{5a\tan(fx+e)^3+3b\tan(\cdots}{\left(a^3-3a^2b+3\cdots\right.}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^2 + 6*a*b + b^2)*(f*x + e)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 +
 b^4) - 4*a*b*tan(f*x + e)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(b*tan(f*x + e)
^2 + a)) - 12*(a^2*b + a*b^2)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan
(b*tan(f*x + e)/sqrt(a*b)))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*sq
rt(a*b)) - (5*a*tan(f*x + e)^3 + 3*b*tan(f*x + e)^3 + 3*a*tan(f*x + e) + 5*
b*tan(f*x + e))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(tan(f*x + e)^2 + 1)^2))/f
```

3.75 $\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=138

$$-\frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2\sqrt{a}f(a-b)^3}-\frac{b\tan(e+fx)}{f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{\sin(e+fx)\cos(e+fx)}{2f(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{x(a+3b)}{2(a-b)^3}$$

```
[Out] ((a + 3*b)*x)/(2*(a - b)^3) - (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a]])/(2*Sqrt[a]*(a - b)^3*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*(a -
 b)*f*(a + b*Tan[e + f*x]^2)) - (b*Tan[e + f*x])/((a - b)^2*f*(a + b*Tan[e
+ f*x]^2))
```

Rubi [A] time = 0.155567, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3663, 471, 527, 522, 203, 205}

$$-\frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2\sqrt{a}f(a-b)^3}-\frac{b\tan(e+fx)}{f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{\sin(e+fx)\cos(e+fx)}{2f(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{x(a+3b)}{2(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```


```
[Out] ((a + 3*b)*x)/(2*(a - b)^3) - (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a]])/(2*Sqrt[a]*(a - b)^3*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*(a -
 b)*f*(a + b*Tan[e + f*x]^2)) - (b*Tan[e + f*x])/((a - b)^2*f*(a + b*Tan[e
+ f*x]^2))
```


Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{a-3bx^2}{\left(1+x^2\right)\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{2(a-b)f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{b\tan(e+fx)}{(a-b)^2f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a(a+b)-4abx^2}{\left(1+x^2\right)\left(a+bx^2\right)} \right.}{4a(a-}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{b\tan(e+fx)}{(a-b)^2f\left(a+b\tan^2(e+fx)\right)} - \frac{(b(3a+b))\,\text{Subst}\left(\int \right.}{2(a-}$$

$$= \frac{(a+3b)x}{2(a-b)^3} - \frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2\sqrt{a}(a-b)^3f} - \frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{b}{(a-b)^2f\,(}$$

Mathematica [A] time = 1.58765, size = 111, normalized size = 0.8

$$-\frac{-2(a+3b)(e+fx)+(a-b)\sin(2(e+fx))+\frac{2\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{2b(a-b)\sin(2(e+fx))}{(a-b)\cos(2(e+fx))+a+b}}{4f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(-2*(a + 3*b)*(e + f*x) + (2*Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tan[e + f*x
])/Sqrt[a]])/Sqrt[a] + (a - b)*Sin[2*(e + f*x)] + (2*(a - b)*b*Sin[2*(e + f
*x)])/(a + b + (a - b)*Cos[2*(e + f*x)]))/(4*(a - b)^3*f)
```

Maple [A] time = 0.077, size = 240, normalized size = 1.7

$$-\frac{\tan\left(fx+e\right)ab}{2f\left(a-b\right)^{3}\left(a+b\left(\tan\left(fx+e\right)\right)^{2}\right)}+\frac{b^{2}\tan\left(fx+e\right)}{2f\left(a-b\right)^{3}\left(a+b\left(\tan\left(fx+e\right)\right)^{2}\right)}-\frac{3ab}{2f\left(a-b\right)^{3}}\arctan\left(b\tan\left(fx+e\right)\frac{1}{\sqrt{ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f/(a-b)^3*b*tan(f*x+e)/(a+b*tan(f*x+e)^2)*a+1/2/f/(a-b)^3*b^2*tan(f*x+
e)/(a+b*tan(f*x+e)^2)-3/2/f/(a-b)^3*b/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)
^(1/2))*a-1/2/f/(a-b)^3*b^2/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-1/
2/f/(a-b)^3*tan(f*x+e)/(1+tan(f*x+e)^2)*a+1/2/f/(a-b)^3*tan(f*x+e)/(1+tan(f
*x+e)^2)*b+1/2/f/(a-b)^3*arctan(tan(f*x+e))*a+3/2/f/(a-b)^3*arctan(tan(f*x+
e))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.2771, size = 1300, normalized size = 9.42

$$\left[\frac{4\left(a^{2}+2ab-3b^{2}\right)fx\cos\left(fx+e\right)^{2}+4\left(ab+3b^{2}\right)fx-\left(\left(3a^{2}-2ab-b^{2}\right)\cos\left(fx+e\right)^{2}+3ab+b^{2}\right)\sqrt{-\frac{b}{a}}\log\left(\frac{\left(a^{2}+6\right.}{}\right.}{8\left(\left(a^{4}-4a^{3}b+6a^{2}b^{2}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^2 + 2*a*b - 3*b^2)*f*x*cos(f*x + e)^2 + 4*(a*b + 3*b^2)*f*x - ((
3*a^2 - 2*a*b - b^2)*cos(f*x + e)^2 + 3*a*b + b^2)*sqrt(-b/a)*log(((a^2 + 6
*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 - 4*((a^2 + a*b
)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*x + e) + b^2)/((a^2 -
 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2)) - 4*((a
^2 - 2*a*b + b^2)*cos(f*x + e)^3 + 2*(a*b - b^2)*cos(f*x + e))*sin(f*x + e)
)/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^3*b -
3*a^2*b^2 + 3*a*b^3 - b^4)*f), 1/4*(2*(a^2 + 2*a*b - 3*b^2)*f*x*cos(f*x + e
)^2 + 2*(a*b + 3*b^2)*f*x + ((3*a^2 - 2*a*b - b^2)*cos(f*x + e)^2 + 3*a*b +
 b^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos(f*
x + e)*sin(f*x + e))) - 2*((a^2 - 2*a*b + b^2)*cos(f*x + e)^3 + 2*(a*b - b^
2)*cos(f*x + e))*sin(f*x + e))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)
*f*cos(f*x + e)^2 + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.56774, size = 892, normalized size = 6.46

$$-\frac{\frac{2\left(2a^3b-2a^2b^2-2ab^3+2b^4+b\left|-a^3+3a^2b-3ab^2+b^3\right|\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan(fx+e)}{\sqrt{\frac{a^3-a^2b-ab^2+b^3+\sqrt{(a^3-a^2b-ab^2+b^3)^2-4(a^3-2a^2b+ab^2)(a^2b-2ab^2+b^3)}}{a^2b-2ab^2+b^3}}}\right)\right)}{a^3\left|-a^3+3a^2b-3ab^2+b^3\right|-a^2b\left|-a^3+3a^2b-3ab^2+b^3\right|-ab^2\left|-a^3+3a^2b-3ab^2+b^3\right|+b^3\left|-a^3+3a^2b-3ab^2+b^3\right|+(a^3-3a^2b+3ab^2-b^3)^2}+\frac{2\left(2\left(a^3-a^2\cdots\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(2*a^3*b - 2*a^2*b^2 - 2*a*b^3 + 2*b^4 + b*abs(-a^3 + 3*a^2*b - 3*a
*b^2 + b^3))*(pi*floor((f*x + e)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(f*x + e
)/sqrt((a^3 - a^2*b - a*b^2 + b^3 + sqrt((a^3 - a^2*b - a*b^2 + b^3)^2 - 4*
(a^3 - 2*a^2*b + a*b^2)*(a^2*b - 2*a*b^2 + b^3)))/(a^2*b - 2*a*b^2 + b^3)))
)/(a^3*abs(-a^3 + 3*a^2*b - 3*a*b^2 + b^3) - a^2*b*abs(-a^3 + 3*a^2*b - 3*a
*b^2 + b^3) - a*b^2*abs(-a^3 + 3*a^2*b - 3*a*b^2 + b^3) + b^3*abs(-a^3 + 3*
a^2*b - 3*a*b^2 + b^3) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)^2) + 2*(2*(a^3 - a
^2*b - a*b^2 + b^3)*sqrt(a*b)*abs(b) - sqrt(a*b)*abs(-a^3 + 3*a^2*b - 3*a*b
^2 + b^3)*abs(b))*(pi*floor((f*x + e)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(f*
x + e)/sqrt((a^3 - a^2*b - a*b^2 + b^3 - sqrt((a^3 - a^2*b - a*b^2 + b^3)^2
 - 4*(a^3 - 2*a^2*b + a*b^2)*(a^2*b - 2*a*b^2 + b^3)))/(a^2*b - 2*a*b^2 + b
^3))))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)^2*b - (a^3*b - a^2*b^2 - a*b^3 + b^
4)*abs(-a^3 + 3*a^2*b - 3*a*b^2 + b^3)) + (2*b*tan(f*x + e)^3 + a*tan(f*x +
 e) + b*tan(f*x + e))/((b*tan(f*x + e)^4 + a*tan(f*x + e)^2 + b*tan(f*x + e
)^2 + a)*(a^2 - 2*a*b + b^2)))/f
```

3.76 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt{b}(3a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{3/2}f(a-b)^2}-\frac{b\tan(e+fx)}{2af(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{x}{(a-b)^2}$$

```
[Out] x/(a - b)^2 - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2
*a^(3/2)*(a - b)^2*f) - (b*Tan[e + f*x])/(2*a*(a - b)*f*(a + b*Tan[e + f*x]
^2))
```

Rubi [A] time = 0.083395, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3661, 414, 522, 203, 205}

$$-\frac{\sqrt{b}(3a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{3/2}f(a-b)^2}-\frac{b\tan(e+fx)}{2af(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-2),x]
```


```
[Out] x/(a - b)^2 - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2
*a^(3/2)*(a - b)^2*f) - (b*Tan[e + f*x])/(2*a*(a - b)*f*(a + b*Tan[e + f*x]
^2))
```


Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-b-bx^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{2a(a-b)f}$$

$$= -\frac{b\tan(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{(a-b)^2 f} - \frac{((3a-b)b)\,\text{Subst}}{}$$

$$= \frac{x}{(a-b)^2} - \frac{(3a-b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2 f} - \frac{b\tan(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)}$$

Mathematica [A] time = 1.02476, size = 88, normalized size = 0.91

$$\frac{\frac{\sqrt{b}(b-3a)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{b(b-a)\tan(e+fx)}{a\left(a+b\tan^2(e+fx)\right)} + 2\tan^{-1}(\tan(e+fx))}{2f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-2),x]
```

```
[Out] (2*ArcTan[Tan[e + f*x]] + (Sqrt[b]*(-3*a + b)*ArcTan[(Sqrt[b]*Tan[e + f*x])
/Sqrt[a]])/a^(3/2) + (b*(-a + b)*Tan[e + f*x])/(a*(a + b*Tan[e + f*x]^2)))/
(2*(a - b)^2*f)
```

Maple [A] time = 0.023, size = 160, normalized size = 1.7

$$-\frac{b\tan\left(fx+e\right)}{2\left(a-b\right)^2 f\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)} + \frac{b^2\tan\left(fx+e\right)}{2\left(a-b\right)^2 fa\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)} - \frac{3b}{2\left(a-b\right)^2 f}\arctan\left(b\tan\left(fx+e\right)\frac{1}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/2*b*tan(f*x+e)/(a-b)^2/f/(a+b*tan(f*x+e)^2)+1/2/f/(a-b)^2*b^2/a*tan(f*x+
e)/(a+b*tan(f*x+e)^2)-3/2/f/(a-b)^2*b/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)
^(1/2))+1/2/f/(a-b)^2*b^2/a/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))+1/
```

```
f/(a-b)^2*arctan(tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85439, size = 865, normalized size = 8.92

$$\left[\frac{8\,abfx\tan\left(fx+e\right)^{2}+8\,a^{2}fx-\left(\left(3\,ab-b^{2}\right)\tan\left(fx+e\right)^{2}+3\,a^{2}-ab\right)\sqrt{-\frac{b}{a}}\log\left(\frac{b^{2}\tan\left(fx+e\right)^{4}-6\,ab\tan\left(fx+e\right)^{2}+a^{2}+4\left(ab\tan\right.}{b^{2}\tan\left(fx+e\right)^{4}+2\,ab\tan\left(\right.}\right.}{8\left(\left(a^{3}b-2\,a^{2}b^{2}+ab^{3}\right)f\tan\left(fx+e\right)^{2}+\left(a^{4}-2\,a^{3}b+a^{2}b^{2}\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*a*b*f*x*tan(f*x + e)^2 + 8*a^2*f*x - ((3*a*b - b^2)*tan(f*x + e)^2
+ 3*a^2 - a*b)*sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 +
a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x +
e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(a*b - b^2)*tan(f*x + e))/((a^3*b -
 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f), 1/4*(4
*a*b*f*x*tan(f*x + e)^2 + 4*a^2*f*x - ((3*a*b - b^2)*tan(f*x + e)^2 + 3*a^2
 - a*b)*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x +
e))) - 2*(a*b - b^2)*tan(f*x + e))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*tan(f*x +
 e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f)]
```

Sympy [A] time = 40.4591, size = 2086, normalized size = 21.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x/tan(e)**4, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x/a**2, Eq(b,
 0)), ((x + 1/(f*tan(e + f*x)) - 1/(3*f*tan(e + f*x)**3))/b**2, Eq(a, 0)),
(x/(a + b*tan(e)**2)**2, Eq(f, 0)), (4*I*a**(5/2)*f*x*sqrt(1/b)/(4*I*a**(9/
2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*
b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2
)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) + 4*I*a
**(3/2)*b*f*x*sqrt(1/b)*tan(e + f*x)**2/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**
(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(
5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I
*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) - 2*I*a**(3/2)*b*sqrt(1/b)*tan(
```

```
e + f*x)/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x
)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f
*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(
e + f*x)**2) + 2*I*sqrt(a)*b**2*sqrt(1/b)*tan(e + f*x)/(4*I*a**(9/2)*f*sqrt
(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(
1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*
sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) - 3*a**2*log(-I*
sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*
b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b
**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3
/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) + 3*a**2*log(I*sqrt(a)*sqrt(1/b) + ta
n(e + f*x))/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e +
f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e
+ f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*t
an(e + f*x)**2) - 3*a*b*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*
x)**2/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**
2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)
**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e +
 f*x)**2) + a*b*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(9/2)*f*sq
rt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqr
t(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*
f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) + 3*a*b*log(I*
sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(9/2)*f*sqrt(1/b)
 + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b)
- 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(
1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) - a*b*log(I*sqrt(a)*s
qrt(1/b) + tan(e + f*x))/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(
1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqr
t(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*
f*sqrt(1/b)*tan(e + f*x)**2) + b**2*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x)
)*tan(e + f*x)**2/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*ta
n(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*
tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(
1/b)*tan(e + f*x)**2) - b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e
+ f*x)**2/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*
x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e +
f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan
(e + f*x)**2), True))
```

Giac [A] time = 1.35426, size = 171, normalized size = 1.76

$$-\frac{\frac{\left(\pi \left\lfloor \frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)\left(3\,ab-b^2\right)}{\left(a^3-2\,a^2b+ab^2\right)\sqrt{ab}}-\frac{2\left(fx+e\right)}{a^2-2\,ab+b^2}+\frac{b\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a\right)\left(a^2-ab\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
)))*(3*a*b - b^2)/((a^3 - 2*a^2*b + a*b^2)*sqrt(a*b)) - 2*(f*x + e)/(a^2 -
2*a*b + b^2) + b*tan(f*x + e)/((b*tan(f*x + e)^2 + a)*(a^2 - a*b)))/f
```

3.77 $\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2} dx$

Optimal. Leaf size=82

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{5/2}f}-\frac{3\cot(e+fx)}{2a^2f}+\frac{\cot(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)}$$

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2*a^(5/2)*f) - (3*Cot[
e + f*x])/(2*a^2*f) + Cot[e + f*x]/(2*a*f*(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.0741918, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3663, 290, 325, 205}

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{5/2}f}-\frac{3\cot(e+fx)}{2a^2f}+\frac{\cot(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2*a^(5/2)*f) - (3*Cot[
e + f*x])/(2*a^2*f) + Cot[e + f*x]/(2*a*f*(a + b*Tan[e + f*x]^2))
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cot(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= -\frac{3\cot(e+fx)}{2a^2f} + \frac{\cot(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(e+fx)\right)}{2a^2f}$$

$$= -\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{5/2}f} - \frac{3\cot(e+fx)}{2a^2f} + \frac{\cot(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)}$$

Mathematica [A] time = 0.569976, size = 83, normalized size = 1.01

$$\frac{\sqrt{a}\left(-\frac{b\sin(2(e+fx))}{(a-b)\cos(2(e+fx))+a+b} - 2\cot(e+fx)\right) - 3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[a]*(-2*Cot[e + f*
x] - (b*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)])))/(2*a^(5/2)*f
)
```

Maple [A] time = 0.08, size = 75, normalized size = 0.9

$$-\frac{1}{fa^2\tan(fx+e)} - \frac{b\tan(fx+e)}{2fa^2\left(a+b\left(\tan(fx+e)\right)^2\right)} - \frac{3b}{2fa^2}\arctan\left(b\tan(fx+e)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/f/a^2/tan(f*x+e)-1/2/f/a^2*b*tan(f*x+e)/(a+b*tan(f*x+e)^2)-3/2/f/a^2*b/(
a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.97383, size = 888, normalized size = 10.83

$$\left[-\frac{4\,(2\,a-3\,b)\cos\left(fx+e\right)^{3}-3\left((a-b)\cos\left(fx+e\right)^{2}+b\right)\sqrt{-\frac{b}{a}}\log\left(\frac{\left(a^{2}+6\,ab+b^{2}\right)\cos\left(fx+e\right)^{4}-2\left(3\,ab+b^{2}\right)\cos\left(fx+e\right)^{2}+4\left(\left(a^{2}+ab\right)c\right.}{\left(a^{2}-2\,ab+b^{2}\right)\cos\left(fx+e\right)^{4}+2\left(ab-b^{2}\right.}\right.}{8\left(a^{2}bf+\left(a^{3}-a^{2}b\right)f\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(2*a - 3*b)*cos(f*x + e)^3 - 3*((a - b)*cos(f*x + e)^2 + b)*sqrt(-
b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)
^2 + 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*x +
 e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)
^2 + b^2))*sin(f*x + e) + 12*b*cos(f*x + e))/((a^2*b*f + (a^3 - a^2*b)*f*co
s(f*x + e)^2)*sin(f*x + e)), -1/4*(2*(2*a - 3*b)*cos(f*x + e)^3 - 3*((a - b
)*cos(f*x + e)^2 + b)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqr
t(b/a)/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 6*b*cos(f*x + e))/((a^
2*b*f + (a^3 - a^2*b)*f*cos(f*x + e)^2)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41373, size = 126, normalized size = 1.54

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)b}{\sqrt{ab}a^{2}}+\frac{3\,b\tan\left(fx+e\right)^{2}+2\,a}{\left(b\tan\left(fx+e\right)^{3}+a\tan\left(fx+e\right)\right)a^{2}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b)))*b/(sqrt(a*b)*a^2) + (3*b*tan(f*x + e)^2 + 2*a)/((b*tan(f*x + e)^3 + a
*tan(f*x + e))*a^2))/f
```

3.78 $\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=116

$$-\frac{\sqrt{b}(3a-5b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{7/2}f}-\frac{b(a-b)\tan(e+fx)}{2a^3f\left(a+b\tan^2(e+fx)\right)}-\frac{(a-2b)\cot(e+fx)}{a^3f}-\frac{\cot^3(e+fx)}{3a^2f}$$

```
[Out] -((3*a - 5*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2*a^(7/2)*f)
 - ((a - 2*b)*Cot[e + f*x])/(a^3*f) - Cot[e + f*x]^3/(3*a^2*f) - ((a - b)*b
*Tan[e + f*x])/(2*a^3*f*(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.145864, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3663, 456, 1261, 205}

$$-\frac{\sqrt{b}(3a-5b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{7/2}f}-\frac{b(a-b)\tan(e+fx)}{2a^3f\left(a+b\tan^2(e+fx)\right)}-\frac{(a-2b)\cot(e+fx)}{a^3f}-\frac{\cot^3(e+fx)}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((3*a - 5*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2*a^(7/2)*f)
 - ((a - 2*b)*Cot[e + f*x])/(a^3*f) - Cot[e + f*x]^3/(3*a^2*f) - ((a - b)*b
*Tan[e + f*x])/(2*a^3*f*(a + b*Tan[e + f*x]^2))
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a-b)b\tan(e+fx)}{2a^3f\left(a+b\tan^2(e+fx)\right)} - \frac{b\,\text{Subst}\left(\int \frac{-\frac{2}{ab}-\frac{2(a-b)x^2}{a^2b}+\frac{(a-b)x^4}{a^3}}{x^4\left(a+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{2f}$$

$$= -\frac{(a-b)b\tan(e+fx)}{2a^3f\left(a+b\tan^2(e+fx)\right)} - \frac{b\,\text{Subst}\left(\int \left(-\frac{2}{a^2bx^4} - \frac{2(a-2b)}{a^3bx^2} + \frac{3a-5b}{a^3\left(a+bx^2\right)}\right) dx, x, \tan(e+f$$

$$= -\frac{(a-2b)\cot(e+fx)}{a^3f} - \frac{\cot^3(e+fx)}{3a^2f} - \frac{(a-b)b\tan(e+fx)}{2a^3f\left(a+b\tan^2(e+fx)\right)} - \frac{((3a-5b)b)\,\text{Subs}}{}$$

$$= -\frac{(3a-5b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{7/2}f} - \frac{(a-2b)\cot(e+fx)}{a^3f} - \frac{\cot^3(e+fx)}{3a^2f} - \frac{(a-b)b}{2a^3f\left(a+b\right.}$$

Mathematica [A] time = 0.762804, size = 112, normalized size = 0.97

$$\frac{3\sqrt{b}(5b-3a)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right) + \sqrt{a}\left(\frac{3b(b-a)\sin(2(e+fx))}{(a-b)\cos(2(e+fx))+a+b} - 2\cot(e+fx)\left(a\csc^2(e+fx)+2a-6b\right)\right)}{6a^{7/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (3*Sqrt[b]*(-3*a + 5*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[a]*(-
2*Cot[e + f*x]*(2*a - 6*b + a*Csc[e + f*x]^2) + (3*b*(-a + b)*Sin[2*(e + f*
x)])/(a + b + (a - b)*Cos[2*(e + f*x)])))/(6*a^(7/2)*f)
```

Maple [A] time = 0.095, size = 169, normalized size = 1.5

$$-\frac{1}{3fa^2\left(\tan(fx+e)\right)^3} - \frac{1}{fa^2\tan(fx+e)} + 2\frac{b}{fa^3\tan(fx+e)} - \frac{b\tan(fx+e)}{2fa^2\left(a+b\left(\tan(fx+e)\right)^2\right)} + \frac{b^2\tan(fx}{2fa^3\left(a+b\left(\tan\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/3/f/a^2/tan(f*x+e)^3-1/f/a^2/tan(f*x+e)+2/f/a^3/tan(f*x+e)*b-1/2/f/a^2*b
*tan(f*x+e)/(a+b*tan(f*x+e)^2)+1/2/f/a^3*b^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)-
3/2/f/a^2*b/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))+5/2/f/a^3*b^2/(a*b
)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.20976, size = 1374, normalized size = 11.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(4*(4*a^2 - 19*a*b + 15*b^2)*cos(f*x + e)^5 - 8*(3*a^2 - 14*a*b + 15
*b^2)*cos(f*x + e)^3 + 3*((3*a^2 - 8*a*b + 5*b^2)*cos(f*x + e)^4 - (3*a^2 -
 11*a*b + 10*b^2)*cos(f*x + e)^2 - 3*a*b + 5*b^2)*sqrt(-b/a)*log(((a^2 + 6*
a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 - 4*((a^2 + a*b)
*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*x + e) + b^2)/((a^2 -
2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2))*sin(f*x
+ e) - 12*(3*a*b - 5*b^2)*cos(f*x + e))/(((a^4 - a^3*b)*f*cos(f*x + e)^4 -
a^3*b*f - (a^4 - 2*a^3*b)*f*cos(f*x + e)^2)*sin(f*x + e)), -1/12*(2*(4*a^2
- 19*a*b + 15*b^2)*cos(f*x + e)^5 - 4*(3*a^2 - 14*a*b + 15*b^2)*cos(f*x + e
)^3 - 3*((3*a^2 - 8*a*b + 5*b^2)*cos(f*x + e)^4 - (3*a^2 - 11*a*b + 10*b^2)
*cos(f*x + e)^2 - 3*a*b + 5*b^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)
^2 - b)*sqrt(b/a)/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(3*a*b -
5*b^2)*cos(f*x + e))/(((a^4 - a^3*b)*f*cos(f*x + e)^4 - a^3*b*f - (a^4 - 2*
a^3*b)*f*cos(f*x + e)^2)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38018, size = 192, normalized size = 1.66

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)(3\,ab-5\,b^2)}{\sqrt{ab}a^3}+\frac{3\left(ab\tan(fx+e)-b^2\tan(fx+e)\right)}{\left(b\tan(fx+e)^2+a\right)a^3}+\frac{2\left(3\,a\tan(fx+e)^2-6\,b\tan(fx+e)^2+a\right)}{a^3\tan(fx+e)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b)))*(3*a*b - 5*b^2)/(sqrt(a*b)*a^3) + 3*(a*b*tan(f*x + e) - b^2*tan(f*x +
 e))/((b*tan(f*x + e)^2 + a)*a^3) + 2*(3*a*tan(f*x + e)^2 - 6*b*tan(f*x + e
)^2 + a)/(a^3*tan(f*x + e)^3))/f
```

3.79 $\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=182

$$-\frac{b\left(5a^2-10ab+7b^2\right)\tan(e+fx)}{10a^4f\left(a+b\tan^2(e+fx)\right)}-\frac{\left(5a^2-20ab+14b^2\right)\cot(e+fx)}{5a^4f}-\frac{\sqrt{b}(3a-7b)(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{9/2}f}-\frac{(10a-$$

```
[Out] -((3*a - 7*b)*(a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2*a^
(9/2)*f) - ((5*a^2 - 20*a*b + 14*b^2)*Cot[e + f*x])/(5*a^4*f) - ((10*a - 7*
b)*Cot[e + f*x]^3)/(15*a^3*f) - Cot[e + f*x]^5/(5*a*f*(a + b*Tan[e + f*x]^2
)) - (b*(5*a^2 - 10*a*b + 7*b^2)*Tan[e + f*x])/(10*a^4*f*(a + b*Tan[e + f*x
]^2))
```

Rubi [A] time = 0.223147, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3663, 462, 456, 1261, 205}

$$-\frac{b\left(5a^2-10ab+7b^2\right)\tan(e+fx)}{10a^4f\left(a+b\tan^2(e+fx)\right)}-\frac{\left(5a^2-20ab+14b^2\right)\cot(e+fx)}{5a^4f}-\frac{\sqrt{b}(3a-7b)(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{9/2}f}-\frac{(10a-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
```


```
[Out] -((3*a - 7*b)*(a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2*a^
(9/2)*f) - ((5*a^2 - 20*a*b + 14*b^2)*Cot[e + f*x])/(5*a^4*f) - ((10*a - 7*
b)*Cot[e + f*x]^3)/(15*a^3*f) - Cot[e + f*x]^5/(5*a*f*(a + b*Tan[e + f*x]^2
)) - (b*(5*a^2 - 10*a*b + 7*b^2)*Tan[e + f*x])/(10*a^4*f*(a + b*Tan[e + f*x
]^2))
```


Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
```

```
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^6\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{10a-7b+5ax^2}{x^4\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{5af}$$

$$= -\frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)} - \frac{b\left(5a^2-10ab+7b^2\right)\tan(e+fx)}{10a^4f\left(a+b\tan^2(e+fx)\right)} - \frac{b\,\text{Subst}\left(\int \frac{2\left(\frac{7}{a}-\frac{10}{b}\right)+2(}{}\right.}{}$$

$$= -\frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)} - \frac{b\left(5a^2-10ab+7b^2\right)\tan(e+fx)}{10a^4f\left(a+b\tan^2(e+fx)\right)} - \frac{b\,\text{Subst}\left(\int \left(-\frac{2(10a-7b}{a^2bx^4}\right.\right.}{}$$

$$= -\frac{\left(5a^2-20ab+14b^2\right)\cot(e+fx)}{5a^4f} - \frac{(10a-7b)\cot^3(e+fx)}{15a^3f} - \frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx\right.}$$

$$= -\frac{(3a-7b)(a-b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{9/2}f} - \frac{\left(5a^2-20ab+14b^2\right)\cot(e+fx)}{5a^4f} - \frac{(10a-7}{}$$

Mathematica [A] time = 1.61669, size = 151, normalized size = 0.83

$$\frac{\sqrt{a}\left(-2\cot(e+fx)\left(3a^2\csc^4(e+fx)+8a^2+2a(2a-5b)\csc^2(e+fx)-50ab+45b^2\right)-\frac{15b(a-b)^2\sin(2(e+fx))}{(a-b)\cos(2(e+fx))+a+b}\right)-15\sqrt{b}}{30a^{9/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-15*Sqrt[b]*(3*a^2 - 10*a*b + 7*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]
] + Sqrt[a]*(-2*Cot[e + f*x]*(8*a^2 - 50*a*b + 45*b^2 + 2*a*(2*a - 5*b)*Csc
[e + f*x]^2 + 3*a^2*Csc[e + f*x]^4) - (15*(a - b)^2*b*Sin[2*(e + f*x)])/(a
+ b + (a - b)*Cos[2*(e + f*x)])))/(30*a^(9/2)*f)
```

Maple [A] time = 0.102, size = 281, normalized size = 1.5

$$-\frac{1}{5\,fa^{2}\left(\tan\left(fx+e\right)\right)^{5}}-\frac{2}{3\,fa^{2}\left(\tan\left(fx+e\right)\right)^{3}}+\frac{2\,b}{3\,fa^{3}\left(\tan\left(fx+e\right)\right)^{3}}-\frac{1}{fa^{2}\tan\left(fx+e\right)}+4\,\frac{b}{fa^{3}\tan\left(fx+e\right)}-3\,\frac{}{fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/5/f/a^2/tan(f*x+e)^5-2/3/f/a^2/tan(f*x+e)^3+2/3/f/a^3/tan(f*x+e)^3*b-1/f
/a^2/tan(f*x+e)+4/f/a^3/tan(f*x+e)*b-3/f/a^4/tan(f*x+e)*b^2-1/2/f/a^2*b*tan
(f*x+e)/(a+b*tan(f*x+e)^2)+1/f/a^3*b^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)-1/2/f*
b^3/a^4*tan(f*x+e)/(a+b*tan(f*x+e)^2)-3/2/f/a^2*b/(a*b)^(1/2)*arctan(b*tan(
f*x+e)/(a*b)^(1/2))+5/f/a^3*b^2/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)
)-7/2/f*b^3/a^4/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.23507, size = 2028, normalized size = 11.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/120*(4*(16*a^3 - 131*a^2*b + 220*a*b^2 - 105*b^3)*cos(f*x + e)^7 - 4*(4
0*a^3 - 321*a^2*b + 590*a*b^2 - 315*b^3)*cos(f*x + e)^5 + 20*(6*a^3 - 47*a^
2*b + 104*a*b^2 - 63*b^3)*cos(f*x + e)^3 - 15*((3*a^3 - 13*a^2*b + 17*a*b^2
 - 7*b^3)*cos(f*x + e)^6 - (6*a^3 - 29*a^2*b + 44*a*b^2 - 21*b^3)*cos(f*x +
 e)^4 + 3*a^2*b - 10*a*b^2 + 7*b^3 + (3*a^3 - 19*a^2*b + 37*a*b^2 - 21*b^3)
*cos(f*x + e)^2)*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*
a*b + b^2)*cos(f*x + e)^2 + 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e
))*sqrt(-b/a)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(
a*b - b^2)*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*(3*a^2*b - 10*a*b^2 + 7
*b^3)*cos(f*x + e))/(((a^5 - a^4*b)*f*cos(f*x + e)^6 + a^4*b*f - (2*a^5 - 3
*a^4*b)*f*cos(f*x + e)^4 + (a^5 - 3*a^4*b)*f*cos(f*x + e)^2)*sin(f*x + e)),
 -1/60*(2*(16*a^3 - 131*a^2*b + 220*a*b^2 - 105*b^3)*cos(f*x + e)^7 - 2*(40
*a^3 - 321*a^2*b + 590*a*b^2 - 315*b^3)*cos(f*x + e)^5 + 10*(6*a^3 - 47*a^2
*b + 104*a*b^2 - 63*b^3)*cos(f*x + e)^3 - 15*((3*a^3 - 13*a^2*b + 17*a*b^2
- 7*b^3)*cos(f*x + e)^6 - (6*a^3 - 29*a^2*b + 44*a*b^2 - 21*b^3)*cos(f*x +
e)^4 + 3*a^2*b - 10*a*b^2 + 7*b^3 + (3*a^3 - 19*a^2*b + 37*a*b^2 - 21*b^3)*
cos(f*x + e)^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b/a)
```

```
/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 30*(3*a^2*b - 10*a*b^2 + 7*b
^3)*cos(f*x + e))/(((a^5 - a^4*b)*f*cos(f*x + e)^6 + a^4*b*f - (2*a^5 - 3*a
^4*b)*f*cos(f*x + e)^4 + (a^5 - 3*a^4*b)*f*cos(f*x + e)^2)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39182, size = 286, normalized size = 1.57

$$-\frac{\frac{15\left(3a^2b-10ab^2+7b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{\sqrt{ab}a^4}+\frac{15\left(a^2b\tan(fx+e)-2ab^2\tan(fx+e)+b^3\tan(fx+e)\right)}{\left(b\tan(fx+e)^2+a\right)a^4}+\frac{2\left(15a^2\tan(fx+e)^4-60ab\right.}{}}{30f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/30*(15*(3*a^2*b - 10*a*b^2 + 7*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b)
 + arctan(b*tan(f*x + e)/sqrt(a*b)))/(sqrt(a*b)*a^4) + 15*(a^2*b*tan(f*x +
e) - 2*a*b^2*tan(f*x + e) + b^3*tan(f*x + e))/((b*tan(f*x + e)^2 + a)*a^4)
+ 2*(15*a^2*tan(f*x + e)^4 - 60*a*b*tan(f*x + e)^4 + 45*b^2*tan(f*x + e)^4
+ 10*a^2*tan(f*x + e)^2 - 10*a*b*tan(f*x + e)^2 + 3*a^2)/(a^4*tan(f*x + e)^
5))/f
```

3.80 $\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=264

$$-\frac{\left(5a^2+20ab+2b^2\right)\cos(e+fx)}{5f(a-b)^5}-\frac{b\left(35a^2+40ab+24b^2\right)\sec(e+fx)}{40f(a-b)^5\left(a+b\sec^2(e+fx)-b\right)}-\frac{b\left(5a^2+4b^2\right)\sec(e+fx)}{20f(a-b)^4\left(a+b\sec^2(e+fx)-b\right)^2}-\frac{\sqrt{b}\,(15$$

```
[Out] -(Sqrt[b]*(15*a^2 + 40*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a -
b]])/(8*(a - b)^(11/2)*f) - ((5*a^2 + 20*a*b + 2*b^2)*Cos[e + f*x])/(5*(a -
 b)^5*f) + ((10*a - b)*Cos[e + f*x]^3)/(15*(a - b)^4*f) - Cos[e + f*x]^5/(5
*(a - b)*f*(a - b + b*Sec[e + f*x]^2)^2) - (b*(5*a^2 + 4*b^2)*Sec[e + f*x])
/(20*(a - b)^4*f*(a - b + b*Sec[e + f*x]^2)^2) - (b*(35*a^2 + 40*a*b + 24*b
^2)*Sec[e + f*x])/(40*(a - b)^5*f*(a - b + b*Sec[e + f*x]^2))
```

Rubi [A] time = 0.410531, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3664, 462, 456, 1259, 1261, 205}

$$-\frac{\left(5a^2+20ab+2b^2\right)\cos(e+fx)}{5f(a-b)^5}-\frac{b\left(35a^2+40ab+24b^2\right)\sec(e+fx)}{40f(a-b)^5\left(a+b\sec^2(e+fx)-b\right)}-\frac{b\left(5a^2+4b^2\right)\sec(e+fx)}{20f(a-b)^4\left(a+b\sec^2(e+fx)-b\right)^2}-\frac{\sqrt{b}\,(15$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```


```
[Out] -(Sqrt[b]*(15*a^2 + 40*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a -
b]])/(8*(a - b)^(11/2)*f) - ((5*a^2 + 20*a*b + 2*b^2)*Cos[e + f*x])/(5*(a -
 b)^5*f) + ((10*a - b)*Cos[e + f*x]^3)/(15*(a - b)^4*f) - Cos[e + f*x]^5/(5
*(a - b)*f*(a - b + b*Sec[e + f*x]^2)^2) - (b*(5*a^2 + 4*b^2)*Sec[e + f*x])
/(20*(a - b)^4*f*(a - b + b*Sec[e + f*x]^2)^2) - (b*(35*a^2 + 40*a*b + 24*b
^2)*Sec[e + f*x])/(40*(a - b)^5*f*(a - b + b*Sec[e + f*x]^2))
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
```

```
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^6\left(a-b+bx^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{-10a+b+5(a-b)x^2}{x^4\left(a-b+bx^2\right)^3}\,dx, x, \sec(e+fx)\right)}{5(a-b)f}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{b\left(5a^2+4b^2\right)\sec(e+fx)}{20(a-b)^4f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{b\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{b\left(5a^2+4b^2\right)\sec(e+fx)}{20(a-b)^4f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{b\left(35a^2+4\right.}{40(a-b)^5f}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{b\left(5a^2+4b^2\right)\sec(e+fx)}{20(a-b)^4f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{b\left(35a^2+4\right.}{40(a-b)^5f}$$

$$= -\frac{\left(5a^2+20ab+2b^2\right)\cos(e+fx)}{5(a-b)^5f} + \frac{(10a-b)\cos^3(e+fx)}{15(a-b)^4f} - \frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(e+\right.}$$

$$= -\frac{\sqrt{b}\left(15a^2+40ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8(a-b)^{11/2}f} - \frac{\left(5a^2+20ab+2b^2\right)\cos(e+fx)}{5(a-b)^5f} + \frac{(10a-}{15}$$

Mathematica [A] time = 5.47999, size = 278, normalized size = 1.05

$$\frac{\frac{(a-b)(5(5a+7b)\cos(3(e+fx))+3(b-a)\cos(5(e+fx)))-30\cos(e+fx)\left(a^2\left(-\frac{8b^2}{((a-b)\cos(2(e+fx))+a+b)^2}+\frac{18b}{(a-b)\cos(2(e+fx))+a+b}+5\right)+16ab\left(\frac{b}{(a-b)\cos(2(e+fx))+a+b}+2\right)+11b^2}{(a-b)^5}}{240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((30*Sqrt[b]*(15*a^2 + 40*a*b + 8*b^2)*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e
 + f*x)/2])/Sqrt[b]])/(a - b)^(11/2) + (30*Sqrt[b]*(15*a^2 + 40*a*b + 8*b^2
)*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a - b)^(11/2)
+ (-30*Cos[e + f*x]*(11*b^2 + 16*a*b*(2 + b/(a + b + (a - b)*Cos[2*(e + f*x
)])) + a^2*(5 - (8*b^2)/(a + b + (a - b)*Cos[2*(e + f*x)])^2 + (18*b)/(a +
b + (a - b)*Cos[2*(e + f*x)]))) + (a - b)*(5*(5*a + 7*b)*Cos[3*(e + f*x)] +
 3*(-a + b)*Cos[5*(e + f*x)]))/(a - b)^5)/(240*f)
```

Maple [B] time = 0.089, size = 844, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -1/5/f/(a^3-3*a^2*b+3*a*b^2-b^3)/(a^2-2*a*b+b^2)*cos(f*x+e)^5*a^2+2/5/f/(a^
3-3*a^2*b+3*a*b^2-b^3)/(a^2-2*a*b+b^2)*cos(f*x+e)^5*a*b-1/5/f/(a^3-3*a^2*b+
3*a*b^2-b^3)/(a^2-2*a*b+b^2)*cos(f*x+e)^5*b^2+2/3/f/(a^3-3*a^2*b+3*a*b^2-b^
3)/(a^2-2*a*b+b^2)*a^2*cos(f*x+e)^3-1/3/f/(a^3-3*a^2*b+3*a*b^2-b^3)/(a^2-2*
a*b+b^2)*cos(f*x+e)^3*a*b-1/3/f/(a^3-3*a^2*b+3*a*b^2-b^3)/(a^2-2*a*b+b^2)*b
^2*cos(f*x+e)^3-1/f/(a^3-3*a^2*b+3*a*b^2-b^3)/(a^2-2*a*b+b^2)*a^2*cos(f*x+e
)-4/f/(a^3-3*a^2*b+3*a*b^2-b^3)/(a^2-2*a*b+b^2)*a*cos(f*x+e)*b-1/f/(a^3-3*a
^2*b+3*a*b^2-b^3)/(a^2-2*a*b+b^2)*cos(f*x+e)*b^2-9/8/f*b/(a-b)^5/(a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)^2*cos(f*x+e)^3*a^3+1/8/f*b^2/(a-b)^5/(a*cos(f*x+e)^
2-cos(f*x+e)^2*b+b)^2*cos(f*x+e)^3*a^2+1/f*b^3/(a-b)^5/(a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)^2*a*cos(f*x+e)^3-7/8/f*b^2/(a-b)^5/(a*cos(f*x+e)^2-cos(f*x+e)
^2*b+b)^2*cos(f*x+e)*a^2-1/f*b^3/(a-b)^5/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^
2*cos(f*x+e)*a+15/8/f*b/(a-b)^5/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*
(a-b))^(1/2))*a^2+5/f*b^2/(a-b)^5/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(
b*(a-b))^(1/2))*a+1/f*b^3/(a-b)^5/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(
b*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.15777, size = 2313, normalized size = 8.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/240*(48*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^9 - 16
*(10*a^4 - 31*a^3*b + 33*a^2*b^2 - 13*a*b^3 + b^4)*cos(f*x + e)^7 + 16*(15*
a^4 + 10*a^3*b - 57*a^2*b^2 + 24*a*b^3 + 8*b^4)*cos(f*x + e)^5 + 50*(15*a^3
*b + 25*a^2*b^2 - 32*a*b^3 - 8*b^4)*cos(f*x + e)^3 + 15*((15*a^4 + 10*a^3*b
 - 57*a^2*b^2 + 24*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 15*a^2*b^2 + 40*a*b^3 +
8*b^4 + 2*(15*a^3*b + 25*a^2*b^2 - 32*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(-
b/(a - b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a - b)*sqrt(-b/(a - b))*cos(f*
x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 30*(15*a^2*b^2 + 40*a*b^3 + 8*b
^4)*cos(f*x + e))/((a^7 - 7*a^6*b + 21*a^5*b^2 - 35*a^4*b^3 + 35*a^3*b^4 -
21*a^2*b^5 + 7*a*b^6 - b^7)*f*cos(f*x + e)^4 + 2*(a^6*b - 6*a^5*b^2 + 15*a^
4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)*f*cos(f*x + e)^2 + (a^5*b^
2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 + 5*a*b^6 - b^7)*f), -1/120*(24*(a^
4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^9 - 8*(10*a^4 - 31*a^
3*b + 33*a^2*b^2 - 13*a*b^3 + b^4)*cos(f*x + e)^7 + 8*(15*a^4 + 10*a^3*b -
57*a^2*b^2 + 24*a*b^3 + 8*b^4)*cos(f*x + e)^5 + 25*(15*a^3*b + 25*a^2*b^2 -
 32*a*b^3 - 8*b^4)*cos(f*x + e)^3 + 15*((15*a^4 + 10*a^3*b - 57*a^2*b^2 + 2
4*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 15*a^2*b^2 + 40*a*b^3 + 8*b^4 + 2*(15*a^3
```

```
*b + 25*a^2*b^2 - 32*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(b/(a - b))*arctan(
-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + 15*(15*a^2*b^2 + 40*a*b^3 + 8*b^
4)*cos(f*x + e))/((a^7 - 7*a^6*b + 21*a^5*b^2 - 35*a^4*b^3 + 35*a^3*b^4 - 2
1*a^2*b^5 + 7*a*b^6 - b^7)*f*cos(f*x + e)^4 + 2*(a^6*b - 6*a^5*b^2 + 15*a^4
*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)*f*cos(f*x + e)^2 + (a^5*b^2
 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 + 5*a*b^6 - b^7)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.67518, size = 1195, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/120*(15*(15*a^2*b + 40*a*b^2 + 8*b^3)*arctan(-(a*cos(f*x + e) - b*cos(f*
x + e) - b)/(sqrt(a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/((a^5 - 5*a^4
*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*sqrt(a*b - b^2)) + 30*(9*a^3*
b + 6*a^2*b^2 + 27*a^3*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 32*a^2*b^2
*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 40*a*b^3*(cos(f*x + e) - 1)/(cos(f
*x + e) + 1) + 27*a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 54*a^2*
b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 24*a*b^3*(cos(f*x + e) - 1)
^2/(cos(f*x + e) + 1)^2 + 48*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
+ 9*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 16*a^2*b^2*(cos(f*x +
 e) - 1)^3/(cos(f*x + e) + 1)^3 - 8*a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e
) + 1)^3)/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*(a + 2
*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x
+ e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)^2) - 16*(8*a^2 + 5
9*a*b + 23*b^2 - 40*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 250*a*b*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) - 70*b^2*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) + 80*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 320*a*b*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 140*b^2*(cos(f*x + e) - 1)^2/(cos(f*x
+ e) + 1)^2 - 270*a*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 90*b^2*(c
os(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 45*a*b*(cos(f*x + e) - 1)^4/(cos(
f*x + e) + 1)^4 + 45*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((a^5 -
 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*((cos(f*x + e) - 1)/(co
s(f*x + e) + 1) - 1)^5))/f
```

3.81 $\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=180

$$\frac{\cos^3(e+fx)}{3f(a-b)^3}-\frac{(a+2b)\cos(e+fx)}{f(a-b)^4}-\frac{b(7a+4b)\sec(e+fx)}{8f(a-b)^4\left(a+b\sec^2(e+fx)-b\right)}-\frac{ab\sec(e+fx)}{4f(a-b)^3\left(a+b\sec^2(e+fx)-b\right)^2}-5$$

```
[Out] (-5*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(8*(a -
 b)^(9/2)*f) - ((a + 2*b)*Cos[e + f*x])/((a - b)^4*f) + Cos[e + f*x]^3/(3*(
a - b)^3*f) - (a*b*Sec[e + f*x])/(4*(a - b)^3*f*(a - b + b*Sec[e + f*x]^2)^
2) - (b*(7*a + 4*b)*Sec[e + f*x])/(8*(a - b)^4*f*(a - b + b*Sec[e + f*x]^2)
)
```

Rubi [A] time = 0.246349, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3664, 456, 1259, 1261, 205}

$$\frac{\cos^3(e+fx)}{3f(a-b)^3}-\frac{(a+2b)\cos(e+fx)}{f(a-b)^4}-\frac{b(7a+4b)\sec(e+fx)}{8f(a-b)^4\left(a+b\sec^2(e+fx)-b\right)}-\frac{ab\sec(e+fx)}{4f(a-b)^3\left(a+b\sec^2(e+fx)-b\right)^2}-5$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```


```
[Out] (-5*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(8*(a -
 b)^(9/2)*f) - ((a + 2*b)*Cos[e + f*x])/((a - b)^4*f) + Cos[e + f*x]^3/(3*(
a - b)^3*f) - (a*b*Sec[e + f*x])/(4*(a - b)^3*f*(a - b + b*Sec[e + f*x]^2)^
2) - (b*(7*a + 4*b)*Sec[e + f*x])/(8*(a - b)^4*f*(a - b + b*Sec[e + f*x]^2)
)
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
```

```
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4\left(a-b+bx^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{ab\sec(e+fx)}{4(a-b)^3 f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{b\,\text{Subst}\left(\int \frac{\frac{4}{(a-b)b}-\frac{4ax^2}{(a-b)^2 b}+\frac{3ax^4}{(a-b)^3}}{x^4\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+fx)\right)}{4f}$$

$$= -\frac{ab\sec(e+fx)}{4(a-b)^3 f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{b(7a+4b)\sec(e+fx)}{8(a-b)^4 f\left(a-b+b\sec^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{8(}{}\right.}{}$$

$$= -\frac{ab\sec(e+fx)}{4(a-b)^3 f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{b(7a+4b)\sec(e+fx)}{8(a-b)^4 f\left(a-b+b\sec^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \left(\right.\right.}{}$$

$$= -\frac{(a+2b)\cos(e+fx)}{(a-b)^4 f} + \frac{\cos^3(e+fx)}{3(a-b)^3 f} - \frac{ab\sec(e+fx)}{4(a-b)^3 f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{b(7a}{8(a-b)^4 f}$$

$$= -\frac{5\sqrt{b}(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8(a-b)^{9/2} f} - \frac{(a+2b)\cos(e+fx)}{(a-b)^4 f} + \frac{\cos^3(e+fx)}{3(a-b)^3 f} - \frac{ab}{4(a-b)^3 f\left(a\right.}$$

Mathematica [A] time = 5.43852, size = 230, normalized size = 1.28

$$\frac{\frac{2\left(3\cos(e+fx)\left(a\left(\frac{4b^2}{((a-b)\cos(2(e+fx))+a+b)^2}-\frac{9b}{(a-b)\cos(2(e+fx))+a+b}-3\right)+b\left(-\frac{4b}{(a-b)\cos(2(e+fx))+a+b}-9\right)\right)+(a-b)\cos(3(e+fx))\right)}{(a-b)^4}+\frac{15\sqrt{b}(3a+4b)\tan^{-1}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}\right.}{\sqrt{b}}\right)}{(a-b)^{9/2}}}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((15*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sq
rt[b]])/(a - b)^(9/2) + (15*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[a - b] + Sqrt[
```

```
a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a - b)^(9/2) + (2*(3*Cos[e + f*x]*(a*(-3 +
(4*b^2)/(a + b + (a - b)*Cos[2*(e + f*x)])^2 - (9*b)/(a + b + (a - b)*Cos[2
*(e + f*x)])) + b*(-9 - (4*b)/(a + b + (a - b)*Cos[2*(e + f*x)]))) + (a - b
)*Cos[3*(e + f*x)]))/(a - b)^4)/(24*f)
```

Maple [B] time = 0.082, size = 504, normalized size = 2.8

$$\frac{a\left(\cos\left(fx+e\right)\right)^{3}}{3f\left(a^{3}-3a^{2}b+3ab^{2}-b^{3}\right)(a-b)}-\frac{b\left(\cos\left(fx+e\right)\right)^{3}}{3f\left(a^{3}-3a^{2}b+3ab^{2}-b^{3}\right)(a-b)}-\frac{\cos\left(fx+e\right)a}{f\left(a^{3}-3a^{2}b+3ab^{2}-b^{3}\right)(a-b)}-2\frac{}{f(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] 1/3/f/(a^3-3*a^2*b+3*a*b^2-b^3)/(a-b)*a*cos(f*x+e)^3-1/3/f/(a^3-3*a^2*b+3*a
*b^2-b^3)/(a-b)*b*cos(f*x+e)^3-1/f/(a^3-3*a^2*b+3*a*b^2-b^3)/(a-b)*a*cos(f*
x+e)-2/f/(a^3-3*a^2*b+3*a*b^2-b^3)/(a-b)*cos(f*x+e)*b-9/8/f*b/(a-b)^4/(a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)^2*a^2*cos(f*x+e)^3+5/8/f*b^2/(a-b)^4/(a*cos(f*
x+e)^2-cos(f*x+e)^2*b+b)^2*cos(f*x+e)^3*a+1/2/f*b^3/(a-b)^4/(a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)^2*cos(f*x+e)^3-7/8/f*b^2/(a-b)^4/(a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)^2*a*cos(f*x+e)-1/2/f*b^3/(a-b)^4/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)^2*cos(f*x+e)+15/8/f*b/(a-b)^4/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b
*(a-b))^(1/2))*a+5/2/f*b^2/(a-b)^4/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/
(b*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.96241, size = 1723, normalized size = 9.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 16*(3*a^3 - 2*a^
2*b - 5*a*b^2 + 4*b^3)*cos(f*x + e)^5 - 50*(3*a^2*b + a*b^2 - 4*b^3)*cos(f*
x + e)^3 + 15*((3*a^3 - 2*a^2*b - 5*a*b^2 + 4*b^3)*cos(f*x + e)^4 + 3*a*b^2
 + 4*b^3 + 2*(3*a^2*b + a*b^2 - 4*b^3)*cos(f*x + e)^2)*sqrt(-b/(a - b))*log
(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/(a - b))*cos(f*x + e) - b)/((a
 - b)*cos(f*x + e)^2 + b)) - 30*(3*a*b^2 + 4*b^3)*cos(f*x + e))/((a^6 - 6*a
^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*f*cos(f*x + e)
^4 + 2*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*f*cos(
```

```
f*x + e)^2 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f), 1/24*(8*
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 8*(3*a^3 - 2*a^2*b - 5*a*b
^2 + 4*b^3)*cos(f*x + e)^5 - 25*(3*a^2*b + a*b^2 - 4*b^3)*cos(f*x + e)^3 -
15*((3*a^3 - 2*a^2*b - 5*a*b^2 + 4*b^3)*cos(f*x + e)^4 + 3*a*b^2 + 4*b^3 +
2*(3*a^2*b + a*b^2 - 4*b^3)*cos(f*x + e)^2)*sqrt(b/(a - b))*arctan(-(a - b)
*sqrt(b/(a - b))*cos(f*x + e)/b) - 15*(3*a*b^2 + 4*b^3)*cos(f*x + e))/((a^6
 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*f*cos(f*
x + e)^4 + 2*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*
f*cos(f*x + e)^2 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.60447, size = 760, normalized size = 4.22

$$\frac{a^6 f^{17} \cos\left(fx+e\right)^3 - 6\,a^5 b f^{17} \cos\left(fx+e\right)^3 + 15\,a^4 b^2 f^{17} \cos\left(fx+e\right)^3 - 20\,a^3 b^3 f^{17} \cos\left(fx+e\right)^3 + 15\,a^2 b^4 f^{17} \cos\left(fx -\right.}{3\left(a^9 f^{18} - 9\,a^8 b f^{18} + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/3*(a^6*f^17*cos(f*x + e)^3 - 6*a^5*b*f^17*cos(f*x + e)^3 + 15*a^4*b^2*f^1
7*cos(f*x + e)^3 - 20*a^3*b^3*f^17*cos(f*x + e)^3 + 15*a^2*b^4*f^17*cos(f*x
 + e)^3 - 6*a*b^5*f^17*cos(f*x + e)^3 + b^6*f^17*cos(f*x + e)^3 - 3*a^6*f^1
7*cos(f*x + e) + 9*a^5*b*f^17*cos(f*x + e) - 30*a^3*b^3*f^17*cos(f*x + e) +
 45*a^2*b^4*f^17*cos(f*x + e) - 27*a*b^5*f^17*cos(f*x + e) + 6*b^6*f^17*cos
(f*x + e))/(a^9*f^18 - 9*a^8*b*f^18 + 36*a^7*b^2*f^18 - 84*a^6*b^3*f^18 + 1
26*a^5*b^4*f^18 - 126*a^4*b^5*f^18 + 84*a^3*b^6*f^18 - 36*a^2*b^7*f^18 + 9*
a*b^8*f^18 - b^9*f^18) + 5/8*(3*a*b + 4*b^2)*arctan((a*cos(f*x + e) - b*cos
(f*x + e))/sqrt(a*b - b^2))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*sq
rt(a*b - b^2)*f) - 1/8*(9*a^2*b*cos(f*x + e)^3/f - 5*a*b^2*cos(f*x + e)^3/f
 - 4*b^3*cos(f*x + e)^3/f + 7*a*b^2*cos(f*x + e)/f + 4*b^3*cos(f*x + e)/f)/
((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(a*cos(f*x + e)^2 - b*cos(f*x
+ e)^2 + b)^2)
```

3.82 $\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=138

$$-\frac{15\cos(e+fx)}{8f(a-b)^3}+\frac{5\cos(e+fx)}{8f(a-b)^2\left(a+b\sec^2(e+fx)-b\right)}+\frac{\cos(e+fx)}{4f(a-b)\left(a+b\sec^2(e+fx)-b\right)^2}-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8f(a-b)^{7/2}}$$

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(8*(a - b)^(7/2)*f
) - (15*Cos[e + f*x])/(8*(a - b)^3*f) + Cos[e + f*x]/(4*(a - b)*f*(a - b +
b*Sec[e + f*x]^2)^2) + (5*Cos[e + f*x])/(8*(a - b)^2*f*(a - b + b*Sec[e + f
*x]^2))
```

Rubi [A] time = 0.0907196, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3664, 290, 325, 205}

$$-\frac{15\cos(e+fx)}{8f(a-b)^3}+\frac{5\cos(e+fx)}{8f(a-b)^2\left(a+b\sec^2(e+fx)-b\right)}+\frac{\cos(e+fx)}{4f(a-b)\left(a+b\sec^2(e+fx)-b\right)^2}-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8f(a-b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(8*(a - b)^(7/2)*f
) - (15*Cos[e + f*x])/(8*(a - b)^3*f) + Cos[e + f*x]/(4*(a - b)*f*(a - b +
b*Sec[e + f*x]^2)^2) + (5*Cos[e + f*x])/(8*(a - b)^2*f*(a - b + b*Sec[e + f
*x]^2))
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a-b+bx^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)}{4(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} + \frac{5\,\text{Subst}\left(\int \frac{1}{x^2\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+fx)\right)}{4(a-b)f}$$

$$= \frac{\cos(e+fx)}{4(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} + \frac{5\cos(e+fx)}{8(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)} + \frac{15\,\text{Subst}\left(\int \frac{}{x^2}\right.}{}$$

$$= -\frac{15\cos(e+fx)}{8(a-b)^3f} + \frac{\cos(e+fx)}{4(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} + \frac{5\cos(e+fx)}{8(a-b)^2f\left(a-b+b\sec^2(e+fx\right.}$$

$$= -\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8(a-b)^{7/2}f} - \frac{15\cos(e+fx)}{8(a-b)^3f} + \frac{\cos(e+fx)}{4(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} + \frac{}{8(a-}$$

Mathematica [A] time = 1.61036, size = 170, normalized size = 1.23

$$\frac{\frac{2\cos(e+fx)\left(\frac{4b^2}{((a-b)\cos(2(e+fx))+a+b)^2}-\frac{9b}{(a-b)\cos(2(e+fx))+a+b}-4\right)}{(a-b)^3}+\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{7/2}}+\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{7/2}}}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((15*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a -
 b)^(7/2) + (15*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sqr
t[b]])/(a - b)^(7/2) + (2*Cos[e + f*x]*(-4 + (4*b^2)/(a + b + (a - b)*Cos[2
*(e + f*x)])^2 - (9*b)/(a + b + (a - b)*Cos[2*(e + f*x)])))/(a - b)^3)/(8*f
)
```

Maple [A] time = 0.073, size = 221, normalized size = 1.6

$$-\frac{\cos\left(fx+e\right)}{f\left(a^3-3a^2b+3ab^2-b^3\right)} - \frac{9ab\left(\cos\left(fx+e\right)\right)^3}{8f(a-b)^3\left(a\left(\cos\left(fx+e\right)\right)^2-\left(\cos\left(fx+e\right)\right)^2b+b\right)^2} + \frac{9b^2\left(\cos\left(fx\right.\right.}{8f(a-b)^3\left(a\left(\cos\left(fx+e\right)\right)^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -1/f/(a^3-3*a^2*b+3*a*b^2-b^3)*cos(f*x+e)-9/8/f*b/(a-b)^3/(a*cos(f*x+e)^2-c
os(f*x+e)^2*b+b)^2*a*cos(f*x+e)^3+9/8/f*b^2/(a-b)^3/(a*cos(f*x+e)^2-cos(f*x
```

```
+e)^2*b+b)^2*cos(f*x+e)^3-7/8/f*b^2/(a-b)^3/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)^2*cos(f*x+e)+15/8/f*b/(a-b)^3/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b
*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.72399, size = 1249, normalized size = 9.05

$$\left[-\frac{16\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^5+50\left(ab-b^2\right)\cos\left(fx+e\right)^3+30\,b^2\cos\left(fx+e\right)+15\left(\left(a^2-2\,ab+b^2\right)\cos\left(fx+\right.\right.}{16\left(\left(a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5\right)f\cos\left(fx+e\right)^4+2\left(a^4b-4\,a^3b^2+6\,a^2b^3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 + 50*(a*b - b^2)*cos(f*x + e)
^3 + 30*b^2*cos(f*x + e) + 15*((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b
- b^2)*cos(f*x + e)^2 + b^2)*sqrt(-b/(a - b))*log(-((a - b)*cos(f*x + e)^2
- 2*(a - b)*sqrt(-b/(a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)
))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*f*cos(f*x + e
)^4 + 2*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cos(f*x + e)^2 +
(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f), -1/8*(8*(a^2 - 2*a*b + b^2)*cos(f
*x + e)^5 + 25*(a*b - b^2)*cos(f*x + e)^3 + 15*b^2*cos(f*x + e) + 15*((a^2
- 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2)*sqrt(b/
(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b))/((a^5 - 5*a^4*b +
 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*f*cos(f*x + e)^4 + 2*(a^4*b - 4*a
^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cos(f*x + e)^2 + (a^3*b^2 - 3*a^2*b^3
 + 3*a*b^4 - b^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.63463, size = 301, normalized size = 2.18

$$-\frac{f^5\cos\left(fx+e\right)}{a^3f^6-3\,a^2bf^6+3\,ab^2f^6-b^3f^6}+\frac{15\,b\arctan\left(\frac{a\cos\left(fx+e\right)-b\cos\left(fx+e\right)}{\sqrt{ab-b^2}}\right)}{8\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\sqrt{ab-b^2}f}-\frac{\frac{9\,ab\cos\left(fx+e\right)^3}{f}-\frac{9\,b^2\cos\left(fx+e\right)^3}{f}+\frac{7\,}{}}{8\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\left(a\cos\left(fx+e\right)^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -f^5*cos(f*x + e)/(a^3*f^6 - 3*a^2*b*f^6 + 3*a*b^2*f^6 - b^3*f^6) + 15/8*b*
arctan((a*cos(f*x + e) - b*cos(f*x + e))/sqrt(a*b - b^2))/((a^3 - 3*a^2*b +
 3*a*b^2 - b^3)*sqrt(a*b - b^2)*f) - 1/8*(9*a*b*cos(f*x + e)^3/f - 9*b^2*co
s(f*x + e)^3/f + 7*b^2*cos(f*x + e)/f)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(a*
cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)^2)
```

3.83 $\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=166

$$-\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^3f(a-b)^{5/2}}-\frac{b(7a-4b)\sec(e+fx)}{8a^2f(a-b)^2\left(a+b\sec^2(e+fx)-b\right)}-\frac{\tanh^{-1}(\cos(e+fx))}{a^3f}-\frac{\cdots}{4af(a\cdots}$$

```
[Out] -(Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a -
b]])/(8*a^3*(a - b)^(5/2)*f) - ArcTanh[Cos[e + f*x]]/(a^3*f) - (b*Sec[e + f
*x])/(4*a*(a - b)*f*(a - b + b*Sec[e + f*x]^2)^2) - ((7*a - 4*b)*b*Sec[e +
f*x])/(8*a^2*(a - b)^2*f*(a - b + b*Sec[e + f*x]^2))
```

Rubi [A] time = 0.21679, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3664, 414, 527, 522, 207, 205}

$$-\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^3f(a-b)^{5/2}}-\frac{b(7a-4b)\sec(e+fx)}{8a^2f(a-b)^2\left(a+b\sec^2(e+fx)-b\right)}-\frac{\tanh^{-1}(\cos(e+fx))}{a^3f}-\frac{\cdots}{4af(a\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -(Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a -
b]])/(8*a^3*(a - b)^(5/2)*f) - ArcTanh[Cos[e + f*x]]/(a^3*f) - (b*Sec[e + f
*x])/(4*a*(a - b)*f*(a - b + b*Sec[e + f*x]^2)^2) - ((7*a - 4*b)*b*Sec[e +
f*x])/(8*a^2*(a - b)^2*f*(a - b + b*Sec[e + f*x]^2))
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)\left(a-b+bx^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{b\sec(e+fx)}{4a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-b-3bx^2}{(-1+x^2)\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+fx)\right)}{4a(a-b)f}$$

$$= -\frac{b\sec(e+fx)}{4a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{(7a-4b)b\sec(e+fx)}{8a^2(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{b\sec(e+fx)}{4a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{(7a-4b)b\sec(e+fx)}{8a^2(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^3(a-b)^{5/2}f} - \frac{\tanh^{-1}(\cos(e+fx))}{a^3f} - \frac{b\sec(e}{4a(a-b)f\left(a-b\right.}$$

Mathematica [A] time = 3.00168, size = 247, normalized size = 1.49

$$\frac{\frac{8a^2b^2\cos(e+fx)}{(a-b)^2((a-b)\cos(2(e+fx))+a+b)^2} + \frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{5/2}} + \frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{5/2}} - \frac{2ab(9a}{(a-b)^2((a-}}{8a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e +
f*x)/2])/Sqrt[b]])/(a - b)^(5/2) + (Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTa
n[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a - b)^(5/2) + (8*a^2
*b^2*Cos[e + f*x])/((a - b)^2*(a + b + (a - b)*Cos[2*(e + f*x)])^2) - (2*a*
```

```
(9*a - 4*b)*b*Cos[e + f*x])/((a - b)^2*(a + b + (a - b)*Cos[2*(e + f*x)]))
- 8*Log[Cos[(e + f*x)/2]] + 8*Log[Sin[(e + f*x)/2]])/(8*a^3*f)
```

Maple [B] time = 0.09, size = 408, normalized size = 2.5

$$-\frac{\ln\left(\cos\left(fx+e\right)+1\right)}{2\,fa^{3}}-\frac{9\,b\left(\cos\left(fx+e\right)\right)^{3}}{8\,fa\left(a\left(\cos\left(fx+e\right)\right)^{2}-\left(\cos\left(fx+e\right)\right)^{2}b+b\right)^{2}(a-b)}+\frac{b^{2}\left(\cos\left(fx+\right.\right.}{2\,fa^{2}\left(a\left(\cos\left(fx+e\right)\right)^{2}-\left(\cos\left(f\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -1/2/f/a^3*ln(cos(f*x+e)+1)-9/8/f*b/a/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2/(
a-b)*cos(f*x+e)^3+1/2/f*b^2/a^2/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2/(a-b)*c
os(f*x+e)^3-7/8/f*b^2/a/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2/(a^2-2*a*b+b^2)
*cos(f*x+e)+1/2/f*b^3/a^2/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2/(a^2-2*a*b+b^
2)*cos(f*x+e)+15/8/f*b/a/(a^2-2*a*b+b^2)/(b*(a-b))^(1/2)*arctan((a-b)*cos(f
*x+e)/(b*(a-b))^(1/2))-5/2/f*b^2/a^2/(a^2-2*a*b+b^2)/(b*(a-b))^(1/2)*arctan
((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))+1/f*b^3/a^3/(a^2-2*a*b+b^2)/(b*(a-b))^(1
/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))+1/2/f/a^3*ln(cos(f*x+e)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.47296, size = 2356, normalized size = 14.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(9*a^3*b - 13*a^2*b^2 + 4*a*b^3)*cos(f*x + e)^3 - ((15*a^4 - 50*a
^3*b + 63*a^2*b^2 - 36*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 15*a^2*b^2 - 20*a*b^
3 + 8*b^4 + 2*(15*a^3*b - 35*a^2*b^2 + 28*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sq
rt(-b/(a - b))*log(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/(a - b))*cos
(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 2*(7*a^2*b^2 - 4*a*b^3)*cos(
f*x + e) + 8*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^4 +
a^2*b^2 - 2*a*b^3 + b^4 + 2*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*cos(f*x + e
)^2)*log(1/2*cos(f*x + e) + 1/2) - 8*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3
+ b^4)*cos(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 2*(a^3*b - 3*a^2*b^2 + 3*
a*b^3 - b^4)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^7 - 4*a^6*b
+ 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^4 + 2*(a^6*b - 3*a^5*b^2
+ 3*a^4*b^3 - a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f
```

```
), -1/8*((9*a^3*b - 13*a^2*b^2 + 4*a*b^3)*cos(f*x + e)^3 + ((15*a^4 - 50*a^
3*b + 63*a^2*b^2 - 36*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 15*a^2*b^2 - 20*a*b^3
 + 8*b^4 + 2*(15*a^3*b - 35*a^2*b^2 + 28*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqr
t(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + (7*a^2*b^2 -
 4*a*b^3)*cos(f*x + e) + 4*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos
(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 2*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^
4)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) - 4*((a^4 - 4*a^3*b + 6*a^2*
b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 2*(a^3*b -
3*a^2*b^2 + 3*a*b^3 - b^4)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((
a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^4 + 2*(a^6*
b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 - 2*a^4*b^
3 + a^3*b^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.64642, size = 717, normalized size = 4.32

$$-\frac{\frac{\left(15\,a^{2}b-20\,ab^{2}+8\,b^{3}\right)\arctan\left(-\frac{a\cos\left(fx+e\right)-b\cos\left(fx+e\right)-b}{\sqrt{ab-b^{2}}\cos\left(fx+e\right)+\sqrt{ab-b^{2}}}\right)}{\left(a^{5}-2\,a^{4}b+a^{3}b^{2}\right)\sqrt{ab-b^{2}}}+\frac{2\left(9\,a^{3}b-6\,a^{2}b^{2}+\frac{27\,a^{3}b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{68\,a^{2}b^{2}\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{32\,ab^{3}\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{27\,a^{3}b\left(\cos\left(fx+e\right)-\right.}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right.}{\left(a^{5}-2\,a^{4}b+a^{3}b^{2}\right)\left(\right.}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b - 20*a*b^2 + 8*b^3)*arctan(-(a*cos(f*x + e) - b*cos(f*x + e
) - b)/(sqrt(a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/((a^5 - 2*a^4*b +
a^3*b^2)*sqrt(a*b - b^2)) + 2*(9*a^3*b - 6*a^2*b^2 + 27*a^3*b*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) - 68*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
 + 32*a*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 27*a^3*b*(cos(f*x + e)
- 1)^2/(cos(f*x + e) + 1)^2 - 90*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2 + 120*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 48*b^4*(cos
(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 9*a^3*b*(cos(f*x + e) - 1)^3/(cos(f
*x + e) + 1)^3 - 28*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 16*
a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)/((a^5 - 2*a^4*b + a^3*b^2)
*(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(c
os(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)^2) - 4*log(
-(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/a^3)/f
```

3.84 $\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3} dx$

Optimal. Leaf size=205

$$-\frac{\sqrt{b}\left(15a^2-40ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^4f(a-b)^{3/2}}-\frac{b(11a-12b)\sec(e+fx)}{8a^3f(a-b)\left(a+b\sec^2(e+fx)-b\right)}-\frac{3b\sec(e+fx)}{4a^2f\left(a+b\sec^2(e+fx)-b\right)^2}$$

```
[Out] -(Sqrt[b]*(15*a^2 - 40*a*b + 24*b^2)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a -
 b]])/(8*a^4*(a - b)^(3/2)*f) - ((a - 6*b)*ArcTanh[Cos[e + f*x]])/(2*a^4*f)
 - (Cot[e + f*x]*Csc[e + f*x])/(2*a*f*(a - b + b*Sec[e + f*x]^2)^2) - (3*b*
Sec[e + f*x])/(4*a^2*f*(a - b + b*Sec[e + f*x]^2)^2) - ((11*a - 12*b)*b*Sec
[e + f*x])/(8*a^3*(a - b)*f*(a - b + b*Sec[e + f*x]^2))
```

Rubi [A] time = 0.292662, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3664, 471, 527, 522, 207, 205}

$$-\frac{\sqrt{b}\left(15a^2-40ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^4f(a-b)^{3/2}}-\frac{b(11a-12b)\sec(e+fx)}{8a^3f(a-b)\left(a+b\sec^2(e+fx)-b\right)}-\frac{3b\sec(e+fx)}{4a^2f\left(a+b\sec^2(e+fx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -(Sqrt[b]*(15*a^2 - 40*a*b + 24*b^2)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a -
 b]])/(8*a^4*(a - b)^(3/2)*f) - ((a - 6*b)*ArcTanh[Cos[e + f*x]])/(2*a^4*f)
 - (Cot[e + f*x]*Csc[e + f*x])/(2*a*f*(a - b + b*Sec[e + f*x]^2)^2) - (3*b*
Sec[e + f*x])/(4*a^2*f*(a - b + b*Sec[e + f*x]^2)^2) - ((11*a - 12*b)*b*Sec
[e + f*x])/(8*a^3*(a - b)*f*(a - b + b*Sec[e + f*x]^2))
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
```

```
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2\left(a-b+bx^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a-b-5bx^2}{\left(-1+x^2\right)\left(a-b+bx^2\right)^3}\,dx, x, \sec(e+fx)\right)}{2af}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{3b\sec(e+fx)}{4a^2f\left(a-b+b\sec^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{2(2a-3b)(a-b)-1}{\left(-1+x^2\right)\left(a-b\right.}\right.}{8a^2}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{3b\sec(e+fx)}{4a^2f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{(11a-12b)b\sec(}{8a^3(a-b)f\left(a-b+b\,\mathrm{s}\right.}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{3b\sec(e+fx)}{4a^2f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{(11a-12b)b\sec(}{8a^3(a-b)f\left(a-b+b\,\mathrm{s}\right.}$$

$$= -\frac{\sqrt{b}\left(15a^2-40ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^4(a-b)^{3/2}f} - \frac{(a-6b)\tanh^{-1}(\cos(e+fx))}{2a^4f} - \frac{\cot(e}{2af\left(a-\right.}$$

Mathematica [A] time = 6.53343, size = 286, normalized size = 1.4

$$\frac{8a^2b^2\cos(e+fx)}{(a-b)((a-b)\cos(2(e+fx))+a+b)^2} + \frac{\sqrt{b}\left(15a^2-40ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{3/2}} + \frac{\sqrt{b}\left(15a^2-40ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{3/2}} - \frac{2ab(9}{(a-b)((a-}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((Sqrt[b]*(15*a^2 - 40*a*b + 24*b^2)*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e +
 f*x)/2])/Sqrt[b]])/(a - b)^(3/2) + (Sqrt[b]*(15*a^2 - 40*a*b + 24*b^2)*Arc
Tan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a - b)^(3/2) + (8*a
^2*b^2*Cos[e + f*x])/((a - b)*(a + b + (a - b)*Cos[2*(e + f*x)])^2) - (2*a*
(9*a - 8*b)*b*Cos[e + f*x])/((a - b)*(a + b + (a - b)*Cos[2*(e + f*x)])) -
a*Csc[(e + f*x)/2]^2 - 4*(a - 6*b)*Log[Cos[(e + f*x)/2]] + 4*(a - 6*b)*Log[
Sin[(e + f*x)/2]] + a*Sec[(e + f*x)/2]^2)/(8*a^4*f)
```

Maple [B] time = 0.109, size = 435, normalized size = 2.1

$$\frac{1}{4fa^3\left(\cos\left(fx+e\right)+1\right)}-\frac{\ln\left(\cos\left(fx+e\right)+1\right)}{4fa^3}+\frac{3\ln\left(\cos\left(fx+e\right)+1\right)b}{2fa^4}-\frac{9b\left(\cos\left(fx+e\right)\right)^3}{8fa^2\left(a\left(\cos\left(fx+e\right)\right)^2-\left(\cos\left(fx+e\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] 1/4/f/a^3/(cos(f*x+e)+1)-1/4/f/a^3*ln(cos(f*x+e)+1)+3/2/f/a^4*ln(cos(f*x+e)
+1)*b-9/8/f*b/a^2/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*cos(f*x+e)^3+1/f*b^2/
a^3/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*cos(f*x+e)^3-7/8/f*b^2/a^2/(a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)^2/(a-b)*cos(f*x+e)+1/f*b^3/a^3/(a*cos(f*x+e)^2-co
s(f*x+e)^2*b+b)^2/(a-b)*cos(f*x+e)+15/8/f*b/a^2/(a-b)/(b*(a-b))^(1/2)*arcta
n((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))-5/f*b^2/a^3/(a-b)/(b*(a-b))^(1/2)*arcta
n((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))+3/f*b^3/a^4/(a-b)/(b*(a-b))^(1/2)*arcta
n((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))+1/4/f/a^3/(cos(f*x+e)-1)+1/4/f/a^3*ln(c
os(f*x+e)-1)-3/2/f/a^4*ln(cos(f*x+e)-1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.79051, size = 3227, normalized size = 15.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(4*a^4 - 21*a^3*b + 29*a^2*b^2 - 12*a*b^3)*cos(f*x + e)^5 + 2*(17*
a^3*b - 40*a^2*b^2 + 24*a*b^3)*cos(f*x + e)^3 - ((15*a^4 - 70*a^3*b + 119*a
^2*b^2 - 88*a*b^3 + 24*b^4)*cos(f*x + e)^6 - (15*a^4 - 100*a^3*b + 229*a^2*
```

```
b^2 - 216*a*b^3 + 72*b^4)*cos(f*x + e)^4 - 15*a^2*b^2 + 40*a*b^3 - 24*b^4 -
 (30*a^3*b - 125*a^2*b^2 + 168*a*b^3 - 72*b^4)*cos(f*x + e)^2)*sqrt(-b/(a -
 b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a - b)*sqrt(-b/(a - b))*cos(f*x + e)
 - b)/((a - b)*cos(f*x + e)^2 + b)) + 2*(11*a^2*b^2 - 12*a*b^3)*cos(f*x + e
) - 4*((a^4 - 9*a^3*b + 21*a^2*b^2 - 19*a*b^3 + 6*b^4)*cos(f*x + e)^6 - (a^
4 - 11*a^3*b + 37*a^2*b^2 - 45*a*b^3 + 18*b^4)*cos(f*x + e)^4 - a^2*b^2 + 7
*a*b^3 - 6*b^4 - (2*a^3*b - 17*a^2*b^2 + 33*a*b^3 - 18*b^4)*cos(f*x + e)^2)
*log(1/2*cos(f*x + e) + 1/2) + 4*((a^4 - 9*a^3*b + 21*a^2*b^2 - 19*a*b^3 +
6*b^4)*cos(f*x + e)^6 - (a^4 - 11*a^3*b + 37*a^2*b^2 - 45*a*b^3 + 18*b^4)*c
os(f*x + e)^4 - a^2*b^2 + 7*a*b^3 - 6*b^4 - (2*a^3*b - 17*a^2*b^2 + 33*a*b^
3 - 18*b^4)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^7 - 3*a^6*b +
 3*a^5*b^2 - a^4*b^3)*f*cos(f*x + e)^6 - (a^7 - 5*a^6*b + 7*a^5*b^2 - 3*a^4
*b^3)*f*cos(f*x + e)^4 - (2*a^6*b - 5*a^5*b^2 + 3*a^4*b^3)*f*cos(f*x + e)^2
 - (a^5*b^2 - a^4*b^3)*f), 1/8*((4*a^4 - 21*a^3*b + 29*a^2*b^2 - 12*a*b^3)*
cos(f*x + e)^5 + (17*a^3*b - 40*a^2*b^2 + 24*a*b^3)*cos(f*x + e)^3 - ((15*a
^4 - 70*a^3*b + 119*a^2*b^2 - 88*a*b^3 + 24*b^4)*cos(f*x + e)^6 - (15*a^4 -
 100*a^3*b + 229*a^2*b^2 - 216*a*b^3 + 72*b^4)*cos(f*x + e)^4 - 15*a^2*b^2
+ 40*a*b^3 - 24*b^4 - (30*a^3*b - 125*a^2*b^2 + 168*a*b^3 - 72*b^4)*cos(f*x
 + e)^2)*sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) +
(11*a^2*b^2 - 12*a*b^3)*cos(f*x + e) - 2*((a^4 - 9*a^3*b + 21*a^2*b^2 - 19*
a*b^3 + 6*b^4)*cos(f*x + e)^6 - (a^4 - 11*a^3*b + 37*a^2*b^2 - 45*a*b^3 + 1
8*b^4)*cos(f*x + e)^4 - a^2*b^2 + 7*a*b^3 - 6*b^4 - (2*a^3*b - 17*a^2*b^2 +
 33*a*b^3 - 18*b^4)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 2*((a^4 -
 9*a^3*b + 21*a^2*b^2 - 19*a*b^3 + 6*b^4)*cos(f*x + e)^6 - (a^4 - 11*a^3*b
+ 37*a^2*b^2 - 45*a*b^3 + 18*b^4)*cos(f*x + e)^4 - a^2*b^2 + 7*a*b^3 - 6*b^
4 - (2*a^3*b - 17*a^2*b^2 + 33*a*b^3 - 18*b^4)*cos(f*x + e)^2)*log(-1/2*cos
(f*x + e) + 1/2))/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f*cos(f*x + e)^6 -
 (a^7 - 5*a^6*b + 7*a^5*b^2 - 3*a^4*b^3)*f*cos(f*x + e)^4 - (2*a^6*b - 5*a^
5*b^2 + 3*a^4*b^3)*f*cos(f*x + e)^2 - (a^5*b^2 - a^4*b^3)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.6436, size = 836, normalized size = 4.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b - 40*a*b^2 + 24*b^3)*arctan(-(a*cos(f*x + e) - b*cos(f*x +
e) - b)/(sqrt(a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/((a^5 - a^4*b)*sq
rt(a*b - b^2)) + 2*(9*a^3*b - 10*a^2*b^2 + 27*a^3*b*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1) - 80*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 56*a*b^
3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 27*a^3*b*(cos(f*x + e) - 1)^2/(co
s(f*x + e) + 1)^2 - 102*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 +
```

```
152*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 80*b^4*(cos(f*x + e)
 - 1)^2/(cos(f*x + e) + 1)^2 + 9*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) +
 1)^3 - 32*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 24*a*b^3*(co
s(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)/((a^5 - a^4*b)*(a + 2*a*(cos(f*x +
e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*
(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)^2) - 2*(a - 6*b)*log(-(cos(f*x +
 e) - 1)/(cos(f*x + e) + 1))/a^4 - (a - 2*a*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) + 12*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(f*x + e) + 1)/(a^
4*(cos(f*x + e) - 1)) + (cos(f*x + e) - 1)/(a^3*(cos(f*x + e) + 1)))/f
```

3.85 $\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=259

$$-\frac{3\left(a^2-12ab+16b^2\right)\tanh^{-1}(\cos(e+fx))}{8a^5f}-\frac{3\sqrt{b}\left(5a^2-20ab+16b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^5f\sqrt{a-b}}-\frac{3b(a-2b)\sec(e+fx)}{2a^4f\left(a+b\sec^2(e+fx)-\right.}$$

```
[Out] (-3*Sqrt[b]*(5*a^2 - 20*a*b + 16*b^2)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a
 - b]])/(8*a^5*Sqrt[a - b]*f) - (3*(a^2 - 12*a*b + 16*b^2)*ArcTanh[Cos[e + f
*x]])/(8*a^5*f) - ((5*a - 8*b)*Cot[e + f*x]*Csc[e + f*x])/(8*a^2*f*(a - b +
 b*Sec[e + f*x]^2)^2) - (Cot[e + f*x]^3*Csc[e + f*x])/(4*a*f*(a - b + b*Sec
[e + f*x]^2)^2) - ((7*a - 12*b)*b*Sec[e + f*x])/(8*a^3*f*(a - b + b*Sec[e +
 f*x]^2)^2) - (3*(a - 2*b)*b*Sec[e + f*x])/(2*a^4*f*(a - b + b*Sec[e + f*x]
^2))
```

Rubi [A] time = 0.376137, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3664, 470, 527, 522, 207, 205}

$$-\frac{3\left(a^2-12ab+16b^2\right)\tanh^{-1}(\cos(e+fx))}{8a^5f}-\frac{3\sqrt{b}\left(5a^2-20ab+16b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^5f\sqrt{a-b}}-\frac{3b(a-2b)\sec(e+fx)}{2a^4f\left(a+b\sec^2(e+fx)-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-3*Sqrt[b]*(5*a^2 - 20*a*b + 16*b^2)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a
 - b]])/(8*a^5*Sqrt[a - b]*f) - (3*(a^2 - 12*a*b + 16*b^2)*ArcTanh[Cos[e + f
*x]])/(8*a^5*f) - ((5*a - 8*b)*Cot[e + f*x]*Csc[e + f*x])/(8*a^2*f*(a - b +
 b*Sec[e + f*x]^2)^2) - (Cot[e + f*x]^3*Csc[e + f*x])/(4*a*f*(a - b + b*Sec
[e + f*x]^2)^2) - ((7*a - 12*b)*b*Sec[e + f*x])/(8*a^3*f*(a - b + b*Sec[e +
 f*x]^2)^2) - (3*(a - 2*b)*b*Sec[e + f*x])/(2*a^4*f*(a - b + b*Sec[e + f*x]
^2))
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^3\left(a-b+bx^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-a+b+(-4a+7b)x^2}{\left(-1+x^2\right)^2\left(a-b+bx^2\right)^3}\,dx, x, \sec(e+fx)\right)}{4af}$$

$$= -\frac{(5a-8b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-(3a-8b}{(-1+}\right.}{}$$

$$= -\frac{(5a-8b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{(7a-12b)b\,\text{se}}{8a^3f\left(a-b+b\,\text{s}\right.}$$

$$= -\frac{(5a-8b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{(7a-12b)b\,\text{se}}{8a^3f\left(a-b+b\,\text{s}\right.}$$

$$= -\frac{(5a-8b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{(7a-12b)b\,\text{se}}{8a^3f\left(a-b+b\,\text{s}\right.}$$

$$= -\frac{3\sqrt{b}\left(5a^2-20ab+16b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^5\sqrt{a-b}f} - \frac{3\left(a^2-12ab+16b^2\right)\tanh^{-1}(\cos(e+}{8a^5f}$$

Mathematica [A] time = 6.56108, size = 468, normalized size = 1.81

$$-\frac{3\left(3ab\cos(e+fx)-4b^2\cos(e+fx)\right)}{4a^4f(a\cos(2(e+fx))+a-b\cos(2(e+fx))+b)}+\frac{b^2\cos(e+fx)}{a^3f(a\cos(2(e+fx))+a-b\cos(2(e+fx))+b)^2}+\frac{3\left(a^2-12ab+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-3*Sqrt[a - b]*Sqrt[b]*(5*a^2 - 20*a*b + 16*b^2)*ArcTan[(Sec[(e + f*x)/2]*
(Sqrt[a - b]*Cos[(e + f*x)/2] - Sqrt[a]*Sin[(e + f*x)/2]))/Sqrt[b]])/(8*a^5
*(-a + b)*f) - (3*Sqrt[a - b]*Sqrt[b]*(5*a^2 - 20*a*b + 16*b^2)*ArcTan[(Sec
[(e + f*x)/2]*(Sqrt[a - b]*Cos[(e + f*x)/2] + Sqrt[a]*Sin[(e + f*x)/2]))/Sq
rt[b]])/(8*a^5*(-a + b)*f) + (b^2*Cos[e + f*x])/(a^3*f*(a + b + a*Cos[2*(e
+ f*x)] - b*Cos[2*(e + f*x)])^2) - (3*(3*a*b*Cos[e + f*x] - 4*b^2*Cos[e + f
*x]))/(4*a^4*f*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])) - (3*(a -
 4*b)*Csc[(e + f*x)/2]^2)/(32*a^4*f) - Csc[(e + f*x)/2]^4/(64*a^3*f) - (3*(
a^2 - 12*a*b + 16*b^2)*Log[Cos[(e + f*x)/2]])/(8*a^5*f) + (3*(a^2 - 12*a*b
+ 16*b^2)*Log[Sin[(e + f*x)/2]])/(8*a^5*f) + (3*(a - 4*b)*Sec[(e + f*x)/2]^
2)/(32*a^4*f) + Sec[(e + f*x)/2]^4/(64*a^3*f)
```

Maple [B] time = 0.112, size = 560, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] 1/16/f/a^3/(cos(f*x+e)+1)^2+3/16/f/a^3/(cos(f*x+e)+1)-3/4/f/a^4/(cos(f*x+e)
+1)*b-3/16/f/a^3*ln(cos(f*x+e)+1)+9/4/f/a^4*ln(cos(f*x+e)+1)*b-3/f/a^5*ln(c
os(f*x+e)+1)*b^2-9/8/f*b/a^2/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*cos(f*x+e)
^3+21/8/f*b^2/a^3/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*cos(f*x+e)^3-3/2/f*b^
3/a^4/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*cos(f*x+e)^3-7/8/f*b^2/a^3/(a*cos
(f*x+e)^2-cos(f*x+e)^2*b+b)^2*cos(f*x+e)+3/2/f*b^3/a^4/(a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)^2*cos(f*x+e)+15/8/f*b/a^3/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*
x+e)/(b*(a-b))^(1/2))-15/2/f*b^2/a^4/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e
)/(b*(a-b))^(1/2))+6/f*b^3/a^5/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(
a-b))^(1/2))-1/16/f/a^3/(cos(f*x+e)-1)^2+3/16/f/a^3/(cos(f*x+e)-1)-3/4/f/a^
4/(cos(f*x+e)-1)*b+3/16/f/a^3*ln(cos(f*x+e)-1)-9/4/f/a^4*ln(cos(f*x+e)-1)*b
+3/f/a^5*ln(cos(f*x+e)-1)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.85351, size = 3969, normalized size = 15.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*(a^4 - 9*a^3*b + 16*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^7 - 2*(5*a^4 -
 46*a^3*b + 108*a^2*b^2 - 72*a*b^3)*cos(f*x + e)^5 - 2*(19*a^3*b - 72*a^2*b
^2 + 72*a*b^3)*cos(f*x + e)^3 + 3*((5*a^4 - 30*a^3*b + 61*a^2*b^2 - 52*a*b^
3 + 16*b^4)*cos(f*x + e)^8 - 2*(5*a^4 - 35*a^3*b + 86*a^2*b^2 - 88*a*b^3 +
32*b^4)*cos(f*x + e)^6 + (5*a^4 - 50*a^3*b + 166*a^2*b^2 - 216*a*b^3 + 96*b
^4)*cos(f*x + e)^4 + 5*a^2*b^2 - 20*a*b^3 + 16*b^4 + 2*(5*a^3*b - 30*a^2*b^
2 + 56*a*b^3 - 32*b^4)*cos(f*x + e)^2)*sqrt(-b/(a - b))*log(((a - b)*cos(f*
x + e)^2 + 2*(a - b)*sqrt(-b/(a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x +
e)^2 + b)) - 24*(a^2*b^2 - 2*a*b^3)*cos(f*x + e) - 3*((a^4 - 14*a^3*b + 41*
a^2*b^2 - 44*a*b^3 + 16*b^4)*cos(f*x + e)^8 - 2*(a^4 - 15*a^3*b + 54*a^2*b^
2 - 72*a*b^3 + 32*b^4)*cos(f*x + e)^6 + (a^4 - 18*a^3*b + 94*a^2*b^2 - 168*
a*b^3 + 96*b^4)*cos(f*x + e)^4 + a^2*b^2 - 12*a*b^3 + 16*b^4 + 2*(a^3*b - 1
4*a^2*b^2 + 40*a*b^3 - 32*b^4)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2)
+ 3*((a^4 - 14*a^3*b + 41*a^2*b^2 - 44*a*b^3 + 16*b^4)*cos(f*x + e)^8 - 2*(
a^4 - 15*a^3*b + 54*a^2*b^2 - 72*a*b^3 + 32*b^4)*cos(f*x + e)^6 + (a^4 - 18
*a^3*b + 94*a^2*b^2 - 168*a*b^3 + 96*b^4)*cos(f*x + e)^4 + a^2*b^2 - 12*a*b
^3 + 16*b^4 + 2*(a^3*b - 14*a^2*b^2 + 40*a*b^3 - 32*b^4)*cos(f*x + e)^2)*lo
g(-1/2*cos(f*x + e) + 1/2))/((a^7 - 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^8 + a
^5*b^2*f - 2*(a^7 - 3*a^6*b + 2*a^5*b^2)*f*cos(f*x + e)^6 + (a^7 - 6*a^6*b
+ 6*a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b - 2*a^5*b^2)*f*cos(f*x + e)^2), 1/
16*(6*(a^4 - 9*a^3*b + 16*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^7 - 2*(5*a^4 - 46
*a^3*b + 108*a^2*b^2 - 72*a*b^3)*cos(f*x + e)^5 - 2*(19*a^3*b - 72*a^2*b^2
+ 72*a*b^3)*cos(f*x + e)^3 - 6*((5*a^4 - 30*a^3*b + 61*a^2*b^2 - 52*a*b^3 +
 16*b^4)*cos(f*x + e)^8 - 2*(5*a^4 - 35*a^3*b + 86*a^2*b^2 - 88*a*b^3 + 32*
b^4)*cos(f*x + e)^6 + (5*a^4 - 50*a^3*b + 166*a^2*b^2 - 216*a*b^3 + 96*b^4)
*cos(f*x + e)^4 + 5*a^2*b^2 - 20*a*b^3 + 16*b^4 + 2*(5*a^3*b - 30*a^2*b^2 +
 56*a*b^3 - 32*b^4)*cos(f*x + e)^2)*sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/
(a - b))*cos(f*x + e)/b) - 24*(a^2*b^2 - 2*a*b^3)*cos(f*x + e) - 3*((a^4 -
14*a^3*b + 41*a^2*b^2 - 44*a*b^3 + 16*b^4)*cos(f*x + e)^8 - 2*(a^4 - 15*a^3
*b + 54*a^2*b^2 - 72*a*b^3 + 32*b^4)*cos(f*x + e)^6 + (a^4 - 18*a^3*b + 94*
a^2*b^2 - 168*a*b^3 + 96*b^4)*cos(f*x + e)^4 + a^2*b^2 - 12*a*b^3 + 16*b^4
+ 2*(a^3*b - 14*a^2*b^2 + 40*a*b^3 - 32*b^4)*cos(f*x + e)^2)*log(1/2*cos(f*
x + e) + 1/2) + 3*((a^4 - 14*a^3*b + 41*a^2*b^2 - 44*a*b^3 + 16*b^4)*cos(f*
x + e)^8 - 2*(a^4 - 15*a^3*b + 54*a^2*b^2 - 72*a*b^3 + 32*b^4)*cos(f*x + e)
^6 + (a^4 - 18*a^3*b + 94*a^2*b^2 - 168*a*b^3 + 96*b^4)*cos(f*x + e)^4 + a^
2*b^2 - 12*a*b^3 + 16*b^4 + 2*(a^3*b - 14*a^2*b^2 + 40*a*b^3 - 32*b^4)*cos(
f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^7 - 2*a^6*b + a^5*b^2)*f*cos(
f*x + e)^8 + a^5*b^2*f - 2*(a^7 - 3*a^6*b + 2*a^5*b^2)*f*cos(f*x + e)^6 + (
a^7 - 6*a^6*b + 6*a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b - 2*a^5*b^2)*f*cos(f
*x + e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.66481, size = 1241, normalized size = 4.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(12*(a^2 - 12*a*b + 16*b^2)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
)/a^5 - 24*(5*a^2*b - 20*a*b^2 + 16*b^3)*arctan(-(a*cos(f*x + e) - b*cos(f*
x + e) - b)/(sqrt(a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/(sqrt(a*b - b
^2)*a^5) - (8*a^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 24*a^2*b*(cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) - a^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)
^2)/a^6 - (a^4 - 4*a^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 16*a^3*b*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) - 20*a^4*(cos(f*x + e) - 1)^2/(cos(f*x +
 e) + 1)^2 + 216*a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 304*a^2*
b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 20*a^4*(cos(f*x + e) - 1)^3
/(cos(f*x + e) + 1)^3 + 360*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3
 - 1024*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 896*a*b^3*(cos(
f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 5*a^4*(cos(f*x + e) - 1)^4/(cos(f*x
+ e) + 1)^4 + 64*a^3*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 - 192*a^2*
b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 256*a*b^3*(cos(f*x + e) - 1
)^4/(cos(f*x + e) + 1)^4 - 256*b^4*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^
4 + 16*a^4*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 - 168*a^3*b*(cos(f*x +
 e) - 1)^5/(cos(f*x + e) + 1)^5 + 384*a^2*b^2*(cos(f*x + e) - 1)^5/(cos(f*x
 + e) + 1)^5 - 256*a*b^3*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 + 6*a^4*
(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 - 72*a^3*b*(cos(f*x + e) - 1)^6/(
cos(f*x + e) + 1)^6 + 96*a^2*b^2*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6)
/(a^5*(a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2*a*(cos(f*x + e) - 1)^2/(
cos(f*x + e) + 1)^2 - 4*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + a*(co
s(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)^2))/f
```

3.86 $\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=250

$$-\frac{3\sqrt{b}\left(5a^2+10ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8\sqrt{a}f(a-b)^5}+\frac{3x\left(a^2+10ab+5b^2\right)}{8(a-b)^5}-\frac{3b(a+b)\tan(e+fx)}{2f(a-b)^4\left(a+b\tan^2(e+fx)\right)}-\frac{b(7a+}{8f(a-b)^3(}$$

```
[Out] (3*(a^2 + 10*a*b + 5*b^2)*x)/(8*(a - b)^5) - (3*Sqrt[b]*(5*a^2 + 10*a*b + b
^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(8*Sqrt[a]*(a - b)^5*f) - ((5*a
 + 3*b)*Cos[e + f*x]*Sin[e + f*x])/(8*(a - b)^2*f*(a + b*Tan[e + f*x]^2)^2)
 + (Cos[e + f*x]^3*Sin[e + f*x])/(4*(a - b)*f*(a + b*Tan[e + f*x]^2)^2) - (
b*(7*a + 5*b)*Tan[e + f*x])/(8*(a - b)^3*f*(a + b*Tan[e + f*x]^2)^2) - (3*b
*(a + b)*Tan[e + f*x])/(2*(a - b)^4*f*(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.330496, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3663, 470, 527, 522, 203, 205}

$$-\frac{3\sqrt{b}\left(5a^2+10ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8\sqrt{a}f(a-b)^5}+\frac{3x\left(a^2+10ab+5b^2\right)}{8(a-b)^5}-\frac{3b(a+b)\tan(e+fx)}{2f(a-b)^4\left(a+b\tan^2(e+fx)\right)}-\frac{b(7a+}{8f(a-b)^3(}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (3*(a^2 + 10*a*b + 5*b^2)*x)/(8*(a - b)^5) - (3*Sqrt[b]*(5*a^2 + 10*a*b + b
^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(8*Sqrt[a]*(a - b)^5*f) - ((5*a
 + 3*b)*Cos[e + f*x]*Sin[e + f*x])/(8*(a - b)^2*f*(a + b*Tan[e + f*x]^2)^2)
 + (Cos[e + f*x]^3*Sin[e + f*x])/(4*(a - b)*f*(a + b*Tan[e + f*x]^2)^2) - (
b*(7*a + 5*b)*Tan[e + f*x])/(8*(a - b)^3*f*(a + b*Tan[e + f*x]^2)^2) - (3*b
*(a + b)*Tan[e + f*x])/(2*(a - b)^4*f*(a + b*Tan[e + f*x]^2))
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^3\left(a+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{a+(-4a-3b)x^2}{\left(1+x^2\right)^2\left(a+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{4(a-b)f}$$

$$= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a(3a+5b)\cdots}{\left(1+x^2\cdots\right.}\right.}{\cdots}$$

$$= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{b(7a+5b)\,\text{ta}\cdots}{8(a-b)^3f\left(a+b\,\text{t}\cdots\right.}$$

$$= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{b(7a+5b)\,\text{ta}\cdots}{8(a-b)^3f\left(a+b\,\text{t}\cdots\right.}$$

$$= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{b(7a+5b)\,\text{ta}\cdots}{8(a-b)^3f\left(a+b\,\text{t}\cdots\right.}$$

$$= \frac{3\left(a^2+10ab+5b^2\right)x}{8(a-b)^5} - \frac{3\sqrt{b}\left(5a^2+10ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8\sqrt{a}(a-b)^5f} - \frac{(5a+3b)\cos(e+f\cdots}{8(a-b)^2f\left(a+b\,\text{ta}\cdots\right.}$$

Mathematica [A] time = 0.921413, size = 194, normalized size = 0.78

$$\frac{12\left(a^2+10ab+5b^2\right)(e+fx)-\frac{12\sqrt{b}\left(5a^2+10ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{16ab^2(a-b)\sin(2(e+fx))}{((a-b)\cos(2(e+fx))+a+b)^2}+(a-b)^2\sin(4(e+fx))-8(}{32f(a-b)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (12*(a^2 + 10*a*b + 5*b^2)*(e + f*x) - (12*Sqrt[b]*(5*a^2 + 10*a*b + b^2)*A
rcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/Sqrt[a] - 8*(a - b)*(a + 2*b)*Sin[2*
(e + f*x)] + (16*a*(a - b)*b^2*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e
+ f*x)])^2 - (4*(a - b)*b*(9*a + 5*b)*Sin[2*(e + f*x)])/(a + b + (a - b)*Co
s[2*(e + f*x)]) + (a - b)^2*Sin[4*(e + f*x)])/(32*(a - b)^5*f)
```

Maple [B] time = 0.082, size = 598, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -7/8/f*b^2/(a-b)^5/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^3*a^2+1/4/f*b^3/(a-b)^5/
(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^3*a+5/8/f*b^4/(a-b)^5/(a+b*tan(f*x+e)^2)^2*
tan(f*x+e)^3-9/8/f*b/(a-b)^5/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)*a^3+3/4/f*b^2/
(a-b)^5/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)*a^2+3/8/f*b^3/(a-b)^5/(a+b*tan(f*x+
e)^2)^2*tan(f*x+e)*a-15/8/f*b/(a-b)^5/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)
^(1/2))*a^2-15/4/f*b^2/(a-b)^5/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))
*a-3/8/f*b^3/(a-b)^5/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-1/4/f/(a-
b)^5/(1+tan(f*x+e)^2)^2*tan(f*x+e)^3*a*b+7/8/f/(a-b)^5/(1+tan(f*x+e)^2)^2*t
an(f*x+e)^3*b^2-5/8/f/(a-b)^5/(1+tan(f*x+e)^2)^2*tan(f*x+e)^3*a^2-3/8/f/(a-
b)^5/(1+tan(f*x+e)^2)^2*tan(f*x+e)*a^2-3/4/f/(a-b)^5/(1+tan(f*x+e)^2)^2*tan
(f*x+e)*a*b+9/8/f/(a-b)^5/(1+tan(f*x+e)^2)^2*tan(f*x+e)*b^2+15/4/f/(a-b)^5*
arctan(tan(f*x+e))*a*b+15/8/f/(a-b)^5*arctan(tan(f*x+e))*b^2+3/8/f/(a-b)^5*
arctan(tan(f*x+e))*a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.26895, size = 2664, normalized size = 10.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(12*(a^4 + 8*a^3*b - 14*a^2*b^2 + 5*b^4)*f*x*cos(f*x + e)^4 + 24*(a^3
*b + 9*a^2*b^2 - 5*a*b^3 - 5*b^4)*f*x*cos(f*x + e)^2 + 12*(a^2*b^2 + 10*a*b
^3 + 5*b^4)*f*x - 3*((5*a^4 - 14*a^2*b^2 + 8*a*b^3 + b^4)*cos(f*x + e)^4 +
5*a^2*b^2 + 10*a*b^3 + b^4 + 2*(5*a^3*b + 5*a^2*b^2 - 9*a*b^3 - b^4)*cos(f*
x + e)^2)*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b
^2)*cos(f*x + e)^2 - 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt
(-b/a)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b
^2)*cos(f*x + e)^2 + b^2)) + 4*(2*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^
4)*cos(f*x + e)^7 - (5*a^4 - 12*a^3*b + 6*a^2*b^2 + 4*a*b^3 - 3*b^4)*cos(f*
x + e)^5 - (19*a^3*b - 21*a^2*b^2 - 15*a*b^3 + 17*b^4)*cos(f*x + e)^3 - 12*
(a^2*b^2 - b^4)*cos(f*x + e))*sin(f*x + e))/((a^7 - 7*a^6*b + 21*a^5*b^2 -
35*a^4*b^3 + 35*a^3*b^4 - 21*a^2*b^5 + 7*a*b^6 - b^7)*f*cos(f*x + e)^4 + 2*
(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)*
f*cos(f*x + e)^2 + (a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 + 5*a*b^6
 - b^7)*f), 1/16*(6*(a^4 + 8*a^3*b - 14*a^2*b^2 + 5*b^4)*f*x*cos(f*x + e)^4
 + 12*(a^3*b + 9*a^2*b^2 - 5*a*b^3 - 5*b^4)*f*x*cos(f*x + e)^2 + 6*(a^2*b^2
 + 10*a*b^3 + 5*b^4)*f*x + 3*((5*a^4 - 14*a^2*b^2 + 8*a*b^3 + b^4)*cos(f*x
+ e)^4 + 5*a^2*b^2 + 10*a*b^3 + b^4 + 2*(5*a^3*b + 5*a^2*b^2 - 9*a*b^3 - b^
4)*cos(f*x + e)^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b
/a)/(b*cos(f*x + e)*sin(f*x + e))) + 2*(2*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*
b^3 + b^4)*cos(f*x + e)^7 - (5*a^4 - 12*a^3*b + 6*a^2*b^2 + 4*a*b^3 - 3*b^4
)*cos(f*x + e)^5 - (19*a^3*b - 21*a^2*b^2 - 15*a*b^3 + 17*b^4)*cos(f*x + e)
^3 - 12*(a^2*b^2 - b^4)*cos(f*x + e))*sin(f*x + e))/((a^7 - 7*a^6*b + 21*a^
5*b^2 - 35*a^4*b^3 + 35*a^3*b^4 - 21*a^2*b^5 + 7*a*b^6 - b^7)*f*cos(f*x + e
)^4 + 2*(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6
 + b^7)*f*cos(f*x + e)^2 + (a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 +
 5*a*b^6 - b^7)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.3016, size = 1793, normalized size = 7.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(6*(3*a^6*b - 2*a^5*b^2 - 19*a^4*b^3 + 36*a^3*b^4 - 19*a^2*b^5 - 2*a*b
^6 + 3*b^7 + 2*a*b*abs(-a^5 + 5*a^4*b - 10*a^3*b^2 + 10*a^2*b^3 - 5*a*b^4 +
 b^5) + 2*b^2*abs(-a^5 + 5*a^4*b - 10*a^3*b^2 + 10*a^2*b^3 - 5*a*b^4 + b^5)
)*(pi*floor((f*x + e)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(f*x + e)/sqrt((a^5
```

```
 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 + sqrt((a^5 - 3*a^4*b +
2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)^2 - 4*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4
*a^2*b^3 + a*b^4)*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)))/(a^4*b
- 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5))))/(a^5*abs(-a^5 + 5*a^4*b - 10*a^
3*b^2 + 10*a^2*b^3 - 5*a*b^4 + b^5) - 3*a^4*b*abs(-a^5 + 5*a^4*b - 10*a^3*b
^2 + 10*a^2*b^3 - 5*a*b^4 + b^5) + 2*a^3*b^2*abs(-a^5 + 5*a^4*b - 10*a^3*b^
2 + 10*a^2*b^3 - 5*a*b^4 + b^5) + 2*a^2*b^3*abs(-a^5 + 5*a^4*b - 10*a^3*b^2
 + 10*a^2*b^3 - 5*a*b^4 + b^5) - 3*a*b^4*abs(-a^5 + 5*a^4*b - 10*a^3*b^2 +
10*a^2*b^3 - 5*a*b^4 + b^5) + b^5*abs(-a^5 + 5*a^4*b - 10*a^3*b^2 + 10*a^2*
b^3 - 5*a*b^4 + b^5) + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 -
 b^5)^2) - 6*(2*sqrt(a*b)*(a + b)*abs(-a^5 + 5*a^4*b - 10*a^3*b^2 + 10*a^2*
b^3 - 5*a*b^4 + b^5)*abs(b) - (3*a^6 - 2*a^5*b - 19*a^4*b^2 + 36*a^3*b^3 -
19*a^2*b^4 - 2*a*b^5 + 3*b^6)*sqrt(a*b)*abs(b))*(pi*floor((f*x + e)/pi + 1/
2) + arctan(2*sqrt(1/2)*tan(f*x + e)/sqrt((a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^
2*b^3 - 3*a*b^4 + b^5 - sqrt((a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b
^4 + b^5)^2 - 4*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*(a^4*b - 4*
a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)))/(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a
*b^4 + b^5))))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)^2
*b - (a^5*b - 3*a^4*b^2 + 2*a^3*b^3 + 2*a^2*b^4 - 3*a*b^5 + b^6)*abs(-a^5 +
 5*a^4*b - 10*a^3*b^2 + 10*a^2*b^3 - 5*a*b^4 + b^5)) + (12*a*b^2*tan(f*x +
e)^7 + 12*b^3*tan(f*x + e)^7 + 19*a^2*b*tan(f*x + e)^5 + 34*a*b^2*tan(f*x +
 e)^5 + 19*b^3*tan(f*x + e)^5 + 5*a^3*tan(f*x + e)^3 + 31*a^2*b*tan(f*x + e
)^3 + 31*a*b^2*tan(f*x + e)^3 + 5*b^3*tan(f*x + e)^3 + 3*a^3*tan(f*x + e) +
 18*a^2*b*tan(f*x + e) + 3*a*b^2*tan(f*x + e))/((b*tan(f*x + e)^4 + a*tan(f
*x + e)^2 + b*tan(f*x + e)^2 + a)^2*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 +
b^4)))/f
```

3.87 $\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=193

$$-\frac{\sqrt{b}\left(15a^2+10ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{3/2}f(a-b)^4}-\frac{b(11a+b)\tan(e+fx)}{8af(a-b)^3\left(a+b\tan^2(e+fx)\right)}-\frac{3b\tan(e+fx)}{4f(a-b)^2\left(a+b\tan^2(e+fx)\right)^2}-\frac{\quad}{2f(\cdots}$$

```
[Out] ((a + 5*b)*x)/(2*(a - b)^4) - (Sqrt[b]*(15*a^2 + 10*a*b - b^2)*ArcTan[(Sqrt
[b]*Tan[e + f*x])/Sqrt[a]])/(8*a^(3/2)*(a - b)^4*f) - (Cos[e + f*x]*Sin[e +
 f*x])/(2*(a - b)*f*(a + b*Tan[e + f*x]^2)^2) - (3*b*Tan[e + f*x])/(4*(a -
b)^2*f*(a + b*Tan[e + f*x]^2)^2) - (b*(11*a + b)*Tan[e + f*x])/(8*a*(a - b)
^3*f*(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.245847, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3663, 471, 527, 522, 203, 205}

$$-\frac{\sqrt{b}\left(15a^2+10ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{3/2}f(a-b)^4}-\frac{b(11a+b)\tan(e+fx)}{8af(a-b)^3\left(a+b\tan^2(e+fx)\right)}-\frac{3b\tan(e+fx)}{4f(a-b)^2\left(a+b\tan^2(e+fx)\right)^2}-\frac{\quad}{2f(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((a + 5*b)*x)/(2*(a - b)^4) - (Sqrt[b]*(15*a^2 + 10*a*b - b^2)*ArcTan[(Sqrt
[b]*Tan[e + f*x])/Sqrt[a]])/(8*a^(3/2)*(a - b)^4*f) - (Cos[e + f*x]*Sin[e +
 f*x])/(2*(a - b)*f*(a + b*Tan[e + f*x]^2)^2) - (3*b*Tan[e + f*x])/(4*(a -
b)^2*f*(a + b*Tan[e + f*x]^2)^2) - (b*(11*a + b)*Tan[e + f*x])/(8*a*(a - b)
^3*f*(a + b*Tan[e + f*x]^2))
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
```

```
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a-5bx^2}{(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{2(a-b)f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{3b\tan(e+fx)}{4(a-b)^2f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{2a(2a+}{(1+x^2}\right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{3b\tan(e+fx)}{4(a-b)^2f\left(a+b\tan^2(e+fx)\right)^2} - \frac{b(11a+b)}{8a(a-b)^3f\,(a+}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{3b\tan(e+fx)}{4(a-b)^2f\left(a+b\tan^2(e+fx)\right)^2} - \frac{b(11a+b)}{8a(a-b)^3f\,(a+}$$

$$= \frac{(a+5b)x}{2(a-b)^4} - \frac{\sqrt{b}\left(15a^2+10ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{3/2}(a-b)^4f} - \frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right.}$$

Mathematica [A] time = 2.59452, size = 164, normalized size = 0.85

$$\frac{\frac{\sqrt{b}\left(-15a^2-10ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{4b^2(a-b)\sin(2(e+fx))}{((a-b)\cos(2(e+fx))+a+b)^2} + 4(a+5b)(e+fx) - 2(a-b)\sin(2(e+fx)) - \frac{b(a-b)(9a+b)\sin(}{a((a-b)\cos(2(e+f}}{8f(a-b)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (4*(a + 5*b)*(e + f*x) + (Sqrt[b]*(-15*a^2 - 10*a*b + b^2)*ArcTan[(Sqrt[b]*
Tan[e + f*x])/Sqrt[a]])/a^(3/2) - 2*(a - b)*Sin[2*(e + f*x)] + (4*(a - b)*b
^2*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)])^2 - ((a - b)*b*(9*a
 + b)*Sin[2*(e + f*x)])/(a*(a + b + (a - b)*Cos[2*(e + f*x)])))/(8*(a - b)^
4*f)
```

Maple [B] time = 0.081, size = 430, normalized size = 2.2

$$-\frac{7ab^2\left(\tan\left(fx+e\right)\right)^3}{8f(a-b)^4\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^2}+\frac{3b^3\left(\tan\left(fx+e\right)\right)^3}{4f(a-b)^4\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^2}+\frac{b^4\left(\tan\left(fx+e\right)\right)^3}{8f(a-b)^4\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^2a}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -7/8/f/(a-b)^4*b^2/(a+b*tan(f*x+e)^2)^2*a*tan(f*x+e)^3+3/4/f/(a-b)^4*b^3/(a
+b*tan(f*x+e)^2)^2*tan(f*x+e)^3+1/8/f/(a-b)^4*b^4/(a+b*tan(f*x+e)^2)^2/a*ta
n(f*x+e)^3-9/8/f/(a-b)^4*b/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)*a^2+5/4/f/(a-b)^
4*b^2/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)*a-1/8/f/(a-b)^4*b^3/(a+b*tan(f*x+e)^2
)^2*tan(f*x+e)-15/8/f/(a-b)^4*b*a/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/
2))-5/4/f/(a-b)^4*b^2/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))+1/8/f/(a
-b)^4*b^3/a/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-1/2/f/(a-b)^4*tan(
f*x+e)/(1+tan(f*x+e)^2)*a+1/2/f/(a-b)^4*tan(f*x+e)/(1+tan(f*x+e)^2)*b+1/2/f
/(a-b)^4*arctan(tan(f*x+e))*a+5/2/f/(a-b)^4*arctan(tan(f*x+e))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.87567, size = 2402, normalized size = 12.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(16*(a^4 + 3*a^3*b - 9*a^2*b^2 + 5*a*b^3)*f*x*cos(f*x + e)^4 + 32*(a^
3*b + 4*a^2*b^2 - 5*a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 5*a*b^3)*f*x
- ((15*a^4 - 20*a^3*b - 6*a^2*b^2 + 12*a*b^3 - b^4)*cos(f*x + e)^4 + 15*a^2
*b^2 + 10*a*b^3 - b^4 + 2*(15*a^3*b - 5*a^2*b^2 - 11*a*b^3 + b^4)*cos(f*x +
 e)^2)*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)
*cos(f*x + e)^2 - 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b
/a)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)
*cos(f*x + e)^2 + b^2)) - 4*(4*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*cos(f*x
```

```
+ e)^5 + (17*a^3*b - 33*a^2*b^2 + 15*a*b^3 + b^4)*cos(f*x + e)^3 + (11*a^2*
b^2 - 10*a*b^3 - b^4)*cos(f*x + e))*sin(f*x + e))/((a^7 - 6*a^6*b + 15*a^5*
b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*f*cos(f*x + e)^4 + 2*(a^
6*b - 5*a^5*b^2 + 10*a^4*b^3 - 10*a^3*b^4 + 5*a^2*b^5 - a*b^6)*f*cos(f*x +
e)^2 + (a^5*b^2 - 4*a^4*b^3 + 6*a^3*b^4 - 4*a^2*b^5 + a*b^6)*f), 1/16*(8*(a
^4 + 3*a^3*b - 9*a^2*b^2 + 5*a*b^3)*f*x*cos(f*x + e)^4 + 16*(a^3*b + 4*a^2*
b^2 - 5*a*b^3)*f*x*cos(f*x + e)^2 + 8*(a^2*b^2 + 5*a*b^3)*f*x + ((15*a^4 -
20*a^3*b - 6*a^2*b^2 + 12*a*b^3 - b^4)*cos(f*x + e)^4 + 15*a^2*b^2 + 10*a*b
^3 - b^4 + 2*(15*a^3*b - 5*a^2*b^2 - 11*a*b^3 + b^4)*cos(f*x + e)^2)*sqrt(b
/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos(f*x + e)*sin(f
*x + e))) - 2*(4*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*cos(f*x + e)^5 + (17*a
^3*b - 33*a^2*b^2 + 15*a*b^3 + b^4)*cos(f*x + e)^3 + (11*a^2*b^2 - 10*a*b^3
 - b^4)*cos(f*x + e))*sin(f*x + e))/((a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b
^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*f*cos(f*x + e)^4 + 2*(a^6*b - 5*a^5*b^
2 + 10*a^4*b^3 - 10*a^3*b^4 + 5*a^2*b^5 - a*b^6)*f*cos(f*x + e)^2 + (a^5*b^
2 - 4*a^4*b^3 + 6*a^3*b^4 - 4*a^2*b^5 + a*b^6)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.61142, size = 381, normalized size = 1.97

$$\frac{\frac{4(fx+e)(a+5b)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}-\frac{(15a^2b+10ab^2-b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{(a^5-4a^4b+6a^3b^2-4a^2b^3+ab^4)\sqrt{ab}}-\frac{4\tan(fx+e)}{(a^3-3a^2b+3ab^2-b^3)\left(\tan(fx+e)^2+1\right)}-\frac{7ab^2\tan(fx+e)}{(a^4}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*(f*x + e)*(a + 5*b)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (1
5*a^2*b + 10*a*b^2 - b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*t
an(f*x + e)/sqrt(a*b)))/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*sq
rt(a*b)) - 4*tan(f*x + e)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(tan(f*x + e)^2
+ 1)) - (7*a*b^2*tan(f*x + e)^3 + b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x + e)
 - a*b^2*tan(f*x + e))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*(b*tan(f*x + e)
^2 + a)^2))/f
```

3.88 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=150

$$-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{5/2}f(a-b)^3}-\frac{b(7a-3b)\tan(e+fx)}{8a^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{b\tan(e+fx)}{4af(a-b)\left(a+b\tan^2(e+fx)\right)^2}+\frac{}{(}$$

```
[Out] x/(a - b)^3 - (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^3*f) - (b*Tan[e + f*x])/(4*a*(a - b)*f*(a
+ b*Tan[e + f*x]^2)^2) - ((7*a - 3*b)*b*Tan[e + f*x])/(8*a^2*(a - b)^2*f*(a
 + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.159388, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3661, 414, 527, 522, 203, 205}

$$-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{5/2}f(a-b)^3}-\frac{b(7a-3b)\tan(e+fx)}{8a^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{b\tan(e+fx)}{4af(a-b)\left(a+b\tan^2(e+fx)\right)^2}+\frac{}{(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-3),x]
```

```
[Out] x/(a - b)^3 - (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^3*f) - (b*Tan[e + f*x])/(4*a*(a - b)*f*(a
+ b*Tan[e + f*x]^2)^2) - ((7*a - 3*b)*b*Tan[e + f*x])/(8*a^2*(a - b)^2*f*(a
 + b*Tan[e + f*x]^2))
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-3b-3bx^2}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4a(a-b)f}$$

$$= -\frac{b\tan(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(7a-3b)b\tan(e+fx)}{8a^2(a-b)^2f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{8a^2}{\cdots}\right.}{\cdots}$$

$$= -\frac{b\tan(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(7a-3b)b\tan(e+fx)}{8a^2(a-b)^2f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x\cdots}\right.}{(\cdots}$$

$$= \frac{x}{(a-b)^3} - \frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3f} - \frac{b\tan(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx\cdots\right.}$$

Mathematica [A] time = 1.9056, size = 138, normalized size = 0.92

$$-\frac{\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}} + \frac{b(7a-3b)(a-b)\tan(e+fx)}{a^2\left(a+b\tan^2(e+fx)\right)} + \frac{2b(a-b)^2\tan(e+fx)}{a\left(a+b\tan^2(e+fx)\right)^2} - 8\tan^{-1}(\tan(e+fx))}{8f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-3),x]
```

```
[Out] -(-8*ArcTan[Tan[e + f*x]] + (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt
[b]*Tan[e + f*x])/Sqrt[a]])/a^(5/2) + (2*(a - b)^2*b*Tan[e + f*x])/(a*(a +
b*Tan[e + f*x]^2)^2) + ((7*a - 3*b)*(a - b)*b*Tan[e + f*x])/(a^2*(a + b*Tan
```

```
[e + f*x]^2)))/(8*(a - b)^3*f)
```

Maple [B] time = 0.024, size = 350, normalized size = 2.3

$$-\frac{7 b^{2} \left(\tan \left(f x+e\right)\right)^{3}}{8 f \left(a-b\right)^{3} \left(a+b \left(\tan \left(f x+e\right)\right)^{2}\right)^{2}}+\frac{5 b^{3} \left(\tan \left(f x+e\right)\right)^{3}}{4 f \left(a-b\right)^{3} \left(a+b \left(\tan \left(f x+e\right)\right)^{2}\right)^{2} a}-\frac{3 b^{4} \left(\tan \left(f x+e\right)\right)^{3}}{8 f \left(a-b\right)^{3} \left(a+b \left(\tan \left(f x+e\right)\right)^{2}\right)^{2} a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -7/8/f*b^2/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^3+5/4/f*b^3/(a-b)^3/(a+b
*tan(f*x+e)^2)^2/a*tan(f*x+e)^3-3/8/f*b^4/(a-b)^3/(a+b*tan(f*x+e)^2)^2/a^2*
tan(f*x+e)^3-9/8/f*b/(a-b)^3/(a+b*tan(f*x+e)^2)^2*a*tan(f*x+e)+7/4/f*b^2/(a
-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)-5/8/f*b^3/(a-b)^3/(a+b*tan(f*x+e)^2)^
2/a*tan(f*x+e)-15/8/f*b/(a-b)^3/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)
)+5/4/f*b^2/(a-b)^3/a/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-3/8/f*b^
3/(a-b)^3/a^2/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))+1/f/(a-b)^3*arct
an(tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.98648, size = 1643, normalized size = 10.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*a^2*b^2*f*x*tan(f*x + e)^4 + 64*a^3*b*f*x*tan(f*x + e)^2 + 32*a^4
*f*x - 4*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(f*x + e)^3 - ((15*a^2*b^2 - 10*
a*b^3 + 3*b^4)*tan(f*x + e)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^3*b
 - 10*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(-b/a)*log((b^2*tan(f*x + e)^4
 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*s
qrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(9*a^3*b
- 14*a^2*b^2 + 5*a*b^3)*tan(f*x + e))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a
^2*b^5)*f*tan(f*x + e)^4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*ta
n(f*x + e)^2 + (a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f), 1/16*(16*a^2*b^2*f
*x*tan(f*x + e)^4 + 32*a^3*b*f*x*tan(f*x + e)^2 + 16*a^4*f*x - 2*(7*a^2*b^2
 - 10*a*b^3 + 3*b^4)*tan(f*x + e)^3 - ((15*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(
f*x + e)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^3*b - 10*a^2*b^2 + 3*a
*b^3)*tan(f*x + e)^2)*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)
```

```
/(b*tan(f*x + e))) - 2*(9*a^3*b - 14*a^2*b^2 + 5*a*b^3)*tan(f*x + e))/((a^5
*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(f*x + e)^4 + 2*(a^6*b - 3*a^5
*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2 + (a^7 - 3*a^6*b + 3*a^5*b^2 -
 a^4*b^3)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29603, size = 288, normalized size = 1.92

$$-\frac{\frac{\left(15\,a^2b-10\,ab^2+3\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)}{\left(a^5-3\,a^4b+3\,a^3b^2-a^2b^3\right)\sqrt{ab}}-\frac{8\left(fx+e\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{7\,ab^2\tan\left(fx+e\right)^3-3\,b^3\tan\left(fx+e\right)^3+9\,a^2b\tan\left(fx+e\right)-5\,ab^2\,\mathrm{ta}}{\left(a^4-2\,a^3b+a^2b^2\right)\left(b\tan\left(fx+e\right)^2+a\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
arctan(b*tan(f*x + e)/sqrt(a*b)))/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sq
rt(a*b)) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (7*a*b^2*tan(f*x +
 e)^3 - 3*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x + e) - 5*a*b^2*tan(f*x + e))
/((a^4 - 2*a^3*b + a^2*b^2)*(b*tan(f*x + e)^2 + a)^2))/f
```

3.89 $\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=112

$$-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{7/2}f}+\frac{5\cot(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)}-\frac{15\cot(e+fx)}{8a^3f}+\frac{\cot(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^2}$$

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(8*a^(7/2)*f) - (15*Co
t[e + f*x])/(8*a^3*f) + Cot[e + f*x]/(4*a*f*(a + b*Tan[e + f*x]^2)^2) + (5*
Cot[e + f*x])/(8*a^2*f*(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.0877426, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3663, 290, 325, 205}

$$-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{7/2}f}+\frac{5\cot(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)}-\frac{15\cot(e+fx)}{8a^3f}+\frac{\cot(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(8*a^(7/2)*f) - (15*Co
t[e + f*x])/(8*a^3*f) + Cot[e + f*x]/(4*a*f*(a + b*Tan[e + f*x]^2)^2) + (5*
Cot[e + f*x])/(8*a^2*f*(a + b*Tan[e + f*x]^2))
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cot(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^2} + \frac{5\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= \frac{\cot(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^2} + \frac{5\cot(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)} + \frac{15\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)}\,dx, x, \mathrm{t}\right.}{8a^2f}$$

$$= -\frac{15\cot(e+fx)}{8a^3f} + \frac{\cot(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^2} + \frac{5\cot(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)} - \frac{(15b)\operatorname{Sub}}{}$$

$$= -\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{7/2}f} - \frac{15\cot(e+fx)}{8a^3f} + \frac{\cot(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^2} + \frac{5\,\mathrm{cc}}{8a^2f\left(a+\right.}$$

Mathematica [A] time = 0.826774, size = 144, normalized size = 1.29

$$\frac{\frac{4a^{3/2}b^2\sin(2(e+fx))}{(a-b)((a-b)\cos(2(e+fx))+a+b)^2} - 15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right) - \frac{\sqrt{a}b(9a-7b)\sin(2(e+fx))}{(a-b)((a-b)\cos(2(e+fx))+a+b)} - 8\sqrt{a}\cot(e+fx)}{8a^{7/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] - 8*Sqrt[a]*Cot[e + f*x
] + (4*a^(3/2)*b^2*Sin[2*(e + f*x)])/((a - b)*(a + b + (a - b)*Cos[2*(e + f
*x)])^2) - (Sqrt[a]*(9*a - 7*b)*b*Sin[2*(e + f*x)])/((a - b)*(a + b + (a -
b)*Cos[2*(e + f*x)])))/(8*a^(7/2)*f)
```

Maple [A] time = 0.086, size = 108, normalized size = 1.

$$-\frac{1}{fa^3\tan\left(fx+e\right)} - \frac{7b^2\left(\tan\left(fx+e\right)\right)^3}{8fa^3\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^2} - \frac{9b\tan\left(fx+e\right)}{8fa^2\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^2} - \frac{15b}{8fa^3}\arctan\left(b\tan\left(fx+e\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -1/f/a^3/tan(f*x+e)-7/8/f/a^3*b^2/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-9/8/f/a
^2*b/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)-15/8/f/a^3*b/(a*b)^(1/2)*arctan(b*tan(
f*x+e)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.34278, size = 1300, normalized size = 11.61

$$\left[-\frac{4\left(8a^2-25ab+15b^2\right)\cos\left(fx+e\right)^5+20\left(5ab-6b^2\right)\cos\left(fx+e\right)^3-15\left(\left(a^2-2ab+b^2\right)\cos\left(fx+e\right)^4+2\left(ab-b^2\right)\right.}{32\left(a^3b^2f+\left(a^5-2a^4b+a^3b^2\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(4*(8*a^2 - 25*a*b + 15*b^2)*cos(f*x + e)^5 + 20*(5*a*b - 6*b^2)*cos
(f*x + e)^3 - 15*((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*
x + e)^2 + b^2)*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a
*b + b^2)*cos(f*x + e)^2 + 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e)
)*sqrt(-b/a)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a
*b - b^2)*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*b^2*cos(f*x + e))/((a^3*
b^2*f + (a^5 - 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^4 + 2*(a^4*b - a^3*b^2)*f*
cos(f*x + e)^2)*sin(f*x + e)), -1/16*(2*(8*a^2 - 25*a*b + 15*b^2)*cos(f*x +
 e)^5 + 10*(5*a*b - 6*b^2)*cos(f*x + e)^3 - 15*((a^2 - 2*a*b + b^2)*cos(f*x
 + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2)*sqrt(b/a)*arctan(1/2*((a + b)
*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e)
+ 30*b^2*cos(f*x + e))/((a^3*b^2*f + (a^5 - 2*a^4*b + a^3*b^2)*f*cos(f*x +
e)^4 + 2*(a^4*b - a^3*b^2)*f*cos(f*x + e)^2)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.62092, size = 147, normalized size = 1.31

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)b}{\sqrt{ab}a^3}+\frac{7b^2\tan\left(fx+e\right)^3+9ab\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a\right)^2a^3}+\frac{8}{a^3\tan\left(fx+e\right)}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b)))*b/(sqrt(a*b)*a^3) + (7*b^2*tan(f*x + e)^3 + 9*a*b*tan(f*x + e))/((b*
tan(f*x + e)^2 + a)^2*a^3) + 8/(a^3*tan(f*x + e)))/f
```

3.90 $\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=154

$$-\frac{5\sqrt{b}(3a-7b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{9/2}f}-\frac{b(7a-11b)\tan(e+fx)}{8a^4f\left(a+b\tan^2(e+fx)\right)}-\frac{b(a-b)\tan(e+fx)}{4a^3f\left(a+b\tan^2(e+fx)\right)^2}-\frac{(a-3b)\cot(e+fx)}{a^4f}-\text{c}$$

```
[Out] (-5*(3*a - 7*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(8*a^(9/2)*
f) - ((a - 3*b)*Cot[e + f*x])/(a^4*f) - Cot[e + f*x]^3/(3*a^3*f) - ((a - b)
*b*Tan[e + f*x])/(4*a^3*f*(a + b*Tan[e + f*x]^2)^2) - ((7*a - 11*b)*b*Tan[e
 + f*x])/(8*a^4*f*(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.205548, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3663, 456, 1259, 1261, 205}

$$-\frac{5\sqrt{b}(3a-7b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{9/2}f}-\frac{b(7a-11b)\tan(e+fx)}{8a^4f\left(a+b\tan^2(e+fx)\right)}-\frac{b(a-b)\tan(e+fx)}{4a^3f\left(a+b\tan^2(e+fx)\right)^2}-\frac{(a-3b)\cot(e+fx)}{a^4f}-\text{c}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```



```
[Out] (-5*(3*a - 7*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(8*a^(9/2)*
f) - ((a - 3*b)*Cot[e + f*x])/(a^4*f) - Cot[e + f*x]^3/(3*a^3*f) - ((a - b)
*b*Tan[e + f*x])/(4*a^3*f*(a + b*Tan[e + f*x]^2)^2) - ((7*a - 11*b)*b*Tan[e
 + f*x])/(8*a^4*f*(a + b*Tan[e + f*x]^2))
```


Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
```

```
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\left(a+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a-b)b\tan(e+fx)}{4a^3f\left(a+b\tan^2(e+fx)\right)^2} - \frac{b\,\text{Subst}\left(\int \frac{-\frac{4}{ab}-\frac{4(a-b)x^2}{a^2b}+\frac{3(a-b)x^4}{a^3}}{x^4\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4f}$$

$$= -\frac{(a-b)b\tan(e+fx)}{4a^3f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(7a-11b)b\tan(e+fx)}{8a^4f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-8ab-8(a-2b)bx^2+(7\ldots}{x^4\left(a+bx^2\right)}\ldots\right)}{8a^{\ldots}}$$

$$= -\frac{(a-b)b\tan(e+fx)}{4a^3f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(7a-11b)b\tan(e+fx)}{8a^4f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int\left(-\frac{8b}{x^4}-\frac{8(a-3b)b}{ax^2}+\ldots\right)\ldots\right)}{8\ldots}$$

$$= -\frac{(a-3b)\cot(e+fx)}{a^4f} - \frac{\cot^3(e+fx)}{3a^3f} - \frac{(a-b)b\tan(e+fx)}{4a^3f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(7a-11b)b\tan\ldots}{8a^4f\left(a+b\tan^2\ldots\right.}$$

$$= -\frac{5(3a-7b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{9/2}f} - \frac{(a-3b)\cot(e+fx)}{a^4f} - \frac{\cot^3(e+fx)}{3a^3f} - \frac{(a-b)\ldots}{4a^3f\left(a+\ldots\right.}$$

Mathematica [A] time = 1.63158, size = 146, normalized size = 0.95

$$\frac{\sqrt{a}\left(-\frac{3b\sin(2(e+fx))\left(\left(9a^2-20ab+11b^2\right)\cos(2(e+fx))+9a^2-6ab-11b^2\right)}{((a-b)\cos(2(e+fx))+a+b)^2} - 8\cot(e+fx)\left(a\csc^2(e+fx)+2a-9b\right)\right)+15\sqrt{b}(7b-3a)\,\mathrm{t}\ldots}{24a^{9/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (15*Sqrt[b]*(-3*a + 7*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[a]*(
-8*Cot[e + f*x]*(2*a - 9*b + a*Csc[e + f*x]^2) - (3*b*(9*a^2 - 6*a*b - 11*b
^2 + (9*a^2 - 20*a*b + 11*b^2)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(a + b +
```

```
(a - b)*Cos[2*(e + f*x)])^2))/(24*a^(9/2)*f)
```

Maple [A] time = 0.099, size = 235, normalized size = 1.5

$$-\frac{1}{3\,fa^{3}\left(\tan\left(fx+e\right)\right)^{3}}-\frac{1}{fa^{3}\tan\left(fx+e\right)}+3\,\frac{b}{fa^{4}\tan\left(fx+e\right)}-\frac{7\,b^{2}\left(\tan\left(fx+e\right)\right)^{3}}{8\,fa^{3}\left(a+b\left(\tan\left(fx+e\right)\right)^{2}\right)^{2}}+\frac{11\,b^{3}\left(\tan\left(fx-\right.\right.}{8\,fa^{4}\left(a+b\left(\tan\left(f\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -1/3/f/a^3/tan(f*x+e)^3-1/f/a^3/tan(f*x+e)+3/f/a^4/tan(f*x+e)*b-7/8/f/a^3*b
^2/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^3+11/8/f/a^4*b^3/(a+b*tan(f*x+e)^2)^2*ta
n(f*x+e)^3-9/8/f/a^2*b/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)+13/8/f/a^3*b^2/(a+b*
tan(f*x+e)^2)^2*tan(f*x+e)-15/8/f/a^3*b/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*
b)^(1/2))+35/8/f/a^4*b^2/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.55194, size = 1999, normalized size = 12.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/96*(4*(16*a^3 - 131*a^2*b + 220*a*b^2 - 105*b^3)*cos(f*x + e)^7 - 4*(24
*a^3 - 206*a^2*b + 485*a*b^2 - 315*b^3)*cos(f*x + e)^5 - 20*(15*a^2*b - 62*
a*b^2 + 63*b^3)*cos(f*x + e)^3 + 15*((3*a^3 - 13*a^2*b + 17*a*b^2 - 7*b^3)*
cos(f*x + e)^6 - (3*a^3 - 19*a^2*b + 37*a*b^2 - 21*b^3)*cos(f*x + e)^4 - 3*
a*b^2 + 7*b^3 - (6*a^2*b - 23*a*b^2 + 21*b^3)*cos(f*x + e)^2)*sqrt(-b/a)*lo
g(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 - 4*
((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*x + e) + b
^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^
2))*sin(f*x + e) - 60*(3*a*b^2 - 7*b^3)*cos(f*x + e))/(((a^6 - 2*a^5*b + a^
4*b^2)*f*cos(f*x + e)^6 - a^4*b^2*f - (a^6 - 4*a^5*b + 3*a^4*b^2)*f*cos(f*x
 + e)^4 - (2*a^5*b - 3*a^4*b^2)*f*cos(f*x + e)^2)*sin(f*x + e)), -1/48*(2*(
16*a^3 - 131*a^2*b + 220*a*b^2 - 105*b^3)*cos(f*x + e)^7 - 2*(24*a^3 - 206*
a^2*b + 485*a*b^2 - 315*b^3)*cos(f*x + e)^5 - 10*(15*a^2*b - 62*a*b^2 + 63*
b^3)*cos(f*x + e)^3 - 15*((3*a^3 - 13*a^2*b + 17*a*b^2 - 7*b^3)*cos(f*x + e
)^6 - (3*a^3 - 19*a^2*b + 37*a*b^2 - 21*b^3)*cos(f*x + e)^4 - 3*a*b^2 + 7*b
^3 - (6*a^2*b - 23*a*b^2 + 21*b^3)*cos(f*x + e)^2)*sqrt(b/a)*arctan(1/2*((a
```

```
 + b)*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x
+ e) - 30*(3*a*b^2 - 7*b^3)*cos(f*x + e))/(((a^6 - 2*a^5*b + a^4*b^2)*f*cos
(f*x + e)^6 - a^4*b^2*f - (a^6 - 4*a^5*b + 3*a^4*b^2)*f*cos(f*x + e)^4 - (2
*a^5*b - 3*a^4*b^2)*f*cos(f*x + e)^2)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.624, size = 236, normalized size = 1.53

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)(3\,ab-7\,b^2)}{\sqrt{ab}a^4}+\frac{3\left(7\,ab^2\tan(fx+e)^3-11\,b^3\tan(fx+e)^3+9\,a^2b\tan(fx+e)-13\,ab^2\tan(fx+e)\right)}{\left(b\tan(fx+e)^2+a\right)^2a^4}+\frac{8\left(3\,a\tan(\ldots\right.}{\ldots}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt
(a*b)))*(3*a*b - 7*b^2)/(sqrt(a*b)*a^4) + 3*(7*a*b^2*tan(f*x + e)^3 - 11*b^
3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x + e) - 13*a*b^2*tan(f*x + e))/((b*tan(f*
x + e)^2 + a)^2*a^4) + 8*(3*a*tan(f*x + e)^2 - 9*b*tan(f*x + e)^2 + a)/(a^4
*tan(f*x + e)^3))/f
```

3.91 $\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=231

$$-\frac{\sqrt{b}\left(15a^2-70ab+63b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{11/2}f}-\frac{b\left(35a^2-110ab+99b^2\right)\tan(e+fx)}{40a^5f\left(a+b\tan^2(e+fx)\right)}-\frac{b\left(5a^2-10ab+9b^2\right)\tan(e+fx)}{20a^4f\left(a+b\tan^2(e+fx)\right)^2}$$

```
[Out] -(Sqrt[b]*(15*a^2 - 70*a*b + 63*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]]
)/(8*a^(11/2)*f) - ((5*a^2 - 30*a*b + 27*b^2)*Cot[e + f*x])/(5*a^5*f) - ((1
0*a - 9*b)*Cot[e + f*x]^3)/(15*a^4*f) - Cot[e + f*x]^5/(5*a*f*(a + b*Tan[e
+ f*x]^2)^2) - (b*(5*a^2 - 10*a*b + 9*b^2)*Tan[e + f*x])/(20*a^4*f*(a + b*T
an[e + f*x]^2)^2) - (b*(35*a^2 - 110*a*b + 99*b^2)*Tan[e + f*x])/(40*a^5*f*
(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.302911, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3663, 462, 456, 1259, 1261, 205}

$$-\frac{\sqrt{b}\left(15a^2-70ab+63b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{11/2}f}-\frac{b\left(35a^2-110ab+99b^2\right)\tan(e+fx)}{40a^5f\left(a+b\tan^2(e+fx)\right)}-\frac{b\left(5a^2-10ab+9b^2\right)\tan(e+fx)}{20a^4f\left(a+b\tan^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```


```
[Out] -(Sqrt[b]*(15*a^2 - 70*a*b + 63*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]]
)/(8*a^(11/2)*f) - ((5*a^2 - 30*a*b + 27*b^2)*Cot[e + f*x])/(5*a^5*f) - ((1
0*a - 9*b)*Cot[e + f*x]^3)/(15*a^4*f) - Cot[e + f*x]^5/(5*a*f*(a + b*Tan[e
+ f*x]^2)^2) - (b*(5*a^2 - 10*a*b + 9*b^2)*Tan[e + f*x])/(20*a^4*f*(a + b*T
an[e + f*x]^2)^2) - (b*(35*a^2 - 110*a*b + 99*b^2)*Tan[e + f*x])/(40*a^5*f*
(a + b*Tan[e + f*x]^2))
```


Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
```

```
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^6\left(a+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{10a-9b+5ax^2}{x^4\left(a+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{5af}$$

$$= -\frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)^2} - \frac{b\left(5a^2-10ab+9b^2\right)\tan(e+fx)}{20a^4f\left(a+b\tan^2(e+fx)\right)^2} - \frac{b\,\text{Subst}\left(\int \frac{4\left(\frac{9}{a}-\frac{10}{b}\right)+4\left(\frac{10}{a}\right.}{}\right.}{}$$

$$= -\frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)^2} - \frac{b\left(5a^2-10ab+9b^2\right)\tan(e+fx)}{20a^4f\left(a+b\tan^2(e+fx)\right)^2} - \frac{b\left(35a^2-110ab+99b^2\right.}{40a^5f\left(a+b\tan^2(\right.}$$

$$= -\frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)^2} - \frac{b\left(5a^2-10ab+9b^2\right)\tan(e+fx)}{20a^4f\left(a+b\tan^2(e+fx)\right)^2} - \frac{b\left(35a^2-110ab+99b^2\right.}{40a^5f\left(a+b\tan^2(\right.}$$

$$= -\frac{\left(5a^2-30ab+27b^2\right)\cot(e+fx)}{5a^5f} - \frac{(10a-9b)\cot^3(e+fx)}{15a^4f} - \frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)^2}$$

$$= -\frac{\sqrt{b}\left(15a^2-70ab+63b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{11/2}f} - \frac{\left(5a^2-30ab+27b^2\right)\cot(e+fx)}{5a^5f} - \frac{(10a}{}$$

Mathematica [A] time = 1.67383, size = 346, normalized size = 1.5

$$\frac{-960\sqrt{b}\left(15a^2-70ab+63b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right) - \frac{2\sqrt{a}\cot(e+fx)\csc^4(e+fx)\left(-8800a^2b^2\cos(6(e+fx))+2479a^2b^2\cos(8(e+fx))+4\left(-1400a^2b\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-960*Sqrt[b]*(15*a^2 - 70*a*b + 63*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt
[a]] - (2*Sqrt[a]*(1600*a^4 - 165*a^3*b + 637*a^2*b^2 - 28875*a*b^3 + 33075
*b^4 + 4*(416*a^4 - 447*a^3*b - 1400*a^2*b^2 + 13125*a*b^3 - 13230*b^4)*Cos
[2*(e + f*x)] - 4*(32*a^4 - 257*a^3*b - 2821*a^2*b^2 + 8925*a*b^3 - 6615*b^
4)*Cos[4*(e + f*x)] - 128*a^4*Cos[6*(e + f*x)] + 1788*a^3*b*Cos[6*(e + f*x)
] - 8800*a^2*b^2*Cos[6*(e + f*x)] + 14700*a*b^3*Cos[6*(e + f*x)] - 7560*b^4
*Cos[6*(e + f*x)] + 64*a^4*Cos[8*(e + f*x)] - 863*a^3*b*Cos[8*(e + f*x)] +
2479*a^2*b^2*Cos[8*(e + f*x)] - 2625*a*b^3*Cos[8*(e + f*x)] + 945*b^4*Cos[8
*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^4)/(a + b + (a - b)*Cos[2*(e + f*x)]
)^2)/(7680*a^(11/2)*f)
```

Maple [A] time = 0.102, size = 380, normalized size = 1.7

$$-\frac{1}{5fa^3\left(\tan\left(fx+e\right)\right)^5} - \frac{2}{3fa^3\left(\tan\left(fx+e\right)\right)^3} + \frac{b}{fa^4\left(\tan\left(fx+e\right)\right)^3} - \frac{1}{fa^3\tan\left(fx+e\right)} + 6\frac{b}{fa^4\tan\left(fx+e\right)} - 6\frac{}{fa^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -1/5/f/a^3/tan(f*x+e)^5-2/3/f/a^3/tan(f*x+e)^3+1/f/a^4/tan(f*x+e)^3*b-1/f/a
^3/tan(f*x+e)+6/f/a^4/tan(f*x+e)*b-6/f/a^5/tan(f*x+e)*b^2-7/8/f/a^3*b^2/(a+
b*tan(f*x+e)^2)^2*tan(f*x+e)^3+11/4/f/a^4*b^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+
e)^3-15/8/f*b^4/a^5/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-9/8/f/a^2*b/(a+b*tan(
f*x+e)^2)^2*tan(f*x+e)+13/4/f/a^3*b^2/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)-17/8/
f*b^3/a^4/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)-15/8/f/a^3*b/(a*b)^(1/2)*arctan(b
*tan(f*x+e)/(a*b)^(1/2))+35/4/f/a^4*b^2/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*
b)^(1/2))-63/8/f*b^3/a^5/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.7423, size = 2931, normalized size = 12.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/480*(4*(64*a^4 - 863*a^3*b + 2479*a^2*b^2 - 2625*a*b^3 + 945*b^4)*cos(f
*x + e)^9 - 4*(160*a^4 - 2173*a^3*b + 7158*a^2*b^2 - 8925*a*b^3 + 3780*b^4)
*cos(f*x + e)^7 + 4*(120*a^4 - 1685*a^3*b + 7104*a^2*b^2 - 11025*a*b^3 + 56
70*b^4)*cos(f*x + e)^5 + 20*(75*a^3*b - 530*a^2*b^2 + 1155*a*b^3 - 756*b^4)
*cos(f*x + e)^3 - 15*((15*a^4 - 100*a^3*b + 218*a^2*b^2 - 196*a*b^3 + 63*b^
4)*cos(f*x + e)^8 - 2*(15*a^4 - 115*a^3*b + 303*a^2*b^2 - 329*a*b^3 + 126*b
^4)*cos(f*x + e)^6 + (15*a^4 - 160*a^3*b + 573*a^2*b^2 - 798*a*b^3 + 378*b^
4)*cos(f*x + e)^4 + 15*a^2*b^2 - 70*a*b^3 + 63*b^4 + 2*(15*a^3*b - 100*a^2*
b^2 + 203*a*b^3 - 126*b^4)*cos(f*x + e)^2)*sqrt(-b/a)*log(((a^2 + 6*a*b + b
^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 + 4*((a^2 + a*b)*cos(f*
x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*x + e) + b^2)/((a^2 - 2*a*b +
 b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2))*sin(f*x + e) +
60*(15*a^2*b^2 - 70*a*b^3 + 63*b^4)*cos(f*x + e))/(((a^7 - 2*a^6*b + a^5*b^
2)*f*cos(f*x + e)^8 + a^5*b^2*f - 2*(a^7 - 3*a^6*b + 2*a^5*b^2)*f*cos(f*x +
 e)^6 + (a^7 - 6*a^6*b + 6*a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b - 2*a^5*b^2
)*f*cos(f*x + e)^2)*sin(f*x + e)), -1/240*(2*(64*a^4 - 863*a^3*b + 2479*a^2
*b^2 - 2625*a*b^3 + 945*b^4)*cos(f*x + e)^9 - 2*(160*a^4 - 2173*a^3*b + 715
8*a^2*b^2 - 8925*a*b^3 + 3780*b^4)*cos(f*x + e)^7 + 2*(120*a^4 - 1685*a^3*b
 + 7104*a^2*b^2 - 11025*a*b^3 + 5670*b^4)*cos(f*x + e)^5 + 10*(75*a^3*b - 5
30*a^2*b^2 + 1155*a*b^3 - 756*b^4)*cos(f*x + e)^3 - 15*((15*a^4 - 100*a^3*b
 + 218*a^2*b^2 - 196*a*b^3 + 63*b^4)*cos(f*x + e)^8 - 2*(15*a^4 - 115*a^3*b
 + 303*a^2*b^2 - 329*a*b^3 + 126*b^4)*cos(f*x + e)^6 + (15*a^4 - 160*a^3*b
+ 573*a^2*b^2 - 798*a*b^3 + 378*b^4)*cos(f*x + e)^4 + 15*a^2*b^2 - 70*a*b^3
```

```
 + 63*b^4 + 2*(15*a^3*b - 100*a^2*b^2 + 203*a*b^3 - 126*b^4)*cos(f*x + e)^2
)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos(f*x +
e)*sin(f*x + e)))*sin(f*x + e) + 30*(15*a^2*b^2 - 70*a*b^3 + 63*b^4)*cos(f*
x + e))/(((a^7 - 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^8 + a^5*b^2*f - 2*(a^7 -
 3*a^6*b + 2*a^5*b^2)*f*cos(f*x + e)^6 + (a^7 - 6*a^6*b + 6*a^5*b^2)*f*cos(
f*x + e)^4 + 2*(a^6*b - 2*a^5*b^2)*f*cos(f*x + e)^2)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.62251, size = 355, normalized size = 1.54

$$-\frac{\frac{15\left(15\,a^2b-70\,ab^2+63\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)}{\sqrt{ab}a^5}+\frac{15\left(7\,a^2b^2\tan\left(fx+e\right)^3-22\,ab^3\tan\left(fx+e\right)^3+15\,b^4\tan\left(fx+e\right)^3+9\,a^3b\tan\left(fx+e\right)-26\,a^2 \cdots\right.}{\left(b\tan\left(fx+e\right)^2+a\right)^2a^5} \cdots}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/120*(15*(15*a^2*b - 70*a*b^2 + 63*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn
(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))/(sqrt(a*b)*a^5) + 15*(7*a^2*b^2*tan
(f*x + e)^3 - 22*a*b^3*tan(f*x + e)^3 + 15*b^4*tan(f*x + e)^3 + 9*a^3*b*tan
(f*x + e) - 26*a^2*b^2*tan(f*x + e) + 17*a*b^3*tan(f*x + e))/((b*tan(f*x +
e)^2 + a)^2*a^5) + 8*(15*a^2*tan(f*x + e)^4 - 90*a*b*tan(f*x + e)^4 + 90*b^
2*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 - 15*a*b*tan(f*x + e)^2 + 3*a^2)/(
a^5*tan(f*x + e)^5))/f
```

3.92 $\int \sin^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=161

$$-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{5f(a-b)}+\frac{2(5a-4b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{15f(a-b)^2}-\frac{\cos(e+fx)\sqrt{a+b\,\mathrm{s}}}{f}$$

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/f
- (Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/f + (2*(5*a - 4*b)*Cos[e +
f*x]^3*(a - b + b*Sec[e + f*x]^2)^(3/2))/(15*(a - b)^2*f) - (Cos[e + f*x]^5
*(a - b + b*Sec[e + f*x]^2)^(3/2))/(5*(a - b)*f)
```

Rubi [A] time = 0.165315, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3664, 462, 451, 277, 217, 206}

$$-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{5f(a-b)}+\frac{2(5a-4b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{15f(a-b)^2}-\frac{\cos(e+fx)\sqrt{a+b\,\mathrm{s}}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```


```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/f
- (Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/f + (2*(5*a - 4*b)*Cos[e +
f*x]^3*(a - b + b*Sec[e + f*x]^2)^(3/2))/(15*(a - b)^2*f) - (Cos[e + f*x]^5
*(a - b + b*Sec[e + f*x]^2)^(3/2))/(5*(a - b)*f)
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sin^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\sqrt{a-b+bx^2}}{x^6}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{5(a-b)f} + \frac{\text{Subst}\left(\int \frac{\left(-2(5a-4b)+5(a-b)x^2\right)\sqrt{a-b+bx}}{x^4}\right.}{5(a-b)f}$$

$$= \frac{2(5a-4b)\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{15(a-b)^2f} - \frac{\cos^5(e+fx)\left(a-b+b\sec\right.}{5(a-b)f}$$

$$= -\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} + \frac{2(5a-4b)\cos^3(e+fx)\left(a-b+b\sec^2\right.}{15(a-b)^2f}$$

$$= -\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} + \frac{2(5a-4b)\cos^3(e+fx)\left(a-b+b\sec^2\right.}{15(a-b)^2f}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} + \frac{2(5a-4\ldots}{}$$

Mathematica [A] time = 3.25176, size = 208, normalized size = 1.29

$$\frac{\cos(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\sqrt{(a-b)\cos(2(e+fx))+a+b}\left(4\left(7a^2-15ab+8b^2\right)\cos(2(e\ldots\right.\right.}{120\sqrt{2}f(a-b)^2\sqrt{(a-b)\cos(\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*(120*Sqrt[2]*(a - b)^2*Sqrt[b]*ArcTanh[Sqrt[a + b + (a - b)*C
os[2*(e + f*x)]]/(Sqrt[2]*Sqrt[b])] + Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]
]*(-89*a^2 + 254*a*b - 149*b^2 + 4*(7*a^2 - 15*a*b + 8*b^2)*Cos[2*(e + f*x)
] - 3*(a - b)^2*Cos[4*(e + f*x)]))*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*
Sec[e + f*x]^2])/(120*Sqrt[2]*(a - b)^2*f*Sqrt[a + b + (a - b)*Cos[2*(e + f
```

```
*x)]])
```

Maple [B] time = 0.752, size = 7044, normalized size = 43.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.36526, size = 941, normalized size = 5.84

$$\left[\frac{15\left(a^2-2\,ab+b^2\right)\sqrt{b}\log\left(-\frac{(a-b)\cos\left(fx+e\right)^2+2\sqrt{b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+2b}{\cos\left(fx+e\right)^2}\right)-2\left(3\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^5-(1\right.}{30\left(a^2-2\,ab+b^2\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(a^2 - 2*a*b + b^2)*sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt
(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/c
os(f*x + e)^2) - 2*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - (10*a^2 - 21*a*b
 + 11*b^2)*cos(f*x + e)^3 + (15*a^2 - 40*a*b + 23*b^2)*cos(f*x + e))*sqrt((
(a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^2 - 2*a*b + b^2)*f), -1/15
*(15*(a^2 - 2*a*b + b^2)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + (3*(a^2 - 2*a*b + b^2)*cos(f*x +
 e)^5 - (10*a^2 - 21*a*b + 11*b^2)*cos(f*x + e)^3 + (15*a^2 - 40*a*b + 23*b
^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^2
- 2*a*b + b^2)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx+e\right)^2+a}\sin\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*sin(f*x + e)^5, x)
```

3.93 $\int \sin^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=113

$$\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{3f(a-b)}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}$$

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/f
- (Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/f + (Cos[e + f*x]^3*(a - b
+ b*Sec[e + f*x]^2)^(3/2))/(3*(a - b)*f)
```

Rubi [A] time = 0.104813, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3664, 451, 277, 217, 206}

$$\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{3f(a-b)}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/f
- (Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/f + (Cos[e + f*x]^3*(a - b
+ b*Sec[e + f*x]^2)^(3/2))/(3*(a - b)*f)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sin^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)\sqrt{a-b+bx^2}}{x^4}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{3(a-b)f} + \frac{\text{Subst}\left(\int \frac{\sqrt{a-b+bx^2}}{x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} + \frac{\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{3(a-b)f}$$

$$= -\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} + \frac{\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{3(a-b)f}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} + \frac{\cos^3(e+}{}$$

Mathematica [A] time = 1.05357, size = 170, normalized size = 1.5

$$\frac{\cos(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\sqrt{(a-b)\cos(2(e+fx))+a+b}((a-b)\cos(2(e+fx))-5a+7\right.}{6\sqrt{2}f(a-b)\sqrt{(a-b)\cos(2(e+fx))+a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*(6*Sqrt[2]*(a - b)*Sqrt[b]*ArcTanh[Sqrt[a + b + (a - b)*Cos[2
*(e + f*x)]]/(Sqrt[2]*Sqrt[b])] + Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]*(-
5*a + 7*b + (a - b)*Cos[2*(e + f*x)]))*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x
)])*Sec[e + f*x]^2])/(6*Sqrt[2]*(a - b)*f*Sqrt[a + b + (a - b)*Cos[2*(e + f
*x)]])
```

Maple [B] time = 0.243, size = 4296, normalized size = 38.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/6/f/b^(1/2)/(a-b)^3/a^(1/2)*(cos(f*x+e)-1)^2*(3*ln(-4/a^(1/2)*(cos(f*x+e)
-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*
```

```
a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f
*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*4^(1/2)*b^(9/2)-3*ln(-2/a^(1/2)*
(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1
)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*4^(1/2)*b^(9/2)-4*((a
*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)*a^(5/2)*b^(1/2)+8*(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)*a^(3/2)*b^(3/2)-4
*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)*a^(1/2)*b^(5/2)
+6*cos(f*x+e)^2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)*
a^(1/2)*b^(5/2)-4*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)
+1)^2)^(3/2)*a^(5/2)*b^(1/2)+8*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b
)/(cos(f*x+e)+1)^2)^(3/2)*a^(3/2)*b^(3/2)-4*cos(f*x+e)*((a*cos(f*x+e)^2-cos
(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)*a^(1/2)*b^(5/2)+6*((a*cos(f*x+e)^2-c
os(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*4^(1/2)*b^(7/2)-9*ln(-4/a^
(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*
x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*4^(1/2)*b^(7/2)
*a+9*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*4^
(1/2)*b^(7/2)*a+3*arctanh(1/8*b^(1/2)*4^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*4^
(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2))*a^(7/2)*4^(1/2)*b+9*ln(-4/a^(1/2)*(cos(f*x+e)-1
)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^
(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x
+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*4^(1/2)*b^(5/2)*a^2-9*ln(-2/a^(1/2
)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)
+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^
2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*4^(1/2)*b^(5/2)*a^2
-9*arctanh(1/8*b^(1/2)*4^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x
+e)-4^(1/2)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+
1)^2)^(1/2))*a^(5/2)*4^(1/2)*b^2-3*ln(-4/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)
*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x
+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/
2)*a^(1/2)+b)/sin(f*x+e)^2)*4^(1/2)*b^(3/2)*a^3+3*ln(-2/a^(1/2)*(cos(f*x+e)
-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*
a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f
*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*4^(1/2)*b^(3/2)*a^3+9*arctanh(1/
8*b^(1/2)*4^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2
)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*
a^(3/2)*4^(1/2)*b^3-3*arctanh(1/8*b^(1/2)*4^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e
)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^
2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*a^(1/2)*4^(1/2)*b^4-3*((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(7/2)*4^(1/2)*b^(1/2)+12*((a*cos(f*
x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(5/2)*4^(1/2)*b^(3/2)-15
*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(3/2)*4^(1/2)
*b^(5/2)+2*cos(f*x+e)^4*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2
)^(3/2)*a^(5/2)*b^(1/2)-4*cos(f*x+e)^4*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(
cos(f*x+e)+1)^2)^(3/2)*a^(3/2)*b^(3/2)+2*cos(f*x+e)^4*((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)*a^(1/2)*b^(5/2)+8*cos(f*x+e)^3*((a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)*a^(5/2)*b^(1/2)-16*cos
(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)*a^(3/2
)*b^(3/2)+8*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^
2)^(3/2)*a^(1/2)*b^(5/2)+3*cos(f*x+e)*ln(-4/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x
+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(
f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^
(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*4^(1/2)*b^(9/2)-3*cos(f*x+e)*ln(-2/a^(1/2)*(
cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)
^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b
```

```
+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*4^(1/2)*b^(9/2)+6*cos(
f*x+e)^2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)*a^(5/2)
*b^(1/2)-12*cos(f*x+e)^2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^
2)^(3/2)*a^(3/2)*b^(3/2)+6*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(c
os(f*x+e)+1)^2)^(1/2)*a^(1/2)*4^(1/2)*b^(7/2)-9*cos(f*x+e)*ln(-4/a^(1/2)*(c
os(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^
2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*4^(1/2)*b^(7/2)*a+9*cos
(f*x+e)*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e
)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(
f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)
*4^(1/2)*b^(7/2)*a+3*cos(f*x+e)*arctanh(1/8*b^(1/2)*4^(1/2)*(cos(f*x+e)-1)*
(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-c
os(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*a^(7/2)*4^(1/2)*b+9*cos(f*x+e)*ln
(-4/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(
cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-c
os(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*4^(1/2)*b
^(5/2)*a^2-9*cos(f*x+e)*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*
x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos
(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+
b)/sin(f*x+e)^2)*4^(1/2)*b^(5/2)*a^2-9*cos(f*x+e)*arctanh(1/8*b^(1/2)*4^(1/
2)*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*a^(5/2)*4^(1/2)
*b^2-3*cos(f*x+e)*ln(-4/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e
)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin
(f*x+e)^2)*4^(1/2)*b^(3/2)*a^3+3*cos(f*x+e)*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(c
os(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2
)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+
1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*4^(1/2)*b^(3/2)*a^3+9*cos(f*x+e)*arcta
nh(1/8*b^(1/2)*4^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1
/2)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1
/2))*a^(3/2)*4^(1/2)*b^3-3*cos(f*x+e)*arctanh(1/8*b^(1/2)*4^(1/2)*(cos(f*x+
e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((a*cos(f*x+
e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*a^(1/2)*4^(1/2)*b^4-6*cos(f
*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(7/2)*4^
(1/2)*b^(1/2)+18*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+
1)^2)^(1/2)*a^(5/2)*4^(1/2)*b^(3/2)-18*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+
e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(3/2)*4^(1/2)*b^(5/2))*cos(f*x+e)*((a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/((a*cos(f*x+e)^2-cos(f*x+
e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)/sin(f*x+e)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 3.45313, size = 674, normalized size = 5.96

$$\left[\frac{3\,(a-b)\sqrt{b}\log\left(-\frac{(a-b)\cos\left(fx+e\right)^2+2\,\sqrt{b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+2\,b}{\cos\left(fx+e\right)^2}\right)+2\left((a-b)\cos\left(fx+e\right)^3-(3\,a-4\,b)\cos\left(fx+e\right)\right)}{6\,(a-b)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a - b)*sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a -
 b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2)
 + 2*((a - b)*cos(f*x + e)^3 - (3*a - 4*b)*cos(f*x + e))*sqrt(((a - b)*cos(
f*x + e)^2 + b)/cos(f*x + e)^2))/((a - b)*f), -1/3*(3*(a - b)*sqrt(-b)*arct
an(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/
b) - ((a - b)*cos(f*x + e)^3 - (3*a - 4*b)*cos(f*x + e))*sqrt(((a - b)*cos(
f*x + e)^2 + b)/cos(f*x + e)^2))/((a - b)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.65597, size = 410, normalized size = 3.63

$$\frac{1}{3}\left(\frac{\left(6\,ab\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)-6\,b^2\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+3\,a\sqrt{-b}\sqrt{b}-4\,\sqrt{-b}b^{\frac{3}{2}}\right)\mathrm{sgn}\left(f\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{a\sqrt{-b}f^2-\sqrt{-b}bf^2}-\sqrt{a\cos\left(fx+e\right)^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((6*a*b*arctan(sqrt(b)/sqrt(-b)) - 6*b^2*arctan(sqrt(b)/sqrt(-b)) + 3*a
*sqrt(-b)*sqrt(b) - 4*sqrt(-b)*b^(3/2))*sgn(f)*sgn(cos(f*x + e))/(a*sqrt(-b
)*f^2 - sqrt(-b)*b*f^2) - sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)*((3
*a*sgn(f)*sgn(cos(f*x + e)) - 4*b*sgn(f)*sgn(cos(f*x + e)))/(a*f^2 - b*f^2)
 - (a*f^2*sgn(f)*sgn(cos(f*x + e)) - b*f^2*sgn(f)*sgn(cos(f*x + e)))*cos(f*
x + e)^2/((a*f^2 - b*f^2)*f^2)) - 6*b*arctan((sqrt(a*cos(f*x + e)^2 - b*cos
(f*x + e)^2 + b) + sqrt(a*f^2 - b*f^2)*cos(f*x + e)/f)/sqrt(-b))*sgn(f)*sgn
(cos(f*x + e))/(sqrt(-b)*f^2))*abs(f)
```

3.94 $\int \sin(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f}$$

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/f
- (Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/f
```

Rubi [A] time = 0.0573515, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3664, 277, 217, 206}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/f
- (Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/f
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sin(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a-b+bx^2}}{x^2}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= -\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx\right.}{f}$$
$$= -\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+f}}\right.}{f}$$
$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f}$$

Mathematica [A] time = 0.544072, size = 140, normalized size = 1.94

$$-\frac{\sin(2(e+fx))\csc(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\sqrt{2}\sqrt{(a-b)\cos(2(e+fx))+a+b}-2\sqrt{b}\,\text{t}\right.}{4f\sqrt{(a-b)\cos(2(e+fx))+a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((-2*Sqrt[b]*ArcTanh[Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]/(Sqrt[2]*Sqrt[
b])] + Sqrt[2]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])*Csc[e + f*x]*Sqrt[(a
 + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*Sin[2*(e + f*x)])/(4*f*Sqr
t[a + b + (a - b)*Cos[2*(e + f*x)]])
```

Maple [B] time = 0.069, size = 144, normalized size = 2.

$$\frac{\cos(fx+e)}{f}\sqrt{\frac{a(\cos(fx+e))^2-(\cos(fx+e))^2b+b}{(\cos(fx+e))^2}}\left(\sqrt{b}\ln\left(2\frac{\sqrt{b}\sqrt{a(\cos(fx+e))^2-(\cos(fx+e))^2b+b}+b}{\cos(fx+e)}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)*(b^(1
/2)*ln(2*(b^(1/2)*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(1/2)+b)/cos(f*x+e))-(a
*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(1/2))/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(1
/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.39196, size = 510, normalized size = 7.08

$$\left[-\frac{2\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)-\sqrt{b}\log\left(-\frac{(a-b)\cos\left(fx+e\right)^2+2\sqrt{b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+2b}{\cos\left(fx+e\right)^2}\right)}{2f},-\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{(a-b)\cdots}{\cdots}}}{\cdots}\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) - s
qrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, -(sqrt(-b)*a
rctan(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e)/b) + sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\tan^2\left(e+fx\right)}\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*sin(e + f*x), x)
```

Giac [B] time = 1.61938, size = 176, normalized size = 2.44

$$-\left(\frac{\left(\frac{b\arctan\left(\frac{\sqrt{a\cos\left(fx+e\right)^2-b\cos\left(fx+e\right)^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+\sqrt{a\cos\left(fx+e\right)^2-b\cos\left(fx+e\right)^2+b}\right)\mathrm{sgn}\left(f\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{f^2}-\frac{\left(b\arctan\left(\frac{\sqrt{b\cdots}}{\sqrt{-\cdots}}\right)\cdots\right.}{\cdots}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -((b*arctan(sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)/sqrt(-b))/sqrt(-b
) + sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b))*sgn(f)*sgn(cos(f*x + e))
/f^2 - (b*arctan(sqrt(b)/sqrt(-b)) + sqrt(-b)*sqrt(b))*sgn(f)*sgn(cos(f*x +
 e))/(sqrt(-b)*f^2))*abs(f)
```

3.95 $\int \csc(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}$$

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/
f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]
])/f
```

Rubi [A] time = 0.0873346, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3664, 402, 217, 206, 377, 207}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/
f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]
])/f
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a-b+bx^2}}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a-b+bx^2}}\,dx, x, \text{se}\right.}{f}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1}{-1+ax^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sec(e+}{\sqrt{a-b+b\sec}}\right.}{f}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f}$$

Mathematica [B] time = 7.38287, size = 295, normalized size = 3.51

$$\frac{\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\left(\tan^2\left(\frac{1}{2}(e+fx)\right)+1\right)}{\sqrt{a\left(\tan^2\left(\frac{1}{2}(e+fx)\right)-1\right)^2+4b\tan^2\left(\frac{1}{2}\right.}}\right.\right.}{2f\sqrt{\sec^4\left(\frac{1}{2}(e+fx)\right)((a-b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((2*Sqrt[b]*ArcTanh[(Sqrt[b]*(1 + Tan[(e + f*x)/2]^2))/Sqrt[4*b*Tan[(e + f*
x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]] - Sqrt[a]*(ArcTanh[(a - (a - 2*b)
*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e
+ f*x)/2]^2)^2])] + ArcTanh[(2*b + a*(-1 + Tan[(e + f*x)/2]^2))/(Sqrt[a]*Sq
rt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])]))*Cos[e + f*x]
*Sec[(e + f*x)/2]^2*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]
)/(2*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4])
```

Maple [B] time = 0.251, size = 719, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/4/f/a^(1/2)/b^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2
)*4^(1/2)*cos(f*x+e)*(cos(f*x+e)-1)*(2*b^(3/2)*ln(-4/a^(1/2)*(cos(f*x+e)-1)
*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(
1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+
e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)-2*arctanh(1/8*b^(1/2)*4^(1/2)*(cos(
f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((a*cos(
f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*a^(1/2)*b-2*ln(-2/a^(1/
2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e
)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)
^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*b^(3/2)+a*ln(-4*(c
os(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2
)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+
1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*b^(1/2)+ln(-2/a^(1/2)*(cos(f*x+e)-1)
*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(
1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+
e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*a*b^(1/2))/sin(f*x+e)^2/((a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \tan\left(fx+e\right)^{2} + a} \csc\left(fx+e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e), x)
```

Fricas [A] time = 2.86341, size = 1319, normalized size = 15.7

$$\left[\frac{\sqrt{a}\log\left(-\frac{2\left((a-b)\cos\left(fx+e\right)^{2}-2\sqrt{a}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)+a+b\right)}{\cos\left(fx+e\right)^{2}-1}\right)+\sqrt{b}\log\left(-\frac{(a-b)\cos\left(fx+e\right)^{2}+2\sqrt{b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)+2b}{\cos\left(fx+e\right)^{2}}\right)}{2f},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1))
 + sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, -1/2*(2*
sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
cos(f*x + e)/b) - sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt((
(a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x
+ e)^2 - 1)))/f, 1/2*(2*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) + sqrt(b)*log(-((a - b)*cos(f*x + e
)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
```

```
e) + 2*b)/cos(f*x + e)^2))/f, (sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) - sqrt(-b)*arctan(sqrt(-b)*
sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\tan^2\left(e+fx\right)}\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*csc(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx+e\right)^2+a}\csc\left(fx+e\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e), x)
```

3.96 $\int \csc^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=127

$$-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2\sqrt{a}f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2f}$$

```
[Out] -((a + b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/(
2*Sqrt[a]*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e
 + f*x]^2]])/f - (Cot[e + f*x]*Csc[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])
/(2*f)
```

Rubi [A] time = 0.139415, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3664, 467, 523, 217, 206, 377, 207}

$$-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2\sqrt{a}f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((a + b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/(
2*Sqrt[a]*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e
 + f*x]^2]])/f - (Cot[e + f*x]*Csc[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])
/(2*f)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a-b+bx^2}}{\left(-1+x^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{a-b+2bx^2}{\left(-1+x^2\right)\sqrt{a-b+bx}}\right.}{2f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a-b+bx^2}}\,dx,\right.}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x,\right.}{f}$$

$$= -\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2\sqrt{a}f} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} - \cot(e$$

Mathematica [B] time = 3.45201, size = 460, normalized size = 3.62

$$-\frac{\cot(e+fx)\csc(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\sqrt{2}\sqrt{a}\sqrt{\sec^4\left(\frac{1}{2}(e+fx)\right)((a-b)\cos(2(e+f\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(Cot[e + f*x]*Csc[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e +
 f*x]^2]*(Sqrt[2]*Sqrt[a]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e +
f*x)/2]^4] - 16*Sqrt[a]*Sqrt[b]*ArcTanh[(Sqrt[b]*(1 + Tan[(e + f*x)/2]^2))/
Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]]*Sin[(e + f*x)
```

```
/2]^2 + 4*(a + b)*ArcTanh[(a - (a - 2*b)*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[
4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])]*Sin[(e + f*x)/2]^
2 + 4*a*ArcTanh[(2*b + a*(-1 + Tan[(e + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(
e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])]*Sin[(e + f*x)/2]^2 + 4*b*A
rcTanh[(2*b + a*(-1 + Tan[(e + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/
2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])]*Sin[(e + f*x)/2]^2))/(4*Sqrt[a]*f*S
qrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4])
```

Maple [B] time = 0.31, size = 2075, normalized size = 16.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/8/f/a^(3/2)/b^(1/2)*(cos(f*x+e)-1)*(4*cos(f*x+e)^2*a^(3/2)*4^(1/2)*b^(1/
2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-4*cos(f*x+e)^
2*a^(3/2)*4^(1/2)*arctanh(1/8*b^(1/2)*4^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*4^
(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2))*b-4*cos(f*x+e)^2*a^(1/2)*4^(1/2)*b^(3/2)*((a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-3*cos(f*x+e)^2*4^(1/2)
*b^(3/2)*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+
e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos
(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2
)*a+cos(f*x+e)^2*4^(1/2)*b^(3/2)*ln(-4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)
-1))*a+4*cos(f*x+e)^2*4^(1/2)*b^(3/2)*ln(-4/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x
+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(
f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^
(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*a-8*cos(f*x+e)^2*a^(1/2)*b^(1/2)*((a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)+cos(f*x+e)^2*4^(1/2)*b^(1/2
)*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)
^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*a^2+c
os(f*x+e)^2*4^(1/2)*b^(1/2)*ln(-4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*
a^2-2*cos(f*x+e)*a^(3/2)*4^(1/2)*b^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)
/(cos(f*x+e)+1)^2)^(1/2)-16*cos(f*x+e)*a^(1/2)*b^(1/2)*((a*cos(f*x+e)^2-cos
(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)-2*a^(3/2)*4^(1/2)*b^(1/2)*((a*cos(f*
x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*a^(3/2)*4^(1/2)*arctanh(
1/8*b^(1/2)*4^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)
-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)
)*b+4*a^(1/2)*4^(1/2)*b^(3/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e
)+1)^2)^(1/2)+3*4^(1/2)*b^(3/2)*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)
*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*
a^(1/2)+b)/sin(f*x+e)^2)*a-4^(1/2)*b^(3/2)*ln(-4*(cos(f*x+e)*((a*cos(f*x+e)
^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x
+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(
cos(f*x+e)-1))*a-4*4^(1/2)*b^(3/2)*ln(-4/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)
*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x
+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/
2)*a^(1/2)+b)/sin(f*x+e)^2)*a-8*a^(1/2)*b^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)
^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)-4^(1/2)*b^(1/2)*ln(-2/a^(1/2)*(cos(f*x+e)-1
```

```
)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^
(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x
+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*a^2-4^(1/2)*b^(1/2)*ln(-4*(cos(f*x
+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(
f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^
(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*a^2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e
)^2*b+b)/cos(f*x+e)^2)^(1/2)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)
+1)^2)^(1/2)/sin(f*x+e)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx+e\right)^{2}+a}\csc\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e)^3, x)
```

Fricas [A] time = 6.42559, size = 2132, normalized size = 16.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
((a + b)*cos(f*x + e)^2 - a - b)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2
*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a
 + b)/(cos(f*x + e)^2 - 1)) + 2*(a*cos(f*x + e)^2 - a)*sqrt(b)*log(-((a - b
)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/(a*f*cos(f*x + e)^2 - a*f), 1/2*((
(a + b)*cos(f*x + e)^2 - a - b)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) + a*sqrt(((a - b)*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + (a*cos(f*x + e)^2 - a)*sqrt(b)*lo
g(-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/(a*f*cos(f*x + e)^2 - a*
f), -1/4*(4*(a*cos(f*x + e)^2 - a)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) - 2*a*sqrt(((a - b)*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) - ((a + b)*cos(f*x + e)^2 - a
- b)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)))
/(a*f*cos(f*x + e)^2 - a*f), 1/2*(((a + b)*cos(f*x + e)^2 - a - b)*sqrt(-a)
*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e)/a) - 2*(a*cos(f*x + e)^2 - a)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + a*sqrt(((a - b)*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/(a*f*cos(f*x + e)^2 - a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\tan^{2}\left(e+fx\right)}\csc^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*csc(e + f*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \tan \left(fx+e\right)^2 + a} \csc \left(fx+e\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e)^3, x)
```

3.97 $\int \csc^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=187

$$-\frac{\left(3a^2+6ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8a^{3/2}f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}-\frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a+b\sec^2(e}{4f}$$

```
[Out] -((3*a^2 + 6*a*b - b^2)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e
 + f*x]^2]])/(8*a^(3/2)*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a
 - b + b*Sec[e + f*x]^2]])/f - ((3*a + b)*Cot[e + f*x]*Csc[e + f*x]*Sqrt[a
- b + b*Sec[e + f*x]^2])/(8*a*f) - (Cot[e + f*x]*Csc[e + f*x]^3*Sqrt[a - b
+ b*Sec[e + f*x]^2])/(4*f)
```

Rubi [A] time = 0.232347, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3664, 467, 578, 523, 217, 206, 377, 207}

$$-\frac{\left(3a^2+6ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8a^{3/2}f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}-\frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a+b\sec^2(e}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((3*a^2 + 6*a*b - b^2)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e
 + f*x]^2]])/(8*a^(3/2)*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a
 - b + b*Sec[e + f*x]^2]])/f - ((3*a + b)*Cot[e + f*x]*Csc[e + f*x]*Sqrt[a
- b + b*Sec[e + f*x]^2])/(8*a*f) - (Cot[e + f*x]*Csc[e + f*x]^3*Sqrt[a - b
+ b*Sec[e + f*x]^2])/(4*f)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a-b+bx^2}}{\left(-1+x^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2\left(3(a-b)+4bx^2\right)}{\left(-1+x^2\right)^2\sqrt{a-b+bx^2}}\right.}{4f}$$

$$= -\frac{(3a+b)\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8af} - \frac{\cot(e+fx)\csc^3(e}{}$$

$$= -\frac{(3a+b)\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8af} - \frac{\cot(e+fx)\csc^3(e}{}$$

$$= -\frac{(3a+b)\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8af} - \frac{\cot(e+fx)\csc^3(e}{}$$

$$= -\frac{\left(3a^2+6ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8a^{3/2}f} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} -$$

Mathematica [B] time = 6.53037, size = 1059, normalized size = 5.66

$$\frac{\sqrt{\frac{\cos(2(e+fx))a+a+b-b\cos(2(e+fx))}{\cos(2(e+fx))+1}}\left(\frac{(-3a\cos(e+fx)-b\cos(e+fx))\csc^2(e+fx)}{8a}-\frac{1}{4}\cot(e+fx)\csc^3(e+fx)\right)}{f}+\frac{(3a^2-2ba-b^2)\left(2\sqrt{a}\tanh^{-1}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x
)])]*(((-3*a*Cos[e + f*x] - b*Cos[e + f*x])*Csc[e + f*x]^2)/(8*a) - (Cot[e
+ f*x]*Csc[e + f*x]^3)/4))/f + (-((3*a^2 + 14*a*b - b^2)*(2*Sqrt[a]*ArcTanh
[(Sqrt[b]*(1 + Tan[(e + f*x)/2]^2))/Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + T
an[(e + f*x)/2]^2)^2]] - Sqrt[b]*(ArcTanh[(a - a*Tan[(e + f*x)/2]^2 + 2*b*T
an[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e +
f*x)/2]^2)^2])] + ArcTanh[(2*b + a*(-1 + Tan[(e + f*x)/2]^2))/(Sqrt[a]*Sqrt
[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])]))*(1 + Cos[e + f
*x])*Sqrt[(1 + Cos[2*(e + f*x)])/(1 + Cos[e + f*x])^2]*Sqrt[(a + b + (a - b
)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(-1 + Tan[(e + f*x)/2]^2)*(1 +
Tan[(e + f*x)/2]^2)*Sqrt[(4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]
^2)^2)/(1 + Tan[(e + f*x)/2]^2)^2])/(4*Sqrt[a]*Sqrt[b]*Sqrt[a + b + (a - b)
*Cos[2*(e + f*x)]]*Sqrt[(-1 + Tan[(e + f*x)/2]^2)^2]*Sqrt[4*b*Tan[(e + f*x)
/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]) + ((3*a^2 - 2*a*b - b^2)*(2*Sqrt[a]
*ArcTanh[(Sqrt[b]*(1 + Tan[(e + f*x)/2]^2))/Sqrt[4*b*Tan[(e + f*x)/2]^2 + a
*(-1 + Tan[(e + f*x)/2]^2)^2]] + Sqrt[b]*(ArcTanh[(a - a*Tan[(e + f*x)/2]^2
 + 2*b*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + T
an[(e + f*x)/2]^2)^2])] + ArcTanh[(2*b + a*(-1 + Tan[(e + f*x)/2]^2))/(Sqrt
[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])]))*(1 + C
os[e + f*x])*Sqrt[(1 + Cos[2*(e + f*x)])/(1 + Cos[e + f*x])^2]*Sqrt[(a + b
+ (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(-1 + Tan[(e + f*x)/2]^
2)*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e +
 f*x)/2]^2)^2)/(1 + Tan[(e + f*x)/2]^2)^2])/(4*Sqrt[a]*Sqrt[b]*Sqrt[a + b +
 (a - b)*Cos[2*(e + f*x)]]*Sqrt[(-1 + Tan[(e + f*x)/2]^2)^2]*Sqrt[4*b*Tan[(
e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]))/(8*a*f)
```

Maple [B] time = 0.296, size = 5378, normalized size = 28.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx+e\right)^2+a}\csc\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e)^5, x)
```

Fricas [A] time = 8.87839, size = 3058, normalized size = 16.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((3*a^2 + 6*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 6*a*b - b^2)*cos
(f*x + e)^2 + 3*a^2 + 6*a*b - b^2)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 +
 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
 a + b)/(cos(f*x + e)^2 - 1)) - 8*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^
2 + a^2)*sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) - 2*((
3*a^2 + a*b)*cos(f*x + e)^3 - (5*a^2 + a*b)*cos(f*x + e))*sqrt(((a - b)*cos
(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x +
 e)^2 + a^2*f), 1/8*(((3*a^2 + 6*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 6*a
*b - b^2)*cos(f*x + e)^2 + 3*a^2 + 6*a*b - b^2)*sqrt(-a)*arctan(sqrt(-a)*sq
rt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) + 4*(a^2*co
s(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(b)*log(-((a - b)*cos(f*x +
e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e) + 2*b)/cos(f*x + e)^2) + ((3*a^2 + a*b)*cos(f*x + e)^3 - (5*a^2 + a*b)
*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^2*f*co
s(f*x + e)^4 - 2*a^2*f*cos(f*x + e)^2 + a^2*f), -1/16*(16*(a^2*cos(f*x + e)
^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + ((3*a^2 + 6*a*b - b^2)*c
os(f*x + e)^4 - 2*(3*a^2 + 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2 + 6*a*b - b^
2)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 + 2*sqrt(a)*sqrt(((a - b)*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)) -
2*((3*a^2 + a*b)*cos(f*x + e)^3 - (5*a^2 + a*b)*cos(f*x + e))*sqrt(((a - b)
*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f
*x + e)^2 + a^2*f), 1/8*(((3*a^2 + 6*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 +
 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2 + 6*a*b - b^2)*sqrt(-a)*arctan(sqrt(-a
)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) - 8*(a^
2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(-b)*arctan(sqrt(-b)*sqr
t(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + ((3*a^2 +
a*b)*cos(f*x + e)^3 - (5*a^2 + a*b)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e
)^2 + b)/cos(f*x + e)^2))/(a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x + e)^2 +
a^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx+e\right)^{2}+a}\csc\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e)^5, x)
```

3.98 $\int \sin^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=189

$$\frac{(3a^2-12ab+8b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f(a-b)^{3/2}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\sin^3(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f}$$

```
[Out] ((3*a^2 - 12*a*b + 8*b^2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[
e + f*x]^2]])/(8*(a - b)^(3/2)*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])
/Sqrt[a + b*Tan[e + f*x]^2]])/f - ((3*a - 4*b)*Cos[e + f*x]*Sin[e + f*x]*Sq
rt[a + b*Tan[e + f*x]^2])/(8*(a - b)*f) - (Cos[e + f*x]*Sin[e + f*x]^3*Sqrt
[a + b*Tan[e + f*x]^2])/(4*f)
```

Rubi [A] time = 0.236631, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3663, 467, 578, 523, 217, 206, 377, 203}

$$\frac{(3a^2-12ab+8b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f(a-b)^{3/2}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\sin^3(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((3*a^2 - 12*a*b + 8*b^2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[
e + f*x]^2]])/(8*(a - b)^(3/2)*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])
/Sqrt[a + b*Tan[e + f*x]^2]])/f - ((3*a - 4*b)*Cos[e + f*x]*Sin[e + f*x]*Sq
rt[a + b*Tan[e + f*x]^2])/(8*(a - b)*f) - (Cos[e + f*x]*Sin[e + f*x]^3*Sqrt
[a + b*Tan[e + f*x]^2])/(4*f)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
```

```
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sin^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2(3a+4bx^2)}{(1+x^2)^2\sqrt{a+bx^2}}\,dx, x, \right.}{4f}$$

$$= -\frac{(3a-4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8(a-b)f} - \frac{\cos(e+fx)\sin^3(e+\ }{}$$

$$= -\frac{(3a-4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8(a-b)f} - \frac{\cos(e+fx)\sin^3(e+\ }{}$$

$$= -\frac{(3a-4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8(a-b)f} - \frac{\cos(e+fx)\sin^3(e+\ }{}$$

$$= \frac{(3a^2-12ab+8b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8(a-b)^{3/2}f} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{(3a}{}$$

Mathematica [C] time = 3.66218, size = 330, normalized size = 1.75

$$\frac{\sin(2(e+fx))\sec^2(e+fx)\left(2\sqrt{2}a\left(3a^2-7ab+4b^2\right)\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+\ }{b}}}{\sqrt{2}}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((-((a - b)*(7*a^2 - 11*b^2 + 6*(a^2 - 3*a*b + 2*b^2)*Cos[2*(e + f*x)] - (a
 - b)^2*Cos[4*(e + f*x)])) + 2*Sqrt[2]*a*(3*a^2 - 7*a*b + 4*b^2)*Sqrt[((a +
 b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[((a
 + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] - 2*Sqrt[2
]*a*(3*a^2 - 12*a*b + 8*b^2)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e
 + f*x]^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*
(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)
])/(32*Sqrt[2]*(a - b)^2*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e +
f*x]^2])
```

Maple [C] time = 0.413, size = 2498, normalized size = 13.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/8/f/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/(a-b)*(2*cos(f*x+e)^5*((2*I
*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2-4*cos(f*x+e)^5*((2*I*b^(1/2)*(a-b)
```

```
^(1/2)+a-2*b)/a)^(1/2)*a*b+2*cos(f*x+e)^5*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/
a)^(1/2)*b^2-2*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2+4
*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b-2*cos(f*x+e)^4*
((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2+6*2^(1/2)*(1/a*(I*cos(f*x+e)*
b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos
(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(
1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x
+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)
*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1
/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a^2*sin(f*x+e)-24*2^(1/2)*(1/a*(I*cos(f*x+
e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(
cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b
)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(
f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1
/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b
^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*b*sin(f*x+e)+16*2^(1/2)*(1/a*(I*cos(f
*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b
)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(
a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((c
os(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b
^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*
I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*b^2*sin(f*x+e)+16*2^(1/2)*(1/a*(I*co
s(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e
)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2
)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi
((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I
*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((
2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*b*sin(f*x+e)-16*2^(1/2)*(1/a*(I*
cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x
+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1
/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*Elliptic
Pi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2
*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/
((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*b^2*sin(f*x+e)-3*2^(1/2)*(1/a*(I
*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*
x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(
1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*Ellipti
cF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*
I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2)
)*a^2*sin(f*x+e)+4*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)
*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*co
s(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e
)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/
2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(
1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*b*sin(f*x+e)-5*cos(f*x+e)^3*((2*I*b^(
1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2+13*cos(f*x+e)^3*((2*I*b^(1/2)*(a-b)^(1
/2)+a-2*b)/a)^(1/2)*a*b-8*cos(f*x+e)^3*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^
(1/2)*b^2+5*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2-13*c
os(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b+8*cos(f*x+e)^2*((
2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2-5*cos(f*x+e)*((2*I*b^(1/2)*(a-b
)^(1/2)+a-2*b)/a)^(1/2)*a*b+6*cos(f*x+e)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a
)^(1/2)*b^2+5*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b-6*((2*I*b^(1/2)
*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2)*cos(f*x+e)*sin(f*x+e)*((a*cos(f*x+e)^2-co
s(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/(cos(f*x+e)-1)/(a*cos(f*x+e)^2-cos(f*x+
e)^2*b+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx+e\right)^{2}+a}\sin\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

Fricas [B] time = 89.2591, size = 4998, normalized size = 26.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*((3*a^2 - 12*a*b + 8*b^2)*sqrt(-a + b)*log(128*(a^4 - 4*a^3*b + 6*a^2
*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a
*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3
 + 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*
b^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 -
 8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b +
 5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)
*cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-
a + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 16
*(a^2 - 2*a*b + b^2)*sqrt(b)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*
(a*b - 2*b^2)*cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x +
e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)
+ 8*b^2)/cos(f*x + e)^4) + 8*(2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - (5*a^2
 - 11*a*b + 6*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e))/((a^2 - 2*a*b + b^2)*f), -1/64*(32*(a^2 - 2*a*b + b^2
)*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b
)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b - b^2)*cos(f*x +
 e)^2 + b^2)*sin(f*x + e))) - (3*a^2 - 12*a*b + 8*b^2)*sqrt(-a + b)*log(128
*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*
a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b
+ 77*a^2*b^2 - 72*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2
*b^2 - 256*a*b^3 + 128*b^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 1
6*b^4)*cos(f*x + e)^2 - 8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^
7 - 24*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2
*b + 48*a*b^2 - 24*b^3)*cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^
3)*cos(f*x + e))*sqrt(-a + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*sin(f*x + e)) - 8*(2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - (5*a^2 - 11*
a*b + 6*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e))/((a^2 - 2*a*b + b^2)*f), 1/32*((3*a^2 - 12*a*b + 8*b^2)*sqr
t(a - b)*arctan(-1/4*(8*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 8*(a^2 - 3*a*b
 + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(a - b)*
sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*(a^3 - 3*a^2*b + 3*a*
b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*a*b^2 - 2*b^3 - (a^3 - 6*a^2*b + 9*a*
b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 8*(a^2 - 2*a*b + b^2)*sqrt(b)
*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2
 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4
*(2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - (5*a^2 - 11*a*b + 6*b^2)*cos(f*x +
```

```
 e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^2
- 2*a*b + b^2)*f), 1/32*((3*a^2 - 12*a*b + 8*b^2)*sqrt(a - b)*arctan(-1/4*(
8*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)
^3 + (a^2 - 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)/((2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e
)^4 - a^2*b + 3*a*b^2 - 2*b^3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x +
 e)^2)*sin(f*x + e))) - 16*(a^2 - 2*a*b + b^2)*sqrt(-b)*arctan(1/2*((a - 2*
b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/(((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e))) +
4*(2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - (5*a^2 - 11*a*b + 6*b^2)*cos(f*x
+ e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^2
 - 2*a*b + b^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\tan^2\left(e + fx\right)}\sin^4\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*sin(e + f*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx + e\right)^2 + a}\sin\left(fx + e\right)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

3.99 $\int \sin^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=128

$$\frac{(a-2b)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f\sqrt{a-b}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

```
[Out] ((a - 2*b)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(
2*Sqrt[a - b]*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e
 + f*x]^2]])/f - (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*
f)
```

Rubi [A] time = 0.134627, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3663, 467, 523, 217, 206, 377, 203}

$$\frac{(a-2b)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f\sqrt{a-b}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```


```
[Out] ((a - 2*b)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(
2*Sqrt[a - b]*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e
 + f*x]^2]])/f - (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*
f)
```


Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sin^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx^2}}{\left(1+x^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{a+2bx^2}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x\right.}{2f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+}}\right.}{2f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\right.}{2f}$$

$$= \frac{(a-2b)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2\sqrt{a-b}f} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx}{}$$

Mathematica [C] time = 3.67552, size = 273, normalized size = 2.13

$$-\frac{\sin(2(e+fx))\sec^2(e+fx)\left(\sqrt{2}a(b-a)\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}}{\sqrt{2}}\right)\right.\right.}{4\sqrt{2}f(a-b)\sqrt{\sec}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(((a - b)*(a + b + (a - b)*Cos[2*(e + f*x)]) + Sqrt[2]*a*(-a + b)*Sqrt[((a
 + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[(
```

```
(a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + Sqrt[2
]*a*(a - 2*b)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*E
llipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc
[e + f*x]^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)])/(4*Sqrt[2]*(
a - b)*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Maple [C] time = 0.193, size = 1343, normalized size = 10.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/f/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*(2*2^(1/2)*(1/a*(I*cos(f*x+
e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(
cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b
)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(
f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1
/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b
^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*sin(f*x+e)-4*2^(1/2)*(1/a*(I*cos(f*x+
e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(
cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b
)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(
f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1
/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b
^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*b*sin(f*x+e)+4*2^(1/2)*(1/a*(I*cos(f*x+
e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(
cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b
)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(
f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/
2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^
(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*b*sin(f*x+e)-2^(1/2)*(1/a*(I*cos(f*x+e)*
b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos
(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(
1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+
e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a
-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*sin(f*x+
e)-cos(f*x+e)^3*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a+cos(f*x+e)^3*((
2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b+cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^
(1/2)+a-2*b)/a)^(1/2)*a-cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1
/2)*b-cos(f*x+e)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b+((2*I*b^(1/2)*
(a-b)^(1/2)+a-2*b)/a)^(1/2)*b)*cos(f*x+e)*sin(f*x+e)*((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/(cos(f*x+e)-1)/(a*cos(f*x+e)^2-cos(f*x+e)^
2*b+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \tan\left(fx+e\right)^2 + a} \sin\left(fx+e\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

Fricas [B] time = 8.63117, size = 4471, normalized size = 34.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
 + e)*sin(f*x + e) - (a - 2*b)*sqrt(-a + b)*log(128*(a^4 - 4*a^3*b + 6*a^2*
b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*
b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3
+ 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*b
^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 -
8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b +
5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*
cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a
 + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 4*(
a - b)*sqrt(b)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2)*
cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sq
rt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f
*x + e)^4))/((a - b)*f), -1/16*(8*(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + 8*(a - b)*sqrt(-b)*arctan(1/2*
((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)/(((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x +
 e))) - (a - 2*b)*sqrt(-a + b)*log(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3
 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*
cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3 + 24*b^4)*cos
(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*b^4 - 32*(a^4
- 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*(a^3 -
3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b
^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*cos(f*x + e)^
3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a + b)*sqrt(((
a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a - b)*f), -1/8
*(4*(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*
sin(f*x + e) - sqrt(a - b)*(a - 2*b)*arctan(-1/4*(8*(a^2 - 2*a*b + b^2)*cos
(f*x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*b + 8*b^2
)*cos(f*x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)/((2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*a*b^2 - 2
*b^3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) - 2
*(a - b)*sqrt(b)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2
)*cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*
sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos
(f*x + e)^4))/((a - b)*f), -1/8*(4*(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b
)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) - sqrt(a - b)*(a - 2*b)*arctan(
-1/4*(8*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*
x + e)^3 + (a^2 - 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(a - b)*sqrt(((a - b)*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f
*x + e)^4 - a^2*b + 3*a*b^2 - 2*b^3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos
(f*x + e)^2)*sin(f*x + e))) + 4*(a - b)*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/(((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e))))/((a - b)
*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\tan^2\left(e + fx\right)}\sin^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*sin(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx+e\right)^2 + a}\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

3.100 $\int \sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])
/f + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f
```

Rubi [A] time = 0.0543786, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3661, 402, 217, 206, 377, 203}

$$\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])
/f + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

$$= \frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

Mathematica [C] time = 0.855987, size = 203, normalized size = 2.39

$$\frac{-i\sqrt{a-b}\log\left(-\frac{4i\left(\sqrt{a-b}\sqrt{a+b\tan^2(e+fx)}+a-ib\tan(e+fx)\right)}{(a-b)^{3/2}(\tan(e+fx)+i)}\right)+i\sqrt{a-b}\log\left(\frac{4i\left(\sqrt{a-b}\sqrt{a+b\tan^2(e+fx)}+a+ib\tan(e+fx)\right)}{(a-b)^{3/2}(\tan(e+fx)-i)}\right)+2\sqrt{b}\log\left(\sqrt{b}\sqrt{a\cdots}\right.}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((-I)*Sqrt[a - b]*Log[((-4*I)*(a - I*b*Tan[e + f*x] + Sqrt[a - b]*Sqrt[a +
b*Tan[e + f*x]^2]))/((a - b)^(3/2)*(I + Tan[e + f*x]))] + I*Sqrt[a - b]*Log
[((4*I)*(a + I*b*Tan[e + f*x] + Sqrt[a - b]*Sqrt[a + b*Tan[e + f*x]^2]))/((
a - b)^(3/2)*(-I + Tan[e + f*x]))] + 2*Sqrt[b]*Log[b*Tan[e + f*x] + Sqrt[b]
*Sqrt[a + b*Tan[e + f*x]^2]])/(2*f)
```

Maple [B] time = 0.04, size = 169, normalized size = 2.

$$\frac{1}{f}\sqrt{b}\ln\left(\sqrt{b}\tan(fx+e)+\sqrt{a+b\left(\tan(fx+e)\right)^2}\right)-\frac{1}{fb(a-b)}\sqrt{b^4(a-b)}\arctan\left((a-b)b^2\tan(fx+e)\frac{1}{\sqrt{b^4(a-b)}}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f*b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-1/f*(b^4*(a-b))
^(1/2)/b/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*
tan(f*x+e))+1/f*a*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^
(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.12744, size = 1046, normalized size = 12.31

$$\left[\frac{\sqrt{b}\log\left(2\,b\tan\left(fx+e\right)^{2}+2\sqrt{b\tan\left(fx+e\right)^{2}+a}\sqrt{b}\tan\left(fx+e\right)+a\right)+\sqrt{-a+b}\log\left(-\frac{(a-2\,b)\tan\left(fx+e\right)^{2}+2\sqrt{b\tan\left(fx\right.}}{\tan\left(fx+e\right)}\right.}{2\,f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)
*tan(f*x + e) + a) + sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b
*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)))/
f, 1/2*(2*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f
*x + e))) + sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*s
qrt(b)*tan(f*x + e) + a))/f, -1/2*(2*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2
+ a)*sqrt(-b)/(b*tan(f*x + e))) - sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)
^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x +
 e)^2 + 1)))/f, (sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b
)*tan(f*x + e))) - sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*t
an(f*x + e))))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\tan^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx+e\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a), x)
```

3.101 $\int \csc^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=66

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f - (C
ot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/f
```

Rubi [A] time = 0.079029, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3663, 277, 217, 206}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f - (C
ot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/f
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

Mathematica [C] time = 2.05935, size = 156, normalized size = 2.36

$$-\frac{\tan(e+fx)\left(\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b) - \sqrt{2}b\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{\csc^2(e}{}}\right.\right.\right.}{\sqrt{2}f\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2 - Sqrt[2]*b*Sqrt[((a
+ b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[((
a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Tan[e +
f*x])/(Sqrt[2]*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]))
```

Maple [C] time = 0.428, size = 1215, normalized size = 18.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^
2*b+b)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)*(-2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*
(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+
1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos
(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((
2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2
)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*b*sin(f*x+e)*cos(f
*x+e)+2*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2
)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^
(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f
*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/
a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b
)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*b*sin(f*
x+e)*cos(f*x+e)-2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a
```

```
-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f
*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b
)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+
a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2
)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*b*sin(f*x+e)+2*2^(1/2)*(1/a*(I*cos(f*x+e)*
b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos
(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(
1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x
+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*
(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/
2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*b*sin(f*x+e)+cos(f*x+e)^2*((2*I*b^(1/2)*(a-
b)^(1/2)+a-2*b)/a)^(1/2)*a-cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)
^(1/2)*b+((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b)/(a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/sin(f*x+e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.33288, size = 834, normalized size = 12.64

$$\left[\frac{\sqrt{b}\log\left(\frac{\left(a^2-8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4+8\left(ab-2\,b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a-2\,b\right)\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\sin\left(fx+e\right)+8\,b^2}{\cos\left(fx+e\right)^4}\right)\sin\left(fx\right.}{4\,f\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2)*c
os(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqr
t(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*
x + e)^4)*sin(f*x + e) - 4*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*cos(f*x + e))/(f*sin(f*x + e)), -1/2*(sqrt(-b)*arctan(1/2*((a - 2*b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/(((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x +
 e) + 2*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/(f*
sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\tan^2\left(e+fx\right)}\csc^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*csc(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \tan\left(fx+e\right)^2 + a} \csc\left(fx+e\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e)^2, x)
```

3.102 $\int \csc^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=100

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3af}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f - (C
ot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/f - (Cot[e + f*x]^3*(a + b*Tan[e +
f*x]^2)^(3/2))/(3*a*f)
```

Rubi [A] time = 0.0954462, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3663, 451, 277, 217, 206}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3af}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f - (C
ot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/f - (Cot[e + f*x]^3*(a + b*Tan[e +
f*x]^2)^(3/2))/(3*a*f)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)\sqrt{a+bx^2}}{x^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3af} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3af} + \frac{b\,\text{S}}{}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3af} + \frac{b\,\text{S}}{}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\cot^3(e+fx)}{}$$

Mathematica [C] time = 4.37492, size = 204, normalized size = 2.04

$$-\frac{\tan(e+fx)\left(\csc^4(e+fx)\left(4\left(a^2-3ab-b^2\right)\cos(2(e+fx))+\left(-2a^2+ab+b^2\right)\cos(4(e+fx))+6a^2+11ab+3b^2\right)-12\right.}{12\sqrt{2}af\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(((6*a^2 + 11*a*b + 3*b^2 + 4*(a^2 - 3*a*b - b^2)*Cos[2*(e + f*x)] + (-2*a
^2 + a*b + b^2)*Cos[4*(e + f*x)])*Csc[e + f*x]^4 - 12*Sqrt[2]*a*b*Sqrt[((a
+ b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[((
a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Tan[e +
f*x])/(12*Sqrt[2]*a*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^
2])
```

Maple [C] time = 0.349, size = 2441, normalized size = 24.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3/f/a/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*(3*EllipticF((cos(f*x+e)
-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b
)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^3
*sin(f*x+e)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+co
s(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2
)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e
)+1))^(1/2)*2^(1/2)*a*b-6*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*
b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/
a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*co
s(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(
a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(
-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/
a)^(1/2))*cos(f*x+e)^3*sin(f*x+e)*a*b+3*EllipticF((cos(f*x+e)-1)*((2*I*b^(1
/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^
(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^2*sin(f*x+e)*(1
/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*c
os(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-
I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*2^
(1/2)*a*b-6*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(
1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1
/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^2
*sin(f*x+e)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+co
s(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2
)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e
)+1))^(1/2)*2^(1/2)*a*b-3*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2
)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1
/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)*sin(f*x+e)*(1/a*(I*cos(f*x+e)
*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(co
s(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^
(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*2^(1/2)*a*b+6*Elli
pticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),
1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1
/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)*sin(f*x+e)*(1/a*(
I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f
*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^
(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*2^(1/2
)*a*b-3*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2
)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^
(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f
*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a
)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-
8*a*b+8*b^2)/a^2)^(1/2))*a*b*sin(f*x+e)+6*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2
)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e
)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-c
os(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)
*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^
(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-
b)^(1/2)+a-2*b)/a)^(1/2))*a*b*sin(f*x+e)-2*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)/a)^(1/2)*a^2+cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)
^(1/2)*a*b+cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2+3*cos
(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2-4*cos(f*x+e)^2*((2*
I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b-2*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b
)^(1/2)+a-2*b)/a)^(1/2)*b^2+3*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b
+((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2)*cos(f*x+e)*((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/si
n(f*x+e)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.2808, size = 1076, normalized size = 10.76

$$\left[\frac{3\left(a\cos\left(fx+e\right)^{2}-a\right)\sqrt{b}\log\left(\frac{\left(a^{2}-8ab+8b^{2}\right)\cos\left(fx+e\right)^{4}+8\left(ab-2b^{2}\right)\cos\left(fx+e\right)^{2}+4\left(\left(a-2b\right)\cos\left(fx+e\right)^{3}+2b\cos\left(fx+e\right)\right)\sqrt{b}\sqrt{\frac{\left(a-b\right)\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\,\mathrm{si}}{\cos\left(fx+e\right)^{4}}\right.}{12\left(af\cos\left(fx+e\right)^{2}-af\right)\sin\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*cos(f*x + e)^2 - a)*sqrt(b)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x
+ e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b
*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*si
n(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((2*a + b)*cos(f*x + e
)^3 - (3*a + b)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e
)^2))/((a*f*cos(f*x + e)^2 - a*f)*sin(f*x + e)), -1/6*(3*(a*cos(f*x + e)^2
- a)*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt
(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b - b^2)*cos(f*
x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) + 2*((2*a + b)*cos(f*x + e)^3 -
 (3*a + b)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
/((a*f*cos(f*x + e)^2 - a*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\tan^{2}\left(e+fx\right)}\csc^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*csc(e + f*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\tan\left(fx+e\right)^{2}+a}\csc\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e)^4, x)
```

3.103 $\int \csc^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=141

$$-\frac{2(5a-b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{15a^2f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{5af}-$$

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f - (C
ot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/f - (2*(5*a - b)*Cot[e + f*x]^3*(a
+ b*Tan[e + f*x]^2)^(3/2))/(15*a^2*f) - (Cot[e + f*x]^5*(a + b*Tan[e + f*x]
^2)^(3/2))/(5*a*f)
```

Rubi [A] time = 0.128221, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3663, 462, 451, 277, 217, 206}

$$-\frac{2(5a-b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{15a^2f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{5af}-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f - (C
ot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/f - (2*(5*a - b)*Cot[e + f*x]^3*(a
+ b*Tan[e + f*x]^2)^(3/2))/(15*a^2*f) - (Cot[e + f*x]^5*(a + b*Tan[e + f*x]
^2)^(3/2))/(5*a*f)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
```

```
Q[m + n, -1]))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2\sqrt{a+bx^2}}{x^6}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{5af} + \frac{\text{Subst}\left(\int \frac{\left(2(5a-b)+5ax^2\right)\sqrt{a+bx^2}}{x^4}\,dx, x,\right.}{5af}$$

$$= -\frac{2(5a-b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{15a^2f} - \frac{\cot^5(e+fx)\left(a+b\tan^2(\right.}{5af}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{2(5a-b)\cot^3(e+fx)\left(a+b\tan^2(e+\right.}{15a^2f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{2(5a-b)\cot^3(e+fx)\left(a+b\tan^2(e+\right.}{15a^2f}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{2(5a-b)}{}$$

Mathematica [C] time = 3.28498, size = 287, normalized size = 2.04

$$-\frac{\tan(e+fx)\left(\csc^6(e+fx)\left(\left(-241a^2b+40a^3-149ab^2+30b^3\right)\cos(2(e+fx))+\left(42a^2b-32a^3+62ab^2-12b^3\right)\cos(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(((80*a^3 + 198*a^2*b + 98*a*b^2 - 20*b^3 + (40*a^3 - 241*a^2*b - 149*a*b^
2 + 30*b^3)*Cos[2*(e + f*x)] + (-32*a^3 + 42*a^2*b + 62*a*b^2 - 12*b^3)*Cos
[4*(e + f*x)] + 8*a^3*Cos[6*(e + f*x)] + a^2*b*Cos[6*(e + f*x)] - 11*a*b^2*
Cos[6*(e + f*x)] + 2*b^3*Cos[6*(e + f*x)])*Csc[e + f*x]^6 - 240*Sqrt[2]*a^2
*b*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[Ar
cSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]],
1])*Tan[e + f*x])/(240*Sqrt[2]*a^2*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)]
)*Sec[e + f*x]^2])
```

Maple [C] time = 0.336, size = 3769, normalized size = 26.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/15/f/a^2/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*(30*2^(1/2)*(1/a*(I*c
os(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+
e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/
2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF
((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*
b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*
cos(f*x+e)^2*sin(f*x+e)*a^2*b+8*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*c
os(f*x+e)^6*a^3+30*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)
*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*co
s(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e
)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1
/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b
^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2
))*cos(f*x+e)*sin(f*x+e)*a^2*b-15*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^
(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1
/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e
)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^
(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*
b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)*sin(f*x+e)*a^
2*b+30*cos(f*x+e)^5*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/
2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)
*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a
+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1
/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b
)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-
2*b)/a)^(1/2))*a^2*b-15*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^
(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*
(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(
f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b
)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a
-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^5*sin(f*x+e)*a^2*b+30*2
^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x
+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-
b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))
^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/
sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a
+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^4*sin(
f*x+e)*a^2*b-15*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a
-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f
*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b
```

```
)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+
a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2
)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^4*sin(f*x+e)*a^2*b-60*cos(f*x+e
)^3*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-
b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*
x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)
/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+
a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/
2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a
^2*b+30*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2
)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^
(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f
*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a
)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-
8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^3*sin(f*x+e)*a^2*b-60*2^(1/2)*(1/a*(I*c
os(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+
e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/
2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticP
i((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*
I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/(
(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^2*sin(f*x+e)*a^2*b+30*
2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*
x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a
-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)
)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)
/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-
a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a^2*b*sin(f*x+e)
-15*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+co
s(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2
)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e
)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1
/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*
b+8*b^2)/a^2)^(1/2))*a^2*b*sin(f*x+e)-2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)
^(1/2)*b^3+15*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a^3+2*
((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^6*b^3-6*((2*I*b^(1/2)*
(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^4*b^3+6*((2*I*b^(1/2)*(a-b)^(1/2)+a-
2*b)/a)^(1/2)*cos(f*x+e)^2*b^3-20*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)
*cos(f*x+e)^4*a^3+15*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2*b+10*((2
*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b^2+((2*I*b^(1/2)*(a-b)^(1/2)+a-2*
b)/a)^(1/2)*cos(f*x+e)^6*a^2*b-11*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)
*cos(f*x+e)^6*a*b^2+9*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^
4*a^2*b+32*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^4*a*b^2-25*
((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a^2*b-31*((2*I*b^(1/
2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a*b^2)*cos(f*x+e)*((a*cos(f*x+e
)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)
/sin(f*x+e)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.09403, size = 1428, normalized size = 10.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(b)*log(((a
^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 + 4*((a
 - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e
) - 4*((8*a^2 + 9*a*b - 2*b^2)*cos(f*x + e)^5 - (20*a^2 + 19*a*b - 4*b^2)*c
os(f*x + e)^3 + (15*a^2 + 10*a*b - 2*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f
*x + e)^2 + b)/cos(f*x + e)^2))/((a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x +
e)^2 + a^2*f)*sin(f*x + e)), -1/30*(15*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x
+ e)^2 + a^2)*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x +
 e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b - b^
2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) + 2*((8*a^2 + 9*a*b -
2*b^2)*cos(f*x + e)^5 - (20*a^2 + 19*a*b - 4*b^2)*cos(f*x + e)^3 + (15*a^2
+ 10*a*b - 2*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2))/((a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x + e)^2 + a^2*f)*sin(f*x +
e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \tan\left(fx+e\right)^2 + a} \csc\left(fx+e\right)^6 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e)^6, x)
```

3.104 $\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=227

$$\frac{b(3a-7b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2f(a-b)}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{5/2}}{5f(a-b)}+\frac{2\cos^3(e+fx)\left(a+b\sec^2\ldots\right.}{3f(a-b)}$$

```
[Out] ((3*a - 7*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e +
f*x]^2]])/(2*f) + ((3*a - 7*b)*b*Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2
])/(2*(a - b)*f) - ((3*a - 7*b)*Cos[e + f*x]*(a - b + b*Sec[e + f*x]^2)^(3/
2))/(3*(a - b)*f) + (2*Cos[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^(5/2))/(3*
(a - b)*f) - (Cos[e + f*x]^5*(a - b + b*Sec[e + f*x]^2)^(5/2))/(5*(a - b)*f
)
```

Rubi [A] time = 0.215731, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3664, 462, 453, 277, 195, 217, 206}

$$\frac{b(3a-7b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2f(a-b)}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{5/2}}{5f(a-b)}+\frac{2\cos^3(e+fx)\left(a+b\sec^2\ldots\right.}{3f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((3*a - 7*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e +
f*x]^2]])/(2*f) + ((3*a - 7*b)*b*Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2
])/(2*(a - b)*f) - ((3*a - 7*b)*Cos[e + f*x]*(a - b + b*Sec[e + f*x]^2)^(3/
2))/(3*(a - b)*f) + (2*Cos[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^(5/2))/(3*
(a - b)*f) - (Cos[e + f*x]^5*(a - b + b*Sec[e + f*x]^2)^(5/2))/(5*(a - b)*f
)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
- a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\left(a-b+bx^2\right)^{3/2}}{x^6}dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{5/2}}{5(a-b)f} + \frac{\text{Subst}\left(\int \frac{\left(-10(a-b)+5(a-b)x^2\right)\left(a-b+b\right.}{x^4}\right.}{5(a-b)f}$$

$$= \frac{2\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{5/2}}{3(a-b)f} - \frac{\cos^5(e+fx)\left(a-b+b\sec^2(e+\right.}{5(a-b)f}$$

$$= -\frac{(3a-7b)\cos(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{3(a-b)f} + \frac{2\cos^3(e+fx)\left(a-b+\right.}{3(a-b)}$$

$$= \frac{(3a-7b)b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2(a-b)f} - \frac{(3a-7b)\cos(e+fx)\left(a-b\right.}{3(a-b)}$$

$$= \frac{(3a-7b)b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2(a-b)f} - \frac{(3a-7b)\cos(e+fx)\left(a-b\right.}{3(a-b)}$$

$$= \frac{(3a-7b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f} + \frac{(3a-7b)b\sec(e+fx)\sqrt{a-b+b\,\mathrm{s}}}{2(a-b)f}$$

Mathematica [A] time = 5.52285, size = 233, normalized size = 1.03

$$\frac{\cos(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(2\sqrt{(a-b)\cos(2(e+fx))+a+b}\left(4\left(7a^2-20ab+13b^2\right)\mathrm{co}\right.\right.}{240\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(120*
Sqrt[2]*Sqrt[b]*(3*a^2 - 10*a*b + 7*b^2)*ArcTanh[Sqrt[a + b + (a - b)*Cos[2
*(e + f*x)]]/(Sqrt[2]*Sqrt[b])] + 2*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]*
(-89*a^2 + 474*a*b - 409*b^2 + 4*(7*a^2 - 20*a*b + 13*b^2)*Cos[2*(e + f*x)]
 - 3*(a - b)^2*Cos[4*(e + f*x)] + 60*(a - b)*b*Sec[e + f*x]^2)))/(240*Sqrt[
2]*(a - b)*f*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])
```

Maple [B] time = 0.453, size = 2399, normalized size = 10.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/60/f/(a-b)/a^(5/2)/b^(1/2)*(cos(f*x+e)-1)^3*(-15*a^(7/2)*b^(3/2)*((a*cos(
f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+15*a^(5/2)*b^(5/2)*((a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+6*a^(9/2)*cos(f*x+e)^
7*b^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-12*cos
(f*x+e)^7*a^(7/2)*b^(3/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)
^2)^(1/2)+6*a^(5/2)*cos(f*x+e)^7*b^(5/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)
/(cos(f*x+e)+1)^2)^(1/2)+6*a^(9/2)*cos(f*x+e)^6*b^(1/2)*((a*cos(f*x+e)^2-co
s(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-12*a^(7/2)*cos(f*x+e)^6*b^(3/2)*((a
*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+6*a^(5/2)*cos(f*x+e
)^6*b^(5/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-45*a
^(9/2)*cos(f*x+e)^2*arctanh(1/8*b^(1/2)*4^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*
4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/(cos(f*x+e)+1)^2)^(1/2))*b-15*cos(f*x+e)^2*b^(9/2)*ln(-2/a^(1/2)*(cos(
f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^
(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/
(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*a+15*cos(f*x+e)^2*b^(9/2)*
ln(-4/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)
/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*a-30*co
s(f*x+e)^2*b^(7/2)*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^
2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+
e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/si
n(f*x+e)^2)*a^2+30*cos(f*x+e)^2*b^(7/2)*ln(-4/a^(1/2)*(cos(f*x+e)-1)*(cos(f
*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-co
s(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2
)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*a^2+45*cos(f*x+e)^2*b^(5/2)*ln(-2/a^(1/2)*
(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1
)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*a^3-45*cos(f*x+e)^2*b
^(5/2)*ln(-4/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)
^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*
```

```
a^3-20*a^(9/2)*cos(f*x+e)^5*b^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos
(f*x+e)+1)^2)^(1/2)+52*cos(f*x+e)^5*a^(7/2)*b^(3/2)*((a*cos(f*x+e)^2-cos(f*
x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-32*a^(5/2)*cos(f*x+e)^5*b^(5/2)*((a*cos
(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-20*a^(9/2)*cos(f*x+e)^4
*b^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+52*a^(7
/2)*cos(f*x+e)^4*b^(3/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^
2)^(1/2)-32*a^(5/2)*cos(f*x+e)^4*b^(5/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)
/(cos(f*x+e)+1)^2)^(1/2)+30*a^(9/2)*cos(f*x+e)^3*b^(1/2)*((a*cos(f*x+e)^2-c
os(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-140*a^(7/2)*cos(f*x+e)^3*b^(3/2)*(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+116*a^(5/2)*cos(f
*x+e)^3*b^(5/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+
30*a^(9/2)*cos(f*x+e)^2*b^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x
+e)+1)^2)^(1/2)-140*a^(7/2)*cos(f*x+e)^2*b^(3/2)*((a*cos(f*x+e)^2-cos(f*x+e
)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+116*a^(5/2)*cos(f*x+e)^2*b^(5/2)*((a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-15*a^(7/2)*cos(f*x+e)*b^(
3/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+15*a^(5/2)*
cos(f*x+e)*b^(5/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/
2)+150*a^(7/2)*cos(f*x+e)^2*arctanh(1/8*b^(1/2)*4^(1/2)*(cos(f*x+e)-1)*(cos
(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*b^2-105*a^(5/2)*cos(f*x+e)^2*arctanh(
1/8*b^(1/2)*4^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)
-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)
)*b^3)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)*4^
(1/2)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)/sin(f*x+e)
^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.67458, size = 1053, normalized size = 4.64

$$\left[-\frac{15\left(3a^2-10ab+7b^2\right)\sqrt{b}\cos\left(fx+e\right)\log\left(-\frac{(a-b)\cos\left(fx+e\right)^2-2\sqrt{b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+2b}{\cos\left(fx+e\right)^2}\right)+2\left(6\left(a^2-2ab+b^2\right)\cos\right.}{60\left(a-b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(3*a^2 - 10*a*b + 7*b^2)*sqrt(b)*cos(f*x + e)*log(-((a - b)*cos(
f*x + e)^2 - 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*co
s(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*(6*(a^2 - 2*a*b + b^2)*cos(f*x + e)^6
```

```
 - 4*(5*a^2 - 13*a*b + 8*b^2)*cos(f*x + e)^4 + 2*(15*a^2 - 70*a*b + 58*b^2)
*cos(f*x + e)^2 - 15*a*b + 15*b^2)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*
x + e)^2))/((a - b)*f*cos(f*x + e)), -1/30*(15*(3*a^2 - 10*a*b + 7*b^2)*sqr
t(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos
(f*x + e)/b)*cos(f*x + e) + (6*(a^2 - 2*a*b + b^2)*cos(f*x + e)^6 - 4*(5*a^
2 - 13*a*b + 8*b^2)*cos(f*x + e)^4 + 2*(15*a^2 - 70*a*b + 58*b^2)*cos(f*x +
 e)^2 - 15*a*b + 15*b^2)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
/((a - b)*f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.105 $\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=186

$$\frac{b(3a-5b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2f(a-b)}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{5/2}}{3f(a-b)}-\frac{(3a-5b)\cos(e+fx)\left(a+b\,\mathrm{s}\right.}{3f(a-b)}$$

```
[Out] ((3*a - 5*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e +
f*x]^2]])/(2*f) + ((3*a - 5*b)*b*Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2
])/(2*(a - b)*f) - ((3*a - 5*b)*Cos[e + f*x]*(a - b + b*Sec[e + f*x]^2)^(3/
2))/(3*(a - b)*f) + (Cos[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^(5/2))/(3*(a
 - b)*f)
```

Rubi [A] time = 0.164599, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3664, 453, 277, 195, 217, 206}

$$\frac{b(3a-5b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2f(a-b)}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{5/2}}{3f(a-b)}-\frac{(3a-5b)\cos(e+fx)\left(a+b\,\mathrm{s}\right.}{3f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((3*a - 5*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e +
f*x]^2]])/(2*f) + ((3*a - 5*b)*b*Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2
])/(2*(a - b)*f) - ((3*a - 5*b)*Cos[e + f*x]*(a - b + b*Sec[e + f*x]^2)^(3/
2))/(3*(a - b)*f) + (Cos[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^(5/2))/(3*(a
 - b)*f)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a-b+bx^2)^{3/2}}{x^4}dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{5/2}}{3(a-b)f} + \frac{(3a-5b)\,\text{Subst}\left(\int \frac{(a-b+bx^2)^{3/2}}{x^2}\right.}{3(a-b)f}$$

$$= -\frac{(3a-5b)\cos(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{3(a-b)f} + \frac{\cos^3(e+fx)\left(a-b\right.}{3(a-}$$

$$= \frac{(3a-5b)b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2(a-b)f} - \frac{(3a-5b)\cos(e+fx)\left(a\right.}{3(a-}$$

$$= \frac{(3a-5b)b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2(a-b)f} - \frac{(3a-5b)\cos(e+fx)\left(a\right.}{3(a-}$$

$$= \frac{(3a-5b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f} + \frac{(3a-5b)b\sec(e+fx)\sqrt{a-b+}}{2(a-b)f}$$

Mathematica [A] time = 1.78038, size = 188, normalized size = 1.01

$$\frac{\sec(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\sqrt{(a-b)\cos(2(e+fx))+a+b}(-8(a-3b)\cos(2(e+fx))\right.}{24\sqrt{2}f\sqrt{(a-b)\cos(2(e+f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((12*Sqrt[2]*(3*a - 5*b)*Sqrt[b]*ArcTanh[Sqrt[a + b + (a - b)*Cos[2*(e + f*
x)]]/(Sqrt[2]*Sqrt[b])]*Cos[e + f*x]^2 + Sqrt[a + b + (a - b)*Cos[2*(e + f*
x)]]*(-9*a + 37*b - 8*(a - 3*b)*Cos[2*(e + f*x)] + (a - b)*Cos[4*(e + f*x)]
))*Sec[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(2
4*Sqrt[2]*f*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])
```

Maple [B] time = 0.183, size = 1104, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/12/f/a^(5/2)/b^(1/2)*(cos(f*x+e)-1)^3*(2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b
+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(7/2)*b^(1/2)*cos(f*x+e)^5-2*((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(5/2)*b^(3/2)*cos(f*x+e)^5+2*(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(7/2)*b^(1/2)*c
os(f*x+e)^4-2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^
(5/2)*b^(3/2)*cos(f*x+e)^4-6*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)
+1)^2)^(1/2)*a^(7/2)*b^(1/2)*cos(f*x+e)^3+14*((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(5/2)*b^(3/2)*cos(f*x+e)^3-6*((a*cos(f*x+e)^
2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(7/2)*b^(1/2)*cos(f*x+e)^2+14
*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(5/2)*b^(3/2)
*cos(f*x+e)^2-6*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-c
os(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f
*x+e)^2)*b^(7/2)*cos(f*x+e)^2*a+6*ln(-4/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+
e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2
)*a^(1/2)+b)/sin(f*x+e)^2)*b^(7/2)*cos(f*x+e)^2*a+9*arctanh(1/8*b^(1/2)*4^(
1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^
2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*a^(7/2)*cos(f
*x+e)^2*b+3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(5
/2)*b^(3/2)*cos(f*x+e)-15*arctanh(1/8*b^(1/2)*4^(1/2)*(cos(f*x+e)-1)*(cos(f
*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*a^(5/2)*cos(f*x+e)^2*b^2+3*((a*cos(f*x+
e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(5/2)*b^(3/2))*cos(f*x+e)*
4^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)/((a*cos(f*x+
e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)/sin(f*x+e)^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.51812, size = 768, normalized size = 4.13

$$\left[-\frac{3\,(3\,a-5\,b)\sqrt{b}\cos\left(fx+e\right)\log\left(-\frac{(a-b)\cos\left(fx+e\right)^{2}-2\sqrt{b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)+2\,b}{\cos\left(fx+e\right)^{2}}\right)-2\left(2\,(a-b)\cos\left(fx+e\right)^{4}-2\,(3\,a\right.}{12\,f\cos\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(3*a - 5*b)*sqrt(b)*cos(f*x + e)*log(-((a - b)*cos(f*x + e)^2 - 2
*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2
*b)/cos(f*x + e)^2) - 2*(2*(a - b)*cos(f*x + e)^4 - 2*(3*a - 7*b)*cos(f*x +
 e)^2 + 3*b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x
+ e)), -1/6*(3*(3*a - 5*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b)*cos(f*x + e) - (2*(a - b)*cos(f*
x + e)^4 - 2*(3*a - 7*b)*cos(f*x + e)^2 + 3*b)*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2))/(f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.106 $\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=113

$$\frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2f}-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{f}+\frac{3\sqrt{b}(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2f}$$

```
[Out] (3*(a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*
x]^2]])/(2*f) + (3*b*Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/(2*f) - (
Cos[e + f*x]*(a - b + b*Sec[e + f*x]^2)^(3/2))/f
```

Rubi [A] time = 0.0819726, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3664, 277, 195, 217, 206}

$$\frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2f}-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{f}+\frac{3\sqrt{b}(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*
x]^2]])/(2*f) + (3*b*Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/(2*f) - (
Cos[e + f*x]*(a - b + b*Sec[e + f*x]^2)^(3/2))/f
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(a-b+bx^2\right)^{3/2}}{x^2}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= -\frac{\cos(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{f} + \frac{(3b)\,\text{Subst}\left(\int \sqrt{a-b+bx^2}\,dx\right.}{f}$$
$$= \frac{3b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} - \frac{\cos(e+fx)\left(a-b+b\sec^2(e+f\right.}{f}$$
$$= \frac{3b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} - \frac{\cos(e+fx)\left(a-b+b\sec^2(e+f\right.}{f}$$
$$= \frac{3(a-b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f} + \frac{3b\sec(e+fx)\sqrt{a-b+b\sec^2(e+}}{2f}$$

Mathematica [A] time = 1.21397, size = 170, normalized size = 1.5

$$\frac{\sec(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(6\sqrt{2}\sqrt{b}(a-b)\cos^2(e+fx)\tanh^{-1}\left(\frac{\sqrt{(a-b)\cos(2(e+fx))+a+b}}{\sqrt{2}\sqrt{b}}\right)-\right.}{4\sqrt{2}f\sqrt{(a-b)\cos(2(e+fx))+a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((6*Sqrt[2]*(a - b)*Sqrt[b]*ArcTanh[Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]/
(Sqrt[2]*Sqrt[b])]*Cos[e + f*x]^2 - 2*(a - 2*b + (a - b)*Cos[2*(e + f*x)])*
Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])*Sec[e + f*x]*Sqrt[(a + b + (a - b)*
Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(4*Sqrt[2]*f*Sqrt[a + b + (a - b)*Cos[2*
(e + f*x)]])
```

Maple [B] time = 0.053, size = 359, normalized size = 3.2

$$-\frac{\cos(fx+e)}{2fb}\left(\frac{a\left(\cos(fx+e)\right)^2-\left(\cos(fx+e)\right)^2b+b}{\left(\cos(fx+e)\right)^2}\right)^{\frac{3}{2}}\left(3b^{5/2}\ln\left(2\frac{\sqrt{b}\sqrt{a\left(\cos(fx+e)\right)^2-\left(\cos(fx+e)\right)^2b+b}}{\cos(fx+e)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)*(3
*b^(5/2)*ln(2*(b^(1/2)*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(1/2)+b)/cos(f*x+e
))*cos(f*x+e)^2-3*b^(3/2)*ln(2*(b^(1/2)*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(
```

```
1/2)+b)/cos(f*x+e))*cos(f*x+e)^2*a+(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(3/2)*
cos(f*x+e)^2*a-(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(3/2)*cos(f*x+e)^2*b-(a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)^(5/2)+3*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(1/2
)*cos(f*x+e)^2*a*b-3*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(1/2)*cos(f*x+e)^2*b
^2)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(3/2)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.90307, size = 667, normalized size = 5.9

$$\left[-\frac{3\,(a-b)\sqrt{b}\cos\left(fx+e\right)\log\left(-\frac{(a-b)\cos\left(fx+e\right)^2-2\sqrt{b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+2b}{\cos\left(fx+e\right)^2}\right)+2\left(2\,(a-b)\cos\left(fx+e\right)^2-b\right)\sqrt{\frac{(a-b)\cos}{\cos}}}{4f\cos\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(3*(a - b)*sqrt(b)*cos(f*x + e)*log(-((a - b)*cos(f*x + e)^2 - 2*sqrt
(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/c
os(f*x + e)^2) + 2*(2*(a - b)*cos(f*x + e)^2 - b)*sqrt(((a - b)*cos(f*x + e
)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)), -1/2*(3*(a - b)*sqrt(-b)*arctan
(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b)
*cos(f*x + e) + (2*(a - b)*cos(f*x + e)^2 - b)*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2))/(f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

<u>**Giac**</u> **[F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.107 $\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=127

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}+\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2f}+\frac{\sqrt{b}(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2f}$$

```
[Out] -((a^(3/2)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/
f) + ((3*a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e
 + f*x]^2]])/(2*f) + (b*Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/(2*f)
```

Rubi [A] time = 0.140676, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3664, 416, 523, 217, 206, 377, 207}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}+\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2f}+\frac{\sqrt{b}(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/
f) + ((3*a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e
 + f*x]^2]])/(2*f) + (b*Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/(2*f)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(a-b+bx^2\right)^{3/2}}{-1+x^2}dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{2a^2-3ab+b^2+(3a-b)bx^2}{\left(-1+x^2\right)\sqrt{a-b+bx^2}}dx, x\right.}{2f}$$

$$= \frac{b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{\left(-1+x^2\right)\sqrt{a-b+bx^2}}dx, x,\right.}{f}$$

$$= \frac{b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{-1+ax^2}dx, x, \frac{\sec(e}{\sqrt{a-b+b\sec}}\right.}{f}$$

$$= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} + \frac{(3a-b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f} +$$

Mathematica [B] time = 4.81574, size = 418, normalized size = 3.29

$$\sec^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(-4a^{3/2}\cos^2(e+fx)\tanh^{-1}\left(\frac{a-(a-}{\sqrt{a}\sqrt{a\left(\tan^2\left(\frac{1}{2}(e\right.\right.}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sec[(e + f*x)/2]^2*(-4*Sqrt[b]*(-3*a + b)*ArcTanh[(Sqrt[b]*(1 + Tan[(e + f
*x)/2]^2))/Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]]*Co
s[e + f*x]^2 - 4*a^(3/2)*ArcTanh[(a - (a - 2*b)*Tan[(e + f*x)/2]^2)/(Sqrt[a
]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])]*Cos[e + f*
x]^2 - 4*a^(3/2)*ArcTanh[(2*b + a*(-1 + Tan[(e + f*x)/2]^2))/(Sqrt[a]*Sqrt[
4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])]*Cos[e + f*x]^2 +
Sqrt[2]*b*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4] + Sqr
```

```
t[2]*b*Cos[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2
]^4])*Sec[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
/(8*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4])
```

Maple [B] time = 0.164, size = 1249, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/4/f/a^(5/2)/b^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/
2)*4^(1/2)*cos(f*x+e)*(cos(f*x+e)-1)^3*(3*arctanh(1/8*b^(1/2)*4^(1/2)*(cos(
f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((a*cos(
f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*a^(7/2)*cos(f*x+e)^2*b+
4*ln(-4/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)
^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*b^(7/
2)*cos(f*x+e)^2*a-4*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)
^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x
+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/s
in(f*x+e)^2)*b^(7/2)*cos(f*x+e)^2*a-arctanh(1/8*b^(1/2)*4^(1/2)*(cos(f*x+e)
-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((a*cos(f*x+e)
^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*a^(5/2)*cos(f*x+e)^2*b^2-6*co
s(f*x+e)^2*ln(-4/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*
x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)
^2)*b^(5/2)*a^2+6*cos(f*x+e)^2*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a
*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*
a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a
^(1/2)+b)/sin(f*x+e)^2)*b^(5/2)*a^2+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos
(f*x+e)+1)^2)^(1/2)*a^(5/2)*b^(3/2)*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^
2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(5/2)*b^(3/2)-cos(f*x+e)^2*ln(-2/a^(1/2)*(
cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)
^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b
+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*b^(1/2)*a^4-cos(f*x+e)
^2*ln(-4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(
1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(
cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*b^(1/2)*a^4)/sin(f*x+e)^6
/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e), x)
```

Fricas [A] time = 7.21928, size = 1905, normalized size = 15.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a^(3/2)*cos(f*x + e)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqr
t(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f
*x + e)^2 - 1)) - (3*a - b)*sqrt(b)*cos(f*x + e)*log(-((a - b)*cos(f*x + e)
^2 - 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e) + 2*b)/cos(f*x + e)^2) + 2*b*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2))/(f*cos(f*x + e)), -1/2*((3*a - b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a
 - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b)*cos(f*x + e) - a^
(3/2)*cos(f*x + e)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2
- 1)) - b*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e
)), 1/4*(4*sqrt(-a)*a*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*cos(f*x + e)/a)*cos(f*x + e) - (3*a - b)*sqrt(b)*cos(f*x + e)*
log(-((a - b)*cos(f*x + e)^2 - 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*b*sqrt(((a - b)*cos
(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)), 1/2*(2*sqrt(-a)*a*arcta
n(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a
)*cos(f*x + e) - (3*a - b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b)*cos(f*x + e) + b*sqrt(((a - b)*c
os(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e), x)
```

3.108 $\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=167

$$\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f}-\frac{\sqrt{a}(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2f}+\frac{\sqrt{b}(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2f}-$$

```
[Out] -(Sqrt[a]*(a + 3*b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f
*x]^2]])/(2*f) + (Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a -
 b + b*Sec[e + f*x]^2]])/(2*f) + (b*Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x
]^2])/f - (Cot[e + f*x]*Csc[e + f*x]*(a - b + b*Sec[e + f*x]^2)^(3/2))/(2*f
)
```

Rubi [A] time = 0.20553, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3664, 467, 528, 523, 217, 206, 377, 207}

$$\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f}-\frac{\sqrt{a}(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2f}+\frac{\sqrt{b}(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2f}-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Sqrt[a]*(a + 3*b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f
*x]^2]])/(2*f) + (Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a -
 b + b*Sec[e + f*x]^2]])/(2*f) + (b*Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x
]^2])/f - (Cot[e + f*x]*Csc[e + f*x]*(a - b + b*Sec[e + f*x]^2)^(3/2))/(2*f
)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a-b+bx^2\right)^{3/2}}{\left(-1+x^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{2f} + \frac{\text{Subst}\left(\int \frac{\sqrt{a-b+bx}}{-}\right.}{}$$

$$= \frac{b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} - \frac{\cot(e+fx)\csc(e+fx)\left(a-b+\right.}{2f}$$

$$= \frac{b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} - \frac{\cot(e+fx)\csc(e+fx)\left(a-b+\right.}{2f}$$

$$= \frac{b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} - \frac{\cot(e+fx)\csc(e+fx)\left(a-b+\right.}{2f}$$

$$= -\frac{\sqrt{a}(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f} + \frac{\sqrt{b}(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+\right.}{\sqrt{a-b+b\sec^2}}}{2f}$$

Mathematica [B] time = 6.59162, size = 1022, normalized size = 6.12

$$\frac{\sqrt{\frac{\cos(2(e+fx))a+a+b-b\cos(2(e+fx))}{\cos(2(e+fx))+1}}\left(\frac{1}{2}b\sec(e+fx)-\frac{1}{2}a\cot(e+fx)\csc(e+fx)\right)}{f}+\frac{(a^2-b^2)\left(2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\left(\tan^2\left(\frac{1}{2}(e+fx)\right)+1\right)}{\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(\tan^2\left(\frac{1}{2}(e+f\right.\right.}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x
)])]*(-(a*Cot[e + f*x]*Csc[e + f*x])/2 + (b*Sec[e + f*x])/2))/f + (-((a^2 +
 6*a*b + b^2)*(2*Sqrt[a]*ArcTanh[(Sqrt[b]*(1 + Tan[(e + f*x)/2]^2))/Sqrt[4*
b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]] - Sqrt[b]*(ArcTanh[(
a - a*Tan[(e + f*x)/2]^2 + 2*b*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*Tan[(e
 + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])] + ArcTanh[(2*b + a*(-1 + Ta
n[(e + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e +
f*x)/2]^2)^2])]))*(1 + Cos[e + f*x])*Sqrt[(1 + Cos[2*(e + f*x)])/(1 + Cos[e
 + f*x])^2]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]
*(-1 + Tan[(e + f*x)/2]^2)*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(4*b*Tan[(e + f*x)
/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2)/(1 + Tan[(e + f*x)/2]^2)^2])/(4*Sqrt
[a]*Sqrt[b]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]*Sqrt[(-1 + Tan[(e + f*x)
/2]^2)^2]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]) + (
(a^2 - b^2)*(2*Sqrt[a]*ArcTanh[(Sqrt[b]*(1 + Tan[(e + f*x)/2]^2))/Sqrt[4*b*
Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]] + Sqrt[b]*(ArcTanh[(a
- a*Tan[(e + f*x)/2]^2 + 2*b*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*Tan[(e +
 f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])] + ArcTanh[(2*b + a*(-1 + Tan[
(e + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*
x)/2]^2)^2])]))*(1 + Cos[e + f*x])*Sqrt[(1 + Cos[2*(e + f*x)])/(1 + Cos[e +
 f*x])^2]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(
-1 + Tan[(e + f*x)/2]^2)*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(4*b*Tan[(e + f*x)/2
]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2)/(1 + Tan[(e + f*x)/2]^2)^2])/(4*Sqrt[a
]*Sqrt[b]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]*Sqrt[(-1 + Tan[(e + f*x)/2
]^2)^2]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]))/(2*f
)
```

Maple [B] time = 0.192, size = 2904, normalized size = 17.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/8/f/a^(5/2)/b^(1/2)*(cos(f*x+e)-1)^2*(-10*cos(f*x+e)^3*ln(-2/a^(1/2)*(co
s(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2
)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b
)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*b^(7/2)*a+10*cos(f*x+e)^
3*ln(-4/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)
^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*b^(7/
2)*a+6*cos(f*x+e)^3*arctanh(1/8*b^(1/2)*4^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*
4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/(cos(f*x+e)+1)^2)^(1/2))*a^(7/2)*b+2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b
```

```
)/(cos(f*x+e)+1)^2)^(1/2)*a^(7/2)*b^(1/2)*cos(f*x+e)^2+2*((a*cos(f*x+e)^2-c
os(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(5/2)*b^(3/2)*cos(f*x+e)^2+18*co
s(f*x+e)^3*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*
x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)
^2)*b^(5/2)*a^2+10*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^
2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+
e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/si
n(f*x+e)^2)*b^(7/2)*cos(f*x+e)^2*a-18*cos(f*x+e)^3*ln(-4/a^(1/2)*(cos(f*x+e
)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)
*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(
f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*b^(5/2)*a^2-10*ln(-4/a^(1/2)*(c
os(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^
2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*b^(7/2)*cos(f*x+e)^2*a+
2*cos(f*x+e)^3*arctanh(1/8*b^(1/2)*4^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/
2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/
(cos(f*x+e)+1)^2)^(1/2))*a^(5/2)*b^2-6*arctanh(1/8*b^(1/2)*4^(1/2)*(cos(f*x
+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*a^(7/2)*cos(f*x+e)^2*b-3*c
os(f*x+e)^3*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e
)^2)*b^(3/2)*a^3-18*cos(f*x+e)^2*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e
)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)
*a^(1/2)+b)/sin(f*x+e)^2)*b^(5/2)*a^2+18*cos(f*x+e)^2*ln(-4/a^(1/2)*(cos(f*
x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1
/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(c
os(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*b^(5/2)*a^2-2*arctanh(1/8*b^
(1/2)*4^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/si
n(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*a^(5
/2)*cos(f*x+e)^2*b^2-3*cos(f*x+e)^3*ln(-4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a
*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x
+e)-1))*b^(3/2)*a^3-2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^
(1/2)*a^(5/2)*b^(3/2)-cos(f*x+e)^3*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)
*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x
+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/
2)*a^(1/2)+b)/sin(f*x+e)^2)*b^(1/2)*a^4+3*cos(f*x+e)^2*ln(-2/a^(1/2)*(cos(f
*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(
1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(
cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*b^(3/2)*a^3-cos(f*x+e)^3*ln
(-4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*
a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f
*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*b^(1/2)*a^4+3*cos(f*x+e)^2*ln(
-4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a
^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*
x+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*b^(3/2)*a^3+cos(f*x+e)^2*ln(-2/
a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(
f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*b^(1/2)*a^4+c
os(f*x+e)^2*ln(-4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e
)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)
^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*b^(1/2)*a^4)*cos
(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)*4^(1/2)/((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)/sin(f*x+e)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^3, x)
```

Fricas [A] time = 7.95492, size = 2515, normalized size = 15.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + 3*b)*cos(f*x + e)^3 - (a + 3*b)*cos(f*x + e))*sqrt(a)*log(-2*((
a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)) + ((3*a + b)*cos(f*x +
 e)^3 - (3*a + b)*cos(f*x + e))*sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sq
rt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)
/cos(f*x + e)^2) + 2*((a + b)*cos(f*x + e)^2 - b)*sqrt(((a - b)*cos(f*x + e
)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), -1/4*(2*((3*
a + b)*cos(f*x + e)^3 - (3*a + b)*cos(f*x + e))*sqrt(-b)*arctan(sqrt(-b)*sq
rt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) - ((a + 3*b
)*cos(f*x + e)^3 - (a + 3*b)*cos(f*x + e))*sqrt(a)*log(-2*((a - b)*cos(f*x
+ e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*
x + e) + a + b)/(cos(f*x + e)^2 - 1)) - 2*((a + b)*cos(f*x + e)^2 - b)*sqrt
(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*
x + e)), 1/4*(2*((a + 3*b)*cos(f*x + e)^3 - (a + 3*b)*cos(f*x + e))*sqrt(-a
)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
 + e)/a) + ((3*a + b)*cos(f*x + e)^3 - (3*a + b)*cos(f*x + e))*sqrt(b)*log(
-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*((a + b)*cos(f*x + e)^2
 - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)^3
 - f*cos(f*x + e)), 1/2*(((a + 3*b)*cos(f*x + e)^3 - (a + 3*b)*cos(f*x + e))
*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*cos(f*x + e)/a) - ((3*a + b)*cos(f*x + e)^3 - (3*a + b)*cos(f*x + e))*sqrt
(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(
f*x + e)/b) + ((a + b)*cos(f*x + e)^2 - b)*sqrt(((a - b)*cos(f*x + e)^2 + b
)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^3, x)
```

3.109 $\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=223

$$-\frac{3\left(a^2+6ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8\sqrt{a}f}+\frac{3(a+3b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{8f}-\frac{3(a+b)\csc^2(e+fx)\sec}{}$$

```
[Out] (-3*(a^2 + 6*a*b + b^2)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e
 + f*x]^2]])/(8*Sqrt[a]*f) + (3*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Sec[e + f*
x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/(2*f) + (3*(a + 3*b)*Sec[e + f*x]*Sqrt
[a - b + b*Sec[e + f*x]^2])/(8*f) - (3*(a + b)*Csc[e + f*x]^2*Sec[e + f*x]*
Sqrt[a - b + b*Sec[e + f*x]^2])/(8*f) - (Cot[e + f*x]*Csc[e + f*x]^3*(a - b
 + b*Sec[e + f*x]^2)^(3/2))/(4*f)
```

Rubi [A] time = 0.36181, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3664, 467, 577, 582, 523, 217, 206, 377, 207}

$$-\frac{3\left(a^2+6ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8\sqrt{a}f}+\frac{3(a+3b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{8f}-\frac{3(a+b)\csc^2(e+fx)\sec}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```


```
[Out] (-3*(a^2 + 6*a*b + b^2)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e
 + f*x]^2]])/(8*Sqrt[a]*f) + (3*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Sec[e + f*
x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/(2*f) + (3*(a + 3*b)*Sec[e + f*x]*Sqrt
[a - b + b*Sec[e + f*x]^2])/(8*f) - (3*(a + b)*Csc[e + f*x]^2*Sec[e + f*x]*
Sqrt[a - b + b*Sec[e + f*x]^2])/(8*f) - (Cot[e + f*x]*Csc[e + f*x]^3*(a - b
 + b*Sec[e + f*x]^2)^(3/2))/(4*f)
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
```

```
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a-b+bx^2\right)^{3/2}}{\left(-1+x^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a-b+bx^2}}{(-1 \cdots}\right.}{\cdots}$$

$$= -\frac{3(a+b)\csc^2(e+fx)\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8f} - \frac{\cot(e+fx)\csc \cdots}{\cdots}$$

$$= \frac{3(a+3b)\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8f} - \frac{3(a+b)\csc^2(e+fx)\sec(e \cdots}{8 \cdots}$$

$$= \frac{3(a+3b)\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8f} - \frac{3(a+b)\csc^2(e+fx)\sec(e \cdots}{8 \cdots}$$

$$= \frac{3(a+3b)\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8f} - \frac{3(a+b)\csc^2(e+fx)\sec(e \cdots}{8 \cdots}$$

$$= -\frac{3\left(a^2+6ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8\sqrt{a}f} + \frac{3\sqrt{b}(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\sec \cdots}{\sqrt{a-b+b\,s \cdots}}\right.}{2f}$$

Mathematica [A] time = 4.85472, size = 409, normalized size = 1.83

$$\frac{\cos(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(-\frac{3\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\cos^2(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)}\left(a^2+6ab+b^2\right)\left(\tanh^{-1}\left(\frac{\cdots}{\sqrt{a}\sqrt{a\left(\tan^2 \cdots\right.}}\right.\right.}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-2*C
sc[e + f*x]^2*(3*a + 5*b + 2*a*Csc[e + f*x]^2) + 8*b*Sec[e + f*x]^2 - (3*(-
8*Sqrt[a]*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*(1 + Tan[(e + f*x)/2]^2))/Sqrt[4
*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]] + (a^2 + 6*a*b + b^
2)*(ArcTanh[(a - (a - 2*b)*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*Tan[(e + f
*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])] + ArcTanh[(2*b + a*(-1 + Tan[(e
 + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)
/2]^2)^2])]))*Sec[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]^2*Sec[(e + f*x)/2]^4])/(
Sqrt[a]*Sqrt[(-1 + Tan[(e + f*x)/2]^2)^2]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(
-1 + Tan[(e + f*x)/2]^2)^2])))/(16*Sqrt[2]*f)
```

Maple [B] time = 0.352, size = 6194, normalized size = 27.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^5, x)
```

Fricas [A] time = 10.1174, size = 3437, normalized size = 15.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*((a^2 + 6*a*b + b^2)*cos(f*x + e)^5 - 2*(a^2 + 6*a*b + b^2)*cos(f*
x + e)^3 + (a^2 + 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*log(-2*((a - b)*cos(f*
x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(
f*x + e) + a + b)/(cos(f*x + e)^2 - 1)) + 12*((a^2 + a*b)*cos(f*x + e)^5 -
2*(a^2 + a*b)*cos(f*x + e)^3 + (a^2 + a*b)*cos(f*x + e))*sqrt(b)*log(-((a -
 b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*(3*(a^2 + 3*a*b)*cos(f*x + e)
^4 - (5*a^2 + 13*a*b)*cos(f*x + e)^2 + 4*a*b)*sqrt(((a - b)*cos(f*x + e)^2
+ b)/cos(f*x + e)^2))/(a*f*cos(f*x + e)^5 - 2*a*f*cos(f*x + e)^3 + a*f*cos(
f*x + e)), 1/8*(3*((a^2 + 6*a*b + b^2)*cos(f*x + e)^5 - 2*(a^2 + 6*a*b + b^
2)*cos(f*x + e)^3 + (a^2 + 6*a*b + b^2)*cos(f*x + e))*sqrt(-a)*arctan(sqrt(
-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) + 6*(
(a^2 + a*b)*cos(f*x + e)^5 - 2*(a^2 + a*b)*cos(f*x + e)^3 + (a^2 + a*b)*cos
(f*x + e))*sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + (3
*(a^2 + 3*a*b)*cos(f*x + e)^4 - (5*a^2 + 13*a*b)*cos(f*x + e)^2 + 4*a*b)*sq
rt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*f*cos(f*x + e)^5 - 2*a*
f*cos(f*x + e)^3 + a*f*cos(f*x + e)), -1/16*(24*((a^2 + a*b)*cos(f*x + e)^5
 - 2*(a^2 + a*b)*cos(f*x + e)^3 + (a^2 + a*b)*cos(f*x + e))*sqrt(-b)*arctan
(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b)
 - 3*((a^2 + 6*a*b + b^2)*cos(f*x + e)^5 - 2*(a^2 + 6*a*b + b^2)*cos(f*x +
e)^3 + (a^2 + 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*log(-2*((a - b)*cos(f*x +
e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e) + a + b)/(cos(f*x + e)^2 - 1)) - 2*(3*(a^2 + 3*a*b)*cos(f*x + e)^4 - (
5*a^2 + 13*a*b)*cos(f*x + e)^2 + 4*a*b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/c
os(f*x + e)^2))/(a*f*cos(f*x + e)^5 - 2*a*f*cos(f*x + e)^3 + a*f*cos(f*x +
e)), 1/8*(3*((a^2 + 6*a*b + b^2)*cos(f*x + e)^5 - 2*(a^2 + 6*a*b + b^2)*cos
(f*x + e)^3 + (a^2 + 6*a*b + b^2)*cos(f*x + e))*sqrt(-a)*arctan(sqrt(-a)*sq
rt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) - 12*((a^2
+ a*b)*cos(f*x + e)^5 - 2*(a^2 + a*b)*cos(f*x + e)^3 + (a^2 + a*b)*cos(f*x
+ e))*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x +
```

```
e)^2)*cos(f*x + e)/b) + (3*(a^2 + 3*a*b)*cos(f*x + e)^4 - (5*a^2 + 13*a*b)*
cos(f*x + e)^2 + 4*a*b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/
(a*f*cos(f*x + e)^5 - 2*a*f*cos(f*x + e)^3 + a*f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^5, x)
```

3.110 $\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=222

$$\frac{3\left(a^2-8ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f\sqrt{a-b}}-\frac{3(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f}+\frac{3(a-2b)\sin^2(e+fx)\tan(e}{8}$$

```
[Out] (3*(a^2 - 8*a*b + 8*b^2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e
 + f*x]^2]])/(8*Sqrt[a - b]*f) + (3*(a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[
e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(2*f) - (3*(a - 4*b)*Tan[e + f*x]*Sq
rt[a + b*Tan[e + f*x]^2])/(8*f) + (3*(a - 2*b)*Sin[e + f*x]^2*Tan[e + f*x]*
Sqrt[a + b*Tan[e + f*x]^2])/(8*f) - (Cos[e + f*x]*Sin[e + f*x]^3*(a + b*Tan
[e + f*x]^2)^(3/2))/(4*f)
```

Rubi [A] time = 0.321516, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3663, 467, 577, 582, 523, 217, 206, 377, 203}

$$\frac{3\left(a^2-8ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f\sqrt{a-b}}-\frac{3(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f}+\frac{3(a-2b)\sin^2(e+fx)\tan(e}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a^2 - 8*a*b + 8*b^2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e
 + f*x]^2]])/(8*Sqrt[a - b]*f) + (3*(a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[
e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(2*f) - (3*(a - 4*b)*Tan[e + f*x]*Sq
rt[a + b*Tan[e + f*x]^2])/(8*f) + (3*(a - 2*b)*Sin[e + f*x]^2*Tan[e + f*x]*
Sqrt[a + b*Tan[e + f*x]^2])/(8*f) - (Cos[e + f*x]*Sin[e + f*x]^3*(a + b*Tan
[e + f*x]^2)^(3/2))/(4*f)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
```

```
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^{3/2}}{\left(1+x^2\right)^3}dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx^2}(3a}{(1+x^2)}\right.}{}$$

$$= \frac{3(a-2b)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} - \frac{\cos(e+fx)\sin}{}$$

$$= -\frac{3(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} + \frac{3(a-2b)\sin^2(e+fx)\tan}{}$$

$$= -\frac{3(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} + \frac{3(a-2b)\sin^2(e+fx)\tan}{}$$

$$= -\frac{3(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} + \frac{3(a-2b)\sin^2(e+fx)\tan}{}$$

$$= \frac{3\left(a^2-8ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8\sqrt{a-b}f} + \frac{3(a-2b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan}{\sqrt{a+b\tan}}\right.}{2f}$$

Mathematica [C] time = 4.74926, size = 278, normalized size = 1.25

$$\frac{\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\frac{3a\sin(2(e+fx))\csc^2(e+fx)\left(\left(a^2-5ab+4b^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}}{\sqrt{2}}\right.\right.\right.}{\sqrt{2}b(a-b)\sqrt{\frac{\csc^2(e+fx)((a-b)\cos}{b}}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*((3*a*Csc[e + f*x]
^2*((a^2 - 5*a*b + 4*b^2)*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e
+ f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] - (a^2 - 8*a*b + 8*b^2)*EllipticPi
[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]
^2)/b]/Sqrt[2]], 1])*Sin[2*(e + f*x)])/(Sqrt[2]*(a - b)*b*Sqrt[((a + b + (a
 - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]) + ((-8*a + 18*b)*Sin[2*(e + f*x
)] + (a - b)*Sin[4*(e + f*x)] + 16*b*Tan[e + f*x])/4))/(8*Sqrt[2]*f)
```

Maple [C] time = 0.416, size = 2630, normalized size = 11.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/8/f/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*(-5*cos(f*x+e)^5*((2*I*b^(1
/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2-4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(
1/2)*b^2-12*cos(f*x+e)^5*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2+2*b^
2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^7-2*b^2*((2*I*b^(1/2
)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^6+2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*
b)/a)^(1/2)*cos(f*x+e)^7*a^2-2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*co
s(f*x+e)^6*a^2-48*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2
)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e
)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-c
os(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)
*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a
-b)^(1/2)+a-2*b)/a)^(1/2))*a*b-48*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/a*(I*c
os(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+
e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/
2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticP
i((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*
I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/(
(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*b^2-3*cos(f*x+e)^2*sin(f*x+e)*2^(
1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e
)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)
^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(
1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin
(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2
)/a^2)^(1/2))*a^2+6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1
/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x
+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)
-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-
1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-
b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*
(a-b)^(1/2)+a-2*b)/a)^(1/2))*a^2+48*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/a*(I
*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*
x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(
1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*Ellipti
cPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/
(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2
)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*b^2+5*cos(f*x+e)^4*((2*I*b^(1/
2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2+12*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2
)+a-2*b)/a)^(1/2)*b^2+6*cos(f*x+e)^3*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1
/2)*b^2-6*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2+4*cos(
f*x+e)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2+12*EllipticF((cos(f*x+
e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a
-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)
^2*sin(f*x+e)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+
cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1
/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x
+e)+1))^(1/2)*2^(1/2)*a*b+24*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^
(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I
*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1
/2))*cos(f*x+e)^2*sin(f*x+e)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/
2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*
cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x
+e)-b)/(cos(f*x+e)+1))^(1/2)*2^(1/2)*a*b-4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)
/a)^(1/2)*cos(f*x+e)^7*a*b+4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(
f*x+e)^6*a*b+17*cos(f*x+e)^5*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b-
17*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b-cos(f*x+e)^3*
((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b+cos(f*x+e)^2*((2*I*b^(1/2)*(a
-b)^(1/2)+a-2*b)/a)^(1/2)*a*b)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b
)/cos(f*x+e)^2)^(3/2)*sin(f*x+e)/(cos(f*x+e)-1)/(a*cos(f*x+e)^2-cos(f*x+e)^
```

```
2*b+b)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

3.111 $\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=165

$$\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{(a-4b)\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}+\frac{\sqrt{b}(3a-4b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}-\text{si}$$

```
[Out] ((a - 4*b)*Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e +
 f*x]^2]])/(2*f) + ((3*a - 4*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt
[a + b*Tan[e + f*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2]
)/f - (Cos[e + f*x]*Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2))/(2*f)
```

Rubi [A] time = 0.201776, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3663, 467, 528, 523, 217, 206, 377, 203}

$$\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{(a-4b)\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}+\frac{\sqrt{b}(3a-4b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}-\text{si}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```


```
[Out] ((a - 4*b)*Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e +
 f*x]^2]])/(2*f) + ((3*a - 4*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt
[a + b*Tan[e + f*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2]
)/f - (Cos[e + f*x]*Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2))/(2*f)
```


Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^{3/2}}{\left(1+x^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{2f} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}\left(a+4b\right.}{1+x^2}\right.}{2}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\cos(e+fx)\sin(e+fx)\left(a+b\tan^2\right.}{2f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\cos(e+fx)\sin(e+fx)\left(a+b\tan^2\right.}{2f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\cos(e+fx)\sin(e+fx)\left(a+b\tan^2\right.}{2f}$$

$$= \frac{(a-4b)\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f} + \frac{(3a-4b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+\right.}{\sqrt{a+b\tan^2(e\right.}}{2f}$$

Mathematica [C] time = 5.17749, size = 324, normalized size = 1.96

$$-\frac{\sin(2(e+fx))\tan(e+fx)\sec^2(e+fx)\left(-4\sqrt{2}a(a-2b)\cot(e+fx)\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\cdots}}{\cdots}\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Sec[e + f*x]^2*(-4*Sqrt[2]*a*(a - 2*b)*Cot[e + f*x]*Sqrt[((a + b + (a - b
)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[((a + b + (a -
 b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + 4*Sqrt[2]*a*(a - 4*
b)*Cot[e + f*x]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]
*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*C
sc[e + f*x]^2)/b]/Sqrt[2]], 1] + (3*a^2 - 6*a*b - 5*b^2 + 4*(a - b)^2*Cos[2
*(e + f*x)] + (a - b)^2*Cos[4*(e + f*x)])*Csc[e + f*x]*Sec[e + f*x])*Sin[2*
(e + f*x)]*Tan[e + f*x])/(16*Sqrt[2]*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x
)])*Sec[e + f*x]^2])
```

Maple [C] time = 0.235, size = 2261, normalized size = 13.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*(cos(f*x+e)^2*sin(f*x+e)*2
^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x
+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-
b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))
^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/s
in(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b
^2)/a^2)^(1/2))*a^2-2*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-
2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*
a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^2*sin(f*x+e)*(1/a*(I*cos(f*x+e)*b
^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(
f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1
/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*2^(1/2)*a*b-2*cos(f*
x+e)^2*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*
(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos
(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)
-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/
2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b
^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2
))*a^2+10*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^
(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1
/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e
)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b
^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a
-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2
)+a-2*b)/a)^(1/2))*a*b-8*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)
*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(co
s(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^
```

```
(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*
x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2
)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(
1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*b^2-6*EllipticPi((cos(f*x+e)-1)*((2*I*b^(
1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*
b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a
-2*b)/a)^(1/2))*cos(f*x+e)^2*sin(f*x+e)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1
/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2
)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*
a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*2^(1/2)*a*b+8*cos(f*x+e)^2*sin(f*x+
e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos
(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)
*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)
+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1
/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/
2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*b^2+cos(f*x+e
)^5*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2-2*cos(f*x+e)^5*((2*I*b^(1
/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b+cos(f*x+e)^5*((2*I*b^(1/2)*(a-b)^(1/2)+
a-2*b)/a)^(1/2)*b^2-cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*
a^2+2*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b-cos(f*x+e)
^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2-cos(f*x+e)*((2*I*b^(1/2)*(
a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2+((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^
2)*cos(f*x+e)*sin(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(
3/2)/(cos(f*x+e)-1)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

Fricas [B] time = 60.2207, size = 4698, normalized size = 28.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((a - 4*b)*sqrt(-a + b)*cos(f*x + e)*log(128*(a^4 - 4*a^3*b + 6*a^2*
b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*
b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3
+ 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*b
^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 +
8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b +
5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*
cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a
 + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 2*(
3*a - 4*b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 +
 8*(a*b - 2*b^2)*cos(f*x + e)^2 - 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x
```

```
 + e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
e) + 8*b^2)/cos(f*x + e)^4) + 8*((a - b)*cos(f*x + e)^2 - b)*sqrt(((a - b)*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), -1/16*(
4*(3*a - 4*b)*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x +
 e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b - b^
2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) + (a - 4*b)*sqrt(-a +
b)*cos(f*x + e)*log(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x
 + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*cos(f*x + e)^6
+ 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a
^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*b^4 - 32*(a^4 - 11*a^3*b + 34
*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 + 8*(16*(a^3 - 3*a^2*b + 3*a*b
^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos(f*x + e
)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*cos(f*x + e)^3 - (a^3 - 10*a
^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a + b)*sqrt(((a - b)*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*((a - b)*cos(f*x + e)^2 - b)*
sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(f*x
+ e)), 1/8*(sqrt(a - b)*(a - 4*b)*arctan(-1/4*(8*(a^2 - 2*a*b + b^2)*cos(f*
x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*b + 8*b^2)*c
os(f*x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/
((2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*a*b^2 - 2*b^
3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*
x + e) - (3*a - 4*b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*
x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 - 4*((a - 2*b)*cos(f*x + e)^3 + 2
*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((a - b)*cos(f*x + e)^2 - b)*sqrt
(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)
), 1/8*(sqrt(a - b)*(a - 4*b)*arctan(-1/4*(8*(a^2 - 2*a*b + b^2)*cos(f*x +
e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*b + 8*b^2)*cos(f
*x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*a*b^2 - 2*b^3 -
(a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x +
e) - 2*(3*a - 4*b)*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(
f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b
 - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - 4*((a - b)*cos(
f*x + e)^2 - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e))/(f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

3.112 $\int \left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=125

$$\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{\sqrt{b}(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}$$

```
[Out] ((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f + ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e
 + f*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Rubi [A] time = 0.099861, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3661, 416, 523, 217, 206, 377, 203}

$$\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{\sqrt{b}(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f + ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e
 + f*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$
$$=\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{\text{Subst}\left(\int \frac{a(2a-b)+(3a-2b)bx^2}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx,x,\tan(e+fx)\right)}{2f}$$
$$=\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx,x,\tan(e+fx)\right)}{f}$$
$$=\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx,x,\frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$
$$=\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{(3a-2b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}+\frac{b\tan(e+\ldots}{\ldots}$$

Mathematica [C] time = 1.33439, size = 233, normalized size = 1.86

$$\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}-i(a-b)^{3/2}\log\left(-\frac{4i\left(\sqrt{a-b}\sqrt{a+b\tan^2(e+fx)}+a-ib\tan(e+fx)\right)}{(a-b)^{5/2}(\tan(e+fx)+i)}\right)+i(a-b)^{3/2}\log\left(\frac{4i\left(\sqrt{a-b}\sqrt{a+b\ldots}\right)}{(a-\ldots}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-I)*(a - b)^(3/2)*Log[((-4*I)*(a - I*b*Tan[e + f*x] + Sqrt[a - b]*Sqrt[a
+ b*Tan[e + f*x]^2]))/((a - b)^(5/2)*(I + Tan[e + f*x]))] + I*(a - b)^(3/2)
*Log[((4*I)*(a + I*b*Tan[e + f*x] + Sqrt[a - b]*Sqrt[a + b*Tan[e + f*x]^2])
```

```
)/((a - b)^(5/2)*(-I + Tan[e + f*x]))] + (3*a - 2*b)*Sqrt[b]*Log[b*Tan[e +
f*x] + Sqrt[b]*Sqrt[a + b*Tan[e + f*x]^2]] + b*Tan[e + f*x]*Sqrt[a + b*Tan[
e + f*x]^2])/(2*f)
```

Maple [B] time = 0.026, size = 297, normalized size = 2.4

$$\frac{b\tan\left(fx+e\right)}{2f}\sqrt{a+b\left(\tan\left(fx+e\right)\right)^{2}}+\frac{3a}{2f}\sqrt{b}\ln\left(\sqrt{b}\tan\left(fx+e\right)+\sqrt{a+b\left(\tan\left(fx+e\right)\right)^{2}}\right)-\frac{1}{f}b^{\frac{3}{2}}\ln\left(\sqrt{b}\tan\left(fx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/2*b*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+3/2/f*b^(1/2)*a*ln(b^(1/2)*tan(
f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-1/f*b^(3/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan
(f*x+e)^2)^(1/2))+1/f*(b^4*(a-b))^(1/2)/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^
(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-2/f*a/b*(b^4*(a-b))^(1/2)/(a-b)*
arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))+1/f
*a^2*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*ta
n(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2), x)
```

Fricas [A] time = 5.67117, size = 1382, normalized size = 11.06

$$\left[-\frac{(3a-2b)\sqrt{b}\log\left(2b\tan\left(fx+e\right)^{2}-2\sqrt{b\tan\left(fx+e\right)^{2}+a}\sqrt{b}\tan\left(fx+e\right)+a\right)+2(a-b)\sqrt{-a+b}\log\left(-\frac{(a-2b)\tan(\ldots}{\ldots}\right.}{4f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((3*a - 2*b)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2
 + a)*sqrt(b)*tan(f*x + e) + a) + 2*(a - b)*sqrt(-a + b)*log(-((a - 2*b)*ta
n(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/
(tan(f*x + e)^2 + 1)) - 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f, -1/
2*((3*a - 2*b)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f
```

```
*x + e))) - (-a + b)^(3/2)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*
x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - sqrt(b
*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f, 1/4*(4*(a - b)^(3/2)*arctan(-sqrt(b
*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (3*a - 2*b)*sqrt(b)*log(
2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a)
 + 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f, 1/2*(2*(a - b)^(3/2)*arc
tan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (3*a - 2*b)*s
qrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + sqrt
(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx + e\right)^2 + a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2), x)
```

3.113 $\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=100

$$\frac{3b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{f}$$

```
[Out] (3*a*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(2
*f) + (3*b*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*f) - (Cot[e + f*x]*(
a + b*Tan[e + f*x]^2)^(3/2))/f
```

Rubi [A] time = 0.0988543, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3663, 277, 195, 217, 206}

$$\frac{3b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```


```
[Out] (3*a*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(2
*f) + (3*b*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*f) - (Cot[e + f*x]*(
a + b*Tan[e + f*x]^2)^(3/2))/f
```


Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^2}dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{f} + \frac{(3b)\,\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \tan\right.}{f}$$

$$= \frac{3b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} - \frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/}}{f}$$

$$= \frac{3b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} - \frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/}}{f}$$

$$= \frac{3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f} + \frac{3b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} - $$

Mathematica [C] time = 2.54962, size = 220, normalized size = 2.2

$$\frac{\csc(e+fx)\sec^3(e+fx)\left(3\sqrt{2}ab\sin^2(2(e+fx))\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+f}{b}}}{\sqrt{2}}\right.\right.\right.}{8\sqrt{2}f\sqrt{\sec^2(e+fx)((a-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Csc[e + f*x]*Sec[e + f*x]^3*(-6*a^2 - a*b + 3*b^2 - 4*(2*a^2 + b^2)*Cos[2*
(e + f*x)] - 2*a^2*Cos[4*(e + f*x)] + a*b*Cos[4*(e + f*x)] + b^2*Cos[4*(e +
 f*x)] + 3*Sqrt[2]*a*b*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x
]^2)/b]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f
*x]^2)/b]/Sqrt[2]], 1]*Sin[2*(e + f*x)]^2))/(8*Sqrt[2]*f*Sqrt[(a + b + (a -
 b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Maple [C] time = 0.203, size = 1355, normalized size = 13.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e
)^2*b+b)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)*(6*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/
2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+
e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-
cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1
)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a
-b)^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^3*sin(f*x+e)*a*b-3*EllipticF((cos(f*x
+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(
a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e
)^3*sin(f*x+e)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)
+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(
1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*
x+e)+1))^(1/2)*2^(1/2)*a*b+6*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^
(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I
*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1
/2))*cos(f*x+e)^2*sin(f*x+e)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/
2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*
cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x
+e)-b)/(cos(f*x+e)+1))^(1/2)*2^(1/2)*a*b-3*EllipticF((cos(f*x+e)-1)*((2*I*b
^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I
*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^2*sin(f*x+e)
*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-
b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/
2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)
*2^(1/2)*a*b+2*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2-c
os(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b-cos(f*x+e)^4*((2*
I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2+cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^
(1/2)+a-2*b)/a)^(1/2)*a*b+2*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a
)^(1/2)*b^2-((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2)/sin(f*x+e)/(a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.4078, size = 964, normalized size = 9.64

$$\left[\frac{3\,a\sqrt{b}\cos\left(fx+e\right)\log\left(\frac{\left(a^2-8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4+8\left(ab-2\,b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a-2\,b\right)\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\sin\left(fx+e\right)+}{\cos\left(fx+e\right)^4}\right.}{8\,f\cos\left(fx+e\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a*sqrt(b)*cos(f*x + e)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 +
8*(a*b - 2*b^2)*cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x
+ e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((2*a + b)*cos(f*x + e)^2 - b)*
sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)*sin(f*x
+ e)), -1/4*(3*a*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b -
 b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)*sin(f*x + e) + 2*((
2*a + b)*cos(f*x + e)^2 - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)
^2))/(f*cos(f*x + e)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^2, x)
```

3.114 $\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=162

$$\frac{b(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af}+\frac{\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^5}{3af}$$

```
[Out] (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]
^2]])/(2*f) + (b*(3*a + 2*b)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*a*
f) - ((3*a + 2*b)*Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2))/(3*a*f) - (Cot
[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(5/2))/(3*a*f)
```

Rubi [A] time = 0.136603, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3663, 453, 277, 195, 217, 206}

$$\frac{b(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af}+\frac{\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^5}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]
^2]])/(2*f) + (b*(3*a + 2*b)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*a*
f) - ((3*a + 2*b)*Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2))/(3*a*f) - (Cot
[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(5/2))/(3*a*f)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\operatorname{Subst}\left(\int \frac{(1+x^2)(a+bx^2)^{3/2}}{x^4}dx,x,\tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{3af} + \frac{(3a+2b)\operatorname{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x^2}dx,\right.}{3af}$$

$$= -\frac{(3a+2b)\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3af} - \frac{\cot^3(e+fx)\left(a+b\tan\right.}{3af}$$

$$= \frac{b(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af} - \frac{(3a+2b)\cot(e+fx)\left(a+\right.}{3af}$$

$$= \frac{b(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af} - \frac{(3a+2b)\cot(e+fx)\left(a+\right.}{3af}$$

$$= \frac{\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f} + \frac{b(3a+2b)\tan(e+fx)\sqrt{a+b\,\mathrm{ta}}}{2af}$$

Mathematica [C] time = 1.73981, size = 177, normalized size = 1.09

$$\frac{\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\frac{3\sqrt{2}(3a+2b)\cot(e+fx)\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}}{\sqrt{2}}\right),1\right)}{\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}} - 4(a+2b\right.}{6\sqrt{2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-4*(a + 2*b)*Cot[
e + f*x] - 2*a*Cot[e + f*x]*Csc[e + f*x]^2 + (3*Sqrt[2]*(3*a + 2*b)*Cot[e +
```

```
 f*x]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x
]^2)/b]/Sqrt[2]], 1])/Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]
^2)/b] + 3*b*Tan[e + f*x]))/(6*Sqrt[2]*f)
```

Maple [C] time = 0.586, size = 4594, normalized size = 28.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x)

[Out] -1/6/f/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*(9*2^(1/2)*(1/a*(I*cos(f*x
+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/
(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-
b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(
f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2
)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*sin(f*
x+e)*cos(f*x+e)^5*a*b-9*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+
a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2
)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^3*sin(f*x+e)*(1/a*(I*cos(f*x+e)
*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(co
s(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^
(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*2^(1/2)*a*b-3*((2*
I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2+11*b^2*((2*I*b^(1/2)*(a-b)^(1/2)+
a-2*b)/a)^(1/2)*cos(f*x+e)^6-4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*co
s(f*x+e)^6*a^2+6*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(
a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(
f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-
b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)
+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/
2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^5*b^2-12*sin(f*x+e)
*cos(f*x+e)^5*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b
)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x
+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/
(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a
-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2
)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*b^
2+6*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-
I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-
2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*
cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*
(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2
)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*b^2-18*sin(f*x+e)*cos(f*x+e)^5
*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f
*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(
a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1
))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2
)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)
-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*b+9*sin(f*x+e
)*cos(f*x+e)^4*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-
b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*
x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)
/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a
-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)
*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*b-18*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/a
*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos
```

```
(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*
b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*Elli
pticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),
1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1
/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*b+12*cos(f*x+e)^2*sin(f*x+
e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos
(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)
*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)
+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1
/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/
2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*b^2+6*cos(f*x
+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2-25*cos(f*x+e)^4*((2*I*b
^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2+17*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^
(1/2)+a-2*b)/a)^(1/2)*b^2-9*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1
/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^
(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^2*sin(f*x+e)*(1/a*(I*cos(f*
x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)
/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a
-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*2^(1/2)*a*b+18
*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*
x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/
a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^2*sin(f*x+e)
*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-
b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/
2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)
*2^(1/2)*a*b+18*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a
-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f
*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b
)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)
+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1
/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*
cos(f*x+e)^3*sin(f*x+e)*a*b-12*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/a*(I*cos(
f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+
b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*
(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((
cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b
^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*
I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*b^2-6*sin(f*x+e)*cos(f*x+e)^3*2^(1/2
)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a
-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1
/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2
)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*
x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a
^2)^(1/2))*b^2+12*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2
)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e
)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-c
os(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)
*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^
(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-
b)^(1/2)+a-2*b)/a)^(1/2))*b^2-6*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/a*(I*cos
(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)
+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)
*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((
cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^
(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*b^
2-7*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^6*a*b+4*cos(f*x+e)
^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b+3*cos(f*x+e)^2*((2*I*b^(1/
2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^
2*b+b)/cos(f*x+e)^2)^(3/2)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2/sin(f*x+e)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 8.76746, size = 1231, normalized size = 7.6

$$\left[\frac{3\left((3a+2b)\cos\left(fx+e\right)^3-(3a+2b)\cos\left(fx+e\right)\right)\sqrt{b}\log\left(\frac{\left(a^2-8ab+8b^2\right)\cos\left(fx+e\right)^4+8\left(ab-2b^2\right)\cos\left(fx+e\right)^2+4\left((a-2b)\cos\left(fx+e\right)^3+\right.}{\cos\left(fx+e\right)^4}\right.}{24\left(f\cos\left(fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*((3*a + 2*b)*cos(f*x + e)^3 - (3*a + 2*b)*cos(f*x + e))*sqrt(b)*lo
g(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 +
4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*
x + e) - 4*((4*a + 11*b)*cos(f*x + e)^4 - 2*(3*a + 7*b)*cos(f*x + e)^2 + 3*
b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x + e)^3 -
f*cos(f*x + e))*sin(f*x + e)), -1/12*(3*((3*a + 2*b)*cos(f*x + e)^3 - (3*a
+ 2*b)*cos(f*x + e))*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*co
s(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a
*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) + 2*((4*a + 11*
b)*cos(f*x + e)^4 - 2*(3*a + 7*b)*cos(f*x + e)^2 + 3*b)*sqrt(((a - b)*cos(f
*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x + e)^3 - f*cos(f*x + e))*sin(f*
x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan \left(fx + e\right)^{2} + a\right)^{\frac{3}{2}} \csc \left(fx + e\right)^{4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^4, x)
```

3.115 $\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=196

$$\frac{b(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af}+\frac{\sqrt{b}(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{5}}{5af}$$

```
[Out] (Sqrt[b]*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]
^2]])/(2*f) + (b*(3*a + 4*b)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*a*
f) - ((3*a + 4*b)*Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2))/(3*a*f) - (2*C
ot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(5/2))/(3*a*f) - (Cot[e + f*x]^5*(a +
b*Tan[e + f*x]^2)^(5/2))/(5*a*f)
```

Rubi [A] time = 0.170512, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3663, 462, 453, 277, 195, 217, 206}

$$\frac{b(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af}+\frac{\sqrt{b}(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{5}}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]
^2]])/(2*f) + (b*(3*a + 4*b)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*a*
f) - ((3*a + 4*b)*Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2))/(3*a*f) - (2*C
ot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(5/2))/(3*a*f) - (Cot[e + f*x]^5*(a +
b*Tan[e + f*x]^2)^(5/2))/(5*a*f)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2(a+bx^2)^{3/2}}{x^6}dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5af} + \frac{\text{Subst}\left(\int \frac{(10a+5ax^2)(a+bx^2)^{3/2}}{x^4}dx, \right.}{5af}$$

$$= -\frac{2\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{3af} - \frac{\cot^5(e+fx)\left(a+b\tan^2(e+f\right.}{5af}$$

$$= -\frac{(3a+4b)\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3af} - \frac{2\cot^3(e+fx)\left(a+b\,\text{ta}\right.}{3af}$$

$$= \frac{b(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af} - \frac{(3a+4b)\cot(e+fx)\left(a+\right.}{3af}$$

$$= \frac{b(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af} - \frac{(3a+4b)\cot(e+fx)\left(a+\right.}{3af}$$

$$= \frac{\sqrt{b}(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f} + \frac{b(3a+4b)\tan(e+fx)\sqrt{a+b\,\text{ta}}}{2af}$$

Mathematica [C] time = 2.04711, size = 213, normalized size = 1.09

$$\frac{\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\frac{15\sqrt{2}(3a+4b)\cot(e+fx)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}}{\sqrt{2}}\right),1\right)}{\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}}-\frac{2(8a^2+34ab+}{}\right.}{30\sqrt{2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*((-2*(8*a^2 + 34*a
*b + 3*b^2)*Cot[e + f*x])/a - 4*(2*a + 3*b)*Cot[e + f*x]*Csc[e + f*x]^2 - 6
*a*Cot[e + f*x]*Csc[e + f*x]^4 + (15*Sqrt[2]*(3*a + 4*b)*Cot[e + f*x]*Ellip
ticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqr
t[2]], 1])/Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b] + 15
*b*Tan[e + f*x]))/(30*Sqrt[2]*f)
```

Maple [C] time = 0.868, size = 6988, normalized size = 35.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 26.6989, size = 1624, normalized size = 8.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/120*(15*((3*a^2 + 4*a*b)*cos(f*x + e)^5 - 2*(3*a^2 + 4*a*b)*cos(f*x + e)
^3 + (3*a^2 + 4*a*b)*cos(f*x + e))*sqrt(b)*log(((a^2 - 8*a*b + 8*b^2)*cos(f
*x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 +
```

```
2*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((16*a^2 + 83*a*b +
 6*b^2)*cos(f*x + e)^6 - (40*a^2 + 193*a*b + 12*b^2)*cos(f*x + e)^4 + (30*a
^2 + 125*a*b + 6*b^2)*cos(f*x + e)^2 - 15*a*b)*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2))/((a*f*cos(f*x + e)^5 - 2*a*f*cos(f*x + e)^3 + a*f*co
s(f*x + e))*sin(f*x + e)), -1/60*(15*((3*a^2 + 4*a*b)*cos(f*x + e)^5 - 2*(3
*a^2 + 4*a*b)*cos(f*x + e)^3 + (3*a^2 + 4*a*b)*cos(f*x + e))*sqrt(-b)*arcta
n(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin
(f*x + e)))*sin(f*x + e) + 2*((16*a^2 + 83*a*b + 6*b^2)*cos(f*x + e)^6 - (4
0*a^2 + 193*a*b + 12*b^2)*cos(f*x + e)^4 + (30*a^2 + 125*a*b + 6*b^2)*cos(f
*x + e)^2 - 15*a*b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a*
f*cos(f*x + e)^5 - 2*a*f*cos(f*x + e)^3 + a*f*cos(f*x + e))*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^6, x)
```

3.116 $\int \frac{\sin^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=144

$$-\frac{\left(15a^2-10ab+3b^2\right)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{15f(a-b)^3}-\frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{5f(a-b)}+\frac{2(5a-3b)\cos^3(e+f}{15}$$

```
[Out] -((15*a^2 - 10*a*b + 3*b^2)*Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/(1
5*(a - b)^3*f) + (2*(5*a - 3*b)*Cos[e + f*x]^3*Sqrt[a - b + b*Sec[e + f*x]^
2])/(15*(a - b)^2*f) - (Cos[e + f*x]^5*Sqrt[a - b + b*Sec[e + f*x]^2])/(5*(
a - b)*f)
```

Rubi [A] time = 0.144111, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3664, 462, 453, 264}

$$-\frac{\left(15a^2-10ab+3b^2\right)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{15f(a-b)^3}-\frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{5f(a-b)}+\frac{2(5a-3b)\cos^3(e+f}{15}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((15*a^2 - 10*a*b + 3*b^2)*Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/(1
5*(a - b)^3*f) + (2*(5*a - 3*b)*Cos[e + f*x]^3*Sqrt[a - b + b*Sec[e + f*x]^
2])/(15*(a - b)^2*f) - (Cos[e + f*x]^5*Sqrt[a - b + b*Sec[e + f*x]^2])/(5*(
a - b)*f)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^6\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{5(a-b)f} + \frac{\text{Subst}\left(\int \frac{-2(5a-3b)+5(a-b)x^2}{x^4\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{5(a-b)f}$$

$$= \frac{2(5a-3b)\cos^3(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{15(a-b)^2 f} - \frac{\cos^5(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{5(a-b)f}$$

$$= -\frac{\left(15a^2-10ab+3b^2\right)\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{15(a-b)^3 f} + \frac{2(5a-3b)\cos^3(e+fx)\sqrt{a}}{15(a-b)}$$

Mathematica [A] time = 2.02296, size = 112, normalized size = 0.78

$$\frac{\cos(e+fx)\left(4\left(7a^2-10ab+3b^2\right)\cos(2(e+fx))-89a^2-3(a-b)^2\cos(4(e+fx))+34ab-9b^2\right)\sqrt{\sec^2(e+fx)((a-}}{120\sqrt{2}f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*(-89*a^2 + 34*a*b - 9*b^2 + 4*(7*a^2 - 10*a*b + 3*b^2)*Cos[2*
(e + f*x)] - 3*(a - b)^2*Cos[4*(e + f*x)])*Sqrt[(a + b + (a - b)*Cos[2*(e +
 f*x)])*Sec[e + f*x]^2])/(120*Sqrt[2]*(a - b)^3*f)
```

Maple [A] time = 0.286, size = 169, normalized size = 1.2

$$-\frac{\left(a\left(\cos\left(fx+e\right)\right)^2-\left(\cos\left(fx+e\right)\right)^2b+b\right)\left(3\left(\cos\left(fx+e\right)\right)^4a^2-6\left(\cos\left(fx+e\right)\right)^4ab+3\left(\cos\left(fx+e\right)\right)^4b^2-10\,(}{15\,f\,(a-b)^3\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/15/f/(a-b)^3*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)*(3*cos(f*x+e)^4*a^2-6*cos
(f*x+e)^4*a*b+3*cos(f*x+e)^4*b^2-10*cos(f*x+e)^2*a^2+16*cos(f*x+e)^2*a*b-6*
cos(f*x+e)^2*b^2+15*a^2-10*a*b+3*b^2)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/co
s(f*x+e)^2)^(1/2)/cos(f*x+e)
```

Maxima [A] time = 1.03985, size = 289, normalized size = 2.01

$$-\frac{\frac{15\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a-b}+\frac{3\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5-10\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}b\cos(fx+e)^3+15\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}b^2\cos(fx+e)}{a^3-3a^2b+3ab^2-b^3}-\frac{10\left(\left(a-b\cdots\right.\right.}{\cdots}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a - b) + (3*(a - b +
 b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 10*(a - b + b/cos(f*x + e)^2)^(3/
2)*b*cos(f*x + e)^3 + 15*sqrt(a - b + b/cos(f*x + e)^2)*b^2*cos(f*x + e))/(
a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 10*((a - b + b/cos(f*x + e)^2)^(3/2)*cos(f
*x + e)^3 - 3*sqrt(a - b + b/cos(f*x + e)^2)*b*cos(f*x + e))/(a^2 - 2*a*b +
 b^2))/f
```

Fricas [A] time = 2.04331, size = 292, normalized size = 2.03

$$-\frac{\left(3\left(a^2-2ab+b^2\right)\cos\left(fx+e\right)^5-2\left(5a^2-8ab+3b^2\right)\cos\left(fx+e\right)^3+\left(15a^2-10ab+3b^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos(f\cdots}{\cos(fx+\cdots}}}{15\left(a^3-3a^2b+3ab^2-b^3\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 2*(5*a^2 - 8*a*b + 3*b^2)*cos
(f*x + e)^3 + (15*a^2 - 10*a*b + 3*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^5}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^5/sqrt(b*tan(f*x + e)^2 + a), x)
```

3.117 $\int \frac{\sin^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=88

$$\frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{3f(a-b)}-\frac{(3a-b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{3f(a-b)^2}$$

```
[Out] -((3*a - b)*Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/(3*(a - b)^2*f) +
(Cos[e + f*x]^3*Sqrt[a - b + b*Sec[e + f*x]^2])/(3*(a - b)*f)
```

Rubi [A] time = 0.101122, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3664, 453, 264}

$$\frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{3f(a-b)}-\frac{(3a-b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{3f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((3*a - b)*Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/(3*(a - b)^2*f) +
(Cos[e + f*x]^3*Sqrt[a - b + b*Sec[e + f*x]^2])/(3*(a - b)*f)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{3(a-b)f} + \frac{(3a-b)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{3(a-b)f}$$

$$= -\frac{(3a-b)\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{3(a-b)^2f} + \frac{\cos^3(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{3(a-b)f}$$

Mathematica [A] time = 1.43294, size = 74, normalized size = 0.84

$$\frac{\cos(e+fx)((a-b)\cos(2(e+fx))-5a+b)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}}{6\sqrt{2}f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*(-5*a + b + (a - b)*Cos[2*(e + f*x)])*Sqrt[(a + b + (a - b)*C
os[2*(e + f*x)])*Sec[e + f*x]^2])/(6*Sqrt[2]*(a - b)^2*f)
```

Maple [A] time = 0.221, size = 104, normalized size = 1.2

$$\frac{\left(a\left(\cos\left(fx+e\right)\right)^2-\left(\cos\left(fx+e\right)\right)^2b+b\right)\left(a\left(\cos\left(fx+e\right)\right)^2-\left(\cos\left(fx+e\right)\right)^2b-3a+b\right)}{3f(a-b)^2\cos\left(fx+e\right)}\frac{1}{\sqrt{\frac{a\left(\cos\left(fx+e\right)\right)^2-\left(\cos\left(fx+e\right)\right)^2b}{\left(\cos\left(fx+e\right)\right)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3/f/(a-b)^2*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)*(a*cos(f*x+e)^2-cos(f*x+e)^
2*b-3*a+b)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/cos(f*x+e
)
```

Maxima [A] time = 1.02708, size = 143, normalized size = 1.62

$$-\frac{\frac{3\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{a-b}-\frac{\left(a-b+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3-3\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}b\cos\left(fx+e\right)}{a^2-2ab+b^2}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a - b) - ((a - b + b/c
os(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 3*sqrt(a - b + b/cos(f*x + e)^2)*b*co
```

```
s(f*x + e))/(a^2 - 2*a*b + b^2))/f
```

Fricas [A] time = 1.85646, size = 174, normalized size = 1.98

$$\frac{\left((a-b)\cos\left(fx+e\right)^{3}-(3\,a-b)\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{3\left(a^{2}-2\,ab+b^{2}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*((a - b)*cos(f*x + e)^3 - (3*a - b)*cos(f*x + e))*sqrt(((a - b)*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a^2 - 2*a*b + b^2)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{3}}{\sqrt{b\tan\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^3/sqrt(b*tan(f*x + e)^2 + a), x)
```

3.118 $\int \frac{\sin(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=37

$$-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f(a-b)}$$

```
[Out] -((Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/((a - b)*f))
```

Rubi [A] time = 0.0465261, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3664, 264}

$$-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]

[Out] -((Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/((a - b)*f))
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}\\ &= -\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{(a-b)f}\end{aligned}$$

Mathematica [A] time = 0.603409, size = 52, normalized size = 1.41

$$\frac{\cos(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}}{\sqrt{2}f(b-a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(Sqr
t[2]*(-a + b)*f)
```

Maple [B] time = 0.069, size = 78, normalized size = 2.1

$$-\frac{a\left(\cos\left(fx+e\right)\right)^{2}-\left(\cos\left(fx+e\right)\right)^{2}b+b}{f\cos\left(fx+e\right)(a-b)}\frac{1}{\sqrt{\frac{a\left(\cos\left(fx+e\right)\right)^{2}-\left(\cos\left(fx+e\right)\right)^{2}b+b}{\left(\cos\left(fx+e\right)\right)^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)*(a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/(a-b)
```

Maxima [A] time = 0.98396, size = 47, normalized size = 1.27

$$-\frac{\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)}{(a-b)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/((a - b)*f)
```

Fricas [A] time = 1.76052, size = 104, normalized size = 2.81

$$-\frac{\sqrt{\frac{(a-b)\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)}{(a-b)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/((a - b)*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(e+fx\right)}{\sqrt{a+b\tan^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)/sqrt(a + b*tan(e + f*x)**2), x)
```

<u>Giac</u> [B] time = 1.7781, size = 119, normalized size = 3.22

$$\frac{\sqrt{b}\operatorname{sgn}\left(f\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{a\left|f\right|-b\left|f\right|}-\frac{\sqrt{a\cos\left(fx+e\right)^{2}-b\cos\left(fx+e\right)^{2}+b}}{a\left|f\right|\operatorname{sgn}\left(f\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)-b\left|f\right|\operatorname{sgn}\left(f\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(b)*sgn(f)*sgn(cos(f*x + e))/(a*abs(f) - b*abs(f)) - sqrt(a*cos(f*x + e
)^2 - b*cos(f*x + e)^2 + b)/(a*abs(f)*sgn(f)*sgn(cos(f*x + e)) - b*abs(f)*s
gn(f)*sgn(cos(f*x + e)))
```

3.119 $\int \frac{\csc(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=42

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{\sqrt{a}f}$$

```
[Out] -(ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]]/(Sqrt[a]*f
))
```

Rubi [A] time = 0.0683289, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3664, 377, 207}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]]/(Sqrt[a]*f
))
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-1+ax^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{\sqrt{a}f}$$

Mathematica [B] time = 2.34934, size = 226, normalized size = 5.38

$$-\frac{\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\tanh^{-1}\left(\frac{a-(a-2b)\tan^2\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a}\sqrt{a\left(\tan^2\left(\frac{1}{2}(e+fx)\right)-1\right)^2+4b\tan^2\left(\frac{1}{2}(e\right.}}\right.\right.}{2\sqrt{a}f\sqrt{\sec^4\left(\frac{1}{2}(e+fx)\right)((a-b)\cos(2(e+fx))+a+b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((ArcTanh[(a - (a - 2*b)*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*
x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])] + ArcTanh[(2*b + a*(-1 + Tan[(e
+ f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/
2]^2)^2])])*Cos[e + f*x]*Sec[(e + f*x)/2]^2*Sqrt[(a + b + (a - b)*Cos[2*(e
+ f*x)])*Sec[e + f*x]^2])/(2*Sqrt[a]*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x
)])*Sec[(e + f*x)/2]^4])
```

Maple [B] time = 0.21, size = 351, normalized size = 8.4

$$\frac{\left(\sin\left(fx+e\right)\right)^2}{2f\cos\left(fx+e\right)\left(\cos\left(fx+e\right)-1\right)}\sqrt{\frac{a\left(\cos\left(fx+e\right)\right)^2-\left(\cos\left(fx+e\right)\right)^2b+b}{\left(\cos\left(fx+e\right)+1\right)^2}}\left(\ln\left(-2\frac{\cos\left(fx+e\right)-1}{\sqrt{a}\left(\sin\left(fx+e\right)\right)^2}\left(\cos\left(fx+e\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/f/a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*(l
n(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/
(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)+ln(-4*(c
os(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2
)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+
1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1)))*sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/(cos(f*x+e)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/sqrt(b*tan(f*x + e)^2 + a), x)
```

Fricas [A] time = 2.45442, size = 346, normalized size = 8.24

$$\left[\frac{\log\left(-\frac{2\left((a-b)\cos\left(fx+e\right)^2-2\sqrt{a}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+a+b\right)}{\cos\left(fx+e\right)^2-1}\right)}{2\sqrt{a}f},\ \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{a}\right)}{af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1))/(sqrt(a)
*f), sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e)/a)/(a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(csc(f*x + e)/sqrt(b*tan(f*x + e)^2 + a), x)
```

3.120 $\int \frac{\csc^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=91

$$-\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2a^{3/2}f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2af}$$

```
[Out] -((a - b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/(
2*a^(3/2)*f) - (Cot[e + f*x]*Csc[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/(
2*a*f)
```

Rubi [A] time = 0.114938, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3664, 471, 12, 377, 207}

$$-\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2a^{3/2}f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((a - b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/(
2*a^(3/2)*f) - (Cot[e + f*x]*Csc[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/(
2*a*f)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2af} + \frac{\text{Subst}\left(\int \frac{a-b}{\left(-1+x^2\right)\sqrt{a-b+bx^2}}\,dx, x, \sec(\right.}{2af}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2af} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{\left(-1+x^2\right)\sqrt{a-b+bx^2}}\,dx,\right.}{2af}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2af} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{-1+ax^2}\,dx, x, \frac{\text{se}}{\sqrt{a-b+}}\right.}{2af}$$

$$= -\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2a^{3/2}f} - \frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2af}$$

Mathematica [B] time = 2.68244, size = 303, normalized size = 3.33

$$-\frac{\cot(e+fx)\csc(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\sqrt{2}\sqrt{a}\sqrt{\sec^4\left(\frac{1}{2}(e+fx)\right)((a-b)\cos(2(e+f\right.}}{4a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(Cot[e + f*x]*Csc[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e +
 f*x]^2]*(Sqrt[2]*Sqrt[a]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e +
f*x)/2]^4] + 4*(a - b)*ArcTanh[(a - (a - 2*b)*Tan[(e + f*x)/2]^2)/(Sqrt[a]*
Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])]*Sin[(e + f*x
)/2]^2 + 4*(a - b)*ArcTanh[(2*b + a*(-1 + Tan[(e + f*x)/2]^2))/(Sqrt[a]*Sqr
t[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])]*Sin[(e + f*x)/2
]^2))/(4*a^(3/2)*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]
^4])
```

Maple [B] time = 0.25, size = 2801, normalized size = 30.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/4/f/a^(5/2)*(2*a^(5/2)*cos(f*x+e)^2-cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*
x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e
)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*
x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1
/2)*a^(1/2)+b)/sin(f*x+e)^2)*a^2+cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a
+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^
(1/2)+b)/sin(f*x+e)^2)*a*b-cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/
(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b
)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^
2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*a^2+
cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(
-4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a
^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*
x+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*a*b-2*a^(3/2)*cos(f*x+e)^2*b-co
s(f*x+e)^2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2
/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos
(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*a^2+cos(f*x+
e)^2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2/a^(1/
2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e
)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)
^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*a*b-cos(f*x+e)^2*(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)
*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x
+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/
2)*a^(1/2)+b)/(cos(f*x+e)-1))*a^2+cos(f*x+e)^2*((a*cos(f*x+e)^2-cos(f*x+e)^
2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e
)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(
f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1
))*a*b+cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2
)*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)
^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*a^2-c
os(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2/
a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(
f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*a*b+cos(f*x+e
)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x
+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(
f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^
(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*a^2-cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)
^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+
e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos
(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-
1))*a*b+2*a^(3/2)*b+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1
/2)*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+
e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*a^2
-((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2/a^(1/2)*(
cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)
^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b
+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*a*b+((a*cos(f*x+e)^2-c
os(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e
)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(co
```

```
s(f*x+e)-1))*a^2-((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)
*ln(-4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/
2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(co
s(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*a*b)*sin(f*x+e)^2/(cos(f*x+
e)-1)^2/cos(f*x+e)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/(
cos(f*x+e)+1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^3}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^3/sqrt(b*tan(f*x + e)^2 + a), x)
```

Fricas [A] time = 2.54162, size = 699, normalized size = 7.68

$$\left[\frac{2\,a\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)-\left((a-b)\cos\left(fx+e\right)^2-a+b\right)\sqrt{a}\log\left(-\frac{2\left((a-b)\cos\left(fx+e\right)^2+2\sqrt{a}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos(f\right.}{\cos\left(fx+e\right)^2-1}\right.}{4\left(a^2f\cos\left(fx+e\right)^2-a^2f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) -
((a - b)*cos(f*x + e)^2 - a + b)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 + 2
*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a
 + b)/(cos(f*x + e)^2 - 1)))/(a^2*f*cos(f*x + e)^2 - a^2*f), 1/2*(((a - b)*
cos(f*x + e)^2 - a + b)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) + a*sqrt(((a - b)*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*cos(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^{3}}{\sqrt{b\tan\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^3/sqrt(b*tan(f*x + e)^2 + a), x)
```

3.121 $\int \frac{\csc^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=143

$$-\frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8a^{5/2}f}-\frac{(5a-3b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{8a^2f}-\frac{\cot^3(e+fx)\csc(e}{}$$

```
[Out] (-3*(a - b)^2*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]
])/(8*a^(5/2)*f) - ((5*a - 3*b)*Cot[e + f*x]*Csc[e + f*x]*Sqrt[a - b + b*Se
c[e + f*x]^2])/(8*a^2*f) - (Cot[e + f*x]^3*Csc[e + f*x]*Sqrt[a - b + b*Sec[
e + f*x]^2])/(4*a*f)
```

Rubi [A] time = 0.160042, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3664, 470, 527, 12, 377, 207}

$$-\frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8a^{5/2}f}-\frac{(5a-3b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{8a^2f}-\frac{\cot^3(e+fx)\csc(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (-3*(a - b)^2*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]
])/(8*a^(5/2)*f) - ((5*a - 3*b)*Cot[e + f*x]*Csc[e + f*x]*Sqrt[a - b + b*Se
c[e + f*x]^2])/(8*a^2*f) - (Cot[e + f*x]^3*Csc[e + f*x]*Sqrt[a - b + b*Sec[
e + f*x]^2])/(4*a*f)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^3\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{4af} - \frac{\text{Subst}\left(\int \frac{-a+b-2(2a-b)x^2}{\left(-1+x^2\right)^2\sqrt{a-b+bx^2}}\,dx, x, \sec(e\right.}{4af}$$

$$= -\frac{(5a-3b)\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8a^2f} - \frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a-}}{4af}$$

$$= -\frac{(5a-3b)\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8a^2f} - \frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a-}}{4af}$$

$$= -\frac{(5a-3b)\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8a^2f} - \frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a-}}{4af}$$

$$= -\frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8a^{5/2}f} - \frac{(5a-3b)\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e}}{8a^2f}$$

Mathematica [A] time = 4.33127, size = 278, normalized size = 1.94

$$\frac{\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(-\sqrt{2}\sqrt{a}\cot(e+fx)\csc(e+fx)\left(2a\csc^2(e+fx)+3a-3b\right)-\frac{3(a-b)^2\cos(e+}{}\right.}{16a^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((-(Sqrt[2]*Sqrt[a]*Cot[e + f*x]*Csc[e + f*x]*(3*a - 3*b + 2*a*Csc[e + f*x]
^2)) - (3*(a - b)^2*(ArcTanh[(a - (a - 2*b)*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sq
rt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])] + ArcTanh[(2*b
 + a*(-1 + Tan[(e + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-
1 + Tan[(e + f*x)/2]^2)^2])])*Cos[e + f*x]*Sec[(e + f*x)/2]^2)/Sqrt[(a + b
+ (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4])*Sqrt[(a + b + (a - b)*Cos[
2*(e + f*x)])*Sec[e + f*x]^2])/(16*a^(5/2)*f)
```

Maple [B] time = 0.238, size = 6334, normalized size = 44.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.69862, size = 1071, normalized size = 7.49

$$\left[\frac{3\left(\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^4-2\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^2+a^2-2\,ab+b^2\right)\sqrt{a}\log\left(-\frac{2\left((a-b)\cos\left(fx+e\right)^2-2\sqrt{a}\sqrt{\cdots}\right.}{\cos(\cdots}\right.}{16\left(a^3f\cos\left(fx+e\right)^4-2\,a^3f\cos\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 - 2*a*b + b^2)*cos(f*
x + e)^2 + a^2 - 2*a*b + b^2)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sq
rt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a +
b)/(cos(f*x + e)^2 - 1)) + 2*(3*(a^2 - a*b)*cos(f*x + e)^3 - (5*a^2 - 3*a*b
)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^3*f*c
os(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f), 1/8*(3*((a^2 - 2*a*b + b^2
)*cos(f*x + e)^4 - 2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^2 + a^2 - 2*a*b + b^2
)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2
```

```
)*cos(f*x + e)/a) + (3*(a^2 - a*b)*cos(f*x + e)^3 - (5*a^2 - 3*a*b)*cos(f*x
 + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^3*f*cos(f*x +
e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^5}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^5/sqrt(b*tan(f*x + e)^2 + a), x)
```

3.122 $\int \frac{\sin^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=146

$$\frac{3a^2\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f(a-b)^{5/2}}+\frac{\sin(e+fx)\cos^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f(a-b)}-\frac{(5a-2b)\sin(e+fx)\cos(e+fx)\sqrt{a+b}}{8f(a-b)^2}$$

```
[Out] (3*a^2*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(8*(a
 - b)^(5/2)*f) - ((5*a - 2*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Tan[e +
f*x]^2])/(8*(a - b)^2*f) + (Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b*Tan[e +
f*x]^2])/(4*(a - b)*f)
```

Rubi [A] time = 0.166636, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3663, 470, 527, 12, 377, 203}

$$\frac{3a^2\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f(a-b)^{5/2}}+\frac{\sin(e+fx)\cos^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f(a-b)}-\frac{(5a-2b)\sin(e+fx)\cos(e+fx)\sqrt{a+b}}{8f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (3*a^2*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(8*(a
 - b)^(5/2)*f) - ((5*a - 2*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Tan[e +
f*x]^2])/(8*(a - b)^2*f) + (Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b*Tan[e +
f*x]^2])/(4*(a - b)*f)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
```

```
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^3\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4(a-b)f} - \frac{\text{Subst}\left(\int \frac{a-2(2a-b)x^2}{\left(1+x^2\right)^2\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{4(a-b)f}$$

$$= -\frac{(5a-2b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8(a-b)^2f} + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4(a-b)f}$$

$$= -\frac{(5a-2b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8(a-b)^2f} + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4(a-b)f}$$

$$= -\frac{(5a-2b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8(a-b)^2f} + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4(a-b)f}$$

$$= \frac{3a^2\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8(a-b)^{5/2}f} - \frac{(5a-2b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8(a-b)^2f} + \cos^3$$

Mathematica [C] time = 4.01079, size = 314, normalized size = 2.15

$$-\frac{\sin(2(e+fx))\sec^2(e+fx)\left(6\sqrt{2}a^2(b-a)\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}}{\sqrt{2}}\right),\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(((a - b)*(7*a^2 + 8*a*b - 3*b^2 + 2*(3*a^2 - 5*a*b + 2*b^2)*Cos[2*(e + f*
x)] - (a - b)^2*Cos[4*(e + f*x)]) + 6*Sqrt[2]*a^2*(-a + b)*Sqrt[((a + b + (
a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[((a + b +
 (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + 6*Sqrt[2]*a^3*
Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticPi[-(b/
(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b
]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)])/(32*Sqrt[2]*(a - b)^3*f*Sq
rt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Maple [C] time = 0.277, size = 1169, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/8/f/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/(a-b)^2*sin(f*x+e)*(2*cos(f
*x+e)^5*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2-4*cos(f*x+e)^5*((2*I*
b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b+2*cos(f*x+e)^5*((2*I*b^(1/2)*(a-b)^
(1/2)+a-2*b)/a)^(1/2)*b^2-2*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a
)^(1/2)*a^2+4*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b-2*
cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2-3*2^(1/2)*(1/a*(
I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f
*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^
(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*Ellipt
icF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8
*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2
))*a^2*sin(f*x+e)+6*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2
)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*c
os(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+
e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(
1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I
*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1
/2))*a^2*sin(f*x+e)-5*cos(f*x+e)^3*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2
)*a^2+9*cos(f*x+e)^3*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b-4*cos(f*
x+e)^3*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2+5*cos(f*x+e)^2*((2*I*b
^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2-9*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(
1/2)+a-2*b)/a)^(1/2)*a*b+4*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)
^(1/2)*b^2-5*cos(f*x+e)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b+2*cos
(f*x+e)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2+5*((2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)/a)^(1/2)*a*b-2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2)
/(cos(f*x+e)-1)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/cos(
f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^4}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)
```

Fricas [B] time = 23.7585, size = 1864, normalized size = 12.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*a^2*sqrt(-a + b)*log(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b
^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*cos(
f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3 + 24*b^4)*cos(f*x
 + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*b^4 - 32*(a^4 - 11
*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 + 8*(16*(a^3 - 3*a^
2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*
cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*cos(f*x + e)^3 -
(a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a + b)*sqrt(((a -
b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(2*(a^2 - 2*a*b +
b^2)*cos(f*x + e)^3 - (5*a^2 - 7*a*b + 2*b^2)*cos(f*x + e))*sqrt(((a - b)*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^3 - 3*a^2*b + 3*a*b^2
- b^3)*f), 1/32*(3*sqrt(a - b)*a^2*arctan(-1/4*(8*(a^2 - 2*a*b + b^2)*cos(f
*x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*b + 8*b^2)*
cos(f*x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/((2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*a*b^2 - 2*b
^3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(
2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - (5*a^2 - 7*a*b + 2*b^2)*cos(f*x + e)
)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^{4}\left(e + fx\right)}{\sqrt{a + b\tan^{2}\left(e + fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**4/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx + e\right)^{4}}{\sqrt{b\tan\left(fx + e\right)^{2} + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)
```

3.123 $\int \frac{\sin^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=93

$$\frac{a\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f(a-b)^{3/2}}-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f(a-b)}$$

```
[Out] (a*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(2*(a - b
)^(3/2)*f) - (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*(a -
 b)*f)
```

Rubi [A] time = 0.107026, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3663, 471, 12, 377, 203}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f(a-b)^{3/2}}-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (a*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(2*(a - b
)^(3/2)*f) - (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*(a -
 b)*f)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)f} + \frac{\text{Subst}\left(\int \frac{a}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{2(a-b)f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)f} + \frac{a\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{2(a-b)f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)f} + \frac{a\,\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2(a-b)f}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2(a-b)^{3/2}f} - \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)f}$$

Mathematica [C] time = 3.00537, size = 270, normalized size = 2.9

$$-\frac{\sin(2(e+fx))\sec^2(e+fx)\left(\sqrt{2}a(b-a)\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}}{\sqrt{2}}\right),1\right)\right.}{4\sqrt{2}f(a-b)^2\sqrt{\sec^2(e+ }$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(((a - b)*(a + b + (a - b)*Cos[2*(e + f*x)]) + Sqrt[2]*a*(-a + b)*Sqrt[((a
 + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[(
(a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + Sqrt[2
]*a^2*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticP
i[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x
]^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)])/(4*Sqrt[2]*(a - b)^2
*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Maple [C] time = 0.206, size = 795, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2/f/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/(a-b)*sin(f*x+e)*(2^(1/2)*
(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b
*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2
)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*
EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+
e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2
)^(1/2))*a*sin(f*x+e)-2*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^
(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*
(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(
f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-
b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-
(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a
)^(1/2))*a*sin(f*x+e)+cos(f*x+e)^3*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2
)*a-cos(f*x+e)^3*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b-cos(f*x+e)^2*(
(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a+cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)/a)^(1/2)*b+cos(f*x+e)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/
2)*b-((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b)/(cos(f*x+e)-1)/((a*cos(f*
x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^2}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

Fricas [B] time = 3.51995, size = 1667, normalized size = 17.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
 + e)*sin(f*x + e) - a*sqrt(-a + b)*log(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*
a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*
b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3 + 24*b^4
)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*b^4 - 32*
(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*(a
^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b + 5*a*b^2
- 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*cos(f*x
+ e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a + b)*sq
rt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a^2 - 2*a*
b + b^2)*f), -1/8*(4*(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e)*sin(f*x + e) - sqrt(a - b)*a*arctan(-1/4*(8*(a^2 - 2*a*b +
 b^2)*cos(f*x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*
```

```
b + 8*b^2)*cos(f*x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)/((2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*
a*b^2 - 2*b^3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x +
 e))))/((a^2 - 2*a*b + b^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2\left(e + fx\right)}{\sqrt{a + b\tan^2\left(e + fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**2/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx + e\right)^2}{\sqrt{b\tan\left(fx + e\right)^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

3.124 $\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]*
f)
```

Rubi [A] time = 0.0340249, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3661, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]*
f)
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}f}$$

Mathematica [A] time = 0.0702021, size = 46, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]*
f)
```

Maple [A] time = 0.029, size = 67, normalized size = 1.5

$$\frac{1}{fb^2(a-b)}\sqrt{b^4(a-b)}\arctan\left((a-b)b^2\tan(fx+e)\frac{1}{\sqrt{b^4(a-b)}}\frac{1}{\sqrt{a+b(\tan(fx+e))^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan
(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07198, size = 311, normalized size = 6.76

$$\left[-\frac{\sqrt{-a+b}\log\left(-\frac{(a-2\,b)\tan\left(fx+e\right)^2-2\,\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{-a+b}\tan\left(fx+e\right)-a}{\tan\left(fx+e\right)^2+1}\right)}{2\,(a-b)f},\ \frac{\arctan\left(-\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{a-b}\tan\left(fx+e\right)}\right)}{\sqrt{a-b}f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2
 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1))/((a - b)*f), arc
tan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e)))/(sqrt(a - b)*f)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*tan(f*x + e)^2 + a), x)
```

3.125 $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=30

$$-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af}$$

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(a*f))
```

Rubi [A] time = 0.0696135, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {3663, 264}

$$-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(a*f))
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\ &= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af}\end{aligned}$$

Mathematica [A] time = 0.248091, size = 49, normalized size = 1.63

$$-\frac{\cot(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}}{\sqrt{2}af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(S
qrt[2]*a*f))
```

Maple [A] time = 0.178, size = 57, normalized size = 1.9

$$-\frac{\cos\left(fx+e\right)}{fa\sin\left(fx+e\right)}\sqrt{\frac{a\left(\cos\left(fx+e\right)\right)^{2}-\left(\cos\left(fx+e\right)\right)^{2}b+b}{\left(\cos\left(fx+e\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f/a*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)/si
n(f*x+e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08909, size = 113, normalized size = 3.77

$$-\frac{\sqrt{\frac{(a-b)\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)}{af\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a*f*sin(f*
x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^{2}\left(e+fx\right)}{\sqrt{a+b\tan^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^{2}}{\sqrt{b\tan\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

3.126 $\int \frac{\csc^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=74

$$-\frac{(3a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af}$$

```
[Out] -((3*a - 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(3*a^2*f) - (Cot[e +
 f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(3*a*f)
```

Rubi [A] time = 0.0900209, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3663, 453, 264}

$$-\frac{(3a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((3*a - 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(3*a^2*f) - (Cot[e +
 f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(3*a*f)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af} + \frac{(3a-2b)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{3af}$$
$$= -\frac{(3a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af}$$

Mathematica [A] time = 0.413838, size = 68, normalized size = 0.92

$$-\frac{\cot(e+fx)\left(a\csc^2(e+fx)+2a-2b\right)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}}{3\sqrt{2}a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(Cot[e + f*x]*(2*a - 2*b + a*Csc[e + f*x]^2)*Sqrt[(a + b + (a - b)*Cos[2*(
e + f*x)])*Sec[e + f*x]^2])/(3*Sqrt[2]*a^2*f)
```

Maple [A] time = 0.194, size = 86, normalized size = 1.2

$$\frac{\left(2a\left(\cos\left(fx+e\right)\right)^2-2\left(\cos\left(fx+e\right)\right)^2b-3a+2b\right)\cos\left(fx+e\right)}{3fa^2\left(\sin\left(fx+e\right)\right)^3}\sqrt{\frac{a\left(\cos\left(fx+e\right)\right)^2-\left(\cos\left(fx+e\right)\right)^2b+b}{\left(\cos\left(fx+e\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3/f/a^2*(2*a*cos(f*x+e)^2-2*cos(f*x+e)^2*b-3*a+2*b)*((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)/sin(f*x+e)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.12206, size = 211, normalized size = 2.85

$$-\frac{\left(2(a-b)\cos\left(fx+e\right)^3-(3a-2b)\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{3\left(a^2f\cos\left(fx+e\right)^2-a^2f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a - b)*cos(f*x + e)^3 - (3*a - 2*b)*cos(f*x + e))*sqrt(((a - b)*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2*f*cos(f*x + e)^2 - a^2*f)*sin(f*x +
 e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^4\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**4/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^4}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)
```

3.127 $\int \frac{\csc^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=123

$$-\frac{\left(15a^2-20ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f}-\frac{2(5a-2b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f}-\frac{\cot^5(e+fx)\sqrt{a}}{5}$$

```
[Out] -((15*a^2 - 20*a*b + 8*b^2)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(15*a^
3*f) - (2*(5*a - 2*b)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(15*a^2*f)
 - (Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(5*a*f)
```

Rubi [A] time = 0.136504, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3663, 462, 453, 264}

$$-\frac{\left(15a^2-20ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f}-\frac{2(5a-2b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f}-\frac{\cot^5(e+fx)\sqrt{a}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```


```
[Out] -((15*a^2 - 20*a*b + 8*b^2)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(15*a^
3*f) - (2*(5*a - 2*b)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(15*a^2*f)
 - (Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(5*a*f)
```


Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^6\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5af} + \frac{\text{Subst}\left(\int \frac{2(5a-2b)+5ax^2}{x^4\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{5af}$$

$$= -\frac{2(5a-2b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f} - \frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5af} + \frac{(1\dots}{\dots}$$

$$= -\frac{\left(15a^2-20ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f} - \frac{2(5a-2b)\cot^3(e+fx)\sqrt{a+b\dots}}{15a^2f}$$

Mathematica [A] time = 1.81809, size = 90, normalized size = 0.73

$$-\frac{\cot(e+fx)\left(3a^2\csc^4(e+fx)+4a(a-b)\csc^2(e+fx)+8(a-b)^2\right)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}}{15\sqrt{2}a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(Cot[e + f*x]*(8*(a - b)^2 + 4*a*(a - b)*Csc[e + f*x]^2 + 3*a^2*Csc[e + f*
x]^4)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(15*Sqrt[2]*
a^3*f)
```

Maple [A] time = 0.214, size = 148, normalized size = 1.2

$$-\frac{\left(8\left(\cos\left(fx+e\right)\right)^4a^2-16\left(\cos\left(fx+e\right)\right)^4ab+8\left(\cos\left(fx+e\right)\right)^4b^2-20\left(\cos\left(fx+e\right)\right)^2a^2+36\left(\cos\left(fx+e\right)\right)^2ab\dots}{15fa^3\left(\sin\left(fx+e\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/15/f/a^3*(8*cos(f*x+e)^4*a^2-16*cos(f*x+e)^4*a*b+8*cos(f*x+e)^4*b^2-20*c
os(f*x+e)^2*a^2+36*cos(f*x+e)^2*a*b-16*cos(f*x+e)^2*b^2+15*a^2-20*a*b+8*b^2
)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)/sin(f*x
+e)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 6.82206, size = 339, normalized size = 2.76

$$-\frac{\left(8\left(a^2-2ab+b^2\right)\cos\left(fx+e\right)^5-4\left(5a^2-9ab+4b^2\right)\cos\left(fx+e\right)^3+\left(15a^2-20ab+8b^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{15\left(a^3f\cos\left(fx+e\right)^4-2a^3f\cos\left(fx+e\right)^2+a^3f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 4*(5*a^2 - 9*a*b + 4*b^2)*cos
(f*x + e)^3 + (15*a^2 - 20*a*b + 8*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^
2 + a^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^6}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^6/sqrt(b*tan(f*x + e)^2 + a), x)
```

3.128 $\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=199

$$-\frac{2b\left(15a^2+10ab-b^2\right)\sec(e+fx)}{15f(a-b)^4\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\left(15a^2+10ab-b^2\right)\cos(e+fx)}{15f(a-b)^3\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\cos^5(e+fx)}{5f(a-b)\sqrt{a+b\sec^2(e+fx)-b}}+\frac{}{15f}$$

```
[Out] -((15*a^2 + 10*a*b - b^2)*Cos[e + f*x])/(15*(a - b)^3*f*Sqrt[a - b + b*Sec[
e + f*x]^2]) + (2*(5*a - 2*b)*Cos[e + f*x]^3)/(15*(a - b)^2*f*Sqrt[a - b +
b*Sec[e + f*x]^2]) - Cos[e + f*x]^5/(5*(a - b)*f*Sqrt[a - b + b*Sec[e + f*x
]^2]) - (2*b*(15*a^2 + 10*a*b - b^2)*Sec[e + f*x])/(15*(a - b)^4*f*Sqrt[a -
 b + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.187187, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3664, 462, 453, 271, 191}

$$-\frac{2b\left(15a^2+10ab-b^2\right)\sec(e+fx)}{15f(a-b)^4\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\left(15a^2+10ab-b^2\right)\cos(e+fx)}{15f(a-b)^3\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\cos^5(e+fx)}{5f(a-b)\sqrt{a+b\sec^2(e+fx)-b}}+\frac{}{15f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((15*a^2 + 10*a*b - b^2)*Cos[e + f*x])/(15*(a - b)^3*f*Sqrt[a - b + b*Sec[
e + f*x]^2]) + (2*(5*a - 2*b)*Cos[e + f*x]^3)/(15*(a - b)^2*f*Sqrt[a - b +
b*Sec[e + f*x]^2]) - Cos[e + f*x]^5/(5*(a - b)*f*Sqrt[a - b + b*Sec[e + f*x
]^2]) - (2*b*(15*a^2 + 10*a*b - b^2)*Sec[e + f*x])/(15*(a - b)^4*f*Sqrt[a -
 b + b*Sec[e + f*x]^2])
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^6\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{-2(5a-2b)+5(a-b)x^2}{x^4\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{5(a-b)f}$$

$$= \frac{2(5a-2b)\cos^3(e+fx)}{15(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{\cos^5(e+fx)}{5(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\left(15a^2+10ab\right.}{}$$

$$= -\frac{\left(15a^2+10ab-b^2\right)\cos(e+fx)}{15(a-b)^3f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{2(5a-2b)\cos^3(e+fx)}{15(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{}{5(a-b)f\sqrt{}}$$

$$= -\frac{\left(15a^2+10ab-b^2\right)\cos(e+fx)}{15(a-b)^3f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{2(5a-2b)\cos^3(e+fx)}{15(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{}{5(a-b)f\sqrt{}}$$

Mathematica [A] time = 1.84405, size = 186, normalized size = 0.93

$$-\frac{\sec(e+fx)\left(\left(169a^2b+125a^3-329ab^2+35b^3\right)\cos(2(e+fx))-9a^2b\cos(6(e+fx))+1078a^2b+3a^3\cos(6(e+fx))+\right.}{240\sqrt{2}f(a-b)^4\sqrt{\sec^2(e+fx)((a-b)\,c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((150*a^3 + 1078*a^2*b + 338*a*b^2 - 30*b^3 + (125*a^3 + 169*a^2*b - 329*a
*b^2 + 35*b^3)*Cos[2*(e + f*x)] - 2*(a - b)^2*(11*a + b)*Cos[4*(e + f*x)] +
 3*a^3*Cos[6*(e + f*x)] - 9*a^2*b*Cos[6*(e + f*x)] + 9*a*b^2*Cos[6*(e + f*x
)] - 3*b^3*Cos[6*(e + f*x)])*Sec[e + f*x])/(240*Sqrt[2]*(a - b)^4*f*Sqrt[(a
 + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Maple [B] time = 3.807, size = 67748, normalized size = 340.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [B] time = 1.14598, size = 525, normalized size = 2.64

$$-\frac{\frac{15\,b^3}{\left(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4\right)\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}+\frac{15\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{a^2-2\,ab+b^2}+\frac{3\left(\left(a-b+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{5}{2}}\cos\left(fx+e\right)^5-5\left(a-b+\frac{b}{\cos\left(fx+e\right)^2}\right.\cdots}{a^4-4\,a^3b+6\,a\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*b^3/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*sqrt(a - b + b/c
os(f*x + e)^2)*cos(f*x + e)) + 15*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x +
e)/(a^2 - 2*a*b + b^2) + 3*((a - b + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5
 - 5*(a - b + b/cos(f*x + e)^2)^(3/2)*b*cos(f*x + e)^3 + 15*sqrt(a - b + b/
cos(f*x + e)^2)*b^2*cos(f*x + e))/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^
4) - 10*((a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 6*sqrt(a - b + b
/cos(f*x + e)^2)*b*cos(f*x + e))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 30*b^2/(
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)
) + 15*b/((a^2 - 2*a*b + b^2)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)))
/f
```

Fricas [A] time = 3.59056, size = 528, normalized size = 2.65

$$-\frac{\left(3\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\cos\left(fx+e\right)^7-2\left(5\,a^3-12\,a^2b+9\,ab^2-2\,b^3\right)\cos\left(fx+e\right)^5+\left(15\,a^3-5\,a^2b-11\,ab^2+\cdots\right.}{15\left(\left(a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5\right)f\cos\left(fx+e\right)^2+\left(a^4b-\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 2*(5*a^3 - 12*a^2
*b + 9*a*b^2 - 2*b^3)*cos(f*x + e)^5 + (15*a^3 - 5*a^2*b - 11*a*b^2 + b^3)*
cos(f*x + e)^3 + 2*(15*a^2*b + 10*a*b^2 - b^3)*cos(f*x + e))*sqrt(((a - b)*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b
^3 + 5*a*b^4 - b^5)*f*cos(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a
*b^4 + b^5)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^5}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^5/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

3.129 $\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=131

$$-\frac{2b(3a+b)\sec(e+fx)}{3f(a-b)^3\sqrt{a+b\sec^2(e+fx)-b}}+\frac{\cos^3(e+fx)}{3f(a-b)\sqrt{a+b\sec^2(e+fx)-b}}-\frac{(3a+b)\cos(e+fx)}{3f(a-b)^2\sqrt{a+b\sec^2(e+fx)-b}}$$

```
[Out] -((3*a + b)*Cos[e + f*x])/(3*(a - b)^2*f*Sqrt[a - b + b*Sec[e + f*x]^2]) +
Cos[e + f*x]^3/(3*(a - b)*f*Sqrt[a - b + b*Sec[e + f*x]^2]) - (2*b*(3*a + b
)*Sec[e + f*x])/(3*(a - b)^3*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.134308, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3664, 453, 271, 191}

$$-\frac{2b(3a+b)\sec(e+fx)}{3f(a-b)^3\sqrt{a+b\sec^2(e+fx)-b}}+\frac{\cos^3(e+fx)}{3f(a-b)\sqrt{a+b\sec^2(e+fx)-b}}-\frac{(3a+b)\cos(e+fx)}{3f(a-b)^2\sqrt{a+b\sec^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((3*a + b)*Cos[e + f*x])/(3*(a - b)^2*f*Sqrt[a - b + b*Sec[e + f*x]^2]) +
Cos[e + f*x]^3/(3*(a - b)*f*Sqrt[a - b + b*Sec[e + f*x]^2]) - (2*b*(3*a + b
)*Sec[e + f*x])/(3*(a - b)^3*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)}{3(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{(3a+b)\,\text{Subst}\left(\int \frac{1}{x^2\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{3(a-b)f}$$

$$= -\frac{(3a+b)\cos(e+fx)}{3(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\cos^3(e+fx)}{3(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{(2b(3a+b))\,\text{S}\cdots}{\cdots}$$

$$= -\frac{(3a+b)\cos(e+fx)}{3(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\cos^3(e+fx)}{3(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{2b(3a+\cdots}{3(a-b)^3f\sqrt{a\cdots}}$$

Mathematica [A] time = 1.13964, size = 106, normalized size = 0.81

$$-\frac{\sec(e+fx)\left(8\left(a^2-b^2\right)\cos(2(e+fx))+9a^2-(a-b)^2\cos(4(e+fx))+46ab+9b^2\right)}{12\sqrt{2}f(a-b)^3\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((9*a^2 + 46*a*b + 9*b^2 + 8*(a^2 - b^2)*Cos[2*(e + f*x)] - (a - b)^2*Cos[
4*(e + f*x)])*Sec[e + f*x])/(12*Sqrt[2]*(a - b)^3*f*Sqrt[(a + b + (a - b)*C
os[2*(e + f*x)])*Sec[e + f*x]^2])
```

Maple [B] time = 1.089, size = 14991, normalized size = 114.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [A] time = 1.09813, size = 292, normalized size = 2.23

$$-\frac{\frac{3\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^2-2ab+b^2} - \frac{\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-6\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)}{a^3-3a^2b+3ab^2-b^3} + \frac{3b^2}{\left(a^3-3a^2b+3ab^2-b^3\right)\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}\cdots}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a^2 - 2*a*b + b^2) - (
(a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 6*sqrt(a - b + b/cos(f*x
+ e)^2)*b*cos(f*x + e))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 3*b^2/((a^3 - 3*a
^2*b + 3*a*b^2 - b^3)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)) + 3*b/((
a^2 - 2*a*b + b^2)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)))/f
```

Fricas [A] time = 2.74309, size = 358, normalized size = 2.73

$$\frac{\left(\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^5-\left(3\,a^2-2\,ab-b^2\right)\cos\left(fx+e\right)^3-2\left(3\,ab+b^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{3\left(\left(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4\right)f\cos\left(fx+e\right)^2+\left(a^3b-3\,a^2b^2+3\,ab^3-b^4\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*((a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - (3*a^2 - 2*a*b - b^2)*cos(f*x + e
)^3 - 2*(3*a*b + b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*f*cos(f*x + e)^2 +
(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^3}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^3/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

3.130 $\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{2b\sec(e+fx)}{f(a-b)^2\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\cos(e+fx)}{f(a-b)\sqrt{a+b\sec^2(e+fx)-b}}$$

```
[Out] -(Cos[e + f*x]/((a - b)*f*Sqrt[a - b + b*Sec[e + f*x]^2])) - (2*b*Sec[e + f
*x])/((a - b)^2*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.0638157, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3664, 271, 191}

$$-\frac{2b\sec(e+fx)}{f(a-b)^2\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\cos(e+fx)}{f(a-b)\sqrt{a+b\sec^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cos[e + f*x]/((a - b)*f*Sqrt[a - b + b*Sec[e + f*x]^2])) - (2*b*Sec[e + f
*x])/((a - b)^2*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)}{(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{(a-b)f}$$

$$= -\frac{\cos(e+fx)}{(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{2b\sec(e+fx)}{(a-b)^2 f\sqrt{a-b+b\sec^2(e+fx)}}$$

Mathematica [A] time = 1.55434, size = 72, normalized size = 0.95

$$-\frac{\sec(e+fx)((a-b)\cos(2(e+fx))+a+3b)}{\sqrt{2}f(a-b)^2\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a + 3*b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x])/(Sqrt[2]*(a - b)^2*f*
Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]))
```

Maple [A] time = 0.053, size = 103, normalized size = 1.4

$$-\frac{\left(a\left(\cos\left(fx+e\right)\right)^2-\left(\cos\left(fx+e\right)\right)^2b+b\right)\left(a\left(\cos\left(fx+e\right)\right)^2-\left(\cos\left(fx+e\right)\right)^2b+2b\right)}{f\left(\cos\left(fx+e\right)\right)^3(a-b)^2}\left(\frac{a\left(\cos\left(fx+e\right)\right)^2-\left(\cos\left(f\right.\right.}{\left(\cos\left(fx+e\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+2*b)/
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)/cos(f*x+e)^3/(a-b)^2
```

Maxima [A] time = 1.02001, size = 112, normalized size = 1.47

$$-\frac{\frac{\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{a^2-2ab+b^2}+\frac{b}{\left(a^2-2ab+b^2\right)\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a^2 - 2*a*b + b^2) + b/((a^2
 - 2*a*b + b^2)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)))/f
```

Fricas [A] time = 2.45921, size = 236, normalized size = 3.11

$$-\frac{\left((a-b)\cos\left(fx+e\right)^{3}+2\,b\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{\left(a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}\right)f\cos\left(fx+e\right)^{2}+\left(a^{2}b-2\,ab^{2}+b^{3}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cos(f*x + e)^2 + (a^
2*b - 2*a*b^2 + b^3)*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(e+fx\right)}{\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

3.131 $\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=84

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{a^{3/2}f}-\frac{b\sec(e+fx)}{af(a-b)\sqrt{a+b\sec^2(e+fx)-b}}$$

```
[Out] -(ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]]/(a^(3/2)*f
)) - (b*Sec[e + f*x])/(a*(a - b)*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.0965029, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3664, 382, 377, 207}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{a^{3/2}f}-\frac{b\sec(e+fx)}{af(a-b)\sqrt{a+b\sec^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```


```
[Out] -(ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]]/(a^(3/2)*f
)) - (b*Sec[e + f*x])/(a*(a - b)*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{b\sec(e+fx)}{a(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{af}$$

$$= -\frac{b\sec(e+fx)}{a(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-1+ax^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{af}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\sec(e+fx)}{a(a-b)f\sqrt{a-b+b\sec^2(e+fx)}}$$

Mathematica [B] time = 4.8542, size = 333, normalized size = 3.96

$$-\frac{\cos(e+fx)\sec^6\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\sqrt{2}\sqrt{a}b(\cos(e+fx)+1)\sqrt{\sec^4\left(\frac{1}{2}(e+fx)\right)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cos[e + f*x]*Sec[(e + f*x)/2]^6*((a - b)*ArcTanh[(a - (a - 2*b)*Tan[(e +
f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^
2)^2])]*(a + b + (a - b)*Cos[2*(e + f*x)]) + (a - b)*ArcTanh[(2*b + a*(-1 +
 Tan[(e + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e
 + f*x)/2]^2)^2])]*(a + b + (a - b)*Cos[2*(e + f*x)]) + Sqrt[2]*Sqrt[a]*b*(
1 + Cos[e + f*x])*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^
4])*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(2*a^(3/2)*(a
- b)*f*((a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4)^(3/2))
```

Maple [B] time = 0.211, size = 3491, normalized size = 41.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f/a^(5/2)/(a-b)*(ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+
e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f
*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)
/sin(f*x+e)^2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*c
os(f*x+e)^3*a^3-2*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e
)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin
(f*x+e)^2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*cos(f
```

```
*x+e)^3*a^2*b+ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos
(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x
+e)^2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e
)^3*a*b^2+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*
(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1
/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e
)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*cos(f*x+e)^3*a^3-2*((a*cos(f*x+e)^
2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*((a*cos(f*x+e
)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*
x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/
(cos(f*x+e)-1))*cos(f*x+e)^3*a^2*b+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(
f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(co
s(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos
(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*cos(f*x+e
)^3*a*b^2+2*cos(f*x+e)^2*a^(5/2)*b+ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)
*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x
+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/
2)*a^(1/2)+b)/sin(f*x+e)^2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+
1)^2)^(1/2)*cos(f*x+e)^2*a^3-2*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a
*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*
a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a
^(1/2)+b)/sin(f*x+e)^2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2
)^(1/2)*cos(f*x+e)^2*a^2*b+ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos
(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*
cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/
2)+b)/sin(f*x+e)^2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1
/2)*cos(f*x+e)^2*a*b^2+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)
^(1/2)*ln(-4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^
2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*cos(f*x+e)^2*a^3-2*((
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+
e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2
)*a^(1/2)+b)/(cos(f*x+e)-1))*cos(f*x+e)^2*a^2*b+((a*cos(f*x+e)^2-cos(f*x+e)
^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+
e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos
(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-
1))*cos(f*x+e)^2*a*b^2-2*cos(f*x+e)^2*a^(3/2)*b^2+ln(-2/a^(1/2)*(cos(f*x+e)
-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*
a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f
*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)*a^2*b-ln(-2/a^(1/2)*(cos(f*x+e)-1)*(c
os(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2
)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+
1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(co
s(f*x+e)+1)^2)^(1/2)*cos(f*x+e)*a*b^2+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(c
os(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/
(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*cos(f*
x+e)*a^2*b-((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4
*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(
1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+
e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*cos(f*x+e)*a*b^2+ln(-2/a^(1/2)*(c
os(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^
2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*((a*cos(f*x+e)^2-cos(f*
x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^2*b-ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos
(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-
```

```
cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)
^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(
f*x+e)+1)^2)^(1/2)*a*b^2+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^
2)^(1/2)*ln(-4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1
)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*a^2*b-((a*cos(f*x+e
)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*((a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(
f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b
)/(cos(f*x+e)-1))*a*b^2+2*b^2*a^(3/2))/cos(f*x+e)^3/((a*cos(f*x+e)^2-cos(f*
x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 3.41413, size = 842, normalized size = 10.02

$$\left[-\frac{2\,ab\sqrt{\frac{(a-b)\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)-\left(\left(a^{2}-2\,ab+b^{2}\right)\cos\left(fx+e\right)^{2}+ab-b^{2}\right)\sqrt{a}\log\left(-\frac{2\left((a-b)\cos\left(fx+e\right)^{2}-2\sqrt{a}\sqrt{\frac{(a-b)\cos(f}{\cos(fx}}\cdots\right.}{\cos\left(fx+e\right)^{2}-\cdots}\right.}{2\left(\left(a^{4}-2\,a^{3}b+a^{2}b^{2}\right)f\cos\left(fx+e\right)^{2}+\left(a^{3}b-a^{2}b^{2}\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*b*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)
 - ((a^2 - 2*a*b + b^2)*cos(f*x + e)^2 + a*b - b^2)*sqrt(a)*log(-2*((a - b)
*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)))/((a^4 - 2*a^3*b + a^2*b^2)*
f*cos(f*x + e)^2 + (a^3*b - a^2*b^2)*f), -(a*b*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*cos(f*x + e) - ((a^2 - 2*a*b + b^2)*cos(f*x + e)^2 +
a*b - b^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*cos(f*x + e)/a))/((a^4 - 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (
a^3*b - a^2*b^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(e+fx\right)}{\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

3.132 $\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{3b\sec(e+fx)}{2a^2f\sqrt{a+b\sec^2(e+fx)-b}}-\frac{(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2a^{5/2}f}-\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a+b\sec^2(e+fx)-b}}$$

```
[Out] -((a - 3*b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])
/(2*a^(5/2)*f) - (Cot[e + f*x]*Csc[e + f*x])/(2*a*f*Sqrt[a - b + b*Sec[e +
f*x]^2]) - (3*b*Sec[e + f*x])/(2*a^2*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.157731, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3664, 471, 527, 12, 377, 207}

$$-\frac{3b\sec(e+fx)}{2a^2f\sqrt{a+b\sec^2(e+fx)-b}}-\frac{(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2a^{5/2}f}-\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a+b\sec^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a - 3*b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])
/(2*a^(5/2)*f) - (Cot[e + f*x]*Csc[e + f*x])/(2*a*f*Sqrt[a - b + b*Sec[e +
f*x]^2]) - (3*b*Sec[e + f*x])/(2*a^2*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-b-2bx^2}{\left(-1+x^2\right)\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{2af}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2a^2f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{(a-3b)(a-b}{\left(-1+x^2\right)\sqrt{a-b}}\right.}{2a^2(a}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2a^2f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{(a-3b)\,\text{Subst}\left(\int \frac{}{(-1}\right.}{}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2a^2f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{(a-3b)\,\text{Subst}\left(\int \frac{}{-1-}\right.}{}$$

$$= -\frac{(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2a^{5/2}f} - \frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{3b\sec(e}{2a^2f\sqrt{a-b+b}}$$

Mathematica [B] time = 4.09678, size = 308, normalized size = 2.43

$$-\frac{\frac{\csc^2(e+fx)\sec(e+fx)((a-3b)\cos(2(e+fx))+a+3b)}{\sqrt{2}a^2\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}} + \frac{(a-3b)\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\tanh^{-1}\left(\frac{a-(a-2b)}{\sqrt{a}\sqrt{a\left(\tan^2\left(\frac{1}{2}(e+f\right.\right.}}\right.\right.}{2a^{5/2}\sqrt{\sec^4\left(\frac{1}{2}(e+fx)\right)((a-b)\cos(2}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((((a + 3*b + (a - 3*b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2*Sec[e + f*x])/(Sqr
t[2]*a^2*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]) + ((a - 3
*b)*(ArcTanh[(a - (a - 2*b)*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*Tan[(e +
```

```
f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])] + ArcTanh[(2*b + a*(-1 + Tan[(
e + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x
)/2]^2)^2])])*Cos[e + f*x]*Sec[(e + f*x)/2]^2*Sqrt[(a + b + (a - b)*Cos[2*(
e + f*x)])*Sec[e + f*x]^2])/(2*a^(5/2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x
)])*Sec[(e + f*x)/2]^4]))/(2*f)
```

Maple [B] time = 0.261, size = 5633, normalized size = 44.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 3.54322, size = 1087, normalized size = 8.56

$$\left[-\frac{\left(\left(a^{2}-4\,ab+3\,b^{2}\right)\cos\left(fx+e\right)^{4}-\left(a^{2}-5\,ab+6\,b^{2}\right)\cos\left(fx+e\right)^{2}-ab+3\,b^{2}\right)\sqrt{a}\log\left(-\frac{2\left((a-b)\cos\left(fx+e\right)^{2}+2\sqrt{a}\sqrt{\frac{(a-b)\cos(f}{\cos(fx}}\right.}{\cos\left(fx+e\right)^{2}-1}\right.}{4\left(\left(a^{4}-a^{3}b\right)f\cos\left(fx+e\right)^{4}-a^{3}bf-\left(a^{4}-2\,a^{3}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((a^2 - 4*a*b + 3*b^2)*cos(f*x + e)^4 - (a^2 - 5*a*b + 6*b^2)*cos(f*
x + e)^2 - a*b + 3*b^2)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 + 2*sqrt(a)*
sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(co
s(f*x + e)^2 - 1)) - 2*((a^2 - 3*a*b)*cos(f*x + e)^3 + 3*a*b*cos(f*x + e))*
sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 - a^3*b)*f*cos(f*x
 + e)^4 - a^3*b*f - (a^4 - 2*a^3*b)*f*cos(f*x + e)^2), 1/2*(((a^2 - 4*a*b +
 3*b^2)*cos(f*x + e)^4 - (a^2 - 5*a*b + 6*b^2)*cos(f*x + e)^2 - a*b + 3*b^2
)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*cos(f*x + e)/a) + ((a^2 - 3*a*b)*cos(f*x + e)^3 + 3*a*b*cos(f*x + e))*sqr
t(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 - a^3*b)*f*cos(f*x +
```

```
e)^4 - a^3*b*f - (a^4 - 2*a^3*b)*f*cos(f*x + e)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(fx+e)^3}{\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^3/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

3.133 $\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=187

$$-\frac{b(13a-15b)\sec(e+fx)}{8a^3f\sqrt{a+b\sec^2(e+fx)-b}}-\frac{3(a-5b)(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8a^{7/2}f}-\frac{5(a-b)\cot(e+fx)\csc(e+fx)}{8a^2f\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\cot^3(e}{4af\sqrt{a}}$$

```
[Out] (-3*(a - 5*b)*(a - b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e +
 f*x]^2]])/(8*a^(7/2)*f) - (5*(a - b)*Cot[e + f*x]*Csc[e + f*x])/(8*a^2*f*S
qrt[a - b + b*Sec[e + f*x]^2]) - (Cot[e + f*x]^3*Csc[e + f*x])/(4*a*f*Sqrt[
a - b + b*Sec[e + f*x]^2]) - ((13*a - 15*b)*b*Sec[e + f*x])/(8*a^3*f*Sqrt[a
 - b + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.236033, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3664, 470, 527, 12, 377, 207}

$$-\frac{b(13a-15b)\sec(e+fx)}{8a^3f\sqrt{a+b\sec^2(e+fx)-b}}-\frac{3(a-5b)(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8a^{7/2}f}-\frac{5(a-b)\cot(e+fx)\csc(e+fx)}{8a^2f\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\cot^3(e}{4af\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*(a - 5*b)*(a - b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e +
 f*x]^2]])/(8*a^(7/2)*f) - (5*(a - b)*Cot[e + f*x]*Csc[e + f*x])/(8*a^2*f*S
qrt[a - b + b*Sec[e + f*x]^2]) - (Cot[e + f*x]^3*Csc[e + f*x])/(4*a*f*Sqrt[
a - b + b*Sec[e + f*x]^2]) - ((13*a - 15*b)*b*Sec[e + f*x])/(8*a^3*f*Sqrt[a
 - b + b*Sec[e + f*x]^2])
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^3\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-a+b-4(a-b)x^2}{\left(-1+x^2\right)^2\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{4af}$$

$$= -\frac{5(a-b)\cot(e+fx)\csc(e+fx)}{8a^2f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-(3a-5b)(}{(-1+x^2}\right.}{}$$

$$= -\frac{5(a-b)\cot(e+fx)\csc(e+fx)}{8a^2f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{(13a-15b)b\sec}{8a^3f\sqrt{a-b+b\sec}}$$

$$= -\frac{5(a-b)\cot(e+fx)\csc(e+fx)}{8a^2f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{(13a-15b)b\sec}{8a^3f\sqrt{a-b+b\sec}}$$

$$= -\frac{5(a-b)\cot(e+fx)\csc(e+fx)}{8a^2f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{(13a-15b)b\sec}{8a^3f\sqrt{a-b+b\sec}}$$

$$= -\frac{3(a-5b)(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8a^{7/2}f} - \frac{5(a-b)\cot(e+fx)\csc(e+fx)}{8a^2f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{\text{co}}{4af}$$

Mathematica [A] time = 4.75673, size = 350, normalized size = 1.87

$$\frac{\frac{\csc^4(e+fx)\sec(e+fx)\left(\left(-8a^2+52ab-60b^2\right)\cos(2(e+fx))+(a-b)(3(a-5b)\cos(4(e+fx))-11a-45b)\right)}{4\sqrt{2}a^3\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}} - \frac{3(a-5b)(a-b)\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec^2(e+fx)((a}}{}}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((((-8*a^2 + 52*a*b - 60*b^2)*Cos[2*(e + f*x)] + (a - b)*(-11*a - 45*b + 3*
(a - 5*b)*Cos[4*(e + f*x)]))*Csc[e + f*x]^4*Sec[e + f*x])/(4*Sqrt[2]*a^3*Sq
rt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]) - (3*(a - 5*b)*(a -
b)*(ArcTanh[(a - (a - 2*b)*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*Tan[(e + f
*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])] + ArcTanh[(2*b + a*(-1 + Tan[(e
 + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)
/2]^2)^2])])*Cos[e + f*x]*Sec[(e + f*x)/2]^2*Sqrt[(a + b + (a - b)*Cos[2*(e
 + f*x)])*Sec[e + f*x]^2])/(2*a^(7/2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)
])*Sec[(e + f*x)/2]^4]))/(8*f)
```

Maple [B] time = 0.228, size = 10582, normalized size = 56.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 3.79066, size = 1656, normalized size = 8.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*((a^3 - 7*a^2*b + 11*a*b^2 - 5*b^3)*cos(f*x + e)^6 - (2*a^3 - 15*a
^2*b + 28*a*b^2 - 15*b^3)*cos(f*x + e)^4 + a^2*b - 6*a*b^2 + 5*b^3 + (a^3 -
 9*a^2*b + 23*a*b^2 - 15*b^3)*cos(f*x + e)^2)*sqrt(a)*log(-2*((a - b)*cos(f
*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos
(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)) + 2*(3*(a^3 - 6*a^2*b + 5*a*b^2)*c
os(f*x + e)^5 - (5*a^3 - 31*a^2*b + 30*a*b^2)*cos(f*x + e)^3 - (13*a^2*b -
15*a*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/
((a^5 - a^4*b)*f*cos(f*x + e)^6 + a^4*b*f - (2*a^5 - 3*a^4*b)*f*cos(f*x + e
)^4 + (a^5 - 3*a^4*b)*f*cos(f*x + e)^2), 1/8*(3*((a^3 - 7*a^2*b + 11*a*b^2
```

```
- 5*b^3)*cos(f*x + e)^6 - (2*a^3 - 15*a^2*b + 28*a*b^2 - 15*b^3)*cos(f*x +
e)^4 + a^2*b - 6*a*b^2 + 5*b^3 + (a^3 - 9*a^2*b + 23*a*b^2 - 15*b^3)*cos(f*
x + e)^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*cos(f*x + e)/a) + (3*(a^3 - 6*a^2*b + 5*a*b^2)*cos(f*x + e)^5 - (
5*a^3 - 31*a^2*b + 30*a*b^2)*cos(f*x + e)^3 - (13*a^2*b - 15*a*b^2)*cos(f*x
 + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^5 - a^4*b)*f*
cos(f*x + e)^6 + a^4*b*f - (2*a^5 - 3*a^4*b)*f*cos(f*x + e)^4 + (a^5 - 3*a^
4*b)*f*cos(f*x + e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^5}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^5/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

3.134 $\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=187

$$\frac{3a(a+4b)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f(a-b)^{7/2}}-\frac{b(13a+2b)\tan(e+fx)}{8f(a-b)^3\sqrt{a+b\tan^2(e+fx)}}+\frac{\sin(e+fx)\cos^3(e+fx)}{4f(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{5a\sin(e+fx)}{8f(a-b)^2\sqrt{a+\cdots}}$$

```
[Out] (3*a*(a + 4*b)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/(8*(a - b)^(7/2)*f) - (5*a*Cos[e + f*x]*Sin[e + f*x])/(8*(a - b)^2*f*Sqr
t[a + b*Tan[e + f*x]^2]) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*(a - b)*f*Sqrt[
a + b*Tan[e + f*x]^2]) - (b*(13*a + 2*b)*Tan[e + f*x])/(8*(a - b)^3*f*Sqrt[
a + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.222787, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3663, 470, 527, 12, 377, 203}

$$\frac{3a(a+4b)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f(a-b)^{7/2}}-\frac{b(13a+2b)\tan(e+fx)}{8f(a-b)^3\sqrt{a+b\tan^2(e+fx)}}+\frac{\sin(e+fx)\cos^3(e+fx)}{4f(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{5a\sin(e+fx)}{8f(a-b)^2\sqrt{a+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*a*(a + 4*b)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/(8*(a - b)^(7/2)*f) - (5*a*Cos[e + f*x]*Sin[e + f*x])/(8*(a - b)^2*f*Sqr
t[a + b*Tan[e + f*x]^2]) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*(a - b)*f*Sqrt[
a + b*Tan[e + f*x]^2]) - (b*(13*a + 2*b)*Tan[e + f*x])/(8*(a - b)^3*f*Sqrt[
a + b*Tan[e + f*x]^2])
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^3\left(a+bx^2\right)^{3/2}}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{a-4ax^2}{\left(1+x^2\right)^2\left(a+bx^2\right)^{3/2}}\,dx,x,\tan(e+fx)\right)}{4(a-b)f}$$

$$= -\frac{5a\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a(3a+2b}{(1+x^2)(}\right.}{}$$

$$= -\frac{5a\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{b(13a+2b)\,\text{t}}{8(a-b)^3f\sqrt{a+}}$$

$$= -\frac{5a\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{b(13a+2b)\,\text{t}}{8(a-b)^3f\sqrt{a+}}$$

$$= -\frac{5a\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{b(13a+2b)\,\text{t}}{8(a-b)^3f\sqrt{a+}}$$

$$= \frac{3a(a+4b)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8(a-b)^{7/2}f} - \frac{5a\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx}{4(a-b)f\sqrt{a+}}$$

Mathematica [C] time = 3.31646, size = 325, normalized size = 1.74

$$\frac{\sin(2(e+fx))\sec^2(e+fx)\left(6\sqrt{2}a\left(a^2+3ab-4b^2\right)\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+f}{b}}}{\sqrt{2}}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-((a - b)*(7*a^2 + 48*a*b + 5*b^2 + (6*a^2 - 2*a*b - 4*b^2)*Cos[2*(e + f*
x)] - (a - b)^2*Cos[4*(e + f*x)])) + 6*Sqrt[2]*a*(a^2 + 3*a*b - 4*b^2)*Sqrt
[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sq
rt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] - 6*
Sqrt[2]*a^2*(a + 4*b)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]
^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*
x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)])/(32*
Sqrt[2]*(a - b)^4*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]
)
```

Maple [C] time = 1.602, size = 6025, normalized size = 32.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^4}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)**4/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(fx+e)^4}{(b\tan(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

3.135 $\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f(a-b)^{5/2}}-\frac{3b\tan(e+fx)}{2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\sin(e+fx)\cos(e+fx)}{2f(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] ((a + 2*b)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(
2*(a - b)^(5/2)*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*(a - b)*f*Sqrt[a + b*Ta
n[e + f*x]^2]) - (3*b*Tan[e + f*x])/(2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^
2])
```

Rubi [A] time = 0.157488, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3663, 471, 527, 12, 377, 203}

$$\frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f(a-b)^{5/2}}-\frac{3b\tan(e+fx)}{2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\sin(e+fx)\cos(e+fx)}{2f(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```


```
[Out] ((a + 2*b)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(
2*(a - b)^(5/2)*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*(a - b)*f*Sqrt[a + b*Ta
n[e + f*x]^2]) - (3*b*Tan[e + f*x])/(2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^
2])
```


Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
```

```
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-2bx^2}{\left(1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{2(a-b)f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a(a+}{(1+x^2)\sqrt{}}\right.}{2a}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{(a+2b)\,\text{Subst}\left(\right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{(a+2b)\,\text{Subst}\left(\right.}{}$$

$$= \frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2(a-b)^{5/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{3b\tan(e+}{2(a-b)^2f\sqrt{a+b}}$$

Mathematica [C] time = 2.77075, size = 282, normalized size = 2.1

$$-\frac{\sin(2(e+fx))\sec^2(e+fx)\left(-\sqrt{2}\left(a^2+ab-2b^2\right)\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(}{b}}}{\sqrt{2}}\right.\right.\right.}{4\sqrt{2}f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)*(a + 5*b + (a - b)*Cos[2*(e + f*x)]) - Sqrt[2]*(a^2 + a*b - 2*b^
2)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[Ar
cSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]],
1] + Sqrt[2]*a*(a + 2*b)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f
*x]^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e +
 f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)])/(
4*Sqrt[2]*(a - b)^3*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^
2])
```

Maple [C] time = 0.218, size = 1614, normalized size = 12.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f/a/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/(a-b)*(a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)*(2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-
b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*
x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)
/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a
-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)
*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*sin(f*x+e)-2*2^(1/2)*(1/a*(I*cos(f*x+e)*b
^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(
f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1
/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+
e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*
(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/
2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*sin(f*x+e)+cos(f*x+e)*((2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)/a)^(1/2)*b-((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b)*sin(f
*x+e)/(cos(f*x+e)-1)/cos(f*x+e)^3/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*
x+e)^2)^(3/2)+1/4/f*(cos(2*f*x+2*e)^2*(a-b)^(3/2)*a^3*b-2*cos(2*f*x+2*e)^2*
(a-b)^(3/2)*a^2*b^2+cos(2*f*x+2*e)^2*(a-b)^(3/2)*a*b^3+2*cos(2*f*x+2*e)*(a-
b)^(3/2)*a^2*b^2+2*cos(2*f*x+2*e)*(a-b)^(3/2)*a*b^3-4*cos(2*f*x+2*e)*(a-b)^
(3/2)*b^4-4*(b^4*(a-b))^(1/2)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)*(-1+cos(2*
f*x+2*e))/sin(2*f*x+2*e)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*
x+2*e)+1))^(1/2))*sin(2*f*x+2*e)*a*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)
/(cos(2*f*x+2*e)+1))^(1/2)*(a-b)^(3/2)+2*sin(2*f*x+2*e)*((a*cos(2*f*x+2*e)-
b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)*arctan((a-b)^(1/2)*(-1+cos(
2*f*x+2*e))/sin(2*f*x+2*e)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*
f*x+2*e)+1))^(1/2))*a^4*b-8*sin(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2
*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)*arctan((a-b)^(1/2)*(-1+cos(2*f*x+2*e))/s
in(2*f*x+2*e)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^
(1/2))*a^3*b^2+10*sin(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(
cos(2*f*x+2*e)+1))^(1/2)*arctan((a-b)^(1/2)*(-1+cos(2*f*x+2*e))/sin(2*f*x+2
*e)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2))*a^2
*b^3-4*sin(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2
*e)+1))^(1/2)*arctan((a-b)^(1/2)*(-1+cos(2*f*x+2*e))/sin(2*f*x+2*e)/((a*cos
(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2))*a*b^4-(a-b)^(3
/2)*a^3*b-3*(a-b)^(3/2)*a*b^3+4*(a-b)^(3/2)*b^4)/b/((a*cos(2*f*x+2*e)-b*cos
(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)/a/(a-b)^(9/2)/sin(2*f*x+2*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^2}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 51.0875, size = 2133, normalized size = 15.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((a^2 + a*b - 2*b^2)*cos(f*x + e)^2 + a*b + 2*b^2)*sqrt(-a + b)*log
(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4
- 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^
3*b + 77*a^2*b^2 - 72*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160
*a^2*b^2 - 256*a*b^3 + 128*b^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3
 + 16*b^4)*cos(f*x + e)^2 + 8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x +
 e)^7 - 24*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29
*a^2*b + 48*a*b^2 - 24*b^3)*cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 1
6*b^3)*cos(f*x + e))*sqrt(-a + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*sin(f*x + e)) + 8*((a^2 - 2*a*b + b^2)*cos(f*x + e)^3 + 3*(a*b - b
^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^3
*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f), 1/8*(((a^2 + a*b - 2*b^2)*cos(f*x + e)^
2 + a*b + 2*b^2)*sqrt(a - b)*arctan(-1/4*(8*(a^2 - 2*a*b + b^2)*cos(f*x + e
)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*b + 8*b^2)*cos(f*
x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*(
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*a*b^2 - 2*b^3 - (
a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) - 4*((a^2 -
 2*a*b + b^2)*cos(f*x + e)^3 + 3*(a*b - b^2)*cos(f*x + e))*sqrt(((a - b)*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^4 - 4*a^3*b + 6*a^2*b^2
 - 4*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)**2/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^2}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

3.136 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{b\tan(e+fx)}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3/2
)*f) - (b*Tan[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.0572698, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3661, 382, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{b\tan(e+fx)}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-3/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3/2
)*f) - (b*Tan[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^{3/2}}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}+\frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx,x,\tan(e+fx)\right)}{(a-b)f}$$

$$= -\frac{b\tan(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}+\frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx,x,\frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f}-\frac{b\tan(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}$$

Mathematica [C] time = 7.09181, size = 214, normalized size = 2.52

$$\frac{4\sin(e+fx)\cos^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\left(a(a-b)\tan^2(e+fx)\text{Hypergeometric2F1}\left(2,2,\frac{7}{2},\frac{(a-b)\sin^2(e+fx)}{a}\right)+\frac{15(3}{\ldots}\right.}{15a^4f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-3/2),x]
```

```
[Out] (4*Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2]*(a*(a - b)*Hyperg
eometric2F1[2, 2, 7/2, ((a - b)*Sin[e + f*x]^2)/a]*Tan[e + f*x]^2 + (15*(3*
a + 2*b*Tan[e + f*x]^2)*(-2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*(a*Cos
[e + f*x]^2 + b*Sin[e + f*x]^2) + a*Sqrt[((a - b)*Sin[2*(e + f*x)]^2*(a + b
*Tan[e + f*x]^2))/a^2]))/(((a - b)*Sin[2*(e + f*x)]^2*(a + b*Tan[e + f*x]^2
))/a^2)^(3/2)))/(15*a^4*f)
```

Maple [A] time = 0.021, size = 104, normalized size = 1.2

$$-\frac{b\tan(fx+e)}{a(a-b)f}\frac{1}{\sqrt{a+b\left(\tan(fx+e)\right)^2}}+\frac{1}{f(a-b)^2b^2}\sqrt{b^4(a-b)}\arctan\left((a-b)b^2\tan(fx+e)\frac{1}{\sqrt{b^4(a-b)}}\frac{1}{\sqrt{a+b(\text{ta}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)+1/f/(a-b)^2*(b^4*(a-b))^(1
/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x
```

+e))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.20957, size = 722, normalized size = 8.49

$$\left[\frac{\left(ab\tan\left(fx+e\right)^2+a^2\right)\sqrt{-a+b}\log\left(-\frac{(a-2b)\tan\left(fx+e\right)^2+2\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{-a+b}\tan\left(fx+e\right)-a}{\tan\left(fx+e\right)^2+1}\right)-2\sqrt{b\tan\left(fx+e\right)^2+a}\left(ab\right.}{2\left(\left(a^3b-2a^2b^2+ab^3\right)f\tan\left(fx+e\right)^2+\left(a^4-2a^3b+a^2b^2\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b*tan(f*x + e)^2 + a^2)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^
2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x +
e)^2 + 1)) - 2*sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan(f*x + e))/((a^3*b
 - 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f), ((a*
b*tan(f*x + e)^2 + a^2)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqr
t(a - b)*tan(f*x + e))) - sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan(f*x +
e))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^
2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(-3/2), x)
```

3.137 $\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2b\tan(e+fx)}{a^2f\sqrt{a+b\tan^2(e+fx)}}-\frac{\cot(e+fx)}{af\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] -(Cot[e + f*x]/(a*f*Sqrt[a + b*Tan[e + f*x]^2])) - (2*b*Tan[e + f*x])/(a^2*
f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.0979908, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3663, 271, 191}

$$-\frac{2b\tan(e+fx)}{a^2f\sqrt{a+b\tan^2(e+fx)}}-\frac{\cot(e+fx)}{af\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cot[e + f*x]/(a*f*Sqrt[a + b*Tan[e + f*x]^2])) - (2*b*Tan[e + f*x])/(a^2*
f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)}{af\sqrt{a+b\tan^2(e+fx)}} - \frac{(2b)\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{af}$$

$$= -\frac{\cot(e+fx)}{af\sqrt{a+b\tan^2(e+fx)}} - \frac{2b\tan(e+fx)}{a^2f\sqrt{a+b\tan^2(e+fx)}}$$

Mathematica [A] time = 0.652551, size = 74, normalized size = 1.19

$$-\frac{\csc(e+fx)\sec(e+fx)((a-2b)\cos(2(e+fx))+a+2b)}{\sqrt{2}a^2f\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a + 2*b + (a - 2*b)*Cos[2*(e + f*x)])*Csc[e + f*x]*Sec[e + f*x])/(Sqrt[
2]*a^2*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]))
```

Maple [A] time = 0.17, size = 109, normalized size = 1.8

$$-\frac{\left(a\left(\cos\left(fx+e\right)\right)^2-2\left(\cos\left(fx+e\right)\right)^2b+2b\right)\left(\cos\left(fx+e\right)\right)^3}{fa^2\left(a\left(\cos\left(fx+e\right)\right)^2-\left(\cos\left(fx+e\right)\right)^2b+b\right)^2\sin\left(fx+e\right)}\left(\frac{a\left(\cos\left(fx+e\right)\right)^2-\left(\cos\left(fx+e\right)\right)^2b+b}{\left(\cos\left(fx+e\right)\right)^2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f/a^2/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*(a*cos(f*x+e)^2-2*cos(f*x+e)^2
*b+2*b)*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)
/sin(f*x+e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.8659, size = 211, normalized size = 3.4

$$-\frac{\left((a-2\,b)\cos\left(fx+e\right)^{3}+2\,b\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{\left(a^{2}bf+\left(a^{3}-a^{2}b\right)f\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/((a^2*b*f + (a^3 - a^2*b)*f*cos(f*x + e)^2)*sin(f*x +
 e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc^{2}\left(e+fx\right)}{\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc\left(fx+e\right)^{2}}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

3.138 $\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{2b(3a-4b)\tan(e+fx)}{3a^3f\sqrt{a+b\tan^2(e+fx)}}-\frac{(3a-4b)\cot(e+fx)}{3a^2f\sqrt{a+b\tan^2(e+fx)}}-\frac{\cot^3(e+fx)}{3af\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] -((3*a - 4*b)*Cot[e + f*x])/(3*a^2*f*Sqrt[a + b*Tan[e + f*x]^2]) - Cot[e +
f*x]^3/(3*a*f*Sqrt[a + b*Tan[e + f*x]^2]) - (2*(3*a - 4*b)*b*Tan[e + f*x])/
(3*a^3*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.121558, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3663, 453, 271, 191}

$$-\frac{2b(3a-4b)\tan(e+fx)}{3a^3f\sqrt{a+b\tan^2(e+fx)}}-\frac{(3a-4b)\cot(e+fx)}{3a^2f\sqrt{a+b\tan^2(e+fx)}}-\frac{\cot^3(e+fx)}{3af\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((3*a - 4*b)*Cot[e + f*x])/(3*a^2*f*Sqrt[a + b*Tan[e + f*x]^2]) - Cot[e +
f*x]^3/(3*a*f*Sqrt[a + b*Tan[e + f*x]^2]) - (2*(3*a - 4*b)*b*Tan[e + f*x])/
(3*a^3*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)}{3af\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-4b)\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3af}$$

$$= -\frac{(3a-4b)\cot(e+fx)}{3a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^3(e+fx)}{3af\sqrt{a+b\tan^2(e+fx)}} - \frac{(2(3a-4b)b)\,\text{Subst}\left(\int \frac{\ldots}{(a+b\ldots}\right.}{3a^2f\ldots}$$

$$= -\frac{(3a-4b)\cot(e+fx)}{3a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^3(e+fx)}{3af\sqrt{a+b\tan^2(e+fx)}} - \frac{2(3a-4b)b\tan(e+fx)}{3a^3f\sqrt{a+b\tan^2(e+fx)}}$$

Mathematica [A] time = 0.868828, size = 119, normalized size = 1.04

$$\frac{\csc^3(e+fx)\sec(e+fx)\left(-2\left(a^2-6ab+8b^2\right)\cos(2(e+fx))+\left(a^2-5ab+4b^2\right)\cos(4(e+fx))-3a^2-7ab+12b^2\right)}{6\sqrt{2}a^3f\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-3*a^2 - 7*a*b + 12*b^2 - 2*(a^2 - 6*a*b + 8*b^2)*Cos[2*(e + f*x)] + (a^2
 - 5*a*b + 4*b^2)*Cos[4*(e + f*x)])*Csc[e + f*x]^3*Sec[e + f*x])/(6*Sqrt[2]
*a^3*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Maple [A] time = 0.194, size = 170, normalized size = 1.5

$$\frac{\left(2\left(\cos\left(fx+e\right)\right)^4a^2-10\left(\cos\left(fx+e\right)\right)^4ab+8\left(\cos\left(fx+e\right)\right)^4b^2-3\left(\cos\left(fx+e\right)\right)^2a^2+16\left(\cos\left(fx+e\right)\right)^2ab-\ldots\right.}{3fa^3\left(a\left(\cos\left(fx+e\right)\right)^2-\left(\cos\left(fx+e\right)\right)^2b+b\right)^2\left(\sin\left(fx+e\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3/f/a^3/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*(2*cos(f*x+e)^4*a^2-10*cos(f*
x+e)^4*a*b+8*cos(f*x+e)^4*b^2-3*cos(f*x+e)^2*a^2+16*cos(f*x+e)^2*a*b-16*cos
(f*x+e)^2*b^2-6*a*b+8*b^2)*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/
cos(f*x+e)^2)^(3/2)/sin(f*x+e)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 27.5993, size = 359, normalized size = 3.15

$$-\frac{\left(2\left(a^{2}-5\,ab+4\,b^{2}\right)\cos\left(fx+e\right)^{5}-\left(3\,a^{2}-16\,ab+16\,b^{2}\right)\cos\left(fx+e\right)^{3}-2\left(3\,ab-4\,b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^{2}+}{\cos\left(fx+e\right)^{2}}}}{3\left(\left(a^{4}-a^{3}b\right)f\cos\left(fx+e\right)^{4}-a^{3}bf-\left(a^{4}-2\,a^{3}b\right)f\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^2 - 5*a*b + 4*b^2)*cos(f*x + e)^5 - (3*a^2 - 16*a*b + 16*b^2)*co
s(f*x + e)^3 - 2*(3*a*b - 4*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/(((a^4 - a^3*b)*f*cos(f*x + e)^4 - a^3*b*f - (a^4 - 2
*a^3*b)*f*cos(f*x + e)^2)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^{4}\left(e+fx\right)}{\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**4/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^{4}}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

3.139 $\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{2b\left(15a^2-40ab+24b^2\right)\tan(e+fx)}{15a^4f\sqrt{a+b\tan^2(e+fx)}}-\frac{\left(15a^2-40ab+24b^2\right)\cot(e+fx)}{15a^3f\sqrt{a+b\tan^2(e+fx)}}-\frac{2(5a-3b)\cot^3(e+fx)}{15a^2f\sqrt{a+b\tan^2(e+fx)}}-\frac{\cot^5}{5af\sqrt{a+}}$$

```
[Out] -((15*a^2 - 40*a*b + 24*b^2)*Cot[e + f*x])/(15*a^3*f*Sqrt[a + b*Tan[e + f*x
]^2]) - (2*(5*a - 3*b)*Cot[e + f*x]^3)/(15*a^2*f*Sqrt[a + b*Tan[e + f*x]^2]
) - Cot[e + f*x]^5/(5*a*f*Sqrt[a + b*Tan[e + f*x]^2]) - (2*b*(15*a^2 - 40*a
*b + 24*b^2)*Tan[e + f*x])/(15*a^4*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.178864, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3663, 462, 453, 271, 191}

$$-\frac{2b\left(15a^2-40ab+24b^2\right)\tan(e+fx)}{15a^4f\sqrt{a+b\tan^2(e+fx)}}-\frac{\left(15a^2-40ab+24b^2\right)\cot(e+fx)}{15a^3f\sqrt{a+b\tan^2(e+fx)}}-\frac{2(5a-3b)\cot^3(e+fx)}{15a^2f\sqrt{a+b\tan^2(e+fx)}}-\frac{\cot^5}{5af\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```


```
[Out] -((15*a^2 - 40*a*b + 24*b^2)*Cot[e + f*x])/(15*a^3*f*Sqrt[a + b*Tan[e + f*x
]^2]) - (2*(5*a - 3*b)*Cot[e + f*x]^3)/(15*a^2*f*Sqrt[a + b*Tan[e + f*x]^2]
) - Cot[e + f*x]^5/(5*a*f*Sqrt[a + b*Tan[e + f*x]^2]) - (2*b*(15*a^2 - 40*a
*b + 24*b^2)*Tan[e + f*x])/(15*a^4*f*Sqrt[a + b*Tan[e + f*x]^2])
```


Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^6\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5af\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{2(5a-3b)+5ax^2}{x^4\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{5af}$$

$$= -\frac{2(5a-3b)\cot^3(e+fx)}{15a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^5(e+fx)}{5af\sqrt{a+b\tan^2(e+fx)}} - \frac{\left(-15a^2+8(5a-3b)b\right)\text{Subst}}{15}$$

$$= -\frac{\left(15a^2-8(5a-3b)b\right)\cot(e+fx)}{15a^3f\sqrt{a+b\tan^2(e+fx)}} - \frac{2(5a-3b)\cot^3(e+fx)}{15a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^5(e+fx)}{5af\sqrt{a+b\tan^2(e+}}$$

$$= -\frac{\left(15a^2-8(5a-3b)b\right)\cot(e+fx)}{15a^3f\sqrt{a+b\tan^2(e+fx)}} - \frac{2(5a-3b)\cot^3(e+fx)}{15a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^5(e+fx)}{5af\sqrt{a+b\tan^2(e+}}$$

Mathematica [A] time = 1.28677, size = 135, normalized size = 0.79

$$-\frac{\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\cot(e+fx)\left(3a^2\csc^4(e+fx)+8a^2+a(4a-9b)\csc^2(e+fx)-41ab+3\right.\right.}{15\sqrt{2}a^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(Cot[e + f*x]*(8*
a^2 - 41*a*b + 33*b^2 + a*(4*a - 9*b)*Csc[e + f*x]^2 + 3*a^2*Csc[e + f*x]^4
) + (15*(a - b)^2*b*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)])))/
(15*Sqrt[2]*a^4*f)
```

Maple [A] time = 0.239, size = 264, normalized size = 1.5

$$-\frac{\left(8\left(\cos\left(fx+e\right)\right)^6a^3-64\left(\cos\left(fx+e\right)\right)^6a^2b+104\left(\cos\left(fx+e\right)\right)^6ab^2-48\left(\cos\left(fx+e\right)\right)^6b^3-20\left(\cos\left(fx+e\right)\right)^4a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/15/f/a^4/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*(8*cos(f*x+e)^6*a^3-64*cos(
f*x+e)^6*a^2*b+104*cos(f*x+e)^6*a*b^2-48*cos(f*x+e)^6*b^3-20*cos(f*x+e)^4*a
^3+164*cos(f*x+e)^4*a^2*b-288*cos(f*x+e)^4*a*b^2+144*cos(f*x+e)^4*b^3+15*co
s(f*x+e)^2*a^3-130*cos(f*x+e)^2*a^2*b+264*cos(f*x+e)^2*a*b^2-144*cos(f*x+e)
^2*b^3+30*a^2*b-80*a*b^2+48*b^3)*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/cos(f*x+e)^2)^(3/2)/sin(f*x+e)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^6}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^6/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

3.140 $\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=248

$$-\frac{8b\left(5a^2+10ab+b^2\right)\sec(e+fx)}{15f(a-b)^5\sqrt{a+b\sec^2(e+fx)-b}}-\frac{4b\left(5a^2+10ab+b^2\right)\sec(e+fx)}{15f(a-b)^4\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{\left(5a^2+10ab+b^2\right)\cos(e+fx)}{5f(a-b)^3\left(a+b\sec^2(e+fx)-b\right)^{3/2}}$$

```
[Out] -((5*a^2 + 10*a*b + b^2)*Cos[e + f*x])/(5*(a - b)^3*f*(a - b + b*Sec[e + f*
x]^2)^(3/2)) + (2*(5*a - b)*Cos[e + f*x]^3)/(15*(a - b)^2*f*(a - b + b*Sec[
e + f*x]^2)^(3/2)) - Cos[e + f*x]^5/(5*(a - b)*f*(a - b + b*Sec[e + f*x]^2)
^(3/2)) - (4*b*(5*a^2 + 10*a*b + b^2)*Sec[e + f*x])/(15*(a - b)^4*f*(a - b
+ b*Sec[e + f*x]^2)^(3/2)) - (8*b*(5*a^2 + 10*a*b + b^2)*Sec[e + f*x])/(15*
(a - b)^5*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.231493, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3664, 462, 453, 271, 192, 191}

$$-\frac{8b\left(5a^2+10ab+b^2\right)\sec(e+fx)}{15f(a-b)^5\sqrt{a+b\sec^2(e+fx)-b}}-\frac{4b\left(5a^2+10ab+b^2\right)\sec(e+fx)}{15f(a-b)^4\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{\left(5a^2+10ab+b^2\right)\cos(e+fx)}{5f(a-b)^3\left(a+b\sec^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```


```
[Out] -((5*a^2 + 10*a*b + b^2)*Cos[e + f*x])/(5*(a - b)^3*f*(a - b + b*Sec[e + f*
x]^2)^(3/2)) + (2*(5*a - b)*Cos[e + f*x]^3)/(15*(a - b)^2*f*(a - b + b*Sec[
e + f*x]^2)^(3/2)) - Cos[e + f*x]^5/(5*(a - b)*f*(a - b + b*Sec[e + f*x]^2)
^(3/2)) - (4*b*(5*a^2 + 10*a*b + b^2)*Sec[e + f*x])/(15*(a - b)^4*f*(a - b
+ b*Sec[e + f*x]^2)^(3/2)) - (8*b*(5*a^2 + 10*a*b + b^2)*Sec[e + f*x])/(15*
(a - b)^5*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^6\left(a-b+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-2(5a-b)+5(a-b)x^2}{x^4\left(a-b+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{5(a-b)f}$$

$$= \frac{2(5a-b)\cos^3(e+fx)}{15(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{\left(5a^2+1\right.}{}$$

$$= -\frac{\left(5a^2+10ab+b^2\right)\cos(e+fx)}{5(a-b)^3f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{2(5a-b)\cos^3(e+fx)}{15(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{}{5(a-}$$

$$= -\frac{\left(5a^2+10ab+b^2\right)\cos(e+fx)}{5(a-b)^3f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{2(5a-b)\cos^3(e+fx)}{15(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{}{5(a-}$$

$$= -\frac{\left(5a^2+10ab+b^2\right)\cos(e+fx)}{5(a-b)^3f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{2(5a-b)\cos^3(e+fx)}{15(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{}{5(a-}$$

Mathematica [A] time = 2.2824, size = 294, normalized size = 1.19

$$-\frac{\cos(e+fx)\left(18a^2b^2\cos(8(e+fx))+48\left(106a^3b+11a^4-106ab^3-11b^4\right)\cos(2(e+fx))+12(a-b)^2\left(7a^2+50ab+7b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(Cos[e + f*x]*(425*a^4 + 4700*a^3*b + 6134*a^2*b^2 + 4700*a*b^3 + 425*b^4
+ 48*(11*a^4 + 106*a^3*b - 106*a*b^3 - 11*b^4)*Cos[2*(e + f*x)] + 12*(a - b
)^2*(7*a^2 + 50*a*b + 7*b^2)*Cos[4*(e + f*x)] - 16*a^4*Cos[6*(e + f*x)] + 3
2*a^3*b*Cos[6*(e + f*x)] - 32*a*b^3*Cos[6*(e + f*x)] + 16*b^4*Cos[6*(e + f*
x)] + 3*a^4*Cos[8*(e + f*x)] - 12*a^3*b*Cos[8*(e + f*x)] + 18*a^2*b^2*Cos[8
*(e + f*x)] - 12*a*b^3*Cos[8*(e + f*x)] + 3*b^4*Cos[8*(e + f*x)])*Sqrt[(a +
 b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(480*Sqrt[2]*(a - b)^5*f*(a
 + b + (a - b)*Cos[2*(e + f*x)])^2)
```

Maple [A] time = 1.559, size = 391, normalized size = 1.6

$$\frac{a^7 (a-b)^2 \left(a \left(\cos \left(fx+e\right)\right)^2-\left(\cos \left(fx+e\right)\right)^2 b+b\right) \left(3 \left(\cos \left(fx+e\right)\right)^8 a^4-12 \left(\cos \left(fx+e\right)\right)^8 a^3 b+18 \left(\cos \left(fx+ \right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/30/f*a^7/((-b*(a-b))^(1/2)+a-b)^7/((-b*(a-b))^(1/2)-a+b)^7*(a-b)^2*(a*cos
(f*x+e)^2-cos(f*x+e)^2*b+b)*(3*cos(f*x+e)^8*a^4-12*cos(f*x+e)^8*a^3*b+18*co
s(f*x+e)^8*a^2*b^2-12*cos(f*x+e)^8*a*b^3+3*cos(f*x+e)^8*b^4-10*cos(f*x+e)^6
*a^4+32*cos(f*x+e)^6*a^3*b-36*cos(f*x+e)^6*a^2*b^2+16*cos(f*x+e)^6*a*b^3-2*
cos(f*x+e)^6*b^4+15*cos(f*x+e)^4*a^4-42*cos(f*x+e)^4*a^2*b^2+24*cos(f*x+e)^
4*a*b^3+3*cos(f*x+e)^4*b^4+60*cos(f*x+e)^2*a^3*b+60*cos(f*x+e)^2*a^2*b^2-10
8*cos(f*x+e)^2*a*b^3-12*cos(f*x+e)^2*b^4+40*a^2*b^2+80*a*b^3+8*b^4)*4^(1/2)
/cos(f*x+e)^5/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(5/2)
```

Maxima [B] time = 1.07457, size = 721, normalized size = 2.91

$$-\frac{\frac{15 \sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}} \cos\left(fx+e\right)}{a^3-3 a^2 b+3 a b^2-b^3}+\frac{3 \left(a-b+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{5}{2}} \cos\left(fx+e\right)^5-20 \left(a-b+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}} b \cos\left(fx+e\right)^3+90 \sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}} b^2 \cos\left(fx+e\right)}{a^5-5 a^4 b+10 a^3 b^2-10 a^2 b^3+5 a b^4-b^5}-\frac{10 \left(\left(\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a^3 - 3*a^2*b + 3*a*
b^2 - b^3) + (3*(a - b + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 20*(a - b
 + b/cos(f*x + e)^2)^(3/2)*b*cos(f*x + e)^3 + 90*sqrt(a - b + b/cos(f*x + e
)^2)*b^2*cos(f*x + e))/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 -
 b^5) - 10*((a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 9*sqrt(a - b
+ b/cos(f*x + e)^2)*b*cos(f*x + e))/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 +
b^4) + 5*(12*(a - b + b/cos(f*x + e)^2)*b^3*cos(f*x + e)^2 - b^4)/((a^5 - 5
*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*(a - b + b/cos(f*x + e)^2
)^(3/2)*cos(f*x + e)^3) + 10*(9*(a - b + b/cos(f*x + e)^2)*b^2*cos(f*x + e)
^2 - b^3)/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(a - b + b/cos(f*x +
```

```
e)^2)^(3/2)*cos(f*x + e)^3) + 5*(6*(a - b + b/cos(f*x + e)^2)*b*cos(f*x +
e)^2 - b^2)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(a - b + b/cos(f*x + e)^2)^(3/
2)*cos(f*x + e)^3))/f
```

Fricas [A] time = 6.25133, size = 829, normalized size = 3.34

$$-\frac{\left(3\left(a^4-4a^3b+6a^2b^2-4ab^3+b^4\right)\cos\left(fx+e\right)^9-2\left(5a^4-16a^3b+18a^2b^2-8ab^3+b^4\right)\cos\left(fx+e\right)^7+3\left(5a^4-14\right.\right.}{15\left(\left(a^7-7a^6b+21a^5b^2-35a^4b^3+35a^3b^4-21a^2b^5+7ab^6-b^7\right)f\cos\left(fx+e\right)^4+2\left(a^6b-6\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^9 - 2*(5*
a^4 - 16*a^3*b + 18*a^2*b^2 - 8*a*b^3 + b^4)*cos(f*x + e)^7 + 3*(5*a^4 - 14
*a^2*b^2 + 8*a*b^3 + b^4)*cos(f*x + e)^5 + 12*(5*a^3*b + 5*a^2*b^2 - 9*a*b^
3 - b^4)*cos(f*x + e)^3 + 8*(5*a^2*b^2 + 10*a*b^3 + b^4)*cos(f*x + e))*sqrt
(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^7 - 7*a^6*b + 21*a^5*b^2
- 35*a^4*b^3 + 35*a^3*b^4 - 21*a^2*b^5 + 7*a*b^6 - b^7)*f*cos(f*x + e)^4 +
2*(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7
)*f*cos(f*x + e)^2 + (a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 + 5*a*b
^6 - b^7)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^5}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^5/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

3.141 $\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=168

$$-\frac{8b(a+b)\sec(e+fx)}{3f(a-b)^4\sqrt{a+b\sec^2(e+fx)-b}}-\frac{4b(a+b)\sec(e+fx)}{3f(a-b)^3\left(a+b\sec^2(e+fx)-b\right)^{3/2}}+\frac{\cos^3(e+fx)}{3f(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-$$

```
[Out] -(((a + b)*Cos[e + f*x])/((a - b)^2*f*(a - b + b*Sec[e + f*x]^2)^(3/2))) +
Cos[e + f*x]^3/(3*(a - b)*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - (4*b*(a + b
)*Sec[e + f*x])/(3*(a - b)^3*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - (8*b*(a
+ b)*Sec[e + f*x])/(3*(a - b)^4*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.159306, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3664, 453, 271, 192, 191}

$$-\frac{8b(a+b)\sec(e+fx)}{3f(a-b)^4\sqrt{a+b\sec^2(e+fx)-b}}-\frac{4b(a+b)\sec(e+fx)}{3f(a-b)^3\left(a+b\sec^2(e+fx)-b\right)^{3/2}}+\frac{\cos^3(e+fx)}{3f(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(((a + b)*Cos[e + f*x])/((a - b)^2*f*(a - b + b*Sec[e + f*x]^2)^(3/2))) +
Cos[e + f*x]^3/(3*(a - b)*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - (4*b*(a + b
)*Sec[e + f*x])/(3*(a - b)^3*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - (8*b*(a
+ b)*Sec[e + f*x])/(3*(a - b)^4*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4\left(a-b+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)}{3(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{x^2\left(a-b+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{(a-b)f}$$

$$= -\frac{(a+b)\cos(e+fx)}{(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)}{3(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(4b(a+b}{}$$

$$= -\frac{(a+b)\cos(e+fx)}{(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)}{3(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{4}{3(a-b)^3}$$

$$= -\frac{(a+b)\cos(e+fx)}{(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)}{3(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{4}{3(a-b)^3}$$

Mathematica [A] time = 1.45017, size = 205, normalized size = 1.22

$$-\frac{\cos(e+fx)\left(3\left(63a^2b+11a^3-31ab^2-43b^3\right)\cos(2(e+fx))+3a^2b\cos(6(e+fx))+186a^2b+a^3(-\cos(6(e+fx)))+2}{24\sqrt{2}f(a-}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(Cos[e + f*x]*(26*a^3 + 186*a^2*b + 190*a*b^2 + 110*b^3 + 3*(11*a^3 + 63*a
^2*b - 31*a*b^2 - 43*b^3)*Cos[2*(e + f*x)] + 6*(a - b)^2*(a + 3*b)*Cos[4*(e
 + f*x)] - a^3*Cos[6*(e + f*x)] + 3*a^2*b*Cos[6*(e + f*x)] - 3*a*b^2*Cos[6*
(e + f*x)] + b^3*Cos[6*(e + f*x)])*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*
Sec[e + f*x]^2])/(24*Sqrt[2]*(a - b)^4*f*(a + b + (a - b)*Cos[2*(e + f*x)])
^2)
```

Maple [A] time = 0.614, size = 262, normalized size = 1.6

$$-\frac{a^5(a-b)\left(a\left(\cos(fx+e)\right)^2-\left(\cos(fx+e)\right)^2b+b\right)\left(\left(\cos(fx+e)\right)^6a^3-3\left(\cos(fx+e)\right)^6a^2b+3\left(\cos(fx+e)\right)^6ab^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/6/f*a^5/((-b*(a-b))^(1/2)+a-b)^5/((-b*(a-b))^(1/2)-a+b)^5*(a-b)*(a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)*(cos(f*x+e)^6*a^3-3*cos(f*x+e)^6*a^2*b+3*cos(f*x+
e)^6*a*b^2-cos(f*x+e)^6*b^3-3*cos(f*x+e)^4*a^3+3*cos(f*x+e)^4*a^2*b+3*cos(f
*x+e)^4*a*b^2-3*cos(f*x+e)^4*b^3-12*cos(f*x+e)^2*a^2*b+12*cos(f*x+e)^2*b^3-
8*a*b^2-8*b^3)*4^(1/2)/cos(f*x+e)^5/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(
f*x+e)^2)^(5/2)
```

Maxima [A] time = 1.06732, size = 416, normalized size = 2.48

$$-\frac{\frac{3\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{a^3-3a^2b+3ab^2-b^3}-\frac{\left(a-b+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3-9\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\,b\cos\left(fx+e\right)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}+\frac{9\left(a-b+\frac{b}{\cos\left(fx+e\right)^2}\right)b^2\cos\left(fx+e\right)^2-\cdots}{\left(a^4-4a^3b+6a^2b^2-4ab^3+b^4\right)\left(a-b+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}\cdots}\cdots}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^
2 - b^3) - ((a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 9*sqrt(a - b
+ b/cos(f*x + e)^2)*b*cos(f*x + e))/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 +
b^4) + (9*(a - b + b/cos(f*x + e)^2)*b^2*cos(f*x + e)^2 - b^3)/((a^4 - 4*a^
3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x +
 e)^3) + (6*(a - b + b/cos(f*x + e)^2)*b*cos(f*x + e)^2 - b^2)/((a^3 - 3*a^
2*b + 3*a*b^2 - b^3)*(a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3))/f
```

Fricas [A] time = 4.4293, size = 598, normalized size = 3.56

$$\frac{\left(\left(a^3-3a^2b+3ab^2-b^3\right)\cos\left(fx+e\right)^7-3\left(a^3-a^2b-ab^2+b^3\right)\cos\left(fx+e\right)^5-12\left(a^2b-b^3\right)\cos\left(fx+\cdots\right.\right.}{3\left(\left(a^6-6a^5b+15a^4b^2-20a^3b^3+15a^2b^4-6ab^5+b^6\right)f\cos\left(fx+e\right)^4+2\left(a^5b-5a^4b^2+10a^3b^3-10a^2b^4+5ab^5\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 3*(a^3 - a^2*b - a*b^
2 + b^3)*cos(f*x + e)^5 - 12*(a^2*b - b^3)*cos(f*x + e)^3 - 8*(a*b^2 + b^3)
*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^6 - 6*
a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*f*cos(f*x + e
)^4 + 2*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*f*cos
(f*x + e)^2 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{3}}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^3/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

3.142 $\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{8b\sec(e+fx)}{3f(a-b)^3\sqrt{a+b\sec^2(e+fx)-b}}-\frac{4b\sec(e+fx)}{3f(a-b)^2\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{\cos(e+fx)}{f(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}$$

```
[Out] -(Cos[e + f*x]/((a - b)*f*(a - b + b*Sec[e + f*x]^2)^(3/2))) - (4*b*Sec[e +
 f*x])/(3*(a - b)^2*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - (8*b*Sec[e + f*x]
)/(3*(a - b)^3*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.0776282, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3664, 271, 192, 191}

$$-\frac{8b\sec(e+fx)}{3f(a-b)^3\sqrt{a+b\sec^2(e+fx)-b}}-\frac{4b\sec(e+fx)}{3f(a-b)^2\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{\cos(e+fx)}{f(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(Cos[e + f*x]/((a - b)*f*(a - b + b*Sec[e + f*x]^2)^(3/2))) - (4*b*Sec[e +
 f*x])/(3*(a - b)^2*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - (8*b*Sec[e + f*x]
)/(3*(a - b)^3*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a-b+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)}{(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(4b)\,\text{Subst}\left(\int \frac{1}{\left(a-b+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{(a-b)f}$$

$$= -\frac{\cos(e+fx)}{(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{4b\sec(e+fx)}{3(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(8b)\,\text{Sub}}{}$$

$$= -\frac{\cos(e+fx)}{(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{4b\sec(e+fx)}{3(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{}{3(a-b)^3}$$

Mathematica [A] time = 1.21265, size = 124, normalized size = 1.05

$$-\frac{\cos(e+fx)\left(12\left(a^2+2ab-3b^2\right)\cos(2(e+fx))+3(a-b)^2\cos(4(e+fx))+(3a+5b)^2\right)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e-}}{6\sqrt{2}f(a-b)^3((a-b)\cos(2(e+fx))+a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(Cos[e + f*x]*((3*a + 5*b)^2 + 12*(a^2 + 2*a*b - 3*b^2)*Cos[2*(e + f*x)] +
 3*(a - b)^2*Cos[4*(e + f*x)])*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[
e + f*x]^2])/(6*Sqrt[2]*(a - b)^3*f*(a + b + (a - b)*Cos[2*(e + f*x)])^2)
```

Maple [A] time = 0.058, size = 147, normalized size = 1.3

$$-\frac{\left(a\left(\cos\left(fx+e\right)\right)^2-\left(\cos\left(fx+e\right)\right)^2b+b\right)\left(3\left(\cos\left(fx+e\right)\right)^4a^2-6\left(\cos\left(fx+e\right)\right)^4ab+3\left(\cos\left(fx+e\right)\right)^4b^2+12\left(\text{co}\right.\right.}{3f\left(\cos\left(fx+e\right)\right)^5(a-b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3/f*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)*(3*cos(f*x+e)^4*a^2-6*cos(f*x+e)^4
*a*b+3*cos(f*x+e)^4*b^2+12*cos(f*x+e)^2*a*b-12*cos(f*x+e)^2*b^2+8*b^2)/((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(5/2)/cos(f*x+e)^5/(a-b)^3
```

Maxima [A] time = 1.03534, size = 182, normalized size = 1.54

$$-\frac{\frac{3\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{a^3-3a^2b+3ab^2-b^3}+\frac{6\left(a-b+\frac{b}{\cos\left(fx+e\right)^2}\right)b\cos\left(fx+e\right)^2-b^2}{\left(a^3-3a^2b+3ab^2-b^3\right)\left(a-b+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^
2 - b^3) + (6*(a - b + b/cos(f*x + e)^2)*b*cos(f*x + e)^2 - b^2)/((a^3 - 3*
a^2*b + 3*a*b^2 - b^3)*(a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3))/f
```

Fricas [A] time = 2.96788, size = 454, normalized size = 3.85

$$-\frac{\left(3\left(a^{2}-2\,ab+b^{2}\right)\cos\left(fx+e\right)^{5}+12\left(ab-b^{2}\right)\cos\left(fx+e\right)^{3}+8\,b^{2}\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\cdots}{\cos(\cdots}}}{3\left(\left(a^{5}-5\,a^{4}b+10\,a^{3}b^{2}-10\,a^{2}b^{3}+5\,ab^{4}-b^{5}\right)f\cos\left(fx+e\right)^{4}+2\left(a^{4}b-4\,a^{3}b^{2}+6\,a^{2}b^{3}-4\,ab^{4}+b^{5}\right)f\cos\left(fx+e\right)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 + 12*(a*b - b^2)*cos(f*x + e)^3
+ 8*b^2*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a
^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*f*cos(f*x + e)^4 +
2*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cos(f*x + e)^2 + (a^3*b
^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

3.143 $\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=136

$$-\frac{b(5a-3b)\sec(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{a^{5/2}f}-\frac{b\sec(e+fx)}{3af(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}$$

```
[Out] -(ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]]/(a^(5/2)*f
)) - (b*Sec[e + f*x])/(3*a*(a - b)*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - ((
5*a - 3*b)*b*Sec[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a - b + b*Sec[e + f*x]^2
])
```

Rubi [A] time = 0.145504, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3664, 414, 527, 12, 377, 207}

$$-\frac{b(5a-3b)\sec(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{a^{5/2}f}-\frac{b\sec(e+fx)}{3af(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```


```
[Out] -(ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]]/(a^(5/2)*f
)) - (b*Sec[e + f*x])/(3*a*(a - b)*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - ((
5*a - 3*b)*b*Sec[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a - b + b*Sec[e + f*x]^2
])
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(-1+x^2\right)\left(a-b+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{b\sec(e+fx)}{3a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-b-2bx^2}{\left(-1+x^2\right)\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec(e+f\right.}{3a(a-b)f}$$

$$= -\frac{b\sec(e+fx)}{3a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(5a-3b)b\sec(e+fx)}{3a^2(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\text{Sub}}{}$$

$$= -\frac{b\sec(e+fx)}{3a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(5a-3b)b\sec(e+fx)}{3a^2(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\text{Sub}}{}$$

$$= -\frac{b\sec(e+fx)}{3a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(5a-3b)b\sec(e+fx)}{3a^2(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\text{Sub}}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\sec(e+fx)}{3a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(5a-3b}{3a^2(a-b)^2f\sqrt{a}}$$

Mathematica [B] time = 4.70739, size = 305, normalized size = 2.24

$$\frac{\cos(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(-\frac{2\sqrt{2}\sqrt{a}b\left(3\left(2a^2-3ab+b^2\right)\cos(2(e+fx))+6a^2+ab-3b^2\right)}{(a-b)^2((a-b)\cos(2(e+fx))+a+b)^2} - \frac{3\sec^2\left(\frac{1}{2}(e+fx)\right)\left(\text{t}\right.}{}\right.}{6a^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cos[e + f*x]*((-2*Sqrt[2]*Sqrt[a]*b*(6*a^2 + a*b - 3*b^2 + 3*(2*a^2 - 3*a*
b + b^2)*Cos[2*(e + f*x)]))/((a - b)^2*(a + b + (a - b)*Cos[2*(e + f*x)])^2
) - (3*(ArcTanh[(a - (a - 2*b)*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*Tan[(e
 + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])] + ArcTanh[(2*b + a*(-1 + Ta
n[(e + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e +
f*x)/2]^2)^2])])*Sec[(e + f*x)/2]^2)/Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)]
)*Sec[(e + f*x)/2]^4])*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]
^2])/(6*a^(5/2)*f)
```

Maple [B] time = 1.415, size = 27448, normalized size = 201.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] time = 3.21182, size = 1561, normalized size = 11.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^4 + a^2*b
^2 - 2*a*b^3 + b^4 + 2*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*cos(f*x + e)^2)*
sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)) - 2*(
3*(2*a^3*b - 3*a^2*b^2 + a*b^3)*cos(f*x + e)^3 + (5*a^2*b^2 - 3*a*b^3)*cos(
f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 - 4*a^6*
b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^4 + 2*(a^6*b - 3*a^5*b^
2 + 3*a^4*b^3 - a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 - 2*a^4*b^3 + a^3*b^4)
*f), 1/3*(3*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^4 + a
^2*b^2 - 2*a*b^3 + b^4 + 2*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*cos(f*x + e)
```

```
^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e)/a) - (3*(2*a^3*b - 3*a^2*b^2 + a*b^3)*cos(f*x + e)^3 + (5*
a^2*b^2 - 3*a*b^3)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2))/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^
4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2
 - 2*a^4*b^3 + a^3*b^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

3.144 $\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{b(13a-15b)\sec(e+fx)}{6a^3f(a-b)\sqrt{a+b\sec^2(e+fx)-b}}-\frac{5b\sec(e+fx)}{6a^2f\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{(a-5b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2a^{7/2}f}-\frac{\cot(e}{2af\left(a+\right.}$$

```
[Out] -((a - 5*b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])
/(2*a^(7/2)*f) - (Cot[e + f*x]*Csc[e + f*x])/(2*a*f*(a - b + b*Sec[e + f*x]
^2)^(3/2)) - (5*b*Sec[e + f*x])/(6*a^2*f*(a - b + b*Sec[e + f*x]^2)^(3/2))
- ((13*a - 15*b)*b*Sec[e + f*x])/(6*a^3*(a - b)*f*Sqrt[a - b + b*Sec[e + f*
x]^2])
```

Rubi [A] time = 0.20946, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3664, 471, 527, 12, 377, 207}

$$-\frac{b(13a-15b)\sec(e+fx)}{6a^3f(a-b)\sqrt{a+b\sec^2(e+fx)-b}}-\frac{5b\sec(e+fx)}{6a^2f\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{(a-5b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2a^{7/2}f}-\frac{\cot(e}{2af\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```



```
[Out] -((a - 5*b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])
/(2*a^(7/2)*f) - (Cot[e + f*x]*Csc[e + f*x])/(2*a*f*(a - b + b*Sec[e + f*x]
^2)^(3/2)) - (5*b*Sec[e + f*x])/(6*a^2*f*(a - b + b*Sec[e + f*x]^2)^(3/2))
- ((13*a - 15*b)*b*Sec[e + f*x])/(6*a^3*(a - b)*f*Sqrt[a - b + b*Sec[e + f*
x]^2])
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
```

```
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2\left(a-b+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a-b-4bx^2}{\left(-1+x^2\right)\left(a-b+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{2af}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec(e+fx)}{6a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{(3a-5b}{(-1+}\right.}{}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec(e+fx)}{6a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(13a-15b}{6a^3(a-b)f\sqrt{a}}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec(e+fx)}{6a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(13a-15b}{6a^3(a-b)f\sqrt{a}}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec(e+fx)}{6a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(13a-15b}{6a^3(a-b)f\sqrt{a}}$$

$$= -\frac{(a-5b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2a^{7/2}f} - \frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec}{6a^2f\left(a-b+\right.}$$

Mathematica [B] time = 4.25862, size = 385, normalized size = 2.18

$$\frac{\frac{\sqrt{\frac{(a-b)\cos(2(e+fx))+a+b}{\cos(2(e+fx))+1}}\left(8ab^2\cos(e+fx)-24b(a-b)\cos(e+fx)((a-b)\cos(2(e+fx))+a+b)-3(a-b)\cot(e+fx)\csc(e+fx)((a-b)\cos(2(e+fx))+a+b)^2\right)}{3a^3(a-b)((a-b)\cos(2(e+fx))+a+b)^2}-\frac{(a-5b)\cos(}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] ((Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(8*a*b^2*
Cos[e + f*x] - 24*(a - b)*b*Cos[e + f*x]*(a + b + (a - b)*Cos[2*(e + f*x)])
 - 3*(a - b)*(a + b + (a - b)*Cos[2*(e + f*x)])^2*Cot[e + f*x]*Csc[e + f*x]
))/(3*a^3*(a - b)*(a + b + (a - b)*Cos[2*(e + f*x)])^2) - ((a - 5*b)*(ArcTa
nh[(a - (a - 2*b)*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2
+ a*(-1 + Tan[(e + f*x)/2]^2)^2])] + ArcTanh[(2*b + a*(-1 + Tan[(e + f*x)/2
]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]
)])*Cos[e + f*x]*Sec[(e + f*x)/2]^2*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])
*Sec[e + f*x]^2])/(2*a^(7/2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e
 + f*x)/2]^4]))/(2*f)
```

Maple [B] time = 2.254, size = 38486, normalized size = 217.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 3.421, size = 2014, normalized size = 11.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((a^4 - 8*a^3*b + 18*a^2*b^2 - 16*a*b^3 + 5*b^4)*cos(f*x + e)^6 -
 (a^4 - 10*a^3*b + 32*a^2*b^2 - 38*a*b^3 + 15*b^4)*cos(f*x + e)^4 - a^2*b^2
 + 6*a*b^3 - 5*b^4 - (2*a^3*b - 15*a^2*b^2 + 28*a*b^3 - 15*b^4)*cos(f*x + e
)^2)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 + 2*sqrt(a)*sqrt(((a - b)*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1))
- 2*(3*(a^4 - 7*a^3*b + 11*a^2*b^2 - 5*a*b^3)*cos(f*x + e)^5 + 2*(9*a^3*b -
 23*a^2*b^2 + 15*a*b^3)*cos(f*x + e)^3 + (13*a^2*b^2 - 15*a*b^3)*cos(f*x +
e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 - 3*a^6*b + 3*
a^5*b^2 - a^4*b^3)*f*cos(f*x + e)^6 - (a^7 - 5*a^6*b + 7*a^5*b^2 - 3*a^4*b^
3)*f*cos(f*x + e)^4 - (2*a^6*b - 5*a^5*b^2 + 3*a^4*b^3)*f*cos(f*x + e)^2 -
(a^5*b^2 - a^4*b^3)*f), 1/6*(3*((a^4 - 8*a^3*b + 18*a^2*b^2 - 16*a*b^3 + 5*
b^4)*cos(f*x + e)^6 - (a^4 - 10*a^3*b + 32*a^2*b^2 - 38*a*b^3 + 15*b^4)*cos
(f*x + e)^4 - a^2*b^2 + 6*a*b^3 - 5*b^4 - (2*a^3*b - 15*a^2*b^2 + 28*a*b^3
- 15*b^4)*cos(f*x + e)^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) + (3*(a^4 - 7*a^3*b + 11*a^2*b^2
- 5*a*b^3)*cos(f*x + e)^5 + 2*(9*a^3*b - 23*a^2*b^2 + 15*a*b^3)*cos(f*x + e
)^3 + (13*a^2*b^2 - 15*a*b^3)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 +
b)/cos(f*x + e)^2))/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f*cos(f*x + e)^6
 - (a^7 - 5*a^6*b + 7*a^5*b^2 - 3*a^4*b^3)*f*cos(f*x + e)^4 - (2*a^6*b - 5*
a^5*b^2 + 3*a^4*b^3)*f*cos(f*x + e)^2 - (a^5*b^2 - a^4*b^3)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^3}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^3/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

3.145 $\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=237

$$-\frac{\left(3a^2-30ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8a^{9/2}f}-\frac{5b(11a-21b)\sec(e+fx)}{24a^4f\sqrt{a+b\sec^2(e+fx)-b}}-\frac{b(23a-35b)\sec(e+fx)}{24a^3f\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-$$

```
[Out] -((3*a^2 - 30*a*b + 35*b^2)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*S
ec[e + f*x]^2]])/(8*a^(9/2)*f) - ((5*a - 7*b)*Cot[e + f*x]*Csc[e + f*x])/(8
*a^2*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - (Cot[e + f*x]^3*Csc[e + f*x])/(4
*a*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - ((23*a - 35*b)*b*Sec[e + f*x])/(24
*a^3*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - (5*(11*a - 21*b)*b*Sec[e + f*x])
/(24*a^4*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.324343, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3664, 470, 527, 12, 377, 207}

$$-\frac{\left(3a^2-30ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8a^{9/2}f}-\frac{5b(11a-21b)\sec(e+fx)}{24a^4f\sqrt{a+b\sec^2(e+fx)-b}}-\frac{b(23a-35b)\sec(e+fx)}{24a^3f\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -((3*a^2 - 30*a*b + 35*b^2)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*S
ec[e + f*x]^2]])/(8*a^(9/2)*f) - ((5*a - 7*b)*Cot[e + f*x]*Csc[e + f*x])/(8
*a^2*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - (Cot[e + f*x]^3*Csc[e + f*x])/(4
*a*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - ((23*a - 35*b)*b*Sec[e + f*x])/(24
*a^3*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - (5*(11*a - 21*b)*b*Sec[e + f*x])
/(24*a^4*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^3\left(a-b+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-a+b-2(2a-3b)x^2}{\left(-1+x^2\right)^2\left(a-b+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{4af}$$

$$= -\frac{(5a-7b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-(3a \cdots}{(\cdots}\right.}{\cdots}$$

$$= -\frac{(5a-7b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(23a-35\cdots}{24a^3f\left(a-b+\cdots\right.}$$

$$= -\frac{(5a-7b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(23a-35\cdots}{24a^3f\left(a-b+\cdots\right.}$$

$$= -\frac{(5a-7b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(23a-35\cdots}{24a^3f\left(a-b+\cdots\right.}$$

$$= -\frac{(5a-7b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(23a-35\cdots}{24a^3f\left(a-b+\cdots\right.}$$

$$= -\frac{\left(3a^2-30ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8a^{9/2}f} - \frac{(5a-7b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} \cdots$$

Mathematica [B] time = 6.81356, size = 1142, normalized size = 4.82

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x
)])]*((4*b^2*Cos[e + f*x])/(3*a^3*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e
+ f*x)])^2) - (2*(2*a*b*Cos[e + f*x] - 3*b^2*Cos[e + f*x]))/(a^4*(a + b + a
*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])) + ((-3*a*Cos[e + f*x] + 11*b*Cos[e
 + f*x])*Csc[e + f*x]^2)/(8*a^4) - (Cot[e + f*x]*Csc[e + f*x]^3)/(4*a^3)))/
f + ((3*a^2 - 30*a*b + 35*b^2)*(-((2*Sqrt[a]*ArcTanh[(Sqrt[b]*(1 + Tan[(e +
 f*x)/2]^2))/Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]]
- Sqrt[b]*(ArcTanh[(a - a*Tan[(e + f*x)/2]^2 + 2*b*Tan[(e + f*x)/2]^2)/(Sqr
t[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])] + ArcTa
nh[(2*b + a*(-1 + Tan[(e + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2
 + a*(-1 + Tan[(e + f*x)/2]^2)^2])]))*(1 + Cos[e + f*x])*Sqrt[(1 + Cos[2*(e
 + f*x)])/(1 + Cos[e + f*x])^2]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1
+ Cos[2*(e + f*x)])]*(-1 + Tan[(e + f*x)/2]^2)*(1 + Tan[(e + f*x)/2]^2)*Sqr
t[(4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2)/(1 + Tan[(e + f*
x)/2]^2)^2])/(4*Sqrt[a]*Sqrt[b]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]*Sqrt
[(-1 + Tan[(e + f*x)/2]^2)^2]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e
+ f*x)/2]^2)^2]) + ((2*Sqrt[a]*ArcTanh[(Sqrt[b]*(1 + Tan[(e + f*x)/2]^2))/S
qrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]] + Sqrt[b]*(Arc
Tanh[(a - a*Tan[(e + f*x)/2]^2 + 2*b*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*
Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])] + ArcTanh[(2*b + a*(-
1 + Tan[(e + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan
[(e + f*x)/2]^2)^2])]))*(1 + Cos[e + f*x])*Sqrt[(1 + Cos[2*(e + f*x)])/(1 +
 Cos[e + f*x])^2]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f
*x)])]*(-1 + Tan[(e + f*x)/2]^2)*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(4*b*Tan[(e
+ f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2)/(1 + Tan[(e + f*x)/2]^2)^2])/(
4*Sqrt[a]*Sqrt[b]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]*Sqrt[(-1 + Tan[(e
+ f*x)/2]^2)^2]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2
])))/(8*a^4*f)
```

Maple [B] time = 4.138, size = 49917, normalized size = 210.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 4.1483, size = 2449, normalized size = 10.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((3*a^4 - 36*a^3*b + 98*a^2*b^2 - 100*a*b^3 + 35*b^4)*cos(f*x + e)
^8 - 2*(3*a^4 - 39*a^3*b + 131*a^2*b^2 - 165*a*b^3 + 70*b^4)*cos(f*x + e)^6
 + (3*a^4 - 48*a^3*b + 233*a^2*b^2 - 390*a*b^3 + 210*b^4)*cos(f*x + e)^4 +
3*a^2*b^2 - 30*a*b^3 + 35*b^4 + 2*(3*a^3*b - 36*a^2*b^2 + 95*a*b^3 - 70*b^4
)*cos(f*x + e)^2)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt((
(a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x
+ e)^2 - 1)) + 2*(3*(3*a^4 - 33*a^3*b + 65*a^2*b^2 - 35*a*b^3)*cos(f*x + e)
^7 - (15*a^4 - 177*a^3*b + 445*a^2*b^2 - 315*a*b^3)*cos(f*x + e)^5 - (78*a^
3*b - 305*a^2*b^2 + 315*a*b^3)*cos(f*x + e)^3 - 5*(11*a^2*b^2 - 21*a*b^3)*c
os(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 - 2*a
^6*b + a^5*b^2)*f*cos(f*x + e)^8 + a^5*b^2*f - 2*(a^7 - 3*a^6*b + 2*a^5*b^2
)*f*cos(f*x + e)^6 + (a^7 - 6*a^6*b + 6*a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*
b - 2*a^5*b^2)*f*cos(f*x + e)^2), 1/24*(3*((3*a^4 - 36*a^3*b + 98*a^2*b^2 -
 100*a*b^3 + 35*b^4)*cos(f*x + e)^8 - 2*(3*a^4 - 39*a^3*b + 131*a^2*b^2 - 1
65*a*b^3 + 70*b^4)*cos(f*x + e)^6 + (3*a^4 - 48*a^3*b + 233*a^2*b^2 - 390*a
*b^3 + 210*b^4)*cos(f*x + e)^4 + 3*a^2*b^2 - 30*a*b^3 + 35*b^4 + 2*(3*a^3*b
 - 36*a^2*b^2 + 95*a*b^3 - 70*b^4)*cos(f*x + e)^2)*sqrt(-a)*arctan(sqrt(-a)
*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) + (3*(3*
a^4 - 33*a^3*b + 65*a^2*b^2 - 35*a*b^3)*cos(f*x + e)^7 - (15*a^4 - 177*a^3*
b + 445*a^2*b^2 - 315*a*b^3)*cos(f*x + e)^5 - (78*a^3*b - 305*a^2*b^2 + 315
*a*b^3)*cos(f*x + e)^3 - 5*(11*a^2*b^2 - 21*a*b^3)*cos(f*x + e))*sqrt(((a -
 b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 - 2*a^6*b + a^5*b^2)*f*cos(f
*x + e)^8 + a^5*b^2*f - 2*(a^7 - 3*a^6*b + 2*a^5*b^2)*f*cos(f*x + e)^6 + (a
^7 - 6*a^6*b + 6*a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b - 2*a^5*b^2)*f*cos(f*
x + e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^5}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^5/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

3.146 $\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=246

$$\frac{\left(3a^2+24ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f(a-b)^{9/2}}-\frac{5b(11a+10b)\tan(e+fx)}{24f(a-b)^4\sqrt{a+b\tan^2(e+fx)}}-\frac{b(23a+12b)\tan(e+fx)}{24f(a-b)^3\left(a+b\tan^2(e+fx)\right)^{3/2}}+$$

```
[Out] ((3*a^2 + 24*a*b + 8*b^2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[
e + f*x]^2]])/(8*(a - b)^(9/2)*f) - ((5*a + 2*b)*Cos[e + f*x]*Sin[e + f*x])
/(8*(a - b)^2*f*(a + b*Tan[e + f*x]^2)^(3/2)) + (Cos[e + f*x]^3*Sin[e + f*x
])/(4*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (b*(23*a + 12*b)*Tan[e + f*
x])/(24*(a - b)^3*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (5*b*(11*a + 10*b)*Tan[
e + f*x])/(24*(a - b)^4*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.333422, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3663, 470, 527, 12, 377, 203}

$$\frac{\left(3a^2+24ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f(a-b)^{9/2}}-\frac{5b(11a+10b)\tan(e+fx)}{24f(a-b)^4\sqrt{a+b\tan^2(e+fx)}}-\frac{b(23a+12b)\tan(e+fx)}{24f(a-b)^3\left(a+b\tan^2(e+fx)\right)^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```


```
[Out] ((3*a^2 + 24*a*b + 8*b^2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[
e + f*x]^2]])/(8*(a - b)^(9/2)*f) - ((5*a + 2*b)*Cos[e + f*x]*Sin[e + f*x])
/(8*(a - b)^2*f*(a + b*Tan[e + f*x]^2)^(3/2)) + (Cos[e + f*x]^3*Sin[e + f*x
])/(4*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (b*(23*a + 12*b)*Tan[e + f*
x])/(24*(a - b)^3*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (5*b*(11*a + 10*b)*Tan[
e + f*x])/(24*(a - b)^4*f*Sqrt[a + b*Tan[e + f*x]^2])
```


Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{a-2(2a+b)x^2}{(1+x^2)^2(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{4(a-b)f}$$

$$= -\frac{(5a+2b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a}{}\right.}{}$$

$$= -\frac{(5a+2b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(23a}{24(a-b)^3f}$$

$$= -\frac{(5a+2b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(23a}{24(a-b)^3f}$$

$$= -\frac{(5a+2b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(23a}{24(a-b)^3f}$$

$$= -\frac{(5a+2b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(23a}{24(a-b)^3f}$$

$$= \frac{\left(3a^2+24ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8(a-b)^{9/2}f} - \frac{(5a+2b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{}{4(a}$$

Mathematica [C] time = 5.65234, size = 378, normalized size = 1.54

$$-\frac{\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(-3\sqrt{2}ab\left(3a^2+24ab+8b^2\right)\sin(2(e+fx))\sin^2(e+fx)\left(\frac{\csc^2(e+fx)((a-b}{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-3*Sqrt[2]*a*b*(
3*a^2 + 24*a*b + 8*b^2)*(((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2
)/b)^(3/2)*(2*(a - b)*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*
x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] - 2*a*EllipticPi[-(b/(a - b)), ArcSin[
Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*S
in[e + f*x]^2*Sin[2*(e + f*x)] - a*(a - b)*(64*a*b^2*Sin[2*(e + f*x)] - 64*
b*(3*a + 2*b)*(a + b + (a - b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)] - 6*(4*a
+ 7*b)*(a + b + (a - b)*Cos[2*(e + f*x)])^2*Sin[2*(e + f*x)] + 3*(a - b)*(a
 + b + (a - b)*Cos[2*(e + f*x)])^2*Sin[4*(e + f*x)])))/(96*Sqrt[2]*a*(a - b
)^5*f*(a + b + (a - b)*Cos[2*(e + f*x)])^2)
```

Maple [B] time = 1.49, size = 7943, normalized size = 32.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^4}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^4}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

3.147 $\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=181

$$\frac{(a+4b)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f(a-b)^{7/2}}-\frac{b(13a+2b)\tan(e+fx)}{6af(a-b)^3\sqrt{a+b\tan^2(e+fx)}}-\frac{5b\tan(e+fx)}{6f(a-b)^2\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\sin(e+fx)}{2f(a-b)\left(a+\right.}$$

```
[Out] ((a + 4*b)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(
2*(a - b)^(7/2)*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*(a - b)*f*(a + b*Tan[e
+ f*x]^2)^(3/2)) - (5*b*Tan[e + f*x])/(6*(a - b)^2*f*(a + b*Tan[e + f*x]^2)
^(3/2)) - (b*(13*a + 2*b)*Tan[e + f*x])/(6*a*(a - b)^3*f*Sqrt[a + b*Tan[e +
 f*x]^2])
```

Rubi [A] time = 0.208875, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3663, 471, 527, 12, 377, 203}

$$\frac{(a+4b)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f(a-b)^{7/2}}-\frac{b(13a+2b)\tan(e+fx)}{6af(a-b)^3\sqrt{a+b\tan^2(e+fx)}}-\frac{5b\tan(e+fx)}{6f(a-b)^2\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\sin(e+fx)}{2f(a-b)\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 4*b)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(
2*(a - b)^(7/2)*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*(a - b)*f*(a + b*Tan[e
+ f*x]^2)^(3/2)) - (5*b*Tan[e + f*x])/(6*(a - b)^2*f*(a + b*Tan[e + f*x]^2)
^(3/2)) - (b*(13*a + 2*b)*Tan[e + f*x])/(6*a*(a - b)^3*f*Sqrt[a + b*Tan[e +
 f*x]^2])
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a-4bx^2}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{2(a-b)f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a \cdots}{(\cdots}\right.}{\cdots}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(13a+\cdots}{6a(a-b)^3f\cdots}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(13a+\cdots}{6a(a-b)^3f\cdots}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(13a+\cdots}{6a(a-b)^3f\cdots}$$

$$= \frac{(a+4b)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2(a-b)^{7/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan\cdots}{6(a-b)^2f\left(a+\cdots\right.}$$

Mathematica [C] time = 4.28657, size = 309, normalized size = 1.71

$$-\frac{\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(-\frac{3ab(a+4b)\sin(2(e+fx))\sin^2(e+fx)\left(\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}\right)^{3/2}\left(2(a-b)\text{EllipticF}\left(\sin\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-((a - b)*(8*a*b
^2 - 4*b*(6*a + b)*(a + b + (a - b)*Cos[2*(e + f*x)]) - 3*a*(a + b + (a - b
)*Cos[2*(e + f*x)])^2)*Sin[2*(e + f*x)]) - (3*a*b*(a + 4*b)*(((a + b + (a -
 b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)^(3/2)*(2*(a - b)*EllipticF[ArcSin[
Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] -
2*a*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)]
)*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sin[e + f*x]^2*Sin[2*(e + f*x)])/Sqrt[2]
))/(12*Sqrt[2]*a*(a - b)^4*f*(a + b + (a - b)*Cos[2*(e + f*x)])^2)
```

Maple [B] time = 0.217, size = 2511, normalized size = 13.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3/f*(-1+cos(2*f*x+2*e))*(-3*cos(2*f*x+2*e)*sin(2*f*x+2*e)*((a*cos(2*f*x+
2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(3/2)*(b^4*(a-b))^(1/2)*arct
an(b^2*(a-b)/(b^4*(a-b))^(1/2)*(-1+cos(2*f*x+2*e))/sin(2*f*x+2*e)/((a*cos(2
*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2))*a^2+6*cos(2*f*x+
2*e)^2*a^3*b^3-14*cos(2*f*x+2*e)^2*a^2*b^4+10*cos(2*f*x+2*e)^2*a*b^5-2*cos(
2*f*x+2*e)^2*b^6-3*(b^4*(a-b))^(1/2)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)*(-1
+cos(2*f*x+2*e))/sin(2*f*x+2*e)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(c
os(2*f*x+2*e)+1))^(1/2))*sin(2*f*x+2*e)*a^2*((a*cos(2*f*x+2*e)-b*cos(2*f*x+
2*e)+a+b)/(cos(2*f*x+2*e)+1))^(3/2)+10*cos(2*f*x+2*e)*a^2*b^4-14*cos(2*f*x+
2*e)*a*b^5+4*cos(2*f*x+2*e)*b^6-6*a^3*b^3+4*a^2*b^4+4*a*b^5-2*b^6)/sin(2*f*
x+2*e)^3/(a-b)^3/a^2/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*
e)+1))^(3/2)/b^2-1/12/f*(-1+cos(2*f*x+2*e))*(6*cos(2*f*x+2*e)*sin(2*f*x+2*e
)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(3/2)*arctan
((a-b)^(1/2)*(-1+cos(2*f*x+2*e))/sin(2*f*x+2*e)/((a*cos(2*f*x+2*e)-b*cos(2*
f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2))*a^5*b+16*cos(2*f*x+2*e)*(a-b)^(3/2
)*b^6-24*cos(2*f*x+2*e)*sin(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+
a+b)/(cos(2*f*x+2*e)+1))^(3/2)*arctan((a-b)^(1/2)*(-1+cos(2*f*x+2*e))/sin(2
*f*x+2*e)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2
))*a^4*b^2-9*cos(2*f*x+2*e)^3*(a-b)^(3/2)*a^4*b^2+3*cos(2*f*x+2*e)^2*(a-b)^
(3/2)*a^5*b+3*cos(2*f*x+2*e)^2*(a-b)^(3/2)*a^4*b^2+3*cos(2*f*x+2*e)^3*(a-b)
^(3/2)*a^5*b-3*(a-b)^(3/2)*a^5*b*cos(2*f*x+2*e)+9*(a-b)^(3/2)*a^4*b^2*cos(2
*f*x+2*e)+9*cos(2*f*x+2*e)^3*(a-b)^(3/2)*a^3*b^3-3*cos(2*f*x+2*e)^3*(a-b)^(
3/2)*a^2*b^4+21*cos(2*f*x+2*e)^2*(a-b)^(3/2)*a^3*b^3-71*cos(2*f*x+2*e)^2*(a
-b)^(3/2)*a^2*b^4+52*cos(2*f*x+2*e)^2*(a-b)^(3/2)*a*b^5+3*cos(2*f*x+2*e)*(a
-b)^(3/2)*a^3*b^3+55*cos(2*f*x+2*e)*(a-b)^(3/2)*a^2*b^4-80*cos(2*f*x+2*e)*(
a-b)^(3/2)*a*b^5-33*(a-b)^(3/2)*a^3*b^3+19*(a-b)^(3/2)*a^2*b^4+28*(a-b)^(3/
```

```
2)*a*b^5-12*cos(2*f*x+2*e)*sin(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*
e)+a+b)/(cos(2*f*x+2*e)+1))^(3/2)*arctan((a-b)^(1/2)*(-1+cos(2*f*x+2*e))/si
n(2*f*x+2*e)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(
1/2))*a^2*b^4-24*(b^4*(a-b))^(1/2)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)*(-1+c
os(2*f*x+2*e))/sin(2*f*x+2*e)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos
(2*f*x+2*e)+1))^(1/2))*sin(2*f*x+2*e)*a^2*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*
e)+a+b)/(cos(2*f*x+2*e)+1))^(3/2)*(a-b)^(3/2)-24*cos(2*f*x+2*e)*sin(2*f*x+2
*e)*(a-b)^(3/2)*(b^4*(a-b))^(1/2)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/
(cos(2*f*x+2*e)+1))^(3/2)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)*(-1+cos(2*f*x+
2*e))/sin(2*f*x+2*e)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*
e)+1))^(1/2))*a^2+6*sin(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)
/(cos(2*f*x+2*e)+1))^(3/2)*arctan((a-b)^(1/2)*(-1+cos(2*f*x+2*e))/sin(2*f*x
+2*e)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2))*a
^5*b-24*sin(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+
2*e)+1))^(3/2)*arctan((a-b)^(1/2)*(-1+cos(2*f*x+2*e))/sin(2*f*x+2*e)/((a*co
s(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2))*a^4*b^2+30*si
n(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(
3/2)*arctan((a-b)^(1/2)*(-1+cos(2*f*x+2*e))/sin(2*f*x+2*e)/((a*cos(2*f*x+2*
e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2))*a^3*b^3-12*sin(2*f*x+2*
e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(3/2)*arcta
n((a-b)^(1/2)*(-1+cos(2*f*x+2*e))/sin(2*f*x+2*e)/((a*cos(2*f*x+2*e)-b*cos(2
*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2))*a^2*b^4-3*(a-b)^(3/2)*a^5*b-3*(a-
b)^(3/2)*a^4*b^2-8*(a-b)^(3/2)*b^6-8*cos(2*f*x+2*e)^2*(a-b)^(3/2)*b^6+30*co
s(2*f*x+2*e)*sin(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2
*f*x+2*e)+1))^(3/2)*arctan((a-b)^(1/2)*(-1+cos(2*f*x+2*e))/sin(2*f*x+2*e)/(
(a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2))*a^3*b^3)
/b/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(3/2)/a^2/(
a-b)^(11/2)/sin(2*f*x+2*e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^2}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{2}}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

3.148 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{b(5a-2b)\tan(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}-\frac{b\tan(e+fx)}{3af(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5/2
)*f) - (b*Tan[e + f*x])/(3*a*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - ((5*
a - 2*b)*b*Tan[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.112636, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3661, 414, 527, 12, 377, 203}

$$-\frac{b(5a-2b)\tan(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}-\frac{b\tan(e+fx)}{3af(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-5/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5/2
)*f) - (b*Tan[e + f*x])/(3*a*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - ((5*
a - 2*b)*b*Tan[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-2b-2bx^2}{(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx,x,\tan(e+fx)\right)}{3a(a-b)f}$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\tan(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{}{(1+x}\right.}{}$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\tan(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{}{(1+x}\right.}{}$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\tan(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{}{1-(-}\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\tan(e+f}{3a^2(a-b)^2f\sqrt{a+b\tan^2(}}$$

Mathematica [C] time = 9.42009, size = 1331, normalized size = 9.93

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-5/2),x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*(1575*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]] -
 (3150*(a - b)*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2)/a +
```

```
 (1575*(a - b)^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^4)/a
^2 + (2100*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^2)/a - (
4200*(a - b)*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2*Tan[
e + f*x]^2)/a^2 + (2100*(a - b)^2*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]
]*Sin[e + f*x]^4*Tan[e + f*x]^2)/a^3 + (840*b^2*ArcSin[Sqrt[((a - b)*Sin[e
+ f*x]^2)/a]]*Tan[e + f*x]^4)/a^2 - (1680*(a - b)*b^2*ArcSin[Sqrt[((a - b)*
Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2*Tan[e + f*x]^4)/a^3 + (840*(a - b)^2*b^2
*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^4*Tan[e + f*x]^4)/a^
4 + 2100*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan
[e + f*x]^2))/a] + 96*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)
/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e +
f*x]^2))/a] + 24*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*
x]^2)/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan
[e + f*x]^2))/a] + (2800*b*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Tan[e + f*x]^
2*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a])/a + (168*b*Hypergeometri
c2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7
/2)*Tan[e + f*x]^2*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a])/a + (48
*b*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*x]^2)/a]*(((a
- b)*Sin[e + f*x]^2)/a)^(7/2)*Tan[e + f*x]^2*Sqrt[(Cos[e + f*x]^2*(a + b*Ta
n[e + f*x]^2))/a])/a + (1120*b^2*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Tan[e +
 f*x]^4*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a])/a^2 + (72*b^2*Hype
rgeometric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*(((a - b)*Sin[e + f*x]
^2)/a)^(7/2)*Tan[e + f*x]^4*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]
)/a^2 + (24*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*x
]^2)/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7/2)*Tan[e + f*x]^4*Sqrt[(Cos[e + f*x
]^2*(a + b*Tan[e + f*x]^2))/a])/a^2 - 1575*Sqrt[((a - b)*Cos[e + f*x]^2*Sin
[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2] - (2100*b*Tan[e + f*x]^2*Sqrt[((a
- b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2])/a - (840*b
^2*Tan[e + f*x]^4*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e
+ f*x]^2))/a^2])/a^2))/(315*a^2*f*(((a - b)*Sin[e + f*x]^2)/a)^(5/2)*Sqrt[a
 + b*Tan[e + f*x]^2]*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]*(1 + (
b*Tan[e + f*x]^2)/a))
```

Maple [A] time = 0.021, size = 176, normalized size = 1.3

$$-\frac{b\tan\left(fx+e\right)}{f\left(a-b\right)^{2}a}\frac{1}{\sqrt{a+b\left(\tan\left(fx+e\right)\right)^{2}}}+\frac{1}{f\left(a-b\right)^{3}b^{2}}\sqrt{b^{4}\left(a-b\right)}\arctan\left(\left(a-b\right)b^{2}\tan\left(fx+e\right)\frac{1}{\sqrt{b^{4}\left(a-b\right)}}\frac{1}{\sqrt{a+b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/f/(a-b)^2*b*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2)+1/f/(a-b)^3*(b^4*(a-b)
)^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan
(f*x+e))-1/3*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)-2/3/f/(a-b)*b/
a^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [B] time = 2.47403, size = 1245, normalized size = 9.29

$$\left[-\frac{3\left(a^2b^2\tan\left(fx+e\right)^4+2\,a^3b\tan\left(fx+e\right)^2+a^4\right)\sqrt{-a+b}\log\left(-\frac{(a-2\,b)\tan\left(fx+e\right)^2-2\,\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{-a+b}\tan\left(fx+e\right)-a}{\tan\left(fx+e\right)^2+1}\right)+2\left(\right.}{6\left(\left(a^5b^2-3\,a^4b^3+3\,a^3b^4-a^2b^5\right)f\tan\left(fx+e\right)^4+2\left(a^6b-3\,a^5b^2+3\,a^4b^3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(a^2*b^2*tan(f*x + e)^4 + 2*a^3*b*tan(f*x + e)^2 + a^4)*sqrt(-a +
b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a +
b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + 2*((5*a^2*b^2 - 7*a*b^3 + 2*b^
4)*tan(f*x + e)^3 + 3*(2*a^3*b - 3*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*ta
n(f*x + e)^2 + a))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(f*x +
 e)^4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2 + (a^7
 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f), 1/3*(3*(a^2*b^2*tan(f*x + e)^4 + 2*a^
3*b*tan(f*x + e)^2 + a^4)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(s
qrt(a - b)*tan(f*x + e))) - ((5*a^2*b^2 - 7*a*b^3 + 2*b^4)*tan(f*x + e)^3 +
 3*(2*a^3*b - 3*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/
((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(f*x + e)^4 + 2*(a^6*b -
3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2 + (a^7 - 3*a^6*b + 3*a^5*
b^2 - a^4*b^3)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(-5/2), x)
```

3.149 $\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}} \, dx$

Optimal. Leaf size=97

$$-\frac{8b\tan(e+fx)}{3a^3f\sqrt{a+b\tan^2(e+fx)}}-\frac{4b\tan(e+fx)}{3a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\cot(e+fx)}{af\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

```
[Out] -(Cot[e + f*x]/(a*f*(a + b*Tan[e + f*x]^2)^(3/2))) - (4*b*Tan[e + f*x])/(3*
a^2*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (8*b*Tan[e + f*x])/(3*a^3*f*Sqrt[a +
b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.105595, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3663, 271, 192, 191}

$$-\frac{8b\tan(e+fx)}{3a^3f\sqrt{a+b\tan^2(e+fx)}}-\frac{4b\tan(e+fx)}{3a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\cot(e+fx)}{af\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(Cot[e + f*x]/(a*f*(a + b*Tan[e + f*x]^2)^(3/2))) - (4*b*Tan[e + f*x])/(3*
a^2*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (8*b*Tan[e + f*x])/(3*a^3*f*Sqrt[a +
b*Tan[e + f*x]^2])
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{5/2}}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)}{af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(4b)\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx,x,\tan(e+fx)\right)}{af}$$

$$= -\frac{\cot(e+fx)}{af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{4b\tan(e+fx)}{3a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(8b)\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx,\right.}{3a^2f}$$

$$= -\frac{\cot(e+fx)}{af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{4b\tan(e+fx)}{3a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{8b\tan(e+fx)}{3a^3f\sqrt{a+b\tan^2(e+fx)}}$$

Mathematica [A] time = 0.95503, size = 133, normalized size = 1.37

$$-\frac{\cot(e+fx)\left(4\left(3a^2-8b^2\right)\cos(2(e+fx))+\left(3a^2-12ab+8b^2\right)\cos(4(e+fx))+3\left(3a^2+4ab+8b^2\right)\right)\sqrt{\sec^2(e+fx)((a}{6\sqrt{2}a^3f((a-b)\cos(2(e+fx))+a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -((3*(3*a^2 + 4*a*b + 8*b^2) + 4*(3*a^2 - 8*b^2)*Cos[2*(e + f*x)] + (3*a^2
- 12*a*b + 8*b^2)*Cos[4*(e + f*x)])*Cot[e + f*x]*Sqrt[(a + b + (a - b)*Cos[
2*(e + f*x)])*Sec[e + f*x]^2])/(6*Sqrt[2]*a^3*f*(a + b + (a - b)*Cos[2*(e +
 f*x)])^2)
```

Maple [A] time = 0.232, size = 153, normalized size = 1.6

$$-\frac{\left(3\left(\cos\left(fx+e\right)\right)^4a^2-12\left(\cos\left(fx+e\right)\right)^4ab+8\left(\cos\left(fx+e\right)\right)^4b^2+12\left(\cos\left(fx+e\right)\right)^2ab-16\left(\cos\left(fx+e\right)\right)^2b^2+8}{3fa^3\left(a\left(\cos\left(fx+e\right)\right)^2-\left(\cos\left(fx+e\right)\right)^2b+b\right)^4\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3/f/a^3/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^4*(3*cos(f*x+e)^4*a^2-12*cos(f
*x+e)^4*a*b+8*cos(f*x+e)^4*b^2+12*cos(f*x+e)^2*a*b-16*cos(f*x+e)^2*b^2+8*b^
2)*cos(f*x+e)^5*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(5/2)/sin(
f*x+e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 47.7678, size = 358, normalized size = 3.69

$$-\frac{\left(\left(3\,a^2-12\,ab+8\,b^2\right)\cos\left(fx+e\right)^5+4\left(3\,ab-4\,b^2\right)\cos\left(fx+e\right)^3+8\,b^2\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{3\left(a^3b^2f+\left(a^5-2\,a^4b+a^3b^2\right)f\cos\left(fx+e\right)^4+2\left(a^4b-a^3b^2\right)f\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*((3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^5 + 4*(3*a*b - 4*b^2)*cos(f*x +
 e)^3 + 8*b^2*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)/((a^3*b^2*f + (a^5 - 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^4 + 2*(a^4*b - a^
3*b^2)*f*cos(f*x + e)^2)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^2}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

3.150 $\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{8b(a-2b)\tan(e+fx)}{3a^4f\sqrt{a+b\tan^2(e+fx)}}-\frac{4b(a-2b)\tan(e+fx)}{3a^3f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{(a-2b)\cot(e+fx)}{a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\cot^3(e+fx)}{3af\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

```
[Out] -(((a - 2*b)*Cot[e + f*x])/(a^2*f*(a + b*Tan[e + f*x]^2)^(3/2))) - Cot[e +
f*x]^3/(3*a*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (4*(a - 2*b)*b*Tan[e + f*x])/
(3*a^3*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (8*(a - 2*b)*b*Tan[e + f*x])/(3*a^
4*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.149029, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3663, 453, 271, 192, 191}

$$-\frac{8b(a-2b)\tan(e+fx)}{3a^4f\sqrt{a+b\tan^2(e+fx)}}-\frac{4b(a-2b)\tan(e+fx)}{3a^3f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{(a-2b)\cot(e+fx)}{a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\cot^3(e+fx)}{3af\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```


```
[Out] -(((a - 2*b)*Cot[e + f*x])/(a^2*f*(a + b*Tan[e + f*x]^2)^(3/2))) - Cot[e +
f*x]^3/(3*a*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (4*(a - 2*b)*b*Tan[e + f*x])/
(3*a^3*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (8*(a - 2*b)*b*Tan[e + f*x])/(3*a^
4*f*Sqrt[a + b*Tan[e + f*x]^2])
```


Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)}{3af\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{af}$$

$$= -\frac{(a-2b)\cot(e+fx)}{a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)}{3af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(4(a-2b)b)\,\text{Subst}\left(\int \right.}{}$$

$$= -\frac{(a-2b)\cot(e+fx)}{a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)}{3af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{4(a-2b)b\tan(e+f}{3a^3f\left(a+b\tan^2(e+fx\right.}$$

$$= -\frac{(a-2b)\cot(e+fx)}{a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)}{3af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{4(a-2b)b\tan(e+f}{3a^3f\left(a+b\tan^2(e+fx\right.}$$

Mathematica [A] time = 1.02165, size = 140, normalized size = 0.96

$$\frac{\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\frac{2b\sin(2(e+fx))\left(\left(-3a^2+7ab-4b^2\right)\cos(2(e+fx))-3a^2+2ab+4b^2\right)}{((a-b)\cos(2(e+fx))+a+b)^2} - \cot(e+fx)\left(a\csc^2(\right.\right.}{3\sqrt{2}a^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-(Cot[e + f*x]*(2
*a - 8*b + a*Csc[e + f*x]^2)) + (2*b*(-3*a^2 + 2*a*b + 4*b^2 + (-3*a^2 + 7*
a*b - 4*b^2)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e
+ f*x)])^2))/(3*Sqrt[2]*a^4*f)
```

Maple [A] time = 0.297, size = 245, normalized size = 1.7

$$\frac{\left(2\left(\cos\left(fx+e\right)\right)^6 a^3 - 18\left(\cos\left(fx+e\right)\right)^6 a^2b + 32\left(\cos\left(fx+e\right)\right)^6 ab^2 - 16\left(\cos\left(fx+e\right)\right)^6 b^3 - 3\left(\cos\left(fx+e\right)\right)^4 a^3\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/f/a^4/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^4*(2*cos(f*x+e)^6*a^3-18*cos(f*
x+e)^6*a^2*b+32*cos(f*x+e)^6*a*b^2-16*cos(f*x+e)^6*b^3-3*cos(f*x+e)^4*a^3+3
0*cos(f*x+e)^4*a^2*b-72*cos(f*x+e)^4*a*b^2+48*cos(f*x+e)^4*b^3-12*cos(f*x+e
)^2*a^2*b+48*cos(f*x+e)^2*a*b^2-48*cos(f*x+e)^2*b^3-8*a*b^2+16*b^3)*cos(f*x
+e)^5*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(5/2)/sin(f*x+e)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^4}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

3.151 $\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=219

$$-\frac{8b\left(5a^2-20ab+16b^2\right)\tan(e+fx)}{15a^5f\sqrt{a+b\tan^2(e+fx)}}-\frac{4b\left(5a^2-20ab+16b^2\right)\tan(e+fx)}{15a^4f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\left(5a^2-20ab+16b^2\right)\cot(e+fx)}{5a^3f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{2}{15a}$$

```
[Out] -((5*a^2 - 20*a*b + 16*b^2)*Cot[e + f*x])/(5*a^3*f*(a + b*Tan[e + f*x]^2)^(
3/2)) - (2*(5*a - 4*b)*Cot[e + f*x]^3)/(15*a^2*f*(a + b*Tan[e + f*x]^2)^(3/
2)) - Cot[e + f*x]^5/(5*a*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (4*b*(5*a^2 - 2
0*a*b + 16*b^2)*Tan[e + f*x])/(15*a^4*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (8*
b*(5*a^2 - 20*a*b + 16*b^2)*Tan[e + f*x])/(15*a^5*f*Sqrt[a + b*Tan[e + f*x]
^2])
```

Rubi [A] time = 0.228843, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3663, 462, 453, 271, 192, 191}

$$-\frac{8b\left(5a^2-20ab+16b^2\right)\tan(e+fx)}{15a^5f\sqrt{a+b\tan^2(e+fx)}}-\frac{4b\left(5a^2-20ab+16b^2\right)\tan(e+fx)}{15a^4f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\left(5a^2-20ab+16b^2\right)\cot(e+fx)}{5a^3f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{2}{15a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -((5*a^2 - 20*a*b + 16*b^2)*Cot[e + f*x])/(5*a^3*f*(a + b*Tan[e + f*x]^2)^(
3/2)) - (2*(5*a - 4*b)*Cot[e + f*x]^3)/(15*a^2*f*(a + b*Tan[e + f*x]^2)^(3/
2)) - Cot[e + f*x]^5/(5*a*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (4*b*(5*a^2 - 2
0*a*b + 16*b^2)*Tan[e + f*x])/(15*a^4*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (8*
b*(5*a^2 - 20*a*b + 16*b^2)*Tan[e + f*x])/(15*a^5*f*Sqrt[a + b*Tan[e + f*x]
^2])
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^6\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\operatorname{Subst}\left(\int \frac{2(5a-4b)+5ax^2}{x^4\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{5af}$$

$$= -\frac{2(5a-4b)\cot^3(e+fx)}{15a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\left(-15a^2+12(5a-4b)b\right)\text{S}}{}$$

$$= -\frac{\left(5a^2-4(5a-4b)b\right)\cot(e+fx)}{5a^3f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{2(5a-4b)\cot^3(e+fx)}{15a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+\right.}$$

$$= -\frac{\left(5a^2-4(5a-4b)b\right)\cot(e+fx)}{5a^3f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{2(5a-4b)\cot^3(e+fx)}{15a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+\right.}$$

$$= -\frac{\left(5a^2-4(5a-4b)b\right)\cot(e+fx)}{5a^3f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{2(5a-4b)\cot^3(e+fx)}{15a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+\right.}$$

Mathematica [A] time = 2.16722, size = 174, normalized size = 0.79

$$\frac{\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\frac{5b(b-a)\sin(2(e+fx))\left(\left(6a^2-17ab+11b^2\right)\cos(2(e+fx))+6a^2-7ab-11b^2\right)}{((a-b)\cos(2(e+fx))+a+b)^2} - \cot(e+fx)\left(3a^2\right.\right.}{15\sqrt{2}a^5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-(Cot[e + f*x]*(8
*a^2 - 66*a*b + 73*b^2 + 2*a*(2*a - 7*b)*Csc[e + f*x]^2 + 3*a^2*Csc[e + f*x
]^4)) + (5*b*(-a + b)*(6*a^2 - 7*a*b - 11*b^2 + (6*a^2 - 17*a*b + 11*b^2)*C
os[2*(e + f*x)])*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)])^2))/(
15*Sqrt[2]*a^5*f)
```

Maple [A] time = 0.228, size = 371, normalized size = 1.7

$$-\frac{\left(8\,\left(\cos\left(fx+e\right)\right)^{8}a^{4}-112\,\left(\cos\left(fx+e\right)\right)^{8}a^{3}b+328\,\left(\cos\left(fx+e\right)\right)^{8}a^{2}b^{2}-352\,\left(\cos\left(fx+e\right)\right)^{8}ab^{3}+128\,\left(\cos\left(f\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/15/f/a^5/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^4*(8*cos(f*x+e)^8*a^4-112*cos
(f*x+e)^8*a^3*b+328*cos(f*x+e)^8*a^2*b^2-352*cos(f*x+e)^8*a*b^3+128*cos(f*x
+e)^8*b^4-20*cos(f*x+e)^6*a^4+292*cos(f*x+e)^6*a^3*b-976*cos(f*x+e)^6*a^2*b
^2+1216*cos(f*x+e)^6*a*b^3-512*cos(f*x+e)^6*b^4+15*cos(f*x+e)^4*a^4-240*cos
(f*x+e)^4*a^3*b+1008*cos(f*x+e)^4*a^2*b^2-1536*cos(f*x+e)^4*a*b^3+768*cos(f
*x+e)^4*b^4+60*cos(f*x+e)^2*a^3*b-400*cos(f*x+e)^2*a^2*b^2+832*cos(f*x+e)^2
*a*b^3-512*cos(f*x+e)^2*b^4+40*a^2*b^2-160*a*b^3+128*b^4)*cos(f*x+e)^5*((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(5/2)/sin(f*x+e)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(fx+e)^6}{\left(b\tan(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^6/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

3.152 $\int (d\sin(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=92

$$\frac{\tan(e+fx)\cos^2(e+fx)^{p+\frac{1}{2}}\left(b\tan^2(e+fx)\right)^p(d\sin(e+fx))^m\text{Hypergeometric2F1}\left(\frac{1}{2}(2p+1),\frac{1}{2}(m+2p+1),\frac{1}{2}(m\right.}{f(m+2p+1)}$$

```
[Out] ((Cos[e + f*x]^2)^(1/2 + p)*Hypergeometric2F1[(1 + 2*p)/2, (1 + m + 2*p)/2,
 (3 + m + 2*p)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^m*Tan[e + f*x]*(b*Tan[e
+ f*x]^2)^p)/(f*(1 + m + 2*p))
```

Rubi [A] time = 0.153309, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3658, 2602, 2577}

$$\frac{\tan(e+fx)\cos^2(e+fx)^{p+\frac{1}{2}}\left(b\tan^2(e+fx)\right)^p(d\sin(e+fx))^m\,{}_2F_1\left(\frac{1}{2}(2p+1),\frac{1}{2}(m+2p+1);\frac{1}{2}(m+2p+3);\sin^2(e+\right.}{f(m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^(1/2 + p)*Hypergeometric2F1[(1 + 2*p)/2, (1 + m + 2*p)/2,
 (3 + m + 2*p)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^m*Tan[e + f*x]*(b*Tan[e
+ f*x]^2)^p)/(f*(1 + m + 2*p))
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (d\sin(e+fx))^m \left(b\tan^2(e+fx)\right)^p dx = \left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int (d\sin(e+fx))^m \tan^{2p}(e+fx)\,dx$$
$$= \left(d\cos^{2p}(e+fx)\sin(e+fx)(d\sin(e+fx))^{-1-2p}\left(b\tan^2(e+fx)\right)^p\right)\int \cos^{-2p}(e$$
$$= \frac{\cos^2(e+fx)^{\frac{1}{2}+p}\,{}_2F_1\left(\frac{1}{2}(1+2p),\frac{1}{2}(1+m+2p);\frac{1}{2}(3+m+2p);\sin^2(e+fx)\right)(d}{f(1+m+2p)}$$

Mathematica [C] time = 2.07811, size = 292, normalized size = 3.17

$$\frac{(m+2p+3)\sin(e+fx)\left(b\tan^2(e+fx\right.}{f(m+2p+1)\left((m+2p+3)F_1\left(\frac{m}{2}+p+\frac{1}{2};2p,m+1;\frac{m}{2}+p+\frac{3}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\tan^2\left(\frac{1}{2}(e+\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((3 + m + 2*p)*AppellF1[1/2 + m/2 + p, 2*p, 1 + m, 3/2 + m/2 + p, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*x]*(d*Sin[e + f*x])^m*(b*Tan[e +
f*x]^2)^p)/(f*(1 + m + 2*p)*((3 + m + 2*p)*AppellF1[1/2 + m/2 + p, 2*p, 1 +
 m, 3/2 + m/2 + p, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*((1 + m)*Ap
pellF1[3/2 + m/2 + p, 2*p, 2 + m, 5/2 + m/2 + p, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] - 2*p*AppellF1[3/2 + m/2 + p, 1 + 2*p, 1 + m, 5/2 + m/2 + p,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))
```

Maple [F] time = 0.777, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^m \left(b\left(\tan\left(fx+e\right)\right)^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*sin(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2\right)^p \left(d\sin\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*sin(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^{2}\right)^{p}\left(d\sin\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2)^p*(d*sin(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(fx+e\right)^{2}\right)^{p}\left(d\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*sin(f*x + e))^m, x)
```

3.153 $\int (d\sin(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p\, dx$

Optimal. Leaf size=121

$$\frac{\tan(e+fx)\sec^2(e+fx)^{m/2}(d\sin(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p} F_1\left(\frac{m+1}{2};\frac{m+2}{2},-p;\frac{m+3}{2};-\tan^2(e+f$$
$$f(m+1)$$

```
[Out] (AppellF1[(1 + m)/2, (2 + m)/2, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/a)]*(Sec[e + f*x]^2)^(m/2)*(d*Sin[e + f*x])^m*Tan[e + f*x]*(a + b
*Tan[e + f*x]^2)^p)/(f*(1 + m)*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.141187, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3667, 511, 510}

$$\frac{\tan(e+fx)\sec^2(e+fx)^{m/2}(d\sin(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p} F_1\left(\frac{m+1}{2};\frac{m+2}{2},-p;\frac{m+3}{2};-\tan^2(e+f$$
$$f(m+1)$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[(1 + m)/2, (2 + m)/2, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/a)]*(Sec[e + f*x]^2)^(m/2)*(d*Sin[e + f*x])^m*Tan[e + f*x]*(a + b
*Tan[e + f*x]^2)^p)/(f*(1 + m)*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 3667

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(f
f*(d*Sin[e + f*x])^m*(Sec[e + f*x]^2)^(m/2))/(f*Tan[e + f*x]^m), Subst[Int[
((ff*x)^m*(a + b*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]
/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (d\sin(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx = \frac{\left(\sec^2(e+fx)^{m/2}(d\sin(e+fx))^m \tan^{-m}(e+fx)\right) \text{Subst}\left(\int x^m \left(1+x^2\right)\right.}{f}$$

$$= \frac{\left(\sec^2(e+fx)^{m/2}(d\sin(e+fx))^m \tan^{-m}(e+fx)\left(a+b\tan^2(e+fx)\right)^p \right(}{}$$

$$= \frac{F_1\left(\frac{1+m}{2}; \frac{2+m}{2}, -p; \frac{3+m}{2}; -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a}\right)\sec^2(e+fx)^{m/2}(d}{f(1+}$$

Mathematica [B] time = 2.3441, size = 275, normalized size = 2.27

$$\frac{a(m+3)\sin(e+fx)\cos(e+fx)(d\sin(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p F_1}{f(m+1)\left(\tan^2(e+fx)\left(2bpF_1\left(\frac{m+3}{2}; \frac{m+2}{2}, 1-p; \frac{m+5}{2}; -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a}\right) - a(m+2)F_1\left(\frac{m+3}{2}; \frac{m+4}{2}, -p; \frac{m+5}{2}; \right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (a*(3 + m)*AppellF1[(1 + m)/2, (2 + m)/2, -p, (3 + m)/2, -Tan[e + f*x]^2, -
((b*Tan[e + f*x]^2)/a)]*Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^m*(a + b
*Tan[e + f*x]^2)^p)/(f*(1 + m)*(a*(3 + m)*AppellF1[(1 + m)/2, (2 + m)/2, -p
, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[(3
 + m)/2, (2 + m)/2, 1 - p, (5 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/a)] - a*(2 + m)*AppellF1[(3 + m)/2, (4 + m)/2, -p, (5 + m)/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2))
```

Maple [F] time = 0.658, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^m \left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p \left(d\sin\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\left(d\sin\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\left(d\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

3.154 $\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=208

$$-\frac{\left(15a^2-20ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}\text{Hypergeometric}}{15f(a-b)^2}$$

```
[Out] ((10*a - 7*b - 2*b*p)*Cos[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^(1 + p))/(1
5*(a - b)^2*f) - (Cos[e + f*x]^5*(a - b + b*Sec[e + f*x]^2)^(1 + p))/(5*(a
- b)*f) - ((15*a^2 - 20*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^2))*Cos[e + f*x]*H
ypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/(a - b))]*(a - b + b*S
ec[e + f*x]^2)^p)/(15*(a - b)^2*f*(1 + (b*Sec[e + f*x]^2)/(a - b))^p)
```

Rubi [A] time = 0.222801, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3664, 462, 453, 365, 364}

$$-\frac{\left(15a^2-20ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b}{}\right.}{15f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]
```


```
[Out] ((10*a - 7*b - 2*b*p)*Cos[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^(1 + p))/(1
5*(a - b)^2*f) - (Cos[e + f*x]^5*(a - b + b*Sec[e + f*x]^2)^(1 + p))/(5*(a
- b)*f) - ((15*a^2 - 20*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^2))*Cos[e + f*x]*H
ypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/(a - b))]*(a - b + b*S
ec[e + f*x]^2)^p)/(15*(a - b)^2*f*(1 + (b*Sec[e + f*x]^2)/(a - b))^p)
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\left(a-b+bx^2\right)^p}{x^6}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{5(a-b)f} + \frac{\text{Subst}\left(\int \frac{\left(-10a+b(7+2p)+5(a-b)x^2\right)\left(a\right.}{x^4}\right.}{5(a-b)}$$

$$= \frac{(10a-7b-2bp)\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{15(a-b)^2f} - \frac{\cos^5(e+fx)\left(a-\right.}{5(}$$

$$= \frac{(10a-7b-2bp)\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{15(a-b)^2f} - \frac{\cos^5(e+fx)\left(a-\right.}{5(}$$

$$= \frac{(10a-7b-2bp)\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{15(a-b)^2f} - \frac{\cos^5(e+fx)\left(a-\right.}{5(}$$

Mathematica [A] time = 7.42739, size = 283, normalized size = 1.36

$$-\frac{2^{p+3}\sin^4(e+fx)\cos(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\left(15a^2-20ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\text{Hypergeometric2F1}\left(-\right.\right.}{15f(a-b)^2\left(-2^{p+2}\cos(2(e+fx))\left(\frac{a+b\tan^2(e+fx)}{a-b}\right)^p+2^p\cos(4(\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -(2^(3 + p)*Cos[e + f*x]*Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p*((15*a^2 -
 20*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^2))*Hypergeometric2F1[-1/2, -p, 1/2, -
((b*Sec[e + f*x]^2)/(a - b))] + ((a + b + (a - b)*Cos[2*(e + f*x)])*(-17*a
+ b*(11 + 4*p) + 3*(a - b)*Cos[2*(e + f*x)])*((a + b*Tan[e + f*x]^2)/(a - b
))^p)/4))/(15*(a - b)^2*f*(3*(((a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f
*x]^2)/(a - b))^p - 2^(2 + p)*Cos[2*(e + f*x)]*((a + b*Tan[e + f*x]^2)/(a -
 b))^p + 2^p*Cos[4*(e + f*x)]*((a + b*Tan[e + f*x]^2)/(a - b))^p))
```

Maple [F] time = 0.885, size = 0, normalized size = 0.

$$\int \left(\sin \left(fx+e\right)\right)^{5} \left(a+b \left(\tan \left(fx+e\right)\right)^{2}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan \left(fx+e\right)^{2}+a\right)^{p} \sin \left(fx+e\right)^{5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\cos \left(fx+e\right)^{4}-2 \cos \left(fx+e\right)^{2}+1\right)\left(b \tan \left(fx+e\right)^{2}+a\right)^{p} \sin \left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*tan(f*x + e)^2 + a)^p*s
in(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan \left(fx+e\right)^{2}+a\right)^{p} \sin \left(fx+e\right)^{5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

3.155 $\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=140

$$\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{p+1}}{3f(a-b)}-\frac{(3a-2b(p+1))\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}\text{H}}{3f(a-b)}$$

```
[Out] (Cos[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^(1 + p))/(3*(a - b)*f) - ((3*a -
 2*b*(1 + p))*Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*
x]^2)/(a - b))]*(a - b + b*Sec[e + f*x]^2)^p)/(3*(a - b)*f*(1 + (b*Sec[e +
f*x]^2)/(a - b))^p)
```

Rubi [A] time = 0.111393, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3664, 453, 365, 364}

$$\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{p+1}}{3f(a-b)}-\frac{(3a-2b(p+1))\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}{}_2F}{3f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Cos[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^(1 + p))/(3*(a - b)*f) - ((3*a -
 2*b*(1 + p))*Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*
x]^2)/(a - b))]*(a - b + b*Sec[e + f*x]^2)^p)/(3*(a - b)*f*(1 + (b*Sec[e +
f*x]^2)/(a - b))^p)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a-b+bx^2)^p}{x^4}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{3(a-b)f} + \frac{(3a-2b(1+p))\,\text{Subst}\left(\int \frac{(a-b+bx^2}{x^2}\right.}{3(a-b)f}$$

$$= \frac{\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{3(a-b)f} + \frac{\left((3a-2b(1+p))\left(a-b+b\sec^2(e\right.\right.}{}$$

$$= \frac{\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{3(a-b)f} - \frac{(3a-2b(1+p))\cos(e+fx)\,{}_2F_1\left(-\right.}{}$$

Mathematica [A] time = 3.87714, size = 184, normalized size = 1.31

$$\frac{\sin(e+fx)\tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left((2b(p+1)-3a)\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a-b}\right)+\left(\frac{a+b\tan^2}{a-}\right.\right.}{f\left(3a\sec^2(e+fx)\left(\frac{a+b\sec^2(e+fx)-b}{a-b}\right)^p-3(a-b)\left(\frac{a+b\tan^2(e+fx)}{a-b}\right)^{p+1}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Sin[e + f*x]*Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p*((-3*a + 2*b*(1 + p))*H
ypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/(a - b))] + (a*Cos[e +
 f*x]^2 + b*Sin[e + f*x]^2)*((a + b*Tan[e + f*x]^2)/(a - b))^p))/(f*(3*a*Se
c[e + f*x]^2*((a - b + b*Sec[e + f*x]^2)/(a - b))^p - 3*(a - b)*((a + b*Tan
[e + f*x]^2)/(a - b))^(1 + p)))
```

Maple [F] time = 0.583, size = 0, normalized size = 0.

$$\int \left(\sin\left(fx+e\right)\right)^3\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\sin\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos\left(fx+e\right)^{2}-1\right)\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*tan(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\sin\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

3.156 $\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=79

$$-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a-b}\right)}{f}$$

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/(a -
b))]*(a - b + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/(a - b))^p))
```

Rubi [A] time = 0.0509318, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3664, 365, 364}

$$-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/(a -
b))]*(a - b + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/(a - b))^p))
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a-b+bx^2)^p}{x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a-b+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a-b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a-b}\right)^p}{x^2}\,dx, x, \sec(e\right.}{f}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a-b}\right)\left(a-b+b\sec^2(e+fx)\right)^p\left(1+\frac{b\,\text{se}}{}\right.}{f}$$

Mathematica [A] time = 0.847632, size = 80, normalized size = 1.01

$$-\frac{\cos(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{a+b\sec^2(e+fx)-b}{a-b}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/(a -
b))]*(a + b*Tan[e + f*x]^2)^p)/(f*((a - b + b*Sec[e + f*x]^2)/(a - b))^p))
```

Maple [F] time = 0.173, size = 0, normalized size = 0.

$$\int \sin(fx+e)\left(a+b\left(\tan(fx+e)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan(fx+e)^2+a\right)^p\sin(fx+e)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan(fx+e)^2+a\right)^p\sin(fx+e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

3.157 $\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=88

$$-\frac{\sec(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a-b}\right)}{f}$$

```
[Out] -((AppellF1[1/2, 1, -p, 3/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/(a - b))]
*Sec[e + f*x]*(a - b + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/(a -
 b))^p))
```

Rubi [A] time = 0.0805196, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3664, 430, 429}

$$-\frac{\sec(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[1/2, 1, -p, 3/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/(a - b))]
*Sec[e + f*x]*(a - b + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/(a -
 b))^p))
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\frac{\left(a-b+bx^2\right)^p}{-1+x^2}\,dx,x,\sec(e+fx)\right)}{f}$$

$$=\frac{\left(\left(a-b+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a-b}\right)^{-p}\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx^2}{a-b}\right)^p}{-1+x^2}\,dx,x,\sec(e+\right.}{f}$$

$$=-\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a-b}\right)\sec(e+fx)\left(a-b+b\sec^2(e+fx)\right)}{f}$$

Mathematica [B] time = 15.0711, size = 1215, normalized size = 13.81

$$\overline{2f\left(bp\sec^2(e+fx)\tan(e+fx)\left(\frac{2F_1\left(-p-\frac{1}{2};-\frac{1}{2},-p;\frac{1}{2}-p;-\cot^2(e+fx),-\frac{a\cot^2(e+fx)}{b}\right)\left(\frac{a\cot^2(e+fx)}{b}+1\right)^{-p}\sqrt{\sec^2(e+fx)}}{(2p+1)\sqrt{\csc^2(e+fx)}}-F_1\left(1;\frac{1}{2},-p;2;-\tan^2\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^(2*p)*((2*AppellF1[-1/2 - p, -1/2, -p,
 1/2 - p, -Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2])/(
(1 + 2*p)*(1 + (a*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) - (AppellF1[1,
 1/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e + f*x]^2)/(1 +
 (b*Tan[e + f*x]^2)/a)^p))/(2*f*(b*p*Sec[e + f*x]^2*Tan[e + f*x]*(a + b*Tan
[e + f*x]^2)^(-1 + p)*((2*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e + f*
x]^2, -((a*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2])/((1 + 2*p)*(1 + (a*Cot
[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) - (AppellF1[1, 1/2, -p, 2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/a
)^p) + ((a + b*Tan[e + f*x]^2)^p*((2*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p,
-Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b)]*Cot[e + f*x]*Sqrt[Sec[e + f*x]^2]
)/((1 + 2*p)*(1 + (a*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) + (4*a*p*Ap
pellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b
)]*Cot[e + f*x]*(1 + (a*Cot[e + f*x]^2)/b)^(-1 - p)*Sqrt[Csc[e + f*x]^2]*Sq
rt[Sec[e + f*x]^2])/(b*(1 + 2*p)) + (2*((-2*a*(-1/2 - p)*p*AppellF1[1/2 - p
, -1/2, 1 - p, 3/2 - p, -Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b)]*Cot[e + f
*x]*Csc[e + f*x]^2)/(b*(1/2 - p)) - ((-1/2 - p)*AppellF1[1/2 - p, 1/2, -p,
3/2 - p, -Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b)]*Cot[e + f*x]*Csc[e + f*x
]^2)/(1/2 - p))*Sqrt[Sec[e + f*x]^2])/((1 + 2*p)*(1 + (a*Cot[e + f*x]^2)/b)
^p*Sqrt[Csc[e + f*x]^2]) + (2*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e
+ f*x]^2, -((a*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2]*Tan[e + f*x])/((1 +
 2*p)*(1 + (a*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) + (2*b*p*AppellF1[
1, 1/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan
[e + f*x]^3*(1 + (b*Tan[e + f*x]^2)/a)^(-1 - p))/a - (2*AppellF1[1, 1/2, -p
, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])
/(1 + (b*Tan[e + f*x]^2)/a)^p - (Tan[e + f*x]^2*((b*p*AppellF1[2, 1/2, 1 -
p, 3, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x]
)/a - (AppellF1[2, 3/2, -p, 3, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Se
```

```
c[e + f*x]^2*Tan[e + f*x])/2))/(1 + (b*Tan[e + f*x]^2)/a)^p))/2))
```

Maple [F] time = 0.214, size = 0, normalized size = 0.

$$\int \csc(fx+e)\left(a+b\left(\tan(fx+e)\right)^{2}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

3.158 $\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=92

$$\frac{\sec^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{3}{2};2,-p;\frac{5}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a-b}\right)}{3f}$$

```
[Out] (AppellF1[3/2, 2, -p, 5/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/(a - b))]*S
ec[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^p)/(3*f*(1 + (b*Sec[e + f*x]^2)/(a
 - b))^p)
```

Rubi [A] time = 0.123787, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3664, 511, 510}

$$\frac{\sec^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{3}{2};2,-p;\frac{5}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a-b}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```


```
[Out] (AppellF1[3/2, 2, -p, 5/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/(a - b))]*S
ec[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^p)/(3*f*(1 + (b*Sec[e + f*x]^2)/(a
 - b))^p)
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a-b+bx^2\right)^p}{\left(-1+x^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a-b+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a-b}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^2\left(1+\frac{bx^2}{a-b}\right)^p}{\left(-1+x^2\right)^2}\,dx, x, \sec(e\right.}{f}$$

$$= \frac{F_1\left(\frac{3}{2};2,-p;\frac{5}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a-b}\right)\sec^3(e+fx)\left(a-b+b\sec^2(e+fx)\right.}{3f}$$

Mathematica [B] time = 2.17901, size = 252, normalized size = 2.74

$$\frac{b(2p-3)\cot(e+fx)\csc(e+fx)\left(a+b\tan^2(e+fx)\right)^p F_1\left(\frac{1}{2}-p;-\frac{1}{2},-p\right.}{f(2p-1)\left(b(2p-3)F_1\left(\frac{1}{2}-p;-\frac{1}{2},-p;\frac{3}{2}-p;-\cot^2(e+fx),-\frac{a\cot^2(e+fx)}{b}\right)-\cot^2(e+fx)\left(2apF_1\left(\frac{3}{2}-p;-\frac{1}{2},1-p;\frac{5}{2}-p;-\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (b*(-3 + 2*p)*AppellF1[1/2 - p, -1/2, -p, 3/2 - p, -Cot[e + f*x]^2, -((a*Co
t[e + f*x]^2)/b)]*Cot[e + f*x]*Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(-
1 + 2*p)*(b*(-3 + 2*p)*AppellF1[1/2 - p, -1/2, -p, 3/2 - p, -Cot[e + f*x]^2
, -((a*Cot[e + f*x]^2)/b)] - (2*a*p*AppellF1[3/2 - p, -1/2, 1 - p, 5/2 - p,
 -Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b)] + b*AppellF1[3/2 - p, 1/2, -p, 5
/2 - p, -Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b)])*Cot[e + f*x]^2))
```

Maple [F] time = 0.228, size = 0, normalized size = 0.

$$\int \left(\csc\left(fx+e\right)\right)^3\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

3.159 $\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};2,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{3f}$$

```
[Out] (AppellF1[3/2, 2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
+ f*x]^3*(a + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.108184, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3663, 511, 510}

$$\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};2,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[3/2, 2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
+ f*x]^3*(a + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^p}{(1+x^2)^2}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^2\left(1+\frac{bx^2}{a}\right)^p}{(1+x^2)^2}dx, x, \tan(e\right.}{f}$$

$$= \frac{F_1\left(\frac{3}{2};2,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan^3(e+fx)\left(a+b\tan^2(e+\right.}{3f}$$

Mathematica [C] time = 19.3409, size = 3698, normalized size = 44.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (3*a*Cos[e + f*x]^3*Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^p*(AppellF1[1/2, 2,
 -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]/(-3*a*AppellF1[1/2, 2,
-p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*(b*p*AppellF1[3/2, 2
, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*a*AppellF1[3/2,
 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) + (
AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e +
 f*x]^2)/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f
*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[
e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e +
 f*x]^2])*Tan[e + f*x]^2))*(-(Cos[2*(e + f*x)]^3*(a + b*Tan[e + f*x]^2)^p)/
4 + (I/4)*Sin[2*(e + f*x)]*(a + b*Tan[e + f*x]^2)^p + (Sin[2*(e + f*x)]^2*(
a + b*Tan[e + f*x]^2)^p)/2 - (I/4)*Sin[2*(e + f*x)]^3*(a + b*Tan[e + f*x]^2
)^p + Cos[2*(e + f*x)]^2*((a + b*Tan[e + f*x]^2)^p/2 - (I/4)*Sin[2*(e + f*x
)]*(a + b*Tan[e + f*x]^2)^p) + Cos[2*(e + f*x)]*(-(a + b*Tan[e + f*x]^2)^p/
4 - (Sin[2*(e + f*x)]^2*(a + b*Tan[e + f*x]^2)^p)/4)))/(f*(6*a*b*p*Sin[e +
f*x]^2*(a + b*Tan[e + f*x]^2)^(-1 + p)*(AppellF1[1/2, 2, -p, 3/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/a)]/(-3*a*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*a*AppellF1[3/2, 3, -p, 5/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) + (AppellF1[1/2, -p, 1
, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2)/(3*a*Appel
lF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*App
ellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*App
ellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f
*x]^2)) + 3*a*Cos[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p*(AppellF1[1/2, 2, -p,
 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]/(-3*a*AppellF1[1/2, 2, -p,
3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*(b*p*AppellF1[3/2, 2, 1
- p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*a*AppellF1[3/2, 3,
-p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) + (Appe
llF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x
]^2)/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^
2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e +
f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x
]^2])*Tan[e + f*x]^2)) - 9*a*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f
*x]^2)^p*(AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a
)]/(-3*a*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)
```

```
] - 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/a)] - 2*a*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/a)])*Tan[e + f*x]^2) + (AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a)
, -Tan[e + f*x]^2]*Sec[e + f*x]^2)/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[
e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b
*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Ta
n[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + 3*a*Cos[e + f*x]^3*S
in[e + f*x]*(a + b*Tan[e + f*x]^2)^p*(((2*b*p*AppellF1[3/2, 2, 1 - p, 5/2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a
) - (4*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*
Sec[e + f*x]^2*Tan[e + f*x])/3)/(-3*a*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*a*AppellF1[3/2, 3, -p, 5/2, -Tan[e
+ f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) + (2*AppellF1[1/2, -p,
1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*
x])/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2
] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f
*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]
^2])*Tan[e + f*x]^2) + (Sec[e + f*x]^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2,
 -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*
a) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]
*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e
 + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*
Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan
[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (AppellF1[1/2, 2, -p,
3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*(-4*(b*p*AppellF1[3/2, 2, 1
- p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*a*AppellF1[3/2, 3,
-p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Sec[e + f*x]^2*Tan[e +
f*x] - 3*a*((2*b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
 + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (4*AppellF1[3/2, 3, -p,
 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x]
)/3) - 2*Tan[e + f*x]^2*(b*p*((-6*b*(1 - p)*AppellF1[5/2, 2, 2 - p, 7/2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/(5*a)
- (12*AppellF1[5/2, 3, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)
]*Sec[e + f*x]^2*Tan[e + f*x])/5) - 2*a*((6*b*p*AppellF1[5/2, 3, 1 - p, 7/2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/(5
*a) - (18*AppellF1[5/2, 4, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a
)]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(-3*a*AppellF1[1/2, 2, -p, 3/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*a*AppellF1[3/2, 3, -p, 5/2, -
Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2)^2 - (AppellF1[1/2
, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*(4*(
b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]
- a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Se
c[e + f*x]^2*Tan[e + f*x] + 3*a*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*T
an[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2
*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e
+ f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*AppellF1[5/2, 1 - p,
 2, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f
*x])/5 - (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -((b*Tan[e + f*x]^2)/a),
 -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*a)) - a*((6*b*p*AppellF1[
5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^
2*Tan[e + f*x])/(5*a) - (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/
a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*a*AppellF1[1/2,
-p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2
, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2
, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)^2)
))
```

Maple [F] time = 0.734, size = 0, normalized size = 0.

$$\int \left(\sin \left(fx+e\right)\right)^{2} \left(a+b \left(\tan \left(fx+e\right)\right)^{2}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan \left(fx+e\right)^{2}+a\right)^{p} \sin \left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(\cos \left(fx+e\right)^{2}-1\right)\left(b \tan \left(fx+e\right)^{2}+a\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*tan(f*x + e)^2 + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan \left(fx+e\right)^{2}+a\right)^{p} \sin \left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

3.160 $\int \left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=78

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
+ f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.064155, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3661, 430, 429}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
+ f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e\cdot}{a}\right.}{f}$$

Mathematica [B] time = 0.552796, size = 192, normalized size = 2.46

$$\frac{3a\sin(2(e+fx))\left(a+b\tan^2(e+fx)\right)^pF_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+f\right.}{4f\tan^2(e+fx)\left(bpF_1\left(\frac{3}{2};1-p,1;\frac{5}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx)\right)-aF_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx)\right)\right)+6af}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Si
n[2*(e + f*x)]*(a + b*Tan[e + f*x]^2)^p)/(6*a*f*AppellF1[1/2, -p, 1, 3/2, -
((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 4*f*(b*p*AppellF1[3/2, 1 - p, 1,
 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/
2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int \left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^{2}+a\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan^{2}\left(e+fx\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p, x)
```

3.161 $\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=68

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*(
a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p))
```

Rubi [A] time = 0.0806688, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3663, 365, 364}

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*(
a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p))
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((a+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2}\,dx, x, \tan(e\right.}{f}$$

$$= -\frac{\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a}\right)\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2}{\right.}}{f}$$

Mathematica [A] time = 0.65354, size = 68, normalized size = 1.

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*(
a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p))
```

Maple [F] time = 0.214, size = 0, normalized size = 0.

$$\int \left(\csc\left(fx+e\right)\right)^2\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

3.162 $\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=120

$$-\frac{(3a-b(1-2p))\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a}\right)}{3af}$$

```
[Out] -(Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(1 + p))/(3*a*f) - ((3*a - b*(1 - 2
*p))*Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]
*(a + b*Tan[e + f*x]^2)^p)/(3*a*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.115567, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3663, 453, 365, 364}

$$-\frac{(3a-b(1-2p))\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a}\right)}{3af}-\frac{\cot^3(e+fx)\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -(Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(1 + p))/(3*a*f) - ((3*a - b*(1 - 2
*p))*Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]
*(a + b*Tan[e + f*x]^2)^p)/(3*a*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)(a+bx^2)^p}{x^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{3af} - \frac{(-3a-b(-3+2(1+p)))\,\text{Subst}\left(\int \right.}{3af}$$

$$= -\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{3af} - \frac{\left((-3a-b(-3+2(1+p)))\left(a+b\tan\right.\right.}{}$$

$$= -\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{3af} - \frac{(3a-b(1-2p))\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2}\right.}{}$$

Mathematica [A] time = 1.46639, size = 111, normalized size = 0.92

$$\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(-(3a+b(2p-1))\tan^2(e+fx)\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\right.\right.}{3af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p*(-a - b*Tan[e + f*x]^2 - ((3*a + b
*(-1 + 2*p))*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*Tan[
e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/a)^p))/(3*a*f)
```

Maple [F] time = 0.234, size = 0, normalized size = 0.

$$\int \left(\csc\left(fx+e\right)\right)^4\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

3.163 $\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=180

$$-\frac{\left(15a^2-b(1-2p)(10a-b(3-2p))\right)\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p\right.}{15a^2f}$$

```
[Out] -((10*a - b*(3 - 2*p))*Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(1 + p))/(15*a
^2*f) - (Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(1 + p))/(5*a*f) - ((15*a^2
- b*(10*a - b*(3 - 2*p))*(1 - 2*p))*Cot[e + f*x]*Hypergeometric2F1[-1/2, -p
, 1/2, -((b*Tan[e + f*x]^2)/a)]*(a + b*Tan[e + f*x]^2)^p)/(15*a^2*f*(1 + (b
*Tan[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.200637, antiderivative size = 176, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3663, 462, 453, 365, 364}

$$-\frac{\left(15-\frac{b(1-2p)(10a-b(3-2p))}{a^2}\right)\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a}\right)}{15f}-\frac{(10a-b(3\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((10*a - b*(3 - 2*p))*Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(1 + p))/(15*a
^2*f) - (Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(1 + p))/(5*a*f) - ((15 - (b
*(10*a - b*(3 - 2*p))*(1 - 2*p))/a^2)*Cot[e + f*x]*Hypergeometric2F1[-1/2,
-p, 1/2, -((b*Tan[e + f*x]^2)/a)]*(a + b*Tan[e + f*x]^2)^p)/(15*f*(1 + (b*T
an[e + f*x]^2)/a)^p)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2(a+bx^2)^p}{x^6}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{5af} + \frac{\text{Subst}\left(\int \frac{(10a-b(3-2p)+5ax^2)(a+bx^2)}{x^4}\right.}{5af}$$

$$= -\frac{(10a-b(3-2p))\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{15a^2f} - \frac{\cot^5(e+fx)\left(a\right.}{}$$

$$= -\frac{(10a-b(3-2p))\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{15a^2f} - \frac{\cot^5(e+fx)\left(a\right.}{}$$

$$= -\frac{(10a-b(3-2p))\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{15a^2f} - \frac{\cot^5(e+fx)\left(a\right.}{}$$

Mathematica [A] time = 1.29323, size = 141, normalized size = 0.78

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\left(15\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a}\right)+3\cot^4(e+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -(Cot[e + f*x]*(3*Cot[e + f*x]^4*Hypergeometric2F1[-5/2, -p, -3/2, -((b*Tan
[e + f*x]^2)/a)] + 10*Cot[e + f*x]^2*Hypergeometric2F1[-3/2, -p, -1/2, -((b
*Tan[e + f*x]^2)/a)] + 15*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x
]^2)/a)])*(a + b*Tan[e + f*x]^2)^p)/(15*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Maple [F] time = 0.258, size = 0, normalized size = 0.

$$\int \left(\csc\left(fx+e\right)\right)^6\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^6,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

3.164 $\int (d\sin(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=98

$$\frac{\tan(e+fx)(d\sin(e+fx))^m\cos^2(e+fx)^{\frac{1}{2}(np+1)}\left(b(c\tan(e+fx))^n\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2}(np+1),\frac{1}{2}(m+np+1\right.}{f(m+np+1)}$$

```
[Out] ((Cos[e + f*x]^2)^((1 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (1 + m + n*p
)/2, (3 + m + n*p)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^m*Tan[e + f*x]*(b*(c
*Tan[e + f*x])^n)^p)/(f*(1 + m + n*p))
```

Rubi [A] time = 0.18114, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3659, 2602, 2577}

$$\frac{\tan(e+fx)(d\sin(e+fx))^m\cos^2(e+fx)^{\frac{1}{2}(np+1)}\left(b(c\tan(e+fx))^n\right)^p\,{}_2F_1\left(\frac{1}{2}(np+1),\frac{1}{2}(m+np+1);\frac{1}{2}(m+np+3);\text{si}\right.}{f(m+np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (1 + m + n*p
)/2, (3 + m + n*p)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^m*Tan[e + f*x]*(b*(c
*Tan[e + f*x])^n)^p)/(f*(1 + m + n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (d\sin(e+fx))^m\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (d\sin(e+fx))^m(c\tan(e+fx))$$
$$= \left(d\cos^{np}(e+fx)\sin(e+fx)(d\sin(e+fx))^{-1-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \mathrm{co}$$
$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(1+m+np);\frac{1}{2}(3+m+np);\sin^2(e+\right.}{f(1+m+np)}$$

Mathematica [C] time = 2.01977, size = 295, normalized size = 3.01

$$\frac{(m+np+3)\sin(e+fx)(d\sin(e}{f(m+np+1)\left((m+np+3)F_1\left(\frac{1}{2}(m+np+1);np,m+1;\frac{1}{2}(m+np+3);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\tan\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((3 + m + n*p)*AppellF1[(1 + m + n*p)/2, n*p, 1 + m, (3 + m + n*p)/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*x]*(d*Sin[e + f*x])^m*(b*(c*T
an[e + f*x])^n)^p)/(f*(1 + m + n*p)*((3 + m + n*p)*AppellF1[(1 + m + n*p)/2
, n*p, 1 + m, (3 + m + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2
*((1 + m)*AppellF1[(3 + m + n*p)/2, n*p, 2 + m, (5 + m + n*p)/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] - n*p*AppellF1[(3 + m + n*p)/2, 1 + n*p, 1 +
 m, (5 + m + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x
)/2]^2))
```

Maple [F] time = 0.414, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^m\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*sin(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\left(d\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*sin(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\left(d\sin\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*(d*sin(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\left(d\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*sin(f*x + e))^m, x)
```

3.165 $\int \sin^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=63

$$\frac{\tan^3(e+fx)\text{Hypergeometric2F1}\left(2,\frac{1}{2}(np+3),\frac{1}{2}(np+5),-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+3)}$$

```
[Out] (Hypergeometric2F1[2, (3 + n*p)/2, (5 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]^3*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 + n*p))
```

Rubi [A] time = 0.112894, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3659, 2591, 364}

$$\frac{\tan^3(e+fx)\,{}_2F_1\left(2,\frac{1}{2}(np+3);\frac{1}{2}(np+5);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[2, (3 + n*p)/2, (5 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]^3*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 + n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \sin^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sin^2(e+fx)(c\tan(e+fx))^{np}\,d$$
$$= \frac{\left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\operatorname{Subst}\left(\int \frac{x^{2+np}}{(c^2+x^2)^2}\,dx, x, c\tan(e+\right.}{f}$$
$$= \frac{{}_2F_1\left(2,\frac{1}{2}(3+np);\frac{1}{2}(5+np);-\tan^2(e+fx)\right)\tan^3(e+fx)\left(b(c\tan(e+fx)\right.}{f(3+np)}$$

Mathematica [C] time = 2.48457, size = 517, normalized size = 8.21

$$\frac{}{f(np+1)\left(2(np+3)\cos^2\left(\frac{1}{2}(e+fx)\right)F_1\left(\frac{1}{2}(np+1);np,2;\frac{1}{2}(np+3);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2(np+\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (8*(6 + 2*n*p)*(AppellF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] - AppellF1[(1 + n*p)/2, n*p, 3, (3 + n*p)/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Cos[(e + f*x)/2]^5*Sin[(e + f*x)/2]^3*
(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p)*(2*(3 + n*p)*AppellF1[(1 + n*p)/2, n
*p, 2, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/
2]^2 - 2*(3 + n*p)*AppellF1[(1 + n*p)/2, n*p, 3, (3 + n*p)/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2*(2*AppellF1[(3 + n*p)/2,
 n*p, 3, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 3*AppellF1
[(3 + n*p)/2, n*p, 4, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + n*p*(-AppellF1[(3 + n*p)/2, 1 + n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + AppellF1[(3 + n*p)/2, 1 + n*p, 3, (5 + n*p)/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*(-1 + Cos[e + f*x])))
```

Maple [F] time = 7.694, size = 0, normalized size = 0.

$$\int \left(\sin\left(fx+e\right)\right)^2\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sin(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos\left(fx+e\right)^2-1\right)\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*((c*tan(f*x + e))^n*b)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^n\right)^p \sin^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*sin(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e)^2, x)
```

3.166 $\int \left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=61

$$\frac{\tan(e+fx)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+1),\frac{1}{2}(np+3),-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rubi [A] time = 0.0483137, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{\tan(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{np}\,dx$$
$$= \frac{\left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \frac{x^{np}}{c^2+x^2}\,dx, x, c\tan(e+fx)\right)}{f}$$
$$= \frac{{}_2F_1\left(1, \frac{1}{2}(1+np); \frac{1}{2}(3+np); -\tan^2(e+fx)\right)\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)}$$

Mathematica [A] time = 0.0533371, size = 59, normalized size = 0.97

$$\frac{\tan(e+fx)\text{Hypergeometric2F1}\left(1, \frac{1}{2}(np+1), \frac{1}{2}(np+3), -\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f + f*n*p)
```

Maple [F] time = 2.855, size = 0, normalized size = 0.

$$\int \left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p, x)
```

3.167 $\int \csc^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=33

$$-\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}$$

```
[Out] -((Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - n*p)))
```

Rubi [A] time = 0.097289, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3659, 2591, 30}

$$-\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - n*p)))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx &= \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \csc^2(e+fx)(c\tan(e+fx))^{np}\,dx\\
&= \frac{\left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\operatorname{Subst}\left(\int x^{-2+np}\,dx, x, c\tan(e+fx)\right)}{f}\\
&= -\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}
\end{aligned}$$

Mathematica [A] time = 0.0540608, size = 31, normalized size = 0.94

$$\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(-1 + n*p))
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(\csc\left(fx+e\right)\right)^2\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(csc(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A] time = 1.16465, size = 50, normalized size = 1.52

$$\frac{b^p(c^n)^p\left(\tan\left(fx+e\right)^n\right)^p}{\left(np-1\right)f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] b^p*(c^n)^p*(tan(f*x + e)^n)^p/((n*p - 1)*f*tan(f*x + e))
```

Fricas [A] time = 1.11238, size = 126, normalized size = 3.82

$$\frac{\cos\left(fx+e\right)e^{\left(np\log\left(\frac{c\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)+p\log\left(b\right)\right)}}{\left(fnp-f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] cos(f*x + e)*e^(n*p*log(c*sin(f*x + e)/cos(f*x + e)) + p*log(b))/((f*n*p -
f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \csc\left(fx+e\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^2, x)
```

3.168 $\int \csc^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=69

$$-\frac{\cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(3-np)}-\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}$$

```
[Out] -((Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - n*p))) - (Cot[e + f*x]^3*
(b*(c*Tan[e + f*x])^n)^p)/(f*(3 - n*p))
```

Rubi [A] time = 0.116107, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3659, 2591, 14}

$$-\frac{\cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(3-np)}-\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - n*p))) - (Cot[e + f*x]^3*
(b*(c*Tan[e + f*x])^n)^p)/(f*(3 - n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \csc^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \csc^4(e+fx)(c\tan(e+fx))^{np}\,dx$$
$$= \frac{\left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int x^{-4+np}\left(c^2+x^2\right)dx,x,c\tan\right.}{f}$$
$$= \frac{\left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int\left(c^2x^{-4+np}+x^{-2+np}\right)dx,x,\right.}{f}$$
$$= -\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)} - \frac{\cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(3-np)}$$

Mathematica [A] time = 0.15638, size = 59, normalized size = 0.86

$$\frac{\cot(e+fx)\csc^2(e+fx)(\cos(2(e+fx))+np-2)\left(b(c\tan(e+fx))^n\right)^p}{f(np-3)(np-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((-2 + n*p + Cos[2*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^2*(b*(c*Tan[e + f*
x])^n)^p)/(f*(-3 + n*p)*(-1 + n*p))
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int\left(\csc\left(fx+e\right)\right)^4\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(csc(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A] time = 1.24483, size = 99, normalized size = 1.43

$$\frac{\frac{b^p(c^n)^p\left(\tan\left(fx+e\right)^n\right)^p}{(np-1)\tan\left(fx+e\right)} + \frac{b^p(c^n)^p\left(\tan\left(fx+e\right)^n\right)^p}{(np-3)\tan\left(fx+e\right)^3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] (b^p*(c^n)^p*(tan(f*x + e)^n)^p/((n*p - 1)*tan(f*x + e)) + b^p*(c^n)^p*(tan
(f*x + e)^n)^p/((n*p - 3)*tan(f*x + e)^3))/f
```

Fricas [A] time = 1.08657, size = 250, normalized size = 3.62

$$\frac{\left(2\,\cos\left(fx+e\right)^{3}+\left(np-3\right)\cos\left(fx+e\right)\right)e^{\left(np\log\left(\frac{c\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)+p\log\left(b\right)\right)}}{\left(fn^{2}p^{2}-4\,fnp-\left(fn^{2}p^{2}-4\,fnp+3\,f\right)\cos\left(fx+e\right)^{2}+3\,f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] (2*cos(f*x + e)^3 + (n*p - 3)*cos(f*x + e))*e^(n*p*log(c*sin(f*x + e)/cos(f
*x + e)) + p*log(b))/((f*n^2*p^2 - 4*f*n*p - (f*n^2*p^2 - 4*f*n*p + 3*f)*co
s(f*x + e)^2 + 3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\csc\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^4, x)
```

3.169 $\int \csc^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=104

$$-\frac{\cot^5(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(5-np)}-\frac{2\cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(3-np)}-\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}$$

```
[Out] -((Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - n*p))) - (2*Cot[e + f*x]^
3*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 - n*p)) - (Cot[e + f*x]^5*(b*(c*Tan[e + f
*x])^n)^p)/(f*(5 - n*p))
```

Rubi [A] time = 0.127677, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3659, 2591, 270}

$$-\frac{\cot^5(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(5-np)}-\frac{2\cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(3-np)}-\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - n*p))) - (2*Cot[e + f*x]^
3*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 - n*p)) - (Cot[e + f*x]^5*(b*(c*Tan[e + f
*x])^n)^p)/(f*(5 - n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \csc^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \csc^6(e+fx)(c\tan(e+fx))^{np}\,d$$
$$= \frac{\left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int x^{-6+np}\left(c^2+x^2\right)^2\,dx, x\right.}{f}$$
$$= \frac{\left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int\left(c^4x^{-6+np}+2c^2x^{-4+np}\right.\right.}{f}$$
$$= -\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)} - \frac{2\cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(3-np)} -$$

Mathematica [A] time = 0.287085, size = 89, normalized size = 0.86

$$\frac{\cot(e+fx)\csc^4(e+fx)\left(2(np-3)\cos(2(e+fx))+\cos(4(e+fx))+n^2p^2-6np+8\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np-5)(np-3)(np-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((8 - 6*n*p + n^2*p^2 + 2*(-3 + n*p)*Cos[2*(e + f*x)] + Cos[4*(e + f*x)])*C
ot[e + f*x]*Csc[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p)/(f*(-5 + n*p)*(-3 + n*
p)*(-1 + n*p))
```

Maple [C] time = 9.345, size = 171293, normalized size = 1647.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 1.2523, size = 146, normalized size = 1.4

$$\frac{\frac{b^p(c^n)^p\left(\tan\left(fx+e\right)^n\right)^p}{(np-1)\tan\left(fx+e\right)} + \frac{2\,b^p(c^n)^p\left(\tan\left(fx+e\right)^n\right)^p}{(np-3)\tan\left(fx+e\right)^3} + \frac{b^p(c^n)^p\left(\tan\left(fx+e\right)^n\right)^p}{(np-5)\tan\left(fx+e\right)^5}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] (b^p*(c^n)^p*(tan(f*x + e)^n)^p/((n*p - 1)*tan(f*x + e)) + 2*b^p*(c^n)^p*(t
an(f*x + e)^n)^p/((n*p - 3)*tan(f*x + e)^3) + b^p*(c^n)^p*(tan(f*x + e)^n)^
p/((n*p - 5)*tan(f*x + e)^5))/f
```

Fricas [A] time = 1.13383, size = 435, normalized size = 4.18

$$\frac{\left(8\cos\left(fx+e\right)^{5}+4\left(np-5\right)\cos\left(fx+e\right)^{3}+\left(n^{2}p^{2}-8\,np+15\right)\cos\left(fx+e\right)\right)e^{\left(np\log\left(\frac{c\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)+\cdots}}{\left(fn^{3}p^{3}-9\,fn^{2}p^{2}+\left(fn^{3}p^{3}-9\,fn^{2}p^{2}+23\,fnp-15\,f\right)\cos\left(fx+e\right)^{4}+23\,fnp-2\left(fn^{3}p^{3}-9\,fn^{2}p^{2}+23\,fnp-15\,f\right)\cos\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] (8*cos(f*x + e)^5 + 4*(n*p - 5)*cos(f*x + e)^3 + (n^2*p^2 - 8*n*p + 15)*cos
(f*x + e))*e^(n*p*log(c*sin(f*x + e)/cos(f*x + e)) + p*log(b))/((f*n^3*p^3
- 9*f*n^2*p^2 + (f*n^3*p^3 - 9*f*n^2*p^2 + 23*f*n*p - 15*f)*cos(f*x + e)^4
+ 23*f*n*p - 2*(f*n^3*p^3 - 9*f*n^2*p^2 + 23*f*n*p - 15*f)*cos(f*x + e)^2 -
 15*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\csc\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^6, x)
```

3.170 $\int \sin^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=93

$$\frac{\sin^3(e+fx)\tan(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\text{Hypergeometric2F1}\left(\frac{1}{2}(np+1),\frac{1}{2}(np+4),\frac{1}{2}(np+6),\sin^2(e+fx)\right)\left(b(\right.}{f(np+4)}$$

```
[Out] ((Cos[e + f*x]^2)^((1 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (4 + n*p)/2,
 (6 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^3*Tan[e + f*x]*(b*(c*Tan[e + f*x
])^n)^p)/(f*(4 + n*p))
```

Rubi [A] time = 0.14207, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3659, 2602, 2577}

$$\frac{\sin^3(e+fx)\tan(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\,{}_2F_1\left(\frac{1}{2}(np+1),\frac{1}{2}(np+4);\frac{1}{2}(np+6);\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (4 + n*p)/2,
 (6 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^3*Tan[e + f*x]*(b*(c*Tan[e + f*x
])^n)^p)/(f*(4 + n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sin^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sin^3(e+fx)(c\tan(e+fx))^{np}\,dx$$
$$= \left(\cos^{np}(e+fx)\sin^{-np}(e+fx)\left(b(c\tan(e+fx))^n\right)^p\right)\int \cos^{-np}(e+fx)\sin^{3+np}(e$$
$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(4+np);\frac{1}{2}(6+np);\sin^2(e+fx)\right)\sin^3(e+}{f(4+np)}$$

Mathematica [C] time = 2.85455, size = 506, normalized size = 5.44

$$\frac{}{f(np+2)\left(2(np+4)\cos^2\left(\frac{1}{2}(e+fx)\right)F_1\left(\frac{np}{2}+1;np,3;\frac{np}{2}+2;\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2(np+4)\cos^2\left(\frac{1}{2}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (4*(4 + n*p)*(AppellF1[1 + (n*p)/2, n*p, 3, 2 + (n*p)/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] - AppellF1[1 + (n*p)/2, n*p, 4, 2 + (n*p)/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Cos[(e + f*x)/2]^3*Sin[(e + f*x)/2]*Sin[
e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p)/(f*(2 + n*p)*(2*(4 + n*p)*AppellF1[1 +
 (n*p)/2, n*p, 3, 2 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos
[(e + f*x)/2]^2 - 2*(4 + n*p)*AppellF1[1 + (n*p)/2, n*p, 4, 2 + (n*p)/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2*(3*AppellF1[2
 + (n*p)/2, n*p, 4, 3 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] -
 4*AppellF1[2 + (n*p)/2, n*p, 5, 3 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + n*p*(-AppellF1[2 + (n*p)/2, 1 + n*p, 3, 3 + (n*p)/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[2 + (n*p)/2, 1 + n*p, 4, 3 + (n
*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*(-1 + Cos[e + f*x])))
```

Maple [F] time = 5.38, size = 0, normalized size = 0.

$$\int \left(\sin\left(fx+e\right)\right)^3\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sin(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \sin\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos\left(fx+e\right)^{2}-1\right)\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*((c*tan(f*x + e))^n*b)^p*sin(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\sin\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e)^3, x)
```

3.171 $\int \sin(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=91

$$\frac{\sin(e+fx)\tan(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\text{Hypergeometric2F1}\left(\frac{1}{2}(np+1),\frac{1}{2}(np+2),\frac{1}{2}(np+4),\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+2)}$$

```
[Out] ((Cos[e + f*x]^2)^((1 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (2 + n*p)/2,
 (4 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*Tan[e + f*x]*(b*(c*Tan[e + f*x])
^n)^p)/(f*(2 + n*p))
```

Rubi [A] time = 0.107261, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3659, 2602, 2577}

$$\frac{\sin(e+fx)\tan(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\,{}_2F_1\left(\frac{1}{2}(np+1),\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```


```
[Out] ((Cos[e + f*x]^2)^((1 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (2 + n*p)/2,
 (4 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*Tan[e + f*x]*(b*(c*Tan[e + f*x])
^n)^p)/(f*(2 + n*p))
```


Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sin(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sin(e+fx)(c\tan(e+fx))^{np}\,dx$$
$$= \left(\cos^{np}(e+fx)\sin^{-np}(e+fx)\left(b(c\tan(e+fx))^n\right)^p\right)\int \cos^{-np}(e+fx)\sin^{1+np}$$
$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(2+np);\frac{1}{2}(4+np);\sin^2(e+fx)\right)\sin(e}{f(2+np)}$$

Mathematica [C] time = 1.30609, size = 284, normalized size = 3.12

$$\frac{8(np+4)\sin^2\left(\frac{1}{2}(e+fx)\right)\cos^4\left(\frac{1}{2}(e+fx)\right)F_1\left(\frac{np}{2}\right.}{f(np+2)\left(2(np+4)\cos^2\left(\frac{1}{2}(e+fx)\right)F_1\left(\frac{np}{2}+1;np,2;\frac{np}{2}+2;\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+2(\cos(e+fx)\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (8*(4 + n*p)*AppellF1[1 + (n*p)/2, n*p, 2, 2 + (n*p)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^4*Sin[(e + f*x)/2]^2*(b*(c*Tan[e + f
*x])^n)^p)/(f*(2 + n*p)*(2*(4 + n*p)*AppellF1[1 + (n*p)/2, n*p, 2, 2 + (n*p
)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2*(2*App
ellF1[2 + (n*p)/2, n*p, 3, 3 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] - n*p*AppellF1[2 + (n*p)/2, 1 + n*p, 2, 3 + (n*p)/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2])*(-1 + Cos[e + f*x])))
```

Maple [F] time = 5.46, size = 0, normalized size = 0.

$$\int \sin\left(fx+e\right)\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*sin(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*sin(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\sin\left(fx+e\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e), x)
```

3.172 $\int \csc(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=81

$$\frac{\sec(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\text{Hypergeometric2F1}\left(\frac{np}{2},\frac{1}{2}(np+1),\frac{1}{2}(np+2),\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp}$$

```
[Out] ((Cos[e + f*x]^2)^((1 + n*p)/2)*Hypergeometric2F1[(n*p)/2, (1 + n*p)/2, (2
+ n*p)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*n*p)
```

Rubi [A] time = 0.123923, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3659, 2601, 2577}

$$\frac{\sec(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\,{}_2F_1\left(\frac{np}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+2);\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n*p)/2)*Hypergeometric2F1[(n*p)/2, (1 + n*p)/2, (2
+ n*p)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*n*p)
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \csc(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \csc(e+fx)(c\tan(e+fx))^{np}\,dx$$
$$= \left(\cos^{np}(e+fx)\sin^{-np}(e+fx)\left(b(c\tan(e+fx))^n\right)^p\right)\int \cos^{-np}(e+fx)\sin^{-1+np}(e$$
$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\,{}_2F_1\left(\frac{np}{2},\frac{1}{2}(1+np);\frac{1}{2}(2+np);\sin^2(e+fx)\right)\sec(e+fx)\left(b(c\right.}{fnp}$$

Mathematica [A] time = 0.201881, size = 77, normalized size = 0.95

$$\frac{\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^{np}\text{Hypergeometric2F1}\left(\frac{np}{2},np,\frac{np}{2}+1,\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[(n*p)/2, n*p, 1 + (n*p)/2, Tan[(e + f*x)/2]^2]*(Cos[e +
f*x]*Sec[(e + f*x)/2]^2)^(n*p)*(b*(c*Tan[e + f*x])^n)^p)/(f*n*p)
```

Maple [F] time = 3.082, size = 0, normalized size = 0.

$$\int \csc\left(fx+e\right)\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(csc(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \csc\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*csc(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*csc(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e), x)
```

3.173 $\int \csc^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=92

$$-\frac{\csc^2(e+fx)\sec(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\text{Hypergeometric2F1}\left(\frac{1}{2}(np-2),\frac{1}{2}(np+1),\frac{np}{2},\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(2-np)}$$

```
[Out] -(((Cos[e + f*x]^2)^((1 + n*p)/2)*Csc[e + f*x]^2*Hypergeometric2F1[(-2 + n*
p)/2, (1 + n*p)/2, (n*p)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(b*(c*Tan[e + f*x]
)^n)^p)/(f*(2 - n*p)))
```

Rubi [A] time = 0.145556, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3659, 2601, 2577}

$$-\frac{\csc^2(e+fx)\sec(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\,{}_2F_1\left(\frac{1}{2}(np-2),\frac{1}{2}(np+1);\frac{np}{2};\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(2-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -(((Cos[e + f*x]^2)^((1 + n*p)/2)*Csc[e + f*x]^2*Hypergeometric2F1[(-2 + n*
p)/2, (1 + n*p)/2, (n*p)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(b*(c*Tan[e + f*x]
)^n)^p)/(f*(2 - n*p)))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \csc^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \csc^3(e+fx)(c\tan(e+fx))^{np} d$$
$$= \left(\cos^{np}(e+fx)\sin^{-np}(e+fx)\left(b(c\tan(e+fx))^n\right)^p\right)\int \cos^{-np}(e+fx)\sin^{-3}$$
$$= -\frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\csc^2(e+fx)\,{}_2F_1\left(\frac{1}{2}(-2+np),\frac{1}{2}(1+np);\frac{np}{2};\sin^2(e+f\right.}{f(2-np)}$$

Mathematica [B] time = 7.30732, size = 217, normalized size = 2.36

$$\frac{\tan^2\left(\frac{1}{2}(e+fx)\right)\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^{np}\left(b(c\tan(e+fx))^n\right)^p\left(2\left(n^2p^2-4\right)\cot^2\left(\frac{1}{2}(e+fx)\right)\text{Hypergeomet}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((2*(-4 + n^2*p^2)*Cot[(e + f*x)/2]^2*Hypergeometric2F1[(n*p)/2, n*p, 1 + (
n*p)/2, Tan[(e + f*x)/2]^2] + n*p*((2 + n*p)*Cot[(e + f*x)/2]^4*Hypergeomet
ric2F1[n*p, -1 + (n*p)/2, (n*p)/2, Tan[(e + f*x)/2]^2] + (-2 + n*p)*Hyperge
ometric2F1[n*p, 1 + (n*p)/2, 2 + (n*p)/2, Tan[(e + f*x)/2]^2]))*(Cos[e + f*
x]*Sec[(e + f*x)/2]^2)^(n*p)*Tan[(e + f*x)/2]^2*(b*(c*Tan[e + f*x])^n)^p)/(
4*f*n*p*(-4 + n^2*p^2))
```

Maple [F] time = 3.131, size = 0, normalized size = 0.

$$\int \left(\csc\left(fx+e\right)\right)^3\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(csc(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \csc\left(fx+e\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \csc\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\csc\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^3, x)
```

3.174 $\int (d\sin(e+fx))^m \left(a+b\tan^n(e+fx)\right)^p\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left((d\sin(e+fx))^m \left(a+b\tan^n(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable[(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^n)^p, x]
```

Rubi [A] time = 0.0537551, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (d\sin(e+fx))^m \left(a+b\tan^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^n)^p,x]
```

```
[Out] Defer[Int][(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^n)^p, x]
```

Rubi steps

$$\int (d\sin(e+fx))^m \left(a+b\tan^n(e+fx)\right)^p\,dx = \int (d\sin(e+fx))^m \left(a+b\tan^n(e+fx)\right)^p\,dx$$

Mathematica [A] time = 2.83118, size = 0, normalized size = 0.

$$\int (d\sin(e+fx))^m \left(a+b\tan^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^n)^p,x]
```

```
[Out] Integrate[(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^n)^p, x]
```

Maple [A] time = 2.056, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^m \left(a+b\left(\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x)
```

```
[Out] int((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^{n}+a\right)^{p}\left(d\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^n + a)^p*(d*sin(f*x + e))^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\tan\left(fx+e\right)^{n}+a\right)^{p}\left(d\sin\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^n + a)^p*(d*sin(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(a+b*tan(f*x+e)**n)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^{n}+a\right)^{p}\left(d\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^n + a)^p*(d*sin(f*x + e))^m, x)
```

3.175 $\int (d\cos(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=99

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p (d\cos(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(-m+2p+1)}\text{Hypergeometric2F1}\left(\frac{1}{2}(2p+1),\frac{1}{2}(-m+2p+\right.}{f(2p+1)}$$

```
[Out] ((d*Cos[e + f*x])^m*(Cos[e + f*x]^2)^((1 - m + 2*p)/2)*Hypergeometric2F1[(1
 + 2*p)/2, (1 - m + 2*p)/2, (3 + 2*p)/2, Sin[e + f*x]^2]*Tan[e + f*x]*(b*Ta
n[e + f*x]^2)^p)/(f*(1 + 2*p))
```

Rubi [A] time = 0.13651, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3658, 2603, 2617}

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p (d\cos(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(-m+2p+1)}\, {}_2F_1\left(\frac{1}{2}(2p+1),\frac{1}{2}(-m+2p+1);\frac{1}{2}(2p+3);\sin^2\right.}{f(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((d*Cos[e + f*x])^m*(Cos[e + f*x]^2)^((1 - m + 2*p)/2)*Hypergeometric2F1[(1
 + 2*p)/2, (1 - m + 2*p)/2, (3 + 2*p)/2, Sin[e + f*x]^2]*Tan[e + f*x]*(b*Ta
n[e + f*x]^2)^p)/(f*(1 + 2*p))
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2603

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cos[e + f*x])^FracPart[m]*(Sec[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sec[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rubi steps

$$\int (d\cos(e+fx))^m \left(b\tan^2(e+fx)\right)^p\, dx = \left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int (d\cos(e+fx))^m \tan^{2p}(e+fx)\, dx$$
$$= \left((d\cos(e+fx))^m \left(\frac{\sec(e+fx)}{d}\right)^m \tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int \left(\frac{\sec(e\right.$$
$$= \frac{(d\cos(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(1-m+2p)}\, {}_2F_1\left(\frac{1}{2}(1+2p), \frac{1}{2}(1-m+2p); \frac{1}{2}(3+2p\right.}{f(1+2p)}$$

Mathematica [A] time = 0.500143, size = 81, normalized size = 0.82

$$\frac{\tan(e+fx)\sec^2(e+fx)^{m/2}\left(b\tan^2(e+fx)\right)^p (d\cos(e+fx))^m \text{Hypergeometric2F1}\left(\frac{m}{2}+1, p+\frac{1}{2}, p+\frac{3}{2}, -\tan^2(e+fx\right.}{f(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((d*Cos[e + f*x])^m*Hypergeometric2F1[1 + m/2, 1/2 + p, 3/2 + p, -Tan[e + f
*x]^2]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(b*Tan[e + f*x]^2)^p)/(f*(1 + 2*
p))
```

Maple [F] time = 0.706, size = 0, normalized size = 0.

$$\int \left(d\cos\left(fx+e\right)\right)^m \left(b\left(\tan\left(fx+e\right)\right)^2\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*cos(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2\right)^p \left(d\cos\left(fx+e\right)\right)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*cos(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2\right)^p \left(d\cos\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2)^p*(d*cos(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan \left(fx+e\right)^{2}\right)^{p} \left(d \cos \left(fx+e\right)\right)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*cos(f*x + e))^m, x)
```

3.176 $\int (d\cos(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=108

$$\frac{\tan(e+fx)\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{m+2}{2},-p;\frac{3}{2};-\tan^2(e+fx),-\right.}{f}$$

```
[Out] (AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)
]*(d*Cos[e + f*x])^m*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(a + b*Tan[e + f*x
]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.144785, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3669, 3679, 430, 429}

$$\frac{\tan(e+fx)\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{m+2}{2},-p;\frac{3}{2};-\tan^2(e+fx),-\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)
]*(d*Cos[e + f*x])^m*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(a + b*Tan[e + f*x
]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 3669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (
f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(d*Cos[e + f*x])^FracPart[m]*(Sec
[e + f*x]/d)^FracPart[m], Int[(a + b*(c*Tan[e + f*x])^n)^p/(Sec[e + f*x]/d)
^m, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m]
```

Rule 3679

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(f
f*(d*Sec[e + f*x])^m)/(f*(Sec[e + f*x]^2)^(m/2)), Subst[Int[(1 + ff^2*x^2)^
(m/2 - 1)*(a + b*ff^2*x^2)^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] && !IntegerQ[m]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (d\cos(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p\,dx = \left((d\cos(e+fx))^m \left(\frac{\sec(e+fx)}{d}\right)^m\right)\int \left(\frac{\sec(e+fx)}{d}\right)^{-m} \left(a+b\tan^2(e+\right.$$

$$= \frac{\left((d\cos(e+fx))^m \sec^2(e+fx)^{m/2}\right)\operatorname{Subst}\left(\int \left(1+x^2\right)^{-1-\frac{m}{2}} \left(a+bx^2\right)^p\, d\right.}{f}$$

$$= \frac{\left((d\cos(e+fx))^m \sec^2(e+fx)^{m/2} \left(a+b\tan^2(e+fx)\right)^p \left(1+\frac{b\tan^2(e+fx}{a}\right.\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};\frac{2+m}{2},-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)(d\cos(e+fx))^m \sec^2(e}{f}$$

Mathematica [B] time = 16.9166, size = 2033, normalized size = 18.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/a)]*(d*Cos[e + f*x])^m*(Sec[e + f*x]^2)^(-1 - m/2)*Tan[e + f*x]*(a + b*Ta
n[e + f*x]^2)^(2*p))/(f*(3*a*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, (2 + m)/2, 1 - p, 5/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - a*(2 + m)*AppellF1[3/2, (4 + m
)/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2)*((
6*a*b*p*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/a)]*Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(-1 + p))/((Sec[e + f*x]^2)^
(m/2)*(3*a*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e +
f*x]^2)/a)] + (2*b*p*AppellF1[3/2, (2 + m)/2, 1 - p, 5/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/a)] - a*(2 + m)*AppellF1[3/2, (4 + m)/2, -p, 5/2, -Tan
[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2)) + (3*a*AppellF1[1/2
, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*(a + b*Tan[
e + f*x]^2)^p)/((Sec[e + f*x]^2)^(m/2)*(3*a*AppellF1[1/2, (2 + m)/2, -p, 3/
2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, (2 + m)
/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - a*(2 + m)*Appel
lF1[3/2, (4 + m)/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan
[e + f*x]^2)) + (6*a*(-1 - m/2)*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/a)]*(Sec[e + f*x]^2)^(-1 - m/2)*Tan[e + f*x]^2
*(a + b*Tan[e + f*x]^2)^p)/(3*a*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, (2 + m)/2, 1 - p, 5
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - a*(2 + m)*AppellF1[3/2, (4
+ m)/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2)
 + (3*a*(Sec[e + f*x]^2)^(-1 - m/2)*Tan[e + f*x]*((2*b*p*AppellF1[3/2, (2 +
 m)/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2
*Tan[e + f*x])/(3*a) - ((2 + m)*AppellF1[3/2, 1 + (2 + m)/2, -p, 5/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/3)*(a + b
*Tan[e + f*x]^2)^p)/(3*a*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2,
 -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, (2 + m)/2, 1 - p, 5/2, -Ta
n[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - a*(2 + m)*AppellF1[3/2, (4 + m)/2,
 -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) - (3*a
*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)
]*(Sec[e + f*x]^2)^(-1 - m/2)*Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p*(2*(2*b
*p*AppellF1[3/2, (2 + m)/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
```

```
2)/a)] - a*(2 + m)*AppellF1[3/2, (4 + m)/2, -p, 5/2, -Tan[e + f*x]^2, -((b*
Tan[e + f*x]^2)/a)])*Sec[e + f*x]^2*Tan[e + f*x] + 3*a*((2*b*p*AppellF1[3/2
, (2 + m)/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e +
f*x]^2*Tan[e + f*x])/(3*a) - ((2 + m)*AppellF1[3/2, 1 + (2 + m)/2, -p, 5/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/3)
+ Tan[e + f*x]^2*(2*b*p*((-6*b*(1 - p)*AppellF1[5/2, (2 + m)/2, 2 - p, 7/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/(5*
a) - (3*(2 + m)*AppellF1[5/2, 1 + (2 + m)/2, 1 - p, 7/2, -Tan[e + f*x]^2, -
((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/5) - a*(2 + m)*((6*b*p
*AppellF1[5/2, (4 + m)/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/a)]*Sec[e + f*x]^2*Tan[e + f*x])/(5*a) - (3*(4 + m)*AppellF1[5/2, 1 + (4 +
 m)/2, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Ta
n[e + f*x])/5))))/(3*a*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -
((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, (2 + m)/2, 1 - p, 5/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - a*(2 + m)*AppellF1[3/2, (4 + m)/2, -
p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2)^2))
```

Maple [F] time = 0.842, size = 0, normalized size = 0.

$$\int \left(d\cos\left(fx+e\right)\right)^{m}\left(a+b\left(\tan\left(fx+e\right)\right)^{2}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\left(d\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*cos(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\left(d\cos\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*(d*cos(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p \left(d\cos\left(fx+e\right)\right)^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*cos(f*x + e))^m, x)
```

3.177 $\int (d\cos(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx$

Optimal. Leaf size=101

$$\frac{\tan(e+fx)(d\cos(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(-m+np+1)} \left(b(c\tan(e+fx))^n\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2}(np+1), \frac{1}{2}(-m+np+\right.}{f(np+1)}$$

```
[Out] ((d*Cos[e + f*x])^m*(Cos[e + f*x]^2)^((1 - m + n*p)/2)*Hypergeometric2F1[(1
 + n*p)/2, (1 - m + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*Tan[e + f*x]*(b*(c
*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rubi [A] time = 0.147662, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3659, 2603, 2617}

$$\frac{\tan(e+fx)(d\cos(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(-m+np+1)} \left(b(c\tan(e+fx))^n\right)^p {}_2F_1\left(\frac{1}{2}(np+1), \frac{1}{2}(-m+np+1); \frac{1}{2}(np+3); \sin^2\right.}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((d*Cos[e + f*x])^m*(Cos[e + f*x]^2)^((1 - m + n*p)/2)*Hypergeometric2F1[(1
 + n*p)/2, (1 - m + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*Tan[e + f*x]*(b*(c
*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2603

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cos[e + f*x])^FracPart[m]*(Sec[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sec[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rubi steps

$$\int (d\cos(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (d\cos(e+fx))^m(c\tan(e+f$$

$$= \left((d\cos(e+fx))^m\left(\frac{\sec(e+fx)}{d}\right)^m (c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)\right.$$

$$= \frac{(d\cos(e+fx))^m\cos^2(e+fx)^{\frac{1}{2}(1-m+np)}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(1-m+np);\frac{1}{2}\right.}{f(1+np)}$$

Mathematica [A] time = 0.463351, size = 91, normalized size = 0.9

$$\frac{\tan(e+fx)\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\left(b(c\tan(e+fx))^n\right)^p \text{Hypergeometric2F1}\left(\frac{m+2}{2},\frac{1}{2}(np+1),\frac{1}{2}(np+3),-\right.}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((d*Cos[e + f*x])^m*Hypergeometric2F1[(2 + m)/2, (1 + n*p)/2, (3 + n*p)/2,
-Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)
^p)/(f*(1 + n*p))
```

Maple [F] time = 0.224, size = 0, normalized size = 0.

$$\int \left(d\cos\left(fx+e\right)\right)^m \left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*cos(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \left(d\cos\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*cos(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \left(d\cos\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*(d*cos(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c \tan \left(fx + e\right)\right)^{n} b\right)^{p} \left(d \cos \left(fx + e\right)\right)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*cos(f*x + e))^m, x)
```

3.178 $\int (d\cos(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=56

$$(d\cos(e+fx))^m\left(\frac{\sec(e+fx)}{d}\right)^m\text{Unintegrable}\left(\left(\frac{\sec(e+fx)}{d}\right)^{-m}\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

```
[Out] (d*Cos[e + f*x])^m*(Sec[e + f*x]/d)^m*Unintegrable[(a + b*(c*Tan[e + f*x])^
n)^p/(Sec[e + f*x]/d)^m, x]
```

Rubi [A] time = 0.127111, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (d\cos(e+fx))^m\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*Cos[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (d*Cos[e + f*x])^m*(Sec[e + f*x]/d)^m*Defer[Int][(a + b*(c*Tan[e + f*x])^n)
^p/(Sec[e + f*x]/d)^m, x]
```

Rubi steps

$$\int (d\cos(e+fx))^m\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\left((d\cos(e+fx))^m\left(\frac{\sec(e+fx)}{d}\right)^m\right)\int\left(\frac{\sec(e+fx)}{d}\right)^{-m}\left(a+b(c\tan\right.$$

Mathematica [A] time = 2.64689, size = 0, normalized size = 0.

$$\int (d\cos(e+fx))^m\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Cos[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Integrate[(d*Cos[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Maple [A] time = 2.256, size = 0, normalized size = 0.

$$\int\left(d\cos\left(fx+e\right)\right)^m\left(a+b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*cos(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b+a\right)^{p}\left(d\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*cos(f*x + e))^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^{n}b+a\right)^{p}\left(d\cos\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*(d*cos(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b+a\right)^{p}\left(d\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*cos(f*x + e))^m, x)
```

3.179 $\int \left(a+a\tan^2(c+dx)\right)^4\,dx$

Optimal. Leaf size=65

$$\frac{a^4\tan^7(c+dx)}{7d}+\frac{3a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^3(c+dx)}{d}+\frac{a^4\tan(c+dx)}{d}$$

```
[Out] (a^4*Tan[c + d*x])/d + (a^4*Tan[c + d*x]^3)/d + (3*a^4*Tan[c + d*x]^5)/(5*d
) + (a^4*Tan[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0350688, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3657, 12, 3767}

$$\frac{a^4\tan^7(c+dx)}{7d}+\frac{3a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^3(c+dx)}{d}+\frac{a^4\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^4,x]
```


```
[Out] (a^4*Tan[c + d*x])/d + (a^4*Tan[c + d*x]^3)/d + (3*a^4*Tan[c + d*x]^5)/(5*d
) + (a^4*Tan[c + d*x]^7)/(7*d)
```


Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+a\tan^2(c+dx)\right)^4\,dx &= \int a^4\sec^8(c+dx)\,dx\\
&= a^4\int \sec^8(c+dx)\,dx\\
&= -\frac{a^4\operatorname{Subst}\left(\int \left(1+3x^2+3x^4+x^6\right)dx,x,-\tan(c+dx)\right)}{d}\\
&= \frac{a^4\tan(c+dx)}{d}+\frac{a^4\tan^3(c+dx)}{d}+\frac{3a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^7(c+dx)}{7d}
\end{aligned}$$

Mathematica [A] time = 0.1956, size = 46, normalized size = 0.71

$$\frac{a^4\left(\frac{1}{7}\tan^7(c+dx)+\frac{3}{5}\tan^5(c+dx)+\tan^3(c+dx)+\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^4,x]
```

```
[Out] (a^4*(Tan[c + d*x] + Tan[c + d*x]^3 + (3*Tan[c + d*x]^5)/5 + Tan[c + d*x]^7
/7))/d
```

Maple [A] time = 0.003, size = 43, normalized size = 0.7

$$\frac{a^4}{d}\left(\frac{(\tan(dx+c))^7}{7}+\frac{3\,(\tan(dx+c))^5}{5}+(\tan(dx+c))^3+\tan(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(d*x+c)^2)^4,x)
```

```
[Out] 1/d*a^4*(1/7*tan(d*x+c)^7+3/5*tan(d*x+c)^5+tan(d*x+c)^3+tan(d*x+c))
```

Maxima [B] time = 1.73162, size = 212, normalized size = 3.26

$$a^4x+\frac{\left(15\,\tan(dx+c)^7-21\,\tan(dx+c)^5+35\,\tan(dx+c)^3+105\,dx+105\,c-105\,\tan(dx+c)\right)a^4}{105\,d}+\frac{4\,(3\,\tan(dx+c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 1/105*(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 +
105*d*x + 105*c - 105*tan(d*x + c))*a^4/d + 4/15*(3*tan(d*x + c)^5 - 5*tan(
d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c))*a^4/d + 2*(tan(d*x + c)^3 + 3
*d*x + 3*c - 3*tan(d*x + c))*a^4/d - 4*(d*x + c - tan(d*x + c))*a^4/d
```

Fricas [A] time = 0.998153, size = 136, normalized size = 2.09

$$\frac{5\,a^4\tan(dx+c)^7+21\,a^4\tan(dx+c)^5+35\,a^4\tan(dx+c)^3+35\,a^4\tan(dx+c)}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/35*(5*a^4*tan(d*x + c)^7 + 21*a^4*tan(d*x + c)^5 + 35*a^4*tan(d*x + c)^3
+ 35*a^4*tan(d*x + c))/d
```

Sympy [A] time = 1.20402, size = 68, normalized size = 1.05

$$\begin{cases}\frac{a^4\tan^7(c+dx)}{7d}+\frac{3a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^3(c+dx)}{d}+\frac{a^4\tan(c+dx)}{d} & \text{for } d\neq 0\\ x\left(a\tan^2(c)+a\right)^4 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)**2)**4,x)
```

```
[Out] Piecewise((a**4*tan(c + d*x)**7/(7*d) + 3*a**4*tan(c + d*x)**5/(5*d) + a**4
*tan(c + d*x)**3/d + a**4*tan(c + d*x)/d, Ne(d, 0)), (x*(a*tan(c)**2 + a)**
4, True))
```

Giac [B] time = 2.31117, size = 701, normalized size = 10.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] -1/35*(35*a^4*tan(d*x)^7*tan(c)^6 + 35*a^4*tan(d*x)^6*tan(c)^7 + 35*a^4*tan
(d*x)^7*tan(c)^4 - 105*a^4*tan(d*x)^6*tan(c)^5 - 105*a^4*tan(d*x)^5*tan(c)^
6 + 35*a^4*tan(d*x)^4*tan(c)^7 + 21*a^4*tan(d*x)^7*tan(c)^2 - 35*a^4*tan(d*
x)^6*tan(c)^3 + 315*a^4*tan(d*x)^5*tan(c)^4 + 315*a^4*tan(d*x)^4*tan(c)^5 -
 35*a^4*tan(d*x)^3*tan(c)^6 + 21*a^4*tan(d*x)^2*tan(c)^7 + 5*a^4*tan(d*x)^7
 - 7*a^4*tan(d*x)^6*tan(c) + 105*a^4*tan(d*x)^5*tan(c)^2 - 315*a^4*tan(d*x)
^4*tan(c)^3 - 315*a^4*tan(d*x)^3*tan(c)^4 + 105*a^4*tan(d*x)^2*tan(c)^5 - 7
*a^4*tan(d*x)*tan(c)^6 + 5*a^4*tan(c)^7 + 21*a^4*tan(d*x)^5 - 35*a^4*tan(d*
x)^4*tan(c) + 315*a^4*tan(d*x)^3*tan(c)^2 + 315*a^4*tan(d*x)^2*tan(c)^3 - 3
5*a^4*tan(d*x)*tan(c)^4 + 21*a^4*tan(c)^5 + 35*a^4*tan(d*x)^3 - 105*a^4*tan
(d*x)^2*tan(c) - 105*a^4*tan(d*x)*tan(c)^2 + 35*a^4*tan(c)^3 + 35*a^4*tan(d
*x) + 35*a^4*tan(c))/(d*tan(d*x)^7*tan(c)^7 - 7*d*tan(d*x)^6*tan(c)^6 + 21*
d*tan(d*x)^5*tan(c)^5 - 35*d*tan(d*x)^4*tan(c)^4 + 35*d*tan(d*x)^3*tan(c)^3
 - 21*d*tan(d*x)^2*tan(c)^2 + 7*d*tan(d*x)*tan(c) - d)
```

3.180 $\int \left(a+a\tan^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=50

$$\frac{a^3\tan^5(c+dx)}{5d}+\frac{2a^3\tan^3(c+dx)}{3d}+\frac{a^3\tan(c+dx)}{d}$$

```
[Out] (a^3*Tan[c + d*x])/d + (2*a^3*Tan[c + d*x]^3)/(3*d) + (a^3*Tan[c + d*x]^5)/
(5*d)
```

Rubi [A] time = 0.0301843, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3657, 12, 3767}

$$\frac{a^3\tan^5(c+dx)}{5d}+\frac{2a^3\tan^3(c+dx)}{3d}+\frac{a^3\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Tan[c + d*x])/d + (2*a^3*Tan[c + d*x]^3)/(3*d) + (a^3*Tan[c + d*x]^5)/
(5*d)
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+a\tan^2(c+dx)\right)^3\,dx &= \int a^3\sec^6(c+dx)\,dx\\ &= a^3\int \sec^6(c+dx)\,dx\\ &= -\frac{a^3\operatorname{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-\tan(c+dx)\right)}{d}\\ &= \frac{a^3\tan(c+dx)}{d}+\frac{2a^3\tan^3(c+dx)}{3d}+\frac{a^3\tan^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.0973905, size = 38, normalized size = 0.76

$$\frac{a^3\left(\frac{1}{5}\tan^5(c+dx)+\frac{2}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^3,x]
```

```
[Out] (a^3*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c + d*x]^5/5))/d
```

Maple [A] time = 0.001, size = 35, normalized size = 0.7

$$\frac{a^3}{d}\left(\frac{(\tan(dx+c))^5}{5}+\frac{2\,(\tan(dx+c))^3}{3}+\tan(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(d*x+c)^2)^3,x)
```

```
[Out] 1/d*a^3*(1/5*tan(d*x+c)^5+2/3*tan(d*x+c)^3+tan(d*x+c))
```

Maxima [B] time = 1.64202, size = 138, normalized size = 2.76

$$a^3x+\frac{\left(3\,\tan(dx+c)^5-5\,\tan(dx+c)^3-15\,dx-15\,c+15\,\tan(dx+c)\right)a^3}{15\,d}+\frac{\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(d\ldots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 1/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(
d*x + c))*a^3/d + (tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^3/d - 3
*(d*x + c - tan(d*x + c))*a^3/d
```

Fricas [A] time = 1.00163, size = 104, normalized size = 2.08

$$\frac{3\,a^3\tan(dx+c)^5+10\,a^3\tan(dx+c)^3+15\,a^3\tan(dx+c)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(3*a^3*tan(d*x + c)^5 + 10*a^3*tan(d*x + c)^3 + 15*a^3*tan(d*x + c))/d
```

Sympy [A] time = 0.650289, size = 54, normalized size = 1.08

$$\begin{cases}\frac{a^3\tan^5(c+dx)}{5d}+\frac{2a^3\tan^3(c+dx)}{3d}+\frac{a^3\tan(c+dx)}{d} & \text{for } d\neq 0\\ x\left(a\tan^2(c)+a\right)^3 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*tan(c + d*x)**5/(5*d) + 2*a**3*tan(c + d*x)**3/(3*d) + a**3
*tan(c + d*x)/d, Ne(d, 0)), (x*(a*tan(c)**2 + a)**3, True))
```

Giac [B] time = 1.56029, size = 401, normalized size = 8.02

$$-\frac{15\,a^3\tan(dx)^5\tan(c)^4+15\,a^3\tan(dx)^4\tan(c)^5+10\,a^3\tan(dx)^5\tan(c)^2-30\,a^3\tan(dx)^4\tan(c)^3-30\,a^3\tan(dx)^3\,\mathrm{t}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/15*(15*a^3*tan(d*x)^5*tan(c)^4 + 15*a^3*tan(d*x)^4*tan(c)^5 + 10*a^3*tan
(d*x)^5*tan(c)^2 - 30*a^3*tan(d*x)^4*tan(c)^3 - 30*a^3*tan(d*x)^3*tan(c)^4
+ 10*a^3*tan(d*x)^2*tan(c)^5 + 3*a^3*tan(d*x)^5 - 5*a^3*tan(d*x)^4*tan(c) +
 60*a^3*tan(d*x)^3*tan(c)^2 + 60*a^3*tan(d*x)^2*tan(c)^3 - 5*a^3*tan(d*x)*t
an(c)^4 + 3*a^3*tan(c)^5 + 10*a^3*tan(d*x)^3 - 30*a^3*tan(d*x)^2*tan(c) - 3
0*a^3*tan(d*x)*tan(c)^2 + 10*a^3*tan(c)^3 + 15*a^3*tan(d*x) + 15*a^3*tan(c)
)/(d*tan(d*x)^5*tan(c)^5 - 5*d*tan(d*x)^4*tan(c)^4 + 10*d*tan(d*x)^3*tan(c)
^3 - 10*d*tan(d*x)^2*tan(c)^2 + 5*d*tan(d*x)*tan(c) - d)
```

3.181 $\int \left(a + a\tan^2(c+dx)\right)^2 \, dx$

Optimal. Leaf size=32

$$\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}$$

```
[Out] (a^2*Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0250517, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3657, 12, 3767}

$$\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3*d)
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+a\tan^2(c+dx)\right)^2 \, dx &= \int a^2\sec^4(c+dx)\,dx\\
&= a^2\int \sec^4(c+dx)\,dx\\
&= -\frac{a^2\,\mathrm{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\tan(c+dx)\right)}{d}\\
&= \frac{a^2\tan(c+dx)}{d}+\frac{a^2\tan^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.0394271, size = 26, normalized size = 0.81

$$\frac{a^2\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] time = 0.003, size = 25, normalized size = 0.8

$$\frac{a^2}{d}\left(\frac{(\tan(dx+c))^3}{3}+\tan(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*a^2*(1/3*tan(d*x+c)^3+tan(d*x+c))
```

Maxima [A] time = 1.49551, size = 80, normalized size = 2.5

$$a^2x+\frac{\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)a^2}{3\,d}-\frac{2\,(dx+c-\tan(dx+c))a^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^2/d - 2*(d*x
+ c - tan(d*x + c))*a^2/d
```

Fricas [A] time = 0.998854, size = 66, normalized size = 2.06

$$\frac{a^2\tan(dx+c)^3+3\,a^2\tan(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*tan(d*x + c)^3 + 3*a^2*tan(d*x + c))/d
```

Sympy [A] time = 0.32683, size = 37, normalized size = 1.16

$$\begin{cases}\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d} & \text{for } d\neq 0\\ x\left(a\tan^2(c)+a\right)^2 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*tan(c + d*x)**3/(3*d) + a**2*tan(c + d*x)/d, Ne(d, 0)), (x*
(a*tan(c)**2 + a)**2, True))
```

Giac [B] time = 1.34174, size = 180, normalized size = 5.62

$$-\frac{3\,a^2\tan(dx)^3\tan(c)^2+3\,a^2\tan(dx)^2\tan(c)^3+a^2\tan(dx)^3-3\,a^2\tan(dx)^2\tan(c)-3\,a^2\tan(dx)\tan(c)^2+a^2\,\mathrm{ta}}{3\left(d\tan(dx)^3\tan(c)^3-3\,d\tan(dx)^2\tan(c)^2+3\,d\tan(dx)\tan(c)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*a^2*tan(d*x)^3*tan(c)^2 + 3*a^2*tan(d*x)^2*tan(c)^3 + a^2*tan(d*x)^
3 - 3*a^2*tan(d*x)^2*tan(c) - 3*a^2*tan(d*x)*tan(c)^2 + a^2*tan(c)^3 + 3*a^
2*tan(d*x) + 3*a^2*tan(c))/(d*tan(d*x)^3*tan(c)^3 - 3*d*tan(d*x)^2*tan(c)^2
 + 3*d*tan(d*x)*tan(c) - d)
```

3.182 $\int \frac{1}{a+a\tan^2(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}+\frac{x}{2a}$$

```
[Out] x/(2*a) + (Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.0217207, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3657, 12, 2635, 8}

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}+\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(-1),x]
```

```
[Out] x/(2*a) + (Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+a\tan^2(c+dx)}\,dx &= \int \frac{\cos^2(c+dx)}{a}\,dx\\
&= \frac{\int \cos^2(c+dx)\,dx}{a}\\
&= \frac{\cos(c+dx)\sin(c+dx)}{2ad}+\frac{\int 1\,dx}{2a}\\
&= \frac{x}{2a}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}
\end{aligned}$$

Mathematica [A] time = 0.0259369, size = 26, normalized size = 0.84

$$\frac{2(c+dx)+\sin(2(c+dx))}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(-1),x]
```

```
[Out] (2*(c + d*x) + Sin[2*(c + d*x)])/(4*a*d)
```

Maple [A] time = 0.018, size = 43, normalized size = 1.4

$$\frac{\tan(dx+c)}{2\,ad\left((\tan(dx+c))^2+1\right)}+\frac{\arctan(\tan(dx+c))}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tan(d*x+c)^2),x)
```

```
[Out] 1/2/a/d*tan(d*x+c)/(tan(d*x+c)^2+1)+1/2/a/d*arctan(tan(d*x+c))
```

Maxima [A] time = 1.54475, size = 49, normalized size = 1.58

$$\frac{\frac{dx+c}{a}+\frac{\tan(dx+c)}{a\tan(dx+c)^2+a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((d*x + c)/a + tan(d*x + c)/(a*tan(d*x + c)^2 + a))/d
```

Fricas [A] time = 1.02715, size = 100, normalized size = 3.23

$$\frac{dx\tan(dx+c)^2+dx+\tan(dx+c)}{2\left(ad\tan(dx+c)^2+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(d*x*tan(d*x + c)^2 + d*x + tan(d*x + c))/(a*d*tan(d*x + c)^2 + a*d)
```

Sympy [A] time = 0.593096, size = 87, normalized size = 2.81

$$\begin{cases} \frac{dx\tan^2(c+dx)}{2ad\tan^2(c+dx)+2ad}+\frac{dx}{2ad\tan^2(c+dx)+2ad}+\frac{\tan(c+dx)}{2ad\tan^2(c+dx)+2ad} & \text{for } d\neq 0 \\ \frac{x}{a\tan^2(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((d*x*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**2 + 2*a*d) + d*x/(2*a*d
*tan(c + d*x)**2 + 2*a*d) + tan(c + d*x)/(2*a*d*tan(c + d*x)**2 + 2*a*d), N
e(d, 0)), (x/(a*tan(c)**2 + a), True))
```

Giac [A] time = 1.29782, size = 50, normalized size = 1.61

$$\frac{\frac{dx+c}{a} + \frac{\tan(dx+c)}{(\tan(dx+c)^2+1)a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*((d*x + c)/a + tan(d*x + c)/((tan(d*x + c)^2 + 1)*a))/d
```

3.183 $\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=55

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}+\frac{3\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{3x}{8a^2}$$

```
[Out] (3*x)/(8*a^2) + (3*Cos[c + d*x]*Sin[c + d*x])/(8*a^2*d) + (Cos[c + d*x]^3*S
in[c + d*x])/(4*a^2*d)
```

Rubi [A] time = 0.0336857, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3657, 12, 2635, 8}

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}+\frac{3\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{3x}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(-2),x]
```

```
[Out] (3*x)/(8*a^2) + (3*Cos[c + d*x]*Sin[c + d*x])/(8*a^2*d) + (Cos[c + d*x]^3*S
in[c + d*x])/(4*a^2*d)
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{(a+a\tan^2(c+dx))^2}\,dx = \int \frac{\cos^4(c+dx)}{a^2}\,dx$$
$$= \frac{\int \cos^4(c+dx)\,dx}{a^2}$$
$$= \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d} + \frac{3\int \cos^2(c+dx)\,dx}{4a^2}$$
$$= \frac{3\cos(c+dx)\sin(c+dx)}{8a^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d} + \frac{3\int 1\,dx}{8a^2}$$
$$= \frac{3x}{8a^2} + \frac{3\cos(c+dx)\sin(c+dx)}{8a^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d}$$

Mathematica [A] time = 0.0443636, size = 36, normalized size = 0.65

$$\frac{12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx))}{32a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(-2),x]
```

```
[Out] (12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)])/(32*a^2*d)
```

Maple [A] time = 0.017, size = 69, normalized size = 1.3

$$\frac{\tan(dx+c)}{4\,da^2\left((\tan(dx+c))^2+1\right)^2} + \frac{3\,\tan(dx+c)}{8\,da^2\left((\tan(dx+c))^2+1\right)} + \frac{3\,\arctan(\tan(dx+c))}{8\,da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/4/d/a^2*tan(d*x+c)/(tan(d*x+c)^2+1)^2+3/8/d/a^2*tan(d*x+c)/(tan(d*x+c)^2+
1)+3/8/d/a^2*arctan(tan(d*x+c))
```

Maxima [A] time = 1.49575, size = 90, normalized size = 1.64

$$\frac{\frac{3\,\tan(dx+c)^3+5\,\tan(dx+c)}{a^2\tan(dx+c)^4+2\,a^2\tan(dx+c)^2+a^2} + \frac{3\,(dx+c)}{a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*((3*tan(d*x + c)^3 + 5*tan(d*x + c))/(a^2*tan(d*x + c)^4 + 2*a^2*tan(d*
x + c)^2 + a^2) + 3*(d*x + c)/a^2)/d
```

Fricas [A] time = 1.04385, size = 204, normalized size = 3.71

$$\frac{3\,dx\tan\left(dx+c\right)^4+6\,dx\tan\left(dx+c\right)^2+3\,\tan\left(dx+c\right)^3+3\,dx+5\,\tan\left(dx+c\right)}{8\left(a^2d\tan\left(dx+c\right)^4+2\,a^2d\tan\left(dx+c\right)^2+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(3*d*x*tan(d*x + c)^4 + 6*d*x*tan(d*x + c)^2 + 3*tan(d*x + c)^3 + 3*d*x
 + 5*tan(d*x + c))/(a^2*d*tan(d*x + c)^4 + 2*a^2*d*tan(d*x + c)^2 + a^2*d)
```

Sympy [A] time = 1.05706, size = 248, normalized size = 4.51

$$\begin{cases}\frac{3dx\tan^4(c+dx)}{8a^2d\tan^4(c+dx)+16a^2d\tan^2(c+dx)+8a^2d}+\frac{6dx\tan^2(c+dx)}{8a^2d\tan^4(c+dx)+16a^2d\tan^2(c+dx)+8a^2d}+\frac{3dx}{8a^2d\tan^4(c+dx)+16a^2d\tan^2(c+dx)+8a^2d}+\frac{}{8a^2d\tan^4(} \\ \frac{x}{\left(a\tan^2(c)+a\right)^2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((3*d*x*tan(c + d*x)**4/(8*a**2*d*tan(c + d*x)**4 + 16*a**2*d*tan(
c + d*x)**2 + 8*a**2*d) + 6*d*x*tan(c + d*x)**2/(8*a**2*d*tan(c + d*x)**4 +
 16*a**2*d*tan(c + d*x)**2 + 8*a**2*d) + 3*d*x/(8*a**2*d*tan(c + d*x)**4 +
16*a**2*d*tan(c + d*x)**2 + 8*a**2*d) + 3*tan(c + d*x)**3/(8*a**2*d*tan(c +
 d*x)**4 + 16*a**2*d*tan(c + d*x)**2 + 8*a**2*d) + 5*tan(c + d*x)/(8*a**2*d
*tan(c + d*x)**4 + 16*a**2*d*tan(c + d*x)**2 + 8*a**2*d), Ne(d, 0)), (x/(a*
tan(c)**2 + a)**2, True))
```

Giac [A] time = 1.29306, size = 69, normalized size = 1.25

$$\frac{\frac{3\,(dx+c)}{a^2}+\frac{3\,\tan(dx+c)^3+5\,\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)^2a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*(d*x + c)/a^2 + (3*tan(d*x + c)^3 + 5*tan(d*x + c))/((tan(d*x + c)^2
 + 1)^2*a^2))/d
```

3.184 $\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=79

$$\frac{\sin(c+dx)\cos^5(c+dx)}{6a^3d}+\frac{5\sin(c+dx)\cos^3(c+dx)}{24a^3d}+\frac{5\sin(c+dx)\cos(c+dx)}{16a^3d}+\frac{5x}{16a^3}$$

```
[Out] (5*x)/(16*a^3) + (5*Cos[c + d*x]*Sin[c + d*x])/(16*a^3*d) + (5*Cos[c + d*x]
^3*Sin[c + d*x])/(24*a^3*d) + (Cos[c + d*x]^5*Sin[c + d*x])/(6*a^3*d)
```

Rubi [A] time = 0.0479145, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3657, 12, 2635, 8}

$$\frac{\sin(c+dx)\cos^5(c+dx)}{6a^3d}+\frac{5\sin(c+dx)\cos^3(c+dx)}{24a^3d}+\frac{5\sin(c+dx)\cos(c+dx)}{16a^3d}+\frac{5x}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(-3),x]
```


```
[Out] (5*x)/(16*a^3) + (5*Cos[c + d*x]*Sin[c + d*x])/(16*a^3*d) + (5*Cos[c + d*x]
^3*Sin[c + d*x])/(24*a^3*d) + (Cos[c + d*x]^5*Sin[c + d*x])/(6*a^3*d)
```


Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^3}\,dx = \int \frac{\cos^6(c+dx)}{a^3}\,dx$$
$$= \frac{\int \cos^6(c+dx)\,dx}{a^3}$$
$$= \frac{\cos^5(c+dx)\sin(c+dx)}{6a^3d} + \frac{5\int \cos^4(c+dx)\,dx}{6a^3}$$
$$= \frac{5\cos^3(c+dx)\sin(c+dx)}{24a^3d} + \frac{\cos^5(c+dx)\sin(c+dx)}{6a^3d} + \frac{5\int \cos^2(c+dx)\,dx}{8a^3}$$
$$= \frac{5\cos(c+dx)\sin(c+dx)}{16a^3d} + \frac{5\cos^3(c+dx)\sin(c+dx)}{24a^3d} + \frac{\cos^5(c+dx)\sin(c+dx)}{6a^3d} + \frac{5}{1}$$
$$= \frac{5x}{16a^3} + \frac{5\cos(c+dx)\sin(c+dx)}{16a^3d} + \frac{5\cos^3(c+dx)\sin(c+dx)}{24a^3d} + \frac{\cos^5(c+dx)\sin(c+d}{6a^3d}$$

Mathematica [A] time = 0.0421645, size = 46, normalized size = 0.58

$$\frac{45\sin(2(c+dx)) + 9\sin(4(c+dx)) + \sin(6(c+dx)) + 60c + 60dx}{192a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(-3),x]
```

```
[Out] (60*c + 60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x)] + Sin[6*(c + d*x)
])/(192*a^3*d)
```

Maple [A] time = 0.016, size = 95, normalized size = 1.2

$$\frac{\tan(dx+c)}{6\,da^3\left((\tan(dx+c))^2+1\right)^3} + \frac{5\,\tan(dx+c)}{24\,da^3\left((\tan(dx+c))^2+1\right)^2} + \frac{5\,\tan(dx+c)}{16\,da^3\left((\tan(dx+c))^2+1\right)} + \frac{5\,\arctan(\tan(dx+c))}{16\,da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tan(d*x+c)^2)^3,x)
```

```
[Out] 1/6/d/a^3*tan(d*x+c)/(tan(d*x+c)^2+1)^3+5/24/d/a^3*tan(d*x+c)/(tan(d*x+c)^2
+1)^2+5/16/d/a^3*tan(d*x+c)/(tan(d*x+c)^2+1)+5/16/d/a^3*arctan(tan(d*x+c))
```

Maxima [A] time = 1.70905, size = 122, normalized size = 1.54

$$\frac{\frac{15\,\tan(dx+c)^5+40\,\tan(dx+c)^3+33\,\tan(dx+c)}{a^3\tan(dx+c)^6+3\,a^3\tan(dx+c)^4+3\,a^3\tan(dx+c)^2+a^3} + \frac{15\,(dx+c)}{a^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/48*((15*tan(d*x + c)^5 + 40*tan(d*x + c)^3 + 33*tan(d*x + c))/(a^3*tan(d*
x + c)^6 + 3*a^3*tan(d*x + c)^4 + 3*a^3*tan(d*x + c)^2 + a^3) + 15*(d*x + c
```

```
)/a^3)/d
```

Fricas [A] time = 1.07574, size = 305, normalized size = 3.86

$$\frac{15\,dx\tan\left(dx+c\right)^{6}+45\,dx\tan\left(dx+c\right)^{4}+15\,\tan\left(dx+c\right)^{5}+45\,dx\tan\left(dx+c\right)^{2}+40\,\tan\left(dx+c\right)^{3}+15\,dx+33\,\tan\left(\ldots\right.}{48\left(a^{3}d\tan\left(dx+c\right)^{6}+3\,a^{3}d\tan\left(dx+c\right)^{4}+3\,a^{3}d\tan\left(dx+c\right)^{2}+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(15*d*x*tan(d*x + c)^6 + 45*d*x*tan(d*x + c)^4 + 15*tan(d*x + c)^5 + 4
5*d*x*tan(d*x + c)^2 + 40*tan(d*x + c)^3 + 15*d*x + 33*tan(d*x + c))/(a^3*d
*tan(d*x + c)^6 + 3*a^3*d*tan(d*x + c)^4 + 3*a^3*d*tan(d*x + c)^2 + a^3*d)
```

Sympy [A] time = 1.80923, size = 454, normalized size = 5.75

$$\begin{cases}\frac{15dx\tan^{6}(c+dx)}{48a^{3}d\tan^{6}(c+dx)+144a^{3}d\tan^{4}(c+dx)+144a^{3}d\tan^{2}(c+dx)+48a^{3}d}+\frac{45dx\tan^{4}(c+dx)}{48a^{3}d\tan^{6}(c+dx)+144a^{3}d\tan^{4}(c+dx)+144a^{3}d\tan^{2}(c+dx)+48a^{3}d}+\frac{\ldots}{48a^{3}d\tan^{6}(c\ldots} \\ \frac{x}{\left(a\tan^{2}(c)+a\right)^{3}}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((15*d*x*tan(c + d*x)**6/(48*a**3*d*tan(c + d*x)**6 + 144*a**3*d*t
an(c + d*x)**4 + 144*a**3*d*tan(c + d*x)**2 + 48*a**3*d) + 45*d*x*tan(c + d
*x)**4/(48*a**3*d*tan(c + d*x)**6 + 144*a**3*d*tan(c + d*x)**4 + 144*a**3*d
*tan(c + d*x)**2 + 48*a**3*d) + 45*d*x*tan(c + d*x)**2/(48*a**3*d*tan(c + d
*x)**6 + 144*a**3*d*tan(c + d*x)**4 + 144*a**3*d*tan(c + d*x)**2 + 48*a**3*
d) + 15*d*x/(48*a**3*d*tan(c + d*x)**6 + 144*a**3*d*tan(c + d*x)**4 + 144*a
**3*d*tan(c + d*x)**2 + 48*a**3*d) + 15*tan(c + d*x)**5/(48*a**3*d*tan(c +
d*x)**6 + 144*a**3*d*tan(c + d*x)**4 + 144*a**3*d*tan(c + d*x)**2 + 48*a**3
*d) + 40*tan(c + d*x)**3/(48*a**3*d*tan(c + d*x)**6 + 144*a**3*d*tan(c + d*
x)**4 + 144*a**3*d*tan(c + d*x)**2 + 48*a**3*d) + 33*tan(c + d*x)/(48*a**3*
d*tan(c + d*x)**6 + 144*a**3*d*tan(c + d*x)**4 + 144*a**3*d*tan(c + d*x)**2
 + 48*a**3*d), Ne(d, 0)), (x/(a*tan(c)**2 + a)**3, True))
```

Giac [A] time = 1.28966, size = 82, normalized size = 1.04

$$\frac{\frac{15\,(dx+c)}{a^{3}}+\frac{15\,\tan(dx+c)^{5}+40\,\tan(dx+c)^{3}+33\,\tan(dx+c)}{\left(\tan(dx+c)^{2}+1\right)^{3}a^{3}}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/48*(15*(d*x + c)/a^3 + (15*tan(d*x + c)^5 + 40*tan(d*x + c)^3 + 33*tan(d*
x + c))/((tan(d*x + c)^2 + 1)^3*a^3))/d
```

3.185 $\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=74

$$\frac{(a-b)\tan^4(e+fx)}{4f}-\frac{(a-b)\tan^2(e+fx)}{2f}-\frac{(a-b)\log(\cos(e+fx))}{f}+\frac{b\tan^6(e+fx)}{6f}$$

```
[Out] -(((a - b)*Log[Cos[e + f*x]])/f) - ((a - b)*Tan[e + f*x]^2)/(2*f) + ((a - b
)*Tan[e + f*x]^4)/(4*f) + (b*Tan[e + f*x]^6)/(6*f)
```

Rubi [A] time = 0.0493532, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3631, 3473, 3475}

$$\frac{(a-b)\tan^4(e+fx)}{4f}-\frac{(a-b)\tan^2(e+fx)}{2f}-\frac{(a-b)\log(\cos(e+fx))}{f}+\frac{b\tan^6(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - b)*Log[Cos[e + f*x]])/f) - ((a - b)*Tan[e + f*x]^2)/(2*f) + ((a - b
)*Tan[e + f*x]^4)/(4*f) + (b*Tan[e + f*x]^6)/(6*f)
```

Rule 3631

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(
m + 1)), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= \frac{b\tan^6(e+fx)}{6f}+(a-b)\int \tan^5(e+fx)\,dx\\
&= \frac{(a-b)\tan^4(e+fx)}{4f}+\frac{b\tan^6(e+fx)}{6f}+(-a+b)\int \tan^3(e+fx)\,dx\\
&= -\frac{(a-b)\tan^2(e+fx)}{2f}+\frac{(a-b)\tan^4(e+fx)}{4f}+\frac{b\tan^6(e+fx)}{6f}+(a-b)\int\\
&= -\frac{(a-b)\log(\cos(e+fx))}{f}-\frac{(a-b)\tan^2(e+fx)}{2f}+\frac{(a-b)\tan^4(e+fx)}{4f}+
\end{aligned}$$

Mathematica [A] time = 0.224886, size = 63, normalized size = 0.85

$$\frac{3(a-b)\tan^4(e+fx)-6(a-b)\tan^2(e+fx)+12(b-a)\log(\cos(e+fx))+2b\tan^6(e+fx)}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (12*(-a + b)*Log[Cos[e + f*x]] - 6*(a - b)*Tan[e + f*x]^2 + 3*(a - b)*Tan[e
 + f*x]^4 + 2*b*Tan[e + f*x]^6)/(12*f)
```

Maple [A] time = 0.006, size = 106, normalized size = 1.4

$$\frac{b\left(\tan\left(fx+e\right)\right)^{6}}{6f}+\frac{\left(\tan\left(fx+e\right)\right)^{4}a}{4f}-\frac{b\left(\tan\left(fx+e\right)\right)^{4}}{4f}-\frac{\left(\tan\left(fx+e\right)\right)^{2}a}{2f}+\frac{b\left(\tan\left(fx+e\right)\right)^{2}}{2f}+\frac{\ln\left(1+\left(\tan\left(fx+\right.\right.\right.}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/6*b*tan(f*x+e)^6/f+1/4/f*tan(f*x+e)^4*a-1/4*b*tan(f*x+e)^4/f-1/2/f*tan(f*
x+e)^2*a+1/2*b*tan(f*x+e)^2/f+1/2/f*ln(1+tan(f*x+e)^2)*a-1/2/f*ln(1+tan(f*x
+e)^2)*b
```

Maxima [A] time = 1.14392, size = 134, normalized size = 1.81

$$-\frac{6\,(a-b)\log\left(\sin\left(fx+e\right)^{2}-1\right)-\frac{6\,(2\,a-3\,b)\sin\left(fx+e\right)^{4}-3\,(7\,a-9\,b)\sin\left(fx+e\right)^{2}+9\,a-11\,b}{\sin\left(fx+e\right)^{6}-3\,\sin\left(fx+e\right)^{4}+3\,\sin\left(fx+e\right)^{2}-1}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/12*(6*(a - b)*log(sin(f*x + e)^2 - 1) - (6*(2*a - 3*b)*sin(f*x + e)^4 -
3*(7*a - 9*b)*sin(f*x + e)^2 + 9*a - 11*b)/(sin(f*x + e)^6 - 3*sin(f*x + e)
^4 + 3*sin(f*x + e)^2 - 1))/f
```

Fricas [A] time = 1.10308, size = 166, normalized size = 2.24

$$\frac{2\,b\tan\left(fx+e\right)^{6}+3\,(a-b)\tan\left(fx+e\right)^{4}-6\,(a-b)\tan\left(fx+e\right)^{2}-6\,(a-b)\log\left(\frac{1}{\tan\left(fx+e\right)^{2}+1}\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b*tan(f*x + e)^6 + 3*(a - b)*tan(f*x + e)^4 - 6*(a - b)*tan(f*x + e
)^2 - 6*(a - b)*log(1/(tan(f*x + e)^2 + 1)))/f
```

Sympy [A] time = 0.884802, size = 116, normalized size = 1.57

$$\begin{cases} \frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} + \frac{a\tan^4\left(e+fx\right)}{4f} - \frac{a\tan^2\left(e+fx\right)}{2f} - \frac{b\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} + \frac{b\tan^6\left(e+fx\right)}{6f} - \frac{b\tan^4\left(e+fx\right)}{4f} + \frac{b\tan^2\left(e+fx\right)}{2f} & \text{for } f \neq \\ x\left(a+b\tan^2\left(e\right)\right)\tan^5\left(e\right) & \text{otherv} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**4/(4*f) - a*t
an(e + f*x)**2/(2*f) - b*log(tan(e + f*x)**2 + 1)/(2*f) + b*tan(e + f*x)**6
/(6*f) - b*tan(e + f*x)**4/(4*f) + b*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(
a + b*tan(e)**2)*tan(e)**5, True))
```

Giac [B] time = 7.18547, size = 2321, normalized size = 31.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/12*(6*a*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
 + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^6*tan
(e)^6 - 6*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
 + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^6*ta
n(e)^6 + 9*a*tan(f*x)^6*tan(e)^6 - 11*b*tan(f*x)^6*tan(e)^6 - 36*a*log(4*(t
an(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)
^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^5*tan(e)^5 + 36*b*log(4*
(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(
e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^5*tan(e)^5 + 6*a*tan(f
*x)^6*tan(e)^4 - 6*b*tan(f*x)^6*tan(e)^4 - 42*a*tan(f*x)^5*tan(e)^5 + 54*b*
tan(f*x)^5*tan(e)^5 + 6*a*tan(f*x)^4*tan(e)^6 - 6*b*tan(f*x)^4*tan(e)^6 + 9
0*a*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f
*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(e)^4 -
 90*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan
(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(e)^4
 - 3*a*tan(f*x)^6*tan(e)^2 + 3*b*tan(f*x)^6*tan(e)^2 - 36*a*tan(f*x)^5*tan(
e)^3 + 36*b*tan(f*x)^5*tan(e)^3 + 69*a*tan(f*x)^4*tan(e)^4 - 99*b*tan(f*x)^
4*tan(e)^4 - 36*a*tan(f*x)^3*tan(e)^5 + 36*b*tan(f*x)^3*tan(e)^5 - 3*a*tan(
f*x)^2*tan(e)^6 + 3*b*tan(f*x)^2*tan(e)^6 - 120*a*log(4*(tan(e)^2 + 1)/(tan
(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 -
 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 + 120*b*log(4*(tan(e)^2 + 1)/(
tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^
2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 - 2*b*tan(f*x)^6 + 6*a*tan(
f*x)^5*tan(e) - 18*b*tan(f*x)^5*tan(e) + 60*a*tan(f*x)^4*tan(e)^2 - 90*b*ta
n(f*x)^4*tan(e)^2 - 72*a*tan(f*x)^3*tan(e)^3 + 72*b*tan(f*x)^3*tan(e)^3 + 6
0*a*tan(f*x)^2*tan(e)^4 - 90*b*tan(f*x)^2*tan(e)^4 + 6*a*tan(f*x)*tan(e)^5
- 18*b*tan(f*x)*tan(e)^5 - 2*b*tan(e)^6 + 90*a*log(4*(tan(e)^2 + 1)/(tan(f*
x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*
tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 - 90*b*log(4*(tan(e)^2 + 1)/(tan(
```

```
f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 -
2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 - 3*a*tan(f*x)^4 + 3*b*tan(f*x)
^4 - 36*a*tan(f*x)^3*tan(e) + 36*b*tan(f*x)^3*tan(e) + 69*a*tan(f*x)^2*tan(
e)^2 - 99*b*tan(f*x)^2*tan(e)^2 - 36*a*tan(f*x)*tan(e)^3 + 36*b*tan(f*x)*ta
n(e)^3 - 3*a*tan(e)^4 + 3*b*tan(e)^4 - 36*a*log(4*(tan(e)^2 + 1)/(tan(f*x)^
4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan
(f*x)*tan(e) + 1))*tan(f*x)*tan(e) + 36*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*
tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f
*x)*tan(e) + 1))*tan(f*x)*tan(e) + 6*a*tan(f*x)^2 - 6*b*tan(f*x)^2 - 42*a*t
an(f*x)*tan(e) + 54*b*tan(f*x)*tan(e) + 6*a*tan(e)^2 - 6*b*tan(e)^2 + 6*a*l
og(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2
*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) - 6*b*log(4*(tan(e)^2 + 1)
/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x
)^2 - 2*tan(f*x)*tan(e) + 1)) + 9*a - 11*b)/(f*tan(f*x)^6*tan(e)^6 - 6*f*ta
n(f*x)^5*tan(e)^5 + 15*f*tan(f*x)^4*tan(e)^4 - 20*f*tan(f*x)^3*tan(e)^3 + 1
5*f*tan(f*x)^2*tan(e)^2 - 6*f*tan(f*x)*tan(e) + f)
```

3.186 $\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)dx$

Optimal. Leaf size=53

$$\frac{(a-b)\tan^2(e+fx)}{2f}+\frac{(a-b)\log(\cos(e+fx))}{f}+\frac{b\tan^4(e+fx)}{4f}$$

```
[Out] ((a - b)*Log[Cos[e + f*x]])/f + ((a - b)*Tan[e + f*x]^2)/(2*f) + (b*Tan[e +
 f*x]^4)/(4*f)
```

Rubi [A] time = 0.0387884, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3631, 3473, 3475}

$$\frac{(a-b)\tan^2(e+fx)}{2f}+\frac{(a-b)\log(\cos(e+fx))}{f}+\frac{b\tan^4(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((a - b)*Log[Cos[e + f*x]])/f + ((a - b)*Tan[e + f*x]^2)/(2*f) + (b*Tan[e +
 f*x]^4)/(4*f)
```

Rule 3631

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(
m + 1)), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)dx &= \frac{b\tan^4(e+fx)}{4f}+(a-b)\int \tan^3(e+fx)\,dx\\ &= \frac{(a-b)\tan^2(e+fx)}{2f}+\frac{b\tan^4(e+fx)}{4f}+(-a+b)\int \tan(e+fx)\,dx\\ &= \frac{(a-b)\log(\cos(e+fx))}{f}+\frac{(a-b)\tan^2(e+fx)}{2f}+\frac{b\tan^4(e+fx)}{4f}\end{aligned}$$

Mathematica [A] time = 0.163594, size = 65, normalized size = 1.23

$$\frac{a\left(\tan^2(e+fx)+2\log(\cos(e+fx))\right)}{2f}-\frac{b\left(-\tan^4(e+fx)+2\tan^2(e+fx)+4\log(\cos(e+fx))\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (a*(2*Log[Cos[e + f*x]] + Tan[e + f*x]^2))/(2*f) - (b*(4*Log[Cos[e + f*x]]
+ 2*Tan[e + f*x]^2 - Tan[e + f*x]^4))/(4*f)
```

Maple [A] time = 0.003, size = 78, normalized size = 1.5

$$\frac{b\left(\tan\left(fx+e\right)\right)^{4}}{4f}+\frac{\left(\tan\left(fx+e\right)\right)^{2}a}{2f}-\frac{b\left(\tan\left(fx+e\right)\right)^{2}}{2f}-\frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^{2}\right)a}{2f}+\frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^{2}\right)b}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/4*b*tan(f*x+e)^4/f+1/2/f*tan(f*x+e)^2*a-1/2*b*tan(f*x+e)^2/f-1/2/f*ln(1+t
an(f*x+e)^2)*a+1/2/f*ln(1+tan(f*x+e)^2)*b
```

Maxima [A] time = 1.14193, size = 95, normalized size = 1.79

$$\frac{2\,(a-b)\log\left(\sin\left(fx+e\right)^{2}-1\right)-\frac{2\,(a-2\,b)\sin\left(fx+e\right)^{2}-2\,a+3\,b}{\sin\left(fx+e\right)^{4}-2\,\sin\left(fx+e\right)^{2}+1}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a - b)*log(sin(f*x + e)^2 - 1) - (2*(a - 2*b)*sin(f*x + e)^2 - 2*a
+ 3*b)/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

Fricas [A] time = 1.10426, size = 126, normalized size = 2.38

$$\frac{b\tan\left(fx+e\right)^{4}+2\,(a-b)\tan\left(fx+e\right)^{2}+2\,(a-b)\log\left(\frac{1}{\tan\left(fx+e\right)^{2}+1}\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(b*tan(f*x + e)^4 + 2*(a - b)*tan(f*x + e)^2 + 2*(a - b)*log(1/(tan(f*x
 + e)^2 + 1)))/f
```

Sympy [A] time = 0.479477, size = 88, normalized size = 1.66

$$\begin{cases} -\frac{a\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^{2}\left(e+fx\right)}{2f}+\frac{b\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{b\tan^{4}\left(e+fx\right)}{4f}-\frac{b\tan^{2}\left(e+fx\right)}{2f} & \text{for}\ f\neq 0\\ x\left(a+b\tan^{2}\left(e\right)\right)\tan^{3}\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((-a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**2/(2*f) + b*
log(tan(e + f*x)**2 + 1)/(2*f) + b*tan(e + f*x)**4/(4*f) - b*tan(e + f*x)**
2/(2*f), Ne(f, 0)), (x*(a + b*tan(e)**2)*tan(e)**3, True))
```

Giac [B] time = 2.99459, size = 1446, normalized size = 27.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/4*(2*a*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) +
tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(e
)^4 - 2*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) +
 tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(
e)^4 + 2*a*tan(f*x)^4*tan(e)^4 - 3*b*tan(f*x)^4*tan(e)^4 - 8*a*log(4*(tan(e
)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 +
 tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 + 8*b*log(4*(tan(
e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2
+ tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 + 2*a*tan(f*x)^4
*tan(e)^2 - 2*b*tan(f*x)^4*tan(e)^2 - 4*a*tan(f*x)^3*tan(e)^3 + 8*b*tan(f*x
)^3*tan(e)^3 + 2*a*tan(f*x)^2*tan(e)^4 - 2*b*tan(f*x)^2*tan(e)^4 + 12*a*log
(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*t
an(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 - 12*b*l
og(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2
*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 + b*ta
n(f*x)^4 - 4*a*tan(f*x)^3*tan(e) + 8*b*tan(f*x)^3*tan(e) + 4*a*tan(f*x)^2*t
an(e)^2 - 4*b*tan(f*x)^2*tan(e)^2 - 4*a*tan(f*x)*tan(e)^3 + 8*b*tan(f*x)*ta
n(e)^3 + b*tan(e)^4 - 8*a*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan
(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))
*tan(f*x)*tan(e) + 8*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*
x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*ta
n(f*x)*tan(e) + 2*a*tan(f*x)^2 - 2*b*tan(f*x)^2 - 4*a*tan(f*x)*tan(e) + 8*b
*tan(f*x)*tan(e) + 2*a*tan(e)^2 - 2*b*tan(e)^2 + 2*a*log(4*(tan(e)^2 + 1)/(
tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^
2 - 2*tan(f*x)*tan(e) + 1)) - 2*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2
 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(
e) + 1)) + 2*a - 3*b)/(f*tan(f*x)^4*tan(e)^4 - 4*f*tan(f*x)^3*tan(e)^3 + 6*
f*tan(f*x)^2*tan(e)^2 - 4*f*tan(f*x)*tan(e) + f)
```

3.187 $\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=34

$$\frac{b\tan^2(e+fx)}{2f}-\frac{(a-b)\log(\cos(e+fx))}{f}$$

```
[Out] -(((a - b)*Log[Cos[e + f*x]])/f) + (b*Tan[e + f*x]^2)/(2*f)
```

Rubi [A] time = 0.0221714, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3631, 3475}

$$\frac{b\tan^2(e+fx)}{2f}-\frac{(a-b)\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - b)*Log[Cos[e + f*x]])/f) + (b*Tan[e + f*x]^2)/(2*f)
```

Rule 3631

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(
m + 1)), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx&=\frac{b\tan^2(e+fx)}{2f}+(a-b)\int \tan(e+fx)\,dx\\&=-\frac{(a-b)\log(\cos(e+fx))}{f}+\frac{b\tan^2(e+fx)}{2f}\end{aligned}$$

Mathematica [A] time = 0.0685205, size = 40, normalized size = 1.18

$$\frac{b\left(\tan^2(e+fx)+2\log(\cos(e+fx))\right)}{2f}-\frac{a\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*Log[Cos[e + f*x]])/f) + (b*(2*Log[Cos[e + f*x]] + Tan[e + f*x]^2))/(2*
f)
```

Maple [A] time = 0.002, size = 50, normalized size = 1.5

$$\frac{b\left(\tan\left(fx+e\right)\right)^{2}}{2f}+\frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^{2}\right)a}{2f}-\frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^{2}\right)b}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/2*b*tan(f*x+e)^2/f+1/2/f*ln(1+tan(f*x+e)^2)*a-1/2/f*ln(1+tan(f*x+e)^2)*b
```

Maxima [A] time = 1.18472, size = 50, normalized size = 1.47

$$-\frac{(a-b)\log\left(\sin\left(fx+e\right)^{2}-1\right)+\frac{b}{\sin\left(fx+e\right)^{2}-1}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*((a - b)*log(sin(f*x + e)^2 - 1) + b/(sin(f*x + e)^2 - 1))/f
```

Fricas [A] time = 1.04847, size = 86, normalized size = 2.53

$$\frac{b\tan\left(fx+e\right)^{2}-(a-b)\log\left(\frac{1}{\tan\left(fx+e\right)^{2}+1}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*tan(f*x + e)^2 - (a - b)*log(1/(tan(f*x + e)^2 + 1)))/f
```

Sympy [A] time = 0.223503, size = 60, normalized size = 1.76

$$\begin{cases}\frac{a\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}-\frac{b\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{b\tan^{2}\left(e+fx\right)}{2f} & \text{for } f\neq 0\\ x\left(a+b\tan^{2}\left(e\right)\right)\tan\left(e\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) - b*log(tan(e + f*x)**2 + 1)/(2
*f) + b*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(a + b*tan(e)**2)*tan(e), True
))
```

Giac [B] time = 1.53251, size = 675, normalized size = 19.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/2*(a*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + t
an(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)
^2 - b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + ta
n(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^
2 - b*tan(f*x)^2*tan(e)^2 - 2*a*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 -
 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e)
 + 1))*tan(f*x)*tan(e) + 2*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*
tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) +
1))*tan(f*x)*tan(e) - b*tan(f*x)^2 - b*tan(e)^2 + a*log(4*(tan(e)^2 + 1)/(t
an(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2
 - 2*tan(f*x)*tan(e) + 1)) - b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 -
2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e)
+ 1)) - b)/(f*tan(f*x)^2*tan(e)^2 - 2*f*tan(f*x)*tan(e) + f)
```

3.188 $\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=26

$$\frac{a\log(\sin(e+fx))}{f}-\frac{b\log(\cos(e+fx))}{f}$$

```
[Out] -((b*Log[Cos[e + f*x]])/f) + (a*Log[Sin[e + f*x]])/f
```

Rubi [A] time = 0.0292719, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3625, 3475}

$$\frac{a\log(\sin(e+fx))}{f}-\frac{b\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((b*Log[Cos[e + f*x]])/f) + (a*Log[Sin[e + f*x]])/f
```

Rule 3625

```
Int[((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/tan[(e_.) + (f_.)*(x_)], x_Sym
bol] :> Dist[A, Int[1/Tan[e + f*x], x], x] + Dist[C, Int[Tan[e + f*x], x],
x] /; FreeQ[{e, f, A, C}, x] && NeQ[A, C]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= a\int \cot(e+fx)\,dx+b\int \tan(e+fx)\,dx\\ &= -\frac{b\log(\cos(e+fx))}{f}+\frac{a\log(\sin(e+fx))}{f}\end{aligned}$$

Mathematica [A] time = 0.039285, size = 34, normalized size = 1.31

$$\frac{a(\log(\tan(e+fx))+\log(\cos(e+fx)))}{f}-\frac{b\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((b*Log[Cos[e + f*x]])/f) + (a*(Log[Cos[e + f*x]] + Log[Tan[e + f*x]]))/f
```

Maple [A] time = 0.043, size = 27, normalized size = 1.

$$-\frac{b\ln\left(\cos\left(fx+e\right)\right)}{f}+\frac{a\ln\left(\sin\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*tan(f*x+e)^2),x)
```

```
[Out] -b*ln(cos(f*x+e))/f+a*ln(sin(f*x+e))/f
```

Maxima [A] time = 1.07368, size = 42, normalized size = 1.62

$$-\frac{b\log\left(\sin\left(fx+e\right)^{2}-1\right)-a\log\left(\sin\left(fx+e\right)^{2}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(b*log(sin(f*x + e)^2 - 1) - a*log(sin(f*x + e)^2))/f
```

Fricas [A] time = 1.12898, size = 113, normalized size = 4.35

$$\frac{a\log\left(\frac{\tan\left(fx+e\right)^{2}}{\tan\left(fx+e\right)^{2}+1}\right)-b\log\left(\frac{1}{\tan\left(fx+e\right)^{2}+1}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) - b*log(1/(tan(f*x + e)^2 +
 1)))/f
```

Sympy [A] time = 0.627207, size = 58, normalized size = 2.23

$$\begin{cases}-\frac{a\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{a\log\left(\tan\left(e+fx\right)\right)}{f}+\frac{b\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f} & \text{for}\ f\neq 0\\ x\left(a+b\tan^{2}\left(e\right)\right)\cot\left(e\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((-a*log(tan(e + f*x)**2 + 1)/(2*f) + a*log(tan(e + f*x))/f + b*lo
g(tan(e + f*x)**2 + 1)/(2*f), Ne(f, 0)), (x*(a + b*tan(e)**2)*cot(e), True)
)
```

Giac [A] time = 1.22549, size = 47, normalized size = 1.81

$$\frac{a\log\left(\sin\left(fx+e\right)^2\right)-b\log\left(-\sin\left(fx+e\right)^2+1\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*(a*log(sin(f*x + e)^2) - b*log(-sin(f*x + e)^2 + 1))/f
```

3.189 $\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=34

$$-\frac{(a-b)\log(\sin(e+fx))}{f}-\frac{a\cot^2(e+fx)}{2f}$$

```
[Out] -(a*Cot[e + f*x]^2)/(2*f) - ((a - b)*Log[Sin[e + f*x]])/f
```

Rubi [A] time = 0.0322388, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3629, 12, 3475}

$$-\frac{(a-b)\log(\sin(e+fx))}{f}-\frac{a\cot^2(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(a*Cot[e + f*x]^2)/(2*f) - ((a - b)*Log[Sin[e + f*x]])/f
```

Rule 3629

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m
 + 1))/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= -\frac{a\cot^2(e+fx)}{2f}-\int (a-b)\cot(e+fx)\,dx\\
&= -\frac{a\cot^2(e+fx)}{2f}-(a-b)\int \cot(e+fx)\,dx\\
&= -\frac{a\cot^2(e+fx)}{2f}-\frac{(a-b)\log(\sin(e+fx))}{f}
\end{aligned}$$

Mathematica [A] time = 0.148674, size = 56, normalized size = 1.65

$$\frac{b(\log(\tan(e+fx))+\log(\cos(e+fx)))}{f}-\frac{a\left(\cot^2(e+fx)+2\log(\tan(e+fx))+2\log(\cos(e+fx))\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (b*(Log[Cos[e + f*x]] + Log[Tan[e + f*x]]))/f - (a*(Cot[e + f*x]^2 + 2*Log[
Cos[e + f*x]] + 2*Log[Tan[e + f*x]]))/(2*f)
```

Maple [A] time = 0.044, size = 41, normalized size = 1.2

$$\frac{b\ln\left(\sin\left(fx+e\right)\right)}{f}-\frac{\left(\cot\left(fx+e\right)\right)^{2}a}{2f}-\frac{a\ln\left(\sin\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*b*ln(sin(f*x+e))-1/2*a*cot(f*x+e)^2/f-a*ln(sin(f*x+e))/f
```

Maxima [A] time = 1.04715, size = 42, normalized size = 1.24

$$-\frac{(a-b)\log\left(\sin\left(fx+e\right)^{2}\right)+\frac{a}{\sin\left(fx+e\right)^{2}}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*((a - b)*log(sin(f*x + e)^2) + a/sin(f*x + e)^2)/f
```

Fricas [A] time = 1.08583, size = 154, normalized size = 4.53

$$-\frac{(a-b)\log\left(\frac{\tan\left(fx+e\right)^{2}}{\tan\left(fx+e\right)^{2}+1}\right)\tan\left(fx+e\right)^{2}+a\tan\left(fx+e\right)^{2}+a}{2f\tan\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*((a - b)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + a*t
an(f*x + e)^2 + a)/(f*tan(f*x + e)^2)
```

Sympy [A] time = 2.68415, size = 100, normalized size = 2.94

$$\begin{cases} \tilde{\infty}ax & \text{for}\ \left(e=0\vee e=-fx\right)\wedge\left(\right. \\ x\left(a+b\tan^{2}\left(e\right)\right)\cot^{3}\left(e\right) & \text{for}\ f=0 \\ \frac{a\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}-\frac{a\log\left(\tan\left(e+fx\right)\right)}{f}-\frac{a}{2f\tan^{2}\left(e+fx\right)}-\frac{b\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{b\log\left(\tan\left(e+fx\right)\right)}{f} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*a*x, (Eq(e, 0) | Eq(e, -f*x)) & (Eq(f, 0) | Eq(e, -f*x))), (
x*(a + b*tan(e)**2)*cot(e)**3, Eq(f, 0)), (a*log(tan(e + f*x)**2 + 1)/(2*f)
 - a*log(tan(e + f*x))/f - a/(2*f*tan(e + f*x)**2) - b*log(tan(e + f*x)**2
+ 1)/(2*f) + b*log(tan(e + f*x))/f, True))
```

Giac [B] time = 1.3396, size = 215, normalized size = 6.32

$$\frac{8\,(a-b)\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1\right)-4\,(a-b)\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)+\frac{\left(a+\frac{4\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{4\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}\right)\left(\cos\left(fx+e\right)+1\right)}{\cos\left(fx+e\right)-1}+\frac{a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/8*(8*(a - b)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1) - 4*(a - b)*
log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + (a + 4*a*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(f*x + e
) + 1)/(cos(f*x + e) - 1) + a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/f
```

3.190 $\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=53

$$\frac{(a-b)\cot^2(e+fx)}{2f}+\frac{(a-b)\log(\sin(e+fx))}{f}-\frac{a\cot^4(e+fx)}{4f}$$

```
[Out] ((a - b)*Cot[e + f*x]^2)/(2*f) - (a*Cot[e + f*x]^4)/(4*f) + ((a - b)*Log[Si
n[e + f*x]])/f
```

Rubi [A] time = 0.0422992, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3629, 12, 3473, 3475}

$$\frac{(a-b)\cot^2(e+fx)}{2f}+\frac{(a-b)\log(\sin(e+fx))}{f}-\frac{a\cot^4(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((a - b)*Cot[e + f*x]^2)/(2*f) - (a*Cot[e + f*x]^4)/(4*f) + ((a - b)*Log[Si
n[e + f*x]])/f
```

Rule 3629

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m
 + 1))/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -\frac{a\cot^4(e+fx)}{4f} - \int (a-b)\cot^3(e+fx)\,dx$$
$$= -\frac{a\cot^4(e+fx)}{4f} - (a-b)\int \cot^3(e+fx)\,dx$$
$$= \frac{(a-b)\cot^2(e+fx)}{2f} - \frac{a\cot^4(e+fx)}{4f} - (-a+b)\int \cot(e+fx)\,dx$$
$$= \frac{(a-b)\cot^2(e+fx)}{2f} - \frac{a\cot^4(e+fx)}{4f} + \frac{(a-b)\log(\sin(e+fx))}{f}$$

Mathematica [A] time = 0.22429, size = 56, normalized size = 1.06

$$\frac{2(a-b)\cot^2(e+fx) + 4(a-b)(\log(\tan(e+fx)) + \log(\cos(e+fx))) - a\cot^4(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (2*(a - b)*Cot[e + f*x]^2 - a*Cot[e + f*x]^4 + 4*(a - b)*(Log[Cos[e + f*x]]
 + Log[Tan[e + f*x]]))/(4*f)
```

Maple [A] time = 0.043, size = 69, normalized size = 1.3

$$-\frac{b\left(\cot\left(fx+e\right)\right)^2}{2f} - \frac{b\ln\left(\sin\left(fx+e\right)\right)}{f} - \frac{a\left(\cot\left(fx+e\right)\right)^4}{4f} + \frac{\left(\cot\left(fx+e\right)\right)^2 a}{2f} + \frac{a\ln\left(\sin\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/2/f*b*cot(f*x+e)^2-1/f*b*ln(sin(f*x+e))-1/4*a*cot(f*x+e)^4/f+1/2*a*cot(f
*x+e)^2/f+a*ln(sin(f*x+e))/f
```

Maxima [A] time = 1.07745, size = 70, normalized size = 1.32

$$\frac{2\,(a-b)\log\left(\sin\left(fx+e\right)^2\right) + \frac{2\,(2\,a-b)\sin\left(fx+e\right)^2 - a}{\sin\left(fx+e\right)^4}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a - b)*log(sin(f*x + e)^2) + (2*(2*a - b)*sin(f*x + e)^2 - a)/sin(f
*x + e)^4)/f
```

Fricas [A] time = 1.0591, size = 205, normalized size = 3.87

$$\frac{2(a-b)\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)\tan(fx+e)^4+(3a-2b)\tan(fx+e)^4+2(a-b)\tan(fx+e)^2-a}{4f\tan(fx+e)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a - b)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 + (3
*a - 2*b)*tan(f*x + e)^4 + 2*(a - b)*tan(f*x + e)^2 - a)/(f*tan(f*x + e)^4)
```

Sympy [A] time = 9.16975, size = 128, normalized size = 2.42

$$\begin{cases} \tilde{\infty}ax \\ x\left(a+b\tan^2(e)\right)\cot^5(e) \\ -\frac{a\log\left(\tan^2(e+fx)+1\right)}{2f}+\frac{a\log\left(\tan(e+fx)\right)}{f}+\frac{a}{2f\tan^2(e+fx)}-\frac{a}{4f\tan^4(e+fx)}+\frac{b\log\left(\tan^2(e+fx)+1\right)}{2f}-\frac{b\log\left(\tan(e+fx)\right)}{f}-\frac{b}{2f\tan^2(e+f} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*a*x, (Eq(e, 0) | Eq(e, -f*x)) & (Eq(f, 0) | Eq(e, -f*x))), (
x*(a + b*tan(e)**2)*cot(e)**5, Eq(f, 0)), (-a*log(tan(e + f*x)**2 + 1)/(2*f
) + a*log(tan(e + f*x))/f + a/(2*f*tan(e + f*x)**2) - a/(4*f*tan(e + f*x)**
4) + b*log(tan(e + f*x)**2 + 1)/(2*f) - b*log(tan(e + f*x))/f - b/(2*f*tan(
e + f*x)**2), True))
```

Giac [B] time = 1.343, size = 350, normalized size = 6.6

$$-\frac{64(a-b)\log\left(-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right)-32(a-b)\log\left(-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+\frac{\left(a+\frac{12a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{8b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{48a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{48b(\cos}{(\cos(f}\right.}{(\cos(fx+e)-1)^2}}{64f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/64*(64*(a - b)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1) - 32*(a -
 b)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + (a + 12*a*(cos(f*x + e) -
 1)/(cos(f*x + e) + 1) - 8*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 48*a*(
cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 48*b*(cos(f*x + e) - 1)^2/(cos(f
*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x + e) - 1)^2 + 12*a*(cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) - 8*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a
*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/f
```

3.191 $\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)dx$

Optimal. Leaf size=80

$$\frac{(a-b)\tan^5(e+fx)}{5f}-\frac{(a-b)\tan^3(e+fx)}{3f}+\frac{(a-b)\tan(e+fx)}{f}-x(a-b)+\frac{b\tan^7(e+fx)}{7f}$$

```
[Out] -((a - b)*x) + ((a - b)*Tan[e + f*x])/f - ((a - b)*Tan[e + f*x]^3)/(3*f) +
((a - b)*Tan[e + f*x]^5)/(5*f) + (b*Tan[e + f*x]^7)/(7*f)
```

Rubi [A] time = 0.0523921, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3631, 3473, 8}

$$\frac{(a-b)\tan^5(e+fx)}{5f}-\frac{(a-b)\tan^3(e+fx)}{3f}+\frac{(a-b)\tan(e+fx)}{f}-x(a-b)+\frac{b\tan^7(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a - b)*x) + ((a - b)*Tan[e + f*x])/f - ((a - b)*Tan[e + f*x]^3)/(3*f) +
((a - b)*Tan[e + f*x]^5)/(5*f) + (b*Tan[e + f*x]^7)/(7*f)
```

Rule 3631

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(
m + 1)), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)dx &= \frac{b\tan^7(e+fx)}{7f}+(a-b)\int \tan^6(e+fx)\,dx\\
&= \frac{(a-b)\tan^5(e+fx)}{5f}+\frac{b\tan^7(e+fx)}{7f}+(-a+b)\int \tan^4(e+fx)\,dx\\
&= -\frac{(a-b)\tan^3(e+fx)}{3f}+\frac{(a-b)\tan^5(e+fx)}{5f}+\frac{b\tan^7(e+fx)}{7f}+(a-b)\int \text{ta}\\
&= \frac{(a-b)\tan(e+fx)}{f}-\frac{(a-b)\tan^3(e+fx)}{3f}+\frac{(a-b)\tan^5(e+fx)}{5f}+\frac{b\tan^7(e}{7f}\\
&= -(a-b)x+\frac{(a-b)\tan(e+fx)}{f}-\frac{(a-b)\tan^3(e+fx)}{3f}+\frac{(a-b)\tan^5(e+fx)}{5f}
\end{aligned}$$

Mathematica [A] time = 0.0473769, size = 129, normalized size = 1.61

$$\frac{a\tan^5(e+fx)}{5f}-\frac{a\tan^3(e+fx)}{3f}-\frac{a\tan^{-1}(\tan(e+fx))}{f}+\frac{a\tan(e+fx)}{f}+\frac{b\tan^7(e+fx)}{7f}-\frac{b\tan^5(e+fx)}{5f}+\frac{b\tan^3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*ArcTan[Tan[e + f*x]])/f) + (b*ArcTan[Tan[e + f*x]])/f + (a*Tan[e + f*x
])/f - (b*Tan[e + f*x])/f - (a*Tan[e + f*x]^3)/(3*f) + (b*Tan[e + f*x]^3)/(
3*f) + (a*Tan[e + f*x]^5)/(5*f) - (b*Tan[e + f*x]^5)/(5*f) + (b*Tan[e + f*x
]^7)/(7*f)
```

Maple [A] time = 0.004, size = 120, normalized size = 1.5

$$\frac{b\left(\tan\left(fx+e\right)\right)^{7}}{7f}+\frac{\left(\tan\left(fx+e\right)\right)^{5}a}{5f}-\frac{b\left(\tan\left(fx+e\right)\right)^{5}}{5f}-\frac{\left(\tan\left(fx+e\right)\right)^{3}a}{3f}+\frac{b\left(\tan\left(fx+e\right)\right)^{3}}{3f}+\frac{a\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/7*b*tan(f*x+e)^7/f+1/5/f*tan(f*x+e)^5*a-1/5*b*tan(f*x+e)^5/f-1/3/f*tan(f*
x+e)^3*a+1/3*b*tan(f*x+e)^3/f+1/f*a*tan(f*x+e)-b*tan(f*x+e)/f-1/f*arctan(ta
n(f*x+e))*a+b/f*arctan(tan(f*x+e))
```

Maxima [A] time = 1.68975, size = 97, normalized size = 1.21

$$\frac{15\,b\tan\left(fx+e\right)^{7}+21\,(a-b)\tan\left(fx+e\right)^{5}-35\,(a-b)\tan\left(fx+e\right)^{3}-105\left(fx+e\right)(a-b)+105\,(a-b)\tan\left(fx+e\right.}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*b*tan(f*x + e)^7 + 21*(a - b)*tan(f*x + e)^5 - 35*(a - b)*tan(f*x
 + e)^3 - 105*(f*x + e)*(a - b) + 105*(a - b)*tan(f*x + e))/f
```

Fricas [A] time = 1.0787, size = 178, normalized size = 2.22

$$\frac{15\,b\tan\left(fx+e\right)^{7}+21\,(a-b)\tan\left(fx+e\right)^{5}-35\,(a-b)\tan\left(fx+e\right)^{3}-105\,(a-b)fx+105\,(a-b)\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b*tan(f*x + e)^7 + 21*(a - b)*tan(f*x + e)^5 - 35*(a - b)*tan(f*x
 + e)^3 - 105*(a - b)*f*x + 105*(a - b)*tan(f*x + e))/f
```

Sympy [A] time = 1.18512, size = 109, normalized size = 1.36

$$\begin{cases} -ax + \frac{a\tan^5(e+fx)}{5f} - \frac{a\tan^3(e+fx)}{3f} + \frac{a\tan(e+fx)}{f} + bx + \frac{b\tan^7(e+fx)}{7f} - \frac{b\tan^5(e+fx)}{5f} + \frac{b\tan^3(e+fx)}{3f} - \frac{b\tan(e+fx)}{f} & \text{for } f \neq 0 \\ x\left(a + b\tan^2(e)\right)\tan^6(e) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((-a*x + a*tan(e + f*x)**5/(5*f) - a*tan(e + f*x)**3/(3*f) + a*tan
(e + f*x)/f + b*x + b*tan(e + f*x)**7/(7*f) - b*tan(e + f*x)**5/(5*f) + b*t
an(e + f*x)**3/(3*f) - b*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e)**2)*ta
n(e)**6, True))
```

Giac [B] time = 6.26445, size = 1467, normalized size = 18.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/105*(105*a*f*x*tan(f*x)^7*tan(e)^7 - 105*b*f*x*tan(f*x)^7*tan(e)^7 - 735
*a*f*x*tan(f*x)^6*tan(e)^6 + 735*b*f*x*tan(f*x)^6*tan(e)^6 + 105*a*tan(f*x)
^7*tan(e)^6 - 105*b*tan(f*x)^7*tan(e)^6 + 105*a*tan(f*x)^6*tan(e)^7 - 105*b
*tan(f*x)^6*tan(e)^7 + 2205*a*f*x*tan(f*x)^5*tan(e)^5 - 2205*b*f*x*tan(f*x)
^5*tan(e)^5 - 35*a*tan(f*x)^7*tan(e)^4 + 35*b*tan(f*x)^7*tan(e)^4 - 735*a*t
an(f*x)^6*tan(e)^5 + 735*b*tan(f*x)^6*tan(e)^5 - 735*a*tan(f*x)^5*tan(e)^6
+ 735*b*tan(f*x)^5*tan(e)^6 - 35*a*tan(f*x)^4*tan(e)^7 + 35*b*tan(f*x)^4*ta
n(e)^7 - 3675*a*f*x*tan(f*x)^4*tan(e)^4 + 3675*b*f*x*tan(f*x)^4*tan(e)^4 +
21*a*tan(f*x)^7*tan(e)^2 - 21*b*tan(f*x)^7*tan(e)^2 + 245*a*tan(f*x)^6*tan(
e)^3 - 245*b*tan(f*x)^6*tan(e)^3 + 2205*a*tan(f*x)^5*tan(e)^4 - 2205*b*tan(
f*x)^5*tan(e)^4 + 2205*a*tan(f*x)^4*tan(e)^5 - 2205*b*tan(f*x)^4*tan(e)^5 +
 245*a*tan(f*x)^3*tan(e)^6 - 245*b*tan(f*x)^3*tan(e)^6 + 21*a*tan(f*x)^2*ta
n(e)^7 - 21*b*tan(f*x)^2*tan(e)^7 + 3675*a*f*x*tan(f*x)^3*tan(e)^3 - 3675*b
*f*x*tan(f*x)^3*tan(e)^3 + 15*b*tan(f*x)^7 - 42*a*tan(f*x)^6*tan(e) + 147*b
*tan(f*x)^6*tan(e) - 420*a*tan(f*x)^5*tan(e)^2 + 735*b*tan(f*x)^5*tan(e)^2
- 3150*a*tan(f*x)^4*tan(e)^3 + 3675*b*tan(f*x)^4*tan(e)^3 - 3150*a*tan(f*x)
^3*tan(e)^4 + 3675*b*tan(f*x)^3*tan(e)^4 - 420*a*tan(f*x)^2*tan(e)^5 + 735*
b*tan(f*x)^2*tan(e)^5 - 42*a*tan(f*x)*tan(e)^6 + 147*b*tan(f*x)*tan(e)^6 +
15*b*tan(e)^7 - 2205*a*f*x*tan(f*x)^2*tan(e)^2 + 2205*b*f*x*tan(f*x)^2*tan(
e)^2 + 21*a*tan(f*x)^5 - 21*b*tan(f*x)^5 + 245*a*tan(f*x)^4*tan(e) - 245*b*
tan(f*x)^4*tan(e) + 2205*a*tan(f*x)^3*tan(e)^2 - 2205*b*tan(f*x)^3*tan(e)^2
 + 2205*a*tan(f*x)^2*tan(e)^3 - 2205*b*tan(f*x)^2*tan(e)^3 + 245*a*tan(f*x)
*tan(e)^4 - 245*b*tan(f*x)*tan(e)^4 + 21*a*tan(e)^5 - 21*b*tan(e)^5 + 735*a
*f*x*tan(f*x)*tan(e) - 735*b*f*x*tan(f*x)*tan(e) - 35*a*tan(f*x)^3 + 35*b*t
an(f*x)^3 - 735*a*tan(f*x)^2*tan(e) + 735*b*tan(f*x)^2*tan(e) - 735*a*tan(f
*x)*tan(e)^2 + 735*b*tan(f*x)*tan(e)^2 - 35*a*tan(e)^3 + 35*b*tan(e)^3 - 10
5*a*f*x + 105*b*f*x + 105*a*tan(f*x) - 105*b*tan(f*x) + 105*a*tan(e) - 105*
b*tan(e))/(f*tan(f*x)^7*tan(e)^7 - 7*f*tan(f*x)^6*tan(e)^6 + 21*f*tan(f*x)^
5*tan(e)^5 - 35*f*tan(f*x)^4*tan(e)^4 + 35*f*tan(f*x)^3*tan(e)^3 - 21*f*tan
(f*x)^2*tan(e)^2 + 7*f*tan(f*x)*tan(e) - f)
```

3.192 $\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=60

$$\frac{(a-b)\tan^3(e+fx)}{3f}-\frac{(a-b)\tan(e+fx)}{f}+x(a-b)+\frac{b\tan^5(e+fx)}{5f}$$

```
[Out] (a - b)*x - ((a - b)*Tan[e + f*x])/f + ((a - b)*Tan[e + f*x]^3)/(3*f) + (b*
Tan[e + f*x]^5)/(5*f)
```

Rubi [A] time = 0.0430068, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3631, 3473, 8}

$$\frac{(a-b)\tan^3(e+fx)}{3f}-\frac{(a-b)\tan(e+fx)}{f}+x(a-b)+\frac{b\tan^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (a - b)*x - ((a - b)*Tan[e + f*x])/f + ((a - b)*Tan[e + f*x]^3)/(3*f) + (b*
Tan[e + f*x]^5)/(5*f)
```

Rule 3631

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(
m + 1)), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= \frac{b\tan^5(e+fx)}{5f}+(a-b)\int \tan^4(e+fx)\,dx\\ &= \frac{(a-b)\tan^3(e+fx)}{3f}+\frac{b\tan^5(e+fx)}{5f}+(-a+b)\int \tan^2(e+fx)\,dx\\ &= -\frac{(a-b)\tan(e+fx)}{f}+\frac{(a-b)\tan^3(e+fx)}{3f}+\frac{b\tan^5(e+fx)}{5f}+(a-b)\int\\ &= (a-b)x-\frac{(a-b)\tan(e+fx)}{f}+\frac{(a-b)\tan^3(e+fx)}{3f}+\frac{b\tan^5(e+fx)}{5f}\end{aligned}$$

Mathematica [A] time = 0.0373539, size = 97, normalized size = 1.62

$$\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan^{-1}(\tan(e+fx))}{f}-\frac{a\tan(e+fx)}{f}+\frac{b\tan^5(e+fx)}{5f}-\frac{b\tan^3(e+fx)}{3f}-\frac{b\tan^{-1}(\tan(e+fx))}{f}+\frac{b\tan}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (a*ArcTan[Tan[e + f*x]])/f - (b*ArcTan[Tan[e + f*x]])/f - (a*Tan[e + f*x])/
f + (b*Tan[e + f*x])/f + (a*Tan[e + f*x]^3)/(3*f) - (b*Tan[e + f*x]^3)/(3*f
) + (b*Tan[e + f*x]^5)/(5*f)
```

Maple [A] time = 0.004, size = 92, normalized size = 1.5

$$\frac{b\left(\tan\left(fx+e\right)\right)^5}{5f}+\frac{\left(\tan\left(fx+e\right)\right)^3a}{3f}-\frac{b\left(\tan\left(fx+e\right)\right)^3}{3f}-\frac{a\tan\left(fx+e\right)}{f}+\frac{b\tan\left(fx+e\right)}{f}+\frac{\arctan\left(\tan\left(fx+e\right)\right)a}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/5*b*tan(f*x+e)^5/f+1/3/f*tan(f*x+e)^3*a-1/3*b*tan(f*x+e)^3/f-1/f*a*tan(f*
x+e)+b*tan(f*x+e)/f+1/f*arctan(tan(f*x+e))*a-b/f*arctan(tan(f*x+e))
```

Maxima [A] time = 1.68851, size = 77, normalized size = 1.28

$$\frac{3\,b\tan\left(fx+e\right)^5+5\,(a-b)\tan\left(fx+e\right)^3+15\left(fx+e\right)(a-b)-15\,(a-b)\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 + 5*(a - b)*tan(f*x + e)^3 + 15*(f*x + e)*(a - b)
- 15*(a - b)*tan(f*x + e))/f
```

Fricas [A] time = 1.0566, size = 134, normalized size = 2.23

$$\frac{3\,b\tan\left(fx+e\right)^5+5\,(a-b)\tan\left(fx+e\right)^3+15\,(a-b)fx-15\,(a-b)\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 + 5*(a - b)*tan(f*x + e)^3 + 15*(a - b)*f*x - 15*(
a - b)*tan(f*x + e))/f
```

Sympy [A] time = 0.663574, size = 82, normalized size = 1.37

$$\begin{cases} ax + \frac{a\tan^3(e+fx)}{3f} - \frac{a\tan(e+fx)}{f} - bx + \frac{b\tan^5(e+fx)}{5f} - \frac{b\tan^3(e+fx)}{3f} + \frac{b\tan(e+fx)}{f} & \text{for } f \neq 0 \\ x\left(a + b\tan^2(e)\right)\tan^4(e) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((a*x + a*tan(e + f*x)**3/(3*f) - a*tan(e + f*x)/f - b*x + b*tan(e
 + f*x)**5/(5*f) - b*tan(e + f*x)**3/(3*f) + b*tan(e + f*x)/f, Ne(f, 0)), (
x*(a + b*tan(e)**2)*tan(e)**4, True))
```

Giac [B] time = 2.63965, size = 855, normalized size = 14.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(15*a*f*x*tan(f*x)^5*tan(e)^5 - 15*b*f*x*tan(f*x)^5*tan(e)^5 - 75*a*f*
x*tan(f*x)^4*tan(e)^4 + 75*b*f*x*tan(f*x)^4*tan(e)^4 + 15*a*tan(f*x)^5*tan(
e)^4 - 15*b*tan(f*x)^5*tan(e)^4 + 15*a*tan(f*x)^4*tan(e)^5 - 15*b*tan(f*x)^
4*tan(e)^5 + 150*a*f*x*tan(f*x)^3*tan(e)^3 - 150*b*f*x*tan(f*x)^3*tan(e)^3
- 5*a*tan(f*x)^5*tan(e)^2 + 5*b*tan(f*x)^5*tan(e)^2 - 75*a*tan(f*x)^4*tan(e
)^3 + 75*b*tan(f*x)^4*tan(e)^3 - 75*a*tan(f*x)^3*tan(e)^4 + 75*b*tan(f*x)^3
*tan(e)^4 - 5*a*tan(f*x)^2*tan(e)^5 + 5*b*tan(f*x)^2*tan(e)^5 - 150*a*f*x*t
an(f*x)^2*tan(e)^2 + 150*b*f*x*tan(f*x)^2*tan(e)^2 - 3*b*tan(f*x)^5 + 10*a*
tan(f*x)^4*tan(e) - 25*b*tan(f*x)^4*tan(e) + 120*a*tan(f*x)^3*tan(e)^2 - 15
0*b*tan(f*x)^3*tan(e)^2 + 120*a*tan(f*x)^2*tan(e)^3 - 150*b*tan(f*x)^2*tan(
e)^3 + 10*a*tan(f*x)*tan(e)^4 - 25*b*tan(f*x)*tan(e)^4 - 3*b*tan(e)^5 + 75*
a*f*x*tan(f*x)*tan(e) - 75*b*f*x*tan(f*x)*tan(e) - 5*a*tan(f*x)^3 + 5*b*tan
(f*x)^3 - 75*a*tan(f*x)^2*tan(e) + 75*b*tan(f*x)^2*tan(e) - 75*a*tan(f*x)*t
an(e)^2 + 75*b*tan(f*x)*tan(e)^2 - 5*a*tan(e)^3 + 5*b*tan(e)^3 - 15*a*f*x +
 15*b*f*x + 15*a*tan(f*x) - 15*b*tan(f*x) + 15*a*tan(e) - 15*b*tan(e))/(f*t
an(f*x)^5*tan(e)^5 - 5*f*tan(f*x)^4*tan(e)^4 + 10*f*tan(f*x)^3*tan(e)^3 - 1
0*f*tan(f*x)^2*tan(e)^2 + 5*f*tan(f*x)*tan(e) - f)
```

3.193 $\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=40

$$\frac{(a-b)\tan(e+fx)}{f}-x(a-b)+\frac{b\tan^3(e+fx)}{3f}$$

```
[Out] -((a - b)*x) + ((a - b)*Tan[e + f*x])/f + (b*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.0336666, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3631, 3473, 8}

$$\frac{(a-b)\tan(e+fx)}{f}-x(a-b)+\frac{b\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a - b)*x) + ((a - b)*Tan[e + f*x])/f + (b*Tan[e + f*x]^3)/(3*f)
```

Rule 3631

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(
m + 1)), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx&=\frac{b\tan^3(e+fx)}{3f}+(a-b)\int \tan^2(e+fx)\,dx\\&=\frac{(a-b)\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}+(-a+b)\int 1\,dx\\&=-(a-b)x+\frac{(a-b)\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A] time = 0.0238519, size = 65, normalized size = 1.62

$$-\frac{a\tan^{-1}(\tan(e+fx))}{f}+\frac{a\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}+\frac{b\tan^{-1}(\tan(e+fx))}{f}-\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*ArcTan[Tan[e + f*x]])/f) + (b*ArcTan[Tan[e + f*x]])/f + (a*Tan[e + f*x
])/f - (b*Tan[e + f*x])/f + (b*Tan[e + f*x]^3)/(3*f)
```

Maple [A] time = 0.003, size = 64, normalized size = 1.6

$$\frac{b\left(\tan\left(fx+e\right)\right)^{3}}{3f}+\frac{a\tan\left(fx+e\right)}{f}-\frac{b\tan\left(fx+e\right)}{f}-\frac{\arctan\left(\tan\left(fx+e\right)\right)a}{f}+\frac{b\arctan\left(\tan\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/3*b*tan(f*x+e)^3/f+1/f*a*tan(f*x+e)-b*tan(f*x+e)/f-1/f*arctan(tan(f*x+e))
*a+b/f*arctan(tan(f*x+e))
```

Maxima [A] time = 1.63988, size = 55, normalized size = 1.38

$$\frac{b\tan\left(fx+e\right)^{3}-3\left(fx+e\right)(a-b)+3\,(a-b)\tan\left(fx+e\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(b*tan(f*x + e)^3 - 3*(f*x + e)*(a - b) + 3*(a - b)*tan(f*x + e))/f
```

Fricas [A] time = 1.02826, size = 90, normalized size = 2.25

$$\frac{b\tan\left(fx+e\right)^{3}-3\,(a-b)fx+3\,(a-b)\tan\left(fx+e\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*tan(f*x + e)^3 - 3*(a - b)*f*x + 3*(a - b)*tan(f*x + e))/f
```

Sympy [A] time = 0.344046, size = 54, normalized size = 1.35

$$\begin{cases} -ax+\frac{a\tan\left(e+fx\right)}{f}+bx+\frac{b\tan^{3}\left(e+fx\right)}{3f}-\frac{b\tan\left(e+fx\right)}{f} & \text{for } f\neq 0\\ x\left(a+b\tan^{2}(e)\right)\tan^{2}(e) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((-a*x + a*tan(e + f*x)/f + b*x + b*tan(e + f*x)**3/(3*f) - b*tan(
e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e)**2)*tan(e)**2, True))
```

Giac [B] time = 1.56232, size = 390, normalized size = 9.75

$$-\frac{3\,afx\tan\left(fx\right)^3\tan\left(e\right)^3-3\,bfx\tan\left(fx\right)^3\tan\left(e\right)^3-9\,afx\tan\left(fx\right)^2\tan\left(e\right)^2+9\,bfx\tan\left(fx\right)^2\tan\left(e\right)^2+3\,a\tan\left(fx\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*a*f*x*tan(f*x)^3*tan(e)^3 - 3*b*f*x*tan(f*x)^3*tan(e)^3 - 9*a*f*x*t
an(f*x)^2*tan(e)^2 + 9*b*f*x*tan(f*x)^2*tan(e)^2 + 3*a*tan(f*x)^3*tan(e)^2
- 3*b*tan(f*x)^3*tan(e)^2 + 3*a*tan(f*x)^2*tan(e)^3 - 3*b*tan(f*x)^2*tan(e)
^3 + 9*a*f*x*tan(f*x)*tan(e) - 9*b*f*x*tan(f*x)*tan(e) + b*tan(f*x)^3 - 6*a
*tan(f*x)^2*tan(e) + 9*b*tan(f*x)^2*tan(e) - 6*a*tan(f*x)*tan(e)^2 + 9*b*ta
n(f*x)*tan(e)^2 + b*tan(e)^3 - 3*a*f*x + 3*b*f*x + 3*a*tan(f*x) - 3*b*tan(f
*x) + 3*a*tan(e) - 3*b*tan(e))/(f*tan(f*x)^3*tan(e)^3 - 3*f*tan(f*x)^2*tan(
e)^2 + 3*f*tan(f*x)*tan(e) - f)
```

3.194 $\int \left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=19

$$ax+\frac{b\tan(e+fx)}{f}-bx$$

```
[Out] a*x - b*x + (b*Tan[e + f*x])/f
```

Rubi [A] time = 0.0119024, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3473, 8}

$$ax+\frac{b\tan(e+fx)}{f}-bx$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Tan[e + f*x]^2,x]
```

```
[Out] a*x - b*x + (b*Tan[e + f*x])/f
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tan^2(e+fx)\right)\,dx &= ax+b\int \tan^2(e+fx)\,dx\\ &= ax+\frac{b\tan(e+fx)}{f}-b\int 1\,dx\\ &= ax-bx+\frac{b\tan(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.006066, size = 28, normalized size = 1.47

$$ax-\frac{b\tan^{-1}(\tan(e+fx))}{f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Tan[e + f*x]^2,x]
```

```
[Out] a*x - (b*ArcTan[Tan[e + f*x]])/f + (b*Tan[e + f*x])/f
```

Maple [A] time = 0., size = 29, normalized size = 1.5

$$ax + \frac{b\tan\left(fx+e\right)}{f} - \frac{b\arctan\left(\tan\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*tan(f*x+e)^2,x)
```

```
[Out] a*x+b*tan(f*x+e)/f-b/f*arctan(tan(f*x+e))
```

Maxima [A] time = 1.61604, size = 31, normalized size = 1.63

$$ax - \frac{\left(fx + e - \tan\left(fx+e\right)\right)b}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] a*x - (f*x + e - tan(f*x + e))*b/f
```

Fricas [A] time = 1.05917, size = 46, normalized size = 2.42

$$\frac{(a-b)fx + b\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] ((a - b)*f*x + b*tan(f*x + e))/f
```

Sympy [A] time = 0.161364, size = 20, normalized size = 1.05

$$ax + b\left(\begin{cases} -x + \frac{\tan\left(e+fx\right)}{f} & \text{for}\ f \neq 0 \\ x\tan^2\left(e\right) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)**2,x)
```

```
[Out] a*x + b*Piecewise((-x + tan(e + f*x)/f, Ne(f, 0)), (x*tan(e)**2, True))
```

Giac [B] time = 1.19552, size = 340, normalized size = 17.89

$$ax + \frac{\left(\pi - 4fx\tan\left(fx\right)\tan\left(e\right) - \pi\text{sgn}\left(2\tan\left(fx\right)^2\tan\left(e\right) + 2\tan\left(fx\right)\tan\left(e\right)^2 - 2\tan\left(fx\right) - 2\tan\left(e\right)\right)\tan\left(fx\right)\text{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] a*x + 1/4*(pi - 4*f*x*tan(f*x)*tan(e) - pi*sgn(2*tan(f*x)^2*tan(e) + 2*tan(
f*x)*tan(e)^2 - 2*tan(f*x) - 2*tan(e))*tan(f*x)*tan(e) - pi*tan(f*x)*tan(e)
 + 2*arctan((tan(f*x)*tan(e) - 1)/(tan(f*x) + tan(e)))*tan(f*x)*tan(e) + 2*
arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)*tan(e) + 4*f*x +
 pi*sgn(2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 - 2*tan(f*x) - 2*tan(e))
- 2*arctan((tan(f*x)*tan(e) - 1)/(tan(f*x) + tan(e))) - 2*arctan((tan(f*x)
+ tan(e))/(tan(f*x)*tan(e) - 1)) - 4*tan(f*x) - 4*tan(e))*b/(f*tan(f*x)*tan
(e) - f)
```

3.195 $\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=21

$$x(-(a-b))-\frac{a\cot(e+fx)}{f}$$

```
[Out] -((a - b)*x) - (a*Cot[e + f*x])/f
```

Rubi [A] time = 0.0253417, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3629, 8}

$$x(-(a-b))-\frac{a\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a - b)*x) - (a*Cot[e + f*x])/f
```

Rule 3629

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m
 + 1))/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= -\frac{a\cot(e+fx)}{f}+\int(-a+b)\,dx\\ &= -(a-b)x-\frac{a\cot(e+fx)}{f}\end{aligned}$$

Mathematica [C] time = 0.0180674, size = 34, normalized size = 1.62

$$bx-\frac{a\cot(e+fx)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] b*x - (a*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[e + f*x]^2])/f
```

Maple [A] time = 0.034, size = 31, normalized size = 1.5

$$\frac{b\left(fx+e\right)+a\left(-\cot\left(fx+e\right)-fx-e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*(b*(f*x+e)+a*(-cot(f*x+e)-f*x-e))
```

Maxima [A] time = 1.66738, size = 36, normalized size = 1.71

$$-\frac{\left(fx+e\right)(a-b)+\frac{a}{\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -((f*x + e)*(a - b) + a/tan(f*x + e))/f
```

Fricas [A] time = 1.02883, size = 68, normalized size = 3.24

$$-\frac{(a-b)fx\tan\left(fx+e\right)+a}{f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -((a - b)*f*x*tan(f*x + e) + a)/(f*tan(f*x + e))
```

Sympy [A] time = 1.40587, size = 46, normalized size = 2.19

$$\begin{cases} \tilde{\infty}ax & \text{for}\ \left(e=0\vee e=-fx\right)\wedge\left(e=-fx\vee f=0\right) \\ x\left(a+b\tan^{2}(e)\right)\cot^{2}(e) & \text{for}\ f=0 \\ -ax-\frac{a}{f\tan\left(e+fx\right)}+bx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*a*x, (Eq(e, 0) | Eq(e, -f*x)) & (Eq(f, 0) | Eq(e, -f*x))), (
x*(a + b*tan(e)**2)*cot(e)**2, Eq(f, 0)), (-a*x - a/(f*tan(e + f*x)) + b*x,
 True))
```

Giac [B] time = 1.32599, size = 62, normalized size = 2.95

$$-\frac{2\left(fx+e\right)(a-b)-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+\frac{a}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*(f*x + e)*(a - b) - a*tan(1/2*f*x + 1/2*e) + a/tan(1/2*f*x + 1/2*e)
)/f
```

3.196 $\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)dx$

Optimal. Leaf size=39

$$\frac{(a-b)\cot(e+fx)}{f}+x(a-b)-\frac{a\cot^3(e+fx)}{3f}$$

```
[Out] (a - b)*x + ((a - b)*Cot[e + f*x])/f - (a*Cot[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.0370697, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3629, 12, 3473, 8}

$$\frac{(a-b)\cot(e+fx)}{f}+x(a-b)-\frac{a\cot^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (a - b)*x + ((a - b)*Cot[e + f*x])/f - (a*Cot[e + f*x]^3)/(3*f)
```

Rule 3629

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m
 + 1))/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)dx &= -\frac{a\cot^3(e+fx)}{3f} - \int (a-b)\cot^2(e+fx)\,dx \\
&= -\frac{a\cot^3(e+fx)}{3f} - (a-b)\int \cot^2(e+fx)\,dx \\
&= \frac{(a-b)\cot(e+fx)}{f} - \frac{a\cot^3(e+fx)}{3f} - (-a+b)\int 1\,dx \\
&= (a-b)x + \frac{(a-b)\cot(e+fx)}{f} - \frac{a\cot^3(e+fx)}{3f}
\end{aligned}
$$

Mathematica [C] time = 0.0371482, size = 65, normalized size = 1.67

$$
-\frac{a\cot^3(e+fx)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(e+fx)\right)}{3f} - \frac{b\cot(e+fx)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan\right.}{f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(a*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e + f*x]^2])/(3*f)
 - (b*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[e + f*x]^2])/f
```

Maple [A] time = 0.04, size = 47, normalized size = 1.2

$$
\frac{1}{f}\left(b\left(-\cot\left(fx+e\right)-fx-e\right)+a\left(-\frac{\left(\cot\left(fx+e\right)\right)^3}{3}+\cot\left(fx+e\right)+fx+e\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*(b*(-cot(f*x+e)-f*x-e)+a*(-1/3*cot(f*x+e)^3+cot(f*x+e)+f*x+e))
```

Maxima [A] time = 1.6165, size = 62, normalized size = 1.59

$$
\frac{3\left(fx+e\right)(a-b)+\frac{3\,(a-b)\tan\left(fx+e\right)^2-a}{\tan\left(fx+e\right)^3}}{3\,f}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*(a - b) + (3*(a - b)*tan(f*x + e)^2 - a)/tan(f*x + e)^3)/f
```

Fricas [A] time = 1.06863, size = 116, normalized size = 2.97

$$\frac{3\,(a-b)fx\tan\left(fx+e\right)^{3}+3\,(a-b)\tan\left(fx+e\right)^{2}-a}{3\,f\tan\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a - b)*f*x*tan(f*x + e)^3 + 3*(a - b)*tan(f*x + e)^2 - a)/(f*tan(f*
x + e)^3)
```

Sympy [A] time = 4.62876, size = 70, normalized size = 1.79

$$\begin{cases} \tilde{\infty}ax & \text{for}\ \left(e=0\vee e=-fx\right)\wedge\left(e=-fx\vee f=0\right) \\ x\left(a+b\tan^{2}(e)\right)\cot^{4}(e) & \text{for}\ f=0 \\ ax+\frac{a}{f\tan\left(e+fx\right)}-\frac{a}{3f\tan^{3}\left(e+fx\right)}-bx-\frac{b}{f\tan\left(e+fx\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*a*x, (Eq(e, 0) | Eq(e, -f*x)) & (Eq(f, 0) | Eq(e, -f*x))), (
x*(a + b*tan(e)**2)*cot(e)**4, Eq(f, 0)), (a*x + a/(f*tan(e + f*x)) - a/(3*
f*tan(e + f*x)**3) - b*x - b/(f*tan(e + f*x)), True))
```

Giac [B] time = 1.32586, size = 143, normalized size = 3.67

$$\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+24\left(fx+e\right)(a-b)-15\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+12\,b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+\frac{15\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-12\,b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-a}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*f*x + 1/2*e)^3 + 24*(f*x + e)*(a - b) - 15*a*tan(1/2*f*x +
1/2*e) + 12*b*tan(1/2*f*x + 1/2*e) + (15*a*tan(1/2*f*x + 1/2*e)^2 - 12*b*ta
n(1/2*f*x + 1/2*e)^2 - a)/tan(1/2*f*x + 1/2*e)^3)/f
```

3.197 $\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=61

$$\frac{(a-b)\cot^3(e+fx)}{3f}-\frac{(a-b)\cot(e+fx)}{f}-x(a-b)-\frac{a\cot^5(e+fx)}{5f}$$

```
[Out] -((a - b)*x) - ((a - b)*Cot[e + f*x])/f + ((a - b)*Cot[e + f*x]^3)/(3*f) -
(a*Cot[e + f*x]^5)/(5*f)
```

Rubi [A] time = 0.0456984, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3629, 12, 3473, 8}

$$\frac{(a-b)\cot^3(e+fx)}{3f}-\frac{(a-b)\cot(e+fx)}{f}-x(a-b)-\frac{a\cot^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a - b)*x) - ((a - b)*Cot[e + f*x])/f + ((a - b)*Cot[e + f*x]^3)/(3*f) -
(a*Cot[e + f*x]^5)/(5*f)
```

Rule 3629

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m
 + 1))/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)dx = -\frac{a\cot^5(e+fx)}{5f} - \int (a-b)\cot^4(e+fx)\,dx$$
$$= -\frac{a\cot^5(e+fx)}{5f} - (a-b)\int \cot^4(e+fx)\,dx$$
$$= \frac{(a-b)\cot^3(e+fx)}{3f} - \frac{a\cot^5(e+fx)}{5f} - (-a+b)\int \cot^2(e+fx)\,dx$$
$$= -\frac{(a-b)\cot(e+fx)}{f} + \frac{(a-b)\cot^3(e+fx)}{3f} - \frac{a\cot^5(e+fx)}{5f} - (a-b)\int 1$$
$$= -(a-b)x - \frac{(a-b)\cot(e+fx)}{f} + \frac{(a-b)\cot^3(e+fx)}{3f} - \frac{a\cot^5(e+fx)}{5f}$$

Mathematica [C] time = 0.0503916, size = 69, normalized size = 1.13

$$-\frac{a\cot^5(e+fx)\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)}{5f} - \frac{b\cot^3(e+fx)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\right.}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(a*Cot[e + f*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2])/(5*f)
 - (b*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e + f*x]^2])/(3*
f)
```

Maple [A] time = 0.042, size = 67, normalized size = 1.1

$$\frac{1}{f}\left(b\left(-\frac{\left(\cot\left(fx+e\right)\right)^3}{3}+\cot\left(fx+e\right)+fx+e\right)+a\left(-\frac{\left(\cot\left(fx+e\right)\right)^5}{5}+\frac{\left(\cot\left(fx+e\right)\right)^3}{3}-\cot\left(fx+e\right)-fx-e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*(b*(-1/3*cot(f*x+e)^3+cot(f*x+e)+f*x+e)+a*(-1/5*cot(f*x+e)^5+1/3*cot(f*
x+e)^3-cot(f*x+e)-f*x-e))
```

Maxima [A] time = 1.67998, size = 82, normalized size = 1.34

$$-\frac{15\left(fx+e\right)(a-b)+\frac{15\,(a-b)\tan\left(fx+e\right)^4-5\,(a-b)\tan\left(fx+e\right)^2+3\,a}{\tan\left(fx+e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(f*x + e)*(a - b) + (15*(a - b)*tan(f*x + e)^4 - 5*(a - b)*tan(f*
x + e)^2 + 3*a)/tan(f*x + e)^5)/f
```

Fricas [A] time = 1.08593, size = 161, normalized size = 2.64

$$-\frac{15\,(a-b)fx\tan\left(fx+e\right)^{5}+15\,(a-b)\tan\left(fx+e\right)^{4}-5\,(a-b)\tan\left(fx+e\right)^{2}+3\,a}{15\,f\tan\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/15*(15*(a - b)*f*x*tan(f*x + e)^5 + 15*(a - b)*tan(f*x + e)^4 - 5*(a - b
)*tan(f*x + e)^2 + 3*a)/(f*tan(f*x + e)^5)
```

Sympy [A] time = 18.2639, size = 97, normalized size = 1.59

$$\begin{cases} \tilde{\infty}ax & \text{for}\ \left(e=0\vee e=-fx\right)\wedge\left(e=-fx\vee\right. \\ x\left(a+b\tan^{2}(e)\right)\cot^{6}(e) & \text{for}\ f=0 \\ -ax-\frac{a}{f\tan\left(e+fx\right)}+\frac{a}{3f\tan^{3}\left(e+fx\right)}-\frac{a}{5f\tan^{5}\left(e+fx\right)}+bx+\frac{b}{f\tan\left(e+fx\right)}-\frac{b}{3f\tan^{3}\left(e+fx\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*a*x, (Eq(e, 0) | Eq(e, -f*x)) & (Eq(f, 0) | Eq(e, -f*x))), (
x*(a + b*tan(e)**2)*cot(e)**6, Eq(f, 0)), (-a*x - a/(f*tan(e + f*x)) + a/(3
*f*tan(e + f*x)**3) - a/(5*f*tan(e + f*x)**5) + b*x + b/(f*tan(e + f*x)) -
b/(3*f*tan(e + f*x)**3), True))
```

Giac [B] time = 1.34912, size = 227, normalized size = 3.72

$$\frac{3\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-35\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+20\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-480\left(fx+e\right)(a-b)+330\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-3}{480\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/480*(3*a*tan(1/2*f*x + 1/2*e)^5 - 35*a*tan(1/2*f*x + 1/2*e)^3 + 20*b*tan(
1/2*f*x + 1/2*e)^3 - 480*(f*x + e)*(a - b) + 330*a*tan(1/2*f*x + 1/2*e) - 3
00*b*tan(1/2*f*x + 1/2*e) - (330*a*tan(1/2*f*x + 1/2*e)^4 - 300*b*tan(1/2*f
*x + 1/2*e)^4 - 35*a*tan(1/2*f*x + 1/2*e)^2 + 20*b*tan(1/2*f*x + 1/2*e)^2 +
 3*a)/tan(1/2*f*x + 1/2*e)^5)/f
```

3.198 $\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=105

$$\frac{b(2a-b)\tan^6(e+fx)}{6f}+\frac{(a-b)^2\tan^4(e+fx)}{4f}-\frac{(a-b)^2\tan^2(e+fx)}{2f}-\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{b^2\tan^8(e+fx)}{8f}$$

```
[Out] -(((a - b)^2*Log[Cos[e + f*x]])/f) - ((a - b)^2*Tan[e + f*x]^2)/(2*f) + ((a
 - b)^2*Tan[e + f*x]^4)/(4*f) + ((2*a - b)*b*Tan[e + f*x]^6)/(6*f) + (b^2*T
an[e + f*x]^8)/(8*f)
```

Rubi [A] time = 0.108878, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 88}

$$\frac{b(2a-b)\tan^6(e+fx)}{6f}+\frac{(a-b)^2\tan^4(e+fx)}{4f}-\frac{(a-b)^2\tan^2(e+fx)}{2f}-\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{b^2\tan^8(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```


```
[Out] -(((a - b)^2*Log[Cos[e + f*x]])/f) - ((a - b)^2*Tan[e + f*x]^2)/(2*f) + ((a
 - b)^2*Tan[e + f*x]^4)/(4*f) + ((2*a - b)*b*Tan[e + f*x]^6)/(6*f) + (b^2*T
an[e + f*x]^8)/(8*f)
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{x^5\left(a+bx^2\right)^2}{1+x^2} dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^2}{1+x} dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(-(a-b)^2+(a-b)^2x+(2a-b)bx^2+b^2x^3+\frac{(a-b)^2}{1+x}\right) dx, x, \tan^2(e\right.}{2f}$$
$$= -\frac{(a-b)^2\log(\cos(e+fx))}{f} - \frac{(a-b)^2\tan^2(e+fx)}{2f} + \frac{(a-b)^2\tan^4(e+fx)}{4f} +$$

Mathematica [A] time = 0.339334, size = 89, normalized size = 0.85

$$\frac{4b(2a-b)\tan^6(e+fx)+6(a-b)^2\tan^4(e+fx)-12(a-b)^2\tan^2(e+fx)-24(a-b)^2\log(\cos(e+fx))+3b^2\tan^8(e+}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-24*(a - b)^2*Log[Cos[e + f*x]] - 12*(a - b)^2*Tan[e + f*x]^2 + 6*(a - b)^
2*Tan[e + f*x]^4 + 4*(2*a - b)*b*Tan[e + f*x]^6 + 3*b^2*Tan[e + f*x]^8)/(24
*f)
```

Maple [B] time = 0.006, size = 198, normalized size = 1.9

$$\frac{b^2\left(\tan\left(fx+e\right)\right)^8}{8f} + \frac{ab\left(\tan\left(fx+e\right)\right)^6}{3f} - \frac{b^2\left(\tan\left(fx+e\right)\right)^6}{6f} + \frac{a^2\left(\tan\left(fx+e\right)\right)^4}{4f} - \frac{\left(\tan\left(fx+e\right)\right)^4 ab}{2f} + \frac{b^2\left(\tan\left(fx+\right.\right.}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/8*b^2*tan(f*x+e)^8/f+1/3/f*a*b*tan(f*x+e)^6-1/6*b^2*tan(f*x+e)^6/f+1/4/f*
a^2*tan(f*x+e)^4-1/2/f*tan(f*x+e)^4*a*b+1/4/f*b^2*tan(f*x+e)^4-1/2/f*tan(f*
x+e)^2*a^2+1/f*tan(f*x+e)^2*a*b-1/2*b^2*tan(f*x+e)^2/f+1/2/f*ln(1+tan(f*x+e
)^2)*a^2-1/f*ln(1+tan(f*x+e)^2)*a*b+1/2/f*ln(1+tan(f*x+e)^2)*b^2
```

Maxima [A] time = 1.11073, size = 219, normalized size = 2.09

$$-\frac{12\left(a^2-2\,ab+b^2\right)\log\left(\sin\left(fx+e\right)^2-1\right) - \frac{24\left(a^2-3\,ab+2\,b^2\right)\sin\left(fx+e\right)^6-6\left(11\,a^2-30\,ab+18\,b^2\right)\sin\left(fx+e\right)^4+4\left(15\,a^2-38\,ab+22\,b^2\right)\sin\left(fx+e\right)}{\sin\left(fx+e\right)^8-4\,\sin\left(fx+e\right)^6+6\,\sin\left(fx+e\right)^4-4\,\sin\left(fx+e\right)^2+1}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/24*(12*(a^2 - 2*a*b + b^2)*log(sin(f*x + e)^2 - 1) - (24*(a^2 - 3*a*b +
2*b^2)*sin(f*x + e)^6 - 6*(11*a^2 - 30*a*b + 18*b^2)*sin(f*x + e)^4 + 4*(15
*a^2 - 38*a*b + 22*b^2)*sin(f*x + e)^2 - 18*a^2 + 44*a*b - 25*b^2)/(sin(f*x
 + e)^8 - 4*sin(f*x + e)^6 + 6*sin(f*x + e)^4 - 4*sin(f*x + e)^2 + 1))/f
```

Fricas [A] time = 1.19546, size = 265, normalized size = 2.52

$$\frac{3\,b^2\tan\left(fx+e\right)^8+4\left(2\,ab-b^2\right)\tan\left(fx+e\right)^6+6\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^4-12\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^2 \cdots}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^2*tan(f*x + e)^8 + 4*(2*a*b - b^2)*tan(f*x + e)^6 + 6*(a^2 - 2*a*
b + b^2)*tan(f*x + e)^4 - 12*(a^2 - 2*a*b + b^2)*tan(f*x + e)^2 - 12*(a^2 -
 2*a*b + b^2)*log(1/(tan(f*x + e)^2 + 1)))/f
```

Sympy [A] time = 1.68333, size = 206, normalized size = 1.96

$$\begin{cases} \frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a^2\tan^4\left(e+fx\right)}{4f}-\frac{a^2\tan^2\left(e+fx\right)}{2f}-\frac{ab\log\left(\tan^2\left(e+fx\right)+1\right)}{f}+\frac{ab\tan^6\left(e+fx\right)}{3f}-\frac{ab\tan^4\left(e+fx\right)}{2f}+\frac{ab\tan^2\left(e+fx\right)}{f}+ \cdots \\ x\left(a+b\tan^2\left(e\right)\right)^2\tan^5\left(e\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*tan(e + f*x)**4/(4*f)
 - a**2*tan(e + f*x)**2/(2*f) - a*b*log(tan(e + f*x)**2 + 1)/f + a*b*tan(e
+ f*x)**6/(3*f) - a*b*tan(e + f*x)**4/(2*f) + a*b*tan(e + f*x)**2/f + b**2*
log(tan(e + f*x)**2 + 1)/(2*f) + b**2*tan(e + f*x)**8/(8*f) - b**2*tan(e +
f*x)**6/(6*f) + b**2*tan(e + f*x)**4/(4*f) - b**2*tan(e + f*x)**2/(2*f), Ne
(f, 0)), (x*(a + b*tan(e)**2)**2*tan(e)**5, True))
```

Giac [B] time = 17.9805, size = 5148, normalized size = 49.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/24*(12*a^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(
e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^8*
tan(e)^8 - 24*a*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*
tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x
)^8*tan(e)^8 + 12*b^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x
)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan
(f*x)^8*tan(e)^8 + 18*a^2*tan(f*x)^8*tan(e)^8 - 44*a*b*tan(f*x)^8*tan(e)^8
+ 25*b^2*tan(f*x)^8*tan(e)^8 - 96*a^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(
```

e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*
tan(e) + 1))*tan(f*x)^7*tan(e)^7 + 192*a*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4
*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(
f*x)*tan(e) + 1))*tan(f*x)^7*tan(e)^7 - 96*b^2*log(4*(tan(e)^2 + 1)/(tan(f*
x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*
tan(f*x)*tan(e) + 1))*tan(f*x)^7*tan(e)^7 + 12*a^2*tan(f*x)^8*tan(e)^6 - 24
*a*b*tan(f*x)^8*tan(e)^6 + 12*b^2*tan(f*x)^8*tan(e)^6 - 120*a^2*tan(f*x)^7*
tan(e)^7 + 304*a*b*tan(f*x)^7*tan(e)^7 - 176*b^2*tan(f*x)^7*tan(e)^7 + 12*a
^2*tan(f*x)^6*tan(e)^8 - 24*a*b*tan(f*x)^6*tan(e)^8 + 12*b^2*tan(f*x)^6*tan
(e)^8 + 336*a^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*ta
n(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^
6*tan(e)^6 - 672*a*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)
^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(
f*x)^6*tan(e)^6 + 336*b^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan
(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))
*tan(f*x)^6*tan(e)^6 - 6*a^2*tan(f*x)^8*tan(e)^4 + 12*a*b*tan(f*x)^8*tan(e)
^4 - 6*b^2*tan(f*x)^8*tan(e)^4 - 96*a^2*tan(f*x)^7*tan(e)^5 + 192*a*b*tan(f
*x)^7*tan(e)^5 - 96*b^2*tan(f*x)^7*tan(e)^5 + 324*a^2*tan(f*x)^6*tan(e)^6 -
 872*a*b*tan(f*x)^6*tan(e)^6 + 520*b^2*tan(f*x)^6*tan(e)^6 - 96*a^2*tan(f*x
)^5*tan(e)^7 + 192*a*b*tan(f*x)^5*tan(e)^7 - 96*b^2*tan(f*x)^5*tan(e)^7 - 6
*a^2*tan(f*x)^4*tan(e)^8 + 12*a*b*tan(f*x)^4*tan(e)^8 - 6*b^2*tan(f*x)^4*ta
n(e)^8 - 672*a^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*t
an(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)
^5*tan(e)^5 + 1344*a*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*
x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*ta
n(f*x)^5*tan(e)^5 - 672*b^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*t
an(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1
))*tan(f*x)^5*tan(e)^5 - 8*a*b*tan(f*x)^8*tan(e)^2 + 4*b^2*tan(f*x)^8*tan(e
)^2 + 24*a^2*tan(f*x)^7*tan(e)^3 - 96*a*b*tan(f*x)^7*tan(e)^3 + 48*b^2*tan(
f*x)^7*tan(e)^3 + 276*a^2*tan(f*x)^6*tan(e)^4 - 672*a*b*tan(f*x)^6*tan(e)^4
 + 336*b^2*tan(f*x)^6*tan(e)^4 - 504*a^2*tan(f*x)^5*tan(e)^5 + 1296*a*b*tan
(f*x)^5*tan(e)^5 - 816*b^2*tan(f*x)^5*tan(e)^5 + 276*a^2*tan(f*x)^4*tan(e)^
6 - 672*a*b*tan(f*x)^4*tan(e)^6 + 336*b^2*tan(f*x)^4*tan(e)^6 + 24*a^2*tan(
f*x)^3*tan(e)^7 - 96*a*b*tan(f*x)^3*tan(e)^7 + 48*b^2*tan(f*x)^3*tan(e)^7 -
 8*a*b*tan(f*x)^2*tan(e)^8 + 4*b^2*tan(f*x)^2*tan(e)^8 + 840*a^2*log(4*(tan
(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2
 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(e)^4 - 1680*a*b*log(
4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*ta
n(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(e)^4 + 840*b^2
log(4(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)
^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(e)^4 - 3*
b^2*tan(f*x)^8 + 16*a*b*tan(f*x)^7*tan(e) - 32*b^2*tan(f*x)^7*tan(e) - 36*a
^2*tan(f*x)^6*tan(e)^2 + 168*a*b*tan(f*x)^6*tan(e)^2 - 168*b^2*tan(f*x)^6*t
an(e)^2 - 384*a^2*tan(f*x)^5*tan(e)^3 + 1008*a*b*tan(f*x)^5*tan(e)^3 - 672*
b^2*tan(f*x)^5*tan(e)^3 + 564*a^2*tan(f*x)^4*tan(e)^4 - 1368*a*b*tan(f*x)^4
*tan(e)^4 + 684*b^2*tan(f*x)^4*tan(e)^4 - 384*a^2*tan(f*x)^3*tan(e)^5 + 100
8*a*b*tan(f*x)^3*tan(e)^5 - 672*b^2*tan(f*x)^3*tan(e)^5 - 36*a^2*tan(f*x)^2
*tan(e)^6 + 168*a*b*tan(f*x)^2*tan(e)^6 - 168*b^2*tan(f*x)^2*tan(e)^6 + 16*
a*b*tan(f*x)*tan(e)^7 - 32*b^2*tan(f*x)*tan(e)^7 - 3*b^2*tan(e)^8 - 672*a^2
log(4(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)
^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 + 13
44*a*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + ta
n(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^
3 - 672*b^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
 + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*ta
n(e)^3 - 8*a*b*tan(f*x)^6 + 4*b^2*tan(f*x)^6 + 24*a^2*tan(f*x)^5*tan(e) - 9
6*a*b*tan(f*x)^5*tan(e) + 48*b^2*tan(f*x)^5*tan(e) + 276*a^2*tan(f*x)^4*tan
(e)^2 - 672*a*b*tan(f*x)^4*tan(e)^2 + 336*b^2*tan(f*x)^4*tan(e)^2 - 504*a^2
*tan(f*x)^3*tan(e)^3 + 1296*a*b*tan(f*x)^3*tan(e)^3 - 816*b^2*tan(f*x)^3*ta

```
n(e)^3 + 276*a^2*tan(f*x)^2*tan(e)^4 - 672*a*b*tan(f*x)^2*tan(e)^4 + 336*b^
2*tan(f*x)^2*tan(e)^4 + 24*a^2*tan(f*x)*tan(e)^5 - 96*a*b*tan(f*x)*tan(e)^5
 + 48*b^2*tan(f*x)*tan(e)^5 - 8*a*b*tan(e)^6 + 4*b^2*tan(e)^6 + 336*a^2*log
(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*t
an(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 - 672*a*
b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x
)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 + 3
36*b^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + ta
n(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^
2 - 6*a^2*tan(f*x)^4 + 12*a*b*tan(f*x)^4 - 6*b^2*tan(f*x)^4 - 96*a^2*tan(f*
x)^3*tan(e) + 192*a*b*tan(f*x)^3*tan(e) - 96*b^2*tan(f*x)^3*tan(e) + 324*a^
2*tan(f*x)^2*tan(e)^2 - 872*a*b*tan(f*x)^2*tan(e)^2 + 520*b^2*tan(f*x)^2*ta
n(e)^2 - 96*a^2*tan(f*x)*tan(e)^3 + 192*a*b*tan(f*x)*tan(e)^3 - 96*b^2*tan(
f*x)*tan(e)^3 - 6*a^2*tan(e)^4 + 12*a*b*tan(e)^4 - 6*b^2*tan(e)^4 - 96*a^2*
log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^
2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) + 192*a*b
*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)
^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) - 96*b^2
*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)
^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) + 12*a^2
*tan(f*x)^2 - 24*a*b*tan(f*x)^2 + 12*b^2*tan(f*x)^2 - 120*a^2*tan(f*x)*tan(
e) + 304*a*b*tan(f*x)*tan(e) - 176*b^2*tan(f*x)*tan(e) + 12*a^2*tan(e)^2 -
24*a*b*tan(e)^2 + 12*b^2*tan(e)^2 + 12*a^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4
*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(
f*x)*tan(e) + 1)) - 24*a*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*ta
n(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)
) + 12*b^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
+ tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) + 18*a^2 - 44*
a*b + 25*b^2)/(f*tan(f*x)^8*tan(e)^8 - 8*f*tan(f*x)^7*tan(e)^7 + 28*f*tan(f
*x)^6*tan(e)^6 - 56*f*tan(f*x)^5*tan(e)^5 + 70*f*tan(f*x)^4*tan(e)^4 - 56*f
*tan(f*x)^3*tan(e)^3 + 28*f*tan(f*x)^2*tan(e)^2 - 8*f*tan(f*x)*tan(e) + f)
```

3.199 $\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=82

$$\frac{b(2a-b)\tan^4(e+fx)}{4f}+\frac{(a-b)^2\tan^2(e+fx)}{2f}+\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{b^2\tan^6(e+fx)}{6f}$$

```
[Out] ((a - b)^2*Log[Cos[e + f*x]])/f + ((a - b)^2*Tan[e + f*x]^2)/(2*f) + ((2*a
- b)*b*Tan[e + f*x]^4)/(4*f) + (b^2*Tan[e + f*x]^6)/(6*f)
```

Rubi [A] time = 0.0947573, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 77}

$$\frac{b(2a-b)\tan^4(e+fx)}{4f}+\frac{(a-b)^2\tan^2(e+fx)}{2f}+\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{b^2\tan^6(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((a - b)^2*Log[Cos[e + f*x]])/f + ((a - b)^2*Tan[e + f*x]^2)/(2*f) + ((2*a
- b)*b*Tan[e + f*x]^4)/(4*f) + (b^2*Tan[e + f*x]^6)/(6*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{x^3(a+bx^2)^2}{1+x^2}dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x(a+bx)^2}{1+x}dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left((a-b)^2+(2a-b)bx+b^2x^2-\frac{(a-b)^2}{1+x}\right)dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{(a-b)^2\log(\cos(e+fx))}{f} + \frac{(a-b)^2\tan^2(e+fx)}{2f} + \frac{(2a-b)b\tan^4(e+fx}{4f}$$

Mathematica [A] time = 0.270865, size = 72, normalized size = 0.88

$$\frac{3b(2a-b)\tan^4(e+fx)+6(a-b)^2\tan^2(e+fx)+12(a-b)^2\log(\cos(e+fx))+2b^2\tan^6(e+fx)}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (12*(a - b)^2*Log[Cos[e + f*x]] + 6*(a - b)^2*Tan[e + f*x]^2 + 3*(2*a - b)*
b*Tan[e + f*x]^4 + 2*b^2*Tan[e + f*x]^6)/(12*f)
```

Maple [A] time = 0.004, size = 151, normalized size = 1.8

$$\frac{b^2\left(\tan\left(fx+e\right)\right)^6}{6f} + \frac{\left(\tan\left(fx+e\right)\right)^4 ab}{2f} - \frac{b^2\left(\tan\left(fx+e\right)\right)^4}{4f} + \frac{\left(\tan\left(fx+e\right)\right)^2 a^2}{2f} - \frac{\left(\tan\left(fx+e\right)\right)^2 ab}{f} + \frac{b^2\left(\tan\left(f\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/6*b^2*tan(f*x+e)^6/f+1/2/f*tan(f*x+e)^4*a*b-1/4/f*b^2*tan(f*x+e)^4+1/2/f*
tan(f*x+e)^2*a^2-1/f*tan(f*x+e)^2*a*b+1/2*b^2*tan(f*x+e)^2/f-1/2/f*ln(1+tan
(f*x+e)^2)*a^2+1/f*ln(1+tan(f*x+e)^2)*a*b-1/2/f*ln(1+tan(f*x+e)^2)*b^2
```

Maxima [A] time = 1.17388, size = 171, normalized size = 2.09

$$\frac{6\left(a^2-2ab+b^2\right)\log\left(\sin\left(fx+e\right)^2-1\right)-\frac{6\left(a^2-4ab+3b^2\right)\sin\left(fx+e\right)^4-3\left(4a^2-14ab+9b^2\right)\sin\left(fx+e\right)^2+6a^2-18ab+11b^2}{\sin\left(fx+e\right)^6-3\sin\left(fx+e\right)^4+3\sin\left(fx+e\right)^2-1}}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(6*(a^2 - 2*a*b + b^2)*log(sin(f*x + e)^2 - 1) - (6*(a^2 - 4*a*b + 3*b
^2)*sin(f*x + e)^4 - 3*(4*a^2 - 14*a*b + 9*b^2)*sin(f*x + e)^2 + 6*a^2 - 18
```

```
*a*b + 11*b^2)/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1))/
f
```

Fricas [A] time = 1.11725, size = 209, normalized size = 2.55

$$\frac{2\,b^2\tan\left(fx+e\right)^6+3\left(2\,ab-b^2\right)\tan\left(fx+e\right)^4+6\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^2+6\left(a^2-2\,ab+b^2\right)\log\left(\frac{1}{\tan\left(fx+e\right)^2+1}\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^2*tan(f*x + e)^6 + 3*(2*a*b - b^2)*tan(f*x + e)^4 + 6*(a^2 - 2*a*
b + b^2)*tan(f*x + e)^2 + 6*(a^2 - 2*a*b + b^2)*log(1/(tan(f*x + e)^2 + 1))
)/f
```

Sympy [A] time = 0.991556, size = 160, normalized size = 1.95

$$\begin{cases} -\frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a^2\tan^2\left(e+fx\right)}{2f}+\frac{ab\log\left(\tan^2\left(e+fx\right)+1\right)}{f}+\frac{ab\tan^4\left(e+fx\right)}{2f}-\frac{ab\tan^2\left(e+fx\right)}{f}-\frac{b^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{b^2\tan^6\left(e+fx\right)}{6f} \\ x\left(a+b\tan^2\left(e\right)\right)^2\tan^3\left(e\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((-a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*tan(e + f*x)**2/(2*f
) + a*b*log(tan(e + f*x)**2 + 1)/f + a*b*tan(e + f*x)**4/(2*f) - a*b*tan(e
+ f*x)**2/f - b**2*log(tan(e + f*x)**2 + 1)/(2*f) + b**2*tan(e + f*x)**6/(6
*f) - b**2*tan(e + f*x)**4/(4*f) + b**2*tan(e + f*x)**2/(2*f), Ne(f, 0)), (
x*(a + b*tan(e)**2)**2*tan(e)**3, True))
```

Giac [B] time = 7.75961, size = 3510, normalized size = 42.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/12*(6*a^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
 + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^6*ta
n(e)^6 - 12*a*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*ta
n(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^
6*tan(e)^6 + 6*b^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3
*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*
x)^6*tan(e)^6 + 6*a^2*tan(f*x)^6*tan(e)^6 - 18*a*b*tan(f*x)^6*tan(e)^6 + 11
*b^2*tan(f*x)^6*tan(e)^6 - 36*a^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2
 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(
e) + 1))*tan(f*x)^5*tan(e)^5 + 72*a*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(
e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*
```

```
tan(e) + 1))*tan(f*x)^5*tan(e)^5 - 36*b^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*
tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f
*x)*tan(e) + 1))*tan(f*x)^5*tan(e)^5 + 6*a^2*tan(f*x)^6*tan(e)^4 - 12*a*b*t
an(f*x)^6*tan(e)^4 + 6*b^2*tan(f*x)^6*tan(e)^4 - 24*a^2*tan(f*x)^5*tan(e)^5
 + 84*a*b*tan(f*x)^5*tan(e)^5 - 54*b^2*tan(f*x)^5*tan(e)^5 + 6*a^2*tan(f*x)
^4*tan(e)^6 - 12*a*b*tan(f*x)^4*tan(e)^6 + 6*b^2*tan(f*x)^4*tan(e)^6 + 90*a
^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*
x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(e)^4 -
180*a*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + t
an(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(e)
^4 + 90*b^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
 + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*ta
n(e)^4 + 6*a*b*tan(f*x)^6*tan(e)^2 - 3*b^2*tan(f*x)^6*tan(e)^2 - 24*a^2*tan
(f*x)^5*tan(e)^3 + 72*a*b*tan(f*x)^5*tan(e)^3 - 36*b^2*tan(f*x)^5*tan(e)^3
+ 42*a^2*tan(f*x)^4*tan(e)^4 - 138*a*b*tan(f*x)^4*tan(e)^4 + 99*b^2*tan(f*x
)^4*tan(e)^4 - 24*a^2*tan(f*x)^3*tan(e)^5 + 72*a*b*tan(f*x)^3*tan(e)^5 - 36
*b^2*tan(f*x)^3*tan(e)^5 + 6*a*b*tan(f*x)^2*tan(e)^6 - 3*b^2*tan(f*x)^2*tan
(e)^6 - 120*a^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*ta
n(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^
3*tan(e)^3 + 240*a*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)
^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(
f*x)^3*tan(e)^3 - 120*b^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan
(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))
*tan(f*x)^3*tan(e)^3 + 2*b^2*tan(f*x)^6 - 12*a*b*tan(f*x)^5*tan(e) + 18*b^2
*tan(f*x)^5*tan(e) + 36*a^2*tan(f*x)^4*tan(e)^2 - 120*a*b*tan(f*x)^4*tan(e)
^2 + 90*b^2*tan(f*x)^4*tan(e)^2 - 48*a^2*tan(f*x)^3*tan(e)^3 + 144*a*b*tan(
f*x)^3*tan(e)^3 - 72*b^2*tan(f*x)^3*tan(e)^3 + 36*a^2*tan(f*x)^2*tan(e)^4 -
 120*a*b*tan(f*x)^2*tan(e)^4 + 90*b^2*tan(f*x)^2*tan(e)^4 - 12*a*b*tan(f*x)
*tan(e)^5 + 18*b^2*tan(f*x)*tan(e)^5 + 2*b^2*tan(e)^6 + 90*a^2*log(4*(tan(e
)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 +
 tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 - 180*a*b*log(4*(
tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e
)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 + 90*b^2*log
(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*t
an(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 + 6*a*b*
tan(f*x)^4 - 3*b^2*tan(f*x)^4 - 24*a^2*tan(f*x)^3*tan(e) + 72*a*b*tan(f*x)^
3*tan(e) - 36*b^2*tan(f*x)^3*tan(e) + 42*a^2*tan(f*x)^2*tan(e)^2 - 138*a*b*
tan(f*x)^2*tan(e)^2 + 99*b^2*tan(f*x)^2*tan(e)^2 - 24*a^2*tan(f*x)*tan(e)^3
 + 72*a*b*tan(f*x)*tan(e)^3 - 36*b^2*tan(f*x)*tan(e)^3 + 6*a*b*tan(e)^4 - 3
*b^2*tan(e)^4 - 36*a^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*
x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*ta
n(f*x)*tan(e) + 72*a*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*
x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*ta
n(f*x)*tan(e) - 36*b^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*
x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*ta
n(f*x)*tan(e) + 6*a^2*tan(f*x)^2 - 12*a*b*tan(f*x)^2 + 6*b^2*tan(f*x)^2 - 2
4*a^2*tan(f*x)*tan(e) + 84*a*b*tan(f*x)*tan(e) - 54*b^2*tan(f*x)*tan(e) + 6
*a^2*tan(e)^2 - 12*a*b*tan(e)^2 + 6*b^2*tan(e)^2 + 6*a^2*log(4*(tan(e)^2 +
1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f
*x)^2 - 2*tan(f*x)*tan(e) + 1)) - 12*a*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*t
an(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*
x)*tan(e) + 1)) + 6*b^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f
*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) +
 6*a^2 - 18*a*b + 11*b^2)/(f*tan(f*x)^6*tan(e)^6 - 6*f*tan(f*x)^5*tan(e)^5
+ 15*f*tan(f*x)^4*tan(e)^4 - 20*f*tan(f*x)^3*tan(e)^3 + 15*f*tan(f*x)^2*tan
(e)^2 - 6*f*tan(f*x)*tan(e) + f)
```

3.200 $\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx$

Optimal. Leaf size=62

$$\frac{b(a-b)\tan^2(e+fx)}{2f}+\frac{\left(a+b\tan^2(e+fx)\right)^2}{4f}-\frac{(a-b)^2\log(\cos(e+fx))}{f}$$

```
[Out] -(((a - b)^2*Log[Cos[e + f*x]])/f) + ((a - b)*b*Tan[e + f*x]^2)/(2*f) + (a
+ b*Tan[e + f*x]^2)^2/(4*f)
```

Rubi [A] time = 0.0604605, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3670, 444, 43}

$$\frac{b(a-b)\tan^2(e+fx)}{2f}+\frac{\left(a+b\tan^2(e+fx)\right)^2}{4f}-\frac{(a-b)^2\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(((a - b)^2*Log[Cos[e + f*x]])/f) + ((a - b)*b*Tan[e + f*x]^2)/(2*f) + (a
+ b*Tan[e + f*x]^2)^2/(4*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x\left(a+bx^2\right)^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left((a-b)b+\frac{(a-b)^2}{1+x}+b(a+bx)\right)dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{(a-b)b\tan^2(e+fx)}{2f}+\frac{\left(a+b\tan^2(e+fx)\right)}{4f}$$

Mathematica [A] time = 0.2131, size = 54, normalized size = 0.87

$$\frac{2b(2a-b)\tan^2(e+fx)-4(a-b)^2\log(\cos(e+fx))+b^2\tan^4(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-4*(a - b)^2*Log[Cos[e + f*x]] + 2*(2*a - b)*b*Tan[e + f*x]^2 + b^2*Tan[e
+ f*x]^4)/(4*f)
```

Maple [A] time = 0.004, size = 104, normalized size = 1.7

$$\frac{b^2\left(\tan\left(fx+e\right)\right)^4}{4f}+\frac{\left(\tan\left(fx+e\right)\right)^2ab}{f}-\frac{b^2\left(\tan\left(fx+e\right)\right)^2}{2f}+\frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^2\right)a^2}{2f}-\frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^2}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/4/f*b^2*tan(f*x+e)^4+1/f*tan(f*x+e)^2*a*b-1/2*b^2*tan(f*x+e)^2/f+1/2/f*ln
(1+tan(f*x+e)^2)*a^2-1/f*ln(1+tan(f*x+e)^2)*a*b+1/2/f*ln(1+tan(f*x+e)^2)*b^
2
```

Maxima [A] time = 1.12846, size = 111, normalized size = 1.79

$$-\frac{2\left(a^2-2ab+b^2\right)\log\left(\sin\left(fx+e\right)^2-1\right)+\frac{4\left(ab-b^2\right)\sin\left(fx+e\right)^2-4ab+3b^2}{\sin\left(fx+e\right)^4-2\sin\left(fx+e\right)^2+1}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(a^2 - 2*a*b + b^2)*log(sin(f*x + e)^2 - 1) + (4*(a*b - b^2)*sin(f*
x + e)^2 - 4*a*b + 3*b^2)/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

Fricas [A] time = 1.21043, size = 153, normalized size = 2.47

$$\frac{b^2\tan\left(fx+e\right)^4 + 2\left(2\,ab-b^2\right)\tan\left(fx+e\right)^2 - 2\left(a^2-2\,ab+b^2\right)\log\left(\frac{1}{\tan\left(fx+e\right)^2+1}\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*tan(f*x + e)^4 + 2*(2*a*b - b^2)*tan(f*x + e)^2 - 2*(a^2 - 2*a*b +
 b^2)*log(1/(tan(f*x + e)^2 + 1)))/f
```

Sympy [A] time = 0.526829, size = 112, normalized size = 1.81

$$\begin{cases} \frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} - \frac{ab\log\left(\tan^2\left(e+fx\right)+1\right)}{f} + \frac{ab\tan^2\left(e+fx\right)}{f} + \frac{b^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} + \frac{b^2\tan^4\left(e+fx\right)}{4f} - \frac{b^2\tan^2\left(e+fx\right)}{2f} & \text{for}\ f \neq 0 \\ x\left(a+b\tan^2\left(e\right)\right)^2\tan\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((a**2*log(tan(e + f*x)**2 + 1)/(2*f) - a*b*log(tan(e + f*x)**2 +
1)/f + a*b*tan(e + f*x)**2/f + b**2*log(tan(e + f*x)**2 + 1)/(2*f) + b**2*t
an(e + f*x)**4/(4*f) - b**2*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(a + b*tan
(e)**2)**2*tan(e), True))
```

Giac [B] time = 3.38934, size = 2039, normalized size = 32.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*a^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
 + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*ta
n(e)^4 - 4*a*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan
(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4
*tan(e)^4 + 2*b^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*
tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x
)^4*tan(e)^4 - 4*a*b*tan(f*x)^4*tan(e)^4 + 3*b^2*tan(f*x)^4*tan(e)^4 - 8*a^
2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x
)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 + 1
6*a*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan
(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3
 - 8*b^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) +
tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e
)^3 - 4*a*b*tan(f*x)^4*tan(e)^2 + 2*b^2*tan(f*x)^4*tan(e)^2 + 8*a*b*tan(f*x
)^3*tan(e)^3 - 8*b^2*tan(f*x)^3*tan(e)^3 - 4*a*b*tan(f*x)^2*tan(e)^4 + 2*b^
2*tan(f*x)^2*tan(e)^4 + 12*a^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 -
```

```
2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e)
+ 1))*tan(f*x)^2*tan(e)^2 - 24*a*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^
2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan
(e) + 1))*tan(f*x)^2*tan(e)^2 + 12*b^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan
(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)
*tan(e) + 1))*tan(f*x)^2*tan(e)^2 - b^2*tan(f*x)^4 + 8*a*b*tan(f*x)^3*tan(e
) - 8*b^2*tan(f*x)^3*tan(e) - 8*a*b*tan(f*x)^2*tan(e)^2 + 4*b^2*tan(f*x)^2*
tan(e)^2 + 8*a*b*tan(f*x)*tan(e)^3 - 8*b^2*tan(f*x)*tan(e)^3 - b^2*tan(e)^4
 - 8*a^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) +
tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e)
+ 16*a*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) +
tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e)
- 8*b^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + t
an(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) -
 4*a*b*tan(f*x)^2 + 2*b^2*tan(f*x)^2 + 8*a*b*tan(f*x)*tan(e) - 8*b^2*tan(f*
x)*tan(e) - 4*a*b*tan(e)^2 + 2*b^2*tan(e)^2 + 2*a^2*log(4*(tan(e)^2 + 1)/(t
an(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2
 - 2*tan(f*x)*tan(e) + 1)) - 4*a*b*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^
2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan
(e) + 1)) + 2*b^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*
tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) - 4*a*b
 + 3*b^2)/(f*tan(f*x)^4*tan(e)^4 - 4*f*tan(f*x)^3*tan(e)^3 + 6*f*tan(f*x)^2
*tan(e)^2 - 4*f*tan(f*x)*tan(e) + f)
```

3.201 $\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx$

Optimal. Leaf size=51

$$\frac{a^2\log(\tan(e+fx))}{f}+\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{b^2\tan^2(e+fx)}{2f}$$

```
[Out] ((a - b)^2*Log[Cos[e + f*x]])/f + (a^2*Log[Tan[e + f*x]])/f + (b^2*Tan[e +
f*x]^2)/(2*f)
```

Rubi [A] time = 0.0635241, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3670, 446, 72}

$$\frac{a^2\log(\tan(e+fx))}{f}+\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{b^2\tan^2(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((a - b)^2*Log[Cos[e + f*x]])/f + (a^2*Log[Tan[e + f*x]])/f + (b^2*Tan[e +
f*x]^2)/(2*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x(1+x^2)}dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{x(1+x)}dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(b^2+\frac{a^2}{x}-\frac{(a-b)^2}{1+x}\right)dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{(a-b)^2\log(\cos(e+fx))}{f} + \frac{a^2\log(\tan(e+fx))}{f} + \frac{b^2\tan^2(e+fx)}{2f}$$

Mathematica [A] time = 0.120248, size = 65, normalized size = 1.27

$$\frac{a^2(\log(\tan(e+fx))+\log(\cos(e+fx)))}{f} - \frac{2ab\log(\cos(e+fx))}{f} + \frac{b^2\left(\tan^2(e+fx)+2\log(\cos(e+fx))\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-2*a*b*Log[Cos[e + f*x]])/f + (a^2*(Log[Cos[e + f*x]] + Log[Tan[e + f*x]])
)/f + (b^2*(2*Log[Cos[e + f*x]] + Tan[e + f*x]^2))/(2*f)
```

Maple [A] time = 0.052, size = 60, normalized size = 1.2

$$\frac{b^2\left(\tan(fx+e)\right)^2}{2f} + \frac{b^2\ln(\cos(fx+e))}{f} - 2\frac{ab\ln(\cos(fx+e))}{f} + \frac{a^2\ln(\sin(fx+e))}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/2*b^2*tan(f*x+e)^2/f+1/f*b^2*ln(cos(f*x+e))-2/f*a*b*ln(cos(f*x+e))+1/f*a^
2*ln(sin(f*x+e))
```

Maxima [A] time = 1.08708, size = 80, normalized size = 1.57

$$\frac{a^2\log\left(\sin(fx+e)^2\right) - \left(2ab-b^2\right)\log\left(\sin(fx+e)^2-1\right) - \frac{b^2}{\sin(fx+e)^2-1}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*log(sin(f*x + e)^2) - (2*a*b - b^2)*log(sin(f*x + e)^2 - 1) - b^2/
(sin(f*x + e)^2 - 1))/f
```

Fricas [A] time = 1.16974, size = 161, normalized size = 3.16

$$\frac{b^2\tan\left(fx+e\right)^2+a^2\log\left(\frac{\tan\left(fx+e\right)^2}{\tan\left(fx+e\right)^2+1}\right)-\left(2\,ab-b^2\right)\log\left(\frac{1}{\tan\left(fx+e\right)^2+1}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*tan(f*x + e)^2 + a^2*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) - (2
*a*b - b^2)*log(1/(tan(f*x + e)^2 + 1)))/f
```

Sympy [A] time = 2.48656, size = 97, normalized size = 1.9

$$\begin{cases} -\frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a^2\log\left(\tan\left(e+fx\right)\right)}{f}+\frac{ab\log\left(\tan^2\left(e+fx\right)+1\right)}{f}-\frac{b^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{b^2\tan^2\left(e+fx\right)}{2f} & \text{for } f\neq 0 \\ x\left(a+b\tan^2\left(e\right)\right)^2\cot\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((-a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*log(tan(e + f*x))/f
+ a*b*log(tan(e + f*x)**2 + 1)/f - b**2*log(tan(e + f*x)**2 + 1)/(2*f) + b*
*2*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(a + b*tan(e)**2)**2*cot(e), True))
```

Giac [A] time = 1.57969, size = 124, normalized size = 2.43

$$\frac{a^2\log\left(\sin\left(fx+e\right)^2\right)-\left(2\,ab-b^2\right)\log\left(-\sin\left(fx+e\right)^2+1\right)+\frac{2\,ab\sin\left(fx+e\right)^2-b^2\sin\left(fx+e\right)^2-2\,ab}{\sin\left(fx+e\right)^2-1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*log(sin(f*x + e)^2) - (2*a*b - b^2)*log(-sin(f*x + e)^2 + 1) + (2*
a*b*sin(f*x + e)^2 - b^2*sin(f*x + e)^2 - 2*a*b)/(sin(f*x + e)^2 - 1))/f
```

3.202 $\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=56

$$-\frac{a^2\cot^2(e+fx)}{2f}-\frac{a(a-2b)\log(\tan(e+fx))}{f}-\frac{(a-b)^2\log(\cos(e+fx))}{f}$$

```
[Out] -(a^2*Cot[e + f*x]^2)/(2*f) - ((a - b)^2*Log[Cos[e + f*x]])/f - (a*(a - 2*b
)*Log[Tan[e + f*x]])/f
```

Rubi [A] time = 0.0816643, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 88}

$$-\frac{a^2\cot^2(e+fx)}{2f}-\frac{a(a-2b)\log(\tan(e+fx))}{f}-\frac{(a-b)^2\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(a^2*Cot[e + f*x]^2)/(2*f) - ((a - b)^2*Log[Cos[e + f*x]])/f - (a*(a - 2*b
)*Log[Tan[e + f*x]])/f
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^3(1+x^2)} dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{x^2(1+x)} dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^2} - \frac{a(a-2b)}{x} + \frac{(a-b)^2}{1+x}\right) dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\cot^2(e+fx)}{2f} - \frac{(a-b)^2\log(\cos(e+fx))}{f} - \frac{a(a-2b)\log(\tan(e+fx))}{f}$$

Mathematica [A] time = 0.243094, size = 51, normalized size = 0.91

$$-\frac{a^2\cot^2(e+fx) + 2a(a-2b)\log(\tan(e+fx)) + 2(a-b)^2\log(\cos(e+fx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(a^2*Cot[e + f*x]^2 + 2*(a - b)^2*Log[Cos[e + f*x]] + 2*a*(a - 2*b)*Log[Ta
n[e + f*x]])/(2*f)
```

Maple [A] time = 0.056, size = 62, normalized size = 1.1

$$-\frac{b^2\ln\left(\cos\left(fx+e\right)\right)}{f} + 2\frac{ab\ln\left(\sin\left(fx+e\right)\right)}{f} - \frac{a^2\left(\cot\left(fx+e\right)\right)^2}{2f} - \frac{a^2\ln\left(\sin\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/f*b^2*ln(cos(f*x+e))+2/f*a*b*ln(sin(f*x+e))-1/2*a^2*cot(f*x+e)^2/f-1/f*a
^2*ln(sin(f*x+e))
```

Maxima [A] time = 1.11063, size = 69, normalized size = 1.23

$$-\frac{b^2\log\left(\sin\left(fx+e\right)^2-1\right) + \left(a^2-2\,ab\right)\log\left(\sin\left(fx+e\right)^2\right) + \frac{a^2}{\sin\left(fx+e\right)^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*log(sin(f*x + e)^2 - 1) + (a^2 - 2*a*b)*log(sin(f*x + e)^2) + a^2
/sin(f*x + e)^2)/f
```

Fricas [A] time = 1.12393, size = 234, normalized size = 4.18

$$-\frac{b^2\log\left(\frac{1}{\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)^2+a^2\tan\left(fx+e\right)^2+\left(a^2-2\,ab\right)\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)^2+a^2}{2\,f\tan\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*log(1/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + a^2*tan(f*x + e)^2 +
 (a^2 - 2*a*b)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + a^
2)/(f*tan(f*x + e)^2)
```

Sympy [A] time = 9.12316, size = 131, normalized size = 2.34

$$\begin{cases} \tilde{\infty}a^2x & \text{fo} \\ x\left(a+b\tan^2\left(e\right)\right)^2\cot^3\left(e\right) & \text{fo} \\ \frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}-\frac{a^2\log\left(\tan\left(e+fx\right)\right)}{f}-\frac{a^2}{2f\tan^2\left(e+fx\right)}-\frac{ab\log\left(\tan^2\left(e+fx\right)+1\right)}{f}+\frac{2ab\log\left(\tan\left(e+fx\right)\right)}{f}+\frac{b^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} & \text{o} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*a**2*x, (Eq(e, 0) | Eq(e, -f*x)) & (Eq(f, 0) | Eq(e, -f*x)))
, (x*(a + b*tan(e)**2)**2*cot(e)**3, Eq(f, 0)), (a**2*log(tan(e + f*x)**2 +
 1)/(2*f) - a**2*log(tan(e + f*x))/f - a**2/(2*f*tan(e + f*x)**2) - a*b*log
(tan(e + f*x)**2 + 1)/f + 2*a*b*log(tan(e + f*x))/f + b**2*log(tan(e + f*x)
**2 + 1)/(2*f), True))
```

Giac [B] time = 1.64026, size = 223, normalized size = 3.98

$$\frac{a^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-4\,b^2\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right)+4\left(a^2-2\,ab+b^2\right)\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+}{\cos(fx+}\right.}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f
*x + e) + 1)) - 4*b^2*log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) - 2) + 4*(a^2 - 2*a*b + b^2)*log(-(cos(f*x +
e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2))/f
```

3.203 $\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=76

$$-\frac{a^2\cot^4(e+fx)}{4f}+\frac{a(a-2b)\cot^2(e+fx)}{2f}+\frac{(a-b)^2\log(\tan(e+fx))}{f}+\frac{(a-b)^2\log(\cos(e+fx))}{f}$$

```
[Out] (a*(a - 2*b)*Cot[e + f*x]^2)/(2*f) - (a^2*Cot[e + f*x]^4)/(4*f) + ((a - b)^
2*Log[Cos[e + f*x]])/f + ((a - b)^2*Log[Tan[e + f*x]])/f
```

Rubi [A] time = 0.0883688, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 88}

$$-\frac{a^2\cot^4(e+fx)}{4f}+\frac{a(a-2b)\cot^2(e+fx)}{2f}+\frac{(a-b)^2\log(\tan(e+fx))}{f}+\frac{(a-b)^2\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (a*(a - 2*b)*Cot[e + f*x]^2)/(2*f) - (a^2*Cot[e + f*x]^4)/(4*f) + ((a - b)^
2*Log[Cos[e + f*x]])/f + ((a - b)^2*Log[Tan[e + f*x]])/f
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{x^5(1+x^2)}dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{x^3(1+x)}dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^3} - \frac{a(a-2b)}{x^2} + \frac{(a-b)^2}{x} - \frac{(a-b)^2}{1+x}\right)dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{a(a-2b)\cot^2(e+fx)}{2f} - \frac{a^2\cot^4(e+fx)}{4f} + \frac{(a-b)^2\log(\cos(e+fx))}{f} + \frac{(a$$

Mathematica [A] time = 0.289288, size = 61, normalized size = 0.8

$$\frac{-a^2\cot^4(e+fx) + 2a(a-2b)\cot^2(e+fx) + 4(a-b)^2(\log(\tan(e+fx)) + \log(\cos(e+fx)))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (2*a*(a - 2*b)*Cot[e + f*x]^2 - a^2*Cot[e + f*x]^4 + 4*(a - b)^2*(Log[Cos[e
 + f*x]] + Log[Tan[e + f*x]]))/(4*f)
```

Maple [A] time = 0.053, size = 91, normalized size = 1.2

$$\frac{b^2\ln\left(\sin\left(fx+e\right)\right)}{f} - \frac{ab\left(\cot\left(fx+e\right)\right)^2}{f} - 2\frac{ab\ln\left(\sin\left(fx+e\right)\right)}{f} - \frac{a^2\left(\cot\left(fx+e\right)\right)^4}{4f} + \frac{a^2\left(\cot\left(fx+e\right)\right)^2}{2f} + \frac{a^2\ln\left(s\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*b^2*ln(sin(f*x+e))-1/f*a*b*cot(f*x+e)^2-2/f*a*b*ln(sin(f*x+e))-1/4*a^2*
cot(f*x+e)^4/f+1/2*a^2*cot(f*x+e)^2/f+1/f*a^2*ln(sin(f*x+e))
```

Maxima [A] time = 1.07737, size = 82, normalized size = 1.08

$$\frac{2\left(a^2 - 2\,ab + b^2\right)\log\left(\sin\left(fx+e\right)^2\right) + \frac{4\left(a^2 - ab\right)\sin\left(fx+e\right)^2 - a^2}{\sin\left(fx+e\right)^4}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a^2 - 2*a*b + b^2)*log(sin(f*x + e)^2) + (4*(a^2 - a*b)*sin(f*x + e
)^2 - a^2)/sin(f*x + e)^4)/f
```

Fricas [A] time = 1.059, size = 238, normalized size = 3.13

$$\frac{2\left(a^{2}-2\,ab+b^{2}\right)\log\left(\frac{\tan\left(fx+e\right)^{2}}{\tan\left(fx+e\right)^{2}+1}\right)\tan\left(fx+e\right)^{4}+\left(3\,a^{2}-4\,ab\right)\tan\left(fx+e\right)^{4}+2\left(a^{2}-2\,ab\right)\tan\left(fx+e\right)^{2}-a^{2}}{4\,f\tan\left(fx+e\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^2 - 2*a*b + b^2)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x
 + e)^4 + (3*a^2 - 4*a*b)*tan(f*x + e)^4 + 2*(a^2 - 2*a*b)*tan(f*x + e)^2 -
 a^2)/(f*tan(f*x + e)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.6882, size = 420, normalized size = 5.53

$$-\frac{\frac{12\,a^{2}\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{16\,ab\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{a^{2}\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+64\left(a^{2}-2\,ab+b^{2}\right)\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1\right)-32\left(a^{2}-2\,ab+b^{2}\right)\mathrm{lo}}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/64*(12*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*a*b*(cos(f*x + e)
- 1)/(cos(f*x + e) + 1) + a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 6
4*(a^2 - 2*a*b + b^2)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1) - 32*
(a^2 - 2*a*b + b^2)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + (a^2 + 12
*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*a*b*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1) + 48*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 96*a*b*
(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 48*b^2*(cos(f*x + e) - 1)^2/(co
s(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x + e) - 1)^2)/f
```

3.204 $\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=113

$$\frac{b(2a-b)\tan^7(e+fx)}{7f}+\frac{(a-b)^2\tan^5(e+fx)}{5f}-\frac{(a-b)^2\tan^3(e+fx)}{3f}+\frac{(a-b)^2\tan(e+fx)}{f}-x(a-b)^2+\frac{b^2\tan^9(e+fx)}{9f}$$

```
[Out] -((a - b)^2*x) + ((a - b)^2*Tan[e + f*x])/f - ((a - b)^2*Tan[e + f*x]^3)/(3
*f) + ((a - b)^2*Tan[e + f*x]^5)/(5*f) + ((2*a - b)*b*Tan[e + f*x]^7)/(7*f)
 + (b^2*Tan[e + f*x]^9)/(9*f)
```

Rubi [A] time = 0.0881873, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 461, 203}

$$\frac{b(2a-b)\tan^7(e+fx)}{7f}+\frac{(a-b)^2\tan^5(e+fx)}{5f}-\frac{(a-b)^2\tan^3(e+fx)}{3f}+\frac{(a-b)^2\tan(e+fx)}{f}-x(a-b)^2+\frac{b^2\tan^9(e+fx)}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a - b)^2*x) + ((a - b)^2*Tan[e + f*x])/f - ((a - b)^2*Tan[e + f*x]^3)/(3
*f) + ((a - b)^2*Tan[e + f*x]^5)/(5*f) + ((2*a - b)*b*Tan[e + f*x]^7)/(7*f)
 + (b^2*Tan[e + f*x]^9)/(9*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{x^6\left(a+bx^2\right)^2}{1+x^2} dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left((a-b)^2-(a-b)^2x^2+(a-b)^2x^4+(2a-b)bx^6+b^2x^8+\frac{-a^2+2ab-b}{1+x^2}\right.\right.}{f}$$

$$= \frac{(a-b)^2\tan(e+fx)}{f} - \frac{(a-b)^2\tan^3(e+fx)}{3f} + \frac{(a-b)^2\tan^5(e+fx)}{5f} + \frac{(2a-}{}$$

$$= -(a-b)^2x + \frac{(a-b)^2\tan(e+fx)}{f} - \frac{(a-b)^2\tan^3(e+fx)}{3f} + \frac{(a-b)^2\tan^5(e+}{5f}$$

Mathematica [B] time = 0.0832041, size = 243, normalized size = 2.15

$$\frac{a^2\tan^5(e+fx)}{5f} - \frac{a^2\tan^3(e+fx)}{3f} - \frac{a^2\tan^{-1}(\tan(e+fx))}{f} + \frac{a^2\tan(e+fx)}{f} + \frac{2ab\tan^7(e+fx)}{7f} - \frac{2ab\tan^5(e+fx)}{5f} +$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a^2*ArcTan[Tan[e + f*x]])/f) + (2*a*b*ArcTan[Tan[e + f*x]])/f - (b^2*Arc
Tan[Tan[e + f*x]])/f + (a^2*Tan[e + f*x])/f - (2*a*b*Tan[e + f*x])/f + (b^2
*Tan[e + f*x])/f - (a^2*Tan[e + f*x]^3)/(3*f) + (2*a*b*Tan[e + f*x]^3)/(3*f
) - (b^2*Tan[e + f*x]^3)/(3*f) + (a^2*Tan[e + f*x]^5)/(5*f) - (2*a*b*Tan[e
+ f*x]^5)/(5*f) + (b^2*Tan[e + f*x]^5)/(5*f) + (2*a*b*Tan[e + f*x]^7)/(7*f)
 - (b^2*Tan[e + f*x]^7)/(7*f) + (b^2*Tan[e + f*x]^9)/(9*f)
```

Maple [B] time = 0.004, size = 226, normalized size = 2.

$$\frac{b^2\left(\tan\left(fx+e\right)\right)^9}{9f} + \frac{2ab\left(\tan\left(fx+e\right)\right)^7}{7f} - \frac{b^2\left(\tan\left(fx+e\right)\right)^7}{7f} + \frac{a^2\left(\tan\left(fx+e\right)\right)^5}{5f} - \frac{2\left(\tan\left(fx+e\right)\right)^5ab}{5f} + \frac{b^2\left(\tan\left(\right.\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/9*b^2*tan(f*x+e)^9/f+2/7/f*a*b*tan(f*x+e)^7-1/7*b^2*tan(f*x+e)^7/f+1/5/f*
a^2*tan(f*x+e)^5-2/5/f*tan(f*x+e)^5*a*b+1/5*b^2*tan(f*x+e)^5/f-1/3/f*tan(f*
x+e)^3*a^2+2/3/f*tan(f*x+e)^3*a*b-1/3*b^2*tan(f*x+e)^3/f+1/f*a^2*tan(f*x+e)
-2*a*b*tan(f*x+e)/f+b^2*tan(f*x+e)/f-1/f*arctan(tan(f*x+e))*a^2+2/f*arctan(
tan(f*x+e))*a*b-1/f*arctan(tan(f*x+e))*b^2
```

Maxima [A] time = 1.60621, size = 159, normalized size = 1.41

$$\frac{35\,b^2\tan\left(fx+e\right)^9 + 45\left(2\,ab-b^2\right)\tan\left(fx+e\right)^7 + 63\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^5 - 105\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/315*(35*b^2*tan(f*x + e)^9 + 45*(2*a*b - b^2)*tan(f*x + e)^7 + 63*(a^2 -
2*a*b + b^2)*tan(f*x + e)^5 - 105*(a^2 - 2*a*b + b^2)*tan(f*x + e)^3 - 315*
(a^2 - 2*a*b + b^2)*(f*x + e) + 315*(a^2 - 2*a*b + b^2)*tan(f*x + e))/f
```

Fricas [A] time = 1.12553, size = 293, normalized size = 2.59

$$\frac{35\,b^2\tan\left(fx+e\right)^9+45\left(2\,ab-b^2\right)\tan\left(fx+e\right)^7+63\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^5-105\left(a^2-2\,ab+b^2\right)\tan\left(fx+\right.}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^2*tan(f*x + e)^9 + 45*(2*a*b - b^2)*tan(f*x + e)^7 + 63*(a^2 -
2*a*b + b^2)*tan(f*x + e)^5 - 105*(a^2 - 2*a*b + b^2)*tan(f*x + e)^3 - 315*
(a^2 - 2*a*b + b^2)*f*x + 315*(a^2 - 2*a*b + b^2)*tan(f*x + e))/f
```

Sympy [A] time = 2.39661, size = 212, normalized size = 1.88

$$\begin{cases} -a^2x+\frac{a^2\tan^5\left(e+fx\right)}{5f}-\frac{a^2\tan^3\left(e+fx\right)}{3f}+\frac{a^2\tan\left(e+fx\right)}{f}+2abx+\frac{2ab\tan^7\left(e+fx\right)}{7f}-\frac{2ab\tan^5\left(e+fx\right)}{5f}+\frac{2ab\tan^3\left(e+fx\right)}{3f}-\frac{2ab\tan\left(e+fx\right)}{f} \\ x\left(a+b\tan^2\left(e\right)\right)^2\tan^6\left(e\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((-a**2*x + a**2*tan(e + f*x)**5/(5*f) - a**2*tan(e + f*x)**3/(3*f
) + a**2*tan(e + f*x)/f + 2*a*b*x + 2*a*b*tan(e + f*x)**7/(7*f) - 2*a*b*tan
(e + f*x)**5/(5*f) + 2*a*b*tan(e + f*x)**3/(3*f) - 2*a*b*tan(e + f*x)/f - b
**2*x + b**2*tan(e + f*x)**9/(9*f) - b**2*tan(e + f*x)**7/(7*f) + b**2*tan(
e + f*x)**5/(5*f) - b**2*tan(e + f*x)**3/(3*f) + b**2*tan(e + f*x)/f, Ne(f,
 0)), (x*(a + b*tan(e)**2)**2*tan(e)**6, True))
```

Giac [B] time = 13.3622, size = 3536, normalized size = 31.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/315*(315*a^2*f*x*tan(f*x)^9*tan(e)^9 - 630*a*b*f*x*tan(f*x)^9*tan(e)^9 +
 315*b^2*f*x*tan(f*x)^9*tan(e)^9 - 2835*a^2*f*x*tan(f*x)^8*tan(e)^8 + 5670*
a*b*f*x*tan(f*x)^8*tan(e)^8 - 2835*b^2*f*x*tan(f*x)^8*tan(e)^8 + 315*a^2*ta
n(f*x)^9*tan(e)^8 - 630*a*b*tan(f*x)^9*tan(e)^8 + 315*b^2*tan(f*x)^9*tan(e)
^8 + 315*a^2*tan(f*x)^8*tan(e)^9 - 630*a*b*tan(f*x)^8*tan(e)^9 + 315*b^2*ta
n(f*x)^8*tan(e)^9 + 11340*a^2*f*x*tan(f*x)^7*tan(e)^7 - 22680*a*b*f*x*tan(f
*x)^7*tan(e)^7 + 11340*b^2*f*x*tan(f*x)^7*tan(e)^7 - 105*a^2*tan(f*x)^9*tan
```

(e)^6 + 210*a*b*tan(f*x)^9*tan(e)^6 - 105*b^2*tan(f*x)^9*tan(e)^6 - 2835*a^
2*tan(f*x)^8*tan(e)^7 + 5670*a*b*tan(f*x)^8*tan(e)^7 - 2835*b^2*tan(f*x)^8*
tan(e)^7 - 2835*a^2*tan(f*x)^7*tan(e)^8 + 5670*a*b*tan(f*x)^7*tan(e)^8 - 28
35*b^2*tan(f*x)^7*tan(e)^8 - 105*a^2*tan(f*x)^6*tan(e)^9 + 210*a*b*tan(f*x)
^6*tan(e)^9 - 105*b^2*tan(f*x)^6*tan(e)^9 - 26460*a^2*f*x*tan(f*x)^6*tan(e)
^6 + 52920*a*b*f*x*tan(f*x)^6*tan(e)^6 - 26460*b^2*f*x*tan(f*x)^6*tan(e)^6
+ 63*a^2*tan(f*x)^9*tan(e)^4 - 126*a*b*tan(f*x)^9*tan(e)^4 + 63*b^2*tan(f*x
)^9*tan(e)^4 + 945*a^2*tan(f*x)^8*tan(e)^5 - 1890*a*b*tan(f*x)^8*tan(e)^5 +
945*b^2*tan(f*x)^8*tan(e)^5 + 11340*a^2*tan(f*x)^7*tan(e)^6 - 22680*a*b*ta
n(f*x)^7*tan(e)^6 + 11340*b^2*tan(f*x)^7*tan(e)^6 + 11340*a^2*tan(f*x)^6*ta
n(e)^7 - 22680*a*b*tan(f*x)^6*tan(e)^7 + 11340*b^2*tan(f*x)^6*tan(e)^7 + 94
5*a^2*tan(f*x)^5*tan(e)^8 - 1890*a*b*tan(f*x)^5*tan(e)^8 + 945*b^2*tan(f*x)
^5*tan(e)^8 + 63*a^2*tan(f*x)^4*tan(e)^9 - 126*a*b*tan(f*x)^4*tan(e)^9 + 63
*b^2*tan(f*x)^4*tan(e)^9 + 39690*a^2*f*x*tan(f*x)^5*tan(e)^5 - 79380*a*b*f*
x*tan(f*x)^5*tan(e)^5 + 39690*b^2*f*x*tan(f*x)^5*tan(e)^5 + 90*a*b*tan(f*x)
^9*tan(e)^2 - 45*b^2*tan(f*x)^9*tan(e)^2 - 252*a^2*tan(f*x)^8*tan(e)^3 + 11
34*a*b*tan(f*x)^8*tan(e)^3 - 567*b^2*tan(f*x)^8*tan(e)^3 - 2835*a^2*tan(f*x
)^7*tan(e)^4 + 7560*a*b*tan(f*x)^7*tan(e)^4 - 3780*b^2*tan(f*x)^7*tan(e)^4
- 24885*a^2*tan(f*x)^6*tan(e)^5 + 52920*a*b*tan(f*x)^6*tan(e)^5 - 26460*b^2
*tan(f*x)^6*tan(e)^5 - 24885*a^2*tan(f*x)^5*tan(e)^6 + 52920*a*b*tan(f*x)^5
*tan(e)^6 - 26460*b^2*tan(f*x)^5*tan(e)^6 - 2835*a^2*tan(f*x)^4*tan(e)^7 +
7560*a*b*tan(f*x)^4*tan(e)^7 - 3780*b^2*tan(f*x)^4*tan(e)^7 - 252*a^2*tan(f
*x)^3*tan(e)^8 + 1134*a*b*tan(f*x)^3*tan(e)^8 - 567*b^2*tan(f*x)^3*tan(e)^8
+ 90*a*b*tan(f*x)^2*tan(e)^9 - 45*b^2*tan(f*x)^2*tan(e)^9 - 39690*a^2*f*x*
tan(f*x)^4*tan(e)^4 + 79380*a*b*f*x*tan(f*x)^4*tan(e)^4 - 39690*b^2*f*x*tan
(f*x)^4*tan(e)^4 + 35*b^2*tan(f*x)^9 - 180*a*b*tan(f*x)^8*tan(e) + 405*b^2*
tan(f*x)^8*tan(e) + 378*a^2*tan(f*x)^7*tan(e)^2 - 2016*a*b*tan(f*x)^7*tan(e
)^2 + 2268*b^2*tan(f*x)^7*tan(e)^2 + 3990*a^2*tan(f*x)^6*tan(e)^3 - 11760*a
*b*tan(f*x)^6*tan(e)^3 + 8820*b^2*tan(f*x)^6*tan(e)^3 + 32130*a^2*tan(f*x)^
5*tan(e)^4 - 70560*a*b*tan(f*x)^5*tan(e)^4 + 39690*b^2*tan(f*x)^5*tan(e)^4
+ 32130*a^2*tan(f*x)^4*tan(e)^5 - 70560*a*b*tan(f*x)^4*tan(e)^5 + 39690*b^2
*tan(f*x)^4*tan(e)^5 + 3990*a^2*tan(f*x)^3*tan(e)^6 - 11760*a*b*tan(f*x)^3*
tan(e)^6 + 8820*b^2*tan(f*x)^3*tan(e)^6 + 378*a^2*tan(f*x)^2*tan(e)^7 - 201
6*a*b*tan(f*x)^2*tan(e)^7 + 2268*b^2*tan(f*x)^2*tan(e)^7 - 180*a*b*tan(f*x)
*tan(e)^8 + 405*b^2*tan(f*x)*tan(e)^8 + 35*b^2*tan(e)^9 + 26460*a^2*f*x*tan
(f*x)^3*tan(e)^3 - 52920*a*b*f*x*tan(f*x)^3*tan(e)^3 + 26460*b^2*f*x*tan(f*
x)^3*tan(e)^3 + 90*a*b*tan(f*x)^7 - 45*b^2*tan(f*x)^7 - 252*a^2*tan(f*x)^6*
tan(e) + 1134*a*b*tan(f*x)^6*tan(e) - 567*b^2*tan(f*x)^6*tan(e) - 2835*a^2*
tan(f*x)^5*tan(e)^2 + 7560*a*b*tan(f*x)^5*tan(e)^2 - 3780*b^2*tan(f*x)^5*ta
n(e)^2 - 24885*a^2*tan(f*x)^4*tan(e)^3 + 52920*a*b*tan(f*x)^4*tan(e)^3 - 26
460*b^2*tan(f*x)^4*tan(e)^3 - 24885*a^2*tan(f*x)^3*tan(e)^4 + 52920*a*b*tan
(f*x)^3*tan(e)^4 - 26460*b^2*tan(f*x)^3*tan(e)^4 - 2835*a^2*tan(f*x)^2*tan(
e)^5 + 7560*a*b*tan(f*x)^2*tan(e)^5 - 3780*b^2*tan(f*x)^2*tan(e)^5 - 252*a^
2*tan(f*x)*tan(e)^6 + 1134*a*b*tan(f*x)*tan(e)^6 - 567*b^2*tan(f*x)*tan(e)^
6 + 90*a*b*tan(e)^7 - 45*b^2*tan(e)^7 - 11340*a^2*f*x*tan(f*x)^2*tan(e)^2 +
22680*a*b*f*x*tan(f*x)^2*tan(e)^2 - 11340*b^2*f*x*tan(f*x)^2*tan(e)^2 + 63
*a^2*tan(f*x)^5 - 126*a*b*tan(f*x)^5 + 63*b^2*tan(f*x)^5 + 945*a^2*tan(f*x)
^4*tan(e) - 1890*a*b*tan(f*x)^4*tan(e) + 945*b^2*tan(f*x)^4*tan(e) + 11340*
a^2*tan(f*x)^3*tan(e)^2 - 22680*a*b*tan(f*x)^3*tan(e)^2 + 11340*b^2*tan(f*x
)^3*tan(e)^2 + 11340*a^2*tan(f*x)^2*tan(e)^3 - 22680*a*b*tan(f*x)^2*tan(e)^
3 + 11340*b^2*tan(f*x)^2*tan(e)^3 + 945*a^2*tan(f*x)*tan(e)^4 - 1890*a*b*ta
n(f*x)*tan(e)^4 + 945*b^2*tan(f*x)*tan(e)^4 + 63*a^2*tan(e)^5 - 126*a*b*tan
(e)^5 + 63*b^2*tan(e)^5 + 2835*a^2*f*x*tan(f*x)*tan(e) - 5670*a*b*f*x*tan(f
*x)*tan(e) + 2835*b^2*f*x*tan(f*x)*tan(e) - 105*a^2*tan(f*x)^3 + 210*a*b*ta
n(f*x)^3 - 105*b^2*tan(f*x)^3 - 2835*a^2*tan(f*x)^2*tan(e) + 5670*a*b*tan(f
*x)^2*tan(e) - 2835*b^2*tan(f*x)^2*tan(e) - 2835*a^2*tan(f*x)*tan(e)^2 + 56
70*a*b*tan(f*x)*tan(e)^2 - 2835*b^2*tan(f*x)*tan(e)^2 - 105*a^2*tan(e)^3 +
210*a*b*tan(e)^3 - 105*b^2*tan(e)^3 - 315*a^2*f*x + 630*a*b*f*x - 315*b^2*f
*x + 315*a^2*tan(f*x) - 630*a*b*tan(f*x) + 315*b^2*tan(f*x) + 315*a^2*tan(e

```
) - 630*a*b*tan(e) + 315*b^2*tan(e))/(f*tan(f*x)^9*tan(e)^9 - 9*f*tan(f*x)^
8*tan(e)^8 + 36*f*tan(f*x)^7*tan(e)^7 - 84*f*tan(f*x)^6*tan(e)^6 + 126*f*ta
n(f*x)^5*tan(e)^5 - 126*f*tan(f*x)^4*tan(e)^4 + 84*f*tan(f*x)^3*tan(e)^3 -
36*f*tan(f*x)^2*tan(e)^2 + 9*f*tan(f*x)*tan(e) - f)
```

3.205 $\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=91

$$\frac{b(2a-b)\tan^5(e+fx)}{5f}+\frac{(a-b)^2\tan^3(e+fx)}{3f}-\frac{(a-b)^2\tan(e+fx)}{f}+x(a-b)^2+\frac{b^2\tan^7(e+fx)}{7f}$$

```
[Out] (a - b)^2*x - ((a - b)^2*Tan[e + f*x])/f + ((a - b)^2*Tan[e + f*x]^3)/(3*f)
 + ((2*a - b)*b*Tan[e + f*x]^5)/(5*f) + (b^2*Tan[e + f*x]^7)/(7*f)
```

Rubi [A] time = 0.0788437, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 461, 203}

$$\frac{b(2a-b)\tan^5(e+fx)}{5f}+\frac{(a-b)^2\tan^3(e+fx)}{3f}-\frac{(a-b)^2\tan(e+fx)}{f}+x(a-b)^2+\frac{b^2\tan^7(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (a - b)^2*x - ((a - b)^2*Tan[e + f*x])/f + ((a - b)^2*Tan[e + f*x]^3)/(3*f)
 + ((2*a - b)*b*Tan[e + f*x]^5)/(5*f) + (b^2*Tan[e + f*x]^7)/(7*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^2}{1+x^2} dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(-(a-b)^2+(a-b)^2x^2+(2a-b)bx^4+b^2x^6+\frac{a^2-2ab+b^2}{1+x^2}\right) dx, x,\right.}{f}$$
$$= -\frac{(a-b)^2\tan(e+fx)}{f}+\frac{(a-b)^2\tan^3(e+fx)}{3f}+\frac{(2a-b)b\tan^5(e+fx)}{5f}+$$
$$= (a-b)^2x-\frac{(a-b)^2\tan(e+fx)}{f}+\frac{(a-b)^2\tan^3(e+fx)}{3f}+\frac{(2a-b)b\tan^5}{5f}$$

Mathematica [B] time = 0.0743482, size = 190, normalized size = 2.09

$$\frac{a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan^{-1}(\tan(e+fx))}{f}-\frac{a^2\tan(e+fx)}{f}+\frac{2ab\tan^5(e+fx)}{5f}-\frac{2ab\tan^3(e+fx)}{3f}-\frac{2ab\tan^{-1}(\tan(}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (a^2*ArcTan[Tan[e + f*x]])/f - (2*a*b*ArcTan[Tan[e + f*x]])/f + (b^2*ArcTan
[Tan[e + f*x]])/f - (a^2*Tan[e + f*x])/f + (2*a*b*Tan[e + f*x])/f - (b^2*Ta
n[e + f*x])/f + (a^2*Tan[e + f*x]^3)/(3*f) - (2*a*b*Tan[e + f*x]^3)/(3*f) +
 (b^2*Tan[e + f*x]^3)/(3*f) + (2*a*b*Tan[e + f*x]^5)/(5*f) - (b^2*Tan[e + f
*x]^5)/(5*f) + (b^2*Tan[e + f*x]^7)/(7*f)
```

Maple [B] time = 0.005, size = 179, normalized size = 2.

$$\frac{b^2\left(\tan\left(fx+e\right)\right)^7}{7f}+\frac{2\left(\tan\left(fx+e\right)\right)^5ab}{5f}-\frac{b^2\left(\tan\left(fx+e\right)\right)^5}{5f}+\frac{\left(\tan\left(fx+e\right)\right)^3a^2}{3f}-\frac{2\left(\tan\left(fx+e\right)\right)^3ab}{3f}+\frac{b^2\left(\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/7*b^2*tan(f*x+e)^7/f+2/5/f*tan(f*x+e)^5*a*b-1/5*b^2*tan(f*x+e)^5/f+1/3/f*
tan(f*x+e)^3*a^2-2/3/f*tan(f*x+e)^3*a*b+1/3*b^2*tan(f*x+e)^3/f-1/f*a^2*tan(
f*x+e)+2*a*b*tan(f*x+e)/f-b^2*tan(f*x+e)/f+1/f*arctan(tan(f*x+e))*a^2-2/f*a
rctan(tan(f*x+e))*a*b+1/f*arctan(tan(f*x+e))*b^2
```

Maxima [A] time = 1.63475, size = 131, normalized size = 1.44

$$\frac{15\,b^2\tan\left(fx+e\right)^7+21\left(2\,ab-b^2\right)\tan\left(fx+e\right)^5+35\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^3+105\left(a^2-2\,ab+b^2\right)\left(fx+e\right)-}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/105*(15*b^2*tan(f*x + e)^7 + 21*(2*a*b - b^2)*tan(f*x + e)^5 + 35*(a^2 -
2*a*b + b^2)*tan(f*x + e)^3 + 105*(a^2 - 2*a*b + b^2)*(f*x + e) - 105*(a^2
- 2*a*b + b^2)*tan(f*x + e))/f
```

Fricas [A] time = 1.17075, size = 238, normalized size = 2.62

$$\frac{15\,b^2\tan\left(fx+e\right)^7+21\left(2\,ab-b^2\right)\tan\left(fx+e\right)^5+35\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^3+105\left(a^2-2\,ab+b^2\right)fx-105\left(a^2\right.}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*tan(f*x + e)^7 + 21*(2*a*b - b^2)*tan(f*x + e)^5 + 35*(a^2 -
2*a*b + b^2)*tan(f*x + e)^3 + 105*(a^2 - 2*a*b + b^2)*f*x - 105*(a^2 - 2*a*
b + b^2)*tan(f*x + e))/f
```

Sympy [A] time = 1.28952, size = 165, normalized size = 1.81

$$\begin{cases} a^2x+\frac{a^2\tan^3\left(e+fx\right)}{3f}-\frac{a^2\tan\left(e+fx\right)}{f}-2abx+\frac{2ab\tan^5\left(e+fx\right)}{5f}-\frac{2ab\tan^3\left(e+fx\right)}{3f}+\frac{2ab\tan\left(e+fx\right)}{f}+b^2x+\frac{b^2\tan^7\left(e+fx\right)}{7f}-\frac{b^2\tan^5\left(e+fx\right)}{5f} \\ x\left(a+b\tan^2\left(e\right)\right)^2\tan^4\left(e\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((a**2*x + a**2*tan(e + f*x)**3/(3*f) - a**2*tan(e + f*x)/f - 2*a*
b*x + 2*a*b*tan(e + f*x)**5/(5*f) - 2*a*b*tan(e + f*x)**3/(3*f) + 2*a*b*tan
(e + f*x)/f + b**2*x + b**2*tan(e + f*x)**7/(7*f) - b**2*tan(e + f*x)**5/(5
*f) + b**2*tan(e + f*x)**3/(3*f) - b**2*tan(e + f*x)/f, Ne(f, 0)), (x*(a +
b*tan(e)**2)**2*tan(e)**4, True))
```

Giac [B] time = 5.31017, size = 2273, normalized size = 24.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/105*(105*a^2*f*x*tan(f*x)^7*tan(e)^7 - 210*a*b*f*x*tan(f*x)^7*tan(e)^7 +
105*b^2*f*x*tan(f*x)^7*tan(e)^7 - 735*a^2*f*x*tan(f*x)^6*tan(e)^6 + 1470*a*
b*f*x*tan(f*x)^6*tan(e)^6 - 735*b^2*f*x*tan(f*x)^6*tan(e)^6 + 105*a^2*tan(f
*x)^7*tan(e)^6 - 210*a*b*tan(f*x)^7*tan(e)^6 + 105*b^2*tan(f*x)^7*tan(e)^6
+ 105*a^2*tan(f*x)^6*tan(e)^7 - 210*a*b*tan(f*x)^6*tan(e)^7 + 105*b^2*tan(f
*x)^6*tan(e)^7 + 2205*a^2*f*x*tan(f*x)^5*tan(e)^5 - 4410*a*b*f*x*tan(f*x)^5
*tan(e)^5 + 2205*b^2*f*x*tan(f*x)^5*tan(e)^5 - 35*a^2*tan(f*x)^7*tan(e)^4 +
 70*a*b*tan(f*x)^7*tan(e)^4 - 35*b^2*tan(f*x)^7*tan(e)^4 - 735*a^2*tan(f*x)
^6*tan(e)^5 + 1470*a*b*tan(f*x)^6*tan(e)^5 - 735*b^2*tan(f*x)^6*tan(e)^5 -
735*a^2*tan(f*x)^5*tan(e)^6 + 1470*a*b*tan(f*x)^5*tan(e)^6 - 735*b^2*tan(f*
```

```
x)^5*tan(e)^6 - 35*a^2*tan(f*x)^4*tan(e)^7 + 70*a*b*tan(f*x)^4*tan(e)^7 - 3
5*b^2*tan(f*x)^4*tan(e)^7 - 3675*a^2*f*x*tan(f*x)^4*tan(e)^4 + 7350*a*b*f*x
*tan(f*x)^4*tan(e)^4 - 3675*b^2*f*x*tan(f*x)^4*tan(e)^4 - 42*a*b*tan(f*x)^7
*tan(e)^2 + 21*b^2*tan(f*x)^7*tan(e)^2 + 140*a^2*tan(f*x)^6*tan(e)^3 - 490*
a*b*tan(f*x)^6*tan(e)^3 + 245*b^2*tan(f*x)^6*tan(e)^3 + 1995*a^2*tan(f*x)^5
*tan(e)^4 - 4410*a*b*tan(f*x)^5*tan(e)^4 + 2205*b^2*tan(f*x)^5*tan(e)^4 + 1
995*a^2*tan(f*x)^4*tan(e)^5 - 4410*a*b*tan(f*x)^4*tan(e)^5 + 2205*b^2*tan(f
*x)^4*tan(e)^5 + 140*a^2*tan(f*x)^3*tan(e)^6 - 490*a*b*tan(f*x)^3*tan(e)^6
+ 245*b^2*tan(f*x)^3*tan(e)^6 - 42*a*b*tan(f*x)^2*tan(e)^7 + 21*b^2*tan(f*x
)^2*tan(e)^7 + 3675*a^2*f*x*tan(f*x)^3*tan(e)^3 - 7350*a*b*f*x*tan(f*x)^3*t
an(e)^3 + 3675*b^2*f*x*tan(f*x)^3*tan(e)^3 - 15*b^2*tan(f*x)^7 + 84*a*b*tan
(f*x)^6*tan(e) - 147*b^2*tan(f*x)^6*tan(e) - 210*a^2*tan(f*x)^5*tan(e)^2 +
840*a*b*tan(f*x)^5*tan(e)^2 - 735*b^2*tan(f*x)^5*tan(e)^2 - 2730*a^2*tan(f*
x)^4*tan(e)^3 + 6300*a*b*tan(f*x)^4*tan(e)^3 - 3675*b^2*tan(f*x)^4*tan(e)^3
 - 2730*a^2*tan(f*x)^3*tan(e)^4 + 6300*a*b*tan(f*x)^3*tan(e)^4 - 3675*b^2*t
an(f*x)^3*tan(e)^4 - 210*a^2*tan(f*x)^2*tan(e)^5 + 840*a*b*tan(f*x)^2*tan(e
)^5 - 735*b^2*tan(f*x)^2*tan(e)^5 + 84*a*b*tan(f*x)*tan(e)^6 - 147*b^2*tan(
f*x)*tan(e)^6 - 15*b^2*tan(e)^7 - 2205*a^2*f*x*tan(f*x)^2*tan(e)^2 + 4410*a
*b*f*x*tan(f*x)^2*tan(e)^2 - 2205*b^2*f*x*tan(f*x)^2*tan(e)^2 - 42*a*b*tan(
f*x)^5 + 21*b^2*tan(f*x)^5 + 140*a^2*tan(f*x)^4*tan(e) - 490*a*b*tan(f*x)^4
*tan(e) + 245*b^2*tan(f*x)^4*tan(e) + 1995*a^2*tan(f*x)^3*tan(e)^2 - 4410*a
*b*tan(f*x)^3*tan(e)^2 + 2205*b^2*tan(f*x)^3*tan(e)^2 + 1995*a^2*tan(f*x)^2
*tan(e)^3 - 4410*a*b*tan(f*x)^2*tan(e)^3 + 2205*b^2*tan(f*x)^2*tan(e)^3 + 1
40*a^2*tan(f*x)*tan(e)^4 - 490*a*b*tan(f*x)*tan(e)^4 + 245*b^2*tan(f*x)*tan
(e)^4 - 42*a*b*tan(e)^5 + 21*b^2*tan(e)^5 + 735*a^2*f*x*tan(f*x)*tan(e) - 1
470*a*b*f*x*tan(f*x)*tan(e) + 735*b^2*f*x*tan(f*x)*tan(e) - 35*a^2*tan(f*x)
^3 + 70*a*b*tan(f*x)^3 - 35*b^2*tan(f*x)^3 - 735*a^2*tan(f*x)^2*tan(e) + 14
70*a*b*tan(f*x)^2*tan(e) - 735*b^2*tan(f*x)^2*tan(e) - 735*a^2*tan(f*x)*tan
(e)^2 + 1470*a*b*tan(f*x)*tan(e)^2 - 735*b^2*tan(f*x)*tan(e)^2 - 35*a^2*tan
(e)^3 + 70*a*b*tan(e)^3 - 35*b^2*tan(e)^3 - 105*a^2*f*x + 210*a*b*f*x - 105
*b^2*f*x + 105*a^2*tan(f*x) - 210*a*b*tan(f*x) + 105*b^2*tan(f*x) + 105*a^2
*tan(e) - 210*a*b*tan(e) + 105*b^2*tan(e))/(f*tan(f*x)^7*tan(e)^7 - 7*f*tan
(f*x)^6*tan(e)^6 + 21*f*tan(f*x)^5*tan(e)^5 - 35*f*tan(f*x)^4*tan(e)^4 + 35
*f*tan(f*x)^3*tan(e)^3 - 21*f*tan(f*x)^2*tan(e)^2 + 7*f*tan(f*x)*tan(e) - f
)
```

3.206 $\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx$

Optimal. Leaf size=69

$$\frac{b(2a-b)\tan^3(e+fx)}{3f}+\frac{(a-b)^2\tan(e+fx)}{f}-x(a-b)^2+\frac{b^2\tan^5(e+fx)}{5f}$$

```
[Out] -((a - b)^2*x) + ((a - b)^2*Tan[e + f*x])/f + ((2*a - b)*b*Tan[e + f*x]^3)/
(3*f) + (b^2*Tan[e + f*x]^5)/(5*f)
```

Rubi [A] time = 0.0737361, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 461, 203}

$$\frac{b(2a-b)\tan^3(e+fx)}{3f}+\frac{(a-b)^2\tan(e+fx)}{f}-x(a-b)^2+\frac{b^2\tan^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a - b)^2*x) + ((a - b)^2*Tan[e + f*x])/f + ((2*a - b)*b*Tan[e + f*x]^3)/
(3*f) + (b^2*Tan[e + f*x]^5)/(5*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left((a-b)^2+(2a-b)bx^2+b^2x^4+\frac{-a^2+2ab-b^2}{1+x^2}\right)dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{(a-b)^2\tan(e+fx)}{f}+\frac{(2a-b)b\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}-\frac{(a-b}{}$$
$$= -(a-b)^2x+\frac{(a-b)^2\tan(e+fx)}{f}+\frac{(2a-b)b\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+}{5f}$$

Mathematica [A] time = 0.0560561, size = 137, normalized size = 1.99

$$-\frac{a^2\tan^{-1}(\tan(e+fx))}{f}+\frac{a^2\tan(e+fx)}{f}+\frac{2ab\tan^3(e+fx)}{3f}+\frac{2ab\tan^{-1}(\tan(e+fx))}{f}-\frac{2ab\tan(e+fx)}{f}+\frac{b^2\tan^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a^2*ArcTan[Tan[e + f*x]])/f) + (2*a*b*ArcTan[Tan[e + f*x]])/f - (b^2*Arc
Tan[Tan[e + f*x]])/f + (a^2*Tan[e + f*x])/f - (2*a*b*Tan[e + f*x])/f + (b^2
*Tan[e + f*x])/f + (2*a*b*Tan[e + f*x]^3)/(3*f) - (b^2*Tan[e + f*x]^3)/(3*f
) + (b^2*Tan[e + f*x]^5)/(5*f)
```

Maple [B] time = 0.004, size = 132, normalized size = 1.9

$$\frac{b^2\left(\tan\left(fx+e\right)\right)^5}{5f}+\frac{2\left(\tan\left(fx+e\right)\right)^3ab}{3f}-\frac{b^2\left(\tan\left(fx+e\right)\right)^3}{3f}+\frac{a^2\tan\left(fx+e\right)}{f}-2\frac{\tan\left(fx+e\right)ab}{f}+\frac{b^2\tan\left(fx+\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/5*b^2*tan(f*x+e)^5/f+2/3/f*tan(f*x+e)^3*a*b-1/3*b^2*tan(f*x+e)^3/f+1/f*a^
2*tan(f*x+e)-2*a*b*tan(f*x+e)/f+b^2*tan(f*x+e)/f-1/f*arctan(tan(f*x+e))*a^2
+2/f*arctan(tan(f*x+e))*a*b-1/f*arctan(tan(f*x+e))*b^2
```

Maxima [A] time = 1.61744, size = 103, normalized size = 1.49

$$\frac{3\,b^2\tan\left(fx+e\right)^5+5\left(2\,ab-b^2\right)\tan\left(fx+e\right)^3-15\left(a^2-2\,ab+b^2\right)\left(fx+e\right)+15\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*b^2*tan(f*x + e)^5 + 5*(2*a*b - b^2)*tan(f*x + e)^3 - 15*(a^2 - 2*a
*b + b^2)*(f*x + e) + 15*(a^2 - 2*a*b + b^2)*tan(f*x + e))/f
```

Fricas [A] time = 1.09442, size = 177, normalized size = 2.57

$$\frac{3\,b^2\tan\left(fx+e\right)^5+5\left(2\,ab-b^2\right)\tan\left(fx+e\right)^3-15\left(a^2-2\,ab+b^2\right)fx+15\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*tan(f*x + e)^5 + 5*(2*a*b - b^2)*tan(f*x + e)^3 - 15*(a^2 - 2*a
*b + b^2)*f*x + 15*(a^2 - 2*a*b + b^2)*tan(f*x + e))/f
```

Sympy [A] time = 0.731504, size = 117, normalized size = 1.7

$$\begin{cases} -a^2x+\frac{a^2\tan\left(e+fx\right)}{f}+2abx+\frac{2ab\tan^3\left(e+fx\right)}{3f}-\frac{2ab\tan\left(e+fx\right)}{f}-b^2x+\frac{b^2\tan^5\left(e+fx\right)}{5f}-\frac{b^2\tan^3\left(e+fx\right)}{3f}+\frac{b^2\tan\left(e+fx\right)}{f} & \text{for } f\neq 0 \\ x\left(a+b\tan^2\left(e\right)\right)^2\tan^2\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((-a**2*x + a**2*tan(e + f*x)/f + 2*a*b*x + 2*a*b*tan(e + f*x)**3/
(3*f) - 2*a*b*tan(e + f*x)/f - b**2*x + b**2*tan(e + f*x)**5/(5*f) - b**2*t
an(e + f*x)**3/(3*f) + b**2*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e)**2)
**2*tan(e)**2, True))
```

Giac [B] time = 2.82322, size = 1265, normalized size = 18.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/15*(15*a^2*f*x*tan(f*x)^5*tan(e)^5 - 30*a*b*f*x*tan(f*x)^5*tan(e)^5 + 15
*b^2*f*x*tan(f*x)^5*tan(e)^5 - 75*a^2*f*x*tan(f*x)^4*tan(e)^4 + 150*a*b*f*x
*tan(f*x)^4*tan(e)^4 - 75*b^2*f*x*tan(f*x)^4*tan(e)^4 + 15*a^2*tan(f*x)^5*t
an(e)^4 - 30*a*b*tan(f*x)^5*tan(e)^4 + 15*b^2*tan(f*x)^5*tan(e)^4 + 15*a^2*
tan(f*x)^4*tan(e)^5 - 30*a*b*tan(f*x)^4*tan(e)^5 + 15*b^2*tan(f*x)^4*tan(e)
^5 + 150*a^2*f*x*tan(f*x)^3*tan(e)^3 - 300*a*b*f*x*tan(f*x)^3*tan(e)^3 + 15
0*b^2*f*x*tan(f*x)^3*tan(e)^3 + 10*a*b*tan(f*x)^5*tan(e)^2 - 5*b^2*tan(f*x)
^5*tan(e)^2 - 60*a^2*tan(f*x)^4*tan(e)^3 + 150*a*b*tan(f*x)^4*tan(e)^3 - 75
*b^2*tan(f*x)^4*tan(e)^3 - 60*a^2*tan(f*x)^3*tan(e)^4 + 150*a*b*tan(f*x)^3*
tan(e)^4 - 75*b^2*tan(f*x)^3*tan(e)^4 + 10*a*b*tan(f*x)^2*tan(e)^5 - 5*b^2*
tan(f*x)^2*tan(e)^5 - 150*a^2*f*x*tan(f*x)^2*tan(e)^2 + 300*a*b*f*x*tan(f*x
)^2*tan(e)^2 - 150*b^2*f*x*tan(f*x)^2*tan(e)^2 + 3*b^2*tan(f*x)^5 - 20*a*b*
tan(f*x)^4*tan(e) + 25*b^2*tan(f*x)^4*tan(e) + 90*a^2*tan(f*x)^3*tan(e)^2 -
 240*a*b*tan(f*x)^3*tan(e)^2 + 150*b^2*tan(f*x)^3*tan(e)^2 + 90*a^2*tan(f*x
)^2*tan(e)^3 - 240*a*b*tan(f*x)^2*tan(e)^3 + 150*b^2*tan(f*x)^2*tan(e)^3 -
20*a*b*tan(f*x)*tan(e)^4 + 25*b^2*tan(f*x)*tan(e)^4 + 3*b^2*tan(e)^5 + 75*a
^2*f*x*tan(f*x)*tan(e) - 150*a*b*f*x*tan(f*x)*tan(e) + 75*b^2*f*x*tan(f*x)*
```

```
tan(e) + 10*a*b*tan(f*x)^3 - 5*b^2*tan(f*x)^3 - 60*a^2*tan(f*x)^2*tan(e) +
150*a*b*tan(f*x)^2*tan(e) - 75*b^2*tan(f*x)^2*tan(e) - 60*a^2*tan(f*x)*tan(
e)^2 + 150*a*b*tan(f*x)*tan(e)^2 - 75*b^2*tan(f*x)*tan(e)^2 + 10*a*b*tan(e)
^3 - 5*b^2*tan(e)^3 - 15*a^2*f*x + 30*a*b*f*x - 15*b^2*f*x + 15*a^2*tan(f*x
) - 30*a*b*tan(f*x) + 15*b^2*tan(f*x) + 15*a^2*tan(e) - 30*a*b*tan(e) + 15*
b^2*tan(e))/(f*tan(f*x)^5*tan(e)^5 - 5*f*tan(f*x)^4*tan(e)^4 + 10*f*tan(f*x
)^3*tan(e)^3 - 10*f*tan(f*x)^2*tan(e)^2 + 5*f*tan(f*x)*tan(e) - f)
```

3.207 $\int \left(a+b\tan^2(e+fx)\right)^2 dx$

Optimal. Leaf size=46

$$\frac{b(2a-b)\tan(e+fx)}{f}+x(a-b)^2+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] (a - b)^2*x + ((2*a - b)*b*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.0310278, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3661, 390, 203}

$$\frac{b(2a-b)\tan(e+fx)}{f}+x(a-b)^2+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (a - b)^2*x + ((2*a - b)*b*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\tan^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{1+x^2} dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left((2a-b)b+b^2x^2+\frac{(a-b)^2}{1+x^2}\right) dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{(2a-b)b\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}+\frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \tan(e+fx)\right)}{f}$$
$$= (a-b)^2x+\frac{(2a-b)b\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Mathematica [A] time = 0.575648, size = 73, normalized size = 1.59

$$\frac{\tan(e+fx)\left(b\left(6a-b\left(3-\tan^2(e+fx)\right)\right)+\frac{3(a-b)^2\tanh^{-1}\left(\sqrt{-\tan^2(e+fx)}\right)}{\sqrt{-\tan^2(e+fx)}}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (Tan[e + f*x]*((3*(a - b)^2*ArcTanh[Sqrt[-Tan[e + f*x]^2]])/Sqrt[-Tan[e + f
*x]^2] + b*(6*a - b*(3 - Tan[e + f*x]^2))))/(3*f)
```

Maple [A] time = 0.003, size = 87, normalized size = 1.9

$$\frac{b^2\left(\tan\left(fx+e\right)\right)^3}{3f}+2\frac{\tan\left(fx+e\right)ab}{f}-\frac{b^2\tan\left(fx+e\right)}{f}+\frac{\arctan\left(\tan\left(fx+e\right)\right)a^2}{f}-2\frac{\arctan\left(\tan\left(fx+e\right)\right)ab}{f}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/3*b^2*tan(f*x+e)^3/f+2*a*b*tan(f*x+e)/f-b^2*tan(f*x+e)/f+1/f*arctan(tan(f
*x+e))*a^2-2/f*arctan(tan(f*x+e))*a*b+1/f*arctan(tan(f*x+e))*b^2
```

Maxima [A] time = 1.6435, size = 78, normalized size = 1.7

$$a^2x-\frac{2\left(fx+e-\tan\left(fx+e\right)\right)ab}{f}+\frac{\left(\tan\left(fx+e\right)^3+3fx+3e-3\tan\left(fx+e\right)\right)b^2}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x - 2*(f*x + e - tan(f*x + e))*a*b/f + 1/3*(tan(f*x + e)^3 + 3*f*x + 3*
e - 3*tan(f*x + e))*b^2/f
```

Fricas [A] time = 1.06408, size = 117, normalized size = 2.54

$$\frac{b^2 \tan\left(fx+e\right)^3 + 3\left(a^2 - 2\,ab + b^2\right)fx + 3\left(2\,ab - b^2\right)\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)*f*x + 3*(2*a*b - b^2)*tan(f
*x + e))/f
```

Sympy [A] time = 0.381832, size = 68, normalized size = 1.48

$$\begin{cases} a^2x - 2abx + \frac{2ab\tan\left(e+fx\right)}{f} + b^2x + \frac{b^2\tan^3\left(e+fx\right)}{3f} - \frac{b^2\tan\left(e+fx\right)}{f} & \text{for } f \neq 0 \\ x\left(a + b\tan^2\left(e\right)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((a**2*x - 2*a*b*x + 2*a*b*tan(e + f*x)/f + b**2*x + b**2*tan(e +
f*x)**3/(3*f) - b**2*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e)**2)**2, Tr
ue))
```

Giac [B] time = 1.46975, size = 516, normalized size = 11.22

$$\frac{3\,a^2fx\tan\left(fx\right)^3\tan\left(e\right)^3 - 6\,abfx\tan\left(fx\right)^3\tan\left(e\right)^3 + 3\,b^2fx\tan\left(fx\right)^3\tan\left(e\right)^3 - 9\,a^2fx\tan\left(fx\right)^2\tan\left(e\right)^2 + 18\,abfx\,\mathrm{t}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*f*x*tan(f*x)^3*tan(e)^3 - 6*a*b*f*x*tan(f*x)^3*tan(e)^3 + 3*b^2*
f*x*tan(f*x)^3*tan(e)^3 - 9*a^2*f*x*tan(f*x)^2*tan(e)^2 + 18*a*b*f*x*tan(f*
x)^2*tan(e)^2 - 9*b^2*f*x*tan(f*x)^2*tan(e)^2 - 6*a*b*tan(f*x)^3*tan(e)^2 +
 3*b^2*tan(f*x)^3*tan(e)^2 - 6*a*b*tan(f*x)^2*tan(e)^3 + 3*b^2*tan(f*x)^2*t
an(e)^3 + 9*a^2*f*x*tan(f*x)*tan(e) - 18*a*b*f*x*tan(f*x)*tan(e) + 9*b^2*f*
x*tan(f*x)*tan(e) - b^2*tan(f*x)^3 + 12*a*b*tan(f*x)^2*tan(e) - 9*b^2*tan(f
*x)^2*tan(e) + 12*a*b*tan(f*x)*tan(e)^2 - 9*b^2*tan(f*x)*tan(e)^2 - b^2*tan
(e)^3 - 3*a^2*f*x + 6*a*b*f*x - 3*b^2*f*x - 6*a*b*tan(f*x) + 3*b^2*tan(f*x)
 - 6*a*b*tan(e) + 3*b^2*tan(e))/(f*tan(f*x)^3*tan(e)^3 - 3*f*tan(f*x)^2*tan
(e)^2 + 3*f*tan(f*x)*tan(e) - f)
```

3.208 $\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=38

$$-\frac{a^2\cot(e+fx)}{f}-x(a-b)^2+\frac{b^2\tan(e+fx)}{f}$$

```
[Out] -((a - b)^2*x) - (a^2*Cot[e + f*x])/f + (b^2*Tan[e + f*x])/f
```

Rubi [A] time = 0.06506, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 461, 203}

$$-\frac{a^2\cot(e+fx)}{f}-x(a-b)^2+\frac{b^2\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a - b)^2*x) - (a^2*Cot[e + f*x])/f + (b^2*Tan[e + f*x])/f
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(b^2+\frac{a^2}{x^2}-\frac{(a-b)^2}{1+x^2}\right)dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{a^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}-\frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -(a-b)^2x-\frac{a^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}$$

Mathematica [C] time = 0.102656, size = 66, normalized size = 1.74

$$-\frac{a^2\cot(e+fx)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(e+fx)\right)}{f}+2abx-\frac{b^2\tan^{-1}(\tan(e+fx))}{f}+\frac{b^2\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] 2*a*b*x - (b^2*ArcTan[Tan[e + f*x]])/f - (a^2*Cot[e + f*x]*Hypergeometric2F
1[-1/2, 1, 1/2, -Tan[e + f*x]^2])/f + (b^2*Tan[e + f*x])/f
```

Maple [A] time = 0.04, size = 53, normalized size = 1.4

$$\frac{b^2\left(\tan\left(fx+e\right)-fx-e\right)+2\left(fx+e\right)ab+a^2\left(-\cot\left(fx+e\right)-fx-e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(b^2*(tan(f*x+e)-f*x-e)+2*(f*x+e)*a*b+a^2*(-cot(f*x+e)-f*x-e))
```

Maxima [A] time = 1.64509, size = 62, normalized size = 1.63

$$\frac{b^2\tan\left(fx+e\right)-\left(a^2-2\,ab+b^2\right)\left(fx+e\right)-\frac{a^2}{\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] (b^2*tan(f*x + e) - (a^2 - 2*a*b + b^2)*(f*x + e) - a^2/tan(f*x + e))/f
```

Fricas [A] time = 1.04018, size = 115, normalized size = 3.03

$$-\frac{\left(a^2-2\,ab+b^2\right)fx\tan\left(fx+e\right)-b^2\tan\left(fx+e\right)^2+a^2}{f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -((a^2 - 2*a*b + b^2)*f*x*tan(f*x + e) - b^2*tan(f*x + e)^2 + a^2)/(f*tan(f
*x + e))
```

Sympy [A] time = 5.50577, size = 73, normalized size = 1.92

$$\begin{cases} \tilde{\infty}a^2x & \text{for}\ \left(e=0\vee e=-fx\right)\wedge\left(e=-fx\vee f=0\right) \\ x\left(a+b\tan^2\left(e\right)\right)^2\cot^2\left(e\right) & \text{for}\ f=0 \\ -a^2x-\frac{a^2}{f\tan\left(e+fx\right)}+2abx-b^2x+\frac{b^2\tan\left(e+fx\right)}{f} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*a**2*x, (Eq(e, 0) | Eq(e, -f*x)) & (Eq(f, 0) | Eq(e, -f*x)))
, (x*(a + b*tan(e)**2)**2*cot(e)**2, Eq(f, 0)), (-a**2*x - a**2/(f*tan(e +
f*x)) + 2*a*b*x - b**2*x + b**2*tan(e + f*x)/f, True))
```

Giac [A] time = 1.6694, size = 66, normalized size = 1.74

$$\frac{b^2\tan\left(fx+e\right)-\left(a^2-2\,ab+b^2\right)\left(fx+e\right)-\frac{a^2}{\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] (b^2*tan(f*x + e) - (a^2 - 2*a*b + b^2)*(f*x + e) - a^2/tan(f*x + e))/f
```

3.209 $\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=44

$$-\frac{a^2\cot^3(e+fx)}{3f}+\frac{a(a-2b)\cot(e+fx)}{f}+x(a-b)^2$$

```
[Out] (a - b)^2*x + (a*(a - 2*b)*Cot[e + f*x])/f - (a^2*Cot[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.0707845, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 461, 203}

$$-\frac{a^2\cot^3(e+fx)}{3f}+\frac{a(a-2b)\cot(e+fx)}{f}+x(a-b)^2$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (a - b)^2*x + (a*(a - 2*b)*Cot[e + f*x])/f - (a^2*Cot[e + f*x]^3)/(3*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^4} - \frac{a(a-2b)}{x^2} + \frac{(a-b)^2}{1+x^2}\right)dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a(a-2b)\cot(e+fx)}{f} - \frac{a^2\cot^3(e+fx)}{3f} + \frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \text{ta}\right.}{f}$$

$$= (a-b)^2x + \frac{a(a-2b)\cot(e+fx)}{f} - \frac{a^2\cot^3(e+fx)}{3f}$$

Mathematica [A] time = 1.2306, size = 71, normalized size = 1.61

$$-\frac{\cot(e+fx)\left(a\left(a\cot^2(e+fx)-3a+6b\right)+3(a-b)^2\sqrt{-\tan^2(e+fx)}\tanh^{-1}\left(\sqrt{-\tan^2(e+fx)}\right)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(Cot[e + f*x]*(a*(-3*a + 6*b + a*Cot[e + f*x]^2) + 3*(a - b)^2*ArcTanh[Sqr
t[-Tan[e + f*x]^2]]*Sqrt[-Tan[e + f*x]^2]))/(3*f)
```

Maple [A] time = 0.047, size = 60, normalized size = 1.4

$$\frac{1}{f}\left(b^2(fx+e)+2ab\left(-\cot(fx+e)-fx-e\right)+a^2\left(-\frac{(\cot(fx+e))^3}{3}+\cot(fx+e)+fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(b^2*(f*x+e)+2*a*b*(-cot(f*x+e)-f*x-e)+a^2*(-1/3*cot(f*x+e)^3+cot(f*x+e
)+f*x+e))
```

Maxima [A] time = 1.61901, size = 77, normalized size = 1.75

$$\frac{3\left(a^2-2ab+b^2\right)(fx+e)+\frac{3\left(a^2-2ab\right)\tan(fx+e)^2-a^2}{\tan(fx+e)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(a^2 - 2*a*b + b^2)*(f*x + e) + (3*(a^2 - 2*a*b)*tan(f*x + e)^2 - a^
2)/tan(f*x + e)^3)/f
```

Fricas [A] time = 1.07965, size = 143, normalized size = 3.25

$$\frac{3\left(a^2-2\,ab+b^2\right)fx\tan\left(fx+e\right)^3+3\left(a^2-2\,ab\right)\tan\left(fx+e\right)^2-a^2}{3\,f\tan\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^2 - 2*a*b + b^2)*f*x*tan(f*x + e)^3 + 3*(a^2 - 2*a*b)*tan(f*x + e
)^2 - a^2)/(f*tan(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.74893, size = 165, normalized size = 3.75

$$\frac{a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-15\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+24\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+24\left(a^2-2\,ab+b^2\right)\left(fx+e\right)+\frac{15\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2}{\tan\left(\frac{1}{2}\,f\right.}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*f*x + 1/2*e)^3 - 15*a^2*tan(1/2*f*x + 1/2*e) + 24*a*b*tan
(1/2*f*x + 1/2*e) + 24*(a^2 - 2*a*b + b^2)*(f*x + e) + (15*a^2*tan(1/2*f*x
+ 1/2*e)^2 - 24*a*b*tan(1/2*f*x + 1/2*e)^2 - a^2)/tan(1/2*f*x + 1/2*e)^3)/f
```

3.210 $\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=68

$$-\frac{a^2\cot^5(e+fx)}{5f}+\frac{a(a-2b)\cot^3(e+fx)}{3f}-\frac{(a-b)^2\cot(e+fx)}{f}-x(a-b)^2$$

```
[Out] -((a - b)^2*x) - ((a - b)^2*Cot[e + f*x])/f + (a*(a - 2*b)*Cot[e + f*x]^3)/
(3*f) - (a^2*Cot[e + f*x]^5)/(5*f)
```

Rubi [A] time = 0.0769123, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 461, 203}

$$-\frac{a^2\cot^5(e+fx)}{5f}+\frac{a(a-2b)\cot^3(e+fx)}{3f}-\frac{(a-b)^2\cot(e+fx)}{f}-x(a-b)^2$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a - b)^2*x) - ((a - b)^2*Cot[e + f*x])/f + (a*(a - 2*b)*Cot[e + f*x]^3)/
(3*f) - (a^2*Cot[e + f*x]^5)/(5*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^6(1+x^2)}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^6} - \frac{a(a-2b)}{x^4} + \frac{(a-b)^2}{x^2} - \frac{(a-b)^2}{1+x^2}\right) dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a-b)^2\cot(e+fx)}{f} + \frac{a(a-2b)\cot^3(e+fx)}{3f} - \frac{a^2\cot^5(e+fx)}{5f} - \frac{(a-b)^2 \text{S}}{}$$

$$= -(a-b)^2x - \frac{(a-b)^2\cot(e+fx)}{f} + \frac{a(a-2b)\cot^3(e+fx)}{3f} - \frac{a^2\cot^5(e+fx)}{5f}$$

Mathematica [C] time = 0.102449, size = 104, normalized size = 1.53

$$-\frac{a^2\cot^5(e+fx)\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)}{5f} - \frac{2ab\cot^3(e+fx)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\right.}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(a^2*Cot[e + f*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2])/(5*
f) - (2*a*b*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e + f*x]^2
])/(3*f) - (b^2*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[e + f*x]^
2])/f
```

Maple [A] time = 0.053, size = 91, normalized size = 1.3

$$\frac{1}{f}\left(b^2\left(-\cot\left(fx+e\right)-fx-e\right)+2\,ab\left(-1/3\,\left(\cot\left(fx+e\right)\right)^3+\cot\left(fx+e\right)+fx+e\right)+a^2\left(-\frac{\left(\cot\left(fx+e\right)\right)^5}{5}+\frac{\left(\cot\left(fx\right.\right.}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(b^2*(-cot(f*x+e)-f*x-e)+2*a*b*(-1/3*cot(f*x+e)^3+cot(f*x+e)+f*x+e)+a^2
*(-1/5*cot(f*x+e)^5+1/3*cot(f*x+e)^3-cot(f*x+e)-f*x-e))
```

Maxima [A] time = 1.62399, size = 105, normalized size = 1.54

$$-\frac{15\left(a^2-2\,ab+b^2\right)\left(fx+e\right)+\frac{15\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^4-5\left(a^2-2\,ab\right)\tan\left(fx+e\right)^2+3\,a^2}{\tan\left(fx+e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/15*(15*(a^2 - 2*a*b + b^2)*(f*x + e) + (15*(a^2 - 2*a*b + b^2)*tan(f*x +
 e)^4 - 5*(a^2 - 2*a*b)*tan(f*x + e)^2 + 3*a^2)/tan(f*x + e)^5)/f
```

Fricas [A] time = 1.09477, size = 204, normalized size = 3.

$$-\frac{15\left(a^2-2\,ab+b^2\right)fx\tan\left(fx+e\right)^5+15\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^4-5\left(a^2-2\,ab\right)\tan\left(fx+e\right)^2+3\,a^2}{15\,f\tan\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(15*(a^2 - 2*a*b + b^2)*f*x*tan(f*x + e)^5 + 15*(a^2 - 2*a*b + b^2)*t
an(f*x + e)^4 - 5*(a^2 - 2*a*b)*tan(f*x + e)^2 + 3*a^2)/(f*tan(f*x + e)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.81102, size = 300, normalized size = 4.41

$$3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-35\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+40\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+330\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-600\,ab\tan\left(\frac{1}{2}\,f\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*f*x + 1/2*e)^5 - 35*a^2*tan(1/2*f*x + 1/2*e)^3 + 40*a*
b*tan(1/2*f*x + 1/2*e)^3 + 330*a^2*tan(1/2*f*x + 1/2*e) - 600*a*b*tan(1/2*f
*x + 1/2*e) + 240*b^2*tan(1/2*f*x + 1/2*e) - 480*(a^2 - 2*a*b + b^2)*(f*x +
 e) - (330*a^2*tan(1/2*f*x + 1/2*e)^4 - 600*a*b*tan(1/2*f*x + 1/2*e)^4 + 24
0*b^2*tan(1/2*f*x + 1/2*e)^4 - 35*a^2*tan(1/2*f*x + 1/2*e)^2 + 40*a*b*tan(1
/2*f*x + 1/2*e)^2 + 3*a^2)/tan(1/2*f*x + 1/2*e)^5)/f
```

3.211 $\int \frac{\tan^5(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=71

$$-\frac{a^2\log\left(a+b\tan^2(e+fx)\right)}{2b^2f(a-b)}-\frac{\log(\cos(e+fx))}{f(a-b)}+\frac{\tan^2(e+fx)}{2bf}$$

```
[Out] -(Log[Cos[e + f*x]]/((a - b)*f)) - (a^2*Log[a + b*Tan[e + f*x]^2])/(2*(a -
b)*b^2*f) + Tan[e + f*x]^2/(2*b*f)
```

Rubi [A] time = 0.0993675, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 72}

$$-\frac{a^2\log\left(a+b\tan^2(e+fx)\right)}{2b^2f(a-b)}-\frac{\log(\cos(e+fx))}{f(a-b)}+\frac{\tan^2(e+fx)}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(Log[Cos[e + f*x]]/((a - b)*f)) - (a^2*Log[a + b*Tan[e + f*x]^2])/(2*(a -
b)*b^2*f) + Tan[e + f*x]^2/(2*b*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)(a+bx)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{b}+\frac{1}{(a-b)(1+x)}-\frac{a^2}{(a-b)b(a+bx)}\right)dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= -\frac{\log(\cos(e+fx))}{(a-b)f} - \frac{a^2\log\left(a+b\tan^2(e+fx)\right)}{2(a-b)b^2f} + \frac{\tan^2(e+fx)}{2bf}$$

Mathematica [A] time = 0.1577, size = 64, normalized size = 0.9

$$\frac{-\frac{a^2\log\left(a+b\tan^2(e+fx)\right)}{b^2(a-b)} - \frac{2\log(\cos(e+fx))}{a-b} + \frac{\tan^2(e+fx)}{b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((-2*Log[Cos[e + f*x]])/(a - b) - (a^2*Log[a + b*Tan[e + f*x]^2])/((a - b)*
b^2) + Tan[e + f*x]^2/b)/(2*f)
```

Maple [A] time = 0.017, size = 72, normalized size = 1.

$$\frac{\left(\tan\left(fx+e\right)\right)^2}{2fb} - \frac{a^2\ln\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)}{(2a-2b)b^2f} + \frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^2\right)}{2f(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/2*tan(f*x+e)^2/b/f-1/2*a^2*ln(a+b*tan(f*x+e)^2)/(a-b)/b^2/f+1/2/f/(a-b)*l
n(1+tan(f*x+e)^2)
```

Maxima [A] time = 1.09973, size = 103, normalized size = 1.45

$$-\frac{\frac{a^2\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{ab^2-b^3} - \frac{(a+b)\log\left(\sin\left(fx+e\right)^2-1\right)}{b^2} + \frac{1}{b\sin\left(fx+e\right)^2-b}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*log(-(a - b)*sin(f*x + e)^2 + a)/(a*b^2 - b^3) - (a + b)*log(sin(
f*x + e)^2 - 1)/b^2 + 1/(b*sin(f*x + e)^2 - b))/f
```

Fricas [A] time = 1.22247, size = 203, normalized size = 2.86

$$-\frac{a^2\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)-(ab-b^2)\tan(fx+e)^2-(a^2-b^2)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{2(ab^2-b^3)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)) - (a*b - b^2)*ta
n(f*x + e)^2 - (a^2 - b^2)*log(1/(tan(f*x + e)^2 + 1)))/((a*b^2 - b^3)*f)
```

Sympy [A] time = 14.1298, size = 348, normalized size = 4.9

$$\begin{cases} \tilde{\infty}x\tan^3(e) & \text{for } a=0\wedge b=(\\ \frac{\frac{\log(\tan^2(e+fx)+1)}{2f}+\frac{\tan^4(e+fx)}{4f}-\frac{\tan^2(e+fx)}{2f}}{a} & \text{for } b=0 \\ -\frac{2\log(\tan^2(e+fx)+1)\tan^2(e+fx)}{2bf\tan^2(e+fx)+2bf}-\frac{2\log(\tan^2(e+fx)+1)}{2bf\tan^2(e+fx)+2bf}+\frac{\tan^4(e+fx)}{2bf\tan^2(e+fx)+2bf}-\frac{2}{2bf\tan^2(e+fx)+2bf} & \text{for } a=b \\ \frac{x\tan^5(e)}{a+b\tan^2(e)} & \text{for } f=0 \\ -\frac{a^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\tan(e+fx)\right)}{2ab^2f-2b^3f}-\frac{a^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\tan(e+fx)\right)}{2ab^2f-2b^3f}+\frac{ab\tan^2(e+fx)}{2ab^2f-2b^3f}+\frac{b^2\log(\tan^2(e+fx)+1)}{2ab^2f-2b^3f}-\frac{b^2\tan^2(e+fx)}{2ab^2f-2b^3f} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x*tan(e)**3, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((log(tan(e +
f*x)**2 + 1)/(2*f) + tan(e + f*x)**4/(4*f) - tan(e + f*x)**2/(2*f))/a, Eq(b
, 0)), (-2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(2*b*f*tan(e + f*x)**2
+ 2*b*f) - 2*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x)**2 + 2*b*f) + tan
(e + f*x)**4/(2*b*f*tan(e + f*x)**2 + 2*b*f) - 2/(2*b*f*tan(e + f*x)**2 + 2
*b*f), Eq(a, b)), (x*tan(e)**5/(a + b*tan(e)**2), Eq(f, 0)), (-a**2*log(-I*
sqrt(a)*sqrt(1/b) + tan(e + f*x))/(2*a*b**2*f - 2*b**3*f) - a**2*log(I*sqrt
(a)*sqrt(1/b) + tan(e + f*x))/(2*a*b**2*f - 2*b**3*f) + a*b*tan(e + f*x)**2
/(2*a*b**2*f - 2*b**3*f) + b**2*log(tan(e + f*x)**2 + 1)/(2*a*b**2*f - 2*b*
*3*f) - b**2*tan(e + f*x)**2/(2*a*b**2*f - 2*b**3*f), True))
```

Giac [B] time = 3.18393, size = 622, normalized size = 8.76

$$\frac{\frac{2(a+b)\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right)}{b^2}-\frac{(a+b)\log\left(\left|a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)^2-4b\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-4a+8b\right|\right)}{b^2}-\frac{(a^2+b^2)\log\left(\left(\frac{\left|-2a\left(\frac{\cos(fx}{\cos(fx}\right.\right.}{\left|-2a\left(\frac{\cos(fx}{\cos(fx}\right.\right.}\right.\right.}{}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/4*(2*(a + b)*log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) -
 1)/(cos(f*x + e) + 1) - 2)/b^2 - (a + b)*log(abs(a*((cos(f*x + e) + 1)/(co
s(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))^2 - 4*b*((cos(f*x
+ e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 4*a
 + 8*b))/b^2 - (a^2 + b^2)*log(abs(-2*a*((cos(f*x + e) + 1)/(cos(f*x + e) -
 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 4*b - 4*abs(a - b))/abs(-2*a
*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e)
+ 1)) + 4*b + 4*abs(a - b)))/(b^2*abs(a - b)) - 2*(a*((cos(f*x + e) + 1)/(c
os(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + b*((cos(f*x + e
) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 2*a +
6*b)/(b^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(
f*x + e) + 1) + 2)))/f
```

3.212 $\int \frac{\tan^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=50

$$\frac{a\log\left(a+b\tan^2(e+fx)\right)}{2bf(a-b)}+\frac{\log(\cos(e+fx))}{f(a-b)}$$

```
[Out] Log[Cos[e + f*x]]/((a - b)*f) + (a*Log[a + b*Tan[e + f*x]^2])/(2*(a - b)*b*
f)
```

Rubi [A] time = 0.0862229, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 72}

$$\frac{a\log\left(a+b\tan^2(e+fx)\right)}{2bf(a-b)}+\frac{\log(\cos(e+fx))}{f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] Log[Cos[e + f*x]]/((a - b)*f) + (a*Log[a + b*Tan[e + f*x]^2])/(2*(a - b)*b*
f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{(1+x)(a+bx)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a-b)(1+x)} + \frac{a}{(a-b)(a+bx)}\right)dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\log(\cos(e+fx))}{(a-b)f} + \frac{a\log\left(a+b\tan^2(e+fx)\right)}{2(a-b)bf}$$

Mathematica [A] time = 0.0328375, size = 41, normalized size = 0.82

$$\frac{a\log\left(a+b\tan^2(e+fx)\right)+2b\log(\cos(e+fx))}{2abf-2b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (2*b*Log[Cos[e + f*x]] + a*Log[a + b*Tan[e + f*x]^2])/(2*a*b*f - 2*b^2*f)
```

Maple [A] time = 0.016, size = 54, normalized size = 1.1

$$\frac{a\ln\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)}{(2a-2b)bf} - \frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^2\right)}{2f(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/2*a*ln(a+b*tan(f*x+e)^2)/(a-b)/b/f-1/2/f/(a-b)*ln(1+tan(f*x+e)^2)
```

Maxima [A] time = 1.06747, size = 72, normalized size = 1.44

$$\frac{\frac{a\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{ab-b^2} - \frac{\log\left(\sin\left(fx+e\right)^2-1\right)}{b}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(a*log(-(a - b)*sin(f*x + e)^2 + a)/(a*b - b^2) - log(sin(f*x + e)^2 -
1)/b)/f
```

Fricas [A] time = 1.17745, size = 151, normalized size = 3.02

$$\frac{a\log\left(\frac{b\tan\left(fx+e\right)^2+a}{\tan\left(fx+e\right)^2+1}\right)-(a-b)\log\left(\frac{1}{\tan\left(fx+e\right)^2+1}\right)}{2\left(ab-b^2\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)) - (a - b)*log(1/(ta
n(f*x + e)^2 + 1)))/((a*b - b^2)*f)
```

Sympy [A] time = 4.97766, size = 240, normalized size = 4.8

$$\begin{cases} \tilde{\infty}x\tan\left(e\right) & \text{for}\: a=0\wedge b=0\wedge f=0 \\ \frac{\log\left(\tan^2\left(e+fx\right)+1\right)\tan^2\left(e+fx\right)}{2bf\tan^2\left(e+fx\right)+2bf}+\frac{\log\left(\tan^2\left(e+fx\right)+1\right)}{2bf\tan^2\left(e+fx\right)+2bf}+\frac{1}{2bf\tan^2\left(e+fx\right)+2bf} & \text{for}\: a=b \\ \frac{-\frac{\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{\tan^2\left(e+fx\right)}{2f}}{a} & \text{for}\: b=0 \\ \frac{x\tan^3\left(e\right)}{a+b\tan^2\left(e\right)} & \text{for}\: f=0 \\ \frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\tan\left(e+fx\right)\right)}{2abf-2b^2f}+\frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\tan\left(e+fx\right)\right)}{2abf-2b^2f}-\frac{b\log\left(\tan^2\left(e+fx\right)+1\right)}{2abf-2b^2f} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x*tan(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (log(tan(e + f*x)
**2 + 1)*tan(e + f*x)**2/(2*b*f*tan(e + f*x)**2 + 2*b*f) + log(tan(e + f*x)
**2 + 1)/(2*b*f*tan(e + f*x)**2 + 2*b*f) + 1/(2*b*f*tan(e + f*x)**2 + 2*b*f
), Eq(a, b)), ((-log(tan(e + f*x)**2 + 1)/(2*f) + tan(e + f*x)**2/(2*f))/a,
 Eq(b, 0)), (x*tan(e)**3/(a + b*tan(e)**2), Eq(f, 0)), (a*log(-I*sqrt(a)*sq
rt(1/b) + tan(e + f*x))/(2*a*b*f - 2*b**2*f) + a*log(I*sqrt(a)*sqrt(1/b) +
tan(e + f*x))/(2*a*b*f - 2*b**2*f) - b*log(tan(e + f*x)**2 + 1)/(2*a*b*f -
2*b**2*f), True))
```

Giac [B] time = 1.97193, size = 252, normalized size = 5.04

$$\frac{\frac{a^2\log\left(\left|-a\left(\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}+\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)-2\,a+4\,b\right|\right)}{a^2b-ab^2}-\frac{\log\left(-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+2\right)}{a-b}-\frac{\log\left(-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}-2\right)}{b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*(a^2*log(abs(-a*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e)
- 1)/(cos(f*x + e) + 1)) - 2*a + 4*b))/(a^2*b - a*b^2) - log(-(cos(f*x + e)
 + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2)/(a -
b) - log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f
*x + e) + 1) - 2)/b)/f
```

3.213 $\int \frac{\tan(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=36

$$-\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)}$$

```
[Out] -Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/(2*(a - b)*f)
```

Rubi [A] time = 0.0530929, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3670, 444, 36, 31}

$$-\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/(2*(a - b)*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2(a-b)f} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, \tan^2(e+fx)\right)}{2(a-b)f}$$
$$= -\frac{\log(\cos(e+fx))}{(a-b)f} - \frac{\log\left(a+b\tan^2(e+fx)\right)}{2(a-b)f}$$

Mathematica [A] time = 0.0283086, size = 37, normalized size = 1.03

$$-\frac{\log\left(a+b\tan^2(e+fx)\right)+2\log(\cos(e+fx))}{2f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(2*Log[Cos[e + f*x]] + Log[a + b*Tan[e + f*x]^2])/(2*(a - b)*f)
```

Maple [A] time = 0.014, size = 50, normalized size = 1.4

$$-\frac{\ln\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)}{2f(a-b)}+\frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^2\right)}{2f(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/2/f/(a-b)*ln(a+b*tan(f*x+e)^2)+1/2/f/(a-b)*ln(1+tan(f*x+e)^2)
```

Maxima [A] time = 1.05478, size = 41, normalized size = 1.14

$$-\frac{\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{2(a-b)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*log(-(a - b)*sin(f*x + e)^2 + a)/((a - b)*f)
```

Fricas [A] time = 1.08921, size = 90, normalized size = 2.5

$$-\frac{\log\left(\frac{b\tan\left(fx+e\right)^{2}+a}{\tan\left(fx+e\right)^{2}+1}\right)}{2\,(a-b)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1))/((a - b)*f)
```

Sympy [A] time = 3.34803, size = 143, normalized size = 3.97

$$\begin{cases} \frac{\tilde{\infty}x}{\tan{(e)}} & \text{for}\: a = 0 \wedge b = 0 \wedge f = 0 \\ -\frac{1}{2bf\tan^{2}{(e+fx)}+2bf} & \text{for}\: a = b \\ \frac{x\tan{(e)}}{a+b\tan^{2}{(e)}} & \text{for}\: f = 0 \\ \frac{\log{\left(\tan^{2}{(e+fx)}+1 \right)}}{2af} & \text{for}\: b = 0 \\ -\frac{\log{\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\tan{(e+fx)} \right)}}{2af-2bf} - \frac{\log{\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\tan{(e+fx)} \right)}}{2af-2bf} + \frac{\log{\left(\tan^{2}{(e+fx)}+1 \right)}}{2af-2bf} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x/tan(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (-1/(2*b*f*tan(e
+ f*x)**2 + 2*b*f), Eq(a, b)), (x*tan(e)/(a + b*tan(e)**2), Eq(f, 0)), (log
(tan(e + f*x)**2 + 1)/(2*a*f), Eq(b, 0)), (-log(-I*sqrt(a)*sqrt(1/b) + tan(
e + f*x))/(2*a*f - 2*b*f) - log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(2*a*f
- 2*b*f) + log(tan(e + f*x)**2 + 1)/(2*a*f - 2*b*f), True))
```

Giac [A] time = 1.48737, size = 72, normalized size = 2.

$$\frac{\frac{2\,\log(2)}{a-b} - \frac{\log\left(\left|a\cos\left(fx+e\right)^{2}-b\cos\left(fx+e\right)^{2}+b\right|\right)}{a-b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*(2*log(2)/(a - b) - log(abs(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b))/(
a - b))/f
```

3.214 $\int \frac{\cot(e+fx)}{a+b\tan^2(e+fx)} \, dx$

Optimal. Leaf size=64

$$\frac{b\log\left(a+b\tan^2(e+fx)\right)}{2af(a-b)}+\frac{\log(\cos(e+fx))}{f(a-b)}+\frac{\log(\tan(e+fx))}{af}$$

```
[Out] Log[Cos[e + f*x]]/((a - b)*f) + Log[Tan[e + f*x]]/(a*f) + (b*Log[a + b*Tan[
e + f*x]^2])/(2*a*(a - b)*f)
```

Rubi [A] time = 0.0823578, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3670, 446, 72}

$$\frac{b\log\left(a+b\tan^2(e+fx)\right)}{2af(a-b)}+\frac{\log(\cos(e+fx))}{f(a-b)}+\frac{\log(\tan(e+fx))}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] Log[Cos[e + f*x]]/((a - b)*f) + Log[Tan[e + f*x]]/(a*f) + (b*Log[a + b*Tan[
e + f*x]^2])/(2*a*(a - b)*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x(1+x)(a+bx)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax} - \frac{1}{(a-b)(1+x)} + \frac{b^2}{a(a-b)(a+bx)}\right)dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\log(\cos(e+fx))}{(a-b)f} + \frac{\log(\tan(e+fx))}{af} + \frac{b\log\left(a+b\tan^2(e+fx)\right)}{2a(a-b)f}$$

Mathematica [A] time = 0.0499152, size = 57, normalized size = 0.89

$$\frac{b\log\left(a+b\tan^2(e+fx)\right)+2(a-b)\log(\tan(e+fx))+2a\log(\cos(e+fx))}{2af(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (2*a*Log[Cos[e + f*x]] + 2*(a - b)*Log[Tan[e + f*x]] + b*Log[a + b*Tan[e +
f*x]^2])/(2*a*(a - b)*f)
```

Maple [A] time = 0.067, size = 76, normalized size = 1.2

$$\frac{\ln\left(\cos\left(fx+e\right)+1\right)}{2fa}+\frac{b\ln\left(a\left(\cos\left(fx+e\right)\right)^2-\left(\cos\left(fx+e\right)\right)^2b+b\right)}{2fa(a-b)}+\frac{\ln\left(\cos\left(fx+e\right)-1\right)}{2fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/2/f/a*ln(cos(f*x+e)+1)+1/2/f*b/a/(a-b)*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b
)+1/2/f/a*ln(cos(f*x+e)-1)
```

Maxima [A] time = 1.05104, size = 66, normalized size = 1.03

$$\frac{\frac{b\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{a^2-ab}+\frac{\log\left(\sin\left(fx+e\right)^2\right)}{a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*log(-(a - b)*sin(f*x + e)^2 + a)/(a^2 - a*b) + log(sin(f*x + e)^2)/a
)/f
```

Fricas [A] time = 1.20025, size = 169, normalized size = 2.64

$$\frac{(a-b)\log\left(\frac{\tan\left(fx+e\right)^{2}}{\tan\left(fx+e\right)^{2}+1}\right)+b\log\left(\frac{b\tan\left(fx+e\right)^{2}+a}{\tan\left(fx+e\right)^{2}+1}\right)}{2\left(a^{2}-ab\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*((a - b)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) + b*log((b*tan(f*x +
e)^2 + a)/(tan(f*x + e)^2 + 1)))/((a^2 - a*b)*f)
```

Sympy [A] time = 16.7503, size = 398, normalized size = 6.22

$$\begin{cases} \frac{\tilde{\infty}x\cot(e)}{\tan^{2}(e)} & \text{for} \\ \frac{-\frac{\log\left(\tan^{2}(e+fx)+1\right)}{2f}+\frac{\log\left(\tan(e+fx)\right)}{f}}{a} & \text{for} \\ \frac{\frac{\log\left(\tan^{2}(e+fx)+1\right)}{2f}-\frac{\log\left(\tan(e+fx)\right)}{f}-\frac{1}{2f\tan^{2}(e+fx)}}{b} & \text{for} \\ -\frac{\log\left(\tan^{2}(e+fx)+1\right)\tan^{2}(e+fx)}{2af\tan^{2}(e+fx)+2af}-\frac{\log\left(\tan^{2}(e+fx)+1\right)}{2af\tan^{2}(e+fx)+2af}+\frac{2\log\left(\tan(e+fx)\right)\tan^{2}(e+fx)}{2af\tan^{2}(e+fx)+2af}+\frac{2\log\left(\tan(e+fx)\right)}{2af\tan^{2}(e+fx)+2af}+\frac{1}{2af\tan^{2}(e+fx)+2af} & \text{for} \\ \frac{x\cot(e)}{a+b\tan^{2}(e)} & \text{for} \\ -\frac{a\log\left(\tan^{2}(e+fx)+1\right)}{2a^{2}f-2abf}+\frac{2a\log\left(\tan(e+fx)\right)}{2a^{2}f-2abf}+\frac{b\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\tan(e+fx)\right)}{2a^{2}f-2abf}+\frac{b\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\tan(e+fx)\right)}{2a^{2}f-2abf}-\frac{2b\log\left(\tan(e+fx)\right)}{2a^{2}f-2abf} & \text{oth} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x*cot(e)/tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-log(
tan(e + f*x)**2 + 1)/(2*f) + log(tan(e + f*x))/f)/a, Eq(b, 0)), ((log(tan(e
 + f*x)**2 + 1)/(2*f) - log(tan(e + f*x))/f - 1/(2*f*tan(e + f*x)**2))/b, E
q(a, 0)), (-log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(2*a*f*tan(e + f*x)**2
 + 2*a*f) - log(tan(e + f*x)**2 + 1)/(2*a*f*tan(e + f*x)**2 + 2*a*f) + 2*lo
g(tan(e + f*x))*tan(e + f*x)**2/(2*a*f*tan(e + f*x)**2 + 2*a*f) + 2*log(tan
(e + f*x))/(2*a*f*tan(e + f*x)**2 + 2*a*f) + 1/(2*a*f*tan(e + f*x)**2 + 2*a
*f), Eq(a, b)), (x*cot(e)/(a + b*tan(e)**2), Eq(f, 0)), (-a*log(tan(e + f*x
)**2 + 1)/(2*a**2*f - 2*a*b*f) + 2*a*log(tan(e + f*x))/(2*a**2*f - 2*a*b*f)
 + b*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(2*a**2*f - 2*a*b*f) + b*log(
I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(2*a**2*f - 2*a*b*f) - 2*b*log(tan(e +
f*x))/(2*a**2*f - 2*a*b*f), True))
```

Giac [A] time = 1.44647, size = 80, normalized size = 1.25

$$\frac{\frac{b\log\left(\left|-a\sin\left(fx+e\right)^{2}+b\sin\left(fx+e\right)^{2}+a\right|\right)}{a^{2}-ab}+\frac{\log\left(\sin\left(fx+e\right)^{2}\right)}{a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*(b*log(abs(-a*sin(f*x + e)^2 + b*sin(f*x + e)^2 + a))/(a^2 - a*b) + log
(sin(f*x + e)^2)/a)/f
```

3.215 $\int \frac{\cot^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=89

$$-\frac{b^2\log\left(a+b\tan^2(e+fx)\right)}{2a^2f(a-b)}-\frac{(a+b)\log(\tan(e+fx))}{a^2f}-\frac{\log(\cos(e+fx))}{f(a-b)}-\frac{\cot^2(e+fx)}{2af}$$

```
[Out] -Cot[e + f*x]^2/(2*a*f) - Log[Cos[e + f*x]]/((a - b)*f) - ((a + b)*Log[Tan[
e + f*x]])/(a^2*f) - (b^2*Log[a + b*Tan[e + f*x]^2])/(2*a^2*(a - b)*f)
```

Rubi [A] time = 0.112835, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 72}

$$-\frac{b^2\log\left(a+b\tan^2(e+fx)\right)}{2a^2f(a-b)}-\frac{(a+b)\log(\tan(e+fx))}{a^2f}-\frac{\log(\cos(e+fx))}{f(a-b)}-\frac{\cot^2(e+fx)}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```


```
[Out] -Cot[e + f*x]^2/(2*a*f) - Log[Cos[e + f*x]]/((a - b)*f) - ((a + b)*Log[Tan[
e + f*x]])/(a^2*f) - (b^2*Log[a + b*Tan[e + f*x]^2])/(2*a^2*(a - b)*f)
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x)(a+bx)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^2} + \frac{-a-b}{a^2x} + \frac{1}{(a-b)(1+x)} - \frac{b^3}{a^2(a-b)(a+bx)}\right)dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= -\frac{\cot^2(e+fx)}{2af} - \frac{\log(\cos(e+fx))}{(a-b)f} - \frac{(a+b)\log(\tan(e+fx))}{a^2f} - \frac{b^2\log\left(a+b\tan^2(e+fx\right.}{2a^2(a-b)f}$$

Mathematica [A] time = 0.249807, size = 63, normalized size = 0.71

$$-\frac{\frac{b^2\log\left(a\cot^2(e+fx)+b\right)}{a^2(a-b)} + \frac{2\log(\sin(e+fx))}{a-b} + \frac{\cot^2(e+fx)}{a}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(Cot[e + f*x]^2/a + (b^2*Log[b + a*Cot[e + f*x]^2])/(a^2*(a - b)) + (2*Log
[Sin[e + f*x]])/(a - b))/(2*f)
```

Maple [A] time = 0.08, size = 150, normalized size = 1.7

$$-\frac{1}{4fa\left(\cos\left(fx+e\right)+1\right)} - \frac{\ln\left(\cos\left(fx+e\right)+1\right)}{2fa} - \frac{\ln\left(\cos\left(fx+e\right)+1\right)b}{2fa^2} - \frac{b^2\ln\left(a\left(\cos\left(fx+e\right)\right)^2 - \left(\cos\left(fx+e\right)\right)^2\right.}{2fa^2(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/4/f/a/(cos(f*x+e)+1)-1/2/f/a*ln(cos(f*x+e)+1)-1/2/f/a^2*ln(cos(f*x+e)+1)
*b-1/2/f*b^2/a^2/(a-b)*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+1/4/f/a/(cos(f*x
+e)-1)-1/2/f/a*ln(cos(f*x+e)-1)-1/2/f/a^2*ln(cos(f*x+e)-1)*b
```

Maxima [A] time = 1.11219, size = 92, normalized size = 1.03

$$-\frac{\frac{b^2\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{a^3-a^2b} + \frac{(a+b)\log\left(\sin\left(fx+e\right)^2\right)}{a^2} + \frac{1}{a\sin\left(fx+e\right)^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*log(-(a - b)*sin(f*x + e)^2 + a)/(a^3 - a^2*b) + (a + b)*log(sin(
f*x + e)^2)/a^2 + 1/(a*sin(f*x + e)^2))/f
```

Fricas [A] time = 1.22708, size = 297, normalized size = 3.34

$$-\frac{b^2\log\left(\frac{b\tan\left(fx+e\right)^2+a}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^2+\left(a^2-b^2\right)\log\left(\frac{\tan\left(fx+e\right)^2}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^2+\left(a^2-ab\right)\tan\left(fx+e\right)^2+a^2-ab}{2\left(a^3-a^2b\right)f\tan\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 +
 (a^2 - b^2)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + (a^2
 - a*b)*tan(f*x + e)^2 + a^2 - a*b)/((a^3 - a^2*b)*f*tan(f*x + e)^2)
```

Sympy [A] time = 68.3657, size = 743, normalized size = 8.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(e, 0) & Eq(f, 0)), ((-log(tan(e
+ f*x)**2 + 1)/(2*f) + log(tan(e + f*x))/f + 1/(2*f*tan(e + f*x)**2) - 1/(4
*f*tan(e + f*x)**4))/b, Eq(a, 0)), (2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)
**4/(2*a*f*tan(e + f*x)**4 + 2*a*f*tan(e + f*x)**2) + 2*log(tan(e + f*x)**2
 + 1)*tan(e + f*x)**2/(2*a*f*tan(e + f*x)**4 + 2*a*f*tan(e + f*x)**2) - 4*l
og(tan(e + f*x))*tan(e + f*x)**4/(2*a*f*tan(e + f*x)**4 + 2*a*f*tan(e + f*x
)**2) - 4*log(tan(e + f*x))*tan(e + f*x)**2/(2*a*f*tan(e + f*x)**4 + 2*a*f*
tan(e + f*x)**2) - 2*tan(e + f*x)**2/(2*a*f*tan(e + f*x)**4 + 2*a*f*tan(e +
 f*x)**2) - 1/(2*a*f*tan(e + f*x)**4 + 2*a*f*tan(e + f*x)**2), Eq(a, b)), (
zoo*x/a, Eq(e, -f*x)), (x*cot(e)**3/(a + b*tan(e)**2), Eq(f, 0)), ((log(tan
(e + f*x)**2 + 1)/(2*f) - log(tan(e + f*x))/f - 1/(2*f*tan(e + f*x)**2))/a,
 Eq(b, 0)), (a**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(2*a**3*f*tan(e
+ f*x)**2 - 2*a**2*b*f*tan(e + f*x)**2) - 2*a**2*log(tan(e + f*x))*tan(e +
f*x)**2/(2*a**3*f*tan(e + f*x)**2 - 2*a**2*b*f*tan(e + f*x)**2) - a**2/(2*a
**3*f*tan(e + f*x)**2 - 2*a**2*b*f*tan(e + f*x)**2) + a*b/(2*a**3*f*tan(e +
 f*x)**2 - 2*a**2*b*f*tan(e + f*x)**2) - b**2*log(-I*sqrt(a)*sqrt(1/b) + ta
n(e + f*x))*tan(e + f*x)**2/(2*a**3*f*tan(e + f*x)**2 - 2*a**2*b*f*tan(e +
f*x)**2) - b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(2*
a**3*f*tan(e + f*x)**2 - 2*a**2*b*f*tan(e + f*x)**2) + 2*b**2*log(tan(e + f
*x))*tan(e + f*x)**2/(2*a**3*f*tan(e + f*x)**2 - 2*a**2*b*f*tan(e + f*x)**2
), True))
```

Giac [B] time = 1.4894, size = 551, normalized size = 6.19

$$\frac{\frac{8\left(2a^5-2a^4b+a^3b^2-a^2b^3\right)\log\left(24|a|^3\right)}{a^6-2a^5b+a^4b^2}-\frac{4\left(2a^5-2a^4b+a^3b^2-a^2b^3\right)\log\left(\left|-12a^4\cos\left(fx+e\right)^2+12a^3b\cos\left(fx+e\right)^2-12a^3b\right|\right)}{a^6-2a^5b+a^4b^2}-\frac{12\left(a+b\right)\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)}{a^2}+\frac{8(\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/24*(8*(2*a^5 - 2*a^4*b + a^3*b^2 - a^2*b^3)*log(24*abs(a)^3)/(a^6 - 2*a^5
*b + a^4*b^2) - 4*(2*a^5 - 2*a^4*b + a^3*b^2 - a^2*b^3)*log(abs(-12*a^4*cos
(f*x + e)^2 + 12*a^3*b*cos(f*x + e)^2 - 12*a^3*b))/(a^6 - 2*a^5*b + a^4*b^2
) - 12*(a + b)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/a^2 + 8*(a + b)*
log(abs(a + a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) - a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 4*b*(c
os(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - a*(cos(f*x + e) - 1)^3/(cos(f*x +
 e) + 1)^3))/a^2 + 3*(a + 4*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 4*b*(
cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(f*x + e) + 1)/(a^2*(cos(f*x + e)
 - 1)) + 3*(cos(f*x + e) - 1)/(a*(cos(f*x + e) + 1)))/f
```

3.216 $\int \frac{\cot^5(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=115

$$\frac{b^3\log\left(a+b\tan^2(e+fx)\right)}{2a^3f(a-b)}+\frac{\left(a^2+ab+b^2\right)\log(\tan(e+fx))}{a^3f}+\frac{(a+b)\cot^2(e+fx)}{2a^2f}+\frac{\log(\cos(e+fx))}{f(a-b)}-\frac{\cot^4(e+fx)}{4af}$$

```
[Out] ((a + b)*Cot[e + f*x]^2)/(2*a^2*f) - Cot[e + f*x]^4/(4*a*f) + Log[Cos[e + f
*x]]/((a - b)*f) + ((a^2 + a*b + b^2)*Log[Tan[e + f*x]])/(a^3*f) + (b^3*Log
[a + b*Tan[e + f*x]^2])/(2*a^3*(a - b)*f)
```

Rubi [A] time = 0.137124, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 72}

$$\frac{b^3\log\left(a+b\tan^2(e+fx)\right)}{2a^3f(a-b)}+\frac{\left(a^2+ab+b^2\right)\log(\tan(e+fx))}{a^3f}+\frac{(a+b)\cot^2(e+fx)}{2a^2f}+\frac{\log(\cos(e+fx))}{f(a-b)}-\frac{\cot^4(e+fx)}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((a + b)*Cot[e + f*x]^2)/(2*a^2*f) - Cot[e + f*x]^4/(4*a*f) + Log[Cos[e + f
*x]]/((a - b)*f) + ((a^2 + a*b + b^2)*Log[Tan[e + f*x]])/(a^3*f) + (b^3*Log
[a + b*Tan[e + f*x]^2])/(2*a^3*(a - b)*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^5(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x)(a+bx)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^3} + \frac{-a-b}{a^2x^2} + \frac{a^2+ab+b^2}{a^3x} - \frac{1}{(a-b)(1+x)} + \frac{b^4}{a^3(a-b)(a+bx)}\right) dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{(a+b)\cot^2(e+fx)}{2a^2f} - \frac{\cot^4(e+fx)}{4af} + \frac{\log(\cos(e+fx))}{(a-b)f} + \frac{(a^2+ab+b^2)\log(\tan(e+fx)}{a^3f}$$

Mathematica [A] time = 0.340275, size = 83, normalized size = 0.72

$$-\frac{-\frac{b^3\log\left(a\cot^2(e+fx)+b\right)}{a^3(a-b)} - \frac{(a+b)\cot^2(e+fx)}{a^2} - \frac{2\log(\sin(e+fx))}{a-b} + \frac{\cot^4(e+fx)}{2a}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(-(((a + b)*Cot[e + f*x]^2)/a^2) + Cot[e + f*x]^4/(2*a) - (b^3*Log[b + a*C
ot[e + f*x]^2])/(a^3*(a - b)) - (2*Log[Sin[e + f*x]])/(a - b))/(2*f)
```

Maple [B] time = 0.082, size = 264, normalized size = 2.3

$$-\frac{1}{16fa\left(\cos\left(fx+e\right)+1\right)^2} + \frac{7}{16fa\left(\cos\left(fx+e\right)+1\right)} + \frac{b}{4fa^2\left(\cos\left(fx+e\right)+1\right)} + \frac{\ln\left(\cos\left(fx+e\right)+1\right)}{2fa} + \frac{\ln\left(\cos\left(\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/16/f/a/(cos(f*x+e)+1)^2+7/16/f/a/(cos(f*x+e)+1)+1/4/f/a^2/(cos(f*x+e)+1)
*b+1/2/f/a*ln(cos(f*x+e)+1)+1/2/f/a^2*ln(cos(f*x+e)+1)*b+1/2/f/a^3*ln(cos(f
*x+e)+1)*b^2+1/2/f*b^3/a^3/(a-b)*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)-1/16/f
/a/(cos(f*x+e)-1)^2-7/16/f/a/(cos(f*x+e)-1)-1/4/f/a^2/(cos(f*x+e)-1)*b+1/2/
f/a*ln(cos(f*x+e)-1)+1/2/f/a^2*ln(cos(f*x+e)-1)*b+1/2/f/a^3*ln(cos(f*x+e)-1
)*b^2
```

Maxima [A] time = 1.07262, size = 130, normalized size = 1.13

$$\frac{\frac{2b^3\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{a^4-a^3b} + \frac{2\left(a^2+ab+b^2\right)\log\left(\sin\left(fx+e\right)^2\right)}{a^3} + \frac{2\left(2a+b\right)\sin\left(fx+e\right)^2-a}{a^2\sin\left(fx+e\right)^4}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b^3*log(-(a - b)*sin(f*x + e)^2 + a)/(a^4 - a^3*b) + 2*(a^2 + a*b +
b^2)*log(sin(f*x + e)^2)/a^3 + (2*(2*a + b)*sin(f*x + e)^2 - a)/(a^2*sin(f*
x + e)^4))/f
```

Fricas [A] time = 1.32564, size = 367, normalized size = 3.19

$$\frac{2\,b^3\log\left(\frac{b\tan\left(fx+e\right)^2+a}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^4+2\left(a^3-b^3\right)\log\left(\frac{\tan\left(fx+e\right)^2}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^4+\left(3\,a^3-a^2b-2\,ab^2\right)\tan\left(fx+e\right)^4-}{4\left(a^4-a^3b\right)f\tan\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4
+ 2*(a^3 - b^3)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 + (
3*a^3 - a^2*b - 2*a*b^2)*tan(f*x + e)^4 - a^3 + a^2*b + 2*(a^3 - a*b^2)*tan
(f*x + e)^2)/((a^4 - a^3*b)*f*tan(f*x + e)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.53353, size = 755, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/192*(64*(2*a^7 - 2*a^6*b + a^4*b^3 - a^3*b^4)*log(24*a^4)/(a^8 - 2*a^7*b
 + a^6*b^2) - 32*(2*a^7 - 2*a^6*b + a^4*b^3 - a^3*b^4)*log(abs(-12*a^5*cos(
f*x + e)^2 + 12*a^4*b*cos(f*x + e)^2 - 12*a^4*b))/(a^8 - 2*a^7*b + a^6*b^2)
 + 3*(12*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 8*b*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^2 - 96*(
a^2 + a*b + b^2)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/a^3 + 64*(a^2
+ a*b + b^2)*log(abs(a + a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos
(f*x + e) - 1)/(cos(f*x + e) + 1) - a*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
1)^2 + 4*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - a*(cos(f*x + e) - 1)
^3/(cos(f*x + e) + 1)^3))/a^3 + 3*(a^2 + 12*a^2*(cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) + 8*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 48*a^2*(cos(f*x
```

```
+ e) - 1)^2/(cos(f*x + e) + 1)^2 + 48*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e
) + 1)^2 + 48*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e)
+ 1)^2/(a^3*(cos(f*x + e) - 1)^2))/f
```

3.217 $\int \frac{\tan^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=85

$$\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{b^{5/2}f(a-b)}-\frac{(a+b)\tan(e+fx)}{b^2f}-\frac{x}{a-b}+\frac{\tan^3(e+fx)}{3bf}$$

```
[Out] -(x/(a - b)) + (a^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/((a - b)*b^
(5/2)*f) - ((a + b)*Tan[e + f*x])/(b^2*f) + Tan[e + f*x]^3/(3*b*f)
```

Rubi [A] time = 0.187996, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 479, 582, 522, 203, 205}

$$\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{b^{5/2}f(a-b)}-\frac{(a+b)\tan(e+fx)}{b^2f}-\frac{x}{a-b}+\frac{\tan^3(e+fx)}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(x/(a - b)) + (a^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/((a - b)*b^
(5/2)*f) - ((a + b)*Tan[e + f*x])/(b^2*f) + Tan[e + f*x]^3/(3*b*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^6(e+fx)}{a+b\tan^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+bx^2)}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\tan^3(e+fx)}{3bf} - \frac{\text{Subst}\left(\int \frac{x^2(3a+3(a+b)x^2)}{(1+x^2)(a+bx^2)}\,dx,x,\tan(e+fx)\right)}{3bf}\\
&= -\frac{(a+b)\tan(e+fx)}{b^2f} + \frac{\tan^3(e+fx)}{3bf} + \frac{\text{Subst}\left(\int \frac{3a(a+b)+3(a^2+ab+b^2)x^2}{(1+x^2)(a+bx^2)}\,dx,x,\tan(e+fx)\right)}{3b^2f}\\
&= -\frac{(a+b)\tan(e+fx)}{b^2f} + \frac{\tan^3(e+fx)}{3bf} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+fx)\right)}{(a-b)f} + \frac{a^3\,\text{Subst}\left(\int\right.}{}\\
&= -\frac{x}{a-b} + \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)b^{5/2}f} - \frac{(a+b)\tan(e+fx)}{b^2f} + \frac{\tan^3(e+fx)}{3bf}
\end{aligned}
$$

Mathematica [A] time = 0.781735, size = 92, normalized size = 1.08

$$
\frac{\sqrt{b}\left((a-b)\tan(e+fx)\left(3a-b\sec^2(e+fx)+4b\right)+3b^2(e+fx)\right)-3a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{3b^{5/2}f(b-a)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-3*a^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[b]*(3*b^2*(e + f*
x) + (a - b)*(3*a + 4*b - b*Sec[e + f*x]^2)*Tan[e + f*x]))/(3*b^(5/2)*(-a +
 b)*f)
```

Maple [A] time = 0.017, size = 102, normalized size = 1.2

$$
\frac{\left(\tan\left(fx+e\right)\right)^3}{3fb} - \frac{a\tan\left(fx+e\right)}{fb^2} - \frac{\tan\left(fx+e\right)}{fb} + \frac{a^3}{fb^2(a-b)}\arctan\left(b\tan\left(fx+e\right)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{\arctan\left(\tan\left(fx\right.\right.}{f(a-b)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/3*tan(f*x+e)^3/b/f-1/f/b^2*a*tan(f*x+e)-tan(f*x+e)/b/f+1/f/b^2*a^3/(a-b)/
(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-1/f/(a-b)*arctan(tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.21668, size = 618, normalized size = 7.27

$$\left[-\frac{12 b^2 f x - 4\left(ab - b^2\right)\tan\left(fx+e\right)^3 + 3a^2\sqrt{-\frac{a}{b}}\log\left(\frac{b^2\tan\left(fx+e\right)^4 - 6ab\tan\left(fx+e\right)^2 + a^2 - 4\left(b^2\tan\left(fx+e\right)^3 - ab\tan\left(fx+e\right)\right)\sqrt{-\frac{a}{b}}}{b^2\tan\left(fx+e\right)^4 + 2ab\tan\left(fx+e\right)^2 + a^2}\right) + 12\,(a}{12\left(ab^2 - b^3\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(12*b^2*f*x - 4*(a*b - b^2)*tan(f*x + e)^3 + 3*a^2*sqrt(-a/b)*log((b
^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(b^2*tan(f*x + e)^3 - a*
b*tan(f*x + e))*sqrt(-a/b))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^
2)) + 12*(a^2 - b^2)*tan(f*x + e))/((a*b^2 - b^3)*f), -1/6*(6*b^2*f*x - 2*(
a*b - b^2)*tan(f*x + e)^3 - 3*a^2*sqrt(a/b)*arctan(1/2*(b*tan(f*x + e)^2 -
a)*sqrt(a/b)/(a*tan(f*x + e))) + 6*(a^2 - b^2)*tan(f*x + e))/((a*b^2 - b^3)
*f)]
```

Sympy [A] time = 20.8982, size = 685, normalized size = 8.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x*tan(e)**4, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x + tan(e +
 f*x)**5/(5*f) - tan(e + f*x)**3/(3*f) + tan(e + f*x)/f)/a, Eq(b, 0)), ((x
+ tan(e + f*x)**3/(3*f) - tan(e + f*x)/f)/b, Eq(a, 0)), (15*f*x*tan(e + f*x
)**2/(6*b*f*tan(e + f*x)**2 + 6*b*f) + 15*f*x/(6*b*f*tan(e + f*x)**2 + 6*b*
f) + 2*tan(e + f*x)**5/(6*b*f*tan(e + f*x)**2 + 6*b*f) - 10*tan(e + f*x)**3
/(6*b*f*tan(e + f*x)**2 + 6*b*f) - 15*tan(e + f*x)/(6*b*f*tan(e + f*x)**2 +
```

```
6*b*f), Eq(a, b)), (x*tan(e)**6/(a + b*tan(e)**2), Eq(f, 0)), (-6*I*a**(5/
2)*b*sqrt(1/b)*tan(e + f*x)/(6*I*a**(3/2)*b**3*f*sqrt(1/b) - 6*I*sqrt(a)*b*
*4*f*sqrt(1/b)) + 2*I*a**(3/2)*b**2*sqrt(1/b)*tan(e + f*x)**3/(6*I*a**(3/2)
*b**3*f*sqrt(1/b) - 6*I*sqrt(a)*b**4*f*sqrt(1/b)) - 6*I*sqrt(a)*b**3*f*x*sq
rt(1/b)/(6*I*a**(3/2)*b**3*f*sqrt(1/b) - 6*I*sqrt(a)*b**4*f*sqrt(1/b)) - 2*
I*sqrt(a)*b**3*sqrt(1/b)*tan(e + f*x)**3/(6*I*a**(3/2)*b**3*f*sqrt(1/b) - 6
*I*sqrt(a)*b**4*f*sqrt(1/b)) + 6*I*sqrt(a)*b**3*sqrt(1/b)*tan(e + f*x)/(6*I
*a**(3/2)*b**3*f*sqrt(1/b) - 6*I*sqrt(a)*b**4*f*sqrt(1/b)) + 3*a**3*log(-I*
sqrt(a)*sqrt(1/b) + tan(e + f*x))/(6*I*a**(3/2)*b**3*f*sqrt(1/b) - 6*I*sqrt
(a)*b**4*f*sqrt(1/b)) - 3*a**3*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(6*I
*a**(3/2)*b**3*f*sqrt(1/b) - 6*I*sqrt(a)*b**4*f*sqrt(1/b)), True))
```

Giac [B] time = 4.15437, size = 521, normalized size = 6.13

$$\frac{\frac{3\left(a^{3}b^{4}+b^{7}+a^{2}b\left|-ab^{3}+b^{4}\right|+ab^{2}\left|-ab^{3}+b^{4}\right|+b^{3}\left|-ab^{3}+b^{4}\right|\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{8\sqrt{\frac{1}{2}}\tan\left(fx+e\right)}{\sqrt{\frac{16\,ab^{3}+16\,b^{4}+\sqrt{-1024\,ab^{7}+256\left(ab^{3}+b^{4}\right)^{2}}}{b^{4}}}}\right)\right)}{ab^{3}\left|-ab^{3}+b^{4}\right|+b^{4}\left|-ab^{3}+b^{4}\right|+\left(ab^{3}-b^{4}\right)^{2}}-\frac{3\left(\left(a^{2}+ab+b^{2}\right)\sqrt{ab}\left|-ab^{3}+b^{4}\right|\left|b\right|-}{}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(a^3*b^4 + b^7 + a^2*b*abs(-a*b^3 + b^4) + a*b^2*abs(-a*b^3 + b^4) +
 b^3*abs(-a*b^3 + b^4))*(pi*floor((f*x + e)/pi + 1/2) + arctan(8*sqrt(1/2)*
tan(f*x + e)/sqrt((16*a*b^3 + 16*b^4 + sqrt(-1024*a*b^7 + 256*(a*b^3 + b^4)
^2))/b^4)))/(a*b^3*abs(-a*b^3 + b^4) + b^4*abs(-a*b^3 + b^4) + (a*b^3 - b^4
)^2) - 3*((a^2 + a*b + b^2)*sqrt(a*b)*abs(-a*b^3 + b^4)*abs(b) - (a^3*b^3 +
 b^6)*sqrt(a*b)*abs(b))*(pi*floor((f*x + e)/pi + 1/2) + arctan(8*sqrt(1/2)*
tan(f*x + e)/sqrt((16*a*b^3 + 16*b^4 - sqrt(-1024*a*b^7 + 256*(a*b^3 + b^4)
^2))/b^4)))/((a*b^3 - b^4)^2*b - (a*b^4 + b^5)*abs(-a*b^3 + b^4)) + (b^2*ta
n(f*x + e)^3 - 3*a*b*tan(f*x + e) - 3*b^2*tan(f*x + e))/b^3)/f
```

3.218 $\int \frac{\tan^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=63

$$-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{b^{3/2}f(a-b)}+\frac{x}{a-b}+\frac{\tan(e+fx)}{bf}$$

```
[Out] x/(a - b) - (a^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/((a - b)*b^(3/
2)*f) + Tan[e + f*x]/(b*f)
```

Rubi [A] time = 0.106324, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3670, 479, 522, 203, 205}

$$-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{b^{3/2}f(a-b)}+\frac{x}{a-b}+\frac{\tan(e+fx)}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] x/(a - b) - (a^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/((a - b)*b^(3/
2)*f) + Tan[e + f*x]/(b*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\tan(e+fx)}{bf} - \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{bf}$$
$$= \frac{\tan(e+fx)}{bf} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{(a-b)f} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(e+fx)\right)}{(a-b)bf}$$
$$= \frac{x}{a-b} - \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)b^{3/2}f} + \frac{\tan(e+fx)}{bf}$$

Mathematica [A] time = 0.274144, size = 70, normalized size = 1.11

$$-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{b^{3/2}f(a-b)} + \frac{e+fx}{f(a-b)} + \frac{\tan(e+fx)}{bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (e + f*x)/((a - b)*f) - (a^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/((
a - b)*b^(3/2)*f) + Tan[e + f*x]/(b*f)
```

Maple [A] time = 0.016, size = 70, normalized size = 1.1

$$\frac{\tan(fx+e)}{fb} - \frac{a^2}{fb(a-b)}\arctan\left(b\tan(fx+e)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{\arctan(\tan(fx+e))}{f(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2),x)
```

```
[Out] tan(f*x+e)/b/f-1/f/b*a^2/(a-b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))
+1/f/(a-b)*arctan(tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.19387, size = 495, normalized size = 7.86

$$\left[\frac{4\,bfx - a\sqrt{-\frac{a}{b}}\log\left(\frac{b^2\tan\left(fx+e\right)^4 - 6\,ab\tan\left(fx+e\right)^2 + a^2 + 4\left(b^2\tan\left(fx+e\right)^3 - ab\tan\left(fx+e\right)\right)\sqrt{-\frac{a}{b}}}{b^2\tan\left(fx+e\right)^4 + 2\,ab\tan\left(fx+e\right)^2 + a^2}\right) + 4\,(a-b)\tan\left(fx+e\right)}{4\left(ab-b^2\right)f}, \frac{2\,bfx - a\sqrt{\frac{a}{b}}\,\mathrm{ar}\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*f*x - a*sqrt(-a/b)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2
 + a^2 + 4*(b^2*tan(f*x + e)^3 - a*b*tan(f*x + e))*sqrt(-a/b))/(b^2*tan(f*x
 + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) + 4*(a - b)*tan(f*x + e))/((a*b - b^
2)*f), 1/2*(2*b*f*x - a*sqrt(a/b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a/
b)/(a*tan(f*x + e))) + 2*(a - b)*tan(f*x + e))/((a*b - b^2)*f)]
```

Sympy [A] time = 9.16737, size = 493, normalized size = 7.83

$$\begin{cases} \tilde{\infty} x\tan^2(e) \\ \frac{-x+\frac{\tan(e+fx)}{f}}{b} \\ -\frac{3fx\tan^2(e+fx)}{2bf\tan^2(e+fx)+2bf} - \frac{3fx}{2bf\tan^2(e+fx)+2bf} + \frac{2\tan^3(e+fx)}{2bf\tan^2(e+fx)+2bf} + \frac{3\tan(e+fx)}{2bf\tan^2(e+fx)+2bf} \\ \frac{x\tan^4(e)}{a+b\tan^2(e)} \\ \frac{x+\frac{\tan^3(e+fx)}{3f}-\frac{\tan(e+fx)}{f}}{a} \\ \frac{2ia^{\frac{3}{2}}b\sqrt{\frac{1}{b}}\tan(e+fx)}{2ia^{\frac{3}{2}}b^2f\sqrt{\frac{1}{b}}-2i\sqrt{a}b^3f\sqrt{\frac{1}{b}}} + \frac{2i\sqrt{a}b^2fx\sqrt{\frac{1}{b}}}{2ia^{\frac{3}{2}}b^2f\sqrt{\frac{1}{b}}-2i\sqrt{a}b^3f\sqrt{\frac{1}{b}}} - \frac{2i\sqrt{a}b^2\sqrt{\frac{1}{b}}\tan(e+fx)}{2ia^{\frac{3}{2}}b^2f\sqrt{\frac{1}{b}}-2i\sqrt{a}b^3f\sqrt{\frac{1}{b}}} - \frac{a^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\tan(e+fx)\right)}{2ia^{\frac{3}{2}}b^2f\sqrt{\frac{1}{b}}-2i\sqrt{a}b^3f\sqrt{\frac{1}{b}}} + \frac{a^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\tan(e+fx)\right)}{2ia^{\frac{3}{2}}b^2f\sqrt{\frac{1}{b}}-2i\sqrt{a}b^3f\sqrt{\frac{1}{b}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x*tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x + tan(e +
 f*x)/f)/b, Eq(a, 0)), (-3*f*x*tan(e + f*x)**2/(2*b*f*tan(e + f*x)**2 + 2*b
*f) - 3*f*x/(2*b*f*tan(e + f*x)**2 + 2*b*f) + 2*tan(e + f*x)**3/(2*b*f*tan(
e + f*x)**2 + 2*b*f) + 3*tan(e + f*x)/(2*b*f*tan(e + f*x)**2 + 2*b*f), Eq(a
, b)), (x*tan(e)**4/(a + b*tan(e)**2), Eq(f, 0)), ((x + tan(e + f*x)**3/(3*
f) - tan(e + f*x)/f)/a, Eq(b, 0)), (2*I*a**(3/2)*b*sqrt(1/b)*tan(e + f*x)/(
2*I*a**(3/2)*b**2*f*sqrt(1/b) - 2*I*sqrt(a)*b**3*f*sqrt(1/b)) + 2*I*sqrt(a)
*b**2*f*x*sqrt(1/b)/(2*I*a**(3/2)*b**2*f*sqrt(1/b) - 2*I*sqrt(a)*b**3*f*sqr
t(1/b)) - 2*I*sqrt(a)*b**2*sqrt(1/b)*tan(e + f*x)/(2*I*a**(3/2)*b**2*f*sqrt
(1/b) - 2*I*sqrt(a)*b**3*f*sqrt(1/b)) - a**2*log(-I*sqrt(a)*sqrt(1/b) + tan
(e + f*x))/(2*I*a**(3/2)*b**2*f*sqrt(1/b) - 2*I*sqrt(a)*b**3*f*sqrt(1/b)) +
```

```
 a**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(2*I*a**(3/2)*b**2*f*sqrt(1/b
) - 2*I*sqrt(a)*b**3*f*sqrt(1/b)), True))
```

Giac [B] time = 2.27763, size = 405, normalized size = 6.43

$$-\frac{\frac{\left(a^2b+b^3+a\left|-ab+b^2\right|+b\left|-ab+b^2\right|\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{4\sqrt{\frac{1}{2}}\tan\left(fx+e\right)}{\sqrt{\frac{4\,ab+4\,b^2+\sqrt{-64\,ab^3+16\,\left(ab+b^2\right)^2}}{b^2}}}\right)\right)}{ab\left|-ab+b^2\right|+b^2\left|-ab+b^2\right|+\left(ab-b^2\right)^2}-\frac{\left(\sqrt{ab}(a+b)\left|-ab+b^2\right||b|-\left(a^2b+b^3\right)\sqrt{ab}|b|\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor+\mathrm{arctan}\cdots\right.}{\left(ab-b^2\right)^2b^2-\left(ab^3+b^4\right)\left|-ab+\cdots\right.}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((a^2*b + b^3 + a*abs(-a*b + b^2) + b*abs(-a*b + b^2))*(pi*floor((f*x + e)
/pi + 1/2) + arctan(4*sqrt(1/2)*tan(f*x + e)/sqrt((4*a*b + 4*b^2 + sqrt(-64
*a*b^3 + 16*(a*b + b^2)^2))/b^2)))/(a*b*abs(-a*b + b^2) + b^2*abs(-a*b + b^
2) + (a*b - b^2)^2) - (sqrt(a*b)*(a + b)*abs(-a*b + b^2)*abs(b) - (a^2*b +
b^3)*sqrt(a*b)*abs(b))*(pi*floor((f*x + e)/pi + 1/2) + arctan(4*sqrt(1/2)*t
an(f*x + e)/sqrt((4*a*b + 4*b^2 - sqrt(-64*a*b^3 + 16*(a*b + b^2)^2))/b^2))
)/((a*b - b^2)^2*b^2 - (a*b^3 + b^4)*abs(-a*b + b^2)) - tan(f*x + e)/b)/f
```

3.219 $\int \frac{\tan^2(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=50

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{b}f(a-b)}-\frac{x}{a-b}$$

```
[Out] -(x/(a - b)) + (Sqrt[a]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/((a - b)*Sq
rt[b]*f)
```

Rubi [A] time = 0.0792586, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3670, 481, 203, 205}

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{b}f(a-b)}-\frac{x}{a-b}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(x/(a - b)) + (Sqrt[a]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/((a - b)*Sq
rt[b]*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+bx^2)}\,dx,x,\tan(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+fx)\right)}{(a-b)f} + \frac{a\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\tan(e+fx)\right)}{(a-b)f}$$
$$= -\frac{x}{a-b} + \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)\sqrt{b}f}$$

Mathematica [A] time = 0.0305413, size = 49, normalized size = 0.98

$$\frac{\tan^{-1}(\tan(e+fx)) - \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{b}}}{bf-af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (ArcTan[Tan[e + f*x]] - (Sqrt[a]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/Sq
rt[b])/(-(a*f) + b*f)
```

Maple [A] time = 0.017, size = 52, normalized size = 1.

$$\frac{a}{f(a-b)}\arctan\left(b\tan(fx+e)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{\arctan(\tan(fx+e))}{f(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*a/(a-b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-1/f/(a-b)*arctan(t
an(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.14065, size = 409, normalized size = 8.18

$$\left[-\frac{4\,fx+\sqrt{-\frac{a}{b}}\log\left(\frac{b^2\tan\left(fx+e\right)^4-6\,ab\tan\left(fx+e\right)^2+a^2-4\left(b^2\tan\left(fx+e\right)^3-ab\tan\left(fx+e\right)\right)\sqrt{-\frac{a}{b}}}{b^2\tan\left(fx+e\right)^4+2\,ab\tan\left(fx+e\right)^2+a^2}\right)}{4\,(a-b)f},-\frac{2\,fx-\sqrt{\frac{a}{b}}\arctan\left(\frac{\left(b\tan\left(fx+e\right)^2-a\right)\sqrt{\frac{a}{b}}}{2\,a\tan\left(fx+e\right)}\right)}{2\,(a-b)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*f*x + sqrt(-a/b)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 +
a^2 - 4*(b^2*tan(f*x + e)^3 - a*b*tan(f*x + e))*sqrt(-a/b))/(b^2*tan(f*x +
e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/((a - b)*f), -1/2*(2*f*x - sqrt(a/b)*a
rctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a/b)/(a*tan(f*x + e))))/((a - b)*f)]
```

Sympy [A] time = 3.60685, size = 292, normalized size = 5.84

$$\begin{cases} \tilde{\infty}x & \text{for}\ a=0\wedge b=0\wedge f=0 \\ \frac{-x+\frac{\tan\left(e+fx\right)}{f}}{a} & \text{for}\ b=0 \\ \frac{x}{b} & \text{for}\ a=0 \\ \frac{fx\tan^2\left(e+fx\right)}{2bf\tan^2\left(e+fx\right)+2bf}+\frac{fx}{2bf\tan^2\left(e+fx\right)+2bf}-\frac{\tan\left(e+fx\right)}{2bf\tan^2\left(e+fx\right)+2bf} & \text{for}\ a=b \\ \frac{x\tan^2\left(e\right)}{a+b\tan^2\left(e\right)} & \text{for}\ f=0 \\ -\frac{2i\sqrt{a}bfx\sqrt{\frac{1}{b}}}{2ia^{\frac{3}{2}}bf\sqrt{\frac{1}{b}}-2i\sqrt{a}b^2f\sqrt{\frac{1}{b}}}+\frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\tan\left(e+fx\right)\right)}{2ia^{\frac{3}{2}}bf\sqrt{\frac{1}{b}}-2i\sqrt{a}b^2f\sqrt{\frac{1}{b}}}-\frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\tan\left(e+fx\right)\right)}{2ia^{\frac{3}{2}}bf\sqrt{\frac{1}{b}}-2i\sqrt{a}b^2f\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x + tan(e + f*x)/f)/a
, Eq(b, 0)), (x/b, Eq(a, 0)), (f*x*tan(e + f*x)**2/(2*b*f*tan(e + f*x)**2 +
 2*b*f) + f*x/(2*b*f*tan(e + f*x)**2 + 2*b*f) - tan(e + f*x)/(2*b*f*tan(e +
 f*x)**2 + 2*b*f), Eq(a, b)), (x*tan(e)**2/(a + b*tan(e)**2), Eq(f, 0)), (-
2*I*sqrt(a)*b*f*x*sqrt(1/b)/(2*I*a**(3/2)*b*f*sqrt(1/b) - 2*I*sqrt(a)*b**2*
f*sqrt(1/b)) + a*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(2*I*a**(3/2)*b*f
*sqrt(1/b) - 2*I*sqrt(a)*b**2*f*sqrt(1/b)) - a*log(I*sqrt(a)*sqrt(1/b) + ta
n(e + f*x))/(2*I*a**(3/2)*b*f*sqrt(1/b) - 2*I*sqrt(a)*b**2*f*sqrt(1/b)), Tr
ue))
```

Giac [B] time = 1.56394, size = 184, normalized size = 3.68

$$\frac{\frac{\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan\left(fx+e\right)}{\sqrt{\frac{a+b+\sqrt{(a+b)^2-4\,ab}}{b}}}\right)}{|-a+b|}-\frac{\sqrt{ab}\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan\left(fx+e\right)}{\sqrt{\frac{a+b-\sqrt{(a+b)^2-4\,ab}}{b}}}\right)\right)|b|}{b^2|-a+b|}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] ((pi*floor((f*x + e)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(f*x + e)/sqrt((a +
b + sqrt((a + b)^2 - 4*a*b))/b)))/abs(-a + b) - sqrt(a*b)*(pi*floor((f*x +
e)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(f*x + e)/sqrt((a + b - sqrt((a + b)^2
 - 4*a*b))/b)))*abs(b)/(b^2*abs(-a + b)))/f
```

3.220 $\int \frac{1}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=50

$$\frac{x}{a-b}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}f(a-b)}$$

```
[Out] x/(a - b) - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*(a -
b)*f)
```

Rubi [A] time = 0.0740666, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3660, 3675, 205}

$$\frac{x}{a-b}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-1),x]
```

```
[Out] x/(a - b) - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*(a -
b)*f)
```

Rule 3660

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/(a -
 b), x] - Dist[b/(a - b), Int[Sec[e + f*x]^2/(a + b*Tan[e + f*x]^2), x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a, b]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+b\tan^2(e+fx)}\,dx = \frac{x}{a-b} - \frac{b\int \frac{\sec^2(e+fx)}{a+b\tan^2(e+fx)}\,dx}{a-b}$$
$$= \frac{x}{a-b} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(e+fx)\right)}{(a-b)f}$$
$$= \frac{x}{a-b} - \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)f}$$

Mathematica [A] time = 0.0470537, size = 49, normalized size = 0.98

$$\frac{\tan^{-1}(\tan(e+fx)) - \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}}}{af-bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-1),x]
```

```
[Out] (ArcTan[Tan[e + f*x]] - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/Sq
rt[a])/(a*f - b*f)
```

Maple [A] time = 0., size = 52, normalized size = 1.

$$-\frac{b}{f(a-b)}\arctan\left(b\tan\left(fx+e\right)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{\arctan\left(\tan\left(fx+e\right)\right)}{f(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/f/(a-b)*b/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))+1/f/(a-b)*arctan(
tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.10647, size = 406, normalized size = 8.12

$$\left[\frac{4\,fx - \sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan\left(fx+e\right)^4 - 6\,ab\tan\left(fx+e\right)^2 + a^2 + 4\left(ab\tan\left(fx+e\right)^3 - a^2\tan\left(fx+e\right)\right)\sqrt{-\frac{b}{a}}}{b^2\tan\left(fx+e\right)^4 + 2\,ab\tan\left(fx+e\right)^2 + a^2}\right)}{4\,(a-b)f}, \frac{2\,fx - \sqrt{\frac{b}{a}}\arctan\left(\frac{\left(b\tan\left(fx+e\right)^2 - a\right)\sqrt{\frac{b}{a}}}{2\,b\tan\left(fx+e\right)}\right)}{2\,(a-b)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*f*x - sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a
^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e
)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/((a - b)*f), 1/2*(2*f*x - sqrt(b/a)*arc
tan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/((a - b)*f)]
```

Sympy [A] time = 3.38359, size = 280, normalized size = 5.6

$$\begin{cases} \frac{\tilde{\infty} x}{\tan^2(e)} & \text{for } a = 0 \wedge b = 0 \wedge f = 0 \\ \frac{x}{a} & \text{for } b = 0 \\ \frac{-x - \frac{1}{f\tan(e+fx)}}{b} & \text{for } a = 0 \\ \frac{fx\tan^2(e+fx)}{2bf\tan^2(e+fx)+2bf} + \frac{fx}{2bf\tan^2(e+fx)+2bf} + \frac{\tan(e+fx)}{2bf\tan^2(e+fx)+2bf} & \text{for } a = b \\ \frac{x}{a+b\tan^2(e)} & \text{for } f = 0 \\ \frac{2i\sqrt{a}fx\sqrt{\frac{1}{b}}}{2ia^{\frac{3}{2}}f\sqrt{\frac{1}{b}}-2i\sqrt{a}bf\sqrt{\frac{1}{b}}} - \frac{\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\tan(e+fx)\right)}{2ia^{\frac{3}{2}}f\sqrt{\frac{1}{b}}-2i\sqrt{a}bf\sqrt{\frac{1}{b}}} + \frac{\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\tan(e+fx)\right)}{2ia^{\frac{3}{2}}f\sqrt{\frac{1}{b}}-2i\sqrt{a}bf\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x/tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x/a, Eq(b, 0)
), ((-x - 1/(f*tan(e + f*x)))/b, Eq(a, 0)), (f*x*tan(e + f*x)**2/(2*b*f*tan
(e + f*x)**2 + 2*b*f) + f*x/(2*b*f*tan(e + f*x)**2 + 2*b*f) + tan(e + f*x)/
(2*b*f*tan(e + f*x)**2 + 2*b*f), Eq(a, b)), (x/(a + b*tan(e)**2), Eq(f, 0))
, (2*I*sqrt(a)*f*x*sqrt(1/b)/(2*I*a**(3/2)*f*sqrt(1/b) - 2*I*sqrt(a)*b*f*sq
rt(1/b)) - log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(2*I*a**(3/2)*f*sqrt(1/
b) - 2*I*sqrt(a)*b*f*sqrt(1/b)) + log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(
2*I*a**(3/2)*f*sqrt(1/b) - 2*I*sqrt(a)*b*f*sqrt(1/b)), True))
```

Giac [B] time = 1.32922, size = 231, normalized size = 4.62

$$-\frac{2\left(\frac{\sqrt{ab}\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan(fx+e)}{\sqrt{\frac{a+b-\sqrt{(a+b)^2-4ab}}{b}}}\right)\right)|b|}{(a-b)^2b-(ab+b^2)|-a+b|} + \frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan(fx+e)}{\sqrt{\frac{a+b+\sqrt{(a+b)^2-4ab}}{b}}}\right)\right)b}{(a-b)^2+a|-a+b|+b|-a+b|}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -2*(sqrt(a*b)*(pi*floor((f*x + e)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(f*x +
e)/sqrt((a + b - sqrt((a + b)^2 - 4*a*b))/b)))*abs(b)/((a - b)^2*b - (a*b +
 b^2)*abs(-a + b)) + (pi*floor((f*x + e)/pi + 1/2) + arctan(2*sqrt(1/2)*tan
(f*x + e)/sqrt((a + b + sqrt((a + b)^2 - 4*a*b))/b)))*b/((a - b)^2 + a*abs(
```

```
-a + b) + b*abs(-a + b)))/f
```

3.221 $\int \frac{\cot^2(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=64

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}f(a-b)}-\frac{x}{a-b}-\frac{\cot(e+fx)}{af}$$

```
[Out] -(x/(a - b)) + (b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(3/2)*(a
 - b)*f) - Cot[e + f*x]/(a*f)
```

Rubi [A] time = 0.11064, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3670, 480, 522, 203, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}f(a-b)}-\frac{x}{a-b}-\frac{\cot(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(x/(a - b)) + (b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(3/2)*(a
 - b)*f) - Cot[e + f*x]/(a*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\cot(e+fx)}{af} + \frac{\text{Subst}\left(\int \frac{-a-b-bx^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{af}$$
$$= -\frac{\cot(e+fx)}{af} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{(a-b)f} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(e+fx)\right)}{a(a-b)f}$$
$$= -\frac{x}{a-b} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}(a-b)f} - \frac{\cot(e+fx)}{af}$$

Mathematica [A] time = 0.23702, size = 68, normalized size = 1.06

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right) - \sqrt{a}((a-b)\cot(e+fx) + a(e+fx))}{a^{3/2}f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] - Sqrt[a]*(a*(e + f*x) + (a
 - b)*Cot[e + f*x]))/(a^(3/2)*(a - b)*f)
```

Maple [A] time = 0.067, size = 73, normalized size = 1.1

$$-\frac{1}{fa\tan(fx+e)} + \frac{b^2}{fa(a-b)}\arctan\left(b\tan(fx+e)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{\arctan(\tan(fx+e))}{f(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/f/a/tan(f*x+e)+1/f/a*b^2/(a-b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/
2))-1/f/(a-b)*arctan(tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.17911, size = 568, normalized size = 8.88

$$\left[-\frac{4\,afx\tan\left(fx+e\right)+b\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan\left(fx+e\right)^4-6\,ab\tan\left(fx+e\right)^2+a^2-4\left(ab\tan\left(fx+e\right)^3-a^2\tan\left(fx+e\right)\right)\sqrt{-\frac{b}{a}}}{b^2\tan\left(fx+e\right)^4+2\,ab\tan\left(fx+e\right)^2+a^2}\right)\tan\left(fx+e\right)+4\,a-4\,b}{4\left(a^2-ab\right)f\tan\left(fx+e\right)},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*f*x*tan(f*x + e) + b*sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*
tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a)
)/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2))*tan(f*x + e) + 4*a - 4
*b)/((a^2 - a*b)*f*tan(f*x + e)), -1/2*(2*a*f*x*tan(f*x + e) - b*sqrt(b/a)*
arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e)))*tan(f*x + e)
+ 2*a - 2*b)/((a^2 - a*b)*f*tan(f*x + e))]
```

Sympy [A] time = 22.1719, size = 570, normalized size = 8.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(e, 0) & Eq(f, 0)), ((x + 1/(f*ta
n(e + f*x)) - 1/(3*f*tan(e + f*x)**3))/b, Eq(a, 0)), (-3*f*x*tan(e + f*x)**
3/(2*b*f*tan(e + f*x)**3 + 2*b*f*tan(e + f*x)) - 3*f*x*tan(e + f*x)/(2*b*f*
tan(e + f*x)**3 + 2*b*f*tan(e + f*x)) - 3*tan(e + f*x)**2/(2*b*f*tan(e + f*
x)**3 + 2*b*f*tan(e + f*x)) - 2/(2*b*f*tan(e + f*x)**3 + 2*b*f*tan(e + f*x)
), Eq(a, b)), (zoo*x/a, Eq(e, -f*x)), (x*cot(e)**2/(a + b*tan(e)**2), Eq(f,
 0)), ((-x - cot(e + f*x)/f)/a, Eq(b, 0)), (-2*I*a**(3/2)*f*x*sqrt(1/b)*tan
(e + f*x)/(2*I*a**(5/2)*f*sqrt(1/b)*tan(e + f*x) - 2*I*a**(3/2)*b*f*sqrt(1/
b)*tan(e + f*x)) - 2*I*a**(3/2)*sqrt(1/b)/(2*I*a**(5/2)*f*sqrt(1/b)*tan(e +
 f*x) - 2*I*a**(3/2)*b*f*sqrt(1/b)*tan(e + f*x)) + 2*I*sqrt(a)*b*sqrt(1/b)/
(2*I*a**(5/2)*f*sqrt(1/b)*tan(e + f*x) - 2*I*a**(3/2)*b*f*sqrt(1/b)*tan(e +
 f*x)) + b*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)/(2*I*a**(5
/2)*f*sqrt(1/b)*tan(e + f*x) - 2*I*a**(3/2)*b*f*sqrt(1/b)*tan(e + f*x)) - b
*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)/(2*I*a**(5/2)*f*sqrt(
1/b)*tan(e + f*x) - 2*I*a**(3/2)*b*f*sqrt(1/b)*tan(e + f*x)), True))
```

Giac [B] time = 1.44382, size = 390, normalized size = 6.09

$$\frac{\frac{\left(a^2b+ab^2-b\left|-a^2+ab\right|\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan\left(fx+e\right)}{\sqrt{\frac{2a^2+2ab+\sqrt{-16a^3b+4\left(a^2+ab\right)^2}}{ab}}}\right)\right)}{a^2\left|-a^2+ab\right|+ab\left|-a^2+ab\right|+\left(a^2-ab\right)^2}+\frac{\left(\left(a^2+ab\right)\sqrt{ab}|b|+\sqrt{ab}\left|-a^2+ab\right||b|\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(f}{\sqrt{\frac{2a^2+2ab-\sqrt{-16}}{ab}}}\right.\right.}{\left(a^2-ab\right)^2b-\left(a^2b+ab^2\right)\left|-a^2+ab\right|}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] ((a^2*b + a*b^2 - b*abs(-a^2 + a*b))*(pi*floor((f*x + e)/pi + 1/2) + arctan
(2*tan(f*x + e)/sqrt((2*a^2 + 2*a*b + sqrt(-16*a^3*b + 4*(a^2 + a*b)^2))/(a
*b))))/(a^2*abs(-a^2 + a*b) + a*b*abs(-a^2 + a*b) + (a^2 - a*b)^2) + ((a^2
+ a*b)*sqrt(a*b)*abs(b) + sqrt(a*b)*abs(-a^2 + a*b)*abs(b))*(pi*floor((f*x
+ e)/pi + 1/2) + arctan(2*tan(f*x + e)/sqrt((2*a^2 + 2*a*b - sqrt(-16*a^3*b
 + 4*(a^2 + a*b)^2))/(a*b))))/((a^2 - a*b)^2*b - (a^2*b + a*b^2)*abs(-a^2 +
 a*b)) - 1/(a*tan(f*x + e)))/f
```

3.222 $\int \frac{\cot^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=84

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}f(a-b)}+\frac{(a+b)\cot(e+fx)}{a^2f}+\frac{x}{a-b}-\frac{\cot^3(e+fx)}{3af}$$

```
[Out] x/(a - b) - (b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(5/2)*(a -
b)*f) + ((a + b)*Cot[e + f*x])/(a^2*f) - Cot[e + f*x]^3/(3*a*f)
```

Rubi [A] time = 0.172941, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 480, 583, 522, 203, 205}

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}f(a-b)}+\frac{(a+b)\cot(e+fx)}{a^2f}+\frac{x}{a-b}-\frac{\cot^3(e+fx)}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] x/(a - b) - (b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(5/2)*(a -
b)*f) + ((a + b)*Cot[e + f*x])/(a^2*f) - Cot[e + f*x]^3/(3*a*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\cot^3(e+fx)}{3af} + \frac{\text{Subst}\left(\int \frac{-3(a+b)-3bx^2}{x^2(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{3af}$$
$$= \frac{(a+b)\cot(e+fx)}{a^2f} - \frac{\cot^3(e+fx)}{3af} - \frac{\text{Subst}\left(\int \frac{-3(a^2+ab+b^2)-3b(a+b)x^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{3a^2f}$$
$$= \frac{(a+b)\cot(e+fx)}{a^2f} - \frac{\cot^3(e+fx)}{3af} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{(a-b)f} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{a+\ldots}\right.}{a^2\ldots}$$
$$= \frac{x}{a-b} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}(a-b)f} + \frac{(a+b)\cot(e+fx)}{a^2f} - \frac{\cot^3(e+fx)}{3af}$$

Mathematica [A] time = 0.647017, size = 92, normalized size = 1.1

$$\frac{\sqrt{a}\left(3a^2(e+fx)-(a-b)\cot(e+fx)\left(a\csc^2(e+fx)-4a-3b\right)\right)-3b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{3a^{5/2}f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-3*b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[a]*(3*a^2*(e + f*
x) - (a - b)*Cot[e + f*x]*(-4*a - 3*b + a*Csc[e + f*x]^2)))/(3*a^(5/2)*(a -
 b)*f)
```

Maple [A] time = 0.079, size = 104, normalized size = 1.2

$$-\frac{1}{3fa\left(\tan\left(fx+e\right)\right)^3} + \frac{1}{fa\tan\left(fx+e\right)} + \frac{b}{fa^2\tan\left(fx+e\right)} - \frac{b^3}{fa^2(a-b)}\arctan\left(b\tan\left(fx+e\right)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{\text{arct}\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/3/f/a/tan(f*x+e)^3+1/f/a/tan(f*x+e)+1/f/a^2/tan(f*x+e)*b-1/f/a^2*b^3/(a-
b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))+1/f/(a-b)*arctan(tan(f*x+e)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.19186, size = 702, normalized size = 8.36

$$\left[\frac{12\,a^2 f x \tan\left(fx+e\right)^3 - 3\,b^2\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan\left(fx+e\right)^4 - 6\,ab\tan\left(fx+e\right)^2 + a^2 + 4\left(ab\tan\left(fx+e\right)^3 - a^2\tan\left(fx+e\right)\right)\sqrt{-\frac{b}{a}}}{b^2\tan\left(fx+e\right)^4 + 2\,ab\tan\left(fx+e\right)^2 + a^2}\right)\tan\left(fx+e\right)^3 + 12\,(a}{12\left(a^3 - a^2 b\right)f\tan\left(fx+e\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/12*(12*a^2*f*x*tan(f*x + e)^3 - 3*b^2*sqrt(-b/a)*log((b^2*tan(f*x + e)^4
 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*s
qrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2))*tan(f*x + e)^
3 + 12*(a^2 - b^2)*tan(f*x + e)^2 - 4*a^2 + 4*a*b)/((a^3 - a^2*b)*f*tan(f*x
 + e)^3), 1/6*(6*a^2*f*x*tan(f*x + e)^3 - 3*b^2*sqrt(b/a)*arctan(1/2*(b*tan
(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e)))*tan(f*x + e)^3 + 6*(a^2 - b^2)
*tan(f*x + e)^2 - 2*a^2 + 2*a*b)/((a^3 - a^2*b)*f*tan(f*x + e)^3)]
```

Sympy [A] time = 89.7141, size = 823, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(e, 0) & Eq(f, 0)), ((-x - 1/(f*t
an(e + f*x)) + 1/(3*f*tan(e + f*x)**3) - 1/(5*f*tan(e + f*x)**5))/b, Eq(a,
0)), (15*f*x*tan(e + f*x)**5/(6*b*f*tan(e + f*x)**5 + 6*b*f*tan(e + f*x)**3
) + 15*f*x*tan(e + f*x)**3/(6*b*f*tan(e + f*x)**5 + 6*b*f*tan(e + f*x)**3)
+ 15*tan(e + f*x)**4/(6*b*f*tan(e + f*x)**5 + 6*b*f*tan(e + f*x)**3) + 10*t
```

```
an(e + f*x)**2/(6*b*f*tan(e + f*x)**5 + 6*b*f*tan(e + f*x)**3) - 2/(6*b*f*t
an(e + f*x)**5 + 6*b*f*tan(e + f*x)**3), Eq(a, b)), (zoo*x/a, Eq(e, -f*x)),
 (x*cot(e)**4/(a + b*tan(e)**2), Eq(f, 0)), ((x - cot(e + f*x)**3/(3*f) + c
ot(e + f*x)/f)/a, Eq(b, 0)), (6*I*a**(5/2)*f*x*sqrt(1/b)*tan(e + f*x)**3/(6
*I*a**(7/2)*f*sqrt(1/b)*tan(e + f*x)**3 - 6*I*a**(5/2)*b*f*sqrt(1/b)*tan(e
+ f*x)**3) + 6*I*a**(5/2)*sqrt(1/b)*tan(e + f*x)**2/(6*I*a**(7/2)*f*sqrt(1/
b)*tan(e + f*x)**3 - 6*I*a**(5/2)*b*f*sqrt(1/b)*tan(e + f*x)**3) - 2*I*a**(
5/2)*sqrt(1/b)/(6*I*a**(7/2)*f*sqrt(1/b)*tan(e + f*x)**3 - 6*I*a**(5/2)*b*f
*sqrt(1/b)*tan(e + f*x)**3) + 2*I*a**(3/2)*b*sqrt(1/b)/(6*I*a**(7/2)*f*sqrt
(1/b)*tan(e + f*x)**3 - 6*I*a**(5/2)*b*f*sqrt(1/b)*tan(e + f*x)**3) - 6*I*s
qrt(a)*b**2*sqrt(1/b)*tan(e + f*x)**2/(6*I*a**(7/2)*f*sqrt(1/b)*tan(e + f*x
)**3 - 6*I*a**(5/2)*b*f*sqrt(1/b)*tan(e + f*x)**3) - 3*b**2*log(-I*sqrt(a)*
sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**3/(6*I*a**(7/2)*f*sqrt(1/b)*tan(e +
 f*x)**3 - 6*I*a**(5/2)*b*f*sqrt(1/b)*tan(e + f*x)**3) + 3*b**2*log(I*sqrt(
a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**3/(6*I*a**(7/2)*f*sqrt(1/b)*tan(
e + f*x)**3 - 6*I*a**(5/2)*b*f*sqrt(1/b)*tan(e + f*x)**3), True))
```

Giac [B] time = 1.60999, size = 504, normalized size = 6.

$$-\frac{\frac{3\left(a^4b+a^2b^3-ab\left|-a^3+a^2b\right|-b^2\left|-a^3+a^2b\right|\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan\left(fx+e\right)}{\sqrt{\frac{2a^3+2a^2b+\sqrt{-16a^5b+4\left(a^3+a^2b\right)^2}}{a^2b}}}\right)\right)}{a^3\left|-a^3+a^2b\right|+a^2b\left|-a^3+a^2b\right|+\left(a^3-a^2b\right)^2}+\frac{3\left(\sqrt{ab}(a+b)\left|-a^3+a^2b\right||b|+\left(a^4+a^2b^2\right)\sqrt{ab}|b|\right)\left(\pi\left\lfloor\frac{fx+\cdots}{\pi}\cdots\right.\right.}{\left(a^3-a^2b\right)^2b-\left(a^3b\cdots\right.}\cdots}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*(a^4*b + a^2*b^3 - a*b*abs(-a^3 + a^2*b) - b^2*abs(-a^3 + a^2*b))*(
pi*floor((f*x + e)/pi + 1/2) + arctan(2*tan(f*x + e)/sqrt((2*a^3 + 2*a^2*b
+ sqrt(-16*a^5*b + 4*(a^3 + a^2*b)^2))/(a^2*b))))/(a^3*abs(-a^3 + a^2*b) +
a^2*b*abs(-a^3 + a^2*b) + (a^3 - a^2*b)^2) + 3*(sqrt(a*b)*(a + b)*abs(-a^3
+ a^2*b)*abs(b) + (a^4 + a^2*b^2)*sqrt(a*b)*abs(b))*(pi*floor((f*x + e)/pi
+ 1/2) + arctan(2*tan(f*x + e)/sqrt((2*a^3 + 2*a^2*b - sqrt(-16*a^5*b + 4*(
a^3 + a^2*b)^2))/(a^2*b))))/((a^3 - a^2*b)^2*b - (a^3*b + a^2*b^2)*abs(-a^3
 + a^2*b)) - (3*a*tan(f*x + e)^2 + 3*b*tan(f*x + e)^2 - a)/(a^2*tan(f*x + e
)^3))/f
```

3.223 $\int \frac{\cot^6(e+fx)}{a+b\tan^2(e+fx)} dx$

Optimal. Leaf size=113

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}f(a-b)}-\frac{\left(a^2+ab+b^2\right)\cot(e+fx)}{a^3f}+\frac{(a+b)\cot^3(e+fx)}{3a^2f}-\frac{x}{a-b}-\frac{\cot^5(e+fx)}{5af}$$

```
[Out] -(x/(a - b)) + (b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(7/2)*(a
 - b)*f) - ((a^2 + a*b + b^2)*Cot[e + f*x])/(a^3*f) + ((a + b)*Cot[e + f*x]
^3)/(3*a^2*f) - Cot[e + f*x]^5/(5*a*f)
```

Rubi [A] time = 0.241209, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 480, 583, 522, 203, 205}

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}f(a-b)}-\frac{\left(a^2+ab+b^2\right)\cot(e+fx)}{a^3f}+\frac{(a+b)\cot^3(e+fx)}{3a^2f}-\frac{x}{a-b}-\frac{\cot^5(e+fx)}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(x/(a - b)) + (b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(7/2)*(a
 - b)*f) - ((a^2 + a*b + b^2)*Cot[e + f*x])/(a^3*f) + ((a + b)*Cot[e + f*x]
^3)/(3*a^2*f) - Cot[e + f*x]^5/(5*a*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5af} + \frac{\text{Subst}\left(\int \frac{-5(a+b)-5bx^2}{x^4(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{5af}$$

$$= \frac{(a+b)\cot^3(e+fx)}{3a^2f} - \frac{\cot^5(e+fx)}{5af} - \frac{\text{Subst}\left(\int \frac{-15(a^2+ab+b^2)-15b(a+b)x^2}{x^2(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{15a^2f}$$

$$= -\frac{(a^2+ab+b^2)\cot(e+fx)}{a^3f} + \frac{(a+b)\cot^3(e+fx)}{3a^2f} - \frac{\cot^5(e+fx)}{5af} + \frac{\text{Subst}\left(\int \frac{-15(a+b)(a^2}{(1}\right.}{}$$

$$= -\frac{(a^2+ab+b^2)\cot(e+fx)}{a^3f} + \frac{(a+b)\cot^3(e+fx)}{3a^2f} - \frac{\cot^5(e+fx)}{5af} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x\right.}{(a-b)}$$

$$= -\frac{x}{a-b} + \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}(a-b)f} - \frac{(a^2+ab+b^2)\cot(e+fx)}{a^3f} + \frac{(a+b)\cot^3(e+fx)}{3a^2f} -$$

Mathematica [A] time = 1.86771, size = 121, normalized size = 1.07

$$\frac{\sqrt{a}\left(-(a-b)\cot(e+fx)\left(3a^2\csc^4(e+fx)+23a^2-a(11a+5b)\csc^2(e+fx)+20ab+15b^2\right)-15a^3(e+fx)\right)+15b^{7/2}}{15a^{7/2}f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (15*b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[a]*(-15*a^3*(e +
f*x) - (a - b)*Cot[e + f*x]*(23*a^2 + 20*a*b + 15*b^2 - a*(11*a + 5*b)*Csc[
e + f*x]^2 + 3*a^2*Csc[e + f*x]^4)))/(15*a^(7/2)*(a - b)*f)
```

Maple [A] time = 0.082, size = 158, normalized size = 1.4

$$-\frac{1}{5fa\left(\tan\left(fx+e\right)\right)^{5}}+\frac{1}{3fa\left(\tan\left(fx+e\right)\right)^{3}}+\frac{b}{3fa^{2}\left(\tan\left(fx+e\right)\right)^{3}}-\frac{1}{fa\tan\left(fx+e\right)}-\frac{b}{fa^{2}\tan\left(fx+e\right)}-\frac{b^{2}}{fa^{3}\tan\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/5/f/a/tan(f*x+e)^5+1/3/f/a/tan(f*x+e)^3+1/3/f/a^2/tan(f*x+e)^3*b-1/f/a/t
an(f*x+e)-1/f/a^2/tan(f*x+e)*b-1/f/a^3/tan(f*x+e)*b^2+1/f/a^3*b^4/(a-b)/(a*
b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-1/f/(a-b)*arctan(tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.23079, size = 811, normalized size = 7.18

$$\left[-\frac{60\,a^{3}fx\tan\left(fx+e\right)^{5}+15\,b^{3}\sqrt{-\frac{b}{a}}\log\left(\frac{b^{2}\tan\left(fx+e\right)^{4}-6\,ab\tan\left(fx+e\right)^{2}+a^{2}-4\left(ab\tan\left(fx+e\right)^{3}-a^{2}\tan\left(fx+e\right)\right)\sqrt{-\frac{b}{a}}}{b^{2}\tan\left(fx+e\right)^{4}+2\,ab\tan\left(fx+e\right)^{2}+a^{2}}\right)\tan\left(fx+e\right)^{5}+60}{60\left(a^{4}-a^{3}b\right)f\tan\left(fx+e\right)^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(60*a^3*f*x*tan(f*x + e)^5 + 15*b^3*sqrt(-b/a)*log((b^2*tan(f*x + e)
^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))
*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2))*tan(f*x + e
)^5 + 60*(a^3 - b^3)*tan(f*x + e)^4 + 12*a^3 - 12*a^2*b - 20*(a^3 - a*b^2)*
tan(f*x + e)^2)/((a^4 - a^3*b)*f*tan(f*x + e)^5), -1/30*(30*a^3*f*x*tan(f*x
 + e)^5 - 15*b^3*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*t
an(f*x + e)))*tan(f*x + e)^5 + 30*(a^3 - b^3)*tan(f*x + e)^4 + 6*a^3 - 6*a^
2*b - 10*(a^3 - a*b^2)*tan(f*x + e)^2)/((a^4 - a^3*b)*f*tan(f*x + e)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.7312, size = 602, normalized size = 5.33

$$\frac{\frac{15\left(a^6b+a^3b^4-a^2b\left|-a^4+a^3b\right|-ab^2\left|-a^4+a^3b\right|-b^3\left|-a^4+a^3b\right|\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan\left(fx+e\right)}{\sqrt{\frac{2a^4+2a^3b+\sqrt{-16a^7b+4\left(a^4+a^3b\right)^2}}{a^3b}}}\right)\right)}{a^4\left|-a^4+a^3b\right|+a^3b\left|-a^4+a^3b\right|+\left(a^4-a^3b\right)^2}+\frac{15\left(\left(a^2+ab+b^2\right)\sqrt{ab}\left|-a^4+a^3b\right|\left|b\right|+\left(a\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(15*(a^6*b + a^3*b^4 - a^2*b*abs(-a^4 + a^3*b) - a*b^2*abs(-a^4 + a^3*
b) - b^3*abs(-a^4 + a^3*b))*(pi*floor((f*x + e)/pi + 1/2) + arctan(2*tan(f*
x + e)/sqrt((2*a^4 + 2*a^3*b + sqrt(-16*a^7*b + 4*(a^4 + a^3*b)^2))/(a^3*b)
)))/(a^4*abs(-a^4 + a^3*b) + a^3*b*abs(-a^4 + a^3*b) + (a^4 - a^3*b)^2) + 1
5*((a^2 + a*b + b^2)*sqrt(a*b)*abs(-a^4 + a^3*b)*abs(b) + (a^6 + a^3*b^3)*s
qrt(a*b)*abs(b))*(pi*floor((f*x + e)/pi + 1/2) + arctan(2*tan(f*x + e)/sqrt
((2*a^4 + 2*a^3*b - sqrt(-16*a^7*b + 4*(a^4 + a^3*b)^2))/(a^3*b))))/((a^4 -
 a^3*b)^2*b - (a^4*b + a^3*b^2)*abs(-a^4 + a^3*b)) - (15*a^2*tan(f*x + e)^4
 + 15*a*b*tan(f*x + e)^4 + 15*b^2*tan(f*x + e)^4 - 5*a^2*tan(f*x + e)^2 - 5
*a*b*tan(f*x + e)^2 + 3*a^2)/(a^3*tan(f*x + e)^5))/f
```

3.224 $\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=90

$$\frac{a^2}{2b^2f(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{a(a-2b)\log\left(a+b\tan^2(e+fx)\right)}{2b^2f(a-b)^2}-\frac{\log(\cos(e+fx))}{f(a-b)^2}$$

```
[Out] -(Log[Cos[e + f*x]]/((a - b)^2*f)) + (a*(a - 2*b)*Log[a + b*Tan[e + f*x]^2]
)/(2*(a - b)^2*b^2*f) + a^2/(2*(a - b)*b^2*f*(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.121305, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 88}

$$\frac{a^2}{2b^2f(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{a(a-2b)\log\left(a+b\tan^2(e+fx)\right)}{2b^2f(a-b)^2}-\frac{\log(\cos(e+fx))}{f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```


```
[Out] -(Log[Cos[e + f*x]]/((a - b)^2*f)) + (a*(a - 2*b)*Log[a + b*Tan[e + f*x]^2]
)/(2*(a - b)^2*b^2*f) + a^2/(2*(a - b)*b^2*f*(a + b*Tan[e + f*x]^2))
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)(a+bx)^2}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a-b)^2(1+x)} - \frac{a^2}{(a-b)b(a+bx)^2} + \frac{a(a-2b)}{(a-b)^2b(a+bx)}\right) dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\log(\cos(e+fx))}{(a-b)^2f} + \frac{a(a-2b)\log\left(a+b\tan^2(e+fx)\right)}{2(a-b)^2b^2f} + \frac{a^2}{2(a-b)b^2f\left(a+b\tan^2(e+\right.}$$

Mathematica [A] time = 0.665275, size = 73, normalized size = 0.81

$$\frac{\frac{a^2(a-b)}{b^2\left(a+b\tan^2(e+fx)\right)} + \frac{a(a-2b)\log\left(a+b\tan^2(e+fx)\right)}{b^2} - 2\log(\cos(e+fx))}{2f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-2*Log[Cos[e + f*x]] + (a*(a - 2*b)*Log[a + b*Tan[e + f*x]^2])/b^2 + (a^2*
(a - b))/(b^2*(a + b*Tan[e + f*x]^2)))/(2*(a - b)^2*f)
```

Maple [A] time = 0.026, size = 149, normalized size = 1.7

$$\frac{a^2\ln\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)}{2f(a-b)^2b^2} - \frac{a\ln\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)}{f(a-b)^2b} + \frac{a^3}{2f(a-b)^2b^2\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)} - \frac{}{2f(a-b)^2b\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/2/f*a^2/(a-b)^2/b^2*ln(a+b*tan(f*x+e)^2)-1/f*a/(a-b)^2/b*ln(a+b*tan(f*x+e
)^2)+1/2/f*a^3/(a-b)^2/b^2/(a+b*tan(f*x+e)^2)-1/2/f*a^2/(a-b)^2/b/(a+b*tan(
f*x+e)^2)+1/2/f/(a-b)^2*ln(1+tan(f*x+e)^2)
```

Maxima [A] time = 1.14079, size = 173, normalized size = 1.92

$$-\frac{\frac{a^2}{a^3b-2a^2b^2+ab^3-\left(a^3b-3a^2b^2+3ab^3-b^4\right)\sin\left(fx+e\right)^2} - \frac{\left(a^2-2ab\right)\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{a^2b^2-2ab^3+b^4} + \frac{\log\left(\sin\left(fx+e\right)^2-1\right)}{b^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2/(a^3*b - 2*a^2*b^2 + a*b^3 - (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*
sin(f*x + e)^2) - (a^2 - 2*a*b)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^2*b^2 -
 2*a*b^3 + b^4) + log(sin(f*x + e)^2 - 1)/b^2)/f
```

Fricas [B] time = 1.26814, size = 423, normalized size = 4.7

$$-\frac{a^2b\tan\left(fx+e\right)^2+a^2b-\left(a^3-2\,a^2b+\left(a^2b-2\,ab^2\right)\tan\left(fx+e\right)^2\right)\log\left(\frac{b\tan\left(fx+e\right)^2+a}{\tan\left(fx+e\right)^2+1}\right)+\left(a^3-2\,a^2b+ab^2+\left(a^2b-2\,a\right.\right.}{2\left(\left(a^2b^3-2\,ab^4+b^5\right)f\tan\left(fx+e\right)^2+\left(a^3b^2-2\,a^2b^3+ab^4\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b*tan(f*x + e)^2 + a^2*b - (a^3 - 2*a^2*b + (a^2*b - 2*a*b^2)*tan
(f*x + e)^2)*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)) + (a^3 - 2*a^
2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x + e)^2)*log(1/(tan(f*x + e)^2
 + 1)))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan(f*x + e)^2 + (a^3*b^2 - 2*a^2*b^3
+ a*b^4)*f)
```

Sympy [A] time = 118.299, size = 1583, normalized size = 17.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x*tan(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (2*log(tan(e + f*
x)**2 + 1)*tan(e + f*x)**4/(4*b**2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x
)**2 + 4*b**2*f) + 4*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(4*b**2*f*tan
(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f) + 2*log(tan(e + f*x)**2
 + 1)/(4*b**2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f) + 4*
tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b*
*2*f) + 3/(4*b**2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f),
 Eq(a, b)), ((log(tan(e + f*x)**2 + 1)/(2*f) + tan(e + f*x)**4/(4*f) - tan(
e + f*x)**2/(2*f))/a**2, Eq(b, 0)), (x*tan(e)**5/(a + b*tan(e)**2)**2, Eq(f
, 0)), (a**3*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(2*a**3*b**2*f + 2*a*
*2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan(e + f*x)**2 + 2*
a*b**4*f + 2*b**5*f*tan(e + f*x)**2) + a**3*log(I*sqrt(a)*sqrt(1/b) + tan(e
 + f*x))/(2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4
*a*b**4*f*tan(e + f*x)**2 + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2) + a**3/(
2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*
tan(e + f*x)**2 + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2) + a**2*b*log(-I*sq
rt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(2*a**3*b**2*f + 2*a**2*b**
3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan(e + f*x)**2 + 2*a*b**4
*f + 2*b**5*f*tan(e + f*x)**2) - 2*a**2*b*log(-I*sqrt(a)*sqrt(1/b) + tan(e
+ f*x))/(2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*
a*b**4*f*tan(e + f*x)**2 + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2) + a**2*b*
log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(2*a**3*b**2*f + 2*
a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan(e + f*x)**2 +
2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2) - 2*a**2*b*log(I*sqrt(a)*sqrt(1/b) +
 tan(e + f*x))/(2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3
```

```
*f - 4*a*b**4*f*tan(e + f*x)**2 + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2) -
a**2*b/(2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a
*b**4*f*tan(e + f*x)**2 + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2) - 2*a*b**2
*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(2*a**3*b**2*f +
2*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan(e + f*x)**2
+ 2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2) - 2*a*b**2*log(I*sqrt(a)*sqrt(1/b)
 + tan(e + f*x))*tan(e + f*x)**2/(2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x
)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan(e + f*x)**2 + 2*a*b**4*f + 2*b**5*f*t
an(e + f*x)**2) + a*b**2*log(tan(e + f*x)**2 + 1)/(2*a**3*b**2*f + 2*a**2*b
**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan(e + f*x)**2 + 2*a*b*
*4*f + 2*b**5*f*tan(e + f*x)**2) + b**3*log(tan(e + f*x)**2 + 1)*tan(e + f*
x)**2/(2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*
b**4*f*tan(e + f*x)**2 + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2), True))
```

Giac [B] time = 2.97375, size = 536, normalized size = 5.96

$$\frac{\frac{(a^3-2a^2b)\log\left(\left|-a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-2a+4b\right|\right)}{a^3b^2-2a^2b^3+ab^4}+\frac{\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right)}{a^2-2ab+b^2}-\frac{a^3\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-2a^2b\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\cdots\right.}{(a^2b^2-2ab^3+b^4)\left(a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\cdots\right.\right.}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((a^3 - 2*a^2*b)*log(abs(-a*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (c
os(f*x + e) - 1)/(cos(f*x + e) + 1)) - 2*a + 4*b))/(a^3*b^2 - 2*a^2*b^3 + a
*b^4) + log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(co
s(f*x + e) + 1) + 2)/(a^2 - 2*a*b + b^2) - (a^3*((cos(f*x + e) + 1)/(cos(f*
x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 2*a^2*b*((cos(f*x +
e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 2*a^3
 - 12*a^2*b + 12*a*b^2)/((a^2*b^2 - 2*a*b^3 + b^4)*(a*((cos(f*x + e) + 1)/(
cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 2*a - 4*b)) -
log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x +
e) + 1) - 2)/b^2)/f
```

3.225 $\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=69

$$\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^2}-\frac{a}{2bf(a-b)\left(a+b\tan^2(e+fx)\right)}$$

```
[Out] Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/(2*(a - b)^2*f) - a/(2*(a - b)*b*f
*(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.100146, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 77}

$$\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^2}-\frac{a}{2bf(a-b)\left(a+b\tan^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/(2*(a - b)^2*f) - a/(2*(a - b)*b*f
*(a + b*Tan[e + f*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(1+x^2)\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{(1+x)(a+bx)^2}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a-b)^2(1+x)}+\frac{a}{(a-b)(a+bx)^2}+\frac{b}{(a-b)^2(a+bx)}\right) dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)^2f} - \frac{a}{2(a-b)bf\left(a+b\tan^2(e+fx)\right)}$$

Mathematica [A] time = 0.54052, size = 61, normalized size = 0.88

$$\frac{\frac{a(b-a)}{b\left(a+b\tan^2(e+fx)\right)}+\log\left(a+b\tan^2(e+fx)\right)+2\log(\cos(e+fx))}{2f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (2*Log[Cos[e + f*x]] + Log[a + b*Tan[e + f*x]^2] + (a*(-a + b))/(b*(a + b*T
an[e + f*x]^2)))/(2*(a - b)^2*f)
```

Maple [A] time = 0.023, size = 109, normalized size = 1.6

$$\frac{\ln\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)}{2f(a-b)^2} - \frac{a^2}{2f(a-b)^2b\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)} + \frac{a}{2f(a-b)^2\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)} - \frac{\ln\left(1+\left(\text{ta}\right.\right.}{2f\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/2/f/(a-b)^2*ln(a+b*tan(f*x+e)^2)-1/2/f*a^2/(a-b)^2/b/(a+b*tan(f*x+e)^2)+1
/2/f/(a-b)^2*a/(a+b*tan(f*x+e)^2)-1/2/f/(a-b)^2*ln(1+tan(f*x+e)^2)
```

Maxima [A] time = 1.16054, size = 119, normalized size = 1.72

$$\frac{\frac{a}{a^3-2a^2b+ab^2-\left(a^3-3a^2b+3ab^2-b^3\right)\sin\left(fx+e\right)^2}+\frac{\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{a^2-2ab+b^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a/(a^3 - 2*a^2*b + a*b^2 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sin(f*x + e
)^2) + log(-(a - b)*sin(f*x + e)^2 + a)/(a^2 - 2*a*b + b^2))/f
```

Fricas [A] time = 1.14962, size = 234, normalized size = 3.39

$$\frac{a\tan\left(fx+e\right)^{2}+\left(b\tan\left(fx+e\right)^{2}+a\right)\log\left(\frac{b\tan\left(fx+e\right)^{2}+a}{\tan\left(fx+e\right)^{2}+1}\right)+a}{2\left(\left(a^{2}b-2\,ab^{2}+b^{3}\right)f\tan\left(fx+e\right)^{2}+\left(a^{3}-2\,a^{2}b+ab^{2}\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*tan(f*x + e)^2 + (b*tan(f*x + e)^2 + a)*log((b*tan(f*x + e)^2 + a)/(
tan(f*x + e)^2 + 1)) + a)/((a^2*b - 2*a*b^2 + b^3)*f*tan(f*x + e)^2 + (a^3
- 2*a^2*b + a*b^2)*f)
```

Sympy [A] time = 48.6895, size = 930, normalized size = 13.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x/tan(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (-2*tan(e + f*x)*
*2/(4*b**2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f) - 1/(4*
b**2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f), Eq(a, b)), (
(-log(tan(e + f*x)**2 + 1)/(2*f) + tan(e + f*x)**2/(2*f))/a**2, Eq(b, 0)),
(x*tan(e)**3/(a + b*tan(e)**2)**2, Eq(f, 0)), (-a**2/(2*a**3*b*f + 2*a**2*b
**2*f*tan(e + f*x)**2 - 4*a**2*b**2*f - 4*a*b**3*f*tan(e + f*x)**2 + 2*a*b*
*3*f + 2*b**4*f*tan(e + f*x)**2) + a*b*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f
*x))/(2*a**3*b*f + 2*a**2*b**2*f*tan(e + f*x)**2 - 4*a**2*b**2*f - 4*a*b**3
*f*tan(e + f*x)**2 + 2*a*b**3*f + 2*b**4*f*tan(e + f*x)**2) + a*b*log(I*sqr
t(a)*sqrt(1/b) + tan(e + f*x))/(2*a**3*b*f + 2*a**2*b**2*f*tan(e + f*x)**2
- 4*a**2*b**2*f - 4*a*b**3*f*tan(e + f*x)**2 + 2*a*b**3*f + 2*b**4*f*tan(e
+ f*x)**2) - a*b*log(tan(e + f*x)**2 + 1)/(2*a**3*b*f + 2*a**2*b**2*f*tan(e
 + f*x)**2 - 4*a**2*b**2*f - 4*a*b**3*f*tan(e + f*x)**2 + 2*a*b**3*f + 2*b*
*4*f*tan(e + f*x)**2) + a*b/(2*a**3*b*f + 2*a**2*b**2*f*tan(e + f*x)**2 - 4
*a**2*b**2*f - 4*a*b**3*f*tan(e + f*x)**2 + 2*a*b**3*f + 2*b**4*f*tan(e + f
*x)**2) + b**2*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(2*
a**3*b*f + 2*a**2*b**2*f*tan(e + f*x)**2 - 4*a**2*b**2*f - 4*a*b**3*f*tan(e
 + f*x)**2 + 2*a*b**3*f + 2*b**4*f*tan(e + f*x)**2) + b**2*log(I*sqrt(a)*sq
rt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(2*a**3*b*f + 2*a**2*b**2*f*tan(e +
 f*x)**2 - 4*a**2*b**2*f - 4*a*b**3*f*tan(e + f*x)**2 + 2*a*b**3*f + 2*b**4
*f*tan(e + f*x)**2) - b**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(2*a**3
*b*f + 2*a**2*b**2*f*tan(e + f*x)**2 - 4*a**2*b**2*f - 4*a*b**3*f*tan(e + f
*x)**2 + 2*a*b**3*f + 2*b**4*f*tan(e + f*x)**2), True))
```

Giac [B] time = 1.73646, size = 397, normalized size = 5.75

$$\frac{\frac{\log\left(a+\frac{2\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{4\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{a\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)}{a^{2}-2\,ab+b^{2}}-\frac{2\,\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1\right)}{a^{2}-2\,ab+b^{2}}-\frac{a+\frac{6\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{8\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{a\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}}{\left(a^{2}-2\,ab+b^{2}\right)\left(a+\frac{2\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{4\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{a\left(\cos\left(fx+e\right)-\right)}{\left(\cos\left(fx+e\right)+1\right)}\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(log(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e)
- 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^2
 - 2*a*b + b^2) - 2*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)/(a^2 -
2*a*b + b^2) - (a + 6*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 8*b*(cos(f*
x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^
2)/((a^2 - 2*a*b + b^2)*(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*
b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x +
 e) + 1)^2)))/f
```

3.226 $\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=65

$$\frac{1}{2f(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^2}$$

```
[Out] -Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/(2*(a - b)^2*f) + 1/(2*(a - b)*f*
(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.0723874, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3670, 444, 44}

$$\frac{1}{2f(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/(2*(a - b)^2*f) + 1/(2*(a - b)*f*
(a + b*Tan[e + f*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^2}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a-b)^2(1+x)} - \frac{b}{(a-b)(a+bx)^2} - \frac{b}{(a-b)^2(a+bx)}\right)dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= -\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)^2f} + \frac{1}{2(a-b)f\left(a+b\tan^2(e+fx)\right)}$$

Mathematica [A] time = 0.60227, size = 57, normalized size = 0.88

$$-\frac{\frac{b-a}{a+b\tan^2(e+fx)} + \log\left(a+b\tan^2(e+fx)\right) + 2\log(\cos(e+fx))}{2f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(2*Log[Cos[e + f*x]] + Log[a + b*Tan[e + f*x]^2] + (-a + b)/(a + b*Tan[e +
 f*x]^2))/(2*(a - b)^2*f)
```

Maple [A] time = 0.024, size = 104, normalized size = 1.6

$$-\frac{\ln\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)}{2f(a-b)^2} + \frac{a}{2f(a-b)^2\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)} - \frac{b}{2f(a-b)^2\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)} + \frac{\ln\left(1+(\text{ta}\right.}{2f(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f/(a-b)^2*ln(a+b*tan(f*x+e)^2)+1/2/f/(a-b)^2*a/(a+b*tan(f*x+e)^2)-1/2/
f/(a-b)^2*b/(a+b*tan(f*x+e)^2)+1/2/f/(a-b)^2*ln(1+tan(f*x+e)^2)
```

Maxima [A] time = 1.10775, size = 119, normalized size = 1.83

$$-\frac{\frac{b}{a^3-2a^2b+ab^2-(a^3-3a^2b+3ab^2-b^3)\sin(fx+e)^2} + \frac{\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^2-2ab+b^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b/(a^3 - 2*a^2*b + a*b^2 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sin(f*x +
e)^2) + log(-(a - b)*sin(f*x + e)^2 + a)/(a^2 - 2*a*b + b^2))/f
```

Fricas [A] time = 1.13793, size = 235, normalized size = 3.62

$$-\frac{b\tan\left(fx+e\right)^{2}+\left(b\tan\left(fx+e\right)^{2}+a\right)\log\left(\frac{b\tan\left(fx+e\right)^{2}+a}{\tan\left(fx+e\right)^{2}+1}\right)+b}{2\left(\left(a^{2}b-2\,ab^{2}+b^{3}\right)f\tan\left(fx+e\right)^{2}+\left(a^{3}-2\,a^{2}b+ab^{2}\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*tan(f*x + e)^2 + (b*tan(f*x + e)^2 + a)*log((b*tan(f*x + e)^2 + a)/
(tan(f*x + e)^2 + 1)) + b)/((a^2*b - 2*a*b^2 + b^3)*f*tan(f*x + e)^2 + (a^3
 - 2*a^2*b + a*b^2)*f)
```

Sympy [A] time = 48.2286, size = 816, normalized size = 12.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x/tan(e)**3, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (-1/(4*b**2*f*
tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f), Eq(a, b)), (x*tan(e
)/(a + b*tan(e)**2)**2, Eq(f, 0)), (log(tan(e + f*x)**2 + 1)/(2*a**2*f), Eq
(b, 0)), (-a*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(2*a**3*f + 2*a**2*b*
f*tan(e + f*x)**2 - 4*a**2*b*f - 4*a*b**2*f*tan(e + f*x)**2 + 2*a*b**2*f +
2*b**3*f*tan(e + f*x)**2) - a*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(2*a*
*3*f + 2*a**2*b*f*tan(e + f*x)**2 - 4*a**2*b*f - 4*a*b**2*f*tan(e + f*x)**2
 + 2*a*b**2*f + 2*b**3*f*tan(e + f*x)**2) + a*log(tan(e + f*x)**2 + 1)/(2*a
**3*f + 2*a**2*b*f*tan(e + f*x)**2 - 4*a**2*b*f - 4*a*b**2*f*tan(e + f*x)**
2 + 2*a*b**2*f + 2*b**3*f*tan(e + f*x)**2) + a/(2*a**3*f + 2*a**2*b*f*tan(e
 + f*x)**2 - 4*a**2*b*f - 4*a*b**2*f*tan(e + f*x)**2 + 2*a*b**2*f + 2*b**3*
f*tan(e + f*x)**2) - b*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x
)**2/(2*a**3*f + 2*a**2*b*f*tan(e + f*x)**2 - 4*a**2*b*f - 4*a*b**2*f*tan(e
 + f*x)**2 + 2*a*b**2*f + 2*b**3*f*tan(e + f*x)**2) - b*log(I*sqrt(a)*sqrt(
1/b) + tan(e + f*x))*tan(e + f*x)**2/(2*a**3*f + 2*a**2*b*f*tan(e + f*x)**2
 - 4*a**2*b*f - 4*a*b**2*f*tan(e + f*x)**2 + 2*a*b**2*f + 2*b**3*f*tan(e +
f*x)**2) + b*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(2*a**3*f + 2*a**2*b*
f*tan(e + f*x)**2 - 4*a**2*b*f - 4*a*b**2*f*tan(e + f*x)**2 + 2*a*b**2*f +
2*b**3*f*tan(e + f*x)**2) - b/(2*a**3*f + 2*a**2*b*f*tan(e + f*x)**2 - 4*a*
*2*b*f - 4*a*b**2*f*tan(e + f*x)**2 + 2*a*b**2*f + 2*b**3*f*tan(e + f*x)**2
), True))
```

<u>**Giac**</u> **[B]** time = 1.39075, size = 413, normalized size = 6.35

$$-\frac{\frac{\log\left(a+\frac{2\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{4\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{a\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)}{a^{2}-2\,ab+b^{2}}-\frac{2\,\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1\right)}{a^{2}-2\,ab+b^{2}}-\frac{a^{2}+\frac{2\,a^{2}\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{4\,b^{2}\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{a^{2}\left(\cos\left(fx+e\right)-1\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}}{\left(a^{3}-2\,a^{2}b+ab^{2}\right)\left(a+\frac{2\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{4\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{a\left(\cos\left(fx\cdots\right.\right.}{\left(\cos\left(fx+\cdots\right.\right.}\right.}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(log(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^
2 - 2*a*b + b^2) - 2*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)/(a^2 -
 2*a*b + b^2) - (a^2 + 2*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b^2*
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a^2*(cos(f*x + e) - 1)^2/(cos(f*x +
 e) + 1)^2)/((a^3 - 2*a^2*b + a*b^2)*(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x +
 e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)
^2/(cos(f*x + e) + 1)^2)))/f
```

3.227 $\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=103

$$\frac{b(2a-b)\log\left(a+b\tan^2(e+fx)\right)}{2a^2f(a-b)^2}+\frac{\log(\tan(e+fx))}{a^2f}-\frac{b}{2af(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{\log(\cos(e+fx))}{f(a-b)^2}$$

```
[Out] Log[Cos[e + f*x]]/((a - b)^2*f) + Log[Tan[e + f*x]]/(a^2*f) + ((2*a - b)*b*
Log[a + b*Tan[e + f*x]^2])/(2*a^2*(a - b)^2*f) - b/(2*a*(a - b)*f*(a + b*Ta
n[e + f*x]^2))
```

Rubi [A] time = 0.120596, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3670, 446, 72}

$$\frac{b(2a-b)\log\left(a+b\tan^2(e+fx)\right)}{2a^2f(a-b)^2}+\frac{\log(\tan(e+fx))}{a^2f}-\frac{b}{2af(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{\log(\cos(e+fx))}{f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] Log[Cos[e + f*x]]/((a - b)^2*f) + Log[Tan[e + f*x]]/(a^2*f) + ((2*a - b)*b*
Log[a + b*Tan[e + f*x]^2])/(2*a^2*(a - b)^2*f) - b/(2*a*(a - b)*f*(a + b*Ta
n[e + f*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(1+x^2)\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x(1+x)(a+bx)^2}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{1}{(a-b)^2(1+x)} + \frac{b^2}{a(a-b)(a+bx)^2} + \frac{(2a-b)b^2}{a^2(a-b)^2(a+bx)}\right) dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\log(\cos(e+fx))}{(a-b)^2 f} + \frac{\log(\tan(e+fx))}{a^2 f} + \frac{(2a-b)b\log\left(a+b\tan^2(e+fx)\right)}{2a^2(a-b)^2 f} - \frac{}{2a(a-b)}$$

Mathematica [A] time = 1.98596, size = 90, normalized size = 0.87

$$\frac{\frac{\frac{b\left(\frac{a(b-a)}{a+b\tan^2(e+fx)}+(2a-b)\log\left(a+b\tan^2(e+fx)\right)\right)}{(a-b)^2}+2\log(\tan(e+fx))}{a^2}+\frac{2\log(\cos(e+fx))}{(a-b)^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((2*Log[Cos[e + f*x]])/(a - b)^2 + (2*Log[Tan[e + f*x]] + (b*((2*a - b)*Log
[a + b*Tan[e + f*x]^2] + (a*(-a + b))/(a + b*Tan[e + f*x]^2)))/(a - b)^2)/a
^2)/(2*f)
```

Maple [A] time = 0.092, size = 160, normalized size = 1.6

$$\frac{\ln\left(\cos\left(fx+e\right)+1\right)}{2fa^2} + \frac{b^2}{2fa\left(a-b\right)^2\left(a\left(\cos\left(fx+e\right)\right)^2-\left(\cos\left(fx+e\right)\right)^2b+b\right)} + \frac{b\ln\left(a\left(\cos\left(fx+e\right)\right)^2-\left(\cos\left(fx\right.\right.\right.}{fa\left(a-b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/2/f/a^2*ln(cos(f*x+e)+1)+1/2/f*b^2/a/(a-b)^2/(a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)+1/f*b/a/(a-b)^2*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)-1/2/f*b^2/a^2/(a-
b)^2*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+1/2/f/a^2*ln(cos(f*x+e)-1)
```

Maxima [A] time = 1.12036, size = 167, normalized size = 1.62

$$\frac{\frac{b^2}{a^4-2a^3b+a^2b^2-\left(a^4-3a^3b+3a^2b^2-ab^3\right)\sin\left(fx+e\right)^2} + \frac{\left(2ab-b^2\right)\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{a^4-2a^3b+a^2b^2} + \frac{\log\left(\sin\left(fx+e\right)^2\right)}{a^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2/(a^4 - 2*a^3*b + a^2*b^2 - (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*sin
(f*x + e)^2) + (2*a*b - b^2)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^4 - 2*a^3*
b + a^2*b^2) + log(sin(f*x + e)^2)/a^2)/f
```

Fricas [A] time = 1.27703, size = 439, normalized size = 4.26

$$\frac{ab^2\tan\left(fx+e\right)^2+ab^2+\left(a^3-2\,a^2b+ab^2+\left(a^2b-2\,ab^2+b^3\right)\tan\left(fx+e\right)^2\right)\log\left(\frac{\tan\left(fx+e\right)^2}{\tan\left(fx+e\right)^2+1}\right)+\left(2\,a^2b-ab^2+\left(2\,ab^2\right.\right.}{2\left(\left(a^4b-2\,a^3b^2+a^2b^3\right)f\tan\left(fx+e\right)^2+\left(a^5-2\,a^4b+a^3b^2\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*b^2*tan(f*x + e)^2 + a*b^2 + (a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b
^2 + b^3)*tan(f*x + e)^2)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) + (2*a^2
*b - a*b^2 + (2*a*b^2 - b^3)*tan(f*x + e)^2)*log((b*tan(f*x + e)^2 + a)/(ta
n(f*x + e)^2 + 1)))/((a^4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^2 + (a^5
- 2*a^4*b + a^3*b^2)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36656, size = 205, normalized size = 1.99

$$\frac{\frac{\left(2\,ab-b^2\right)\log\left(\left|-a\sin\left(fx+e\right)^2+b\sin\left(fx+e\right)^2+a\right|\right)}{a^4-2\,a^3b+a^2b^2}-\frac{2\,ab\sin\left(fx+e\right)^2-b^2\sin\left(fx+e\right)^2-2\,ab}{\left(a^3-a^2b\right)\left(a\sin\left(fx+e\right)^2-b\sin\left(fx+e\right)^2-a\right)}+\frac{\log\left(\sin\left(fx+e\right)^2\right)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*a*b - b^2)*log(abs(-a*sin(f*x + e)^2 + b*sin(f*x + e)^2 + a))/(a^4
- 2*a^3*b + a^2*b^2) - (2*a*b*sin(f*x + e)^2 - b^2*sin(f*x + e)^2 - 2*a*b)/
((a^3 - a^2*b)*(a*sin(f*x + e)^2 - b*sin(f*x + e)^2 - a)) + log(sin(f*x + e
)^2)/a^2)/f
```

3.228 $\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2} dx$

Optimal. Leaf size=132

$$\frac{b^2}{2a^2f(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{b^2(3a-2b)\log\left(a+b\tan^2(e+fx)\right)}{2a^3f(a-b)^2}-\frac{(a+2b)\log(\tan(e+fx))}{a^3f}-\frac{\cot^2(e+fx)}{2a^2f}-$$

```
[Out] -Cot[e + f*x]^2/(2*a^2*f) - Log[Cos[e + f*x]]/((a - b)^2*f) - ((a + 2*b)*Lo
g[Tan[e + f*x]])/(a^3*f) - ((3*a - 2*b)*b^2*Log[a + b*Tan[e + f*x]^2])/(2*a
^3*(a - b)^2*f) + b^2/(2*a^2*(a - b)*f*(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.155696, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 88}

$$\frac{b^2}{2a^2f(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{b^2(3a-2b)\log\left(a+b\tan^2(e+fx)\right)}{2a^3f(a-b)^2}-\frac{(a+2b)\log(\tan(e+fx))}{a^3f}-\frac{\cot^2(e+fx)}{2a^2f}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```


```
[Out] -Cot[e + f*x]^2/(2*a^2*f) - Log[Cos[e + f*x]]/((a - b)^2*f) - ((a + 2*b)*Lo
g[Tan[e + f*x]])/(a^3*f) - ((3*a - 2*b)*b^2*Log[a + b*Tan[e + f*x]^2])/(2*a
^3*(a - b)^2*f) + b^2/(2*a^2*(a - b)*f*(a + b*Tan[e + f*x]^2))
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x)(a+bx)^2}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x^2} + \frac{-a-2b}{a^3x} + \frac{1}{(a-b)^2(1+x)} - \frac{b^3}{a^2(a-b)(a+bx)^2} - \frac{(3a-2b)b^3}{a^3(a-b)^2(a+bx)}\right) dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)}{2a^2f} - \frac{\log(\cos(e+fx))}{(a-b)^2f} - \frac{(a+2b)\log(\tan(e+fx))}{a^3f} - \frac{(3a-2b)b^2\log\left(a+b\tan\right.}{2a^3(a-b)^2f}$$

Mathematica [A] time = 0.825586, size = 98, normalized size = 0.74

$$-\frac{\frac{b^3}{a^3(a-b)\left(a\cot^2(e+fx)+b\right)} + \frac{b^2(3a-2b)\log\left(a\cot^2(e+fx)+b\right)}{a^3(a-b)^2} + \frac{\cot^2(e+fx)}{a^2} + \frac{2\log(\sin(e+fx))}{(a-b)^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(Cot[e + f*x]^2/a^2 + b^3/(a^3*(a - b)*(b + a*Cot[e + f*x]^2)) + ((3*a - 2
*b)*b^2*Log[b + a*Cot[e + f*x]^2])/(a^3*(a - b)^2) + (2*Log[Sin[e + f*x]])/
(a - b)^2)/(2*f)
```

Maple [A] time = 0.101, size = 234, normalized size = 1.8

$$-\frac{1}{4fa^2(\cos(fx+e)+1)} - \frac{\ln(\cos(fx+e)+1)}{2fa^2} - \frac{\ln(\cos(fx+e)+1)b}{fa^3} - \frac{b^3}{2fa^2(a-b)^2\left(a(\cos(fx+e))^2 - (\cos(fx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/4/f/a^2/(cos(f*x+e)+1)-1/2/f/a^2*ln(cos(f*x+e)+1)-1/f/a^3*ln(cos(f*x+e)+
1)*b-1/2/f*b^3/a^2/(a-b)^2/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)-3/2/f*b^2/a^2/
(a-b)^2*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+1/f*b^3/a^3/(a-b)^2*ln(a*cos(f*
x+e)^2-cos(f*x+e)^2*b+b)+1/4/f/a^2/(cos(f*x+e)-1)-1/2/f/a^2*ln(cos(f*x+e)-1
)-1/f/a^3*ln(cos(f*x+e)-1)*b
```

Maxima [A] time = 1.16446, size = 252, normalized size = 1.91

$$-\frac{\frac{(3ab^2-2b^3)\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^5-2a^4b+a^3b^2} - \frac{a^3-2a^2b+ab^2-(a^3-3a^2b+3ab^2-2b^3)\sin(fx+e)^2}{(a^5-3a^4b+3a^3b^2-a^2b^3)\sin(fx+e)^4-(a^5-2a^4b+a^3b^2)\sin(fx+e)^2} + \frac{(a+2b)\log\left(\sin(fx+e)^2\right)}{a^3}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*a*b^2 - 2*b^3)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^5 - 2*a^4*b + a
^3*b^2) - (a^3 - 2*a^2*b + a*b^2 - (a^3 - 3*a^2*b + 3*a*b^2 - 2*b^3)*sin(f*
x + e)^2)/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sin(f*x + e)^4 - (a^5 - 2*
a^4*b + a^3*b^2)*sin(f*x + e)^2) + (a + 2*b)*log(sin(f*x + e)^2)/a^3)/f
```

Fricas [B] time = 1.38986, size = 648, normalized size = 4.91

$$-\frac{(a^3b - 2a^2b^2 + 2ab^3)\tan(fx+e)^4 + a^4 - 2a^3b + a^2b^2 + (a^4 - a^3b - a^2b^2 + 2ab^3)\tan(fx+e)^2 + ((a^3b - 3ab^3 +}{2((a^5b - 2a^4b^2 -}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a^3*b - 2*a^2*b^2 + 2*a*b^3)*tan(f*x + e)^4 + a^4 - 2*a^3*b + a^2*b^
2 + (a^4 - a^3*b - a^2*b^2 + 2*a*b^3)*tan(f*x + e)^2 + ((a^3*b - 3*a*b^3 +
2*b^4)*tan(f*x + e)^4 + (a^4 - 3*a^2*b^2 + 2*a*b^3)*tan(f*x + e)^2)*log(tan
(f*x + e)^2/(tan(f*x + e)^2 + 1)) + ((3*a*b^3 - 2*b^4)*tan(f*x + e)^4 + (3*
a^2*b^2 - 2*a*b^3)*tan(f*x + e)^2)*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)
^2 + 1)))/((a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^4 + (a^6 - 2*a^5*b
+ a^4*b^2)*f*tan(f*x + e)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.46413, size = 918, normalized size = 6.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/24*(12*(3*a*b^2 - 2*b^3)*log(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) +
1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(co
s(f*x + e) + 1)^2)/(a^5 - 2*a^4*b + a^3*b^2) - 24*log(-(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) + 1)/(a^2 - 2*a*b + b^2) - (3*a^4 - 6*a^3*b + 3*a^2*b^2 +
 10*a^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 24*a^3*b*(cos(f*x + e) - 1)
/(cos(f*x + e) + 1) + 42*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 20
*a*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 11*a^4*(cos(f*x + e) - 1)^2/
```

```
(cos(f*x + e) + 1)^2 - 22*a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 +
 27*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 16*a*b^3*(cos(f*x +
 e) - 1)^2/(cos(f*x + e) + 1)^2 - 16*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2 + 4*a^4*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 12*a^2*b^2*(cos
(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 8*a*b^3*(cos(f*x + e) - 1)^3/(cos(f
*x + e) + 1)^3)/((a^5 - 2*a^4*b + a^3*b^2)*(a*(cos(f*x + e) - 1)/(cos(f*x +
 e) + 1) + 2*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 4*b*(cos(f*x + e
) - 1)^2/(cos(f*x + e) + 1)^2 + a*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3
)) + 12*(a + 2*b)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/a^3 - 3*(cos(
f*x + e) - 1)/(a^2*(cos(f*x + e) + 1)))/f
```

3.229 $\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=161

$$-\frac{b^3}{2a^3f(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{b^3(4a-3b)\log\left(a+b\tan^2(e+fx)\right)}{2a^4f(a-b)^2}+\frac{\left(a^2+2ab+3b^2\right)\log(\tan(e+fx))}{a^4f}+\frac{(a+$$

```
[Out] ((a + 2*b)*Cot[e + f*x]^2)/(2*a^3*f) - Cot[e + f*x]^4/(4*a^2*f) + Log[Cos[e
 + f*x]]/((a - b)^2*f) + ((a^2 + 2*a*b + 3*b^2)*Log[Tan[e + f*x]])/(a^4*f)
+ ((4*a - 3*b)*b^3*Log[a + b*Tan[e + f*x]^2])/(2*a^4*(a - b)^2*f) - b^3/(2*
a^3*(a - b)*f*(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.179065, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 88}

$$-\frac{b^3}{2a^3f(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{b^3(4a-3b)\log\left(a+b\tan^2(e+fx)\right)}{2a^4f(a-b)^2}+\frac{\left(a^2+2ab+3b^2\right)\log(\tan(e+fx))}{a^4f}+\frac{(a+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b)*Cot[e + f*x]^2)/(2*a^3*f) - Cot[e + f*x]^4/(4*a^2*f) + Log[Cos[e
 + f*x]]/((a - b)^2*f) + ((a^2 + 2*a*b + 3*b^2)*Log[Tan[e + f*x]])/(a^4*f)
+ ((4*a - 3*b)*b^3*Log[a + b*Tan[e + f*x]^2])/(2*a^4*(a - b)^2*f) - b^3/(2*
a^3*(a - b)*f*(a + b*Tan[e + f*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^5(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x)(a+bx)^2}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x^3} + \frac{-a-2b}{a^3x^2} + \frac{a^2+2ab+3b^2}{a^4x} - \frac{1}{(a-b)^2(1+x)} + \frac{b^4}{a^3(a-b)(a+bx)^2} + \frac{(4a-3b)b^4}{a^4(a-b)^2(a+bx)}\right) dx, x, \tan^2\right.}{2f}$$
$$= \frac{(a+2b)\cot^2(e+fx)}{2a^3f} - \frac{\cot^4(e+fx)}{4a^2f} + \frac{\log(\cos(e+fx))}{(a-b)^2f} + \frac{\left(a^2+2ab+3b^2\right)\log(\tan(e+\right.}{a^4f}$$

Mathematica [A] time = 1.01116, size = 121, normalized size = 0.75

$$-\frac{-\frac{b^4}{a^4(a-b)\left(a\cot^2(e+fx)+b\right)} - \frac{b^3(4a-3b)\log\left(a\cot^2(e+fx)+b\right)}{a^4(a-b)^2} - \frac{(a+2b)\cot^2(e+fx)}{a^3} + \frac{\cot^4(e+fx)}{2a^2} - \frac{2\log(\sin(e+fx))}{(a-b)^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(-(((a + 2*b)*Cot[e + f*x]^2)/a^3) + Cot[e + f*x]^4/(2*a^2) - b^4/(a^4*(a
- b)*(b + a*Cot[e + f*x]^2)) - ((4*a - 3*b)*b^3*Log[b + a*Cot[e + f*x]^2])/
(a^4*(a - b)^2) - (2*Log[Sin[e + f*x]])/(a - b)^2)/(2*f)
```

Maple [B] time = 0.109, size = 347, normalized size = 2.2

$$-\frac{1}{16\,fa^2\left(\cos\left(fx+e\right)+1\right)^2} + \frac{7}{16\,fa^2\left(\cos\left(fx+e\right)+1\right)} + \frac{b}{2\,fa^3\left(\cos\left(fx+e\right)+1\right)} + \frac{\ln\left(\cos\left(fx+e\right)+1\right)}{2\,fa^2} + \frac{\ln\left(\cos\left(f\right.\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/16/f/a^2/(cos(f*x+e)+1)^2+7/16/f/a^2/(cos(f*x+e)+1)+1/2/f/a^3/(cos(f*x+e
)+1)*b+1/2/f/a^2*ln(cos(f*x+e)+1)+1/f/a^3*ln(cos(f*x+e)+1)*b+3/2/f/a^4*ln(c
os(f*x+e)+1)*b^2+1/2/f*b^4/a^3/(a-b)^2/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+2/
f*b^3/a^3/(a-b)^2*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)-3/2/f*b^4/a^4/(a-b)^2
*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)-1/16/f/a^2/(cos(f*x+e)-1)^2-7/16/f/a^2
/(cos(f*x+e)-1)-1/2/f/a^3/(cos(f*x+e)-1)*b+1/2/f/a^2*ln(cos(f*x+e)-1)+1/f/a
^3*ln(cos(f*x+e)-1)*b+3/2/f/a^4*ln(cos(f*x+e)-1)*b^2
```

Maxima [A] time = 1.15325, size = 319, normalized size = 1.98

$$\frac{\frac{2\left(4\,ab^3-3\,b^4\right)\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{a^6-2\,a^5b+a^4b^2} + \frac{2\left(2\,a^4-4\,a^3b+4\,ab^3-3\,b^4\right)\sin\left(fx+e\right)^4+a^4-2\,a^3b+a^2b^2-\left(5\,a^4-7\,a^3b-a^2b^2+3\,ab^3\right)\sin\left(fx+e\right)^2}{\left(a^6-3\,a^5b+3\,a^4b^2-a^3b^3\right)\sin\left(fx+e\right)^6-\left(a^6-2\,a^5b+a^4b^2\right)\sin\left(fx+e\right)^4} + \frac{2\left(a^2+2\,ab+3\,b^2\right)\mathrm{l}}{a^4}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(4*a*b^3 - 3*b^4)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^6 - 2*a^5*b +
a^4*b^2) + (2*(2*a^4 - 4*a^3*b + 4*a*b^3 - 3*b^4)*sin(f*x + e)^4 + a^4 - 2*
a^3*b + a^2*b^2 - (5*a^4 - 7*a^3*b - a^2*b^2 + 3*a*b^3)*sin(f*x + e)^2)/((a
^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*sin(f*x + e)^6 - (a^6 - 2*a^5*b + a^4*b
^2)*sin(f*x + e)^4) + 2*(a^2 + 2*a*b + 3*b^2)*log(sin(f*x + e)^2)/a^4)/f
```

Fricas [B] time = 1.47444, size = 759, normalized size = 4.71

$$\left(3\,a^4b-2\,a^3b^2-5\,a^2b^3+6\,ab^4\right)\tan\left(fx+e\right)^6-a^5+2\,a^4b-a^3b^2+\left(3\,a^5-5\,a^3b^2-2\,a^2b^3+6\,ab^4\right)\tan\left(fx+e\right)^4+\big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((3*a^4*b - 2*a^3*b^2 - 5*a^2*b^3 + 6*a*b^4)*tan(f*x + e)^6 - a^5 + 2*a
^4*b - a^3*b^2 + (3*a^5 - 5*a^3*b^2 - 2*a^2*b^3 + 6*a*b^4)*tan(f*x + e)^4 +
 (2*a^5 - a^4*b - 4*a^3*b^2 + 3*a^2*b^3)*tan(f*x + e)^2 + 2*((a^4*b - 4*a*b
^4 + 3*b^5)*tan(f*x + e)^6 + (a^5 - 4*a^2*b^3 + 3*a*b^4)*tan(f*x + e)^4)*lo
g(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) + 2*((4*a*b^4 - 3*b^5)*tan(f*x + e)^
6 + (4*a^2*b^3 - 3*a*b^4)*tan(f*x + e)^4)*log((b*tan(f*x + e)^2 + a)/(tan(f
*x + e)^2 + 1)))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*f*tan(f*x + e)^6 + (a^7 - 2
*a^6*b + a^5*b^2)*f*tan(f*x + e)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.49274, size = 919, normalized size = 5.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(32*(4*a*b^3 - 3*b^4)*log(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1
) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos
(f*x + e) + 1)^2)/(a^6 - 2*a^5*b + a^4*b^2) - 64*log(-(cos(f*x + e) - 1)/(c
```

```
os(f*x + e) + 1) + 1)/(a^2 - 2*a*b + b^2) - 32*(4*a^2*b^3 - 3*a*b^4 + 8*a^2
*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 18*a*b^4*(cos(f*x + e) - 1)/(c
os(f*x + e) + 1) + 8*b^5*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 4*a^2*b^3*
(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 3*a*b^4*(cos(f*x + e) - 1)^2/(c
os(f*x + e) + 1)^2)/((a^6 - 2*a^5*b + a^4*b^2)*(a + 2*a*(cos(f*x + e) - 1)/
(cos(f*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x
 + e) - 1)^2/(cos(f*x + e) + 1)^2)) + 32*(a^2 + 2*a*b + 3*b^2)*log(-(cos(f*
x + e) - 1)/(cos(f*x + e) + 1))/a^4 - (12*a^2*(cos(f*x + e) - 1)/(cos(f*x +
 e) + 1) + 16*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a^2*(cos(f*x + e)
 - 1)^2/(cos(f*x + e) + 1)^2)/a^4 - (a^2 + 12*a^2*(cos(f*x + e) - 1)/(cos(f
*x + e) + 1) + 16*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 48*a^2*(cos(f
*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 96*a*b*(cos(f*x + e) - 1)^2/(cos(f*x
+ e) + 1)^2 + 144*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x +
 e) + 1)^2/(a^4*(cos(f*x + e) - 1)^2))/f
```

3.230 $\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=130

$$-\frac{a^{3/2}(3a-5b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2b^{5/2}f(a-b)^2}+\frac{(3a-2b)\tan(e+fx)}{2b^2f(a-b)}-\frac{a\tan^3(e+fx)}{2bf(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{x}{(a-b)^2}$$

```
[Out] -(x/(a - b)^2) - (a^(3/2)*(3*a - 5*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]
])/(2*(a - b)^2*b^(5/2)*f) + ((3*a - 2*b)*Tan[e + f*x])/(2*(a - b)*b^2*f) -
 (a*Tan[e + f*x]^3)/(2*(a - b)*b*f*(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.196597, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 470, 582, 522, 203, 205}

$$-\frac{a^{3/2}(3a-5b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2b^{5/2}f(a-b)^2}+\frac{(3a-2b)\tan(e+fx)}{2b^2f(a-b)}-\frac{a\tan^3(e+fx)}{2bf(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
```



```
[Out] -(x/(a - b)^2) - (a^(3/2)*(3*a - 5*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]
])/(2*(a - b)^2*b^(5/2)*f) + ((3*a - 2*b)*Tan[e + f*x])/(2*(a - b)*b^2*f) -
 (a*Tan[e + f*x]^3)/(2*(a - b)*b*f*(a + b*Tan[e + f*x]^2))
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
```

```
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan^3(e+fx)}{2(a-b)bf\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(3a+(3a-2b)x^2\right)}{(1+x^2)\left(a+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{2(a-b)bf}$$

$$= \frac{(3a-2b)\tan(e+fx)}{2(a-b)b^2f} - \frac{a\tan^3(e+fx)}{2(a-b)bf\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{a(3a-2b)+\left(3a^2-2ab-2b^2\right)x^{\cdots}}{(1+x^2)\left(a+bx^2\right)} \cdots\right)}{2(a-b)b^2f \cdots}$$

$$= \frac{(3a-2b)\tan(e+fx)}{2(a-b)b^2f} - \frac{a\tan^3(e+fx)}{2(a-b)bf\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+f\cdots\right)}{(a-b)^2f}$$

$$= -\frac{x}{(a-b)^2} - \frac{a^{3/2}(3a-5b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2(a-b)^2b^{5/2}f} + \frac{(3a-2b)\tan(e+fx)}{2(a-b)b^2f} - \frac{a\tan^3(e\cdots}{2(a-b)bf\left(a+b\cdots\right.}$$

Mathematica [A] time = 1.22763, size = 118, normalized size = 0.91

$$\frac{-\frac{a^{3/2}(3a-5b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{b^{5/2}(a-b)^2} + \frac{a^2\sin(2(e+fx))}{b^2(a-b)((a-b)\cos(2(e+fx))+a+b)} - \frac{2(e+fx)}{(a-b)^2} + \frac{2\tan(e+fx)}{b^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((-2*(e + f*x))/(a - b)^2 - (a^(3/2)*(3*a - 5*b)*ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a]])/((a - b)^2*b^(5/2)) + (a^2*Sin[2*(e + f*x)])/((a - b)*b^2*(a
+ b + (a - b)*Cos[2*(e + f*x)])) + (2*Tan[e + f*x])/b^2)/(2*f)
```

Maple [A] time = 0.023, size = 184, normalized size = 1.4

$$\frac{\tan\left(fx+e\right)}{f b^{2}}+\frac{a^{3}\tan\left(fx+e\right)}{2 f \left(a-b\right)^{2} b^{2}\left(a+b\left(\tan\left(fx+e\right)\right)^{2}\right)}-\frac{a^{2}\tan\left(fx+e\right)}{2 f \left(a-b\right)^{2} b\left(a+b\left(\tan\left(fx+e\right)\right)^{2}\right)}-\frac{3 a^{3}}{2 f \left(a-b\right)^{2} b^{2}}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f/b^2*tan(f*x+e)+1/2/f*a^3/(a-b)^2/b^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)-1/2/
f*a^2/(a-b)^2/b*tan(f*x+e)/(a+b*tan(f*x+e)^2)-3/2/f*a^3/(a-b)^2/b^2/(a*b)^(
1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))+5/2/f*a^2/(a-b)^2/b/(a*b)^(1/2)*arcta
n(b*tan(f*x+e)/(a*b)^(1/2))-1/f/(a-b)^2*arctan(tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.28956, size = 1065, normalized size = 8.19

$$\left[-\frac{8 b^{3} f x \tan\left(fx+e\right)^{2}+8 a b^{2} f x-8\left(a^{2} b-2 a b^{2}+b^{3}\right)\tan\left(fx+e\right)^{3}+\left(3 a^{3}-5 a^{2} b+\left(3 a^{2} b-5 a b^{2}\right)\tan\left(fx+e\right)^{2}\right)}{8\left(\left(a^{2} b^{3}-2 a b^{4}+b^{5}\right)f\tan\left(fx+e\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(8*b^3*f*x*tan(f*x + e)^2 + 8*a*b^2*f*x - 8*(a^2*b - 2*a*b^2 + b^3)*t
an(f*x + e)^3 + (3*a^3 - 5*a^2*b + (3*a^2*b - 5*a*b^2)*tan(f*x + e)^2)*sqrt
(-a/b)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(b^2*tan(f*
x + e)^3 - a*b*tan(f*x + e))*sqrt(-a/b))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*
x + e)^2 + a^2)) - 4*(3*a^3 - 5*a^2*b + 2*a*b^2)*tan(f*x + e))/((a^2*b^3 -
2*a*b^4 + b^5)*f*tan(f*x + e)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*f), -1/4*(4
*b^3*f*x*tan(f*x + e)^2 + 4*a*b^2*f*x - 4*(a^2*b - 2*a*b^2 + b^3)*tan(f*x +
 e)^3 + (3*a^3 - 5*a^2*b + (3*a^2*b - 5*a*b^2)*tan(f*x + e)^2)*sqrt(a/b)*ar
ctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a/b)/(a*tan(f*x + e))) - 2*(3*a^3 - 5*
a^2*b + 2*a*b^2)*tan(f*x + e))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan(f*x + e)^2
+ (a^3*b^2 - 2*a^2*b^3 + a*b^4)*f)]
```

Sympy [A] time = 147.662, size = 2914, normalized size = 22.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x*tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x + tan(e +
 f*x)**5/(5*f) - tan(e + f*x)**3/(3*f) + tan(e + f*x)/f)/a**2, Eq(b, 0)), (
(-x + tan(e + f*x)/f)/b**2, Eq(a, 0)), (x*tan(e)**6/(a + b*tan(e)**2)**2, E
q(f, 0)), (6*I*a**(7/2)*b*sqrt(1/b)*tan(e + f*x)/(4*I*a**(7/2)*b**3*f*sqrt(
1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**4*f*
sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b*
*5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f*x)**2) + 4*I*a**(5/
2)*b**2*sqrt(1/b)*tan(e + f*x)**3/(4*I*a**(7/2)*b**3*f*sqrt(1/b) + 4*I*a**(
5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**4*f*sqrt(1/b) - 8*I
*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**5*f*sqrt(1/b)
+ 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f*x)**2) - 10*I*a**(5/2)*b**2*sqrt(1
/b)*tan(e + f*x)/(4*I*a**(7/2)*b**3*f*sqrt(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(
1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**4*f*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*
sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**
6*f*sqrt(1/b)*tan(e + f*x)**2) - 4*I*a**(3/2)*b**3*f*x*sqrt(1/b)/(4*I*a**(7
/2)*b**3*f*sqrt(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*
a**(5/2)*b**4*f*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 +
 4*I*a**(3/2)*b**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f*x)*
*2) - 8*I*a**(3/2)*b**3*sqrt(1/b)*tan(e + f*x)**3/(4*I*a**(7/2)*b**3*f*sqrt
(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**4*f
*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b
**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f*x)**2) + 4*I*a**(3
/2)*b**3*sqrt(1/b)*tan(e + f*x)/(4*I*a**(7/2)*b**3*f*sqrt(1/b) + 4*I*a**(5/
2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**4*f*sqrt(1/b) - 8*I*a
**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**5*f*sqrt(1/b) +
4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f*x)**2) - 4*I*sqrt(a)*b**4*f*x*sqrt(1
/b)*tan(e + f*x)**2/(4*I*a**(7/2)*b**3*f*sqrt(1/b) + 4*I*a**(5/2)*b**4*f*sq
rt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**4*f*sqrt(1/b) - 8*I*a**(3/2)*b**5
*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**5*f*sqrt(1/b) + 4*I*sqrt(a)*
b**6*f*sqrt(1/b)*tan(e + f*x)**2) + 4*I*sqrt(a)*b**4*sqrt(1/b)*tan(e + f*x)
**3/(4*I*a**(7/2)*b**3*f*sqrt(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e +
f*x)**2 - 8*I*a**(5/2)*b**4*f*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*tan
(e + f*x)**2 + 4*I*a**(3/2)*b**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b)
*tan(e + f*x)**2) - 3*a**4*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a*
*(7/2)*b**3*f*sqrt(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 8
*I*a**(5/2)*b**4*f*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**
2 + 4*I*a**(3/2)*b**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f*
x)**2) + 3*a**4*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(7/2)*b**3*
f*sqrt(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*
b**4*f*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(
3/2)*b**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f*x)**2) - 3*a
**3*b*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(7/2
)*b**3*f*sqrt(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a*
*(5/2)*b**4*f*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 4
*I*a**(3/2)*b**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f*x)**2
) + 5*a**3*b*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(7/2)*b**3*f*
sqrt(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b*
*4*f*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/
2)*b**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f*x)**2) + 3*a**
3*b*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(7/2)*b
**3*f*sqrt(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5
```

```
/2)*b**4*f*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*
a**(3/2)*b**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f*x)**2) -
 5*a**3*b*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(7/2)*b**3*f*sqrt
(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**4*f
*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b
**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f*x)**2) + 5*a**2*b*
*2*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(7/2)*b
**3*f*sqrt(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5
/2)*b**4*f*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*
a**(3/2)*b**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f*x)**2) -
 5*a**2*b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a
**(7/2)*b**3*f*sqrt(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 -
8*I*a**(5/2)*b**4*f*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)*
*2 + 4*I*a**(3/2)*b**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f
*x)**2), True))
```

Giac [A] time = 4.35374, size = 201, normalized size = 1.55

$$\frac{\frac{a^2\tan\left(fx+e\right)}{\left(ab^2-b^3\right)\left(b\tan\left(fx+e\right)^2+a\right)}-\frac{\left(3\,a^3-5\,a^2b\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)}{\left(a^2b^2-2\,ab^3+b^4\right)\sqrt{ab}}-\frac{2\left(fx+e\right)}{a^2-2\,ab+b^2}+\frac{2\,\tan\left(fx+e\right)}{b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*tan(f*x + e)/((a*b^2 - b^3)*(b*tan(f*x + e)^2 + a)) - (3*a^3 - 5*a
^2*b)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
)))/((a^2*b^2 - 2*a*b^3 + b^4)*sqrt(a*b)) - 2*(f*x + e)/(a^2 - 2*a*b + b^2)
 + 2*tan(f*x + e)/b^2)/f
```

3.231 $\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=95

$$\frac{\sqrt{a}(a-3b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2b^{3/2}f(a-b)^2}-\frac{a\tan(e+fx)}{2bf(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{x}{(a-b)^2}$$

```
[Out] x/(a - b)^2 + (Sqrt[a]*(a - 3*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2
*(a - b)^2*b^(3/2)*f) - (a*Tan[e + f*x])/(2*(a - b)*b*f*(a + b*Tan[e + f*x]
^2))
```

Rubi [A] time = 0.117503, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3670, 470, 522, 203, 205}

$$\frac{\sqrt{a}(a-3b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2b^{3/2}f(a-b)^2}-\frac{a\tan(e+fx)}{2bf(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```


```
[Out] x/(a - b)^2 + (Sqrt[a]*(a - 3*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2
*(a - b)^2*b^(3/2)*f) - (a*Tan[e + f*x])/(2*(a - b)*b*f*(a + b*Tan[e + f*x]
^2))
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+bx^2\right)^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan(e+fx)}{2(a-b)bf\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{a+(a-2b)x^2}{(1+x^2)\left(a+bx^2\right)}\,dx,x,\tan(e+fx)\right)}{2(a-b)bf}$$

$$= -\frac{a\tan(e+fx)}{2(a-b)bf\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+fx)\right)}{(a-b)^2f} + \frac{(a(a-3b))\,\text{Su}}{}$$

$$= \frac{x}{(a-b)^2} + \frac{\sqrt{a}(a-3b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2(a-b)^2b^{3/2}f} - \frac{a\tan(e+fx)}{2(a-b)bf\left(a+b\tan^2(e+fx)\right)}$$

Mathematica [A] time = 0.751754, size = 94, normalized size = 0.99

$$\frac{\frac{\sqrt{a}(a-3b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{b^{3/2}} - \frac{a(a-b)\sin(2(e+fx))}{b((a-b)\cos(2(e+fx))+a+b)} + 2(e+fx)}{2f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (2*(e + f*x) + (Sqrt[a]*(a - 3*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/b
^(3/2) - (a*(a - b)*Sin[2*(e + f*x)])/(b*(a + b + (a - b)*Cos[2*(e + f*x)])
))/(2*(a - b)^2*f)
```

Maple [A] time = 0.023, size = 160, normalized size = 1.7

$$-\frac{a^2\tan\left(fx+e\right)}{2f\left(a-b\right)^2b\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)} + \frac{a\tan\left(fx+e\right)}{2f\left(a-b\right)^2\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)} + \frac{a^2}{2f\left(a-b\right)^2b}\arctan\left(b\tan\left(fx+e\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*a^2/(a-b)^2/b*tan(f*x+e)/(a+b*tan(f*x+e)^2)+1/2/f*a/(a-b)^2*tan(f*x+
e)/(a+b*tan(f*x+e)^2)+1/2/f*a^2/(a-b)^2/b/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(
```

```
a*b)^(1/2))-3/2/f*a/(a-b)^2/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))+1/
f/(a-b)^2*arctan(tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.25805, size = 865, normalized size = 9.11

$$\left[\frac{8\,b^2 f x \tan\left(fx+e\right)^2 + 8\,abfx - \left(\left(ab - 3\,b^2\right)\tan\left(fx+e\right)^2 + a^2 - 3\,ab\right)\sqrt{-\frac{a}{b}}\log\left(\frac{b^2\tan\left(fx+e\right)^4 - 6\,ab\tan\left(fx+e\right)^2 + a^2 - 4\left(b^2\tan\left(fx\right.\right.}{b^2\tan\left(fx+e\right)^4 + 2\,ab\tan\left(fx+e\right.}\right.}{8\left(\left(a^2b^2 - 2\,ab^3 + b^4\right)f\tan\left(fx+e\right)^2 + \left(a^3b - 2\,a^2b^2 + ab^3\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*b^2*f*x*tan(f*x + e)^2 + 8*a*b*f*x - ((a*b - 3*b^2)*tan(f*x + e)^2
+ a^2 - 3*a*b)*sqrt(-a/b)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 +
a^2 - 4*(b^2*tan(f*x + e)^3 - a*b*tan(f*x + e))*sqrt(-a/b))/(b^2*tan(f*x +
e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(a^2 - a*b)*tan(f*x + e))/((a^2*b^2
 - 2*a*b^3 + b^4)*f*tan(f*x + e)^2 + (a^3*b - 2*a^2*b^2 + a*b^3)*f), 1/4*(4
*b^2*f*x*tan(f*x + e)^2 + 4*a*b*f*x + ((a*b - 3*b^2)*tan(f*x + e)^2 + a^2 -
 3*a*b)*sqrt(a/b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a/b)/(a*tan(f*x +
e))) - 2*(a^2 - a*b)*tan(f*x + e))/((a^2*b^2 - 2*a*b^3 + b^4)*f*tan(f*x + e
)^2 + (a^3*b - 2*a^2*b^2 + a*b^3)*f)]
```

Sympy [A] time = 52.0489, size = 2179, normalized size = 22.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((x + tan(e + f*x)**3/(3
*f) - tan(e + f*x)/f)/a**2, Eq(b, 0)), (x/b**2, Eq(a, 0)), (x*tan(e)**4/(a
+ b*tan(e)**2)**2, Eq(f, 0)), (-2*I*a**(5/2)*b*sqrt(1/b)*tan(e + f*x)/(4*I*
a**(7/2)*b**2*f*sqrt(1/b) + 4*I*a**(5/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 -
 8*I*a**(5/2)*b**3*f*sqrt(1/b) - 8*I*a**(3/2)*b**4*f*sqrt(1/b)*tan(e + f*x)
**2 + 4*I*a**(3/2)*b**4*f*sqrt(1/b) + 4*I*sqrt(a)*b**5*f*sqrt(1/b)*tan(e +
f*x)**2) + 4*I*a**(3/2)*b**2*f*x*sqrt(1/b)/(4*I*a**(7/2)*b**2*f*sqrt(1/b) +
 4*I*a**(5/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**3*f*sqrt(1
/b) - 8*I*a**(3/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**4*f*s
```

```
qrt(1/b) + 4*I*sqrt(a)*b**5*f*sqrt(1/b)*tan(e + f*x)**2) + 2*I*a**(3/2)*b**
2*sqrt(1/b)*tan(e + f*x)/(4*I*a**(7/2)*b**2*f*sqrt(1/b) + 4*I*a**(5/2)*b**3
*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**3*f*sqrt(1/b) - 8*I*a**(3/2)
*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**4*f*sqrt(1/b) + 4*I*sqr
t(a)*b**5*f*sqrt(1/b)*tan(e + f*x)**2) + 4*I*sqrt(a)*b**3*f*x*sqrt(1/b)*tan
(e + f*x)**2/(4*I*a**(7/2)*b**2*f*sqrt(1/b) + 4*I*a**(5/2)*b**3*f*sqrt(1/b)
*tan(e + f*x)**2 - 8*I*a**(5/2)*b**3*f*sqrt(1/b) - 8*I*a**(3/2)*b**4*f*sqrt
(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**4*f*sqrt(1/b) + 4*I*sqrt(a)*b**5*f*
sqrt(1/b)*tan(e + f*x)**2) + a**3*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/
(4*I*a**(7/2)*b**2*f*sqrt(1/b) + 4*I*a**(5/2)*b**3*f*sqrt(1/b)*tan(e + f*x)
**2 - 8*I*a**(5/2)*b**3*f*sqrt(1/b) - 8*I*a**(3/2)*b**4*f*sqrt(1/b)*tan(e +
 f*x)**2 + 4*I*a**(3/2)*b**4*f*sqrt(1/b) + 4*I*sqrt(a)*b**5*f*sqrt(1/b)*tan
(e + f*x)**2) - a**3*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(7/2)*
b**2*f*sqrt(1/b) + 4*I*a**(5/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(
5/2)*b**3*f*sqrt(1/b) - 8*I*a**(3/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I
*a**(3/2)*b**4*f*sqrt(1/b) + 4*I*sqrt(a)*b**5*f*sqrt(1/b)*tan(e + f*x)**2)
+ a**2*b*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(
7/2)*b**2*f*sqrt(1/b) + 4*I*a**(5/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I
*a**(5/2)*b**3*f*sqrt(1/b) - 8*I*a**(3/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2
+ 4*I*a**(3/2)*b**4*f*sqrt(1/b) + 4*I*sqrt(a)*b**5*f*sqrt(1/b)*tan(e + f*x)
**2) - 3*a**2*b*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(7/2)*b**2
*f*sqrt(1/b) + 4*I*a**(5/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)
*b**3*f*sqrt(1/b) - 8*I*a**(3/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**
(3/2)*b**4*f*sqrt(1/b) + 4*I*sqrt(a)*b**5*f*sqrt(1/b)*tan(e + f*x)**2) - a*
*2*b*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(7/2)*
b**2*f*sqrt(1/b) + 4*I*a**(5/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(
5/2)*b**3*f*sqrt(1/b) - 8*I*a**(3/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I
*a**(3/2)*b**4*f*sqrt(1/b) + 4*I*sqrt(a)*b**5*f*sqrt(1/b)*tan(e + f*x)**2)
+ 3*a**2*b*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(7/2)*b**2*f*sqr
t(1/b) + 4*I*a**(5/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**3*
f*sqrt(1/b) - 8*I*a**(3/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*
b**4*f*sqrt(1/b) + 4*I*sqrt(a)*b**5*f*sqrt(1/b)*tan(e + f*x)**2) - 3*a*b**2
*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(7/2)*b**
2*f*sqrt(1/b) + 4*I*a**(5/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2
)*b**3*f*sqrt(1/b) - 8*I*a**(3/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a*
*(3/2)*b**4*f*sqrt(1/b) + 4*I*sqrt(a)*b**5*f*sqrt(1/b)*tan(e + f*x)**2) + 3
*a*b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(7/
2)*b**2*f*sqrt(1/b) + 4*I*a**(5/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a
**(5/2)*b**3*f*sqrt(1/b) - 8*I*a**(3/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 +
4*I*a**(3/2)*b**4*f*sqrt(1/b) + 4*I*sqrt(a)*b**5*f*sqrt(1/b)*tan(e + f*x)**
2), True))
```

Giac [A] time = 2.20204, size = 171, normalized size = 1.8

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)\left(a^2-3\,ab\right)}{\left(a^2b-2\,ab^2+b^3\right)\sqrt{ab}}+\frac{2\left(fx+e\right)}{a^2-2\,ab+b^2}-\frac{a\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a\right)\left(ab-b^2\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)
))*(a^2 - 3*a*b)/((a^2*b - 2*a*b^2 + b^3)*sqrt(a*b)) + 2*(f*x + e)/(a^2 - 2
*a*b + b^2) - a*tan(f*x + e)/((b*tan(f*x + e)^2 + a)*(a*b - b^2)))/f
```

3.232 $\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=90

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}f(a-b)^2}+\frac{\tan(e+fx)}{2f(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{x}{(a-b)^2}$$

```
[Out] -(x/(a - b)^2) + ((a + b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2*Sqrt[a
]*(a - b)^2*Sqrt[b]*f) + Tan[e + f*x]/(2*(a - b)*f*(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.103679, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3670, 471, 522, 203, 205}

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}f(a-b)^2}+\frac{\tan(e+fx)}{2f(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(x/(a - b)^2) + ((a + b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2*Sqrt[a
]*(a - b)^2*Sqrt[b]*f) + Tan[e + f*x]/(2*(a - b)*f*(a + b*Tan[e + f*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1-x^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{2(a-b)f}$$

$$= \frac{\tan(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{(a-b)^2f} + \frac{(a+b)\,\text{Subst}\left(\int\right.}{2}$$

$$= -\frac{x}{(a-b)^2} + \frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2\sqrt{a}(a-b)^2\sqrt{b}f} + \frac{\tan(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)}$$

Mathematica [A] time = 0.507534, size = 87, normalized size = 0.97

$$\frac{\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} + \frac{(a-b)\sin(2(e+fx))}{(a-b)\cos(2(e+fx))+a+b} - 2(e+fx)}{2f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-2*(e + f*x) + ((a + b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*S
qrt[b]) + ((a - b)*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)]))/(2
*(a - b)^2*f)
```

Maple [A] time = 0.022, size = 151, normalized size = 1.7

$$\frac{a\tan\left(fx+e\right)}{2f(a-b)^2\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)} - \frac{b\tan\left(fx+e\right)}{2f(a-b)^2\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)} + \frac{a}{2f(a-b)^2}\arctan\left(b\tan\left(fx+e\right)\frac{1}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/2/f*a/(a-b)^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)-1/2*b*tan(f*x+e)/(a-b)^2/f/(a
+b*tan(f*x+e)^2)+1/2/f*a/(a-b)^2/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2
))+1/2/f/(a-b)^2*b/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-1/f/(a-b)^2
*arctan(tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.55971, size = 892, normalized size = 9.91

$$\left[-\frac{8\,ab^2fx\tan\left(fx+e\right)^2+8\,a^2bfx+\left(\left(ab+b^2\right)\tan\left(fx+e\right)^2+a^2+ab\right)\sqrt{-ab}\log\left(\frac{b^2\tan\left(fx+e\right)^4-6\,ab\tan\left(fx+e\right)^2+a^2-4\left(b\tan\left(f\right.\right.}{b^2\tan\left(fx+e\right)^4+2\,ab\tan\left(fx+\right.}\right.}{8\left(\left(a^3b^2-2\,a^2b^3+ab^4\right)f\tan\left(fx+e\right)^2+\left(a^4b-2\,a^3b^2+a^2b^3\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(8*a*b^2*f*x*tan(f*x + e)^2 + 8*a^2*b*f*x + ((a*b + b^2)*tan(f*x + e)
^2 + a^2 + a*b)*sqrt(-a*b)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 +
 a^2 - 4*(b*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(-a*b))/(b^2*tan(f*x + e)^
4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(a^2*b - a*b^2)*tan(f*x + e))/((a^3*b^
2 - 2*a^2*b^3 + a*b^4)*f*tan(f*x + e)^2 + (a^4*b - 2*a^3*b^2 + a^2*b^3)*f),
 -1/4*(4*a*b^2*f*x*tan(f*x + e)^2 + 4*a^2*b*f*x - ((a*b + b^2)*tan(f*x + e)
^2 + a^2 + a*b)*sqrt(a*b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a*b)/(a*b*
tan(f*x + e))) - 2*(a^2*b - a*b^2)*tan(f*x + e))/((a^3*b^2 - 2*a^2*b^3 + a*
b^4)*f*tan(f*x + e)^2 + (a^4*b - 2*a^3*b^2 + a^2*b^3)*f)]
```

Sympy [A] time = 50.9058, size = 2145, normalized size = 23.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x/tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x + tan(e +
 f*x)/f)/a**2, Eq(b, 0)), ((-x - 1/(f*tan(e + f*x)))/b**2, Eq(a, 0)), (x*ta
n(e)**2/(a + b*tan(e)**2)**2, Eq(f, 0)), (-4*I*a**(3/2)*b*f*x*sqrt(1/b)/(4*
I*a**(7/2)*b*f*sqrt(1/b) + 4*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 -
8*I*a**(5/2)*b**2*f*sqrt(1/b) - 8*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)*
*2 + 4*I*a**(3/2)*b**3*f*sqrt(1/b) + 4*I*sqrt(a)*b**4*f*sqrt(1/b)*tan(e + f
*x)**2) + 2*I*a**(3/2)*b*sqrt(1/b)*tan(e + f*x)/(4*I*a**(7/2)*b*f*sqrt(1/b)
 + 4*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**2*f*sqrt
(1/b) - 8*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**3*f
*sqrt(1/b) + 4*I*sqrt(a)*b**4*f*sqrt(1/b)*tan(e + f*x)**2) - 4*I*sqrt(a)*b*
*2*f*x*sqrt(1/b)*tan(e + f*x)**2/(4*I*a**(7/2)*b*f*sqrt(1/b) + 4*I*a**(5/2)
*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**2*f*sqrt(1/b) - 8*I*a**
```

```
(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**3*f*sqrt(1/b) + 4*
I*sqrt(a)*b**4*f*sqrt(1/b)*tan(e + f*x)**2) - 2*I*sqrt(a)*b**2*sqrt(1/b)*ta
n(e + f*x)/(4*I*a**(7/2)*b*f*sqrt(1/b) + 4*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(
e + f*x)**2 - 8*I*a**(5/2)*b**2*f*sqrt(1/b) - 8*I*a**(3/2)*b**3*f*sqrt(1/b)
*tan(e + f*x)**2 + 4*I*a**(3/2)*b**3*f*sqrt(1/b) + 4*I*sqrt(a)*b**4*f*sqrt(
1/b)*tan(e + f*x)**2) + a**2*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*
a**(7/2)*b*f*sqrt(1/b) + 4*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 8*
I*a**(5/2)*b**2*f*sqrt(1/b) - 8*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2
 + 4*I*a**(3/2)*b**3*f*sqrt(1/b) + 4*I*sqrt(a)*b**4*f*sqrt(1/b)*tan(e + f*x
)**2) - a**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(7/2)*b*f*sqrt
(1/b) + 4*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**2*f
*sqrt(1/b) - 8*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b
**3*f*sqrt(1/b) + 4*I*sqrt(a)*b**4*f*sqrt(1/b)*tan(e + f*x)**2) + a*b*log(-
I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(7/2)*b*f*sqrt(
1/b) + 4*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**2*f*
sqrt(1/b) - 8*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b*
*3*f*sqrt(1/b) + 4*I*sqrt(a)*b**4*f*sqrt(1/b)*tan(e + f*x)**2) + a*b*log(-I
*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(7/2)*b*f*sqrt(1/b) + 4*I*a**(5/
2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**2*f*sqrt(1/b) - 8*I*a
**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**3*f*sqrt(1/b) +
4*I*sqrt(a)*b**4*f*sqrt(1/b)*tan(e + f*x)**2) - a*b*log(I*sqrt(a)*sqrt(1/b)
 + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(7/2)*b*f*sqrt(1/b) + 4*I*a**(5/2)
*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**2*f*sqrt(1/b) - 8*I*a**
(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**3*f*sqrt(1/b) + 4*
I*sqrt(a)*b**4*f*sqrt(1/b)*tan(e + f*x)**2) - a*b*log(I*sqrt(a)*sqrt(1/b) +
 tan(e + f*x))/(4*I*a**(7/2)*b*f*sqrt(1/b) + 4*I*a**(5/2)*b**2*f*sqrt(1/b)*
tan(e + f*x)**2 - 8*I*a**(5/2)*b**2*f*sqrt(1/b) - 8*I*a**(3/2)*b**3*f*sqrt(
1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**3*f*sqrt(1/b) + 4*I*sqrt(a)*b**4*f*s
qrt(1/b)*tan(e + f*x)**2) + b**2*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*t
an(e + f*x)**2/(4*I*a**(7/2)*b*f*sqrt(1/b) + 4*I*a**(5/2)*b**2*f*sqrt(1/b)*
tan(e + f*x)**2 - 8*I*a**(5/2)*b**2*f*sqrt(1/b) - 8*I*a**(3/2)*b**3*f*sqrt(
1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**3*f*sqrt(1/b) + 4*I*sqrt(a)*b**4*f*s
qrt(1/b)*tan(e + f*x)**2) - b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*ta
n(e + f*x)**2/(4*I*a**(7/2)*b*f*sqrt(1/b) + 4*I*a**(5/2)*b**2*f*sqrt(1/b)*t
an(e + f*x)**2 - 8*I*a**(5/2)*b**2*f*sqrt(1/b) - 8*I*a**(3/2)*b**3*f*sqrt(1
/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**3*f*sqrt(1/b) + 4*I*sqrt(a)*b**4*f*sq
rt(1/b)*tan(e + f*x)**2), True))
```

Giac [A] time = 1.56828, size = 151, normalized size = 1.68

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)(a+b)}{\left(a^2-2\,ab+b^2\right)\sqrt{ab}}-\frac{2\left(fx+e\right)}{a^2-2\,ab+b^2}+\frac{\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a\right)(a-b)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)
))*(a + b)/((a^2 - 2*a*b + b^2)*sqrt(a*b)) - 2*(f*x + e)/(a^2 - 2*a*b + b^2
) + tan(f*x + e)/((b*tan(f*x + e)^2 + a)*(a - b)))/f
```

3.233 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt{b}(3a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{3/2}f(a-b)^2}-\frac{b\tan(e+fx)}{2af(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{x}{(a-b)^2}$$

```
[Out] x/(a - b)^2 - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2
*a^(3/2)*(a - b)^2*f) - (b*Tan[e + f*x])/(2*a*(a - b)*f*(a + b*Tan[e + f*x]
^2))
```

Rubi [A] time = 0.0801249, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3661, 414, 522, 203, 205}

$$-\frac{\sqrt{b}(3a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{3/2}f(a-b)^2}-\frac{b\tan(e+fx)}{2af(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-2),x]
```

```
[Out] x/(a - b)^2 - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2
*a^(3/2)*(a - b)^2*f) - (b*Tan[e + f*x])/(2*a*(a - b)*f*(a + b*Tan[e + f*x]
^2))
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-b-bx^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{2a(a-b)f}$$

$$= -\frac{b\tan(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{(a-b)^2 f} - \frac{((3a-b)b)\,\text{Su}}{}$$

$$= \frac{x}{(a-b)^2} - \frac{(3a-b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2 f} - \frac{b\tan(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)}$$

Mathematica [A] time = 0.996943, size = 88, normalized size = 0.91

$$\frac{\frac{\sqrt{b}(b-3a)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{b(b-a)\tan(e+fx)}{a\left(a+b\tan^2(e+fx)\right)} + 2\tan^{-1}(\tan(e+fx))}{2f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-2),x]
```

```
[Out] (2*ArcTan[Tan[e + f*x]] + (Sqrt[b]*(-3*a + b)*ArcTan[(Sqrt[b]*Tan[e + f*x])
/Sqrt[a]])/a^(3/2) + (b*(-a + b)*Tan[e + f*x])/(a*(a + b*Tan[e + f*x]^2)))/
(2*(a - b)^2*f)
```

Maple [A] time = 0., size = 160, normalized size = 1.7

$$-\frac{b\tan\left(fx+e\right)}{2\left(a-b\right)^2 f\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)} + \frac{b^2\tan\left(fx+e\right)}{2\left(a-b\right)^2 fa\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)} - \frac{3b}{2\left(a-b\right)^2 f}\arctan\left(b\tan\left(fx+e\right)\frac{1}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/2*b*tan(f*x+e)/(a-b)^2/f/(a+b*tan(f*x+e)^2)+1/2/f/(a-b)^2*b^2/a*tan(f*x+
e)/(a+b*tan(f*x+e)^2)-3/2/f/(a-b)^2*b/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)
^(1/2))+1/2/f/(a-b)^2*b^2/a/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))+1/
```

```
f/(a-b)^2*arctan(tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57699, size = 865, normalized size = 8.92

$$\left[\frac{8\,abfx\tan\left(fx+e\right)^{2}+8\,a^{2}fx-\left(\left(3\,ab-b^{2}\right)\tan\left(fx+e\right)^{2}+3\,a^{2}-ab\right)\sqrt{-\frac{b}{a}}\log\left(\frac{b^{2}\tan\left(fx+e\right)^{4}-6\,ab\tan\left(fx+e\right)^{2}+a^{2}+4\left(ab\tan\left(fx\right.\right.}{b^{2}\tan\left(fx+e\right)^{4}+2\,ab\tan\left(fx+e\right.}\right.}{8\left(\left(a^{3}b-2\,a^{2}b^{2}+ab^{3}\right)f\tan\left(fx+e\right)^{2}+\left(a^{4}-2\,a^{3}b+a^{2}b^{2}\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*a*b*f*x*tan(f*x + e)^2 + 8*a^2*f*x - ((3*a*b - b^2)*tan(f*x + e)^2
+ 3*a^2 - a*b)*sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 +
a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x +
e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(a*b - b^2)*tan(f*x + e))/((a^3*b -
 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f), 1/4*(4
*a*b*f*x*tan(f*x + e)^2 + 4*a^2*f*x - ((3*a*b - b^2)*tan(f*x + e)^2 + 3*a^2
 - a*b)*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x +
e))) - 2*(a*b - b^2)*tan(f*x + e))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*tan(f*x +
 e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f)]
```

Sympy [A] time = 37.2045, size = 2086, normalized size = 21.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x/tan(e)**4, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x/a**2, Eq(b,
 0)), ((x + 1/(f*tan(e + f*x)) - 1/(3*f*tan(e + f*x)**3))/b**2, Eq(a, 0)),
(x/(a + b*tan(e)**2)**2, Eq(f, 0)), (4*I*a**(5/2)*f*x*sqrt(1/b)/(4*I*a**(9/
2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*
b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2
)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) + 4*I*a
**(3/2)*b*f*x*sqrt(1/b)*tan(e + f*x)**2/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**
(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(
5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I
*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) - 2*I*a**(3/2)*b*sqrt(1/b)*tan(
```

```
e + f*x)/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x
)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f
*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(
e + f*x)**2) + 2*I*sqrt(a)*b**2*sqrt(1/b)*tan(e + f*x)/(4*I*a**(9/2)*f*sqrt
(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(
1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*
sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) - 3*a**2*log(-I*
sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*
b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b
**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3
/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) + 3*a**2*log(I*sqrt(a)*sqrt(1/b) + ta
n(e + f*x))/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e +
f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e
+ f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*t
an(e + f*x)**2) - 3*a*b*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*
x)**2/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**
2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)
**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e +
 f*x)**2) + a*b*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(9/2)*f*sq
rt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqr
t(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*
f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) + 3*a*b*log(I*
sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(9/2)*f*sqrt(1/b)
 + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b)
- 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(
1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) - a*b*log(I*sqrt(a)*s
qrt(1/b) + tan(e + f*x))/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(
1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqr
t(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*
f*sqrt(1/b)*tan(e + f*x)**2) + b**2*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x)
)*tan(e + f*x)**2/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*ta
n(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*
tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(
1/b)*tan(e + f*x)**2) - b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e
+ f*x)**2/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*
x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e +
f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan
(e + f*x)**2), True))
```

Giac [A] time = 1.31223, size = 171, normalized size = 1.76

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)(3\,ab-b^2)}{(a^3-2\,a^2b+ab^2)\sqrt{ab}}-\frac{2\,(fx+e)}{a^2-2\,ab+b^2}+\frac{b\tan(fx+e)}{\left(b\tan(fx+e)^2+a\right)(a^2-ab)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
)))*(3*a*b - b^2)/((a^3 - 2*a^2*b + a*b^2)*sqrt(a*b)) - 2*(f*x + e)/(a^2 -
2*a*b + b^2) + b*tan(f*x + e)/((b*tan(f*x + e)^2 + a)*(a^2 - a*b)))/f
```

3.234 $\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=128

$$\frac{b^{3/2}(5a-3b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{5/2}f(a-b)^2}-\frac{(2a-3b)\cot(e+fx)}{2a^2f(a-b)}-\frac{b\cot(e+fx)}{2af(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{x}{(a-b)^2}$$

```
[Out] -(x/(a - b)^2) + ((5*a - 3*b)*b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]
])/(2*a^(5/2)*(a - b)^2*f) - ((2*a - 3*b)*Cot[e + f*x])/(2*a^2*(a - b)*f) -
 (b*Cot[e + f*x])/(2*a*(a - b)*f*(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.191909, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 472, 583, 522, 203, 205}

$$\frac{b^{3/2}(5a-3b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{5/2}f(a-b)^2}-\frac{(2a-3b)\cot(e+fx)}{2a^2f(a-b)}-\frac{b\cot(e+fx)}{2af(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(x/(a - b)^2) + ((5*a - 3*b)*b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]
])/(2*a^(5/2)*(a - b)^2*f) - ((2*a - 3*b)*Cot[e + f*x])/(2*a^2*(a - b)*f) -
 (b*Cot[e + f*x])/(2*a*(a - b)*f*(a + b*Tan[e + f*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
```

```
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-3b-3bx^2}{x^2(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{2a(a-b)f}$$

$$= -\frac{(2a-3b)\cot(e+fx)}{2a^2(a-b)f} - \frac{b\cot(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{2a^2+2ab-3b^2+(2a-3b \ldots}{(1+x^2)(a+bx^2)} \ldots\right)}{2a^2(a- \ldots}$$

$$= -\frac{(2a-3b)\cot(e+fx)}{2a^2(a-b)f} - \frac{b\cot(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e \ldots\right.}{(a-b)^2f}$$

$$= -\frac{x}{(a-b)^2} + \frac{(5a-3b)b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{5/2}(a-b)^2f} - \frac{(2a-3b)\cot(e+fx)}{2a^2(a-b)f} - \frac{b\cot \ldots}{2a(a-b)f\left(a \ldots\right.}$$

Mathematica [A] time = 2.79902, size = 117, normalized size = 0.91

$$\frac{\frac{b^{3/2}(5a-3b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}(a-b)^2} + \frac{\frac{b^2(a-b)\sin(2(e+fx))}{a^2((a-b)\cos(2(e+fx))+a+b)} - 2(e+fx)}{(a-b)^2} - \frac{2\cot(e+fx)}{a^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (((5*a - 3*b)*b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(5/2)*(a -
 b)^2) - (2*Cot[e + f*x])/a^2 + (-2*(e + f*x) + ((a - b)*b^2*Sin[2*(e + f*x
)])/(a^2*(a + b + (a - b)*Cos[2*(e + f*x)])))/(a - b)^2)/(2*f)
```

Maple [A] time = 0.087, size = 187, normalized size = 1.5

$$-\frac{1}{fa^{2}\tan\left(fx+e\right)}+\frac{b^{2}\tan\left(fx+e\right)}{2f\left(a-b\right)^{2}a\left(a+b\left(\tan\left(fx+e\right)\right)^{2}\right)}-\frac{b^{3}\tan\left(fx+e\right)}{2fa^{2}\left(a-b\right)^{2}\left(a+b\left(\tan\left(fx+e\right)\right)^{2}\right)}+\frac{5b^{2}}{2f\left(a-b\right)^{2}a}\operatorname{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/f/a^2/tan(f*x+e)+1/2/f/(a-b)^2*b^2/a*tan(f*x+e)/(a+b*tan(f*x+e)^2)-1/2/f
*b^3/a^2/(a-b)^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)+5/2/f/(a-b)^2*b^2/a/(a*b)^(1
/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-3/2/f*b^3/a^2/(a-b)^2/(a*b)^(1/2)*arct
an(b*tan(f*x+e)/(a*b)^(1/2))-1/f/(a-b)^2*arctan(tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68502, size = 1142, normalized size = 8.92

$$\left[-\frac{8a^{2}bfx\tan\left(fx+e\right)^{3}+8a^{3}fx\tan\left(fx+e\right)+8a^{3}-16a^{2}b+8ab^{2}+4\left(2a^{2}b-5ab^{2}+3b^{3}\right)\tan\left(fx+e\right)^{2}+\left(\left(5ab^{2}-\right.\right.}{8\left(\left(a^{4}b-2a^{3}b^{2}+a^{2}b^{3}\right)f\tan\left(fx+e\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(8*a^2*b*f*x*tan(f*x + e)^3 + 8*a^3*f*x*tan(f*x + e) + 8*a^3 - 16*a^2
*b + 8*a*b^2 + 4*(2*a^2*b - 5*a*b^2 + 3*b^3)*tan(f*x + e)^2 + ((5*a*b^2 - 3
*b^3)*tan(f*x + e)^3 + (5*a^2*b - 3*a*b^2)*tan(f*x + e))*sqrt(-b/a)*log((b^
2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3 - a^2
*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2
)))/((a^4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^3 + (a^5 - 2*a^4*b + a^3*
b^2)*f*tan(f*x + e)), -1/4*(4*a^2*b*f*x*tan(f*x + e)^3 + 4*a^3*f*x*tan(f*x
+ e) + 4*a^3 - 8*a^2*b + 4*a*b^2 + 2*(2*a^2*b - 5*a*b^2 + 3*b^3)*tan(f*x +
e)^2 - ((5*a*b^2 - 3*b^3)*tan(f*x + e)^3 + (5*a^2*b - 3*a*b^2)*tan(f*x + e)
)*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/
((a^4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^3 + (a^5 - 2*a^4*b + a^3*b^2)
*f*tan(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41246, size = 231, normalized size = 1.8

$$\frac{\frac{\left(5\,ab^2-3\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{\left(a^4-2\,a^3b+a^2b^2\right)\sqrt{ab}}-\frac{2\,(fx+e)}{a^2-2\,ab+b^2}-\frac{2\,ab\tan(fx+e)^2-3\,b^2\tan(fx+e)^2+2\,a^2-2\,ab}{\left(b\tan(fx+e)^3+a\tan(fx+e)\right)\left(a^3-a^2b\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((5*a*b^2 - 3*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(
f*x + e)/sqrt(a*b)))/((a^4 - 2*a^3*b + a^2*b^2)*sqrt(a*b)) - 2*(f*x + e)/(a
^2 - 2*a*b + b^2) - (2*a*b*tan(f*x + e)^2 - 3*b^2*tan(f*x + e)^2 + 2*a^2 -
2*a*b)/((b*tan(f*x + e)^3 + a*tan(f*x + e))*(a^3 - a^2*b)))/f
```

3.235 $\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=169

$$-\frac{b^{5/2}(7a-5b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{7/2}f(a-b)^2}+\frac{\left(2a^2+2ab-5b^2\right)\cot(e+fx)}{2a^3f(a-b)}-\frac{(2a-5b)\cot^3(e+fx)}{6a^2f(a-b)}-\frac{b\cot^3(e+fx)}{2af(a-b)\left(a+b\tan^2(e\right.}$$

```
[Out] x/(a - b)^2 - ((7*a - 5*b)*b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/
(2*a^(7/2)*(a - b)^2*f) + ((2*a^2 + 2*a*b - 5*b^2)*Cot[e + f*x])/(2*a^3*(a
- b)*f) - ((2*a - 5*b)*Cot[e + f*x]^3)/(6*a^2*(a - b)*f) - (b*Cot[e + f*x]^
3)/(2*a*(a - b)*f*(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.287193, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 472, 583, 522, 203, 205}

$$-\frac{b^{5/2}(7a-5b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{7/2}f(a-b)^2}+\frac{\left(2a^2+2ab-5b^2\right)\cot(e+fx)}{2a^3f(a-b)}-\frac{(2a-5b)\cot^3(e+fx)}{6a^2f(a-b)}-\frac{b\cot^3(e+fx)}{2af(a-b)\left(a+b\tan^2(e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```


```
[Out] x/(a - b)^2 - ((7*a - 5*b)*b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/
(2*a^(7/2)*(a - b)^2*f) + ((2*a^2 + 2*a*b - 5*b^2)*Cot[e + f*x])/(2*a^3*(a
- b)*f) - ((2*a - 5*b)*Cot[e + f*x]^3)/(6*a^2*(a - b)*f) - (b*Cot[e + f*x]^
3)/(2*a*(a - b)*f*(a + b*Tan[e + f*x]^2))
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
```

```
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-5b-5bx^2}{x^4(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{2a(a-b)f}$$

$$= -\frac{(2a-5b)\cot^3(e+fx)}{6a^2(a-b)f} - \frac{b\cot^3(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{3\left(2a^2+2ab-5b^2\right)+3(}{x^2(1+x^2)(a+}\right.}{6a^2(}$$

$$= \frac{\left(2a^2+2ab-5b^2\right)\cot(e+fx)}{2a^3(a-b)f} - \frac{(2a-5b)\cot^3(e+fx)}{6a^2(a-b)f} - \frac{b\cot^3(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+f\right.}$$

$$= \frac{\left(2a^2+2ab-5b^2\right)\cot(e+fx)}{2a^3(a-b)f} - \frac{(2a-5b)\cot^3(e+fx)}{6a^2(a-b)f} - \frac{b\cot^3(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+f\right.}$$

$$= \frac{x}{(a-b)^2} - \frac{(7a-5b)b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{7/2}(a-b)^2f} + \frac{\left(2a^2+2ab-5b^2\right)\cot(e+fx)}{2a^3(a-b)f} - \frac{(2a-5}{6}$$

Mathematica [A] time = 3.0002, size = 137, normalized size = 0.81

$$\frac{\frac{3b^{5/2}(5b-7a)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}(a-b)^2} + \frac{3\left(2(e+fx)-\frac{b^3(a-b)\sin(2(e+fx))}{a^3((a-b)\cos(2(e+fx))+a+b)}\right)}{(a-b)^2} - \frac{2\cot(e+fx)\left(a\csc^2(e+fx)-4a-6b\right)}{a^3}}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((3*b^(5/2)*(-7*a + 5*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(7/2)*(
a - b)^2) - (2*Cot[e + f*x]*(-4*a - 6*b + a*Csc[e + f*x]^2))/a^3 + (3*(2*(e
 + f*x) - ((a - b)*b^3*Sin[2*(e + f*x)])/(a^3*(a + b + (a - b)*Cos[2*(e + f
*x)]))))/(a - b)^2)/(6*f)
```

Maple [A] time = 0.098, size = 218, normalized size = 1.3

$$-\frac{1}{3fa^2\left(\tan\left(fx+e\right)\right)^3}+\frac{1}{fa^2\tan\left(fx+e\right)}+2\frac{b}{fa^3\tan\left(fx+e\right)}-\frac{b^3\tan\left(fx+e\right)}{2fa^2\left(a-b\right)^2\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)}+\frac{b^4}{2fa^3\left(a-b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/3/f/a^2/tan(f*x+e)^3+1/f/a^2/tan(f*x+e)+2/f/a^3/tan(f*x+e)*b-1/2/f*b^3/a
^2/(a-b)^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)+1/2/f*b^4/a^3/(a-b)^2*tan(f*x+e)/(
a+b*tan(f*x+e)^2)-7/2/f*b^3/a^2/(a-b)^2/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*
b)^(1/2))+5/2/f*b^4/a^3/(a-b)^2/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)
)+1/f/(a-b)^2*arctan(tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.80331, size = 1335, normalized size = 7.9

$$\left[\frac{24\,a^3bfx\tan\left(fx+e\right)^5+24\,a^4fx\tan\left(fx+e\right)^3+12\left(2\,a^3b-7\,ab^3+5\,b^4\right)\tan\left(fx+e\right)^4-8\,a^4+16\,a^3b-8\,a^2b^2+8\,(3}{24\left(\left(a^5b-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/24*(24*a^3*b*f*x*tan(f*x + e)^5 + 24*a^4*f*x*tan(f*x + e)^3 + 12*(2*a^3*
b - 7*a*b^3 + 5*b^4)*tan(f*x + e)^4 - 8*a^4 + 16*a^3*b - 8*a^2*b^2 + 8*(3*a
^4 - a^3*b - 7*a^2*b^2 + 5*a*b^3)*tan(f*x + e)^2 - 3*((7*a*b^3 - 5*b^4)*tan
(f*x + e)^5 + (7*a^2*b^2 - 5*a*b^3)*tan(f*x + e)^3)*sqrt(-b/a)*log((b^2*tan
(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(
f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/(
(a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^5 + (a^6 - 2*a^5*b + a^4*b^2)*
f*tan(f*x + e)^3), 1/12*(12*a^3*b*f*x*tan(f*x + e)^5 + 12*a^4*f*x*tan(f*x +
 e)^3 + 6*(2*a^3*b - 7*a*b^3 + 5*b^4)*tan(f*x + e)^4 - 4*a^4 + 8*a^3*b - 4*
```

```
a^2*b^2 + 4*(3*a^4 - a^3*b - 7*a^2*b^2 + 5*a*b^3)*tan(f*x + e)^2 - 3*((7*a*
b^3 - 5*b^4)*tan(f*x + e)^5 + (7*a^2*b^2 - 5*a*b^3)*tan(f*x + e)^3)*sqrt(b/
a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/((a^5*b -
 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^5 + (a^6 - 2*a^5*b + a^4*b^2)*f*tan(f*
x + e)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43984, size = 242, normalized size = 1.43

$$-\frac{\frac{3\,b^3\tan\left(fx+e\right)}{\left(a^4-a^3b\right)\left(b\tan\left(fx+e\right)^2+a\right)}+\frac{3\left(7\,ab^3-5\,b^4\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)}{\left(a^5-2\,a^4b+a^3b^2\right)\sqrt{ab}}-\frac{6\left(fx+e\right)}{a^2-2\,ab+b^2}-\frac{2\left(3\,a\tan\left(fx+e\right)^2+6\,b\tan\left(fx+e\right)^2-a\right)}{a^3\tan\left(fx+e\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*b^3*tan(f*x + e)/((a^4 - a^3*b)*(b*tan(f*x + e)^2 + a)) + 3*(7*a*b^
3 - 5*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqr
t(a*b)))/((a^5 - 2*a^4*b + a^3*b^2)*sqrt(a*b)) - 6*(f*x + e)/(a^2 - 2*a*b +
 b^2) - 2*(3*a*tan(f*x + e)^2 + 6*b*tan(f*x + e)^2 - a)/(a^3*tan(f*x + e)^3
))/f
```

3.236 $\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=218

$$\frac{b^{7/2}(9a-7b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{9/2}f(a-b)^2}+\frac{\left(2a^2+2ab-7b^2\right)\cot^3(e+fx)}{6a^3f(a-b)}-\frac{\left(2a^2b+2a^3+2ab^2-7b^3\right)\cot(e+fx)}{2a^4f(a-b)}-\frac{(2a-7b)}{10a^2}$$

```
[Out] -(x/(a - b)^2) + ((9*a - 7*b)*b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]
])/(2*a^(9/2)*(a - b)^2*f) - ((2*a^3 + 2*a^2*b + 2*a*b^2 - 7*b^3)*Cot[e + f
*x])/(2*a^4*(a - b)*f) + ((2*a^2 + 2*a*b - 7*b^2)*Cot[e + f*x]^3)/(6*a^3*(a
 - b)*f) - ((2*a - 7*b)*Cot[e + f*x]^5)/(10*a^2*(a - b)*f) - (b*Cot[e + f*x
]^5)/(2*a*(a - b)*f*(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.341666, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 472, 583, 522, 203, 205}

$$\frac{b^{7/2}(9a-7b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{9/2}f(a-b)^2}+\frac{\left(2a^2+2ab-7b^2\right)\cot^3(e+fx)}{6a^3f(a-b)}-\frac{\left(2a^2b+2a^3+2ab^2-7b^3\right)\cot(e+fx)}{2a^4f(a-b)}-\frac{(2a-7b)}{10a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(x/(a - b)^2) + ((9*a - 7*b)*b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]
])/(2*a^(9/2)*(a - b)^2*f) - ((2*a^3 + 2*a^2*b + 2*a*b^2 - 7*b^3)*Cot[e + f
*x])/(2*a^4*(a - b)*f) + ((2*a^2 + 2*a*b - 7*b^2)*Cot[e + f*x]^3)/(6*a^3*(a
 - b)*f) - ((2*a - 7*b)*Cot[e + f*x]^5)/(10*a^2*(a - b)*f) - (b*Cot[e + f*x
]^5)/(2*a*(a - b)*f*(a + b*Tan[e + f*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6\left(1+x^2\right)\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-7b-7bx^2}{x^6\left(1+x^2\right)\left(a+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{2a(a-b)f}$$

$$= -\frac{(2a-7b)\cot^5(e+fx)}{10a^2(a-b)f} - \frac{b\cot^5(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{5\left(2a^2+2ab-7b^2\right)+5(}{x^4\left(1+x^2\right)\left(a+\right.}\right.}{10a^2(}$$

$$= \frac{\left(2a^2+2ab-7b^2\right)\cot^3(e+fx)}{6a^3(a-b)f} - \frac{(2a-7b)\cot^5(e+fx)}{10a^2(a-b)f} - \frac{b\cot^5(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+\right.}$$

$$= -\frac{\left(2a^3+2a^2b+2ab^2-7b^3\right)\cot(e+fx)}{2a^4(a-b)f} + \frac{\left(2a^2+2ab-7b^2\right)\cot^3(e+fx)}{6a^3(a-b)f} - \frac{(2a-7b)}{10a^2}$$

$$= -\frac{\left(2a^3+2a^2b+2ab^2-7b^3\right)\cot(e+fx)}{2a^4(a-b)f} + \frac{\left(2a^2+2ab-7b^2\right)\cot^3(e+fx)}{6a^3(a-b)f} - \frac{(2a-7b)}{10a^2}$$

$$= -\frac{x}{(a-b)^2} + \frac{(9a-7b)b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{9/2}(a-b)^2f} - \frac{\left(2a^3+2a^2b+2ab^2-7b^3\right)\cot(e+fx)}{2a^4(a-b)f}$$

Mathematica [A] time = 5.02335, size = 165, normalized size = 0.76

$$\frac{\frac{15b^{7/2}(9a-7b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{9/2}(a-b)^2} + \frac{15\left(\frac{b^4(a-b)\sin(2(e+fx))}{a^4((a-b)\cos(2(e+fx))+a+b)}-2(e+fx)\right)}{(a-b)^2} - \frac{2\cot(e+fx)\left(3a^2\csc^4(e+fx)+23a^2-a(11a+10b)\csc^2(e+fx)+40ab+45b^2\right)}{a^4}}{30f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((15*(9*a - 7*b)*b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(9/2)*(
a - b)^2) - (2*Cot[e + f*x]*(23*a^2 + 40*a*b + 45*b^2 - a*(11*a + 10*b)*Csc
[e + f*x]^2 + 3*a^2*Csc[e + f*x]^4))/a^4 + (15*(-2*(e + f*x) + ((a - b)*b^4
*Sin[2*(e + f*x)])/(a^4*(a + b + (a - b)*Cos[2*(e + f*x)]))))/(a - b)^2)/(3
0*f)
```

Maple [A] time = 0.102, size = 272, normalized size = 1.3

$$-\frac{1}{5\,fa^2\left(\tan\left(fx+e\right)\right)^5}+\frac{1}{3\,fa^2\left(\tan\left(fx+e\right)\right)^3}+\frac{2\,b}{3\,fa^3\left(\tan\left(fx+e\right)\right)^3}-\frac{1}{fa^2\tan\left(fx+e\right)}-2\,\frac{b}{fa^3\tan\left(fx+e\right)}-3\,\frac{}{fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/5/f/a^2/tan(f*x+e)^5+1/3/f/a^2/tan(f*x+e)^3+2/3/f/a^3/tan(f*x+e)^3*b-1/f
/a^2/tan(f*x+e)-2/f/a^3/tan(f*x+e)*b-3/f/a^4/tan(f*x+e)*b^2+1/2/f*b^4/a^3/(
a-b)^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)-1/2/f*b^5/a^4/(a-b)^2*tan(f*x+e)/(a+b*
tan(f*x+e)^2)+9/2/f*b^4/a^3/(a-b)^2/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(
1/2))-7/2/f*b^5/a^4/(a-b)^2/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-1/
f/(a-b)^2*arctan(tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.92831, size = 1520, normalized size = 6.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/120*(120*a^4*b*f*x*tan(f*x + e)^7 + 120*a^5*f*x*tan(f*x + e)^5 + 60*(2*
a^4*b - 9*a*b^4 + 7*b^5)*tan(f*x + e)^6 + 24*a^5 - 48*a^4*b + 24*a^3*b^2 +
40*(3*a^5 - a^4*b - 9*a^2*b^3 + 7*a*b^4)*tan(f*x + e)^4 - 8*(5*a^5 - 3*a^4*
b - 9*a^3*b^2 + 7*a^2*b^3)*tan(f*x + e)^2 + 15*((9*a*b^4 - 7*b^5)*tan(f*x +
 e)^7 + (9*a^2*b^3 - 7*a*b^4)*tan(f*x + e)^5)*sqrt(-b/a)*log((b^2*tan(f*x +
 e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x +
```

```
e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/((a^6*b
 - 2*a^5*b^2 + a^4*b^3)*f*tan(f*x + e)^7 + (a^7 - 2*a^6*b + a^5*b^2)*f*tan(
f*x + e)^5), -1/60*(60*a^4*b*f*x*tan(f*x + e)^7 + 60*a^5*f*x*tan(f*x + e)^5
 + 30*(2*a^4*b - 9*a*b^4 + 7*b^5)*tan(f*x + e)^6 + 12*a^5 - 24*a^4*b + 12*a
^3*b^2 + 20*(3*a^5 - a^4*b - 9*a^2*b^3 + 7*a*b^4)*tan(f*x + e)^4 - 4*(5*a^5
 - 3*a^4*b - 9*a^3*b^2 + 7*a^2*b^3)*tan(f*x + e)^2 - 15*((9*a*b^4 - 7*b^5)*
tan(f*x + e)^7 + (9*a^2*b^3 - 7*a*b^4)*tan(f*x + e)^5)*sqrt(b/a)*arctan(1/2
*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/((a^6*b - 2*a^5*b^2 +
a^4*b^3)*f*tan(f*x + e)^7 + (a^7 - 2*a^6*b + a^5*b^2)*f*tan(f*x + e)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.44007, size = 304, normalized size = 1.39

$$\frac{\frac{15\,b^4\tan\left(fx+e\right)}{\left(a^5-a^4b\right)\left(b\tan\left(fx+e\right)^2+a\right)}+\frac{15\left(9\,ab^4-7\,b^5\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)}{\left(a^6-2\,a^5b+a^4b^2\right)\sqrt{ab}}-\frac{30\left(fx+e\right)}{a^2-2\,ab+b^2}-\frac{2\left(15\,a^2\tan\left(fx+e\right)^4+30\,ab\tan\left(fx+e\right)^4+45\,b^2}{a^4}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/30*(15*b^4*tan(f*x + e)/((a^5 - a^4*b)*(b*tan(f*x + e)^2 + a)) + 15*(9*a*
b^4 - 7*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/s
qrt(a*b)))/((a^6 - 2*a^5*b + a^4*b^2)*sqrt(a*b)) - 30*(f*x + e)/(a^2 - 2*a*
b + b^2) - 2*(15*a^2*tan(f*x + e)^4 + 30*a*b*tan(f*x + e)^4 + 45*b^2*tan(f*
x + e)^4 - 5*a^2*tan(f*x + e)^2 - 10*a*b*tan(f*x + e)^2 + 3*a^2)/(a^4*tan(f
*x + e)^5))/f
```

3.237 $\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3} dx$

Optimal. Leaf size=108

$$\frac{a^2}{4b^2f(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{a(a-2b)}{2b^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^3}$$

```
[Out] -Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/(2*(a - b)^3*f) + a^2/(4*(a - b)*
b^2*f*(a + b*Tan[e + f*x]^2)^2) - (a*(a - 2*b))/(2*(a - b)^2*b^2*f*(a + b*T
an[e + f*x]^2))
```

Rubi [A] time = 0.152464, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 88}

$$\frac{a^2}{4b^2f(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{a(a-2b)}{2b^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/(2*(a - b)^3*f) + a^2/(4*(a - b)*
b^2*f*(a + b*Tan[e + f*x]^2)^2) - (a*(a - 2*b))/(2*(a - b)^2*b^2*f*(a + b*T
an[e + f*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)(a+bx)^3}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a-b)^3(1+x)} - \frac{a^2}{(a-b)b(a+bx)^3} + \frac{a(a-2b)}{(a-b)^2b(a+bx)^2} + \frac{b}{(-a+b)^3(a+bx)}\right) dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)^3f} + \frac{a^2}{4(a-b)b^2f\left(a+b\tan^2(e+fx)\right)^2} - \frac{}{2(a-b)^2}$$

Mathematica [A] time = 1.0693, size = 97, normalized size = 0.9

$$\frac{\frac{a^2(a-b)^2}{b^2\left(a+b\tan^2(e+fx)\right)^2} - \frac{2a(a-2b)(a-b)}{b^2\left(a+b\tan^2(e+fx)\right)} - 2\log\left(a+b\tan^2(e+fx)\right) - 4\log(\cos(e+fx))}{4f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-4*Log[Cos[e + f*x]] - 2*Log[a + b*Tan[e + f*x]^2] + (a^2*(a - b)^2)/(b^2*
(a + b*Tan[e + f*x]^2)^2) - (2*a*(a - 2*b)*(a - b))/(b^2*(a + b*Tan[e + f*x
]^2)))/(4*(a - b)^3*f)
```

Maple [B] time = 0.026, size = 234, normalized size = 2.2

$$-\frac{\ln\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)}{2f(a-b)^3} + \frac{a^4}{4f(a-b)^3b^2\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^2} - \frac{a^3}{2f(a-b)^3b\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^2} + \frac{}{4f(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -1/2/f/(a-b)^3*ln(a+b*tan(f*x+e)^2)+1/4/f/(a-b)^3*a^4/b^2/(a+b*tan(f*x+e)^2
)^2-1/2/f/(a-b)^3*a^3/b/(a+b*tan(f*x+e)^2)^2+1/4/f/(a-b)^3*a^2/(a+b*tan(f*x
+e)^2)^2-1/2/f/(a-b)^3*a^3/b^2/(a+b*tan(f*x+e)^2)+3/2/f/(a-b)^3*a^2/b/(a+b*
tan(f*x+e)^2)-1/f/(a-b)^3*a/(a+b*tan(f*x+e)^2)+1/2/f/(a-b)^3*ln(1+tan(f*x+e
)^2)
```

Maxima [A] time = 1.12832, size = 255, normalized size = 2.36

$$\frac{\frac{4\left(a^2-ab\right)\sin\left(fx+e\right)^2-3a^2}{a^5-3a^4b+3a^3b^2-a^2b^3+\left(a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5\right)\sin\left(fx+e\right)^4-2\left(a^5-4a^4b+6a^3b^2-4a^2b^3+ab^4\right)\sin\left(fx+e\right)^2} - \frac{2\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{a^3-3a^2b+3ab^2-b^3}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((4*(a^2 - a*b)*sin(f*x + e)^2 - 3*a^2)/(a^5 - 3*a^4*b + 3*a^3*b^2 - a^
2*b^3 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*sin(f*x +
 e)^4 - 2*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*sin(f*x + e)^2) -
 2*log(-(a - b)*sin(f*x + e)^2 + a)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

Fricas [B] time = 1.56294, size = 459, normalized size = 4.25

$$\frac{\left(a^2-4\,ab\right)\tan\left(fx+e\right)^4-2\left(a^2+2\,ab\right)\tan\left(fx+e\right)^2-3\,a^2-2\left(b^2\tan\left(fx+e\right)^4+2\,ab\tan\left(fx+e\right)^2+a^2\right)\log\left(\frac{b\tan(f\cdots}{\tan(fx\cdots}\right.}{4\left(\left(a^3b^2-3\,a^2b^3+3\,ab^4-b^5\right)f\tan\left(fx+e\right)^4+2\left(a^4b-3\,a^3b^2+3\,a^2b^3-ab^4\right)f\tan\left(fx+e\right)^2+\left(a^5-3\,a^4b+3\,a^3b^2-\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((a^2 - 4*a*b)*tan(f*x + e)^4 - 2*(a^2 + 2*a*b)*tan(f*x + e)^2 - 3*a^2
- 2*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*log((b*tan(f*x + e)^2
 + a)/(tan(f*x + e)^2 + 1)))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*tan(f
*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*tan(f*x + e)^2 + (a
^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)
```

Sympy [A] time = 173.463, size = 3225, normalized size = 29.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Piecewise((zoo*x/tan(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((log(tan(e + f*x
)**2 + 1)/(2*f) + tan(e + f*x)**4/(4*f) - tan(e + f*x)**2/(2*f))/a**3, Eq(b
, 0)), (tan(e + f*x)**6/(6*b**3*f*tan(e + f*x)**6 + 18*b**3*f*tan(e + f*x)*
*4 + 18*b**3*f*tan(e + f*x)**2 + 6*b**3*f), Eq(a, b)), (x*tan(e)**5/(a + b*
tan(e)**2)**3, Eq(f, 0)), (-a**4/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x
)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(
e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**
5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b**
6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x)**4) - 2*a**3*b*tan(e + f*x)**2/
(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b*
*4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*a**3*b**4*f - 12
*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5
*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*ta
n(e + f*x)**4) + 4*a**3*b/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x)**2 -
12*a**4*b**3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x
)**2 + 12*a**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan
(e + f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan
(e + f*x)**2 - 4*b**7*f*tan(e + f*x)**4) - 2*a**2*b**2*log(-I*sqrt(a)*sqrt(
1/b) + tan(e + f*x))/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a*
*4*b**3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**2
+ 12*a**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e +
f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e +
```

```
f*x)**2 - 4*b**7*f*tan(e + f*x)**4) - 2*a**2*b**2*log(I*sqrt(a)*sqrt(1/b) +
 tan(e + f*x))/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b**
3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*a
**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e + f*x)**
2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e + f*x)**
2 - 4*b**7*f*tan(e + f*x)**4) + 2*a**2*b**2*log(tan(e + f*x)**2 + 1)/(4*a**
5*b**2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*t
an(e + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*
b**5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 1
2*a*b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e +
f*x)**4) + 6*a**2*b**2*tan(e + f*x)**2/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e
 + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*
f*tan(e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a*
*2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8
*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x)**4) - 3*a**2*b**2/(4*a**5
*b**2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*ta
n(e + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b
**5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12
*a*b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e + f
*x)**4) - 4*a*b**3*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2
/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b
**4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*a**3*b**4*f - 1
2*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**
5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*t
an(e + f*x)**4) - 4*a*b**3*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e +
f*x)**2/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b**3*f + 4
*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*a**3*b**
4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e + f*x)**2 - 4*a
**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e + f*x)**2 - 4*b
**7*f*tan(e + f*x)**4) + 4*a*b**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/
(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b*
*4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*a**3*b**4*f - 12
*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5
*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*ta
n(e + f*x)**4) - 4*a*b**3*tan(e + f*x)**2/(4*a**5*b**2*f + 8*a**4*b**3*f*ta
n(e + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b*
*4*f*tan(e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24
*a**2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4
- 8*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x)**4) - 2*b**4*log(-I*sq
rt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**4/(4*a**5*b**2*f + 8*a**4*b**
3*f*tan(e + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a
**3*b**4*f*tan(e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**
4 + 24*a**2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*
x)**4 - 8*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x)**4) - 2*b**4*log
(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**4/(4*a**5*b**2*f + 8*a**
4*b**3*f*tan(e + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e + f*x)**4 -
 24*a**3*b**4*f*tan(e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5*f*tan(e + f
*x)**4 + 24*a**2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e
 + f*x)**4 - 8*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x)**4) + 2*b**
4*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**4/(4*a**5*b**2*f + 8*a**4*b**3*f*t
an(e + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b
**4*f*tan(e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 2
4*a**2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4
 - 8*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x)**4), True))
```

Giac [B] time = 3.33438, size = 632, normalized size = 5.85

$$-\frac{\frac{2\log\left(a+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^3-3a^2b+3ab^2-b^3}-\frac{4\log\left(-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right)}{a^3-3a^2b+3ab^2-b^3}-\frac{3a^2+\frac{20a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{32ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{50a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\cdots}{(a^3-3a^2b+3ab^2-b^3)\cdots}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*log(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x +
e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(
a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 4*log(-(cos(f*x + e) - 1)/(cos(f*x + e) +
1) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (3*a^2 + 20*a^2*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) - 32*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 50*a^
2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 128*a*b*(cos(f*x + e) - 1)^2/
(cos(f*x + e) + 1)^2 + 96*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 2
0*a^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 32*a*b*(cos(f*x + e) - 1)
^3/(cos(f*x + e) + 1)^3 + 3*a^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/
((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e)
+ 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(
cos(f*x + e) + 1)^2)^2))/f
```

3.238 $\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=97

$$-\frac{a}{4bf(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{1}{2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^3}$$

```
[Out] Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/(2*(a - b)^3*f) - a/(4*(a - b)*b*f
*(a + b*Tan[e + f*x]^2)^2) - 1/(2*(a - b)^2*f*(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.11735, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 77}

$$-\frac{a}{4bf(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{1}{2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/(2*(a - b)^3*f) - a/(4*(a - b)*b*f
*(a + b*Tan[e + f*x]^2)^2) - 1/(2*(a - b)^2*f*(a + b*Tan[e + f*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{(1+x)(a+bx)^3}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a-b)^3(1+x)} + \frac{a}{(a-b)(a+bx)^3} + \frac{b}{(a-b)^2(a+bx)^2} + \frac{b}{(a-b)^3(a+bx)}\right) dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)^3f} - \frac{a}{4(a-b)bf\left(a+b\tan^2(e+fx)\right)^2} - \frac{\quad}{2(a-b)^2f\left(a+\right.}$$

Mathematica [A] time = 0.749997, size = 87, normalized size = 0.9

$$\frac{-\frac{a(a-b)^2}{b\left(a+b\tan^2(e+fx)\right)^2} - \frac{2(a-b)}{a+b\tan^2(e+fx)} + 2\log\left(a+b\tan^2(e+fx)\right) + 4\log(\cos(e+fx))}{4f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (4*Log[Cos[e + f*x]] + 2*Log[a + b*Tan[e + f*x]^2] - (a*(a - b)^2)/(b*(a +
b*Tan[e + f*x]^2)^2) - (2*(a - b))/(a + b*Tan[e + f*x]^2))/(4*(a - b)^3*f)
```

Maple [B] time = 0.023, size = 193, normalized size = 2.

$$\frac{\ln\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)}{2f(a-b)^3} - \frac{a^3}{4f(a-b)^3b\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^2} + \frac{a^2}{2f(a-b)^3\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^2} - \frac{\quad}{4f(a-b)^3\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] 1/2/f/(a-b)^3*ln(a+b*tan(f*x+e)^2)-1/4/f/(a-b)^3*a^3/b/(a+b*tan(f*x+e)^2)^2
+1/2/f/(a-b)^3*a^2/(a+b*tan(f*x+e)^2)^2-1/4/f/(a-b)^3*a*b/(a+b*tan(f*x+e)^2
)^2-1/2/f/(a-b)^3*a/(a+b*tan(f*x+e)^2)+1/2/f/(a-b)^3/(a+b*tan(f*x+e)^2)*b-1
/2/f/(a-b)^3*ln(1+tan(f*x+e)^2)
```

Maxima [B] time = 1.17617, size = 262, normalized size = 2.7

$$-\frac{\frac{2\left(a^2-b^2\right)\sin\left(fx+e\right)^2-2a^2-ab}{a^5-3a^4b+3a^3b^2-a^2b^3+\left(a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5\right)\sin\left(fx+e\right)^4-2\left(a^5-4a^4b+6a^3b^2-4a^2b^3+ab^4\right)\sin\left(fx+e\right)^2} - \frac{2\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{a^3-3a^2b+3ab^2-b^3}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((2*(a^2 - b^2)*sin(f*x + e)^2 - 2*a^2 - a*b)/(a^5 - 3*a^4*b + 3*a^3*b
^2 - a^2*b^3 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*si
n(f*x + e)^4 - 2*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*sin(f*x +
e)^2) - 2*log(-(a - b)*sin(f*x + e)^2 + a)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))
/f
```

Fricas [B] time = 1.57221, size = 473, normalized size = 4.88

$$\frac{(ab+2b^2)\tan(fx+e)^4+2(a^2+ab+b^2)\tan(fx+e)^2+2a^2+ab+2\left(b^2\tan(fx+e)^4+2ab\tan(fx+e)^2+a^2\right.}{4\left((a^3b^2-3a^2b^3+3ab^4-b^5)f\tan(fx+e)^4+2(a^4b-3a^3b^2+3a^2b^3-ab^4)f\tan(fx+e)^2+(a^5-3a^4b+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((a*b + 2*b^2)*tan(f*x + e)^4 + 2*(a^2 + a*b + b^2)*tan(f*x + e)^2 + 2*
a^2 + a*b + 2*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*log((b*tan(
f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^
5)*f*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*tan(f*x +
 e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)
```

Sympy [A] time = 170.769, size = 2849, normalized size = 29.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Piecewise((zoo*x/tan(e)**3, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (-3*tan(e + f*
x)**2/(12*b**3*f*tan(e + f*x)**6 + 36*b**3*f*tan(e + f*x)**4 + 36*b**3*f*ta
n(e + f*x)**2 + 12*b**3*f) - 1/(12*b**3*f*tan(e + f*x)**6 + 36*b**3*f*tan(e
 + f*x)**4 + 36*b**3*f*tan(e + f*x)**2 + 12*b**3*f), Eq(a, b)), ((-log(tan(
e + f*x)**2 + 1)/(2*f) + tan(e + f*x)**2/(2*f))/a**3, Eq(b, 0)), (x*tan(e)*
*3/(a + b*tan(e)**2)**3, Eq(f, 0)), (-a**3/(4*a**5*b*f + 8*a**4*b**2*f*tan(
e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 24*a**3*b**3
*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x)**4 + 24*a
**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e + f*x)**4 -
8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**4) + 2*a**2*b*log(-I*sq
rt(a)*sqrt(1/b) + tan(e + f*x))/(4*a**5*b*f + 8*a**4*b**2*f*tan(e + f*x)**2
 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 24*a**3*b**3*f*tan(e +
f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x)**4 + 24*a**2*b**4*f*
tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e + f*x)**4 - 8*a*b**5*f*
tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**4) + 2*a**2*b*log(I*sqrt(a)*sqrt(1
/b) + tan(e + f*x))/(4*a**5*b*f + 8*a**4*b**2*f*tan(e + f*x)**2 - 12*a**4*b
**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 24*a**3*b**3*f*tan(e + f*x)**2 + 12
*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x)**4 + 24*a**2*b**4*f*tan(e + f*x)
**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e + f*x)**4 - 8*a*b**5*f*tan(e + f*x)
**2 - 4*b**6*f*tan(e + f*x)**4) - 2*a**2*b*log(tan(e + f*x)**2 + 1)/(4*a**5
*b*f + 8*a**4*b**2*f*tan(e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e
 + f*x)**4 - 24*a**3*b**3*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**4
*f*tan(e + f*x)**4 + 24*a**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a*
```

```
b**5*f*tan(e + f*x)**4 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x)
**4) + 4*a*b**2*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4
*a**5*b*f + 8*a**4*b**2*f*tan(e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**3*f*
tan(e + f*x)**4 - 24*a**3*b**3*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*a**2
*b**4*f*tan(e + f*x)**4 + 24*a**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*f +
12*a*b**5*f*tan(e + f*x)**4 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e +
 f*x)**4) + 4*a*b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**
2/(4*a**5*b*f + 8*a**4*b**2*f*tan(e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**
3*f*tan(e + f*x)**4 - 24*a**3*b**3*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*
a**2*b**4*f*tan(e + f*x)**4 + 24*a**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*
f + 12*a*b**5*f*tan(e + f*x)**4 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan
(e + f*x)**4) - 4*a*b**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(4*a**5*b
*f + 8*a**4*b**2*f*tan(e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e +
 f*x)**4 - 24*a**3*b**3*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f
*tan(e + f*x)**4 + 24*a**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b*
*5*f*tan(e + f*x)**4 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**
4) - 2*a*b**2*tan(e + f*x)**2/(4*a**5*b*f + 8*a**4*b**2*f*tan(e + f*x)**2 -
 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 24*a**3*b**3*f*tan(e + f*
x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x)**4 + 24*a**2*b**4*f*ta
n(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e + f*x)**4 - 8*a*b**5*f*ta
n(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**4) + a*b**2/(4*a**5*b*f + 8*a**4*b**
2*f*tan(e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 24*a
**3*b**3*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x)**
4 + 24*a**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e + f*
x)**4 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**4) + 2*b**3*log
(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**4/(4*a**5*b*f + 8*a**4*
b**2*f*tan(e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 2
4*a**3*b**3*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x
)**4 + 24*a**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e +
 f*x)**4 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**4) + 2*b**3*
log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**4/(4*a**5*b*f + 8*a**
4*b**2*f*tan(e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 -
 24*a**3*b**3*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f
*x)**4 + 24*a**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e
 + f*x)**4 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**4) - 2*b**
3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**4/(4*a**5*b*f + 8*a**4*b**2*f*tan(
e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 24*a**3*b**3
*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x)**4 + 24*a
**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e + f*x)**4 -
8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**4) + 2*b**3*tan(e + f*x
)**2/(4*a**5*b*f + 8*a**4*b**2*f*tan(e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*
b**3*f*tan(e + f*x)**4 - 24*a**3*b**3*f*tan(e + f*x)**2 + 12*a**3*b**3*f -
12*a**2*b**4*f*tan(e + f*x)**4 + 24*a**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b*
*4*f + 12*a*b**5*f*tan(e + f*x)**4 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*
tan(e + f*x)**4), True))
```

Giac [B] time = 2.00028, size = 682, normalized size = 7.03

$$\frac{\frac{2\,\log\left(a+\frac{2\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{4\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{a\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}-\frac{4\,\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1\right)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}-\frac{3\,a^{3}+\frac{20\,a^{3}\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{32\,a^{2}b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{34\,a^{3}\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\cdots}{\left(a^{4}-3\,a^{3}b+\cdots\right)}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*log(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e
) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a
^3 - 3*a^2*b + 3*a*b^2 - b^3) - 4*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1
) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (3*a^3 + 20*a^3*(cos(f*x + e) - 1)
/(cos(f*x + e) + 1) - 32*a^2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 34*a
^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 80*a^2*b*(cos(f*x + e) - 1)^
2/(cos(f*x + e) + 1)^2 + 48*a*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
 + 16*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 20*a^3*(cos(f*x + e)
- 1)^3/(cos(f*x + e) + 1)^3 - 32*a^2*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) +
 1)^3 + 3*a^3*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((a^4 - 3*a^3*b +
3*a^2*b^2 - a*b^3)*(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2)^2))/f
```

3.239 $\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=93

$$\frac{1}{2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{1}{4f(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^3}$$

```
[Out] -Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/(2*(a - b)^3*f) + 1/(4*(a - b)*f*
(a + b*Tan[e + f*x]^2)^2) + 1/(2*(a - b)^2*f*(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.0881894, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3670, 444, 44}

$$\frac{1}{2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{1}{4f(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/(2*(a - b)^3*f) + 1/(4*(a - b)*f*
(a + b*Tan[e + f*x]^2)^2) + 1/(2*(a - b)^2*f*(a + b*Tan[e + f*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule s steps

$$\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^3}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a-b)^3(1+x)} - \frac{b}{(a-b)(a+bx)^3} - \frac{b}{(a-b)^2(a+bx)^2} + \frac{b}{(-a+b)^3(a+bx)}\right) dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= -\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)^3 f} + \frac{1}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\ }{2(a-b)^2 f}$$

Mathematica [A] time = 0.65667, size = 82, normalized size = 0.88

$$\frac{\frac{(a-b)^2}{\left(a+b\tan^2(e+fx)\right)^2} + \frac{2(a-b)}{a+b\tan^2(e+fx)} - 2\log\left(a+b\tan^2(e+fx)\right) - 4\log(\cos(e+fx))}{4f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-4*Log[Cos[e + f*x]] - 2*Log[a + b*Tan[e + f*x]^2] + (a - b)^2/(a + b*Tan[
e + f*x]^2)^2 + (2*(a - b))/(a + b*Tan[e + f*x]^2))/(4*(a - b)^3*f)
```

Maple [B] time = 0.026, size = 190, normalized size = 2.

$$-\frac{\ln\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)}{2f(a-b)^3} + \frac{a^2}{4f(a-b)^3\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^2} - \frac{ab}{2f(a-b)^3\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^2} + \frac{\ }{4f(a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -1/2/f/(a-b)^3*ln(a+b*tan(f*x+e)^2)+1/4/f/(a-b)^3*a^2/(a+b*tan(f*x+e)^2)^2-
1/2/f/(a-b)^3*a*b/(a+b*tan(f*x+e)^2)^2+1/4/f*b^2/(a-b)^3/(a+b*tan(f*x+e)^2)
^2+1/2/f/(a-b)^3*a/(a+b*tan(f*x+e)^2)-1/2/f/(a-b)^3/(a+b*tan(f*x+e)^2)*b+1/
2/f/(a-b)^3*ln(1+tan(f*x+e)^2)
```

Maxima [B] time = 1.1013, size = 259, normalized size = 2.78

$$\frac{\frac{4\left(ab-b^2\right)\sin\left(fx+e\right)^2-4\,ab+b^2}{a^5-3\,a^4b+3\,a^3b^2-a^2b^3+\left(a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5\right)\sin\left(fx+e\right)^4-2\left(a^5-4\,a^4b+6\,a^3b^2-4\,a^2b^3+ab^4\right)\sin\left(fx+e\right)^2} - \frac{2\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{a^3-3\,a^2b+3\,ab^2-b^3}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((4*(a*b - b^2)*sin(f*x + e)^2 - 4*a*b + b^2)/(a^5 - 3*a^4*b + 3*a^3*b^
2 - a^2*b^3 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*sin
(f*x + e)^4 - 2*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*sin(f*x + e
)^2) - 2*log(-(a - b)*sin(f*x + e)^2 + a)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/
f
```

Fricas [B] time = 1.58334, size = 458, normalized size = 4.92

$$-\frac{3\,b^2\tan\left(fx+e\right)^4+2\left(2\,ab+b^2\right)\tan\left(fx+e\right)^2+4\,ab-b^2+2\left(b^2\tan\left(fx+e\right)^4+2\,ab\tan\left(fx+e\right)^2+a^2\right)\log\left(\frac{b\tan(f}{\tan(fx}\right.}{4\left(\left(a^3b^2-3\,a^2b^3+3\,ab^4-b^5\right)f\tan\left(fx+e\right)^4+2\left(a^4b-3\,a^3b^2+3\,a^2b^3-ab^4\right)f\tan\left(fx+e\right)^2+\left(a^5-3\,a^4b+3\,a^3b^2-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(3*b^2*tan(f*x + e)^4 + 2*(2*a*b + b^2)*tan(f*x + e)^2 + 4*a*b - b^2 +
 2*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*log((b*tan(f*x + e)^2
+ a)/(tan(f*x + e)^2 + 1)))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*tan(f*
x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*tan(f*x + e)^2 + (a^
5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)
```

Sympy [A] time = 171.352, size = 2876, normalized size = 30.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Piecewise((zoo*x/tan(e)**5, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (log(tan(e + f
*x)**2 + 1)/(2*a**3*f), Eq(b, 0)), (-1/(6*b**3*f*tan(e + f*x)**6 + 18*b**3*
f*tan(e + f*x)**4 + 18*b**3*f*tan(e + f*x)**2 + 6*b**3*f), Eq(a, b)), (x*ta
n(e)/(a + b*tan(e)**2)**3, Eq(f, 0)), (-2*a**2*log(-I*sqrt(a)*sqrt(1/b) + t
an(e + f*x))/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**3*
b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f -
12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b*
*3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*f*
tan(e + f*x)**4) - 2*a**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*a**5*f
 + 8*a**4*b*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4
 - 24*a**3*b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e +
 f*x)**4 + 24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan
(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) + 2*a
**2*log(tan(e + f*x)**2 + 1)/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 - 12*a*
*4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)**2 + 1
2*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e + f*x
)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x
)**2 - 4*b**5*f*tan(e + f*x)**4) + 3*a**2/(4*a**5*f + 8*a**4*b*f*tan(e + f*
x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e
+ f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*
f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*
f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) - 4*a*b*log(-I*sqrt(a)*sqrt(1
/b) + tan(e + f*x))*tan(e + f*x)**2/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2
```

```
- 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)
**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(
e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(
e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) - 4*a*b*log(I*sqrt(a)*sqrt(1/b) + t
an(e + f*x))*tan(e + f*x)**2/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 - 12*a*
*4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)**2 + 1
2*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e + f*x
)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x
)**2 - 4*b**5*f*tan(e + f*x)**4) + 4*a*b*log(tan(e + f*x)**2 + 1)*tan(e + f
*x)**2/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**3*b**2*f
*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f - 12*a**
2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f +
 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*f*tan(e
+ f*x)**4) + 2*a*b*tan(e + f*x)**2/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 -
 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)*
*2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e
 + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e
 + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) - 4*a*b/(4*a**5*f + 8*a**4*b*f*tan(e
 + f*x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*t
an(e + f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*
b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*
b**4*f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) - 2*b**2*log(-I*sqrt(a)*
sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**4/(4*a**5*f + 8*a**4*b*f*tan(e + f*
x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e
+ f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*
f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*
f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) - 2*b**2*log(I*sqrt(a)*sqrt(1
/b) + tan(e + f*x))*tan(e + f*x)**4/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2
- 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)
**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(
e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(
e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) + 2*b**2*log(tan(e + f*x)**2 + 1)*t
an(e + f*x)**4/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**
3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f
- 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*
b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*
f*tan(e + f*x)**4) - 2*b**2*tan(e + f*x)**2/(4*a**5*f + 8*a**4*b*f*tan(e +
f*x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(
e + f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**
3*f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**
4*f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) + b**2/(4*a**5*f + 8*a**4*b
*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*
b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 +
24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**
4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4), True))
```

Giac [B] time = 1.68757, size = 876, normalized size = 9.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*log(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x +
e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(
a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 4*log(-(cos(f*x + e) - 1)/(cos(f*x + e) +
```

```
1) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (3*a^4 + 12*a^4*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) - 8*a^3*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 24*a
^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 8*a*b^3*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) + 18*a^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 16*a
^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 48*a^2*b^2*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2 + 80*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
1)^2 - 16*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 12*a^4*(cos(f*x +
 e) - 1)^3/(cos(f*x + e) + 1)^3 - 8*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e
) + 1)^3 - 24*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 8*a*b^3*(
cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 3*a^4*(cos(f*x + e) - 1)^4/(cos(
f*x + e) + 1)^4)/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*(a + 2*a*(cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a
*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)^2))/f
```

3.240 $\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=148

$$\frac{b\left(3a^2-3ab+b^2\right)\log\left(a+b\tan^2(e+fx)\right)}{2a^3f(a-b)^3}-\frac{b(2a-b)}{2a^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{\log(\tan(e+fx))}{a^3f}-\frac{}{4af(a-b)\left(a+\right.}$$

```
[Out] Log[Cos[e + f*x]]/((a - b)^3*f) + Log[Tan[e + f*x]]/(a^3*f) + (b*(3*a^2 - 3
*a*b + b^2)*Log[a + b*Tan[e + f*x]^2])/(2*a^3*(a - b)^3*f) - b/(4*a*(a - b)
*f*(a + b*Tan[e + f*x]^2)^2) - ((2*a - b)*b)/(2*a^2*(a - b)^2*f*(a + b*Tan[
e + f*x]^2))
```

Rubi [A] time = 0.165032, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3670, 446, 72}

$$\frac{b\left(3a^2-3ab+b^2\right)\log\left(a+b\tan^2(e+fx)\right)}{2a^3f(a-b)^3}-\frac{b(2a-b)}{2a^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{\log(\tan(e+fx))}{a^3f}-\frac{}{4af(a-b)\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```


```
[Out] Log[Cos[e + f*x]]/((a - b)^3*f) + Log[Tan[e + f*x]]/(a^3*f) + (b*(3*a^2 - 3
*a*b + b^2)*Log[a + b*Tan[e + f*x]^2])/(2*a^3*(a - b)^3*f) - b/(4*a*(a - b)
*f*(a + b*Tan[e + f*x]^2)^2) - ((2*a - b)*b)/(2*a^2*(a - b)^2*f*(a + b*Tan[
e + f*x]^2))
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(1+x^2)\left(a+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x(1+x)(a+bx)^3}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x} - \frac{1}{(a-b)^3(1+x)} + \frac{b^2}{a(a-b)(a+bx)^3} + \frac{(2a-b)b^2}{a^2(a-b)^2(a+bx)^2} + \frac{b^2\left(3a^2-3ab+b^2\right)}{a^3(a-b)^3(a+bx)}\right) dx, x, \tan^2(e+ \ldots}{2f}$$
$$= \frac{\log(\cos(e+fx))}{(a-b)^3 f} + \frac{\log(\tan(e+fx))}{a^3 f} + \frac{b\left(3a^2-3ab+b^2\right)\log\left(a+b\tan^2(e+fx)\right)}{2a^3(a-b)^3 f} - \frac{\ldots}{4a(\ldots}$$

Mathematica [A] time = 1.6598, size = 126, normalized size = 0.85

$$\frac{\frac{\frac{b\left(2\left(3a^2-3ab+b^2\right)\log\left(a+b\tan^2(e+fx)\right)-\frac{a(a-b)\left(2b(2a-b)\tan^2(e+fx)+a(5a-3b)\right)}{\left(a+b\tan^2(e+fx)\right)^2}\right)}{(a-b)^3}+4\log(\tan(e+fx))}{a^3}+\frac{4\log(\cos(e+fx))}{(a-b)^3}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((4*Log[Cos[e + f*x]])/(a - b)^3 + (4*Log[Tan[e + f*x]] + (b*(2*(3*a^2 - 3*
a*b + b^2)*Log[a + b*Tan[e + f*x]^2] - (a*(a - b)*(a*(5*a - 3*b) + 2*(2*a -
 b)*b*Tan[e + f*x]^2))/(a + b*Tan[e + f*x]^2)^2))/(a - b)^3)/a^3)/(4*f)
```

Maple [B] time = 0.098, size = 289, normalized size = 2.

$$\frac{\ln\left(\cos\left(fx+e\right)+1\right)}{2fa^3} + \frac{3b^2}{2fa(a-b)^3\left(a\left(\cos\left(fx+e\right)\right)^2-\left(\cos\left(fx+e\right)\right)^2b+b\right)} - \frac{b^3}{2fa^2(a-b)^3\left(a\left(\cos\left(fx+e\right)\right)^2-\left(\cos \ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] 1/2/f/a^3*ln(cos(f*x+e)+1)+3/2/f*b^2/a/(a-b)^3/(a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)-1/2/f*b^3/a^2/(a-b)^3/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+3/2/f*b/a/(a-
b)^3*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)-3/2/f*b^2/a^2/(a-b)^3*ln(a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)+1/2/f*b^3/a^3/(a-b)^3*ln(a*cos(f*x+e)^2-cos(f*x+e)^
2*b+b)-1/4/f*b^3/a/(a-b)^3/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2+1/2/f/a^3*ln
(cos(f*x+e)-1)
```

Maxima [A] time = 1.11028, size = 338, normalized size = 2.28

$$\frac{\frac{2\left(3a^2b-3ab^2+b^3\right)\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{a^6-3a^5b+3a^4b^2-a^3b^3} + \frac{6a^2b^2-3ab^3-2\left(3a^2b^2-4ab^3+b^4\right)\sin\left(fx+e\right)^2}{a^7-3a^6b+3a^5b^2-a^4b^3+\left(a^7-5a^6b+10a^5b^2-10a^4b^3+5a^3b^4-a^2b^5\right)\sin\left(fx+e\right)^4-2\left(a^7-4a^6b+6a^5b^2-4a^4b^3+a^3 \ldots\right.}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(3*a^2*b - 3*a*b^2 + b^3)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^6 - 3*
a^5*b + 3*a^4*b^2 - a^3*b^3) + (6*a^2*b^2 - 3*a*b^3 - 2*(3*a^2*b^2 - 4*a*b^
3 + b^4)*sin(f*x + e)^2)/(a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3 + (a^7 - 5*a^
6*b + 10*a^5*b^2 - 10*a^4*b^3 + 5*a^3*b^4 - a^2*b^5)*sin(f*x + e)^4 - 2*(a^
7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*sin(f*x + e)^2) + 2*log(sin(
f*x + e)^2)/a^3)/f
```

Fricas [B] time = 1.89519, size = 895, normalized size = 6.05

$$\frac{6\,a^{3}b^{2}-3\,a^{2}b^{3}+\left(5\,a^{2}b^{3}-2\,ab^{4}\right)\tan\left(fx+e\right)^{4}+2\left(3\,a^{3}b^{2}+a^{2}b^{3}-ab^{4}\right)\tan\left(fx+e\right)^{2}+2\left(a^{5}-3\,a^{4}b+3\,a^{3}b^{2}-a^{2}b^{3}\cdots\right.}{4\left(\left(a^{6}\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(6*a^3*b^2 - 3*a^2*b^3 + (5*a^2*b^3 - 2*a*b^4)*tan(f*x + e)^4 + 2*(3*a^
3*b^2 + a^2*b^3 - a*b^4)*tan(f*x + e)^2 + 2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^
2*b^3 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3
*a^3*b^2 + 3*a^2*b^3 - a*b^4)*tan(f*x + e)^2)*log(tan(f*x + e)^2/(tan(f*x +
 e)^2 + 1)) + 2*(3*a^4*b - 3*a^3*b^2 + a^2*b^3 + (3*a^2*b^3 - 3*a*b^4 + b^5
)*tan(f*x + e)^4 + 2*(3*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(f*x + e)^2)*log((b
*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b
^4 - a^3*b^5)*f*tan(f*x + e)^4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4
)*f*tan(f*x + e)^2 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.63073, size = 360, normalized size = 2.43

$$\frac{\frac{2\left(3\,a^{2}b-3\,ab^{2}+b^{3}\right)\log\left(\left|-a\sin\left(fx+e\right)^{2}+b\sin\left(fx+e\right)^{2}+a\right|\right)}{a^{6}-3\,a^{5}b+3\,a^{4}b^{2}-a^{3}b^{3}}-\frac{9\,a^{3}b\sin\left(fx+e\right)^{4}-18\,a^{2}b^{2}\sin\left(fx+e\right)^{4}+12\,ab^{3}\sin\left(fx+e\right)^{4}-3\,b^{4}\sin\left(fx+e\right)^{4}-18\,a^{3}b\sin\left(fx+e\right)^{2}\cdots}{\left(a^{5}-2\,a^{4}b+a^{3}b^{2}\right)\left(a\sin\left(fx+e\right)^{2}-b\sin\left(fx+e\right)^{2}\cdots\right.}\cdots}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*(3*a^2*b - 3*a*b^2 + b^3)*log(abs(-a*sin(f*x + e)^2 + b*sin(f*x + e)
^2 + a))/(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3) - (9*a^3*b*sin(f*x + e)^4 -
18*a^2*b^2*sin(f*x + e)^4 + 12*a*b^3*sin(f*x + e)^4 - 3*b^4*sin(f*x + e)^4
- 18*a^3*b*sin(f*x + e)^2 + 24*a^2*b^2*sin(f*x + e)^2 - 8*a*b^3*sin(f*x + e
)^2 + 9*a^3*b - 6*a^2*b^2)/((a^5 - 2*a^4*b + a^3*b^2)*(a*sin(f*x + e)^2 - b
*sin(f*x + e)^2 - a)^2) + 2*log(sin(f*x + e)^2)/a^3)/f
```

3.241 $\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=181

$$\frac{b^2(3a-2b)}{2a^3f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{b^2}{4a^2f(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{b^2\left(6a^2-8ab+3b^2\right)\log\left(a+b\tan^2(e+fx)\right)}{2a^4f(a-b)^3}$$

```
[Out] -Cot[e + f*x]^2/(2*a^3*f) - Log[Cos[e + f*x]]/((a - b)^3*f) - ((a + 3*b)*Lo
g[Tan[e + f*x]])/(a^4*f) - (b^2*(6*a^2 - 8*a*b + 3*b^2)*Log[a + b*Tan[e + f
*x]^2])/(2*a^4*(a - b)^3*f) + b^2/(4*a^2*(a - b)*f*(a + b*Tan[e + f*x]^2)^2
) + ((3*a - 2*b)*b^2)/(2*a^3*(a - b)^2*f*(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.213556, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 88}

$$\frac{b^2(3a-2b)}{2a^3f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{b^2}{4a^2f(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{b^2\left(6a^2-8ab+3b^2\right)\log\left(a+b\tan^2(e+fx)\right)}{2a^4f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -Cot[e + f*x]^2/(2*a^3*f) - Log[Cos[e + f*x]]/((a - b)^3*f) - ((a + 3*b)*Lo
g[Tan[e + f*x]])/(a^4*f) - (b^2*(6*a^2 - 8*a*b + 3*b^2)*Log[a + b*Tan[e + f
*x]^2])/(2*a^4*(a - b)^3*f) + b^2/(4*a^2*(a - b)*f*(a + b*Tan[e + f*x]^2)^2
) + ((3*a - 2*b)*b^2)/(2*a^3*(a - b)^2*f*(a + b*Tan[e + f*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x^2)\left(a+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x)(a+bx)^3}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x^2} + \frac{-a-3b}{a^4x} + \frac{1}{(a-b)^3(1+x)} - \frac{b^3}{a^2(a-b)(a+bx)^3} - \frac{(3a-2b)b^3}{a^3(a-b)^2(a+bx)^2} - \frac{b^3\left(6a^2-8ab+3b^2\right)}{a^4(a-b)^3(a+bx)}\right) dx, x,\right.}{2f}$$

$$= -\frac{\cot^2(e+fx)}{2a^3f} - \frac{\log(\cos(e+fx))}{(a-b)^3f} - \frac{(a+3b)\log(\tan(e+fx))}{a^4f} - \frac{b^2\left(6a^2-8ab+3b^2\right)\log}{2a^4(a-}$$

Mathematica [A] time = 1.92266, size = 144, normalized size = 0.8

$$-\frac{-\frac{b^4}{2a^4(a-b)\left(a\cot^2(e+fx)+b\right)^2} + \frac{b^3(4a-3b)}{a^4(a-b)^2\left(a\cot^2(e+fx)+b\right)} + \frac{b^2\left(6a^2-8ab+3b^2\right)\log\left(a\cot^2(e+fx)+b\right)}{a^4(a-b)^3} + \frac{\cot^2(e+fx)}{a^3} + \frac{2\log(\sin(e+fx))}{(a-b)^3}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -(Cot[e + f*x]^2/a^3 - b^4/(2*a^4*(a - b)*(b + a*Cot[e + f*x]^2)^2) + ((4*a
 - 3*b)*b^3)/(a^4*(a - b)^2*(b + a*Cot[e + f*x]^2)) + (b^2*(6*a^2 - 8*a*b +
 3*b^2)*Log[b + a*Cot[e + f*x]^2])/(a^4*(a - b)^3) + (2*Log[Sin[e + f*x]])/
(a - b)^3)/(2*f)
```

Maple [B] time = 0.105, size = 362, normalized size = 2.

$$-\frac{1}{4fa^3\left(\cos\left(fx+e\right)+1\right)} - \frac{\ln\left(\cos\left(fx+e\right)+1\right)}{2fa^3} - \frac{3\ln\left(\cos\left(fx+e\right)+1\right)b}{2fa^4} - 3\frac{b^2\ln\left(a\left(\cos\left(fx+e\right)\right)^2 - \left(\cos\left(fx+e\right)\right.\right.}{fa^2(a-b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -1/4/f/a^3/(cos(f*x+e)+1)-1/2/f/a^3*ln(cos(f*x+e)+1)-3/2/f/a^4*ln(cos(f*x+e
)+1)*b-3/f*b^2/a^2/(a-b)^3*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+4/f*b^3/a^3/
(a-b)^3*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)-3/2/f*b^4/a^4/(a-b)^3*ln(a*cos(
f*x+e)^2-cos(f*x+e)^2*b+b)-2/f*b^3/a^2/(a-b)^3/(a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)+1/f*b^4/a^3/(a-b)^3/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+1/4/f*b^4/a^2/(
a-b)^3/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2+1/4/f/a^3/(cos(f*x+e)-1)-1/2/f/a
^3*ln(cos(f*x+e)-1)-3/2/f/a^4*ln(cos(f*x+e)-1)*b
```

Maxima [A] time = 1.13208, size = 466, normalized size = 2.57

$$-\frac{\frac{2\left(6a^2b^2-8ab^3+3b^4\right)\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{a^7-3a^6b+3a^5b^2-a^4b^3} + \frac{2a^5-6a^4b+6a^3b^2-2a^2b^3+2\left(a^5-5a^4b+10a^3b^2-14a^2b^3+11ab^4-3b^5\right)\sin\left(fx+e\right)^4-\left(4a^5-16a^4b+24a\right.}{\left(a^8-5a^7b+10a^6b^2-10a^5b^3+5a^4b^4-a^3b^5\right)\sin\left(fx+e\right)^6-2\left(a^8-4a^7b+6a^6b^2-4a^5b^3+a^4b^4\right)\sin\left(fx+e\right)^4+\left(a\right.}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(6*a^2*b^2 - 8*a*b^3 + 3*b^4)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^7
 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3) + (2*a^5 - 6*a^4*b + 6*a^3*b^2 - 2*a^2*b^
3 + 2*(a^5 - 5*a^4*b + 10*a^3*b^2 - 14*a^2*b^3 + 11*a*b^4 - 3*b^5)*sin(f*x
+ e)^4 - (4*a^5 - 16*a^4*b + 24*a^3*b^2 - 24*a^2*b^3 + 9*a*b^4)*sin(f*x + e
)^2)/((a^8 - 5*a^7*b + 10*a^6*b^2 - 10*a^5*b^3 + 5*a^4*b^4 - a^3*b^5)*sin(f
*x + e)^6 - 2*(a^8 - 4*a^7*b + 6*a^6*b^2 - 4*a^5*b^3 + a^4*b^4)*sin(f*x + e
)^4 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*sin(f*x + e)^2) + 2*(a + 3*b)*l
og(sin(f*x + e)^2)/a^4)/f
```

Fricas [B] time = 2.05262, size = 1175, normalized size = 6.49

$$-\frac{\left(2\,a^4b^2-6\,a^3b^3+13\,a^2b^4-6\,ab^5\right)\tan\left(fx+e\right)^6+2\,a^6-6\,a^5b+6\,a^4b^2-2\,a^3b^3+2\left(2\,a^5b-5\,a^4b^2+7\,a^3b^3+2\,a^2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((2*a^4*b^2 - 6*a^3*b^3 + 13*a^2*b^4 - 6*a*b^5)*tan(f*x + e)^6 + 2*a^6
 - 6*a^5*b + 6*a^4*b^2 - 2*a^3*b^3 + 2*(2*a^5*b - 5*a^4*b^2 + 7*a^3*b^3 + 2
*a^2*b^4 - 3*a*b^5)*tan(f*x + e)^4 + (2*a^6 - 2*a^5*b - 6*a^4*b^2 + 18*a^3*
b^3 - 9*a^2*b^4)*tan(f*x + e)^2 + 2*((a^4*b^2 - 6*a^2*b^4 + 8*a*b^5 - 3*b^6
)*tan(f*x + e)^6 + 2*(a^5*b - 6*a^3*b^3 + 8*a^2*b^4 - 3*a*b^5)*tan(f*x + e)
^4 + (a^6 - 6*a^4*b^2 + 8*a^3*b^3 - 3*a^2*b^4)*tan(f*x + e)^2)*log(tan(f*x
+ e)^2/(tan(f*x + e)^2 + 1)) + 2*((6*a^2*b^4 - 8*a*b^5 + 3*b^6)*tan(f*x + e
)^6 + 2*(6*a^3*b^3 - 8*a^2*b^4 + 3*a*b^5)*tan(f*x + e)^4 + (6*a^4*b^2 - 8*a
^3*b^3 + 3*a^2*b^4)*tan(f*x + e)^2)*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e
)^2 + 1)))/((a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*f*tan(f*x + e)^6 +
2*(a^8*b - 3*a^7*b^2 + 3*a^6*b^3 - a^5*b^4)*f*tan(f*x + e)^4 + (a^9 - 3*a^8
*b + 3*a^7*b^2 - a^6*b^3)*f*tan(f*x + e)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.71084, size = 1218, normalized size = 6.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*(6*a^2*b^2 - 8*a*b^3 + 3*b^4)*log(a + 2*a*(cos(f*x + e) - 1)/(cos(f
*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e)
- 1)^2/(cos(f*x + e) + 1)^2)/(a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3) - 8*log(
-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3)
 - 2*(18*a^4*b^2 - 24*a^3*b^3 + 9*a^2*b^4 + 72*a^4*b^2*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) - 208*a^3*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 172
*a^2*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 48*a*b^5*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) + 108*a^4*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^
2 - 368*a^3*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 502*a^2*b^4*(co
s(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 288*a*b^5*(cos(f*x + e) - 1)^2/(co
s(f*x + e) + 1)^2 + 64*b^6*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 72*a
^4*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 208*a^3*b^3*(cos(f*x + e
) - 1)^3/(cos(f*x + e) + 1)^3 + 172*a^2*b^4*(cos(f*x + e) - 1)^3/(cos(f*x +
 e) + 1)^3 - 48*a*b^5*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 18*a^4*b^
2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 - 24*a^3*b^3*(cos(f*x + e) - 1)
^4/(cos(f*x + e) + 1)^4 + 9*a^2*b^4*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)
^4)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*(a + 2*a*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e
) - 1)^2/(cos(f*x + e) + 1)^2)^2) + 4*(a + 3*b)*log(-(cos(f*x + e) - 1)/(co
s(f*x + e) + 1))/a^4 - (a + 4*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 12*
b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(f*x + e) + 1)/(a^4*(cos(f*x +
 e) - 1)) - (cos(f*x + e) - 1)/(a^3*(cos(f*x + e) + 1)))/f
```

3.242 $\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=210

$$-\frac{b^3(4a-3b)}{2a^4f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{b^3}{4a^3f(a-b)\left(a+b\tan^2(e+fx)\right)^2}+\frac{b^3\left(10a^2-15ab+6b^2\right)\log\left(a+b\tan^2(e+\right.}{2a^5f(a-b)^3}$$

```
[Out] ((a + 3*b)*Cot[e + f*x]^2)/(2*a^4*f) - Cot[e + f*x]^4/(4*a^3*f) + Log[Cos[e
 + f*x]]/((a - b)^3*f) + ((a^2 + 3*a*b + 6*b^2)*Log[Tan[e + f*x]])/(a^5*f)
+ (b^3*(10*a^2 - 15*a*b + 6*b^2)*Log[a + b*Tan[e + f*x]^2])/(2*a^5*(a - b)^
3*f) - b^3/(4*a^3*(a - b)*f*(a + b*Tan[e + f*x]^2)^2) - ((4*a - 3*b)*b^3)/(
2*a^4*(a - b)^2*f*(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.237839, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 88}

$$-\frac{b^3(4a-3b)}{2a^4f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{b^3}{4a^3f(a-b)\left(a+b\tan^2(e+fx)\right)^2}+\frac{b^3\left(10a^2-15ab+6b^2\right)\log\left(a+b\tan^2(e+\right.}{2a^5f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((a + 3*b)*Cot[e + f*x]^2)/(2*a^4*f) - Cot[e + f*x]^4/(4*a^3*f) + Log[Cos[e
 + f*x]]/((a - b)^3*f) + ((a^2 + 3*a*b + 6*b^2)*Log[Tan[e + f*x]])/(a^5*f)
+ (b^3*(10*a^2 - 15*a*b + 6*b^2)*Log[a + b*Tan[e + f*x]^2])/(2*a^5*(a - b)^
3*f) - b^3/(4*a^3*(a - b)*f*(a + b*Tan[e + f*x]^2)^2) - ((4*a - 3*b)*b^3)/(
2*a^4*(a - b)^2*f*(a + b*Tan[e + f*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^5(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x)(a+bx)^3}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x^3} + \frac{-a-3b}{a^4x^2} + \frac{a^2+3ab+6b^2}{a^5x} - \frac{1}{(a-b)^3(1+x)} + \frac{b^4}{a^3(a-b)(a+bx)^3} + \frac{(4a-3b)b^4}{a^4(a-b)^2(a+bx)^2} + \frac{b^4(10a^2-1}{a^5(a-b)^3}\right.\right.}{2f}$$

$$= \frac{(a+3b)\cot^2(e+fx)}{2a^4f} - \frac{\cot^4(e+fx)}{4a^3f} + \frac{\log(\cos(e+fx))}{(a-b)^3f} + \frac{\left(a^2+3ab+6b^2\right)\log(\tan(e+}{a^5f}$$

Mathematica [A] time = 2.4705, size = 178, normalized size = 0.85

$$\frac{\frac{\frac{b^3\left(2\left(10a^2-15ab+6b^2\right)\log\left(a+b\tan^2(e+fx)\right)-\frac{a(a-b)\left(2b(4a-3b)\tan^2(e+fx)+a(9a-7b)\right)}{\left(a+b\tan^2(e+fx)\right)^2}\right)}{(a-b)^3}+4\left(a^2+3ab+6b^2\right)\log(\tan(e+fx))}{2a^5} + \frac{(a+3b)\cot^2(e+fx)}{a^4} - \frac{\cot^4(e+fx)}{2a^3} + \frac{2\log(\text{co}}{(a-}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (((a + 3*b)*Cot[e + f*x]^2)/a^4 - Cot[e + f*x]^4/(2*a^3) + (2*Log[Cos[e + f
*x]])/(a - b)^3 + (4*(a^2 + 3*a*b + 6*b^2)*Log[Tan[e + f*x]] + (b^3*(2*(10*
a^2 - 15*a*b + 6*b^2)*Log[a + b*Tan[e + f*x]^2] - (a*(a - b)*(a*(9*a - 7*b)
 + 2*(4*a - 3*b)*b*Tan[e + f*x]^2))/(a + b*Tan[e + f*x]^2)^2))/(a - b)^3)/(
2*a^5))/(2*f)
```

Maple [B] time = 0.109, size = 477, normalized size = 2.3

$$-\frac{1}{16fa^3\left(\cos(fx+e)+1\right)^2} + \frac{7}{16fa^3\left(\cos(fx+e)+1\right)} + \frac{3b}{4fa^4\left(\cos(fx+e)+1\right)} + \frac{\ln\left(\cos(fx+e)+1\right)}{2fa^3} + \frac{3\ln\left(\cos\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -1/16/f/a^3/(cos(f*x+e)+1)^2+7/16/f/a^3/(cos(f*x+e)+1)+3/4/f/a^4/(cos(f*x+e
)+1)*b+1/2/f/a^3*ln(cos(f*x+e)+1)+3/2/f/a^4*ln(cos(f*x+e)+1)*b+3/f/a^5*ln(c
os(f*x+e)+1)*b^2+5/f*b^3/a^3/(a-b)^3*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)-15
/2/f*b^4/a^4/(a-b)^3*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+3/f*b^5/a^5/(a-b)^
3*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+5/2/f*b^4/a^3/(a-b)^3/(a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)-3/2/f*b^5/a^4/(a-b)^3/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)-
1/4/f*b^5/a^3/(a-b)^3/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2-1/16/f/a^3/(cos(f
*x+e)-1)^2-7/16/f/a^3/(cos(f*x+e)-1)-3/4/f/a^4/(cos(f*x+e)-1)*b+1/2/f/a^3*l
n(cos(f*x+e)-1)+3/2/f/a^4*ln(cos(f*x+e)-1)*b+3/f/a^5*ln(cos(f*x+e)-1)*b^2
```

Maxima [B] time = 1.18268, size = 562, normalized size = 2.68

$$\frac{\frac{2\left(10\,a^2b^3-15\,ab^4+6\,b^5\right)\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{a^8-3\,a^7b+3\,a^6b^2-a^5b^3}+\frac{2\left(2\,a^6-7\,a^5b+5\,a^4b^2+10\,a^3b^3-25\,a^2b^4+21\,ab^5-6\,b^6\right)\sin\left(fx+e\right)^6-a^6+3\,a^5b-3\,a^4b^2+a^3b^3-\left(9\,a^6-25\,a\right.}{\left(a^9-5\,a^8b+10\,a^7b^2-10\,a^6b^3+5\,a^5b^4-a^4b^5\right)\sin\left(fx+e\right)^8-2\left(a^9-4\,a^8b+6\right.}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(10*a^2*b^3 - 15*a*b^4 + 6*b^5)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^
8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3) + (2*(2*a^6 - 7*a^5*b + 5*a^4*b^2 + 10*a
^3*b^3 - 25*a^2*b^4 + 21*a*b^5 - 6*b^6)*sin(f*x + e)^6 - a^6 + 3*a^5*b - 3*
a^4*b^2 + a^3*b^3 - (9*a^6 - 25*a^5*b + 10*a^4*b^2 + 30*a^3*b^3 - 45*a^2*b^
4 + 18*a*b^5)*sin(f*x + e)^4 + 2*(3*a^6 - 7*a^5*b + 3*a^4*b^2 + 3*a^3*b^3 -
 2*a^2*b^4)*sin(f*x + e)^2)/((a^9 - 5*a^8*b + 10*a^7*b^2 - 10*a^6*b^3 + 5*a
^5*b^4 - a^4*b^5)*sin(f*x + e)^8 - 2*(a^9 - 4*a^8*b + 6*a^7*b^2 - 4*a^6*b^3
 + a^5*b^4)*sin(f*x + e)^6 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^6*b^3)*sin(f*x
+ e)^4) + 2*(a^2 + 3*a*b + 6*b^2)*log(sin(f*x + e)^2)/a^5)/f
```

Fricas [B] time = 2.32967, size = 1324, normalized size = 6.3

$$\frac{3\left(a^5b^2-a^4b^3-3\,a^3b^4+8\,a^2b^5-4\,ab^6\right)\tan\left(fx+e\right)^8-a^7+3\,a^6b-3\,a^5b^2+a^4b^3+2\left(3\,a^6b-2\,a^5b^2-9\,a^4b^3+14\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*(a^5*b^2 - a^4*b^3 - 3*a^3*b^4 + 8*a^2*b^5 - 4*a*b^6)*tan(f*x + e)^8
 - a^7 + 3*a^6*b - 3*a^5*b^2 + a^4*b^3 + 2*(3*a^6*b - 2*a^5*b^2 - 9*a^4*b^3
 + 14*a^3*b^4 + 3*a^2*b^5 - 6*a*b^6)*tan(f*x + e)^6 + (3*a^7 + a^6*b - 10*a
^5*b^2 - 6*a^4*b^3 + 33*a^3*b^4 - 18*a^2*b^5)*tan(f*x + e)^4 + 2*(a^7 - a^6
*b - 3*a^5*b^2 + 5*a^4*b^3 - 2*a^3*b^4)*tan(f*x + e)^2 + 2*((a^5*b^2 - 10*a
^2*b^5 + 15*a*b^6 - 6*b^7)*tan(f*x + e)^8 + 2*(a^6*b - 10*a^3*b^4 + 15*a^2*
b^5 - 6*a*b^6)*tan(f*x + e)^6 + (a^7 - 10*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5)
*tan(f*x + e)^4)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) + 2*((10*a^2*b^5
- 15*a*b^6 + 6*b^7)*tan(f*x + e)^8 + 2*(10*a^3*b^4 - 15*a^2*b^5 + 6*a*b^6)*
tan(f*x + e)^6 + (10*a^4*b^3 - 15*a^3*b^4 + 6*a^2*b^5)*tan(f*x + e)^4)*log(
(b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)))/((a^8*b^2 - 3*a^7*b^3 + 3*a^6
*b^4 - a^5*b^5)*f*tan(f*x + e)^8 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*b^3 - a^6*b
^4)*f*tan(f*x + e)^6 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f*tan(f*x + e
)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.77064, size = 2016, normalized size = 9.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(32*(10*a^2*b^3 - 15*a*b^4 + 6*b^5)*log(a + 2*a*(cos(f*x + e) - 1)/(co
s(f*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x +
e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3) - 64*
log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 -
b^3) - (a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3 + 16*a^7*(cos(f*x + e) - 1)/(co
s(f*x + e) + 1) - 32*a^6*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 32*a^4*b
^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*a^3*b^4*(cos(f*x + e) - 1)/(c
os(f*x + e) + 1) + 70*a^7*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 178*a
^6*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 34*a^5*b^2*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2 + 586*a^4*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2 - 752*a^3*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 272*a^2*b
^5*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 140*a^7*(cos(f*x + e) - 1)^3
/(cos(f*x + e) + 1)^3 - 412*a^6*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3
 + 204*a^5*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 1356*a^4*b^3*(co
s(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 3272*a^3*b^4*(cos(f*x + e) - 1)^3/
(cos(f*x + e) + 1)^3 + 2496*a^2*b^5*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)
^3 - 640*a*b^6*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 145*a^7*(cos(f*x
 + e) - 1)^4/(cos(f*x + e) + 1)^4 - 403*a^6*b*(cos(f*x + e) - 1)^4/(cos(f*x
 + e) + 1)^4 + 211*a^5*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 1487
*a^4*b^3*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 - 3296*a^3*b^4*(cos(f*x
+ e) - 1)^4/(cos(f*x + e) + 1)^4 + 2560*a^2*b^5*(cos(f*x + e) - 1)^4/(cos(f
*x + e) + 1)^4 - 256*a*b^6*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 - 256*
b^7*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 76*a^7*(cos(f*x + e) - 1)^5
/(cos(f*x + e) + 1)^5 - 140*a^6*b*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5
 - 36*a^5*b^2*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 + 700*a^4*b^3*(cos(
f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 - 1624*a^3*b^4*(cos(f*x + e) - 1)^5/(c
os(f*x + e) + 1)^5 + 1152*a^2*b^5*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5
 - 256*a*b^6*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 + 16*a^7*(cos(f*x +
e) - 1)^6/(cos(f*x + e) + 1)^6 + 160*a^4*b^3*(cos(f*x + e) - 1)^6/(cos(f*x
+ e) + 1)^6 - 240*a^3*b^4*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 + 96*a^
2*b^5*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6)/((a^8 - 3*a^7*b + 3*a^6*b^
2 - a^5*b^3)*(a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2*a*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2 - 4*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
+ a*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)^2) + 32*(a^2 + 3*a*b + 6*b^2
)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/a^5 - (12*a^3*(cos(f*x + e) -
 1)/(cos(f*x + e) + 1) + 24*a^2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a
^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^6)/f
```

3.243 $\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=153

$$\frac{\sqrt{a}\left(3a^2-10ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8b^{5/2}f(a-b)^3}-\frac{a(3a-7b)\tan(e+fx)}{8b^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{a\tan^3(e+fx)}{4bf(a-b)\left(a+b\tan^2(e+fx)\right)^2}-$$

```
[Out] -(x/(a - b)^3) + (Sqrt[a]*(3*a^2 - 10*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tan[e +
 f*x])/Sqrt[a]])/(8*(a - b)^3*b^(5/2)*f) - (a*Tan[e + f*x]^3)/(4*(a - b)*b*
f*(a + b*Tan[e + f*x]^2)^2) - (a*(3*a - 7*b)*Tan[e + f*x])/(8*(a - b)^2*b^2
*f*(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.229099, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 470, 578, 522, 203, 205}

$$\frac{\sqrt{a}\left(3a^2-10ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8b^{5/2}f(a-b)^3}-\frac{a(3a-7b)\tan(e+fx)}{8b^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{a\tan^3(e+fx)}{4bf(a-b)\left(a+b\tan^2(e+fx)\right)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -(x/(a - b)^3) + (Sqrt[a]*(3*a^2 - 10*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tan[e +
 f*x])/Sqrt[a]])/(8*(a - b)^3*b^(5/2)*f) - (a*Tan[e + f*x]^3)/(4*(a - b)*b*
f*(a + b*Tan[e + f*x]^2)^2) - (a*(3*a - 7*b)*Tan[e + f*x])/(8*(a - b)^2*b^2
*f*(a + b*Tan[e + f*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
```

```
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan^3(e+fx)}{4(a-b)bf\left(a+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2(3a+(3a-4b)x^2)}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4(a-b)bf}$$

$$= -\frac{a\tan^3(e+fx)}{4(a-b)bf\left(a+b\tan^2(e+fx)\right)^2} - \frac{a(3a-7b)\tan(e+fx)}{8(a-b)^2b^2f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-a(3a-7}{}\right.}{}$$

$$= -\frac{a\tan^3(e+fx)}{4(a-b)bf\left(a+b\tan^2(e+fx)\right)^2} - \frac{a(3a-7b)\tan(e+fx)}{8(a-b)^2b^2f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,d\right.}{(a-}$$

$$= -\frac{x}{(a-b)^3} + \frac{\sqrt{a}\left(3a^2-10ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8(a-b)^3b^{5/2}f} - \frac{a\tan^3(e+fx)}{4(a-b)bf\left(a+b\tan^2(e+fx)\right.}$$

Mathematica [A] time = 2.17079, size = 142, normalized size = 0.93

$$\frac{\frac{\sqrt{a}\left(3a^2-10ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{b^{5/2}} - \frac{a(a-b)\sin(2(e+fx))\left(3\left(a^2-4ab+3b^2\right)\cos(2(e+fx))+3a^2-2ab-9b^2\right)}{b^2((a-b)\cos(2(e+fx))+a+b)^2} - 8(e+fx)}{8f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-8*(e + f*x) + (Sqrt[a]*(3*a^2 - 10*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tan[e +
f*x])/Sqrt[a]])/b^(5/2) - (a*(a - b)*(3*a^2 - 2*a*b - 9*b^2 + 3*(a^2 - 4*a*
```

```
b + 3*b^2)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(b^2*(a + b + (a - b)*Cos[2*
(e + f*x)])^2))/(8*(a - b)^3*f)
```

Maple [B] time = 0.026, size = 351, normalized size = 2.3

$$-\frac{5a^3(\tan(fx+e))^3}{8f(a-b)^3(a+b(\tan(fx+e))^2)^2 b}+\frac{7a^2(\tan(fx+e))^3}{4f(a-b)^3(a+b(\tan(fx+e))^2)^2}-\frac{9ab(\tan(fx+e))^3}{8f(a-b)^3(a+b(\tan(fx+e))^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -5/8/f*a^3/(a-b)^3/(a+b*tan(f*x+e)^2)^2/b*tan(f*x+e)^3+7/4/f*a^2/(a-b)^3/(a
+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-9/8/f*a/(a-b)^3/(a+b*tan(f*x+e)^2)^2*b*tan(
f*x+e)^3-3/8/f*a^4/(a-b)^3/(a+b*tan(f*x+e)^2)^2/b^2*tan(f*x+e)+5/4/f*a^3/(a
-b)^3/(a+b*tan(f*x+e)^2)^2/b*tan(f*x+e)-7/8/f*a^2/(a-b)^3/(a+b*tan(f*x+e)^2
)^2*tan(f*x+e)+3/8/f*a^3/(a-b)^3/b^2/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^
(1/2))-5/4/f*a^2/(a-b)^3/b/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))+15/
8/f*a/(a-b)^3/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-1/f/(a-b)^3*arct
an(tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.7163, size = 1646, normalized size = 10.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(32*b^4*f*x*tan(f*x + e)^4 + 64*a*b^3*f*x*tan(f*x + e)^2 + 32*a^2*b^
2*f*x + 4*(5*a^3*b - 14*a^2*b^2 + 9*a*b^3)*tan(f*x + e)^3 + ((3*a^2*b^2 - 1
0*a*b^3 + 15*b^4)*tan(f*x + e)^4 + 3*a^4 - 10*a^3*b + 15*a^2*b^2 + 2*(3*a^3
*b - 10*a^2*b^2 + 15*a*b^3)*tan(f*x + e)^2)*sqrt(-a/b)*log((b^2*tan(f*x + e
)^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(b^2*tan(f*x + e)^3 - a*b*tan(f*x + e)
)*sqrt(-a/b))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) + 4*(3*a^4
 - 10*a^3*b + 7*a^2*b^2)*tan(f*x + e))/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^
7)*f*tan(f*x + e)^4 + 2*(a^4*b^3 - 3*a^3*b^4 + 3*a^2*b^5 - a*b^6)*f*tan(f*x
 + e)^2 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f), -1/16*(16*b^4*f*x
*tan(f*x + e)^4 + 32*a*b^3*f*x*tan(f*x + e)^2 + 16*a^2*b^2*f*x + 2*(5*a^3*b
 - 14*a^2*b^2 + 9*a*b^3)*tan(f*x + e)^3 - ((3*a^2*b^2 - 10*a*b^3 + 15*b^4)*
tan(f*x + e)^4 + 3*a^4 - 10*a^3*b + 15*a^2*b^2 + 2*(3*a^3*b - 10*a^2*b^2 +
```

```
15*a*b^3)*tan(f*x + e)^2)*sqrt(a/b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(
a/b)/(a*tan(f*x + e))) + 2*(3*a^4 - 10*a^3*b + 7*a^2*b^2)*tan(f*x + e))/((a
^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*f*tan(f*x + e)^4 + 2*(a^4*b^3 - 3*a^3*b
^4 + 3*a^2*b^5 - a*b^6)*f*tan(f*x + e)^2 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4
 - a^2*b^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 4.32386, size = 290, normalized size = 1.9

$$\frac{\frac{(3a^3-10a^2b+15ab^2)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{(a^3b^2-3a^2b^3+3ab^4-b^5)\sqrt{ab}}-\frac{8(fx+e)}{a^3-3a^2b+3ab^2-b^3}-\frac{5a^2b\tan(fx+e)^3-9ab^2\tan(fx+e)^3+3a^3\tan(fx+e)-7a^2b\tan(fx+}{(a^2b^2-2ab^3+b^4)\left(b\tan(fx+e)^2+a\right)^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((3*a^3 - 10*a^2*b + 15*a*b^2)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + a
rctan(b*tan(f*x + e)/sqrt(a*b)))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*sqr
t(a*b)) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (5*a^2*b*tan(f*x +
e)^3 - 9*a*b^2*tan(f*x + e)^3 + 3*a^3*tan(f*x + e) - 7*a^2*b*tan(f*x + e))/
((a^2*b^2 - 2*a*b^3 + b^4)*(b*tan(f*x + e)^2 + a)^2))/f
```

3.244 $\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=145

$$\frac{\left(a^2-6ab-3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8\sqrt{a}b^{3/2}f(a-b)^3}+\frac{(a-5b)\tan(e+fx)}{8bf(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{a\tan(e+fx)}{4bf(a-b)\left(a+b\tan^2(e+fx)\right)^2}+\frac{x}{(a-b)}$$

```
[Out] x/(a - b)^3 + ((a^2 - 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]]
)/(8*Sqrt[a]*(a - b)^3*b^(3/2)*f) - (a*Tan[e + f*x])/(4*(a - b)*b*f*(a + b*
Tan[e + f*x]^2)^2) + ((a - 5*b)*Tan[e + f*x])/(8*(a - b)^2*b*f*(a + b*Tan[e
 + f*x]^2))
```

Rubi [A] time = 0.180763, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 470, 527, 522, 203, 205}

$$\frac{\left(a^2-6ab-3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8\sqrt{a}b^{3/2}f(a-b)^3}+\frac{(a-5b)\tan(e+fx)}{8bf(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{a\tan(e+fx)}{4bf(a-b)\left(a+b\tan^2(e+fx)\right)^2}+\frac{x}{(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] x/(a - b)^3 + ((a^2 - 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]]
)/(8*Sqrt[a]*(a - b)^3*b^(3/2)*f) - (a*Tan[e + f*x])/(4*(a - b)*b*f*(a + b*
Tan[e + f*x]^2)^2) + ((a - 5*b)*Tan[e + f*x])/(8*(a - b)^2*b*f*(a + b*Tan[e
 + f*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
```

```
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan(e+fx)}{4(a-b)bf\left(a+b\tan^2(e+fx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{a+(a-4b)x^2}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4(a-b)bf}$$

$$= -\frac{a\tan(e+fx)}{4(a-b)bf\left(a+b\tan^2(e+fx)\right)^2} + \frac{(a-5b)\tan(e+fx)}{8(a-b)^2bf\left(a+b\tan^2(e+fx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{a(a+3b)+}{(1+x^2)}\right.}{8}$$

$$= -\frac{a\tan(e+fx)}{4(a-b)bf\left(a+b\tan^2(e+fx)\right)^2} + \frac{(a-5b)\tan(e+fx)}{8(a-b)^2bf\left(a+b\tan^2(e+fx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx\right.}{(a-}$$

$$= \frac{x}{(a-b)^3} + \frac{\left(a^2-6ab-3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8\sqrt{a}(a-b)^3b^{3/2}f} - \frac{a\tan(e+fx)}{4(a-b)bf\left(a+b\tan^2(e+fx)\right)^2} + \frac{}{8(a}$$

Mathematica [A] time = 1.89498, size = 136, normalized size = 0.94

$$\frac{\frac{\left(a^2-6ab-3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}} - \frac{(a-b)\sin(2(e+fx))\left(\left(a^2+4ab-5b^2\right)\cos(2(e+fx))+a^2+2ab+5b^2\right)}{b((a-b)\cos(2(e+fx))+a+b)^2} + 8(e+fx)}{8f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (8*(e + f*x) + ((a^2 - 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]
])/(Sqrt[a]*b^(3/2)) - ((a - b)*(a^2 + 2*a*b + 5*b^2 + (a^2 + 4*a*b - 5*b^2
```

```
)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(b*(a + b + (a - b)*Cos[2*(e + f*x)])
^2))/(8*(a - b)^3*f)
```

Maple [B] time = 0.025, size = 338, normalized size = 2.3

$$\frac{a^2 \left(\tan\left(fx+e\right)\right)^3}{8f(a-b)^3 \left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^2} - \frac{3ab\left(\tan\left(fx+e\right)\right)^3}{4f(a-b)^3 \left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^2} + \frac{5b^2\left(\tan\left(fx+e\right)\right)^3}{8f(a-b)^3 \left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^2} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] 1/8/f*a^2/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-3/4/f*a/(a-b)^3/(a+b*ta
n(f*x+e)^2)^2*b*tan(f*x+e)^3+5/8/f*b^2/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x
+e)^3-1/8/f*a^3/(a-b)^3/(a+b*tan(f*x+e)^2)^2/b*tan(f*x+e)-1/4/f*a^2/(a-b)^3
/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)+3/8/f*b/(a-b)^3/(a+b*tan(f*x+e)^2)^2*a*tan
(f*x+e)+1/8/f*a^2/(a-b)^3/b/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-3/
4/f*a/(a-b)^3/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-3/8/f*b/(a-b)^3/
(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))+1/f/(a-b)^3*arctan(tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.71704, size = 1635, normalized size = 11.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*a*b^4*f*x*tan(f*x + e)^4 + 64*a^2*b^3*f*x*tan(f*x + e)^2 + 32*a^3
*b^2*f*x + 4*(a^3*b^2 - 6*a^2*b^3 + 5*a*b^4)*tan(f*x + e)^3 - ((a^2*b^2 - 6
*a*b^3 - 3*b^4)*tan(f*x + e)^4 + a^4 - 6*a^3*b - 3*a^2*b^2 + 2*(a^3*b - 6*a
^2*b^2 - 3*a*b^3)*tan(f*x + e)^2)*sqrt(-a*b)*log((b^2*tan(f*x + e)^4 - 6*a*
b*tan(f*x + e)^2 + a^2 - 4*(b*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(-a*b))/
(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(a^4*b + 2*a^3*b^2 -
 3*a^2*b^3)*tan(f*x + e))/((a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*f*tan(
f*x + e)^4 + 2*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*f*tan(f*x + e)^2
 + (a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f), 1/16*(16*a*b^4*f*x*tan(f
*x + e)^4 + 32*a^2*b^3*f*x*tan(f*x + e)^2 + 16*a^3*b^2*f*x + 2*(a^3*b^2 - 6
*a^2*b^3 + 5*a*b^4)*tan(f*x + e)^3 + ((a^2*b^2 - 6*a*b^3 - 3*b^4)*tan(f*x +
 e)^4 + a^4 - 6*a^3*b - 3*a^2*b^2 + 2*(a^3*b - 6*a^2*b^2 - 3*a*b^3)*tan(f*x
 + e)^2)*sqrt(a*b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a*b)/(a*b*tan(f*x
```

```
 + e))) - 2*(a^4*b + 2*a^3*b^2 - 3*a^2*b^3)*tan(f*x + e))/((a^4*b^4 - 3*a^3
*b^5 + 3*a^2*b^6 - a*b^7)*f*tan(f*x + e)^4 + 2*(a^5*b^3 - 3*a^4*b^4 + 3*a^3
*b^5 - a^2*b^6)*f*tan(f*x + e)^2 + (a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b
^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.41633, size = 269, normalized size = 1.86

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)\left(a^2-6\,ab-3\,b^2\right)}{\left(a^3b-3\,a^2b^2+3\,ab^3-b^4\right)\sqrt{ab}}+\frac{8\left(fx+e\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{ab\tan\left(fx+e\right)^3-5\,b^2\tan\left(fx+e\right)^3-a^2\tan\left(fx+e\right)-3\,ab\tan\left(fx+e\right)}{\left(a^2b-2\,ab^2+b^3\right)\left(b\tan\left(fx+e\right)^2+a\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)
))*(a^2 - 6*a*b - 3*b^2)/((a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*sqrt(a*b)) +
8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (a*b*tan(f*x + e)^3 - 5*b^2*t
an(f*x + e)^3 - a^2*tan(f*x + e) - 3*a*b*tan(f*x + e))/((a^2*b - 2*a*b^2 +
b^3)*(b*tan(f*x + e)^2 + a)^2))/f
```

3.245 $\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=144

$$\frac{\left(3a^2+6ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{3/2}\sqrt{b}f(a-b)^3}+\frac{(3a+b)\tan(e+fx)}{8af(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{\tan(e+fx)}{4f(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{x}{(a-b)^3}$$

```
[Out] -(x/(a - b)^3) + ((3*a^2 + 6*a*b - b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[
a]])/(8*a^(3/2)*(a - b)^3*Sqrt[b]*f) + Tan[e + f*x]/(4*(a - b)*f*(a + b*Tan
[e + f*x]^2)^2) + ((3*a + b)*Tan[e + f*x])/(8*a*(a - b)^2*f*(a + b*Tan[e +
f*x]^2))
```

Rubi [A] time = 0.15395, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 471, 527, 522, 203, 205}

$$\frac{\left(3a^2+6ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{3/2}\sqrt{b}f(a-b)^3}+\frac{(3a+b)\tan(e+fx)}{8af(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{\tan(e+fx)}{4f(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{x}{(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -(x/(a - b)^3) + ((3*a^2 + 6*a*b - b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[
a]])/(8*a^(3/2)*(a - b)^3*Sqrt[b]*f) + Tan[e + f*x]/(4*(a - b)*f*(a + b*Tan
[e + f*x]^2)^2) + ((3*a + b)*Tan[e + f*x])/(8*a*(a - b)^2*f*(a + b*Tan[e +
f*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
```

```
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1+x^2)\left(a+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{1-3x^2}{(1+x^2)\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4(a-b)f}$$

$$= \frac{\tan(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{(3a+b)\tan(e+fx)}{8a(a-b)^2f\left(a+b\tan^2(e+fx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{5a-b+(-3a-}{(1+x^2)(a+b}\right.}{8a(a}$$

$$= \frac{\tan(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{(3a+b)\tan(e+fx)}{8a(a-b)^2f\left(a+b\tan^2(e+fx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x,\right.}{(a-b)^3}$$

$$= -\frac{x}{(a-b)^3} + \frac{\left(3a^2+6ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{3/2}(a-b)^3\sqrt{b}f} + \frac{\tan(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{}{8a(}$$

Mathematica [A] time = 1.9767, size = 139, normalized size = 0.97

$$\frac{\frac{\left(3a^2+6ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}} + \frac{(a-b)\sin(2(e+fx))\left(\left(5a^2-4ab-b^2\right)\cos(2(e+fx))+5a^2+2ab+b^2\right)}{a((a-b)\cos(2(e+fx))+a+b)^2} - 8(e+fx)}{8f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-8*(e + f*x) + ((3*a^2 + 6*a*b - b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a
]])/(a^(3/2)*Sqrt[b]) + ((a - b)*(5*a^2 + 2*a*b + b^2 + (5*a^2 - 4*a*b - b^
2)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(a*(a + b + (a - b)*Cos[2*(e + f*x)]
```

```
)^2))/(8*(a - b)^3*f)
```

Maple [B] time = 0.023, size = 339, normalized size = 2.4

$$\frac{3ab\left(\tan\left(fx+e\right)\right)^{3}}{8f\left(a-b\right)^{3}\left(a+b\left(\tan\left(fx+e\right)\right)^{2}\right)^{2}}-\frac{b^{2}\left(\tan\left(fx+e\right)\right)^{3}}{4f\left(a-b\right)^{3}\left(a+b\left(\tan\left(fx+e\right)\right)^{2}\right)^{2}}-\frac{b^{3}\left(\tan\left(fx+e\right)\right)^{3}}{8f\left(a-b\right)^{3}\left(a+b\left(\tan\left(fx+e\right)\right)^{2}\right)^{2}a}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] 3/8/f*a/(a-b)^3/(a+b*tan(f*x+e)^2)^2*b*tan(f*x+e)^3-1/4/f*b^2/(a-b)^3/(a+b*
tan(f*x+e)^2)^2*tan(f*x+e)^3-1/8/f*b^3/(a-b)^3/(a+b*tan(f*x+e)^2)^2/a*tan(f
*x+e)^3+5/8/f*a^2/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)-3/4/f*b/(a-b)^3/(
a+b*tan(f*x+e)^2)^2*a*tan(f*x+e)+1/8/f*b^2/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan
(f*x+e)+3/8/f*a/(a-b)^3/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))+3/4/f*
b/(a-b)^3/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-1/8/f*b^2/(a-b)^3/a/
(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-1/f/(a-b)^3*arctan(tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.62202, size = 1638, normalized size = 11.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(32*a^2*b^3*f*x*tan(f*x + e)^4 + 64*a^3*b^2*f*x*tan(f*x + e)^2 + 32*
a^4*b*f*x - 4*(3*a^3*b^2 - 2*a^2*b^3 - a*b^4)*tan(f*x + e)^3 + ((3*a^2*b^2
+ 6*a*b^3 - b^4)*tan(f*x + e)^4 + 3*a^4 + 6*a^3*b - a^2*b^2 + 2*(3*a^3*b +
6*a^2*b^2 - a*b^3)*tan(f*x + e)^2)*sqrt(-a*b)*log((b^2*tan(f*x + e)^4 - 6*a
*b*tan(f*x + e)^2 + a^2 - 4*(b*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(-a*b))
/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(5*a^4*b - 6*a^3*b^
2 + a^2*b^3)*tan(f*x + e))/((a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*f*t
an(f*x + e)^4 + 2*(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e
)^2 + (a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f), -1/16*(16*a^2*b^3*f*x*t
an(f*x + e)^4 + 32*a^3*b^2*f*x*tan(f*x + e)^2 + 16*a^4*b*f*x - 2*(3*a^3*b^2
 - 2*a^2*b^3 - a*b^4)*tan(f*x + e)^3 - ((3*a^2*b^2 + 6*a*b^3 - b^4)*tan(f*x
 + e)^4 + 3*a^4 + 6*a^3*b - a^2*b^2 + 2*(3*a^3*b + 6*a^2*b^2 - a*b^3)*tan(f
*x + e)^2)*sqrt(a*b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a*b)/(a*b*tan(f
*x + e))) - 2*(5*a^4*b - 6*a^3*b^2 + a^2*b^3)*tan(f*x + e))/((a^5*b^3 - 3*a
```

```
^4*b^4 + 3*a^3*b^5 - a^2*b^6)*f*tan(f*x + e)^4 + 2*(a^6*b^2 - 3*a^5*b^3 + 3
*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^2 + (a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4
*b^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.75252, size = 270, normalized size = 1.88

$$\frac{\frac{\left(\pi \left\lfloor \frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)(3a^2+6ab-b^2)}{(a^4-3a^3b+3a^2b^2-ab^3)\sqrt{ab}}-\frac{8(fx+e)}{a^3-3a^2b+3ab^2-b^3}+\frac{3ab\tan(fx+e)^3+b^2\tan(fx+e)^3+5a^2\tan(fx+e)-ab\tan(fx+e)}{(a^3-2a^2b+ab^2)\left(b\tan(fx+e)^2+a\right)^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)
))*(3*a^2 + 6*a*b - b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*sqrt(a*b)) -
8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (3*a*b*tan(f*x + e)^3 + b^2*t
an(f*x + e)^3 + 5*a^2*tan(f*x + e) - a*b*tan(f*x + e))/((a^3 - 2*a^2*b + a*
b^2)*(b*tan(f*x + e)^2 + a)^2))/f
```

3.246 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=150

$$-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{5/2}f(a-b)^3}-\frac{b(7a-3b)\tan(e+fx)}{8a^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{b\tan(e+fx)}{4af(a-b)\left(a+b\tan^2(e+fx)\right)^2}$$

```
[Out] x/(a - b)^3 - (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^3*f) - (b*Tan[e + f*x])/(4*a*(a - b)*f*(a
+ b*Tan[e + f*x]^2)^2) - ((7*a - 3*b)*b*Tan[e + f*x])/(8*a^2*(a - b)^2*f*(a
 + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.151704, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3661, 414, 527, 522, 203, 205}

$$-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{5/2}f(a-b)^3}-\frac{b(7a-3b)\tan(e+fx)}{8a^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{b\tan(e+fx)}{4af(a-b)\left(a+b\tan^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-3),x]
```

```
[Out] x/(a - b)^3 - (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^3*f) - (b*Tan[e + f*x])/(4*a*(a - b)*f*(a
+ b*Tan[e + f*x]^2)^2) - ((7*a - 3*b)*b*Tan[e + f*x])/(8*a^2*(a - b)^2*f*(a
 + b*Tan[e + f*x]^2))
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3}\,dx &= \frac{\mathrm{Subst}\left(\int \frac{1}{(1+x^2)\left(a+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{b\tan(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\mathrm{Subst}\left(\int \frac{4a-3b-3bx^2}{(1+x^2)\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4a(a-b)f} \\
&= -\frac{b\tan(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(7a-3b)b\tan(e+fx)}{8a^2(a-b)^2f\left(a+b\tan^2(e+fx)\right)} + \frac{\mathrm{Subst}\left(\int \frac{8a^2-7a\ldots}{(1\ldots}\right.}{\ldots} \\
&= -\frac{b\tan(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(7a-3b)b\tan(e+fx)}{8a^2(a-b)^2f\left(a+b\tan^2(e+fx)\right)} + \frac{\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,d\ldots\right.}{(a-\ldots} \\
&= \frac{x}{(a-b)^3} - \frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3f} - \frac{b\tan(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2}
\end{aligned}
$$

Mathematica [A] time = 1.78068, size = 138, normalized size = 0.92

$$
-\frac{\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}} + \frac{b(7a-3b)(a-b)\tan(e+fx)}{a^2\left(a+b\tan^2(e+fx)\right)} + \frac{2b(a-b)^2\tan(e+fx)}{a\left(a+b\tan^2(e+fx)\right)^2} - 8\tan^{-1}(\tan(e+fx))}{8f(a-b)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-3),x]
```

```
[Out] -(-8*ArcTan[Tan[e + f*x]] + (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt
[b]*Tan[e + f*x])/Sqrt[a]])/a^(5/2) + (2*(a - b)^2*b*Tan[e + f*x])/(a*(a +
b*Tan[e + f*x]^2)^2) + ((7*a - 3*b)*(a - b)*b*Tan[e + f*x])/(a^2*(a + b*Tan
```

```
[e + f*x]^2)))/(8*(a - b)^3*f)
```

Maple [B] time = 0., size = 350, normalized size = 2.3

$$-\frac{7 b^{2} \left(\tan \left(f x + e\right)\right)^{3}}{8 f \left(a - b\right)^{3} \left(a + b \left(\tan \left(f x + e\right)\right)^{2}\right)^{2}} + \frac{5 b^{3} \left(\tan \left(f x + e\right)\right)^{3}}{4 f \left(a - b\right)^{3} \left(a + b \left(\tan \left(f x + e\right)\right)^{2}\right)^{2} a} - \frac{3 b^{4} \left(\tan \left(f x + e\right)\right)^{3}}{8 f \left(a - b\right)^{3} \left(a + b \left(\tan \left(f x + e\right)\right)^{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -7/8/f*b^2/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^3+5/4/f*b^3/(a-b)^3/(a+b
*tan(f*x+e)^2)^2/a*tan(f*x+e)^3-3/8/f*b^4/(a-b)^3/(a+b*tan(f*x+e)^2)^2/a^2*
tan(f*x+e)^3-9/8/f*b/(a-b)^3/(a+b*tan(f*x+e)^2)^2*a*tan(f*x+e)+7/4/f*b^2/(a
-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)-5/8/f*b^3/(a-b)^3/(a+b*tan(f*x+e)^2)^
2/a*tan(f*x+e)-15/8/f*b/(a-b)^3/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)
)+5/4/f*b^2/(a-b)^3/a/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-3/8/f*b^
3/(a-b)^3/a^2/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))+1/f/(a-b)^3*arct
an(tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.71621, size = 1643, normalized size = 10.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*a^2*b^2*f*x*tan(f*x + e)^4 + 64*a^3*b*f*x*tan(f*x + e)^2 + 32*a^4
*f*x - 4*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(f*x + e)^3 - ((15*a^2*b^2 - 10*
a*b^3 + 3*b^4)*tan(f*x + e)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^3*b
 - 10*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(-b/a)*log((b^2*tan(f*x + e)^4
 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*s
qrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(9*a^3*b
- 14*a^2*b^2 + 5*a*b^3)*tan(f*x + e))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a
^2*b^5)*f*tan(f*x + e)^4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*ta
n(f*x + e)^2 + (a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f), 1/16*(16*a^2*b^2*f
*x*tan(f*x + e)^4 + 32*a^3*b*f*x*tan(f*x + e)^2 + 16*a^4*f*x - 2*(7*a^2*b^2
 - 10*a*b^3 + 3*b^4)*tan(f*x + e)^3 - ((15*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(
f*x + e)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^3*b - 10*a^2*b^2 + 3*a
*b^3)*tan(f*x + e)^2)*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)
```

```
/(b*tan(f*x + e))) - 2*(9*a^3*b - 14*a^2*b^2 + 5*a*b^3)*tan(f*x + e))/((a^5
*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(f*x + e)^4 + 2*(a^6*b - 3*a^5
*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2 + (a^7 - 3*a^6*b + 3*a^5*b^2 -
 a^4*b^3)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2679, size = 288, normalized size = 1.92

$$-\frac{\frac{\left(15\,a^2b-10\,ab^2+3\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)}{\left(a^5-3\,a^4b+3\,a^3b^2-a^2b^3\right)\sqrt{ab}}-\frac{8\left(fx+e\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{7\,ab^2\tan\left(fx+e\right)^3-3\,b^3\tan\left(fx+e\right)^3+9\,a^2b\tan\left(fx+e\right)-5\,ab^2\tan\left(f\right.}{\left(a^4-2\,a^3b+a^2b^2\right)\left(b\tan\left(fx+e\right)^2+a\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
arctan(b*tan(f*x + e)/sqrt(a*b)))/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sq
rt(a*b)) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (7*a*b^2*tan(f*x +
 e)^3 - 3*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x + e) - 5*a*b^2*tan(f*x + e))
/((a^4 - 2*a^3*b + a^2*b^2)*(b*tan(f*x + e)^2 + a)^2))/f
```

3.247 $\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=189

$$\frac{b^{3/2}\left(35a^2-42ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{7/2}f(a-b)^3}-\frac{\left(8a^2-27ab+15b^2\right)\cot(e+fx)}{8a^3f(a-b)^2}-\frac{b(9a-5b)\cot(e+fx)}{8a^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}$$

```
[Out] -(x/(a - b)^3) + (b^(3/2)*(35*a^2 - 42*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tan[e
+ f*x])/Sqrt[a]])/(8*a^(7/2)*(a - b)^3*f) - ((8*a^2 - 27*a*b + 15*b^2)*Cot[
e + f*x])/(8*a^3*(a - b)^2*f) - (b*Cot[e + f*x])/(4*a*(a - b)*f*(a + b*Tan[
e + f*x]^2)^2) - ((9*a - 5*b)*b*Cot[e + f*x])/(8*a^2*(a - b)^2*f*(a + b*Tan
[e + f*x]^2))
```

Rubi [A] time = 0.290321, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3670, 472, 579, 583, 522, 203, 205}

$$\frac{b^{3/2}\left(35a^2-42ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{7/2}f(a-b)^3}-\frac{\left(8a^2-27ab+15b^2\right)\cot(e+fx)}{8a^3f(a-b)^2}-\frac{b(9a-5b)\cot(e+fx)}{8a^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```


```
[Out] -(x/(a - b)^3) + (b^(3/2)*(35*a^2 - 42*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tan[e
+ f*x])/Sqrt[a]])/(8*a^(7/2)*(a - b)^3*f) - ((8*a^2 - 27*a*b + 15*b^2)*Cot[
e + f*x])/(8*a^3*(a - b)^2*f) - (b*Cot[e + f*x])/(4*a*(a - b)*f*(a + b*Tan[
e + f*x]^2)^2) - ((9*a - 5*b)*b*Cot[e + f*x])/(8*a^2*(a - b)^2*f*(a + b*Tan
[e + f*x]^2))
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
```

```
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-5b-5bx^2}{x^2(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4a(a-b)f}$$

$$= -\frac{b\cot(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(9a-5b)b\cot(e+fx)}{8a^2(a-b)^2f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{8a^2}{}\right.}{}$$

$$= -\frac{\left(8a^2-27ab+15b^2\right)\cot(e+fx)}{8a^3(a-b)^2f} - \frac{b\cot(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(9a-5b)}{8a^2(a-b)^2f\,(a}$$

$$= -\frac{\left(8a^2-27ab+15b^2\right)\cot(e+fx)}{8a^3(a-b)^2f} - \frac{b\cot(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(9a-5b)}{8a^2(a-b)^2f\,(a}$$

$$= -\frac{x}{(a-b)^3} + \frac{b^{3/2}\left(35a^2-42ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{7/2}(a-b)^3f} - \frac{\left(8a^2-27ab+15b^2\right)\cot(e}{8a^3(a-b)^2f}$$

Mathematica [A] time = 2.01344, size = 174, normalized size = 0.92

$$\frac{\frac{b^{3/2}\left(35a^2-42ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}(a-b)^3} - \frac{4b^3\sin(2(e+fx))}{a^2(a-b)^2((a-b)\cos(2(e+fx))+a+b)^2} + \frac{b^2(13a-7b)\sin(2(e+fx))}{a^3(a-b)^2((a-b)\cos(2(e+fx))+a+b)} - \frac{8\cot(e+fx)}{a^3} + \frac{8(e+fx)}{(b-a)^3}}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((8*(e + f*x))/(-a + b)^3 + (b^(3/2)*(35*a^2 - 42*a*b + 15*b^2)*ArcTan[(Sqr
t[b]*Tan[e + f*x])/Sqrt[a]])/(a^(7/2)*(a - b)^3) - (8*Cot[e + f*x])/a^3 - (
4*b^3*Sin[2*(e + f*x)])/(a^2*(a - b)^2*(a + b + (a - b)*Cos[2*(e + f*x)])^2
) + ((13*a - 7*b)*b^2*Sin[2*(e + f*x)])/(a^3*(a - b)^2*(a + b + (a - b)*Cos
[2*(e + f*x)])))/(8*f)
```

Maple [B] time = 0.09, size = 379, normalized size = 2.

$$-\frac{1}{fa^3\tan(fx+e)} + \frac{11\,b^3\left(\tan(fx+e)\right)^3}{8\,f\,(a-b)^3\left(a+b\left(\tan(fx+e)\right)^2\right)^2a} - \frac{9\,b^4\left(\tan(fx+e)\right)^3}{4\,f\,(a-b)^3\left(a+b\left(\tan(fx+e)\right)^2\right)^2a^2} + \frac{7\,b^5}{8\,fa^3\,(a-b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -1/f/a^3/tan(f*x+e)+11/8/f*b^3/(a-b)^3/(a+b*tan(f*x+e)^2)^2/a*tan(f*x+e)^3-
9/4/f*b^4/(a-b)^3/(a+b*tan(f*x+e)^2)^2/a^2*tan(f*x+e)^3+7/8/f*b^5/a^3/(a-b)
^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^3+13/8/f*b^2/(a-b)^3/(a+b*tan(f*x+e)^2)^
```

```
2*tan(f*x+e)-11/4/f*b^3/(a-b)^3/(a+b*tan(f*x+e)^2)^2/a*tan(f*x+e)+9/8/f*b^4
/a^2/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)+35/8/f*b^2/(a-b)^3/a/(a*b)^(1/
2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-21/4/f*b^3/(a-b)^3/a^2/(a*b)^(1/2)*arct
an(b*tan(f*x+e)/(a*b)^(1/2))+15/8/f*b^4/a^3/(a-b)^3/(a*b)^(1/2)*arctan(b*ta
n(f*x+e)/(a*b)^(1/2))-1/f/(a-b)^3*arctan(tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.93673, size = 1989, normalized size = 10.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(32*a^3*b^2*f*x*tan(f*x + e)^5 + 64*a^4*b*f*x*tan(f*x + e)^3 + 32*a^
5*f*x*tan(f*x + e) + 32*a^5 - 96*a^4*b + 96*a^3*b^2 - 32*a^2*b^3 + 4*(8*a^3
*b^2 - 35*a^2*b^3 + 42*a*b^4 - 15*b^5)*tan(f*x + e)^4 + 4*(16*a^4*b - 61*a^
3*b^2 + 70*a^2*b^3 - 25*a*b^4)*tan(f*x + e)^2 + ((35*a^2*b^3 - 42*a*b^4 + 1
5*b^5)*tan(f*x + e)^5 + 2*(35*a^3*b^2 - 42*a^2*b^3 + 15*a*b^4)*tan(f*x + e)
^3 + (35*a^4*b - 42*a^3*b^2 + 15*a^2*b^3)*tan(f*x + e))*sqrt(-b/a)*log((b^2
*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3 - a^2*
tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)
))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^5 + 2*(a^7*b
 - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x + e)^3 + (a^8 - 3*a^7*b + 3*a
^6*b^2 - a^5*b^3)*f*tan(f*x + e)), -1/16*(16*a^3*b^2*f*x*tan(f*x + e)^5 + 3
2*a^4*b*f*x*tan(f*x + e)^3 + 16*a^5*f*x*tan(f*x + e) + 16*a^5 - 48*a^4*b +
48*a^3*b^2 - 16*a^2*b^3 + 2*(8*a^3*b^2 - 35*a^2*b^3 + 42*a*b^4 - 15*b^5)*ta
n(f*x + e)^4 + 2*(16*a^4*b - 61*a^3*b^2 + 70*a^2*b^3 - 25*a*b^4)*tan(f*x +
e)^2 - ((35*a^2*b^3 - 42*a*b^4 + 15*b^5)*tan(f*x + e)^5 + 2*(35*a^3*b^2 - 4
2*a^2*b^3 + 15*a*b^4)*tan(f*x + e)^3 + (35*a^4*b - 42*a^3*b^2 + 15*a^2*b^3)
*tan(f*x + e))*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan
(f*x + e))))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^5
+ 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x + e)^3 + (a^8 - 3*a
^7*b + 3*a^6*b^2 - a^5*b^3)*f*tan(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.6405, size = 312, normalized size = 1.65

$$\frac{\frac{\left(35\,a^{2}b^{2}-42\,ab^{3}+15\,b^{4}\right)\left(\pi\left\lfloor \frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{\left(a^{6}-3\,a^{5}b+3\,a^{4}b^{2}-a^{3}b^{3}\right)\sqrt{ab}}-\frac{8\,(fx+e)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}+\frac{11\,ab^{3}\tan(fx+e)^{3}-7\,b^{4}\tan(fx+e)^{3}+13\,a^{2}b^{2}\tan(fx+e)-9\,ab}{\left(a^{5}-2\,a^{4}b+a^{3}b^{2}\right)\left(b\tan(fx+e)^{2}+a\right)^{2}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((35*a^2*b^2 - 42*a*b^3 + 15*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b)
+ arctan(b*tan(f*x + e)/sqrt(a*b)))/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*
sqrt(a*b)) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (11*a*b^3*tan(f*
x + e)^3 - 7*b^4*tan(f*x + e)^3 + 13*a^2*b^2*tan(f*x + e) - 9*a*b^3*tan(f*x
 + e))/((a^5 - 2*a^4*b + a^3*b^2)*(b*tan(f*x + e)^2 + a)^2) - 8/(a^3*tan(f*
x + e)))/f
```

3.248 $\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=240

$$-\frac{b^{5/2}\left(63a^2-90ab+35b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{9/2}f(a-b)^3}-\frac{\left(8a^2-55ab+35b^2\right)\cot^3(e+fx)}{24a^3f(a-b)^2}+\frac{\left(8a^2b+8a^3-55ab^2+35b^3\right)\cot(}{8a^4f(a-b)^2}$$

```
[Out] x/(a - b)^3 - (b^(5/2)*(63*a^2 - 90*a*b + 35*b^2)*ArcTan[(Sqrt[b]*Tan[e + f
*x])/Sqrt[a]])/(8*a^(9/2)*(a - b)^3*f) + ((8*a^3 + 8*a^2*b - 55*a*b^2 + 35*
b^3)*Cot[e + f*x])/(8*a^4*(a - b)^2*f) - ((8*a^2 - 55*a*b + 35*b^2)*Cot[e +
 f*x]^3)/(24*a^3*(a - b)^2*f) - (b*Cot[e + f*x]^3)/(4*a*(a - b)*f*(a + b*Ta
n[e + f*x]^2)^2) - ((11*a - 7*b)*b*Cot[e + f*x]^3)/(8*a^2*(a - b)^2*f*(a +
b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.364925, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3670, 472, 579, 583, 522, 203, 205}

$$-\frac{b^{5/2}\left(63a^2-90ab+35b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{9/2}f(a-b)^3}-\frac{\left(8a^2-55ab+35b^2\right)\cot^3(e+fx)}{24a^3f(a-b)^2}+\frac{\left(8a^2b+8a^3-55ab^2+35b^3\right)\cot(}{8a^4f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] x/(a - b)^3 - (b^(5/2)*(63*a^2 - 90*a*b + 35*b^2)*ArcTan[(Sqrt[b]*Tan[e + f
*x])/Sqrt[a]])/(8*a^(9/2)*(a - b)^3*f) + ((8*a^3 + 8*a^2*b - 55*a*b^2 + 35*
b^3)*Cot[e + f*x])/(8*a^4*(a - b)^2*f) - ((8*a^2 - 55*a*b + 35*b^2)*Cot[e +
 f*x]^3)/(24*a^3*(a - b)^2*f) - (b*Cot[e + f*x]^3)/(4*a*(a - b)*f*(a + b*Ta
n[e + f*x]^2)^2) - ((11*a - 7*b)*b*Cot[e + f*x]^3)/(8*a^2*(a - b)^2*f*(a +
b*Tan[e + f*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-7b-7bx^2}{x^4(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4a(a-b)f}$$

$$= -\frac{b\cot^3(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(11a-7b)b\cot^3(e+fx)}{8a^2(a-b)^2f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{8a^2-55\cdots}{x\cdots}\cdots\right)}{\cdots}$$

$$= -\frac{\left(8a^2-55ab+35b^2\right)\cot^3(e+fx)}{24a^3(a-b)^2f} - \frac{b\cot^3(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(11a-7b)b\cdots}{8a^2(a-b)^2f\left(a-\cdots\right.}$$

$$= \frac{\left(8a^3+8a^2b-55ab^2+35b^3\right)\cot(e+fx)}{8a^4(a-b)^2f} - \frac{\left(8a^2-55ab+35b^2\right)\cot^3(e+fx)}{24a^3(a-b)^2f} - \frac{b\cdots}{4a(a-b)\cdots}$$

$$= \frac{\left(8a^3+8a^2b-55ab^2+35b^3\right)\cot(e+fx)}{8a^4(a-b)^2f} - \frac{\left(8a^2-55ab+35b^2\right)\cot^3(e+fx)}{24a^3(a-b)^2f} - \frac{b\cdots}{4a(a-b)\cdots}$$

$$= \frac{x}{(a-b)^3} - \frac{b^{5/2}\left(63a^2-90ab+35b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{9/2}(a-b)^3f} + \frac{\left(8a^3+8a^2b-55ab^2+35b^3\right)\text{c}\cdots}{8a^4(a-b)^2f}$$

Mathematica [A] time = 4.25921, size = 184, normalized size = 0.77

$$\frac{-\frac{3b^{5/2}\left(63a^2-90ab+35b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{9/2}(a-b)^3} + \frac{3\left(8(e+fx)-\frac{b^3(a-b)\sin(2(e+fx))\left(\left(17a^2-28ab+11b^2\right)\cos(2(e+fx))+17a^2+2ab-11b^2\right)}{a^4((a-b)\cos(2(e+fx))+a+b)^2}\right)}{(a-b)^3} - \frac{8\cot(e+fx)\left(a\csc^2(e+fx)-4a\cdots\right.}{a^4}\cdots}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((-3*b^(5/2)*(63*a^2 - 90*a*b + 35*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[
a]])/(a^(9/2)*(a - b)^3) - (8*Cot[e + f*x]*(-4*a - 9*b + a*Csc[e + f*x]^2))
/a^4 + (3*(8*(e + f*x) - ((a - b)*b^3*(17*a^2 + 2*a*b - 11*b^2 + (17*a^2 -
28*a*b + 11*b^2)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(a^4*(a + b + (a - b)*
Cos[2*(e + f*x)])^2)))/(a - b)^3)/(24*f)
```

Maple [A] time = 0.101, size = 413, normalized size = 1.7

$$-\frac{1}{3fa^3\left(\tan\left(fx+e\right)\right)^3} + \frac{1}{fa^3\tan\left(fx+e\right)} + 3\frac{b}{fa^4\tan\left(fx+e\right)} - \frac{15b^4\left(\tan\left(fx+e\right)\right)^3}{8f(a-b)^3\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^2a^2} + \frac{13\cdots}{4fa^3(a-b\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -1/3/f/a^3/tan(f*x+e)^3+1/f/a^3/tan(f*x+e)+3/f/a^4/tan(f*x+e)*b-15/8/f*b^4/
(a-b)^3/(a+b*tan(f*x+e)^2)^2/a^2*tan(f*x+e)^3+13/4/f*b^5/a^3/(a-b)^3/(a+b*t
an(f*x+e)^2)^2*tan(f*x+e)^3-11/8/f*b^6/a^4/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan
(f*x+e)^3-17/8/f*b^3/(a-b)^3/(a+b*tan(f*x+e)^2)^2/a*tan(f*x+e)+15/4/f*b^4/a
^2/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)-13/8/f*b^5/a^3/(a-b)^3/(a+b*tan(
f*x+e)^2)^2*tan(f*x+e)-63/8/f*b^3/(a-b)^3/a^2/(a*b)^(1/2)*arctan(b*tan(f*x+
e)/(a*b)^(1/2))+45/4/f*b^4/a^3/(a-b)^3/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b
)^(1/2))-35/8/f*b^5/a^4/(a-b)^3/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)
)+1/f/(a-b)^3*arctan(tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.12939, size = 2261, normalized size = 9.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/96*(96*a^4*b^2*f*x*tan(f*x + e)^7 + 192*a^5*b*f*x*tan(f*x + e)^5 + 96*a^
6*f*x*tan(f*x + e)^3 + 12*(8*a^4*b^2 - 63*a^2*b^4 + 90*a*b^5 - 35*b^6)*tan(
f*x + e)^6 - 32*a^6 + 96*a^5*b - 96*a^4*b^2 + 32*a^3*b^3 + 4*(48*a^5*b - 8*
a^4*b^2 - 315*a^3*b^3 + 450*a^2*b^4 - 175*a*b^5)*tan(f*x + e)^4 + 32*(3*a^6
 - 2*a^5*b - 12*a^4*b^2 + 18*a^3*b^3 - 7*a^2*b^4)*tan(f*x + e)^2 - 3*((63*a
^2*b^4 - 90*a*b^5 + 35*b^6)*tan(f*x + e)^7 + 2*(63*a^3*b^3 - 90*a^2*b^4 + 3
5*a*b^5)*tan(f*x + e)^5 + (63*a^4*b^2 - 90*a^3*b^3 + 35*a^2*b^4)*tan(f*x +
e)^3)*sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(
a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*
a*b*tan(f*x + e)^2 + a^2)))/((a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*f*
tan(f*x + e)^7 + 2*(a^8*b - 3*a^7*b^2 + 3*a^6*b^3 - a^5*b^4)*f*tan(f*x + e)
^5 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^6*b^3)*f*tan(f*x + e)^3), 1/48*(48*a^4*
b^2*f*x*tan(f*x + e)^7 + 96*a^5*b*f*x*tan(f*x + e)^5 + 48*a^6*f*x*tan(f*x +
 e)^3 + 6*(8*a^4*b^2 - 63*a^2*b^4 + 90*a*b^5 - 35*b^6)*tan(f*x + e)^6 - 16*
a^6 + 48*a^5*b - 48*a^4*b^2 + 16*a^3*b^3 + 2*(48*a^5*b - 8*a^4*b^2 - 315*a^
3*b^3 + 450*a^2*b^4 - 175*a*b^5)*tan(f*x + e)^4 + 16*(3*a^6 - 2*a^5*b - 12*
a^4*b^2 + 18*a^3*b^3 - 7*a^2*b^4)*tan(f*x + e)^2 - 3*((63*a^2*b^4 - 90*a*b^
5 + 35*b^6)*tan(f*x + e)^7 + 2*(63*a^3*b^3 - 90*a^2*b^4 + 35*a*b^5)*tan(f*x
 + e)^5 + (63*a^4*b^2 - 90*a^3*b^3 + 35*a^2*b^4)*tan(f*x + e)^3)*sqrt(b/a)*
arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/((a^7*b^2 -
3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*f*tan(f*x + e)^7 + 2*(a^8*b - 3*a^7*b^2 +
3*a^6*b^3 - a^5*b^4)*f*tan(f*x + e)^5 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^6*b^
3)*f*tan(f*x + e)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.63909, size = 352, normalized size = 1.47

$$-\frac{\frac{3\left(63\,a^2b^3-90\,ab^4+35\,b^5\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)}{\left(a^7-3\,a^6b+3\,a^5b^2-a^4b^3\right)\sqrt{ab}}-\frac{24\left(fx+e\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{3\left(15\,ab^4\tan\left(fx+e\right)^3-11\,b^5\tan\left(fx+e\right)^3+17\,a^2b^3\tan\left(fx+e\right)-\right.}{\left(a^6-2\,a^5b+a^4b^2\right)\left(b\tan\left(fx+e\right)^2+a\right)^2}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(3*(63*a^2*b^3 - 90*a*b^4 + 35*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn
(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b
^3)*sqrt(a*b)) - 24*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 3*(15*a*b^4
*tan(f*x + e)^3 - 11*b^5*tan(f*x + e)^3 + 17*a^2*b^3*tan(f*x + e) - 13*a*b^
4*tan(f*x + e))/((a^6 - 2*a^5*b + a^4*b^2)*(b*tan(f*x + e)^2 + a)^2) - 8*(3
*a*tan(f*x + e)^2 + 9*b*tan(f*x + e)^2 - a)/(a^4*tan(f*x + e)^3))/f
```

3.249 $\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=297

$$\frac{b^{7/2}\left(99a^2-154ab+63b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{11/2}f(a-b)^3}-\frac{\left(8a^2-91ab+63b^2\right)\cot^5(e+fx)}{40a^3f(a-b)^2}+\frac{\left(8a^2b+8a^3-91ab^2+63b^3\right)\mathrm{cc}}{24a^4f(a-b)^2}$$

```
[Out] -(x/(a - b)^3) + (b^(7/2)*(99*a^2 - 154*a*b + 63*b^2)*ArcTan[(Sqrt[b]*Tan[e
 + f*x])/Sqrt[a]])/(8*a^(11/2)*(a - b)^3*f) - ((8*a^4 + 8*a^3*b + 8*a^2*b^2
 - 91*a*b^3 + 63*b^4)*Cot[e + f*x])/(8*a^5*(a - b)^2*f) + ((8*a^3 + 8*a^2*b
 - 91*a*b^2 + 63*b^3)*Cot[e + f*x]^3)/(24*a^4*(a - b)^2*f) - ((8*a^2 - 91*a
*b + 63*b^2)*Cot[e + f*x]^5)/(40*a^3*(a - b)^2*f) - (b*Cot[e + f*x]^5)/(4*a
*(a - b)*f*(a + b*Tan[e + f*x]^2)^2) - ((13*a - 9*b)*b*Cot[e + f*x]^5)/(8*a
^2*(a - b)^2*f*(a + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.468399, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3670, 472, 579, 583, 522, 203, 205}

$$\frac{b^{7/2}\left(99a^2-154ab+63b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{11/2}f(a-b)^3}-\frac{\left(8a^2-91ab+63b^2\right)\cot^5(e+fx)}{40a^3f(a-b)^2}+\frac{\left(8a^2b+8a^3-91ab^2+63b^3\right)\mathrm{cc}}{24a^4f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -(x/(a - b)^3) + (b^(7/2)*(99*a^2 - 154*a*b + 63*b^2)*ArcTan[(Sqrt[b]*Tan[e
 + f*x])/Sqrt[a]])/(8*a^(11/2)*(a - b)^3*f) - ((8*a^4 + 8*a^3*b + 8*a^2*b^2
 - 91*a*b^3 + 63*b^4)*Cot[e + f*x])/(8*a^5*(a - b)^2*f) + ((8*a^3 + 8*a^2*b
 - 91*a*b^2 + 63*b^3)*Cot[e + f*x]^3)/(24*a^4*(a - b)^2*f) - ((8*a^2 - 91*a
*b + 63*b^2)*Cot[e + f*x]^5)/(40*a^3*(a - b)^2*f) - (b*Cot[e + f*x]^5)/(4*a
*(a - b)*f*(a + b*Tan[e + f*x]^2)^2) - ((13*a - 9*b)*b*Cot[e + f*x]^5)/(8*a
^2*(a - b)^2*f*(a + b*Tan[e + f*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-9b-9bx^2}{x^6(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4a(a-b)f}$$

$$= -\frac{b\cot^5(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(13a-9b)b\cot^5(e+fx)}{8a^2(a-b)^2f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{8a^2}{}\right.}{}$$

$$= -\frac{\left(8a^2-91ab+63b^2\right)\cot^5(e+fx)}{40a^3(a-b)^2f} - \frac{b\cot^5(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(13a-9b}{8a^2(a-b)^2f\,(}$$

$$= \frac{\left(8a^3+8a^2b-91ab^2+63b^3\right)\cot^3(e+fx)}{24a^4(a-b)^2f} - \frac{\left(8a^2-91ab+63b^2\right)\cot^5(e+fx)}{40a^3(a-b)^2f} - \frac{}{4a(a-}$$

$$= -\frac{\left(8a^4+8a^3b+8a^2b^2-91ab^3+63b^4\right)\cot(e+fx)}{8a^5(a-b)^2f} + \frac{\left(8a^3+8a^2b-91ab^2+63b^3\right)\cot^3}{24a^4(a-b)^2f}$$

$$= -\frac{\left(8a^4+8a^3b+8a^2b^2-91ab^3+63b^4\right)\cot(e+fx)}{8a^5(a-b)^2f} + \frac{\left(8a^3+8a^2b-91ab^2+63b^3\right)\cot^3}{24a^4(a-b)^2f}$$

$$= -\frac{x}{(a-b)^3} + \frac{b^{7/2}\left(99a^2-154ab+63b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{11/2}(a-b)^3f} - \frac{\left(8a^4+8a^3b+8a^2b^2-9\right.}{8a^5(a}$$

Mathematica [B] time = 6.30532, size = 949, normalized size = 3.2

$$\frac{\left(-3184\cos(e+fx)a^7 - 1536\cos(3(e+fx))a^7 - 704\cos(5(e+fx))a^7 - 536\cos(7(e+fx))a^7 - 184\cos(9(e+fx))a^7 \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (b^(7/2)*(99*a^2 - 154*a*b + 63*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]]
)/(8*a^(11/2)*(a - b)^3*f) + (Csc[e + f*x]^5*(-3184*a^7*Cos[e + f*x] + 7440
*a^6*b*Cos[e + f*x] - 12000*a^5*b^2*Cos[e + f*x] + 10240*a^4*b^3*Cos[e + f*
x] + 6450*a^3*b^4*Cos[e + f*x] + 714*a^2*b^5*Cos[e + f*x] - 22890*a*b^6*Cos
[e + f*x] + 13230*b^7*Cos[e + f*x] - 1536*a^7*Cos[3*(e + f*x)] + 7648*a^6*b
*Cos[3*(e + f*x)] - 2912*a^5*b^2*Cos[3*(e + f*x)] - 1152*a^4*b^3*Cos[3*(e +
 f*x)] - 14872*a^3*b^4*Cos[3*(e + f*x)] - 12796*a^2*b^5*Cos[3*(e + f*x)] +
52080*a*b^6*Cos[3*(e + f*x)] - 26460*b^7*Cos[3*(e + f*x)] - 704*a^7*Cos[5*(
e + f*x)] + 2656*a^6*b*Cos[5*(e + f*x)] - 4128*a^5*b^2*Cos[5*(e + f*x)] - 3
712*a^4*b^3*Cos[5*(e + f*x)] + 5504*a^3*b^4*Cos[5*(e + f*x)] + 27684*a^2*b^
5*Cos[5*(e + f*x)] - 46200*a*b^6*Cos[5*(e + f*x)] + 18900*b^7*Cos[5*(e + f*
x)] - 536*a^7*Cos[7*(e + f*x)] + 248*a^6*b*Cos[7*(e + f*x)] + 768*a^5*b^2*C
os[7*(e + f*x)] + 128*a^4*b^3*Cos[7*(e + f*x)] + 6553*a^3*b^4*Cos[7*(e + f*
x)] - 21441*a^2*b^5*Cos[7*(e + f*x)] + 20895*a*b^6*Cos[7*(e + f*x)] - 6615*
b^7*Cos[7*(e + f*x)] - 184*a^7*Cos[9*(e + f*x)] + 440*a^6*b*Cos[9*(e + f*x)
```

```
] - 160*a^5*b^2*Cos[9*(e + f*x)] + 640*a^4*b^3*Cos[9*(e + f*x)] - 3635*a^3*
b^4*Cos[9*(e + f*x)] + 5839*a^2*b^5*Cos[9*(e + f*x)] - 3885*a*b^6*Cos[9*(e
+ f*x)] + 945*b^7*Cos[9*(e + f*x)] - 720*a^7*(e + f*x)*Sin[e + f*x] - 3360*
a^6*b*(e + f*x)*Sin[e + f*x] - 15120*a^5*b^2*(e + f*x)*Sin[e + f*x] - 480*a
^7*(e + f*x)*Sin[3*(e + f*x)] + 10080*a^5*b^2*(e + f*x)*Sin[3*(e + f*x)] +
480*a^7*(e + f*x)*Sin[5*(e + f*x)] + 1920*a^6*b*(e + f*x)*Sin[5*(e + f*x)]
- 4320*a^5*b^2*(e + f*x)*Sin[5*(e + f*x)] + 120*a^7*(e + f*x)*Sin[7*(e + f*
x)] - 1200*a^6*b*(e + f*x)*Sin[7*(e + f*x)] + 1080*a^5*b^2*(e + f*x)*Sin[7*
(e + f*x)] - 120*a^7*(e + f*x)*Sin[9*(e + f*x)] + 240*a^6*b*(e + f*x)*Sin[9
*(e + f*x)] - 120*a^5*b^2*(e + f*x)*Sin[9*(e + f*x)]))/(7680*a^5*(a - b)^3*
f*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])^2)
```

Maple [A] time = 0.107, size = 466, normalized size = 1.6

$$-\frac{1}{5fa^3(\tan(fx+e))^5}+\frac{1}{3fa^3(\tan(fx+e))^3}+\frac{b}{fa^4(\tan(fx+e))^3}-\frac{1}{fa^3\tan(fx+e)}-3\frac{b}{fa^4\tan(fx+e)}-6\frac{}{fa^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -1/5/f/a^3/tan(f*x+e)^5+1/3/f/a^3/tan(f*x+e)^3+1/f/a^4/tan(f*x+e)^3*b-1/f/a
^3/tan(f*x+e)-3/f/a^4/tan(f*x+e)*b-6/f/a^5/tan(f*x+e)*b^2+19/8/f*b^5/a^3/(a
-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-17/4/f*b^6/a^4/(a-b)^3/(a+b*tan(f*x
+e)^2)^2*tan(f*x+e)^3+15/8/f*b^7/a^5/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e
)^3+21/8/f*b^4/a^2/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)-19/4/f*b^5/a^3/(
a-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)+17/8/f*b^6/a^4/(a-b)^3/(a+b*tan(f*x+
e)^2)^2*tan(f*x+e)+99/8/f*b^4/a^3/(a-b)^3/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(
a*b)^(1/2))-77/4/f*b^5/a^4/(a-b)^3/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1
/2))+63/8/f*b^6/a^5/(a-b)^3/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-1/
f/(a-b)^3*arctan(tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.45771, size = 2547, normalized size = 8.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/480*(480*a^5*b^2*f*x*tan(f*x + e)^9 + 960*a^6*b*f*x*tan(f*x + e)^7 + 48
0*a^7*f*x*tan(f*x + e)^5 + 60*(8*a^5*b^2 - 99*a^2*b^5 + 154*a*b^6 - 63*b^7)
*tan(f*x + e)^8 + 96*a^7 - 288*a^6*b + 288*a^5*b^2 - 96*a^4*b^3 + 20*(48*a^
6*b - 8*a^5*b^2 - 495*a^3*b^4 + 770*a^2*b^5 - 315*a*b^6)*tan(f*x + e)^6 + 3
2*(15*a^7 - 10*a^6*b + 3*a^5*b^2 - 99*a^4*b^3 + 154*a^3*b^4 - 63*a^2*b^5)*t
an(f*x + e)^4 - 32*(5*a^7 - 6*a^6*b - 12*a^5*b^2 + 22*a^4*b^3 - 9*a^3*b^4)*
tan(f*x + e)^2 + 15*((99*a^2*b^5 - 154*a*b^6 + 63*b^7)*tan(f*x + e)^9 + 2*(
99*a^3*b^4 - 154*a^2*b^5 + 63*a*b^6)*tan(f*x + e)^7 + (99*a^4*b^3 - 154*a^3
*b^4 + 63*a^2*b^5)*tan(f*x + e)^5)*sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a
*b*tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b
/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/((a^8*b^2 - 3*a^7*
b^3 + 3*a^6*b^4 - a^5*b^5)*f*tan(f*x + e)^9 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*
b^3 - a^6*b^4)*f*tan(f*x + e)^7 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f*
tan(f*x + e)^5), -1/240*(240*a^5*b^2*f*x*tan(f*x + e)^9 + 480*a^6*b*f*x*tan
(f*x + e)^7 + 240*a^7*f*x*tan(f*x + e)^5 + 30*(8*a^5*b^2 - 99*a^2*b^5 + 154
*a*b^6 - 63*b^7)*tan(f*x + e)^8 + 48*a^7 - 144*a^6*b + 144*a^5*b^2 - 48*a^4
*b^3 + 10*(48*a^6*b - 8*a^5*b^2 - 495*a^3*b^4 + 770*a^2*b^5 - 315*a*b^6)*ta
n(f*x + e)^6 + 16*(15*a^7 - 10*a^6*b + 3*a^5*b^2 - 99*a^4*b^3 + 154*a^3*b^4
 - 63*a^2*b^5)*tan(f*x + e)^4 - 16*(5*a^7 - 6*a^6*b - 12*a^5*b^2 + 22*a^4*b
^3 - 9*a^3*b^4)*tan(f*x + e)^2 - 15*((99*a^2*b^5 - 154*a*b^6 + 63*b^7)*tan(
f*x + e)^9 + 2*(99*a^3*b^4 - 154*a^2*b^5 + 63*a*b^6)*tan(f*x + e)^7 + (99*a
^4*b^3 - 154*a^3*b^4 + 63*a^2*b^5)*tan(f*x + e)^5)*sqrt(b/a)*arctan(1/2*(b*
tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/((a^8*b^2 - 3*a^7*b^3 + 3*
a^6*b^4 - a^5*b^5)*f*tan(f*x + e)^9 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*b^3 - a^
6*b^4)*f*tan(f*x + e)^7 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f*tan(f*x
+ e)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.63939, size = 414, normalized size = 1.39

$$\frac{\frac{15\left(99\,a^2b^4-154\,ab^5+63\,b^6\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{\left(a^8-3\,a^7b+3\,a^6b^2-a^5b^3\right)\sqrt{ab}}-\frac{120\,(fx+e)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{15\left(19\,ab^5\tan(fx+e)^3-15\,b^6\tan(fx+e)^3+21\,a^2b^4\tan(f\cdots\right.}{\left(a^7-2\,a^6b+a^5b^2\right)\left(b\tan(fx+e)^2+a\right)\cdots}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(99*a^2*b^4 - 154*a*b^5 + 63*b^6)*(pi*floor((f*x + e)/pi + 1/2)*s
gn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))/((a^8 - 3*a^7*b + 3*a^6*b^2 - a^5
*b^3)*sqrt(a*b)) - 120*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 15*(19*a
*b^5*tan(f*x + e)^3 - 15*b^6*tan(f*x + e)^3 + 21*a^2*b^4*tan(f*x + e) - 17*
a*b^5*tan(f*x + e))/((a^7 - 2*a^6*b + a^5*b^2)*(b*tan(f*x + e)^2 + a)^2) -
8*(15*a^2*tan(f*x + e)^4 + 45*a*b*tan(f*x + e)^4 + 90*b^2*tan(f*x + e)^4 -
```

```
5*a^2*tan(f*x + e)^2 - 15*a*b*tan(f*x + e)^2 + 3*a^2)/(a^5*tan(f*x + e)^5))
/f
```

3.250 $\int \left(a+b\tan^2(c+dx)\right)^4 \, dx$

Optimal. Leaf size=115

$$\frac{b^2\left(6a^2-4ab+b^2\right)\tan^3(c+dx)}{3d}+\frac{b(2a-b)\left(2a^2-2ab+b^2\right)\tan(c+dx)}{d}+\frac{b^3(4a-b)\tan^5(c+dx)}{5d}+x(a-b)^4+\frac{b^4}{}$$

```
[Out] (a - b)^4*x + ((2*a - b)*b*(2*a^2 - 2*a*b + b^2)*Tan[c + d*x])/d + (b^2*(6*
a^2 - 4*a*b + b^2)*Tan[c + d*x]^3)/(3*d) + ((4*a - b)*b^3*Tan[c + d*x]^5)/(
5*d) + (b^4*Tan[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0732737, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3661, 390, 203}

$$\frac{b^2\left(6a^2-4ab+b^2\right)\tan^3(c+dx)}{3d}+\frac{b(2a-b)\left(2a^2-2ab+b^2\right)\tan(c+dx)}{d}+\frac{b^3(4a-b)\tan^5(c+dx)}{5d}+x(a-b)^4+\frac{b^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^2)^4,x]
```

```
[Out] (a - b)^4*x + ((2*a - b)*b*(2*a^2 - 2*a*b + b^2)*Tan[c + d*x])/d + (b^2*(6*
a^2 - 4*a*b + b^2)*Tan[c + d*x]^3)/(3*d) + ((4*a - b)*b^3*Tan[c + d*x]^5)/(
5*d) + (b^4*Tan[c + d*x]^7)/(7*d)
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\tan^2(c+dx)\right)^4 dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^4}{1+x^2} dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left((2a-b)b\left(2a^2-2ab+b^2\right)+b^2\left(6a^2-4ab+b^2\right)x^2+(4a-b)b^3x^4+b^4x^6+\frac{(a-b}{1+}\right.\right.}{d}$$
$$= \frac{(2a-b)b\left(2a^2-2ab+b^2\right)\tan(c+dx)}{d}+\frac{b^2\left(6a^2-4ab+b^2\right)\tan^3(c+dx)}{3d}+\frac{(4a-b)b^3\text{ta}}{5d}$$
$$= (a-b)^4x+\frac{(2a-b)b\left(2a^2-2ab+b^2\right)\tan(c+dx)}{d}+\frac{b^2\left(6a^2-4ab+b^2\right)\tan^3(c+dx)}{3d}+\frac{(4}{}$$

Mathematica [A] time = 1.88495, size = 137, normalized size = 1.19

$$\frac{\tan(c+dx)\left(b\left(35b\left(6a^2-4ab+b^2\right)\tan^2(c+dx)+105\left(-6a^2b+4a^3+4ab^2-b^3\right)+21b^2(4a-b)\tan^4(c+dx)+15b^3\text{ta}\right.\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^2)^4,x]
```

```
[Out] (Tan[c + d*x]*((105*(a - b)^4*ArcTanh[Sqrt[-Tan[c + d*x]^2]])/Sqrt[-Tan[c +
 d*x]^2] + b*(105*(4*a^3 - 6*a^2*b + 4*a*b^2 - b^3) + 35*b*(6*a^2 - 4*a*b +
 b^2)*Tan[c + d*x]^2 + 21*(4*a - b)*b^2*Tan[c + d*x]^4 + 15*b^3*Tan[c + d*x
]^6)))/(105*d)
```

Maple [B] time = 0.006, size = 242, normalized size = 2.1

$$\frac{b^4(\tan(dx+c))^7}{7d}+\frac{4(\tan(dx+c))^5ab^3}{5d}-\frac{(\tan(dx+c))^5b^4}{5d}+2\frac{(\tan(dx+c))^3a^2b^2}{d}-\frac{4(\tan(dx+c))^3ab^3}{3d}+\frac{(\tan(d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^2)^4,x)
```

```
[Out] 1/7*b^4*tan(d*x+c)^7/d+4/5/d*tan(d*x+c)^5*a*b^3-1/5/d*tan(d*x+c)^5*b^4+2/d*
tan(d*x+c)^3*a^2*b^2-4/3/d*tan(d*x+c)^3*a*b^3+1/3/d*tan(d*x+c)^3*b^4+4/d*ta
n(d*x+c)*a^3*b-6/d*a^2*b^2*tan(d*x+c)+4/d*a*b^3*tan(d*x+c)-1/d*b^4*tan(d*x+
c)+1/d*arctan(tan(d*x+c))*a^4-4/d*arctan(tan(d*x+c))*a^3*b+6/d*arctan(tan(d
*x+c))*a^2*b^2-4/d*arctan(tan(d*x+c))*a*b^3+1/d*arctan(tan(d*x+c))*b^4
```

Maxima [A] time = 1.69935, size = 219, normalized size = 1.9

$$a^4x-\frac{4(dx+c-\tan(dx+c))a^3b}{d}+\frac{2\left(\tan(dx+c)^3+3dx+3c-3\tan(dx+c)\right)a^2b^2}{d}+\frac{4\left(3\tan(dx+c)^5-5\tan(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] a^4*x - 4*(d*x + c - tan(d*x + c))*a^3*b/d + 2*(tan(d*x + c)^3 + 3*d*x + 3*
c - 3*tan(d*x + c))*a^2*b^2/d + 4/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 -
 15*d*x - 15*c + 15*tan(d*x + c))*a*b^3/d + 1/105*(15*tan(d*x + c)^7 - 21*t
an(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x + 105*c - 105*tan(d*x + c))*b^4
/d
```

Fricas [A] time = 1.41045, size = 308, normalized size = 2.68

$$\frac{15\,b^4\tan\left(dx+c\right)^7+21\left(4\,ab^3-b^4\right)\tan\left(dx+c\right)^5+35\left(6\,a^2b^2-4\,ab^3+b^4\right)\tan\left(dx+c\right)^3+105\left(a^4-4\,a^3b+6\,a^2b^2-\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^4*tan(d*x + c)^7 + 21*(4*a*b^3 - b^4)*tan(d*x + c)^5 + 35*(6*a^
2*b^2 - 4*a*b^3 + b^4)*tan(d*x + c)^3 + 105*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*
a*b^3 + b^4)*d*x + 105*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*tan(d*x + c))/
d
```

Sympy [A] time = 1.54244, size = 209, normalized size = 1.82

$$\begin{cases} a^4x - 4a^3bx + \frac{4a^3b\tan\left(c+dx\right)}{d} + 6a^2b^2x + \frac{2a^2b^2\tan^3\left(c+dx\right)}{d} - \frac{6a^2b^2\tan\left(c+dx\right)}{d} - 4ab^3x + \frac{4ab^3\tan^5\left(c+dx\right)}{5d} - \frac{4ab^3\tan^3\left(c+dx\right)}{3d} + \frac{4ab^3}{} \\ x\left(a+b\tan^2\left(c\right)\right)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)**2)**4,x)
```

```
[Out] Piecewise((a**4*x - 4*a**3*b*x + 4*a**3*b*tan(c + d*x)/d + 6*a**2*b**2*x +
2*a**2*b**2*tan(c + d*x)**3/d - 6*a**2*b**2*tan(c + d*x)/d - 4*a*b**3*x + 4
*a*b**3*tan(c + d*x)**5/(5*d) - 4*a*b**3*tan(c + d*x)**3/(3*d) + 4*a*b**3*t
an(c + d*x)/d + b**4*x + b**4*tan(c + d*x)**7/(7*d) - b**4*tan(c + d*x)**5/
(5*d) + b**4*tan(c + d*x)**3/(3*d) - b**4*tan(c + d*x)/d, Ne(d, 0)), (x*(a
+ b*tan(c)**2)**4, True))
```

Giac [B] time = 4.9932, size = 2982, normalized size = 25.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/105*(105*a^4*d*x*tan(d*x)^7*tan(c)^7 - 420*a^3*b*d*x*tan(d*x)^7*tan(c)^7
+ 630*a^2*b^2*d*x*tan(d*x)^7*tan(c)^7 - 420*a*b^3*d*x*tan(d*x)^7*tan(c)^7 +
 105*b^4*d*x*tan(d*x)^7*tan(c)^7 - 735*a^4*d*x*tan(d*x)^6*tan(c)^6 + 2940*a
^3*b*d*x*tan(d*x)^6*tan(c)^6 - 4410*a^2*b^2*d*x*tan(d*x)^6*tan(c)^6 + 2940*
a*b^3*d*x*tan(d*x)^6*tan(c)^6 - 735*b^4*d*x*tan(d*x)^6*tan(c)^6 - 420*a^3*b
*tan(d*x)^7*tan(c)^6 + 630*a^2*b^2*tan(d*x)^7*tan(c)^6 - 420*a*b^3*tan(d*x)
^7*tan(c)^6 + 105*b^4*tan(d*x)^7*tan(c)^6 - 420*a^3*b*tan(d*x)^6*tan(c)^7 +
```

```
630*a^2*b^2*tan(d*x)^6*tan(c)^7 - 420*a*b^3*tan(d*x)^6*tan(c)^7 + 105*b^4*
tan(d*x)^6*tan(c)^7 + 2205*a^4*d*x*tan(d*x)^5*tan(c)^5 - 8820*a^3*b*d*x*tan
(d*x)^5*tan(c)^5 + 13230*a^2*b^2*d*x*tan(d*x)^5*tan(c)^5 - 8820*a*b^3*d*x*t
an(d*x)^5*tan(c)^5 + 2205*b^4*d*x*tan(d*x)^5*tan(c)^5 - 210*a^2*b^2*tan(d*x
)^7*tan(c)^4 + 140*a*b^3*tan(d*x)^7*tan(c)^4 - 35*b^4*tan(d*x)^7*tan(c)^4 +
 2520*a^3*b*tan(d*x)^6*tan(c)^5 - 4410*a^2*b^2*tan(d*x)^6*tan(c)^5 + 2940*a
*b^3*tan(d*x)^6*tan(c)^5 - 735*b^4*tan(d*x)^6*tan(c)^5 + 2520*a^3*b*tan(d*x
)^5*tan(c)^6 - 4410*a^2*b^2*tan(d*x)^5*tan(c)^6 + 2940*a*b^3*tan(d*x)^5*tan
(c)^6 - 735*b^4*tan(d*x)^5*tan(c)^6 - 210*a^2*b^2*tan(d*x)^4*tan(c)^7 + 140
*a*b^3*tan(d*x)^4*tan(c)^7 - 35*b^4*tan(d*x)^4*tan(c)^7 - 3675*a^4*d*x*tan(
d*x)^4*tan(c)^4 + 14700*a^3*b*d*x*tan(d*x)^4*tan(c)^4 - 22050*a^2*b^2*d*x*t
an(d*x)^4*tan(c)^4 + 14700*a*b^3*d*x*tan(d*x)^4*tan(c)^4 - 3675*b^4*d*x*tan
(d*x)^4*tan(c)^4 - 84*a*b^3*tan(d*x)^7*tan(c)^2 + 21*b^4*tan(d*x)^7*tan(c)^
2 + 840*a^2*b^2*tan(d*x)^6*tan(c)^3 - 980*a*b^3*tan(d*x)^6*tan(c)^3 + 245*b
^4*tan(d*x)^6*tan(c)^3 - 6300*a^3*b*tan(d*x)^5*tan(c)^4 + 11970*a^2*b^2*tan
(d*x)^5*tan(c)^4 - 8820*a*b^3*tan(d*x)^5*tan(c)^4 + 2205*b^4*tan(d*x)^5*tan
(c)^4 - 6300*a^3*b*tan(d*x)^4*tan(c)^5 + 11970*a^2*b^2*tan(d*x)^4*tan(c)^5
- 8820*a*b^3*tan(d*x)^4*tan(c)^5 + 2205*b^4*tan(d*x)^4*tan(c)^5 + 840*a^2*b
^2*tan(d*x)^3*tan(c)^6 - 980*a*b^3*tan(d*x)^3*tan(c)^6 + 245*b^4*tan(d*x)^3
*tan(c)^6 - 84*a*b^3*tan(d*x)^2*tan(c)^7 + 21*b^4*tan(d*x)^2*tan(c)^7 + 367
5*a^4*d*x*tan(d*x)^3*tan(c)^3 - 14700*a^3*b*d*x*tan(d*x)^3*tan(c)^3 + 22050
*a^2*b^2*d*x*tan(d*x)^3*tan(c)^3 - 14700*a*b^3*d*x*tan(d*x)^3*tan(c)^3 + 36
75*b^4*d*x*tan(d*x)^3*tan(c)^3 - 15*b^4*tan(d*x)^7 + 168*a*b^3*tan(d*x)^6*t
an(c) - 147*b^4*tan(d*x)^6*tan(c) - 1260*a^2*b^2*tan(d*x)^5*tan(c)^2 + 1680
*a*b^3*tan(d*x)^5*tan(c)^2 - 735*b^4*tan(d*x)^5*tan(c)^2 + 8400*a^3*b*tan(d
*x)^4*tan(c)^3 - 16380*a^2*b^2*tan(d*x)^4*tan(c)^3 + 12600*a*b^3*tan(d*x)^4
*tan(c)^3 - 3675*b^4*tan(d*x)^4*tan(c)^3 + 8400*a^3*b*tan(d*x)^3*tan(c)^4 -
 16380*a^2*b^2*tan(d*x)^3*tan(c)^4 + 12600*a*b^3*tan(d*x)^3*tan(c)^4 - 3675
*b^4*tan(d*x)^3*tan(c)^4 - 1260*a^2*b^2*tan(d*x)^2*tan(c)^5 + 1680*a*b^3*ta
n(d*x)^2*tan(c)^5 - 735*b^4*tan(d*x)^2*tan(c)^5 + 168*a*b^3*tan(d*x)*tan(c)
^6 - 147*b^4*tan(d*x)*tan(c)^6 - 15*b^4*tan(c)^7 - 2205*a^4*d*x*tan(d*x)^2*
tan(c)^2 + 8820*a^3*b*d*x*tan(d*x)^2*tan(c)^2 - 13230*a^2*b^2*d*x*tan(d*x)^
2*tan(c)^2 + 8820*a*b^3*d*x*tan(d*x)^2*tan(c)^2 - 2205*b^4*d*x*tan(d*x)^2*t
an(c)^2 - 84*a*b^3*tan(d*x)^5 + 21*b^4*tan(d*x)^5 + 840*a^2*b^2*tan(d*x)^4*
tan(c) - 980*a*b^3*tan(d*x)^4*tan(c) + 245*b^4*tan(d*x)^4*tan(c) - 6300*a^3
*b*tan(d*x)^3*tan(c)^2 + 11970*a^2*b^2*tan(d*x)^3*tan(c)^2 - 8820*a*b^3*tan
(d*x)^3*tan(c)^2 + 2205*b^4*tan(d*x)^3*tan(c)^2 - 6300*a^3*b*tan(d*x)^2*tan
(c)^3 + 11970*a^2*b^2*tan(d*x)^2*tan(c)^3 - 8820*a*b^3*tan(d*x)^2*tan(c)^3
+ 2205*b^4*tan(d*x)^2*tan(c)^3 + 840*a^2*b^2*tan(d*x)*tan(c)^4 - 980*a*b^3*
tan(d*x)*tan(c)^4 + 245*b^4*tan(d*x)*tan(c)^4 - 84*a*b^3*tan(c)^5 + 21*b^4*
tan(c)^5 + 735*a^4*d*x*tan(d*x)*tan(c) - 2940*a^3*b*d*x*tan(d*x)*tan(c) + 4
410*a^2*b^2*d*x*tan(d*x)*tan(c) - 2940*a*b^3*d*x*tan(d*x)*tan(c) + 735*b^4*
d*x*tan(d*x)*tan(c) - 210*a^2*b^2*tan(d*x)^3 + 140*a*b^3*tan(d*x)^3 - 35*b^
4*tan(d*x)^3 + 2520*a^3*b*tan(d*x)^2*tan(c) - 4410*a^2*b^2*tan(d*x)^2*tan(c
) + 2940*a*b^3*tan(d*x)^2*tan(c) - 735*b^4*tan(d*x)^2*tan(c) + 2520*a^3*b*t
an(d*x)*tan(c)^2 - 4410*a^2*b^2*tan(d*x)*tan(c)^2 + 2940*a*b^3*tan(d*x)*tan
(c)^2 - 735*b^4*tan(d*x)*tan(c)^2 - 210*a^2*b^2*tan(c)^3 + 140*a*b^3*tan(c)
^3 - 35*b^4*tan(c)^3 - 105*a^4*d*x + 420*a^3*b*d*x - 630*a^2*b^2*d*x + 420*
a*b^3*d*x - 105*b^4*d*x - 420*a^3*b*tan(d*x) + 630*a^2*b^2*tan(d*x) - 420*a
*b^3*tan(d*x) + 105*b^4*tan(d*x) - 420*a^3*b*tan(c) + 630*a^2*b^2*tan(c) -
420*a*b^3*tan(c) + 105*b^4*tan(c))/(d*tan(d*x)^7*tan(c)^7 - 7*d*tan(d*x)^6*
tan(c)^6 + 21*d*tan(d*x)^5*tan(c)^5 - 35*d*tan(d*x)^4*tan(c)^4 + 35*d*tan(d
*x)^3*tan(c)^3 - 21*d*tan(d*x)^2*tan(c)^2 + 7*d*tan(d*x)*tan(c) - d)
```

3.251 $\int \left(a+b\tan^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=77

$$\frac{b\left(3a^2-3ab+b^2\right)\tan(c+dx)}{d}+\frac{b^2(3a-b)\tan^3(c+dx)}{3d}+x(a-b)^3+\frac{b^3\tan^5(c+dx)}{5d}$$

```
[Out] (a - b)^3*x + (b*(3*a^2 - 3*a*b + b^2)*Tan[c + d*x])/d + ((3*a - b)*b^2*Tan
[c + d*x]^3)/(3*d) + (b^3*Tan[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.048896, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3661, 390, 203}

$$\frac{b\left(3a^2-3ab+b^2\right)\tan(c+dx)}{d}+\frac{b^2(3a-b)\tan^3(c+dx)}{3d}+x(a-b)^3+\frac{b^3\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^2)^3,x]
```

```
[Out] (a - b)^3*x + (b*(3*a^2 - 3*a*b + b^2)*Tan[c + d*x])/d + ((3*a - b)*b^2*Tan
[c + d*x]^3)/(3*d) + (b^3*Tan[c + d*x]^5)/(5*d)
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\tan^2(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{1+x^2} dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(b\left(3a^2-3ab+b^2\right)+(3a-b)b^2x^2+b^3x^4+\frac{(a-b)^3}{1+x^2}\right) dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{b\left(3a^2-3ab+b^2\right)\tan(c+dx)}{d}+\frac{(3a-b)b^2\tan^3(c+dx)}{3d}+\frac{b^3\tan^5(c+dx)}{5d}+\frac{(a-b)^3\,\text{Sul}}{}$$
$$= (a-b)^3x+\frac{b\left(3a^2-3ab+b^2\right)\tan(c+dx)}{d}+\frac{(3a-b)b^2\tan^3(c+dx)}{3d}+\frac{b^3\tan^5(c+dx)}{5d}$$

Mathematica [A] time = 0.921627, size = 102, normalized size = 1.32

$$\frac{\tan(c+dx)\left(b\left(45a^2-15ab\left(3-\tan^2(c+dx)\right)+b^2\left(3\tan^4(c+dx)-5\tan^2(c+dx)+15\right)\right)+\frac{15(a-b)^3\tanh^{-1}\left(\sqrt{-\tan^2(c+dx)}\right)}{\sqrt{-\tan^2(c+dx)}}\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^2)^3,x]
```

```
[Out] (Tan[c + d*x]*((15*(a - b)^3*ArcTanh[Sqrt[-Tan[c + d*x]^2]])/Sqrt[-Tan[c +
d*x]^2] + b*(45*a^2 - 15*a*b*(3 - Tan[c + d*x]^2) + b^2*(15 - 5*Tan[c + d*x
]^2 + 3*Tan[c + d*x]^4))))/(15*d)
```

Maple [B] time = 0.003, size = 154, normalized size = 2.

$$\frac{b^3(\tan(dx+c))^5}{5d}+\frac{ab^2(\tan(dx+c))^3}{d}-\frac{(\tan(dx+c))^3b^3}{3d}+3\frac{\tan(dx+c)a^2b}{d}-3\frac{ab^2\tan(dx+c)}{d}+\frac{b^3\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^2)^3,x)
```

```
[Out] 1/5*b^3*tan(d*x+c)^5/d+a*b^2*tan(d*x+c)^3/d-1/3/d*tan(d*x+c)^3*b^3+3/d*tan(
d*x+c)*a^2*b-3*a*b^2*tan(d*x+c)/d+1/d*b^3*tan(d*x+c)+1/d*arctan(tan(d*x+c))
*a^3-3/d*arctan(tan(d*x+c))*a^2*b+3/d*arctan(tan(d*x+c))*a*b^2-1/d*arctan(t
an(d*x+c))*b^3
```

Maxima [A] time = 1.6906, size = 140, normalized size = 1.82

$$a^3x-\frac{3(dx+c-\tan(dx+c))a^2b}{d}+\frac{\left(\tan(dx+c)^3+3dx+3c-3\tan(dx+c)\right)ab^2}{d}+\frac{\left(3\tan(dx+c)^5-5\tan(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 3*(d*x + c - tan(d*x + c))*a^2*b/d + (tan(d*x + c)^3 + 3*d*x + 3*c
- 3*tan(d*x + c))*a*b^2/d + 1/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*
```

```
d*x - 15*c + 15*tan(d*x + c))*b^3/d
```

Fricas [A] time = 1.40855, size = 204, normalized size = 2.65

$$\frac{3\,b^{3}\tan\left(dx+c\right)^{5}+5\left(3\,ab^{2}-b^{3}\right)\tan\left(dx+c\right)^{3}+15\left(a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}\right)dx+15\left(3\,a^{2}b-3\,ab^{2}+b^{3}\right)\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^3*tan(d*x + c)^5 + 5*(3*a*b^2 - b^3)*tan(d*x + c)^3 + 15*(a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*d*x + 15*(3*a^2*b - 3*a*b^2 + b^3)*tan(d*x + c))/d
```

Sympy [A] time = 0.838256, size = 126, normalized size = 1.64

$$\begin{cases} a^{3}x-3a^{2}bx+\frac{3a^{2}b\tan\left(c+dx\right)}{d}+3ab^{2}x+\frac{ab^{2}\tan^{3}\left(c+dx\right)}{d}-\frac{3ab^{2}\tan\left(c+dx\right)}{d}-b^{3}x+\frac{b^{3}\tan^{5}\left(c+dx\right)}{5d}-\frac{b^{3}\tan^{3}\left(c+dx\right)}{3d}+\frac{b^{3}\tan\left(c+dx\right)}{d} \\ x\left(a+b\tan^{2}\left(c\right)\right)^{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x - 3*a**2*b*x + 3*a**2*b*tan(c + d*x)/d + 3*a*b**2*x + a*b
**2*tan(c + d*x)**3/d - 3*a*b**2*tan(c + d*x)/d - b**3*x + b**3*tan(c + d*x
)**5/(5*d) - b**3*tan(c + d*x)**3/(3*d) + b**3*tan(c + d*x)/d, Ne(d, 0)), (
x*(a + b*tan(c)**2)**3, True))
```

Giac [B] time = 2.33848, size = 1386, normalized size = 18.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*a^3*d*x*tan(d*x)^5*tan(c)^5 - 45*a^2*b*d*x*tan(d*x)^5*tan(c)^5 + 4
5*a*b^2*d*x*tan(d*x)^5*tan(c)^5 - 15*b^3*d*x*tan(d*x)^5*tan(c)^5 - 75*a^3*d
*x*tan(d*x)^4*tan(c)^4 + 225*a^2*b*d*x*tan(d*x)^4*tan(c)^4 - 225*a*b^2*d*x*
tan(d*x)^4*tan(c)^4 + 75*b^3*d*x*tan(d*x)^4*tan(c)^4 - 45*a^2*b*tan(d*x)^5*
tan(c)^4 + 45*a*b^2*tan(d*x)^5*tan(c)^4 - 15*b^3*tan(d*x)^5*tan(c)^4 - 45*a
^2*b*tan(d*x)^4*tan(c)^5 + 45*a*b^2*tan(d*x)^4*tan(c)^5 - 15*b^3*tan(d*x)^4
*tan(c)^5 + 150*a^3*d*x*tan(d*x)^3*tan(c)^3 - 450*a^2*b*d*x*tan(d*x)^3*tan(
c)^3 + 450*a*b^2*d*x*tan(d*x)^3*tan(c)^3 - 150*b^3*d*x*tan(d*x)^3*tan(c)^3
- 15*a*b^2*tan(d*x)^5*tan(c)^2 + 5*b^3*tan(d*x)^5*tan(c)^2 + 180*a^2*b*tan(
d*x)^4*tan(c)^3 - 225*a*b^2*tan(d*x)^4*tan(c)^3 + 75*b^3*tan(d*x)^4*tan(c)^
3 + 180*a^2*b*tan(d*x)^3*tan(c)^4 - 225*a*b^2*tan(d*x)^3*tan(c)^4 + 75*b^3*
tan(d*x)^3*tan(c)^4 - 15*a*b^2*tan(d*x)^2*tan(c)^5 + 5*b^3*tan(d*x)^2*tan(c
)^5 - 150*a^3*d*x*tan(d*x)^2*tan(c)^2 + 450*a^2*b*d*x*tan(d*x)^2*tan(c)^2 -
 450*a*b^2*d*x*tan(d*x)^2*tan(c)^2 + 150*b^3*d*x*tan(d*x)^2*tan(c)^2 - 3*b^
3*tan(d*x)^5 + 30*a*b^2*tan(d*x)^4*tan(c) - 25*b^3*tan(d*x)^4*tan(c) - 270*
```

```
a^2*b*tan(d*x)^3*tan(c)^2 + 360*a*b^2*tan(d*x)^3*tan(c)^2 - 150*b^3*tan(d*x
)^3*tan(c)^2 - 270*a^2*b*tan(d*x)^2*tan(c)^3 + 360*a*b^2*tan(d*x)^2*tan(c)^
3 - 150*b^3*tan(d*x)^2*tan(c)^3 + 30*a*b^2*tan(d*x)*tan(c)^4 - 25*b^3*tan(d
*x)*tan(c)^4 - 3*b^3*tan(c)^5 + 75*a^3*d*x*tan(d*x)*tan(c) - 225*a^2*b*d*x*
tan(d*x)*tan(c) + 225*a*b^2*d*x*tan(d*x)*tan(c) - 75*b^3*d*x*tan(d*x)*tan(c
) - 15*a*b^2*tan(d*x)^3 + 5*b^3*tan(d*x)^3 + 180*a^2*b*tan(d*x)^2*tan(c) -
225*a*b^2*tan(d*x)^2*tan(c) + 75*b^3*tan(d*x)^2*tan(c) + 180*a^2*b*tan(d*x)
*tan(c)^2 - 225*a*b^2*tan(d*x)*tan(c)^2 + 75*b^3*tan(d*x)*tan(c)^2 - 15*a*b
^2*tan(c)^3 + 5*b^3*tan(c)^3 - 15*a^3*d*x + 45*a^2*b*d*x - 45*a*b^2*d*x + 1
5*b^3*d*x - 45*a^2*b*tan(d*x) + 45*a*b^2*tan(d*x) - 15*b^3*tan(d*x) - 45*a^
2*b*tan(c) + 45*a*b^2*tan(c) - 15*b^3*tan(c))/(d*tan(d*x)^5*tan(c)^5 - 5*d*
tan(d*x)^4*tan(c)^4 + 10*d*tan(d*x)^3*tan(c)^3 - 10*d*tan(d*x)^2*tan(c)^2 +
 5*d*tan(d*x)*tan(c) - d)
```

3.252 $\int \left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=46

$$\frac{b(2a-b)\tan(c+dx)}{d}+x(a-b)^2+\frac{b^2\tan^3(c+dx)}{3d}$$

```
[Out] (a - b)^2*x + ((2*a - b)*b*Tan[c + d*x])/d + (b^2*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0312722, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3661, 390, 203}

$$\frac{b(2a-b)\tan(c+dx)}{d}+x(a-b)^2+\frac{b^2\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a - b)^2*x + ((2*a - b)*b*Tan[c + d*x])/d + (b^2*Tan[c + d*x]^3)/(3*d)
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tan^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left((2a-b)b+b^2x^2+\frac{(a-b)^2}{1+x^2}\right)dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{(2a-b)b\tan(c+dx)}{d}+\frac{b^2\tan^3(c+dx)}{3d}+\frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}\\ &= (a-b)^2x+\frac{(2a-b)b\tan(c+dx)}{d}+\frac{b^2\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.573146, size = 73, normalized size = 1.59

$$\frac{\tan(c+dx)\left(b\left(6a-b\left(3-\tan^2(c+dx)\right)\right)+\frac{3(a-b)^2\tanh^{-1}\left(\sqrt{-\tan^2(c+dx)}\right)}{\sqrt{-\tan^2(c+dx)}}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (Tan[c + d*x]*((3*(a - b)^2*ArcTanh[Sqrt[-Tan[c + d*x]^2]])/Sqrt[-Tan[c + d
*x]^2] + b*(6*a - b*(3 - Tan[c + d*x]^2))))/(3*d)
```

Maple [A] time = 0.004, size = 87, normalized size = 1.9

$$\frac{b^2(\tan(dx+c))^3}{3d}+2\frac{a\tan(dx+c)\,b}{d}-\frac{b^2\tan(dx+c)}{d}+\frac{\arctan(\tan(dx+c))\,a^2}{d}-2\frac{\arctan(\tan(dx+c))\,ab}{d}+\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/3*b^2*tan(d*x+c)^3/d+2/d*tan(d*x+c)*a*b-b^2*tan(d*x+c)/d+1/d*arctan(tan(d
*x+c))*a^2-2/d*arctan(tan(d*x+c))*a*b+1/d*arctan(tan(d*x+c))*b^2
```

Maxima [A] time = 1.69442, size = 78, normalized size = 1.7

$$a^2x-\frac{2(dx+c-\tan(dx+c))ab}{d}+\frac{\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)b^2}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x - 2*(d*x + c - tan(d*x + c))*a*b/d + 1/3*(tan(d*x + c)^3 + 3*d*x + 3*
c - 3*tan(d*x + c))*b^2/d
```

Fricas [A] time = 1.33751, size = 117, normalized size = 2.54

$$\frac{b^2\tan(dx+c)^3+3\left(a^2-2\,ab+b^2\right)dx+3\left(2\,ab-b^2\right)\tan(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*tan(d*x + c)^3 + 3*(a^2 - 2*a*b + b^2)*d*x + 3*(2*a*b - b^2)*tan(d
*x + c))/d
```

Sympy [A] time = 0.396985, size = 68, normalized size = 1.48

$$\begin{cases} a^2x - 2abx + \frac{2ab\tan(c+dx)}{d} + b^2x + \frac{b^2\tan^3(c+dx)}{3d} - \frac{b^2\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a + b\tan^2(c)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x - 2*a*b*x + 2*a*b*tan(c + d*x)/d + b**2*x + b**2*tan(c +
d*x)**3/(3*d) - b**2*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c)**2)**2, Tr
ue))
```

Giac [B] time = 1.46966, size = 485, normalized size = 10.54

$$\frac{3\,a^2dx\tan(dx)^3\tan(c)^3 - 6\,abdx\tan(dx)^3\tan(c)^3 + 3\,b^2dx\tan(dx)^3\tan(c)^3 - 9\,a^2dx\tan(dx)^2\tan(c)^2 + 18\,abdx\,\mathrm{t}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*d*x*tan(d*x)^3*tan(c)^3 - 6*a*b*d*x*tan(d*x)^3*tan(c)^3 + 3*b^2*
d*x*tan(d*x)^3*tan(c)^3 - 9*a^2*d*x*tan(d*x)^2*tan(c)^2 + 18*a*b*d*x*tan(d*
x)^2*tan(c)^2 - 9*b^2*d*x*tan(d*x)^2*tan(c)^2 - 6*a*b*tan(d*x)^3*tan(c)^2 +
 3*b^2*tan(d*x)^3*tan(c)^2 - 6*a*b*tan(d*x)^2*tan(c)^3 + 3*b^2*tan(d*x)^2*t
an(c)^3 + 9*a^2*d*x*tan(d*x)*tan(c) - 18*a*b*d*x*tan(d*x)*tan(c) + 9*b^2*d*
x*tan(d*x)*tan(c) - b^2*tan(d*x)^3 + 12*a*b*tan(d*x)^2*tan(c) - 9*b^2*tan(d
*x)^2*tan(c) + 12*a*b*tan(d*x)*tan(c)^2 - 9*b^2*tan(d*x)*tan(c)^2 - b^2*tan
(c)^3 - 3*a^2*d*x + 6*a*b*d*x - 3*b^2*d*x - 6*a*b*tan(d*x) + 3*b^2*tan(d*x)
 - 6*a*b*tan(c) + 3*b^2*tan(c))/(d*tan(d*x)^3*tan(c)^3 - 3*d*tan(d*x)^2*tan
(c)^2 + 3*d*tan(d*x)*tan(c) - d)
```

3.253 $\int \left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=19

$$ax+\frac{b\tan(c+dx)}{d}-bx$$

```
[Out] a*x - b*x + (b*Tan[c + d*x])/d
```

Rubi [A] time = 0.0129866, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3473, 8}

$$ax+\frac{b\tan(c+dx)}{d}-bx$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Tan[c + d*x]^2,x]
```

```
[Out] a*x - b*x + (b*Tan[c + d*x])/d
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tan^2(c+dx)\right)\,dx &= ax+b\int \tan^2(c+dx)\,dx\\ &= ax+\frac{b\tan(c+dx)}{d}-b\int 1\,dx\\ &= ax-bx+\frac{b\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0063722, size = 28, normalized size = 1.47

$$ax-\frac{b\tan^{-1}(\tan(c+dx))}{d}+\frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Tan[c + d*x]^2,x]
```

```
[Out] a*x - (b*ArcTan[Tan[c + d*x]])/d + (b*Tan[c + d*x])/d
```

Maple [A] time = 0.001, size = 29, normalized size = 1.5

$$ax+\frac{b\tan(dx+c)}{d}-\frac{b\arctan(\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*tan(d*x+c)^2,x)
```

```
[Out] a*x+b*tan(d*x+c)/d-b/d*arctan(tan(d*x+c))
```

Maxima [A] time = 1.63292, size = 31, normalized size = 1.63

$$ax-\frac{(dx+c-\tan(dx+c))b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] a*x - (d*x + c - tan(d*x + c))*b/d
```

Fricas [A] time = 1.38478, size = 46, normalized size = 2.42

$$\frac{(a-b)dx+b\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] ((a - b)*d*x + b*tan(d*x + c))/d
```

Sympy [A] time = 0.156409, size = 20, normalized size = 1.05

$$ax+b\left(\begin{cases}-x+\frac{\tan(c+dx)}{d} & \text{for } d\neq 0\\ x\tan^2(c) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)**2,x)
```

```
[Out] a*x + b*Piecewise((-x + tan(c + d*x)/d, Ne(d, 0)), (x*tan(c)**2, True))
```

Giac [B] time = 1.26503, size = 312, normalized size = 16.42

$$ax+\frac{\left(\pi-4\,dx\tan(dx)\tan(c)-\pi\mathrm{sgn}\left(2\,\tan(dx)^2\tan(c)+2\,\tan(dx)\tan(c)^2-2\,\tan(dx)-2\,\tan(c)\right)\tan(dx)\,\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] a*x + 1/4*(pi - 4*d*x*tan(d*x)*tan(c) - pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(
d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c) - pi*tan(d*x)*tan(c)
 + 2*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c) + 2*
arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)*tan(c) + 4*d*x +
 pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))
- 2*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c))) - 2*arctan((tan(d*x)
+ tan(c))/(tan(d*x)*tan(c) - 1)) - 4*tan(d*x) - 4*tan(c))*b/(d*tan(d*x)*tan
(c) - d)
```

3.254 $\int \frac{1}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=50

$$\frac{x}{a-b}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)}$$

```
[Out] x/(a - b) - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a -
b)*d)
```

Rubi [A] time = 0.0736336, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3660, 3675, 205}

$$\frac{x}{a-b}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^2)^(-1),x]
```

```
[Out] x/(a - b) - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a -
b)*d)
```

Rule 3660

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/(a -
 b), x] - Dist[b/(a - b), Int[Sec[e + f*x]^2/(a + b*Tan[e + f*x]^2), x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a, b]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+b\tan^2(c+dx)}\,dx = \frac{x}{a-b} - \frac{b\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx}{a-b}$$
$$= \frac{x}{a-b} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{(a-b)d}$$
$$= \frac{x}{a-b} - \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)d}$$

Mathematica [A] time = 0.0702576, size = 49, normalized size = 0.98

$$\frac{\tan^{-1}(\tan(c+dx)) - \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}}}{ad-bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^2)^(-1),x]
```

```
[Out] (ArcTan[Tan[c + d*x]] - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/Sq
rt[a])/(a*d - b*d)
```

Maple [A] time = 0.02, size = 52, normalized size = 1.

$$-\frac{b}{d(a-b)}\arctan\left(b\tan(dx+c)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{\arctan(\tan(dx+c))}{d(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^2),x)
```

```
[Out] -1/d/(a-b)*b/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))+1/d/(a-b)*arctan(
tan(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49304, size = 406, normalized size = 8.12

$$\left[\frac{4\,dx - \sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(dx+c)^4 - 6\,ab\tan(dx+c)^2 + a^2 + 4\left(ab\tan(dx+c)^3 - a^2\tan(dx+c)\right)\sqrt{-\frac{b}{a}}}{b^2\tan(dx+c)^4 + 2\,ab\tan(dx+c)^2 + a^2}\right)}{4(a-b)d}, \frac{2\,dx - \sqrt{\frac{b}{a}}\arctan\left(\frac{\left(b\tan(dx+c)^2 - a\right)\sqrt{\frac{b}{a}}}{2\,b\tan(dx+c)}\right)}{2(a-b)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*d*x - sqrt(-b/a)*log((b^2*tan(d*x + c)^4 - 6*a*b*tan(d*x + c)^2 + a
^2 + 4*(a*b*tan(d*x + c)^3 - a^2*tan(d*x + c))*sqrt(-b/a))/(b^2*tan(d*x + c
)^4 + 2*a*b*tan(d*x + c)^2 + a^2)))/((a - b)*d), 1/2*(2*d*x - sqrt(b/a)*arc
tan(1/2*(b*tan(d*x + c)^2 - a)*sqrt(b/a)/(b*tan(d*x + c))))/((a - b)*d)]
```

Sympy [A] time = 3.16271, size = 280, normalized size = 5.6

$$\begin{cases} \frac{\tilde{\infty}x}{\tan^2(c)} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{x}{a} & \text{for } b=0 \\ \frac{-x-\frac{1}{d\tan(c+dx)}}{b} & \text{for } a=0 \\ \frac{dx\tan^2(c+dx)}{2bd\tan^2(c+dx)+2bd}+\frac{dx}{2bd\tan^2(c+dx)+2bd}+\frac{\tan(c+dx)}{2bd\tan^2(c+dx)+2bd} & \text{for } a=b \\ \frac{x}{a+b\tan^2(c)} & \text{for } d=0 \\ \frac{2i\sqrt{a}dx\sqrt{\frac{1}{b}}}{2ia^{\frac{3}{2}}d\sqrt{\frac{1}{b}}-2i\sqrt{a}bd\sqrt{\frac{1}{b}}}-\frac{\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\tan(c+dx)\right)}{2ia^{\frac{3}{2}}d\sqrt{\frac{1}{b}}-2i\sqrt{a}bd\sqrt{\frac{1}{b}}}+\frac{\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\tan(c+dx)\right)}{2ia^{\frac{3}{2}}d\sqrt{\frac{1}{b}}-2i\sqrt{a}bd\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x/tan(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a, Eq(b, 0)
), ((-x - 1/(d*tan(c + d*x)))/b, Eq(a, 0)), (d*x*tan(c + d*x)**2/(2*b*d*tan
(c + d*x)**2 + 2*b*d) + d*x/(2*b*d*tan(c + d*x)**2 + 2*b*d) + tan(c + d*x)/
(2*b*d*tan(c + d*x)**2 + 2*b*d), Eq(a, b)), (x/(a + b*tan(c)**2), Eq(d, 0))
, (2*I*sqrt(a)*d*x*sqrt(1/b)/(2*I*a**(3/2)*d*sqrt(1/b) - 2*I*sqrt(a)*b*d*sq
rt(1/b)) - log(-I*sqrt(a)*sqrt(1/b) + tan(c + d*x))/(2*I*a**(3/2)*d*sqrt(1/
b) - 2*I*sqrt(a)*b*d*sqrt(1/b)) + log(I*sqrt(a)*sqrt(1/b) + tan(c + d*x))/(
2*I*a**(3/2)*d*sqrt(1/b) - 2*I*sqrt(a)*b*d*sqrt(1/b)), True))
```

Giac [B] time = 1.31467, size = 225, normalized size = 4.5

$$-\frac{2\left(\frac{\sqrt{ab}\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan(dx+c)}{\sqrt{\frac{a+b-\sqrt{(a+b)^2-4ab}}{b}}}\right)\right)|b|}{(a-b)^2b-\left(ab+b^2\right)|-a+b|}+\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan(dx+c)}{\sqrt{\frac{a+b+\sqrt{(a+b)^2-4ab}}{b}}}\right)\right)b}{(a-b)^2+a|-a+b|+b|-a+b|}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2*(sqrt(a*b)*(pi*floor((d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(d*x +
c)/sqrt((a + b - sqrt((a + b)^2 - 4*a*b))/b)))*abs(b)/((a - b)^2*b - (a*b +
 b^2)*abs(-a + b)) + (pi*floor((d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan
(d*x + c)/sqrt((a + b + sqrt((a + b)^2 - 4*a*b))/b)))*b/((a - b)^2 + a*abs(
-a + b) + b*abs(-a + b)))/d
```

3.255 $\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt{b}(3a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^2}-\frac{b\tan(c+dx)}{2ad(a-b)\left(a+b\tan^2(c+dx)\right)}+\frac{x}{(a-b)^2}$$

```
[Out] x/(a - b)^2 - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(2
*a^(3/2)*(a - b)^2*d) - (b*Tan[c + d*x])/(2*a*(a - b)*d*(a + b*Tan[c + d*x]
^2))
```

Rubi [A] time = 0.0920903, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3661, 414, 522, 203, 205}

$$-\frac{\sqrt{b}(3a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^2}-\frac{b\tan(c+dx)}{2ad(a-b)\left(a+b\tan^2(c+dx)\right)}+\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^2)^(-2),x]
```

```
[Out] x/(a - b)^2 - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(2
*a^(3/2)*(a - b)^2*d) - (b*Tan[c + d*x])/(2*a*(a - b)*d*(a + b*Tan[c + d*x]
^2))
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+bx^2\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b\tan(c+dx)}{2a(a-b)d\left(a+b\tan^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-b-bx^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(c+dx)\right)}{2a(a-b)d}$$

$$= -\frac{b\tan(c+dx)}{2a(a-b)d\left(a+b\tan^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{(a-b)^2d} - \frac{((3a-b)b)\,\text{Su}}{}$$

$$= \frac{x}{(a-b)^2} - \frac{(3a-b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2d} - \frac{b\tan(c+dx)}{2a(a-b)d\left(a+b\tan^2(c+dx)\right)}$$

Mathematica [A] time = 0.993233, size = 88, normalized size = 0.91

$$\frac{\frac{\sqrt{b}(b-3a)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{b(b-a)\tan(c+dx)}{a\left(a+b\tan^2(c+dx)\right)} + 2\tan^{-1}(\tan(c+dx))}{2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^2)^(-2),x]
```

```
[Out] (2*ArcTan[Tan[c + d*x]] + (Sqrt[b]*(-3*a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])
/Sqrt[a]])/a^(3/2) + (b*(-a + b)*Tan[c + d*x])/(a*(a + b*Tan[c + d*x]^2)))/
(2*(a - b)^2*d)
```

Maple [A] time = 0.027, size = 160, normalized size = 1.7

$$-\frac{b\tan(dx+c)}{2\,d\,(a-b)^2\left(a+b\,(\tan(dx+c))^2\right)} + \frac{b^2\tan(dx+c)}{2\,d\,(a-b)^2\,a\left(a+b\,(\tan(dx+c))^2\right)} - \frac{3\,b}{2\,d\,(a-b)^2}\arctan\left(b\tan(dx+c)\frac{1}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d/(a-b)^2*b*tan(d*x+c)/(a+b*tan(d*x+c)^2)+1/2/d/(a-b)^2*b^2/a*tan(d*x+
c)/(a+b*tan(d*x+c)^2)-3/2/d/(a-b)^2*b/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)
^(1/2))+1/2/d/(a-b)^2*b^2/a/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))+1/
```

```
d/(a-b)^2*arctan(tan(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52268, size = 865, normalized size = 8.92

$$\left[\frac{8\,abdx\tan\left(dx+c\right)^2+8\,a^2dx-\left(\left(3\,ab-b^2\right)\tan\left(dx+c\right)^2+3\,a^2-ab\right)\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(dx+c)^4-6\,ab\tan(dx+c)^2+a^2+4\left(ab\tan(dx+c\right.}{b^2\tan(dx+c)^4+2\,ab\tan(dx+c)^2+}\right.}{8\left(\left(a^3b-2\,a^2b^2+ab^3\right)d\tan\left(dx+c\right)^2+\left(a^4-2\,a^3b+a^2b^2\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*a*b*d*x*tan(d*x + c)^2 + 8*a^2*d*x - ((3*a*b - b^2)*tan(d*x + c)^2
+ 3*a^2 - a*b)*sqrt(-b/a)*log((b^2*tan(d*x + c)^4 - 6*a*b*tan(d*x + c)^2 +
a^2 + 4*(a*b*tan(d*x + c)^3 - a^2*tan(d*x + c))*sqrt(-b/a))/(b^2*tan(d*x +
c)^4 + 2*a*b*tan(d*x + c)^2 + a^2)) - 4*(a*b - b^2)*tan(d*x + c))/((a^3*b -
 2*a^2*b^2 + a*b^3)*d*tan(d*x + c)^2 + (a^4 - 2*a^3*b + a^2*b^2)*d), 1/4*(4
*a*b*d*x*tan(d*x + c)^2 + 4*a^2*d*x - ((3*a*b - b^2)*tan(d*x + c)^2 + 3*a^2
 - a*b)*sqrt(b/a)*arctan(1/2*(b*tan(d*x + c)^2 - a)*sqrt(b/a)/(b*tan(d*x +
c))) - 2*(a*b - b^2)*tan(d*x + c))/((a^3*b - 2*a^2*b^2 + a*b^3)*d*tan(d*x +
 c)^2 + (a^4 - 2*a^3*b + a^2*b^2)*d)]
```

Sympy [A] time = 36.543, size = 2086, normalized size = 21.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((zoo*x/tan(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x + 1/(d*tan
(c + d*x)) - 1/(3*d*tan(c + d*x)**3))/b**2, Eq(a, 0)), (x/(a + b*tan(c)**2)
**2, Eq(d, 0)), (x/a**2, Eq(b, 0)), (4*I*a**(5/2)*d*x*sqrt(1/b)/(4*I*a**(9/
2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 8*I*a**(7/2)*
b*d*sqrt(1/b) - 8*I*a**(5/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 4*I*a**(5/2
)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2) + 4*I*a
**(3/2)*b*d*x*sqrt(1/b)*tan(c + d*x)**2/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**
(7/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 8*I*a**(7/2)*b*d*sqrt(1/b) - 8*I*a**(
5/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I
*a**(3/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2) - 2*I*a**(3/2)*b*sqrt(1/b)*tan(
```

```
c + d*x)/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*tan(c + d*x
)**2 - 8*I*a**(7/2)*b*d*sqrt(1/b) - 8*I*a**(5/2)*b**2*d*sqrt(1/b)*tan(c + d
*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*tan(
c + d*x)**2) + 2*I*sqrt(a)*b**2*sqrt(1/b)*tan(c + d*x)/(4*I*a**(9/2)*d*sqrt
(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 8*I*a**(7/2)*b*d*sqrt(
1/b) - 8*I*a**(5/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*
sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2) - 3*a**2*log(-I*
sqrt(a)*sqrt(1/b) + tan(c + d*x))/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*
b*d*sqrt(1/b)*tan(c + d*x)**2 - 8*I*a**(7/2)*b*d*sqrt(1/b) - 8*I*a**(5/2)*b
**2*d*sqrt(1/b)*tan(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3
/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2) + 3*a**2*log(I*sqrt(a)*sqrt(1/b) + ta
n(c + d*x))/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*tan(c +
d*x)**2 - 8*I*a**(7/2)*b*d*sqrt(1/b) - 8*I*a**(5/2)*b**2*d*sqrt(1/b)*tan(c
+ d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*t
an(c + d*x)**2) - 3*a*b*log(-I*sqrt(a)*sqrt(1/b) + tan(c + d*x))*tan(c + d*
x)**2/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*tan(c + d*x)**
2 - 8*I*a**(7/2)*b*d*sqrt(1/b) - 8*I*a**(5/2)*b**2*d*sqrt(1/b)*tan(c + d*x)
**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*tan(c +
 d*x)**2) + a*b*log(-I*sqrt(a)*sqrt(1/b) + tan(c + d*x))/(4*I*a**(9/2)*d*sq
rt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 8*I*a**(7/2)*b*d*sqr
t(1/b) - 8*I*a**(5/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 4*I*a**(5/2)*b**2*
d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2) + 3*a*b*log(I*
sqrt(a)*sqrt(1/b) + tan(c + d*x))*tan(c + d*x)**2/(4*I*a**(9/2)*d*sqrt(1/b)
 + 4*I*a**(7/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 8*I*a**(7/2)*b*d*sqrt(1/b)
- 8*I*a**(5/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(
1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2) - a*b*log(I*sqrt(a)*s
qrt(1/b) + tan(c + d*x))/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(
1/b)*tan(c + d*x)**2 - 8*I*a**(7/2)*b*d*sqrt(1/b) - 8*I*a**(5/2)*b**2*d*sqr
t(1/b)*tan(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*
d*sqrt(1/b)*tan(c + d*x)**2) + b**2*log(-I*sqrt(a)*sqrt(1/b) + tan(c + d*x)
)*tan(c + d*x)**2/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*ta
n(c + d*x)**2 - 8*I*a**(7/2)*b*d*sqrt(1/b) - 8*I*a**(5/2)*b**2*d*sqrt(1/b)*
tan(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(
1/b)*tan(c + d*x)**2) - b**2*log(I*sqrt(a)*sqrt(1/b) + tan(c + d*x))*tan(c
+ d*x)**2/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*tan(c + d*
x)**2 - 8*I*a**(7/2)*b*d*sqrt(1/b) - 8*I*a**(5/2)*b**2*d*sqrt(1/b)*tan(c +
d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*tan
(c + d*x)**2), True))
```

Giac [A] time = 1.32069, size = 165, normalized size = 1.7

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)(3\,ab-b^2)}{(a^3-2\,a^2b+ab^2)\sqrt{ab}}-\frac{2\,(dx+c)}{a^2-2\,ab+b^2}+\frac{b\tan(dx+c)}{\left(b\tan(dx+c)^2+a\right)(a^2-ab)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b
)))*(3*a*b - b^2)/((a^3 - 2*a^2*b + a*b^2)*sqrt(a*b)) - 2*(d*x + c)/(a^2 -
2*a*b + b^2) + b*tan(d*x + c)/((b*tan(d*x + c)^2 + a)*(a^2 - a*b)))/d
```

3.256 $\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=150

$$-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^3}-\frac{b(7a-3b)\tan(c+dx)}{8a^2d(a-b)^2\left(a+b\tan^2(c+dx)\right)}-\frac{b\tan(c+dx)}{4ad(a-b)\left(a+b\tan^2(c+dx)\right)^2}+$$

```
[Out] x/(a - b)^3 - (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tan[c + d*
x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^3*d) - (b*Tan[c + d*x])/(4*a*(a - b)*d*(a
+ b*Tan[c + d*x]^2)^2) - ((7*a - 3*b)*b*Tan[c + d*x])/(8*a^2*(a - b)^2*d*(a
 + b*Tan[c + d*x]^2))
```

Rubi [A] time = 0.144264, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3661, 414, 527, 522, 203, 205}

$$-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^3}-\frac{b(7a-3b)\tan(c+dx)}{8a^2d(a-b)^2\left(a+b\tan^2(c+dx)\right)}-\frac{b\tan(c+dx)}{4ad(a-b)\left(a+b\tan^2(c+dx)\right)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^2)^(-3),x]
```

```
[Out] x/(a - b)^3 - (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tan[c + d*
x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^3*d) - (b*Tan[c + d*x])/(4*a*(a - b)*d*(a
+ b*Tan[c + d*x]^2)^2) - ((7*a - 3*b)*b*Tan[c + d*x])/(8*a^2*(a - b)^2*d*(a
 + b*Tan[c + d*x]^2))
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+bx^2\right)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b\tan(c+dx)}{4a(a-b)d\left(a+b\tan^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-3b-3bx^2}{(1+x^2)\left(a+bx^2\right)^2}\,dx, x, \tan(c+dx)\right)}{4a(a-b)d}$$

$$= -\frac{b\tan(c+dx)}{4a(a-b)d\left(a+b\tan^2(c+dx)\right)^2} - \frac{(7a-3b)b\tan(c+dx)}{8a^2(a-b)^2d\left(a+b\tan^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{8a^2-\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{b\tan(c+dx)}{4a(a-b)d\left(a+b\tan^2(c+dx)\right)^2} - \frac{(7a-3b)b\tan(c+dx)}{8a^2(a-b)^2d\left(a+b\tan^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\cdots\right.}{(a\cdots}$$

$$= \frac{x}{(a-b)^3} - \frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3d} - \frac{b\tan(c+dx)}{4a(a-b)d\left(a+b\tan^2(c+dx\cdots\right.}$$

Mathematica [A] time = 1.87812, size = 138, normalized size = 0.92

$$-\frac{\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}} + \frac{b(7a-3b)(a-b)\tan(c+dx)}{a^2\left(a+b\tan^2(c+dx)\right)} + \frac{2b(a-b)^2\tan(c+dx)}{a\left(a+b\tan^2(c+dx)\right)^2} - 8\tan^{-1}(\tan(c+dx))}{8d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^2)^(-3),x]
```

```
[Out] -(-8*ArcTan[Tan[c + d*x]] + (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt
[b]*Tan[c + d*x])/Sqrt[a]])/a^(5/2) + (2*(a - b)^2*b*Tan[c + d*x])/(a*(a +
b*Tan[c + d*x]^2)^2) + ((7*a - 3*b)*(a - b)*b*Tan[c + d*x])/(a^2*(a + b*Tan
```

```
[c + d*x]^2)))/(8*(a - b)^3*d)
```

Maple [B] time = 0.027, size = 350, normalized size = 2.3

$$-\frac{7b^2(\tan(dx+c))^3}{8d(a-b)^3\left(a+b(\tan(dx+c))^2\right)^2}+\frac{5b^3(\tan(dx+c))^3}{4d(a-b)^3\left(a+b(\tan(dx+c))^2\right)^2a}-\frac{3b^4(\tan(dx+c))^3}{8d(a-b)^3\left(a+b(\tan(dx+c))^2\right)^2a^2}-\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^2)^3,x)
```

```
[Out] -7/8/d*b^2/(a-b)^3/(a+b*tan(d*x+c)^2)^2*tan(d*x+c)^3+5/4/d*b^3/(a-b)^3/(a+b
*tan(d*x+c)^2)^2/a*tan(d*x+c)^3-3/8/d*b^4/(a-b)^3/(a+b*tan(d*x+c)^2)^2/a^2*
tan(d*x+c)^3-9/8/d*b/(a-b)^3/(a+b*tan(d*x+c)^2)^2*a*tan(d*x+c)+7/4/d*b^2/(a
-b)^3/(a+b*tan(d*x+c)^2)^2*tan(d*x+c)-5/8/d*b^3/(a-b)^3/(a+b*tan(d*x+c)^2)^
2/a*tan(d*x+c)-15/8/d*b/(a-b)^3/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2)
)+5/4/d*b^2/(a-b)^3/a/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))-3/8/d*b^
3/(a-b)^3/a^2/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))+1/d/(a-b)^3*arct
an(tan(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.68687, size = 1643, normalized size = 10.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*a^2*b^2*d*x*tan(d*x + c)^4 + 64*a^3*b*d*x*tan(d*x + c)^2 + 32*a^4
*d*x - 4*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(d*x + c)^3 - ((15*a^2*b^2 - 10*
a*b^3 + 3*b^4)*tan(d*x + c)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^3*b
 - 10*a^2*b^2 + 3*a*b^3)*tan(d*x + c)^2)*sqrt(-b/a)*log((b^2*tan(d*x + c)^4
 - 6*a*b*tan(d*x + c)^2 + a^2 + 4*(a*b*tan(d*x + c)^3 - a^2*tan(d*x + c))*s
qrt(-b/a))/(b^2*tan(d*x + c)^4 + 2*a*b*tan(d*x + c)^2 + a^2)) - 4*(9*a^3*b
- 14*a^2*b^2 + 5*a*b^3)*tan(d*x + c))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a
^2*b^5)*d*tan(d*x + c)^4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d*ta
n(d*x + c)^2 + (a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d), 1/16*(16*a^2*b^2*d
*x*tan(d*x + c)^4 + 32*a^3*b*d*x*tan(d*x + c)^2 + 16*a^4*d*x - 2*(7*a^2*b^2
 - 10*a*b^3 + 3*b^4)*tan(d*x + c)^3 - ((15*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(
d*x + c)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^3*b - 10*a^2*b^2 + 3*a
*b^3)*tan(d*x + c)^2)*sqrt(b/a)*arctan(1/2*(b*tan(d*x + c)^2 - a)*sqrt(b/a)
```

```
/(b*tan(d*x + c))) - 2*(9*a^3*b - 14*a^2*b^2 + 5*a*b^3)*tan(d*x + c))/((a^5
*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*tan(d*x + c)^4 + 2*(a^6*b - 3*a^5
*b^2 + 3*a^4*b^3 - a^3*b^4)*d*tan(d*x + c)^2 + (a^7 - 3*a^6*b + 3*a^5*b^2 -
 a^4*b^3)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31306, size = 277, normalized size = 1.85

$$-\frac{\frac{(15a^2b-10ab^2+3b^3)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)}{(a^5-3a^4b+3a^3b^2-a^2b^3)\sqrt{ab}}-\frac{8(dx+c)}{a^3-3a^2b+3ab^2-b^3}+\frac{7ab^2\tan(dx+c)^3-3b^3\tan(dx+c)^3+9a^2b\tan(dx+c)-5ab^2\tan(dx+c)}{(a^4-2a^3b+a^2b^2)\left(b\tan(dx+c)^2+a\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*(pi*floor((d*x + c)/pi + 1/2)*sgn(b) +
arctan(b*tan(d*x + c)/sqrt(a*b)))/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sq
rt(a*b)) - 8*(d*x + c)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (7*a*b^2*tan(d*x +
 c)^3 - 3*b^3*tan(d*x + c)^3 + 9*a^2*b*tan(d*x + c) - 5*a*b^2*tan(d*x + c))
/((a^4 - 2*a^3*b + a^2*b^2)*(b*tan(d*x + c)^2 + a)^2))/d
```

3.257 $\int \tan^4(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal. Leaf size=54

$$\frac{1}{4}\tan^3(x)\sqrt{a\sec^2(x)}-\frac{3}{8}\tan(x)\sqrt{a\sec^2(x)}+\frac{3}{8}\cos(x)\sqrt{a\sec^2(x)}\tanh^{-1}(\sin(x))$$

```
[Out] (3*ArcTanh[Sin[x]]*Cos[x]*Sqrt[a*Sec[x]^2])/8 - (3*Sqrt[a*Sec[x]^2]*Tan[x])
/8 + (Sqrt[a*Sec[x]^2]*Tan[x]^3)/4
```

Rubi [A] time = 0.105229, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3657, 4125, 2611, 3770}

$$\frac{1}{4}\tan^3(x)\sqrt{a\sec^2(x)}-\frac{3}{8}\tan(x)\sqrt{a\sec^2(x)}+\frac{3}{8}\cos(x)\sqrt{a\sec^2(x)}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^4*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] (3*ArcTanh[Sin[x]]*Cos[x]*Sqrt[a*Sec[x]^2])/8 - (3*Sqrt[a*Sec[x]^2]*Tan[x])
/8 + (Sqrt[a*Sec[x]^2]*Tan[x]^3)/4
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4125

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sec[e + f*x]^
n)^FracPart[p])/(Sec[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tan^4(x)\sqrt{a+a\tan^2(x)}\,dx &= \int \sqrt{a\sec^2(x)}\tan^4(x)\,dx\\ &= \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int \sec(x)\tan^4(x)\,dx\\ &= \frac{1}{4}\sqrt{a\sec^2(x)}\tan^3(x)-\frac{1}{4}\left(3\cos(x)\sqrt{a\sec^2(x)}\right)\int \sec(x)\tan^2(x)\,dx\\ &= -\frac{3}{8}\sqrt{a\sec^2(x)}\tan(x)+\frac{1}{4}\sqrt{a\sec^2(x)}\tan^3(x)+\frac{1}{8}\left(3\cos(x)\sqrt{a\sec^2(x)}\right)\int \sec(x)\,dx\\ &= \frac{3}{8}\tanh^{-1}(\sin(x))\cos(x)\sqrt{a\sec^2(x)}-\frac{3}{8}\sqrt{a\sec^2(x)}\tan(x)+\frac{1}{4}\sqrt{a\sec^2(x)}\tan^3(x)\end{aligned}$$

Mathematica [A] time = 0.0757117, size = 32, normalized size = 0.59

$$\frac{1}{8}\sqrt{a\sec^2(x)}\left(2\tan^3(x)-3\tan(x)+3\cos(x)\tanh^{-1}(\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^4*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] (Sqrt[a*Sec[x]^2]*(3*ArcTanh[Sin[x]]*Cos[x] - 3*Tan[x] + 2*Tan[x]^3))/8
```

Maple [A] time = 0.052, size = 56, normalized size = 1.

$$\frac{\tan(x)}{4a}\left(a+a(\tan(x))^2\right)^{\frac{3}{2}}-\frac{5\tan(x)}{8}\sqrt{a+a(\tan(x))^2}+\frac{3}{8}\sqrt{a}\ln\left(\sqrt{a}\tan(x)+\sqrt{a+a(\tan(x))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(x)^2)^(1/2)*tan(x)^4,x)
```

```
[Out] 1/4*tan(x)*(a+a*tan(x)^2)^(3/2)/a-5/8*(a+a*tan(x)^2)^(1/2)*tan(x)+3/8*a^(1/
2)*ln(a^(1/2)*tan(x)+(a+a*tan(x)^2)^(1/2))
```

Maxima [B] time = 2.84765, size = 1161, normalized size = 21.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^4,x, algorithm="maxima")
```

```
[Out] -1/16*(4*(5*sin(7*x) - 3*sin(5*x) + 3*sin(3*x) - 5*sin(x))*cos(8*x) - 40*(2
*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*cos(7*x) - 16*(3*sin(5*x) - 3*sin(3*x)
 + 5*sin(x))*cos(6*x) + 24*(3*sin(4*x) + 2*sin(2*x))*cos(5*x) + 24*(3*sin(3
*x) - 5*sin(x))*cos(4*x) - 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) + 1)*
cos(8*x) + cos(8*x)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6
*x)^2 + 12*(4*cos(2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2
*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(4*x)
 + 2*sin(2*x))*sin(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*sin(2
*x) + 16*sin(2*x)^2 + 8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) +
1) + 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) + 1)*cos(8*x) + cos(8*x)^2
```

```
+ 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6*x)^2 + 12*(4*cos(2*x)
 + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2*sin(6*x) + 3*sin(4*x)
 + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(4*x) + 2*sin(2*x))*sin(6*x
) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*x) + 16*sin(2*x)^2 +
8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - 4*(5*cos(7*x) - 3
*cos(5*x) + 3*cos(3*x) - 5*cos(x))*sin(8*x) + 20*(4*cos(6*x) + 6*cos(4*x) +
 4*cos(2*x) + 1)*sin(7*x) + 16*(3*cos(5*x) - 3*cos(3*x) + 5*cos(x))*sin(6*x
) - 12*(6*cos(4*x) + 4*cos(2*x) + 1)*sin(5*x) - 24*(3*cos(3*x) - 5*cos(x))*
sin(4*x) + 12*(4*cos(2*x) + 1)*sin(3*x) - 48*cos(3*x)*sin(2*x) + 80*cos(x)*
sin(2*x) - 80*cos(2*x)*sin(x) - 20*sin(x))*sqrt(a)/(2*(4*cos(6*x) + 6*cos(4
*x) + 4*cos(2*x) + 1)*cos(8*x) + cos(8*x)^2 + 8*(6*cos(4*x) + 4*cos(2*x) +
1)*cos(6*x) + 16*cos(6*x)^2 + 12*(4*cos(2*x) + 1)*cos(4*x) + 36*cos(4*x)^2
+ 16*cos(2*x)^2 + 4*(2*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*sin(8*x) + sin(8
*x)^2 + 16*(3*sin(4*x) + 2*sin(2*x))*sin(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)
^2 + 48*sin(4*x)*sin(2*x) + 16*sin(2*x)^2 + 8*cos(2*x) + 1)
```

Fricas [A] time = 1.48208, size = 171, normalized size = 3.17

$$\frac{1}{8}\sqrt{a\tan(x)^2+a}\left(2\tan(x)^3-3\tan(x)\right)+\frac{3}{16}\sqrt{a}\log\left(2a\tan(x)^2+2\sqrt{a\tan(x)^2+a}\sqrt{a}\tan(x)+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^4,x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(a*tan(x)^2 + a)*(2*tan(x)^3 - 3*tan(x)) + 3/16*sqrt(a)*log(2*a*tan
(x)^2 + 2*sqrt(a*tan(x)^2 + a)*sqrt(a)*tan(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\tan^2(x)+1\right)}\tan^4(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)**2)**(1/2)*tan(x)**4,x)
```

```
[Out] Integral(sqrt(a*(tan(x)**2 + 1))*tan(x)**4, x)
```

Giac [A] time = 1.14118, size = 65, normalized size = 1.2

$$\frac{1}{8}\sqrt{a\tan(x)^2+a}\left(2\tan(x)^2-3\right)\tan(x)-\frac{3}{8}\sqrt{a}\log\left(\left|-\sqrt{a}\tan(x)+\sqrt{a\tan(x)^2+a}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^4,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(a*tan(x)^2 + a)*(2*tan(x)^2 - 3)*tan(x) - 3/8*sqrt(a)*log(abs(-sqr
t(a)*tan(x) + sqrt(a*tan(x)^2 + a)))
```

3.258 $\int \tan^3(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal. Leaf size=30

$$\frac{\left(a\sec^2(x)\right)^{3/2}}{3a}-\sqrt{a\sec^2(x)}$$

```
[Out] -Sqrt[a*Sec[x]^2] + (a*Sec[x]^2)^(3/2)/(3*a)
```

Rubi [A] time = 0.0870712, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3657, 4124, 43}

$$\frac{\left(a\sec^2(x)\right)^{3/2}}{3a}-\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -Sqrt[a*Sec[x]^2] + (a*Sec[x]^2)^(3/2)/(3*a)
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \tan^3(x)\sqrt{a+a\tan^2(x)}\,dx &= \int \sqrt{a\sec^2(x)}\tan^3(x)\,dx\\
&= \frac{1}{2}a\,\text{Subst}\left(\int \frac{-1+x}{\sqrt{ax}}\,dx,x,\sec^2(x)\right)\\
&= \frac{1}{2}a\,\text{Subst}\left(\int \left(-\frac{1}{\sqrt{ax}}+\frac{\sqrt{ax}}{a}\right)dx,x,\sec^2(x)\right)\\
&= -\sqrt{a\sec^2(x)}+\frac{\left(a\sec^2(x)\right)^{3/2}}{3a}
\end{aligned}$$

Mathematica [A] time = 0.0288181, size = 20, normalized size = 0.67

$$\frac{1}{3}\left(\sec^2(x)-3\right)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] (Sqrt[a*Sec[x]^2]*(-3 + Sec[x]^2))/3
```

Maple [A] time = 0.03, size = 29, normalized size = 1.

$$\frac{1}{3a}\left(a+a(\tan(x))^2\right)^{\frac{3}{2}}-\sqrt{a+a(\tan(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(x)^2)^(1/2)*tan(x)^3,x)
```

```
[Out] 1/3/a*(a+a*tan(x)^2)^(3/2)-(a+a*tan(x)^2)^(1/2)
```

Maxima [B] time = 1.93065, size = 373, normalized size = 12.43

$$-\frac{2\left((3\cos(5x)+2\cos(3x)+3\cos(x))\cos(6x)+3(3\cos(4x)+3\cos(2x)+1)\cos(5x)+3(2\cos(3x)+3\cos(x)\right.}{3\left(2(3\cos(4x)+3\cos(2x)+1)\cos(6x)+\cos(6x)^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^3,x, algorithm="maxima")
```

```
[Out] -2/3*((3*cos(5*x) + 2*cos(3*x) + 3*cos(x))*cos(6*x) + 3*(3*cos(4*x) + 3*cos
(2*x) + 1)*cos(5*x) + 3*(2*cos(3*x) + 3*cos(x))*cos(4*x) + 2*(3*cos(2*x) +
1)*cos(3*x) + 9*cos(2*x)*cos(x) + (3*sin(5*x) + 2*sin(3*x) + 3*sin(x))*sin(
6*x) + 9*(sin(4*x) + sin(2*x))*sin(5*x) + 3*(2*sin(3*x) + 3*sin(x))*sin(4*x
) + 6*sin(3*x)*sin(2*x) + 9*sin(2*x)*sin(x) + 3*cos(x))*sqrt(a)/(2*(3*cos(4
*x) + 3*cos(2*x) + 1)*cos(6*x) + cos(6*x)^2 + 6*(3*cos(2*x) + 1)*cos(4*x) +
 9*cos(4*x)^2 + 9*cos(2*x)^2 + 6*(sin(4*x) + sin(2*x))*sin(6*x) + sin(6*x)^
2 + 9*sin(4*x)^2 + 18*sin(4*x)*sin(2*x) + 9*sin(2*x)^2 + 6*cos(2*x) + 1)
```

Fricas [A] time = 1.35013, size = 55, normalized size = 1.83

$$\frac{1}{3}\sqrt{a\tan(x)^2+a}\left(\tan(x)^2-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(a*tan(x)^2 + a)*(tan(x)^2 - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\tan^2\left(x\right)+1\right)}\tan^3\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)**2)**(1/2)*tan(x)**3,x)
```

```
[Out] Integral(sqrt(a*(tan(x)**2 + 1))*tan(x)**3, x)
```

Giac [A] time = 1.15969, size = 39, normalized size = 1.3

$$\frac{\left(a\tan\left(x\right)^2+a\right)^{\frac{3}{2}}-3\sqrt{a\tan\left(x\right)^2+a}a}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^3,x, algorithm="giac")
```

```
[Out] 1/3*((a*tan(x)^2 + a)^(3/2) - 3*sqrt(a*tan(x)^2 + a)*a)/a
```

3.259 $\int \tan^2(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal. Leaf size=36

$$\frac{1}{2}\tan(x)\sqrt{a\sec^2(x)}-\frac{1}{2}\cos(x)\sqrt{a\sec^2(x)}\tanh^{-1}(\sin(x))$$

```
[Out] -(ArcTanh[Sin[x]]*Cos[x]*Sqrt[a*Sec[x]^2])/2 + (Sqrt[a*Sec[x]^2]*Tan[x])/2
```

Rubi [A] time = 0.0926689, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3657, 4125, 2611, 3770}

$$\frac{1}{2}\tan(x)\sqrt{a\sec^2(x)}-\frac{1}{2}\cos(x)\sqrt{a\sec^2(x)}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -(ArcTanh[Sin[x]]*Cos[x]*Sqrt[a*Sec[x]^2])/2 + (Sqrt[a*Sec[x]^2]*Tan[x])/2
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4125

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sec[e + f*x]^
n)^FracPart[p])/(Sec[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tan^2(x)\sqrt{a+a\tan^2(x)}\,dx = \int \sqrt{a\sec^2(x)}\tan^2(x)\,dx$$
$$= \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int \sec(x)\tan^2(x)\,dx$$
$$= \frac{1}{2}\sqrt{a\sec^2(x)}\tan(x) - \frac{1}{2}\left(\cos(x)\sqrt{a\sec^2(x)}\right)\int \sec(x)\,dx$$
$$= -\frac{1}{2}\tanh^{-1}(\sin(x))\cos(x)\sqrt{a\sec^2(x)} + \frac{1}{2}\sqrt{a\sec^2(x)}\tan(x)$$

Mathematica [A] time = 0.0490207, size = 24, normalized size = 0.67

$$\frac{1}{2}\sqrt{a\sec^2(x)}\left(\tan(x) - \cos(x)\tanh^{-1}(\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] (Sqrt[a*Sec[x]^2]*(-(ArcTanh[Sin[x]]*Cos[x]) + Tan[x]))/2
```

Maple [A] time = 0.029, size = 39, normalized size = 1.1

$$\frac{\tan(x)}{2}\sqrt{a+a(\tan(x))^2} - \frac{1}{2}\sqrt{a}\ln\left(\sqrt{a}\tan(x) + \sqrt{a+a(\tan(x))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(x)^2)^(1/2)*tan(x)^2,x)
```

```
[Out] 1/2*(a+a*tan(x)^2)^(1/2)*tan(x)-1/2*a^(1/2)*ln(a^(1/2)*tan(x)+(a+a*tan(x)^2
)^(1/2))
```

Maxima [B] time = 1.87742, size = 398, normalized size = 11.06

$$\frac{\left(4(\sin(3x) - \sin(x))\cos(4x) - \left(2(2\cos(2x)+1)\cos(4x) + \cos(4x)^2 + 4\cos(2x)^2 + \sin(4x)^2 + 4\sin(4x)\sin(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(sin(3*x) - sin(x))*cos(4*x) - (2*(2*cos(2*x) + 1)*cos(4*x) + cos(4*
x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4*c
os(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + (2*(2*cos(2*x) + 1)*
cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4
*sin(2*x)^2 + 4*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - 4*(
cos(3*x) - cos(x))*sin(4*x) + 4*(2*cos(2*x) + 1)*sin(3*x) - 8*cos(3*x)*sin(
2*x) + 8*cos(x)*sin(2*x) - 8*cos(2*x)*sin(x) - 4*sin(x))*sqrt(a)/(2*(2*cos(
2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*si
n(2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)
```

Fricas [A] time = 1.41434, size = 147, normalized size = 4.08

$$\frac{1}{4}\sqrt{a}\log\left(2\,a\tan\left(x\right)^{2}-2\,\sqrt{a\tan\left(x\right)^{2}+a}\sqrt{a}\tan\left(x\right)+a\right)+\frac{1}{2}\,\sqrt{a\tan\left(x\right)^{2}+a}\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(a)*log(2*a*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a)*sqrt(a)*tan(x) + a) +
 1/2*sqrt(a*tan(x)^2 + a)*tan(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\tan^{2}\left(x\right)+1\right)}\tan^{2}\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)**2)**(1/2)*tan(x)**2,x)
```

```
[Out] Integral(sqrt(a*(tan(x)**2 + 1))*tan(x)**2, x)
```

Giac [A] time = 1.17488, size = 54, normalized size = 1.5

$$\frac{1}{2}\sqrt{a}\log\left(\left|-\sqrt{a}\tan\left(x\right)+\sqrt{a\tan\left(x\right)^{2}+a}\right|\right)+\frac{1}{2}\,\sqrt{a\tan\left(x\right)^{2}+a}\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a)*log(abs(-sqrt(a)*tan(x) + sqrt(a*tan(x)^2 + a))) + 1/2*sqrt(a*t
an(x)^2 + a)*tan(x)
```

3.260 $\int \tan(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal. Leaf size=10

$$\sqrt{a\sec^2(x)}$$

[Out] Sqrt[a*Sec[x]^2]

Rubi [A] time = 0.0431939, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3657, 4124, 32}

$$\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

[In] Int[Tan[x]*Sqrt[a + a*Tan[x]^2],x]

[Out] Sqrt[a*Sec[x]^2]

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \tan(x)\sqrt{a+a\tan^2(x)}\,dx &= \int \sqrt{a\sec^2(x)}\tan(x)\,dx\\ &= \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{\sqrt{ax}}\,dx, x, \sec^2(x)\right)\\ &= \sqrt{a\sec^2(x)}\end{aligned}$$

Mathematica [A] time = 0.0088662, size = 10, normalized size = 1.

$$\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] Sqrt[a*Sec[x]^2]
```

Maple [A] time = 0.023, size = 11, normalized size = 1.1

$$\sqrt{a + a(\tan(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(x)^2)^(1/2)*tan(x),x)
```

```
[Out] (a+a*tan(x)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\tan(x)^2 + a}\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*tan(x)^2 + a)*tan(x), x)
```

Fricas [A] time = 1.31103, size = 30, normalized size = 3.

$$\sqrt{a\tan(x)^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x),x, algorithm="fricas")
```

```
[Out] sqrt(a*tan(x)^2 + a)
```

Sympy [A] time = 0.671388, size = 10, normalized size = 1.

$$\sqrt{a\tan^2(x) + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)**2)**(1/2)*tan(x),x)
```

```
[Out] sqrt(a*tan(x)**2 + a)
```

Giac [A] time = 1.1183, size = 14, normalized size = 1.4

$$\sqrt{a\tan(x)^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x),x, algorithm="giac")
```

```
[Out] sqrt(a*tan(x)^2 + a)
```

3.261 $\int \cot(x)\sqrt{a + a\tan^2(x)}\,dx$

Optimal. Leaf size=24

$$-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)$$

```
[Out] -(Sqrt[a]*ArcTanh[Sqrt[a*Sec[x]^2]/Sqrt[a]])
```

Rubi [A] time = 0.0729868, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3657, 4124, 63, 207}

$$-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Sqrt[a + a*Tan[x]^2],x]

[Out] -(Sqrt[a]*ArcTanh[Sqrt[a*Sec[x]^2]/Sqrt[a]])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cot(x)\sqrt{a+a\tan^2(x)}\,dx &= \int \cot(x)\sqrt{a\sec^2(x)}\,dx \\
&= \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{ax}}\,dx, x, \sec^2(x)\right) \\
&= \text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a\sec^2(x)}\right) \\
&= -\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0177689, size = 30, normalized size = 1.25

$$
\cos(x)\sqrt{a\sec^2(x)}\left(\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] Cos[x]*(-Log[Cos[x/2]] + Log[Sin[x/2]])*Sqrt[a*Sec[x]^2]
```

Maple [A] time = 0.096, size = 23, normalized size = 1.

$$
\cos(x)\sqrt{\frac{a}{(\cos(x))^2}}\ln\left(-\frac{\cos(x)-1}{\sin(x)}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+a*tan(x)^2)^(1/2),x)
```

```
[Out] cos(x)*(a/cos(x)^2)^(1/2)*ln(-(cos(x)-1)/sin(x))
```

Maxima [B] time = 1.86701, size = 51, normalized size = 2.12

$$
-\frac{1}{2}\sqrt{a}\left(\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)-\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a)*(log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - log(cos(x)^2 + sin(
x)^2 - 2*cos(x) + 1))
```

Fricas [A] time = 1.39649, size = 180, normalized size = 7.5

$$
\left[\frac{1}{2}\sqrt{a}\log\left(\frac{a\tan(x)^2-2\sqrt{a\tan(x)^2+a}\sqrt{a}+2a}{\tan(x)^2}\right), \sqrt{-a}\arctan\left(\frac{\sqrt{a\tan(x)^2+a}\sqrt{-a}}{a}\right)\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((a*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a)*sqrt(a) + 2*a)/tan(x)
^2), sqrt(-a)*arctan(sqrt(a*tan(x)^2 + a)*sqrt(-a)/a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\tan^{2}(x)+1\right)}\cot(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(tan(x)**2 + 1))*cot(x), x)
```

Giac [A] time = 1.09717, size = 32, normalized size = 1.33

$$\frac{a\arctan\left(\frac{\sqrt{a\tan(x)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] a*arctan(sqrt(a*tan(x)^2 + a)/sqrt(-a))/sqrt(-a)
```

3.262 $\int \cot^2(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal. Leaf size=14

$$-\cot(x)\sqrt{a\sec^2(x)}$$

```
[Out] -(Cot[x]*Sqrt[a*Sec[x]^2])
```

Rubi [A] time = 0.0804953, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3657, 4125, 2606, 8}

$$-\cot(x)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -(Cot[x]*Sqrt[a*Sec[x]^2])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4125

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sec[e + f*x]^
n)^FracPart[p])/(Sec[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \cot^2(x)\sqrt{a+a\tan^2(x)}\,dx &= \int \cot^2(x)\sqrt{a\sec^2(x)}\,dx\\
&= \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int \cot(x)\csc(x)\,dx\\
&= -\left(\left(\cos(x)\sqrt{a\sec^2(x)}\right)\mathrm{Subst}\left(\int 1\,dx, x, \csc(x)\right)\right)\\
&= -\cot(x)\sqrt{a\sec^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.0141342, size = 14, normalized size = 1.

$$-\cot(x)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -(Cot[x]*Sqrt[a*Sec[x]^2])
```

Maple [A] time = 0.083, size = 17, normalized size = 1.2

$$-\frac{\cos(x)}{\sin(x)}\sqrt{\frac{a}{(\cos(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2*(a+a*tan(x)^2)^(1/2),x)
```

```
[Out] -cos(x)*(a/cos(x)^2)^(1/2)/sin(x)
```

Maxima [A] time = 1.65049, size = 23, normalized size = 1.64

$$-\frac{\sqrt{\tan(x)^2+1}\sqrt{a}}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(tan(x)^2 + 1)*sqrt(a)/tan(x)
```

Fricas [A] time = 1.28554, size = 41, normalized size = 2.93

$$-\frac{\sqrt{a\tan(x)^2+a}}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(a*tan(x)^2 + a)/tan(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\tan^2(x)+1\right)}\cot^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2*(a+a*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(tan(x)**2 + 1))*cot(x)**2, x)
```

Giac [B] time = 1.1129, size = 43, normalized size = 3.07

$$\frac{2\,a^{\frac{3}{2}}}{\left(\sqrt{a}\tan\left(x\right) - \sqrt{a\tan\left(x\right)^{2} + a}\right)^{2} - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*a^(3/2)/((sqrt(a)*tan(x) - sqrt(a*tan(x)^2 + a))^2 - a)
```

3.263 $\int \cot^3(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal. Leaf size=45

$$\frac{1}{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)-\frac{1}{2}\cot^2(x)\sqrt{a\sec^2(x)}$$

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a*Sec[x]^2/Sqrt[a]])/2 - (Cot[x]^2*Sqrt[a*Sec[x]^2])
/2
```

Rubi [A] time = 0.089685, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3657, 4124, 51, 63, 207}

$$\frac{1}{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)-\frac{1}{2}\cot^2(x)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a*Sec[x]^2/Sqrt[a]])/2 - (Cot[x]^2*Sqrt[a*Sec[x]^2])
/2
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cot^3(x)\sqrt{a+a\tan^2(x)}\,dx &= \int \cot^3(x)\sqrt{a\sec^2(x)}\,dx\\ &= \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{(-1+x)^2\sqrt{ax}}\,dx,x,\sec^2(x)\right)\\ &= -\frac{1}{2}\cot^2(x)\sqrt{a\sec^2(x)}-\frac{1}{4}a\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{ax}}\,dx,x,\sec^2(x)\right)\\ &= -\frac{1}{2}\cot^2(x)\sqrt{a\sec^2(x)}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx,x,\sqrt{a\sec^2(x)}\right)\\ &= \frac{1}{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)-\frac{1}{2}\cot^2(x)\sqrt{a\sec^2(x)}\end{aligned}$$

Mathematica [A] time = 0.0815088, size = 38, normalized size = 0.84

$$-\frac{1}{2}\cos(x)\sqrt{a\sec^2(x)}\left(\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)+\cot(x)\csc(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -(Cos[x]*(Cot[x]*Csc[x] - Log[Cos[x/2]] + Log[Sin[x/2]])*Sqrt[a*Sec[x]^2])/
2
```

Maple [A] time = 0.089, size = 51, normalized size = 1.1

$$\frac{\cos(x)}{2(\sin(x))^2}\left((\cos(x))^2\ln\left(-\frac{\cos(x)-1}{\sin(x)}\right)-\cos(x)-\ln\left(-\frac{\cos(x)-1}{\sin(x)}\right)\right)\sqrt{\frac{a}{(\cos(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3*(a+a*tan(x)^2)^(1/2),x)
```

```
[Out] 1/2*(cos(x)^2*ln(-(cos(x)-1)/sin(x))-cos(x)-ln(-(cos(x)-1)/sin(x)))*cos(x)*
(a/cos(x)^2)^(1/2)/sin(x)^2
```

Maxima [B] time = 1.92552, size = 409, normalized size = 9.09

$$-\frac{(4(\cos(3x)+\cos(x))\cos(4x)-4(2\cos(2x)-1)\cos(3x)-8\cos(2x)\cos(x)-(2(2\cos(2x)-1)\cos(4x)-\text{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(cos(3*x) + cos(x))*cos(4*x) - 4*(2*cos(2*x) - 1)*cos(3*x) - 8*cos(
2*x)*cos(x) - (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - si
n(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*log(cos(x)^
2 + sin(x)^2 + 2*cos(x) + 1) + (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 -
4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x)
 - 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 4*(sin(3*x) + sin(x))*sin(4
*x) - 8*sin(3*x)*sin(2*x) - 8*sin(2*x)*sin(x) + 4*cos(x))*sqrt(a)/(2*(2*cos
(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*s
in(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)
```

Fricas [A] time = 1.41154, size = 167, normalized size = 3.71

$$\frac{\sqrt{a}\log\left(\frac{a\tan(x)^2+2\sqrt{a\tan(x)^2+a}\sqrt{a}+2a}{\tan(x)^2}\right)\tan\left(x\right)^2-2\sqrt{a\tan\left(x\right)^2+a}}{4\tan\left(x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(a)*log((a*tan(x)^2 + 2*sqrt(a*tan(x)^2 + a)*sqrt(a) + 2*a)/tan(x)
^2)*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a))/tan(x)^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\tan^2\left(x\right)+1\right)}\cot^3\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3*(a+a*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(tan(x)**2 + 1))*cot(x)**3, x)
```

Giac [A] time = 1.10714, size = 68, normalized size = 1.51

$$-\frac{1}{2}a^2\left(\frac{\arctan\left(\frac{\sqrt{a\tan(x)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{\sqrt{a\tan\left(x\right)^2+a}}{a^2\tan\left(x\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*a^2*(arctan(sqrt(a*tan(x)^2 + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(a*tan(x
)^2 + a)/(a^2*tan(x)^2))
```

3.264 $\int \cot^4(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal. Leaf size=34

$$\cot(x)\sqrt{a\sec^2(x)}-\frac{1}{3}\cot(x)\csc^2(x)\sqrt{a\sec^2(x)}$$

```
[Out] Cot[x]*Sqrt[a*Sec[x]^2] - (Cot[x]*Csc[x]^2*Sqrt[a*Sec[x]^2])/3
```

Rubi [A] time = 0.0996947, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3657, 4125, 2606}

$$\cot(x)\sqrt{a\sec^2(x)}-\frac{1}{3}\cot(x)\csc^2(x)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^4*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] Cot[x]*Sqrt[a*Sec[x]^2] - (Cot[x]*Csc[x]^2*Sqrt[a*Sec[x]^2])/3
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4125

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sec[e + f*x]^
n)^FracPart[p])/(Sec[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}
\int \cot^4(x)\sqrt{a+a\tan^2(x)}\,dx &= \int \cot^4(x)\sqrt{a\sec^2(x)}\,dx\\
&= \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int \cot^3(x)\csc(x)\,dx\\
&= -\left(\left(\cos(x)\sqrt{a\sec^2(x)}\right)\operatorname{Subst}\left(\int \left(-1+x^2\right)\,dx,x,\csc(x)\right)\right)\\
&= \cot(x)\sqrt{a\sec^2(x)}-\frac{1}{3}\cot(x)\csc^2(x)\sqrt{a\sec^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.0235239, size = 22, normalized size = 0.65

$$-\frac{1}{3}\cot(x)\left(\csc^2(x)-3\right)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^4*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -(Cot[x]*(-3 + Csc[x]^2)*Sqrt[a*Sec[x]^2])/3
```

Maple [A] time = 0.084, size = 25, normalized size = 0.7

$$-\frac{\left(3\left(\cos(x)\right)^2-2\right)\cos(x)}{3\left(\sin(x)\right)^3}\sqrt{\frac{a}{\left(\cos(x)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4*(a+a*tan(x)^2)^(1/2),x)
```

```
[Out] -1/3*(3*cos(x)^2-2)*cos(x)*(a/cos(x)^2)^(1/2)/sin(x)^3
```

Maxima [A] time = 1.64409, size = 39, normalized size = 1.15

$$\frac{\left(2\sqrt{a}\tan(x)^2-\sqrt{a}\right)\sqrt{\tan(x)^2+1}}{3\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4*(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*sqrt(a)*tan(x)^2 - sqrt(a))*sqrt(tan(x)^2 + 1)/tan(x)^3
```

Fricas [A] time = 1.29801, size = 70, normalized size = 2.06

$$\frac{\sqrt{a\tan(x)^2+a}\left(2\tan(x)^2-1\right)}{3\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4*(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(a*tan(x)^2 + a)*(2*tan(x)^2 - 1)/tan(x)^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\tan^2(x)+1\right)}\cot^4(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**4*(a+a*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(tan(x)**2 + 1))*cot(x)**4, x)
```

Giac [B] time = 1.09745, size = 80, normalized size = 2.35

$$\frac{4\left(3\left(\sqrt{a}\tan\left(x\right)-\sqrt{a\tan\left(x\right)^{2}+a}\right)^{2}-a\right)a^{\frac{5}{2}}}{3\left(\left(\sqrt{a}\tan\left(x\right)-\sqrt{a\tan\left(x\right)^{2}+a}\right)^{2}-a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4*(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(a)*tan(x) - sqrt(a*tan(x)^2 + a))^2 - a)*a^(5/2)/((sqrt(a)*tan
(x) - sqrt(a*tan(x)^2 + a))^2 - a)^3
```

3.265 $\int \sqrt{a+a\tan^2(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{d}$$

```
[Out] (Sqrt[a]*ArcTanh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a*Sec[c + d*x]^2]])/d
```

Rubi [A] time = 0.0349328, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3657, 4122, 217, 206}

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Tan[c + d*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTanh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a*Sec[c + d*x]^2]])/d
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\tan^2(c+dx)}\,dx = \int \sqrt{a\sec^2(c+dx)}\,dx$$
$$= \frac{a\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+ax^2}}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{a\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{d}$$
$$= \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{d}$$

Mathematica [A] time = 0.0314185, size = 31, normalized size = 0.86

$$\frac{\cos(c+dx)\sqrt{a\sec^2(c+dx)}\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Tan[c + d*x]^2],x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Cos[c + d*x]*Sqrt[a*Sec[c + d*x]^2])/d
```

Maple [A] time = 0.065, size = 34, normalized size = 0.9

$$\frac{1}{d}\sqrt{a}\ln\left(\sqrt{a}\tan(dx+c)+\sqrt{a+a(\tan(dx+c))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(d*x+c)^2)^(1/2),x)
```

```
[Out] 1/d*a^(1/2)*ln(a^(1/2)*tan(d*x+c)+(a+a*tan(d*x+c)^2)^(1/2))
```

Maxima [B] time = 1.6991, size = 88, normalized size = 2.44

$$\frac{\sqrt{a}\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - lo
g(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A] time = 1.37526, size = 234, normalized size = 6.5

$$\left[\frac{\sqrt{a}\log\left(2a\tan(dx+c)^2+2\sqrt{a\tan(dx+c)^2+a}\sqrt{a}\tan(dx+c)+a\right)}{2d}, -\frac{\sqrt{-a}\arctan\left(\frac{\sqrt{a\tan(dx+c)^2+a}\sqrt{-a}}{a\tan(dx+c)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log(2*a*tan(d*x + c)^2 + 2*sqrt(a*tan(d*x + c)^2 + a)*sqrt(a)*
tan(d*x + c) + a)/d, -sqrt(-a)*arctan(sqrt(a*tan(d*x + c)^2 + a)*sqrt(-a)/(
a*tan(d*x + c)))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\tan^2(c+dx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*tan(c + d*x)**2 + a), x)
```

Giac [B] time = 1.43536, size = 89, normalized size = 2.47

$$-\frac{\left(\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1\right)-\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1\right)\right)\sqrt{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(log(abs(tan(1/2*d*x + 1/2*c) + 1))*sgn(tan(1/2*d*x + 1/2*c)^4 - 1) - log(
abs(tan(1/2*d*x + 1/2*c) - 1))*sgn(tan(1/2*d*x + 1/2*c)^4 - 1))*sqrt(a)/d
```

3.266 $\int \tan^3(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=32

$$\frac{\left(a\sec^2(x)\right)^{5/2}}{5a}-\frac{1}{3}\left(a\sec^2(x)\right)^{3/2}$$

```
[Out] -(a*Sec[x]^2)^(3/2)/3 + (a*Sec[x]^2)^(5/2)/(5*a)
```

Rubi [A] time = 0.0999202, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3657, 4124, 43}

$$\frac{\left(a\sec^2(x)\right)^{5/2}}{5a}-\frac{1}{3}\left(a\sec^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] -(a*Sec[x]^2)^(3/2)/3 + (a*Sec[x]^2)^(5/2)/(5*a)
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \tan^3(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx&=\int \left(a\sec^2(x)\right)^{3/2}\tan^3(x)\,dx\\&=\frac{1}{2}a\,\text{Subst}\left(\int (-1+x)\sqrt{ax}\,dx,x,\sec^2(x)\right)\\&=\frac{1}{2}a\,\text{Subst}\left(\int \left(-\sqrt{ax}+\frac{(ax)^{3/2}}{a}\right)dx,x,\sec^2(x)\right)\\&=-\frac{1}{3}\left(a\sec^2(x)\right)^{3/2}+\frac{\left(a\sec^2(x)\right)^{5/2}}{5a}\end{aligned}$$

Mathematica [A] time = 0.0495771, size = 22, normalized size = 0.69

$$\frac{1}{15}\left(3\sec^2(x)-5\right)\left(a\sec^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] ((a*Sec[x]^2)^(3/2)*(-5 + 3*Sec[x]^2))/15
```

Maple [A] time = 0.018, size = 29, normalized size = 0.9

$$\frac{1}{5a}\left(a+a(\tan(x))^2\right)^{\frac{5}{2}}-\frac{1}{3}\left(a+a(\tan(x))^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3*(a+a*tan(x)^2)^(3/2),x)
```

```
[Out] 1/5/a*(a+a*tan(x)^2)^(5/2)-1/3*(a+a*tan(x)^2)^(3/2)
```

Maxima [B] time = 1.93163, size = 755, normalized size = 23.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3*(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -8/15*(50*a*cos(4*x)*cos(3*x) + 50*a*sin(4*x)*sin(3*x) + 25*a*sin(3*x)*sin(
2*x) + (5*a*cos(7*x) - 2*a*cos(5*x) + 5*a*cos(3*x))*cos(10*x) + 5*(5*a*cos(
7*x) - 2*a*cos(5*x) + 5*a*cos(3*x))*cos(8*x) + 5*(10*a*cos(6*x) + 10*a*cos(
4*x) + 5*a*cos(2*x) + a)*cos(7*x) - 10*(2*a*cos(5*x) - 5*a*cos(3*x))*cos(6*
x) - 2*(10*a*cos(4*x) + 5*a*cos(2*x) + a)*cos(5*x) + 5*(5*a*cos(2*x) + a)*c
os(3*x) + (5*a*sin(7*x) - 2*a*sin(5*x) + 5*a*sin(3*x))*sin(10*x) + 5*(5*a*s
in(7*x) - 2*a*sin(5*x) + 5*a*sin(3*x))*sin(8*x) + 25*(2*a*sin(6*x) + 2*a*si
n(4*x) + a*sin(2*x))*sin(7*x) - 10*(2*a*sin(5*x) - 5*a*sin(3*x))*sin(6*x) -
 10*(2*a*sin(4*x) + a*sin(2*x))*sin(5*x))*sqrt(a)/(2*(5*cos(8*x) + 10*cos(6
*x) + 10*cos(4*x) + 5*cos(2*x) + 1)*cos(10*x) + cos(10*x)^2 + 10*(10*cos(6*
x) + 10*cos(4*x) + 5*cos(2*x) + 1)*cos(8*x) + 25*cos(8*x)^2 + 20*(10*cos(4*
x) + 5*cos(2*x) + 1)*cos(6*x) + 100*cos(6*x)^2 + 20*(5*cos(2*x) + 1)*cos(4*
x) + 100*cos(4*x)^2 + 25*cos(2*x)^2 + 10*(sin(8*x) + 2*sin(6*x) + 2*sin(4*x
) + sin(2*x))*sin(10*x) + sin(10*x)^2 + 50*(2*sin(6*x) + 2*sin(4*x) + sin(2
*x))*sin(8*x) + 25*sin(8*x)^2 + 100*(2*sin(4*x) + sin(2*x))*sin(6*x) + 100*
sin(6*x)^2 + 100*sin(4*x)^2 + 100*sin(4*x)*sin(2*x) + 25*sin(2*x)^2 + 10*co
s(2*x) + 1)
```

Fricas [A] time = 1.39101, size = 82, normalized size = 2.56

$$\frac{1}{15}\left(3a\tan(x)^4+a\tan(x)^2-2a\right)\sqrt{a\tan(x)^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3*(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a*tan(x)^4 + a*tan(x)^2 - 2*a)*sqrt(a*tan(x)^2 + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\tan^2(x)+1\right)\right)^{\frac{3}{2}}\tan^3(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3*(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] Integral((a*(tan(x)**2 + 1))**(3/2)*tan(x)**3, x)
```

Giac [A] time = 1.13284, size = 42, normalized size = 1.31

$$\frac{3\left(a\tan(x)^2+a\right)^{\frac{5}{2}}-5\left(a\tan(x)^2+a\right)^{\frac{3}{2}}a}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3*(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*(3*(a*tan(x)^2 + a)^(5/2) - 5*(a*tan(x)^2 + a)^(3/2)*a)/a
```

3.267 $\int \tan^2(x) \left(a + a\tan^2(x)\right)^{3/2} dx$

Optimal. Leaf size=59

$$\frac{1}{4}a\tan(x)\sec^2(x)\sqrt{a\sec^2(x)} - \frac{1}{8}a\tan(x)\sqrt{a\sec^2(x)} - \frac{1}{8}a\cos(x)\sqrt{a\sec^2(x)}\tanh^{-1}(\sin(x))$$

```
[Out] -(a*ArcTanh[Sin[x]]*Cos[x]*Sqrt[a*Sec[x]^2])/8 - (a*Sqrt[a*Sec[x]^2]*Tan[x]
)/8 + (a*Sec[x]^2*Sqrt[a*Sec[x]^2]*Tan[x])/4
```

Rubi [A] time = 0.119628, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3657, 4125, 2611, 3768, 3770}

$$\frac{1}{4}a\tan(x)\sec^2(x)\sqrt{a\sec^2(x)} - \frac{1}{8}a\tan(x)\sqrt{a\sec^2(x)} - \frac{1}{8}a\cos(x)\sqrt{a\sec^2(x)}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] -(a*ArcTanh[Sin[x]]*Cos[x]*Sqrt[a*Sec[x]^2])/8 - (a*Sqrt[a*Sec[x]^2]*Tan[x]
)/8 + (a*Sec[x]^2*Sqrt[a*Sec[x]^2]*Tan[x])/4
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4125

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sec[e + f*x]^
n)^FracPart[p])/(Sec[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tan^2(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx &= \int \left(a\sec^2(x)\right)^{3/2}\tan^2(x)\,dx\\ &= \left(a\cos(x)\sqrt{a\sec^2(x)}\right)\int \sec^3(x)\tan^2(x)\,dx\\ &= \frac{1}{4}a\sec^2(x)\sqrt{a\sec^2(x)}\tan(x)-\frac{1}{4}\left(a\cos(x)\sqrt{a\sec^2(x)}\right)\int \sec^3(x)\,dx\\ &= -\frac{1}{8}a\sqrt{a\sec^2(x)}\tan(x)+\frac{1}{4}a\sec^2(x)\sqrt{a\sec^2(x)}\tan(x)-\frac{1}{8}\left(a\cos(x)\sqrt{a\sec^2(x)}\right)\int\\ &= -\frac{1}{8}a\tanh^{-1}(\sin(x))\cos(x)\sqrt{a\sec^2(x)}-\frac{1}{8}a\sqrt{a\sec^2(x)}\tan(x)+\frac{1}{4}a\sec^2(x)\sqrt{a\sec^2}\end{aligned}$$

Mathematica [A] time = 0.0634659, size = 34, normalized size = 0.58

$$\frac{1}{8}\left(a\sec^2(x)\right)^{3/2}\left(2\tan(x)-\sin(x)\cos(x)+\cos^3(x)\left(-\tanh^{-1}(\sin(x))\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] ((a*Sec[x]^2)^(3/2)*(-(ArcTanh[Sin[x]]*Cos[x]^3) - Cos[x]*Sin[x] + 2*Tan[x]
))/8
```

Maple [A] time = 0.016, size = 54, normalized size = 0.9

$$\frac{\tan(x)}{4}\left(a+a(\tan(x))^2\right)^{\frac{3}{2}}-\frac{a\tan(x)}{8}\sqrt{a+a(\tan(x))^2}-\frac{1}{8}a^{\frac{3}{2}}\ln\left(\sqrt{a}\tan(x)+\sqrt{a+a(\tan(x))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2*(a+a*tan(x)^2)^(3/2),x)
```

```
[Out] 1/4*tan(x)*(a+a*tan(x)^2)^(3/2)-1/8*a*tan(x)*(a+a*tan(x)^2)^(1/2)-1/8*a^(3/
2)*ln(a^(1/2)*tan(x)+(a+a*tan(x)^2)^(1/2))
```

Maxima [B] time = 2.86221, size = 1261, normalized size = 21.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*(112*a*cos(3*x)*sin(2*x) - 16*a*cos(x)*sin(2*x) + 16*a*cos(2*x)*sin(x)
 - 4*(a*sin(7*x) - 7*a*sin(5*x) + 7*a*sin(3*x) - a*sin(x))*cos(8*x) + 8*(2*
a*sin(6*x) + 3*a*sin(4*x) + 2*a*sin(2*x))*cos(7*x) + 16*(7*a*sin(5*x) - 7*a
*sin(3*x) + a*sin(x))*cos(6*x) - 56*(3*a*sin(4*x) + 2*a*sin(2*x))*cos(5*x)
```

```
- 24*(7*a*sin(3*x) - a*sin(x))*cos(4*x) - (a*cos(8*x)^2 + 16*a*cos(6*x)^2 +
 36*a*cos(4*x)^2 + 16*a*cos(2*x)^2 + a*sin(8*x)^2 + 16*a*sin(6*x)^2 + 36*a*
sin(4*x)^2 + 48*a*sin(4*x)*sin(2*x) + 16*a*sin(2*x)^2 + 2*(4*a*cos(6*x) + 6
*a*cos(4*x) + 4*a*cos(2*x) + a)*cos(8*x) + 8*(6*a*cos(4*x) + 4*a*cos(2*x) +
 a)*cos(6*x) + 12*(4*a*cos(2*x) + a)*cos(4*x) + 8*a*cos(2*x) + 4*(2*a*sin(6
*x) + 3*a*sin(4*x) + 2*a*sin(2*x))*sin(8*x) + 16*(3*a*sin(4*x) + 2*a*sin(2*
x))*sin(6*x) + a)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + (a*cos(8*x)^2 +
 16*a*cos(6*x)^2 + 36*a*cos(4*x)^2 + 16*a*cos(2*x)^2 + a*sin(8*x)^2 + 16*a*
sin(6*x)^2 + 36*a*sin(4*x)^2 + 48*a*sin(4*x)*sin(2*x) + 16*a*sin(2*x)^2 + 2
*(4*a*cos(6*x) + 6*a*cos(4*x) + 4*a*cos(2*x) + a)*cos(8*x) + 8*(6*a*cos(4*x
) + 4*a*cos(2*x) + a)*cos(6*x) + 12*(4*a*cos(2*x) + a)*cos(4*x) + 8*a*cos(2
*x) + 4*(2*a*sin(6*x) + 3*a*sin(4*x) + 2*a*sin(2*x))*sin(8*x) + 16*(3*a*sin
(4*x) + 2*a*sin(2*x))*sin(6*x) + a)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
 + 4*(a*cos(7*x) - 7*a*cos(5*x) + 7*a*cos(3*x) - a*cos(x))*sin(8*x) - 4*(4*
a*cos(6*x) + 6*a*cos(4*x) + 4*a*cos(2*x) + a)*sin(7*x) - 16*(7*a*cos(5*x) -
 7*a*cos(3*x) + a*cos(x))*sin(6*x) + 28*(6*a*cos(4*x) + 4*a*cos(2*x) + a)*s
in(5*x) + 24*(7*a*cos(3*x) - a*cos(x))*sin(4*x) - 28*(4*a*cos(2*x) + a)*sin
(3*x) + 4*a*sin(x))*sqrt(a)/(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) + 1)*c
os(8*x) + cos(8*x)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6*
x)^2 + 12*(4*cos(2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2*
sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(4*x)
+ 2*sin(2*x))*sin(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*
x) + 16*sin(2*x)^2 + 8*cos(2*x) + 1)
```

Fricas [A] time = 1.37776, size = 174, normalized size = 2.95

$$\frac{1}{16}\,a^{\frac{3}{2}}\log\left(2\,a\tan\left(x\right)^{2}-2\,\sqrt{a\tan\left(x\right)^{2}+a}\sqrt{a}\tan\left(x\right)+a\right)+\frac{1}{8}\left(2\,a\tan\left(x\right)^{3}+a\tan\left(x\right)\right)\sqrt{a\tan\left(x\right)^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*a^(3/2)*log(2*a*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a)*sqrt(a)*tan(x) + a)
+ 1/8*(2*a*tan(x)^3 + a*tan(x))*sqrt(a*tan(x)^2 + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\left(\tan^{2}\left(x\right)+1\right)\right)^{\frac{3}{2}}\tan^{2}\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2*(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] Integral((a*(tan(x)**2 + 1))**(3/2)*tan(x)**2, x)
```

Giac [A] time = 1.11016, size = 66, normalized size = 1.12

$$\frac{1}{8}\left(\sqrt{a\tan\left(x\right)^{2}+a}\left(2\,\tan\left(x\right)^{2}+1\right)\tan\left(x\right)+\sqrt{a}\log\left(\left|-\sqrt{a}\tan\left(x\right)+\sqrt{a\tan\left(x\right)^{2}+a}\right|\right)\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(a*tan(x)^2 + a)*(2*tan(x)^2 + 1)*tan(x) + sqrt(a)*log(abs(-sqrt(a
)*tan(x) + sqrt(a*tan(x)^2 + a))))*a
```

3.268 $\int \tan(x) \left(a + a\tan^2(x)\right)^{3/2} dx$

Optimal. Leaf size=14

$$\frac{1}{3}\left(a\sec^2(x)\right)^{3/2}$$

```
[Out] (a*Sec[x]^2)^(3/2)/3
```

Rubi [A] time = 0.0511205, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3657, 4124, 32}

$$\frac{1}{3}\left(a\sec^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] (a*Sec[x]^2)^(3/2)/3
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \tan(x)\left(a+a\tan^2(x)\right)^{3/2} dx &= \int \left(a\sec^2(x)\right)^{3/2}\tan(x)\,dx\\ &= \frac{1}{2}a\,\text{Subst}\left(\int \sqrt{ax}\,dx, x, \sec^2(x)\right)\\ &= \frac{1}{3}\left(a\sec^2(x)\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0149399, size = 14, normalized size = 1.

$$\frac{1}{3}\left(a\sec^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] (a*Sec[x]^2)^(3/2)/3
```

Maple [A] time = 0.01, size = 13, normalized size = 0.9

$$\frac{1}{3}\left(a+a\left(\tan\left(x\right)\right)^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a+a*tan(x)^2)^(3/2),x)
```

```
[Out] 1/3*(a+a*tan(x)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\tan\left(x\right)^{2}+a\right)^{\frac{3}{2}}\tan\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*tan(x)^2 + a)^(3/2)*tan(x), x)
```

Fricas [A] time = 1.33693, size = 38, normalized size = 2.71

$$\frac{1}{3}\left(a\tan\left(x\right)^{2}+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a*tan(x)^2 + a)^(3/2)
```

Sympy [A] time = 3.30181, size = 12, normalized size = 0.86

$$\frac{\left(a\tan^{2}\left(x\right)+a\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] (a*tan(x)**2 + a)**(3/2)/3
```

Giac [A] time = 1.05332, size = 16, normalized size = 1.14

$$\frac{1}{3}\left(a\tan\left(x\right)^{2}+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(a*tan(x)^2 + a)^(3/2)
```

3.269 $\int \cot(x) \left(a + a\tan^2(x)\right)^{3/2} dx$

Optimal. Leaf size=37

$$a\sqrt{a\sec^2(x)} - a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)$$

```
[Out] -(a^(3/2)*ArcTanh[Sqrt[a*Sec[x]^2]/Sqrt[a]]) + a*Sqrt[a*Sec[x]^2]
```

Rubi [A] time = 0.0905474, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3657, 4124, 50, 63, 207}

$$a\sqrt{a\sec^2(x)} - a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*(a + a*Tan[x]^2)^(3/2),x]

[Out] -(a^(3/2)*ArcTanh[Sqrt[a*Sec[x]^2]/Sqrt[a]]) + a*Sqrt[a*Sec[x]^2]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \cot(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx &= \int \cot(x)\left(a\sec^2(x)\right)^{3/2}\,dx\\
&= \frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{\sqrt{ax}}{-1+x}\,dx,x,\sec^2(x)\right)\\
&= a\sqrt{a\sec^2(x)}+\frac{1}{2}a^2\,\mathrm{Subst}\left(\int \frac{1}{(-1+x)\sqrt{ax}}\,dx,x,\sec^2(x)\right)\\
&= a\sqrt{a\sec^2(x)}+a\,\mathrm{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx,x,\sqrt{a\sec^2(x)}\right)\\
&= -a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)+a\sqrt{a\sec^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.0380756, size = 34, normalized size = 0.92

$$a\sqrt{a\sec^2(x)}\left(\cos(x)\left(\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] a*(1 + Cos[x]*(-Log[Cos[x/2]] + Log[Sin[x/2]]))*Sqrt[a*Sec[x]^2]
```

Maple [A] time = 0.069, size = 32, normalized size = 0.9

$$\left(\cos(x)\ln\left(-\frac{\cos(x)-1}{\sin(x)}\right)+\cos(x)+1\right)(\cos(x))^2\left(\frac{a}{(\cos(x))^2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+a*tan(x)^2)^(3/2),x)
```

```
[Out] (cos(x)*ln(-(cos(x)-1)/sin(x))+cos(x)+1)*cos(x)^2*(a/cos(x)^2)^(3/2)
```

Maxima [B] time = 1.90549, size = 181, normalized size = 4.89

$$\frac{\left(4a\cos(2x)\cos(x)+4a\sin(2x)\sin(x)+4a\cos(x)-\left(a\cos(2x)^2+a\sin(2x)^2+2a\cos(2x)+a\right)\log\left(\cos(x)^2+\sin\right.\right.}{2\left(\cos(2x)^2+\sin(2x)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*cos(2*x)*cos(x) + 4*a*sin(2*x)*sin(x) + 4*a*cos(x) - (a*cos(2*x)^2
 + a*sin(2*x)^2 + 2*a*cos(2*x) + a)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)
```

```
+ (a*cos(2*x)^2 + a*sin(2*x)^2 + 2*a*cos(2*x) + a)*log(cos(x)^2 + sin(x)^2
- 2*cos(x) + 1))*sqrt(a)/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Fricas [A] time = 1.43828, size = 140, normalized size = 3.78

$$\frac{1}{2}\,a^{\frac{3}{2}}\log\left(\frac{a\tan\left(x\right)^{2}-2\,\sqrt{a\tan\left(x\right)^{2}+a}\sqrt{a}+2\,a}{\tan\left(x\right)^{2}}\right)+\sqrt{a\tan\left(x\right)^{2}+a}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*a^(3/2)*log((a*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a)*sqrt(a) + 2*a)/tan(x)^
2) + sqrt(a*tan(x)^2 + a)*a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\left(\tan^{2}\left(x\right)+1\right)\right)^{\frac{3}{2}}\cot\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] Integral((a*(tan(x)**2 + 1))**(3/2)*cot(x), x)
```

Giac [A] time = 1.08167, size = 50, normalized size = 1.35

$$\left(\frac{a\arctan\left(\frac{\sqrt{a\tan\left(x\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{a\tan\left(x\right)^{2}+a}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] (a*arctan(sqrt(a*tan(x)^2 + a)/sqrt(-a))/sqrt(-a) + sqrt(a*tan(x)^2 + a))*a
```

3.270 $\int \cot^2(x) \left(a + a\tan^2(x)\right)^{3/2} dx$

Optimal. Leaf size=33

$$a\cos(x)\sqrt{a\sec^2(x)}\tanh^{-1}(\sin(x)) - a\cot(x)\sqrt{a\sec^2(x)}$$

```
[Out] a*ArcTanh[Sin[x]]*Cos[x]*Sqrt[a*Sec[x]^2] - a*Cot[x]*Sqrt[a*Sec[x]^2]
```

Rubi [A] time = 0.109094, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3657, 4125, 2621, 321, 207}

$$a\cos(x)\sqrt{a\sec^2(x)}\tanh^{-1}(\sin(x)) - a\cot(x)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] a*ArcTanh[Sin[x]]*Cos[x]*Sqrt[a*Sec[x]^2] - a*Cot[x]*Sqrt[a*Sec[x]^2]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4125

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sec[e + f*x]^
n)^FracPart[p])/(Sec[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^2(x)\left(a+a\tan^2(x)\right)^{3/2}dx=\int \cot^2(x)\left(a\sec^2(x)\right)^{3/2}dx$$
$$=\left(a\cos(x)\sqrt{a\sec^2(x)}\right)\int \csc^2(x)\sec(x)\,dx$$
$$=-\left(\left(a\cos(x)\sqrt{a\sec^2(x)}\right)\mathrm{Subst}\left(\int \frac{x^2}{-1+x^2}dx,x,\csc(x)\right)\right)$$
$$=-a\cot(x)\sqrt{a\sec^2(x)}-\left(a\cos(x)\sqrt{a\sec^2(x)}\right)\mathrm{Subst}\left(\int \frac{1}{-1+x^2}dx,x,\csc(x)\right)$$
$$=a\tanh^{-1}(\sin(x))\cos(x)\sqrt{a\sec^2(x)}-a\cot(x)\sqrt{a\sec^2(x)}$$

Mathematica [C] time = 0.0267249, size = 27, normalized size = 0.82

$$-a\cot(x)\sqrt{a\sec^2(x)}\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\sin^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] -(a*Cot[x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[x]^2]*Sqrt[a*Sec[x]^2])
```

Maple [A] time = 0.089, size = 54, normalized size = 1.6

$$\frac{(\cos(x))^3}{\sin(x)}\left(\frac{a}{(\cos(x))^2}\right)^{\frac{3}{2}}\left(\ln\left(\frac{1-\cos(x)+\sin(x)}{\sin(x)}\right)\sin(x)-\ln\left(-\frac{\cos(x)-1+\sin(x)}{\sin(x)}\right)\sin(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2*(a+a*tan(x)^2)^(3/2),x)
```

```
[Out] (a/cos(x)^2)^(3/2)*cos(x)^3*(ln((1-cos(x)+sin(x))/sin(x))*sin(x)-ln(-(cos(x
)-1+sin(x))/sin(x))*sin(x)-1)/sin(x)
```

Maxima [B] time = 1.98608, size = 181, normalized size = 5.48

$$-\frac{\left(4\,a\cos(x)\sin(2\,x)-4\,a\cos(2\,x)\sin(x)-\left(a\cos(2\,x)^2+a\sin(2\,x)^2-2\,a\cos(2\,x)+a\right)\log\left(\cos(x)^2+\sin(x)^2+2\right.}{2\left(\cos(2\,x)^2+\sin(2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(4*a*cos(x)*sin(2*x) - 4*a*cos(2*x)*sin(x) - (a*cos(2*x)^2 + a*sin(2*x
)^2 - 2*a*cos(2*x) + a)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + (a*cos(2*
x)^2 + a*sin(2*x)^2 - 2*a*cos(2*x) + a)*log(cos(x)^2 + sin(x)^2 - 2*sin(x)
+ 1) + 4*a*sin(x))*sqrt(a)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)
```

Fricas [A] time = 1.32379, size = 159, normalized size = 4.82

$$\frac{a^{\frac{3}{2}} \log\left(2\,a \tan\left(x\right)^{2} + 2\,\sqrt{a \tan\left(x\right)^{2} + a}\sqrt{a} \tan\left(x\right) + a\right) \tan\left(x\right) - 2\,\sqrt{a \tan\left(x\right)^{2} + a}a}{2\,\tan\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(a^(3/2)*log(2*a*tan(x)^2 + 2*sqrt(a*tan(x)^2 + a)*sqrt(a)*tan(x) + a)*
tan(x) - 2*sqrt(a*tan(x)^2 + a)*a)/tan(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \left(\tan^{2}{\left(x \right)} + 1\right)\right)^{\frac{3}{2}} \cot^{2}{\left(x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2*(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] Integral((a*(tan(x)**2 + 1))**(3/2)*cot(x)**2, x)
```

Giac [B] time = 1.10418, size = 84, normalized size = 2.55

$$-\frac{1}{2} \left(\sqrt{a} \log\left(\left(\sqrt{a} \tan\left(x\right) - \sqrt{a \tan\left(x\right)^{2} + a}\right)^{2}\right) - \frac{4\,a^{\frac{3}{2}}}{\left(\sqrt{a} \tan\left(x\right) - \sqrt{a \tan\left(x\right)^{2} + a}\right)^{2} - a}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(a)*log((sqrt(a)*tan(x) - sqrt(a*tan(x)^2 + a))^2) - 4*a^(3/2)/((
sqrt(a)*tan(x) - sqrt(a*tan(x)^2 + a))^2 - a))*a
```

3.271 $\int \left(a + a\tan^2(c + dx)\right)^{3/2} dx$

Optimal. Leaf size=68

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{2d} + \frac{a\tan(c+dx)\sqrt{a\sec^2(c+dx)}}{2d}$$

```
[Out] (a^(3/2)*ArcTanh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a*Sec[c + d*x]^2]])/(2*d) + (a
*Sqrt[a*Sec[c + d*x]^2]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0390246, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3657, 4122, 195, 217, 206}

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{2d} + \frac{a\tan(c+dx)\sqrt{a\sec^2(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTanh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a*Sec[c + d*x]^2]])/(2*d) + (a
*Sqrt[a*Sec[c + d*x]^2]*Tan[c + d*x])/(2*d)
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+a\tan^2(c+dx)\right)^{3/2}\,dx = \int \left(a\sec^2(c+dx)\right)^{3/2}\,dx$$
$$= \frac{a\,\text{Subst}\left(\int \sqrt{a+ax^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{a\sqrt{a\sec^2(c+dx)}\tan(c+dx)}{2d} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+ax^2}}\,dx, x, \tan(c+dx)\right)}{2d}$$
$$= \frac{a\sqrt{a\sec^2(c+dx)}\tan(c+dx)}{2d} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{2d}$$
$$= \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{2d} + \frac{a\sqrt{a\sec^2(c+dx)}\tan(c+dx)}{2d}$$

Mathematica [A] time = 0.0675036, size = 43, normalized size = 0.63

$$\frac{a\sqrt{a\sec^2(c+dx)}\left(\tan(c+dx)+\cos(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(3/2),x]
```

```
[Out] (a*Sqrt[a*Sec[c + d*x]^2]*(ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + Tan[c + d*x
]))/(2*d)
```

Maple [A] time = 0.02, size = 62, normalized size = 0.9

$$\frac{a\tan(dx+c)}{2d}\sqrt{a+a(\tan(dx+c))^2}+\frac{1}{2d}a^{\frac{3}{2}}\ln\left(\sqrt{a}\tan(dx+c)+\sqrt{a+a(\tan(dx+c))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(d*x+c)^2)^(3/2),x)
```

```
[Out] 1/2/d*a*tan(d*x+c)*(a+a*tan(d*x+c)^2)^(1/2)+1/2/d*a^(3/2)*ln(a^(1/2)*tan(d*
x+c)+(a+a*tan(d*x+c)^2)^(1/2))
```

Maxima [B] time = 1.9083, size = 751, normalized size = 11.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(8*a*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) - 8*a*cos(d*x + c)*sin(2*d*x +
2*c) + 8*a*cos(2*d*x + 2*c)*sin(d*x + c) - 4*(a*sin(3*d*x + 3*c) - a*sin(d*
x + c))*cos(4*d*x + 4*c) - (a*cos(4*d*x + 4*c)^2 + 4*a*cos(2*d*x + 2*c)^2 +
 a*sin(4*d*x + 4*c)^2 + 4*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*sin(2*d
```

```
*x + 2*c)^2 + 2*(2*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 4*a*cos(2*d*x
 + 2*c) + a)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + (a
*cos(4*d*x + 4*c)^2 + 4*a*cos(2*d*x + 2*c)^2 + a*sin(4*d*x + 4*c)^2 + 4*a*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*sin(2*d*x + 2*c)^2 + 2*(2*a*cos(2*d*
x + 2*c) + a)*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*log(cos(d*x + c)
^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 4*(a*cos(3*d*x + 3*c) - a*cos(d
*x + c))*sin(4*d*x + 4*c) - 4*(2*a*cos(2*d*x + 2*c) + a)*sin(3*d*x + 3*c) +
 4*a*sin(d*x + c))*sqrt(a)/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*
d)
```

Fricas [A] time = 1.29946, size = 193, normalized size = 2.84

$$\frac{a^{\frac{3}{2}} \log\left(2\, a \tan\left(dx+c\right)^2 + 2\, \sqrt{a \tan\left(dx+c\right)^2 + a}\sqrt{a} \tan\left(dx+c\right) + a\right) + 2\, \sqrt{a \tan\left(dx+c\right)^2 + a}\, a a \tan\left(dx+c\right)}{4\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(a^(3/2)*log(2*a*tan(d*x + c)^2 + 2*sqrt(a*tan(d*x + c)^2 + a)*sqrt(a)*
tan(d*x + c) + a) + 2*sqrt(a*tan(d*x + c)^2 + a)*a*tan(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \tan^2\left(c+dx\right) + a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((a*tan(c + d*x)**2 + a)**(3/2), x)
```

Giac [B] time = 3.1976, size = 2869, normalized size = 42.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)
^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*
c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*c) - a^(3/2)*s
gn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*
x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*
c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1))*log(abs(-tan(1/2*d*x)*tan(1/2*c) + t
an(1/2*d*x) + tan(1/2*c) + 1))/(tan(1/2*c) - 1) - (a^(3/2)*sgn(tan(1/2*d*x)
^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/
```

2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/
2*d*x)*tan(1/2*c) + 1)*tan(1/2*c) + a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4
 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/
2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*
c) + 1))*log(abs(-tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) + 1))
/(tan(1/2*c) + 1) - 2*(a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*
d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(
1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1
/2*d*x)^3*tan(1/2*c)^8 + 6*a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(
1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*
tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*t
an(1/2*d*x)^3*tan(1/2*c)^6 + 2*a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*
tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)
 - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) +
1)*tan(1/2*d*x)^2*tan(1/2*c)^7 + a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 -
4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c
) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c)
+ 1)*tan(1/2*d*x)*tan(1/2*c)^8 - 18*a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4
 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/
2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*
c) + 1)*tan(1/2*d*x)^2*tan(1/2*c)^5 - 10*a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2
*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*t
an(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan
(1/2*c) + 1)*tan(1/2*d*x)*tan(1/2*c)^6 - 2*a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1
/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3
*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*t
an(1/2*c) + 1)*tan(1/2*c)^7 - 6*a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4
*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)
 - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) +
 1)*tan(1/2*d*x)^3*tan(1/2*c)^2 - 18*a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^
4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1
/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2
*c) + 1)*tan(1/2*d*x)^2*tan(1/2*c)^3 + 2*a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2
*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*t
an(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan
(1/2*c) + 1)*tan(1/2*c)^5 - a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan
(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4
*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*
tan(1/2*d*x)^3 + 2*a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)
^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*
d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*d
*x)^2*tan(1/2*c) + 10*a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d
*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1
/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/
2*d*x)*tan(1/2*c)^2 + 2*a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2
*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan
(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(
1/2*c)^3 - a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1
/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan
(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*d*x) - 2*
a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - t
an(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 -
 tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*c))/((tan(1/2*d*x)^2
*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 +
 1)^2*(tan(1/2*c)^4 - 2*tan(1/2*c)^2 + 1)))/d

3.272 $\int \left(a+a\tan^2(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=98

$$\frac{3a^2\tan(c+dx)\sqrt{a\sec^2(c+dx)}}{8d}+\frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{8d}+\frac{a\tan(c+dx)\left(a\sec^2(c+dx)\right)^{3/2}}{4d}$$

```
[Out] (3*a^(5/2)*ArcTanh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a*Sec[c + d*x]^2]])/(8*d) +
(3*a^2*Sqrt[a*Sec[c + d*x]^2]*Tan[c + d*x])/(8*d) + (a*(a*Sec[c + d*x]^2)^(
3/2)*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.048974, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3657, 4122, 195, 217, 206}

$$\frac{3a^2\tan(c+dx)\sqrt{a\sec^2(c+dx)}}{8d}+\frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{8d}+\frac{a\tan(c+dx)\left(a\sec^2(c+dx)\right)^{3/2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(5/2),x]
```

```
[Out] (3*a^(5/2)*ArcTanh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a*Sec[c + d*x]^2]])/(8*d) +
(3*a^2*Sqrt[a*Sec[c + d*x]^2]*Tan[c + d*x])/(8*d) + (a*(a*Sec[c + d*x]^2)^(
3/2)*Tan[c + d*x])/(4*d)
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+a\tan^2(c+dx)\right)^{5/2}dx=\int \left(a\sec^2(c+dx)\right)^{5/2}dx$$
$$=\frac{a\,\mathrm{Subst}\left(\int \left(a+ax^2\right)^{3/2}dx,x,\tan(c+dx)\right)}{d}$$
$$=\frac{a\left(a\sec^2(c+dx)\right)^{3/2}\tan(c+dx)}{4d}+\frac{\left(3a^2\right)\mathrm{Subst}\left(\int \sqrt{a+ax^2}\,dx,x,\tan(c+dx)\right)}{4d}$$
$$=\frac{3a^2\sqrt{a\sec^2(c+dx)}\tan(c+dx)}{8d}+\frac{a\left(a\sec^2(c+dx)\right)^{3/2}\tan(c+dx)}{4d}+\frac{\left(3a^3\right)\mathrm{Subst}\left(\int \frac{}{\sqrt{a}}\right.}{}$$
$$=\frac{3a^2\sqrt{a\sec^2(c+dx)}\tan(c+dx)}{8d}+\frac{a\left(a\sec^2(c+dx)\right)^{3/2}\tan(c+dx)}{4d}+\frac{\left(3a^3\right)\mathrm{Subst}\left(\int \frac{}{1-}\right.}{}$$
$$=\frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{8d}+\frac{3a^2\sqrt{a\sec^2(c+dx)}\tan(c+dx)}{8d}+\frac{a\left(a\sec^2(c+dx)\right)^{3/2}\tan}{4d}$$

Mathematica [A] time = 0.215845, size = 65, normalized size = 0.66

$$\frac{a^2\cos(c+dx)\sqrt{a\sec^2(c+dx)}\left(3\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\left(2\sec^2(c+dx)+3\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(5/2),x]
```

```
[Out] (a^2*Cos[c + d*x]*Sqrt[a*Sec[c + d*x]^2]*(3*ArcTanh[Sin[c + d*x]] + Sec[c +
 d*x]*(3 + 2*Sec[c + d*x]^2)*Tan[c + d*x]))/(8*d)
```

Maple [A] time = 0.021, size = 90, normalized size = 0.9

$$\frac{a\tan(dx+c)}{4d}\left(a+a(\tan(dx+c))^2\right)^{\frac{3}{2}}+\frac{3a^2\tan(dx+c)}{8d}\sqrt{a+a(\tan(dx+c))^2}+\frac{3}{8d}a^{\frac{5}{2}}\ln\left(\sqrt{a}\tan(dx+c)+\sqrt{a+a(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(d*x+c)^2)^(5/2),x)
```

```
[Out] 1/4/d*a*tan(d*x+c)*(a+a*tan(d*x+c)^2)^(3/2)+3/8/d*a^2*tan(d*x+c)*(a+a*tan(d
*x+c)^2)^(1/2)+3/8/d*a^(5/2)*ln(a^(1/2)*tan(d*x+c)+(a+a*tan(d*x+c)^2)^(1/2)
)
```

Maxima [B] time = 2.87545, size = 2388, normalized size = 24.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/16*(176*a^2*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 48*a^2*cos(d*x + c)*sin(2
*d*x + 2*c) - 48*a^2*cos(2*d*x + 2*c)*sin(d*x + c) - 12*a^2*sin(d*x + c) +
4*(3*a^2*sin(7*d*x + 7*c) + 11*a^2*sin(5*d*x + 5*c) - 11*a^2*sin(3*d*x + 3*
c) - 3*a^2*sin(d*x + c))*cos(8*d*x + 8*c) - 24*(2*a^2*sin(6*d*x + 6*c) + 3*
a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*cos(7*d*x + 7*c) + 16*(11*a^
2*sin(5*d*x + 5*c) - 11*a^2*sin(3*d*x + 3*c) - 3*a^2*sin(d*x + c))*cos(6*d*
x + 6*c) - 88*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*cos(5*d*x +
 5*c) - 24*(11*a^2*sin(3*d*x + 3*c) + 3*a^2*sin(d*x + c))*cos(4*d*x + 4*c)
+ 3*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x
+ 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(
6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(
4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) +
a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c)
+ a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c
) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x +
2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c
))*sin(6*d*x + 6*c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) +
 1) - 3*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*
d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*
sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 +
 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c
) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2
*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x +
 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*
x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x +
 2*c))*sin(6*d*x + 6*c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x +
c) + 1) - 4*(3*a^2*cos(7*d*x + 7*c) + 11*a^2*cos(5*d*x + 5*c) - 11*a^2*cos(
3*d*x + 3*c) - 3*a^2*cos(d*x + c))*sin(8*d*x + 8*c) + 12*(4*a^2*cos(6*d*x +
 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*sin(7*d*x +
7*c) - 16*(11*a^2*cos(5*d*x + 5*c) - 11*a^2*cos(3*d*x + 3*c) - 3*a^2*cos(d*
x + c))*sin(6*d*x + 6*c) + 44*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2
*c) + a^2)*sin(5*d*x + 5*c) + 24*(11*a^2*cos(3*d*x + 3*c) + 3*a^2*cos(d*x +
 c))*sin(4*d*x + 4*c) - 44*(4*a^2*cos(2*d*x + 2*c) + a^2)*sin(3*d*x + 3*c))
*sqrt(a)/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c)
+ 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(
2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*
x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c
)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(
8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x +
2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x +
2*c) + 1)*d)
```

Fricas [A] time = 1.42479, size = 236, normalized size = 2.41

$$\frac{3\, a^{\frac{5}{2}} \log\left(2\, a \tan\left(dx+c\right)^{2} + 2\, \sqrt{a \tan\left(dx+c\right)^{2} + a}\sqrt{a} \tan\left(dx+c\right) + a\right) + 2\, \left(2\, a^{2} \tan\left(dx+c\right)^{3} + 5\, a^{2} \tan\left(dx+c\right)\right)\sqrt{\ldots}}{16\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*a^(5/2)*log(2*a*tan(d*x + c)^2 + 2*sqrt(a*tan(d*x + c)^2 + a)*sqrt(
a)*tan(d*x + c) + a) + 2*(2*a^2*tan(d*x + c)^3 + 5*a^2*tan(d*x + c))*sqrt(a
*tan(d*x + c)^2 + a))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\tan^2\left(c+dx\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((a*tan(c + d*x)**2 + a)**(5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.273 $\int \frac{\tan^3(x)}{\sqrt{a+a\tan^2(x)}}\,dx$

Optimal. Leaf size=25

$$\frac{\sqrt{a\sec^2(x)}}{a}+\frac{1}{\sqrt{a\sec^2(x)}}$$

```
[Out] 1/Sqrt[a*Sec[x]^2] + Sqrt[a*Sec[x]^2]/a
```

Rubi [A] time = 0.0922876, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3657, 4124, 43}

$$\frac{\sqrt{a\sec^2(x)}}{a}+\frac{1}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3/Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] 1/Sqrt[a*Sec[x]^2] + Sqrt[a*Sec[x]^2]/a
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\tan^3(x)}{\sqrt{a+a\tan^2(x)}}\,dx = \int \frac{\tan^3(x)}{\sqrt{a\sec^2(x)}}\,dx$$
$$= \frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{-1+x}{(ax)^{3/2}}\,dx, x, \sec^2(x)\right)$$
$$= \frac{1}{2}a\,\mathrm{Subst}\left(\int \left(-\frac{1}{(ax)^{3/2}}+\frac{1}{a\sqrt{ax}}\right)dx, x, \sec^2(x)\right)$$
$$= \frac{1}{\sqrt{a\sec^2(x)}}+\frac{\sqrt{a\sec^2(x)}}{a}$$

Mathematica [A] time = 0.0238331, size = 17, normalized size = 0.68

$$\frac{\sec^2(x)+1}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3/Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] (1 + Sec[x]^2)/Sqrt[a*Sec[x]^2]
```

Maple [A] time = 0.03, size = 26, normalized size = 1.

$$\frac{1}{a}\sqrt{a+a(\tan(x))^2}+\frac{1}{\sqrt{a+a(\tan(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a+a*tan(x)^2)^(1/2),x)
```

```
[Out] 1/a*(a+a*tan(x)^2)^(1/2)+1/(a+a*tan(x)^2)^(1/2)
```

Maxima [A] time = 1.13729, size = 50, normalized size = 2.

$$\frac{\left(\sin(x)^2-2\right)\sqrt{\sin(x)+1}\sqrt{-\sin(x)+1}}{\sqrt{a}\sin(x)^2-\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (sin(x)^2 - 2)*sqrt(sin(x) + 1)*sqrt(-sin(x) + 1)/(sqrt(a)*sin(x)^2 - sqrt(
a))
```

Fricas [A] time = 1.33972, size = 50, normalized size = 2.

$$\frac{\tan(x)^2+2}{\sqrt{a\tan(x)^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (tan(x)^2 + 2)/sqrt(a*tan(x)^2 + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^{3}(x)}{\sqrt{a\left(\tan^{2}(x)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3/(a+a*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(tan(x)**3/sqrt(a*(tan(x)**2 + 1)), x)
```

Giac [A] time = 1.10193, size = 36, normalized size = 1.44

$$\frac{\sqrt{a\tan(x)^{2}+a}+\frac{a}{\sqrt{a\tan(x)^{2}+a}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (sqrt(a*tan(x)^2 + a) + a/sqrt(a*tan(x)^2 + a))/a
```

3.274 $\int \frac{\tan^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx$

Optimal. Leaf size=31

$$\frac{\sec(x)\tanh^{-1}(\sin(x))}{\sqrt{a\sec^2(x)}}-\frac{\tan(x)}{\sqrt{a\sec^2(x)}}$$

```
[Out] (ArcTanh[Sin[x]]*Sec[x])/Sqrt[a*Sec[x]^2] - Tan[x]/Sqrt[a*Sec[x]^2]
```

Rubi [A] time = 0.0929046, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3657, 4125, 2592, 321, 206}

$$\frac{\sec(x)\tanh^{-1}(\sin(x))}{\sqrt{a\sec^2(x)}}-\frac{\tan(x)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2/Sqrt[a + a*Tan[x]^2],x]

[Out] (ArcTanh[Sin[x]]*Sec[x])/Sqrt[a*Sec[x]^2] - Tan[x]/Sqrt[a*Sec[x]^2]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4125

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sec[e + f*x]^
n)^FracPart[p])/(Sec[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx &= \int \frac{\tan^2(x)}{\sqrt{a\sec^2(x)}}\,dx \\
&= \frac{\sec(x)\int \sin(x)\tan(x)\,dx}{\sqrt{a\sec^2(x)}} \\
&= \frac{\sec(x)\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(x)\right)}{\sqrt{a\sec^2(x)}} \\
&= -\frac{\tan(x)}{\sqrt{a\sec^2(x)}} + \frac{\sec(x)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)}{\sqrt{a\sec^2(x)}} \\
&= \frac{\tanh^{-1}(\sin(x))\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{\tan(x)}{\sqrt{a\sec^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.0373315, size = 49, normalized size = 1.58

$$-\frac{\sec(x)\left(\sin(x)+\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2/Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -((Sec[x]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]] + Sin[x]))/S
qrt[a*Sec[x]^2])
```

Maple [A] time = 0.028, size = 38, normalized size = 1.2

$$\ln\left(\sqrt{a}\tan(x)+\sqrt{a+a(\tan(x))^2}\right)\frac{1}{\sqrt{a}}-\tan(x)\frac{1}{\sqrt{a+a(\tan(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a+a*tan(x)^2)^(1/2),x)
```

```
[Out] ln(a^(1/2)*tan(x)+(a+a*tan(x)^2)^(1/2))/a^(1/2)-tan(x)/(a+a*tan(x)^2)^(1/2)
```

Maxima [A] time = 1.8584, size = 57, normalized size = 1.84

$$\frac{\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right)-\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)-2\sin(x)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2*
sin(x) + 1) - 2*sin(x))/sqrt(a)
```

Fricas [B] time = 1.27321, size = 190, normalized size = 6.13

$$\frac{\left(\tan\left(x\right)^{2}+1\right)\sqrt{a}\log\left(2\,a\tan\left(x\right)^{2}+2\,\sqrt{a\tan\left(x\right)^{2}+a}\sqrt{a}\tan\left(x\right)+a\right)-2\,\sqrt{a\tan\left(x\right)^{2}+a}\tan\left(x\right)}{2\left(a\tan\left(x\right)^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((tan(x)^2 + 1)*sqrt(a)*log(2*a*tan(x)^2 + 2*sqrt(a*tan(x)^2 + a)*sqrt(
a)*tan(x) + a) - 2*sqrt(a*tan(x)^2 + a)*tan(x))/(a*tan(x)^2 + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan^{2}\left(x\right)}{\sqrt{a\left(\tan^{2}\left(x\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2/(a+a*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(tan(x)**2/sqrt(a*(tan(x)**2 + 1)), x)
```

Giac [A] time = 1.10569, size = 54, normalized size = 1.74

$$-\frac{\log\left(\left|-\sqrt{a}\tan\left(x\right)+\sqrt{a\tan\left(x\right)^{2}+a}\right|\right)}{\sqrt{a}}-\frac{\tan\left(x\right)}{\sqrt{a\tan\left(x\right)^{2}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(a)*tan(x) + sqrt(a*tan(x)^2 + a)))/sqrt(a) - tan(x)/sqrt(a*t
an(x)^2 + a)
```

3.275 $\int \frac{\tan(x)}{\sqrt{a+a\tan^2(x)}}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{\sqrt{a\sec^2(x)}}$$

```
[Out] -(1/Sqrt[a*Sec[x]^2])
```

Rubi [A] time = 0.0464586, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3657, 4124, 32}

$$-\frac{1}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -(1/Sqrt[a*Sec[x]^2])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(x)}{\sqrt{a+a\tan^2(x)}}\,dx &= \int \frac{\tan(x)}{\sqrt{a\sec^2(x)}}\,dx\\ &= \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{(ax)^{3/2}}\,dx, x, \sec^2(x)\right)\\ &= -\frac{1}{\sqrt{a\sec^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.0128336, size = 12, normalized size = 1.

$$-\frac{1}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -(1/Sqrt[a*Sec[x]^2])
```

Maple [A] time = 0.023, size = 13, normalized size = 1.1

$$-\frac{1}{\sqrt{a+a\left(\tan\left(x\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+a*tan(x)^2)^(1/2),x)
```

```
[Out] -1/(a+a*tan(x)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan\left(x\right)}{\sqrt{a\tan\left(x\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)/sqrt(a*tan(x)^2 + a), x)
```

Fricas [A] time = 1.37088, size = 34, normalized size = 2.83

$$-\frac{1}{\sqrt{a\tan\left(x\right)^{2}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/sqrt(a*tan(x)^2 + a)
```

Sympy [A] time = 0.609833, size = 14, normalized size = 1.17

$$-\frac{1}{\sqrt{a\tan^{2}\left(x\right)+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)**2)**(1/2),x)
```

```
[Out] -1/sqrt(a*tan(x)**2 + a)
```

Giac [A] time = 1.08739, size = 16, normalized size = 1.33

$$-\frac{1}{\sqrt{a\tan\left(x\right)^{2}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/sqrt(a*tan(x)^2 + a)
```

3.276 $\int \frac{\cot(x)}{\sqrt{a+a\tan^2(x)}}\,dx$

Optimal. Leaf size=35

$$\frac{1}{\sqrt{a\sec^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -(ArcTanh[Sqrt[a*Sec[x]^2]/Sqrt[a]]/Sqrt[a]) + 1/Sqrt[a*Sec[x]^2]
```

Rubi [A] time = 0.0796105, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3657, 4124, 51, 63, 207}

$$\frac{1}{\sqrt{a\sec^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a*Sec[x]^2]/Sqrt[a]]/Sqrt[a]) + 1/Sqrt[a*Sec[x]^2]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(x)}{\sqrt{a+a\tan^2(x)}}\,dx &= \int \frac{\cot(x)}{\sqrt{a\sec^2(x)}}\,dx \\
&= \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{(-1+x)(ax)^{3/2}}\,dx, x, \sec^2(x)\right) \\
&= \frac{1}{\sqrt{a\sec^2(x)}} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{ax}}\,dx, x, \sec^2(x)\right) \\
&= \frac{1}{\sqrt{a\sec^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a\sec^2(x)}\right)}{a} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{1}{\sqrt{a\sec^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.0328115, size = 32, normalized size = 0.91

$$\frac{\sec(x)\left(\cos(x)+\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)\right)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] ((Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]])*Sec[x])/Sqrt[a*Sec[x]^2]
```

Maple [A] time = 0.079, size = 29, normalized size = 0.8

$$\frac{1}{\cos(x)}\left(\cos(x)+\ln\left(-\frac{\cos(x)-1}{\sin(x)}\right)+1\right)\frac{1}{\sqrt{\frac{a}{(\cos(x))^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+a*tan(x)^2)^(1/2),x)
```

```
[Out] (cos(x)+ln(-(cos(x)-1)/sin(x))+1)/(a/cos(x)^2)^(1/2)/cos(x)
```

Maxima [A] time = 1.92286, size = 57, normalized size = 1.63

$$\frac{2\cos(x)-\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)+\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*cos(x) - log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1))/sqrt(a)
```

Fricas [B] time = 1.40237, size = 186, normalized size = 5.31

$$\frac{\left(\tan\left(x\right)^{2}+1\right)\sqrt{a}\log\left(\frac{a\tan(x)^{2}-2\sqrt{a\tan(x)^{2}+a}\sqrt{a}+2a}{\tan(x)^{2}}\right)+2\sqrt{a\tan\left(x\right)^{2}+a}}{2\left(a\tan\left(x\right)^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((tan(x)^2 + 1)*sqrt(a)*log((a*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a)*sqrt(a
) + 2*a)/tan(x)^2) + 2*sqrt(a*tan(x)^2 + a))/(a*tan(x)^2 + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cot\left(x\right)}{\sqrt{a\left(\tan^{2}\left(x\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(cot(x)/sqrt(a*(tan(x)**2 + 1)), x)
```

Giac [A] time = 1.0731, size = 58, normalized size = 1.66

$$a\left(\frac{\arctan\left(\frac{\sqrt{a\tan(x)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{1}{\sqrt{a\tan\left(x\right)^{2}+a}a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] a*(arctan(sqrt(a*tan(x)^2 + a)/sqrt(-a))/(sqrt(-a)*a) + 1/(sqrt(a*tan(x)^2
+ a)*a))
```

3.277 $\int \frac{\cot^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx$

Optimal. Leaf size=31

$$-\frac{\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}-\frac{\tan(x)}{\sqrt{a\sec^2(x)}}$$

```
[Out] -((Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2]) - Tan[x]/Sqrt[a*Sec[x]^2]
```

Rubi [A] time = 0.0923178, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3657, 4125, 2590, 14}

$$-\frac{\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}-\frac{\tan(x)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2/Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -((Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2]) - Tan[x]/Sqrt[a*Sec[x]^2]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4125

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sec[e + f*x]^
n)^FracPart[p])/(Sec[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\cot^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx = \int \frac{\cot^2(x)}{\sqrt{a\sec^2(x)}}\,dx$$
$$= \frac{\sec(x)\int \cos(x)\cot^2(x)\,dx}{\sqrt{a\sec^2(x)}}$$
$$= -\frac{\sec(x)\,\mathrm{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, -\sin(x)\right)}{\sqrt{a\sec^2(x)}}$$
$$= -\frac{\sec(x)\,\mathrm{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx, x, -\sin(x)\right)}{\sqrt{a\sec^2(x)}}$$
$$= -\frac{\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{\tan(x)}{\sqrt{a\sec^2(x)}}$$

Mathematica [A] time = 0.0270374, size = 22, normalized size = 0.71

$$\frac{-\tan(x)-\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2/Sqrt[a + a*Tan[x]^2],x]

[Out] (-(Csc[x]*Sec[x]) - Tan[x])/Sqrt[a*Sec[x]^2]
```

Maple [A] time = 0.086, size = 24, normalized size = 0.8

$$\frac{(\cos(x))^2-2}{\sin(x)\cos(x)}\frac{1}{\sqrt{\frac{a}{(\cos(x))^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a+a*tan(x)^2)^(1/2),x)

[Out] (cos(x)^2-2)/sin(x)/cos(x)/(a/cos(x)^2)^(1/2)
```

Maxima [B] time = 1.87953, size = 173, normalized size = 5.58

$$\frac{((\sin(3x)-\sin(x))\cos(4x)-(\cos(3x)-\cos(x))\sin(4x)-(6\cos(2x)-1)\sin(3x)+6\cos(3x)\sin(2x)-6\cos(x)}{2\left(a\cos(3x)^2-2a\cos(3x)\cos(x)+a\cos(x)^2+a\sin(3x)^2-2a\sin(3x)\sin(x)+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")

[Out] 1/2*((sin(3*x) - sin(x))*cos(4*x) - (cos(3*x) - cos(x))*sin(4*x) - (6*cos(2
*x) - 1)*sin(3*x) + 6*cos(3*x)*sin(2*x) - 6*cos(x)*sin(2*x) + 6*cos(2*x)*si
n(x) - sin(x))*sqrt(a)/(a*cos(3*x)^2 - 2*a*cos(3*x)*cos(x) + a*cos(x)^2 + a
```

```
*sin(3*x)^2 - 2*a*sin(3*x)*sin(x) + a*sin(x)^2)
```

Fricas [A] time = 1.402, size = 86, normalized size = 2.77

$$-\frac{\sqrt{a\tan(x)^2+a}\left(2\tan(x)^2+1\right)}{a\tan(x)^3+a\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(a*tan(x)^2 + a)*(2*tan(x)^2 + 1)/(a*tan(x)^3 + a*tan(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cot^2(x)}{\sqrt{a\left(\tan^2(x)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2/(a+a*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(cot(x)**2/sqrt(a*(tan(x)**2 + 1)), x)
```

Giac [A] time = 1.08203, size = 63, normalized size = 2.03

$$-\frac{\tan(x)}{\sqrt{a\tan(x)^2+a}}+\frac{2\sqrt{a}}{\left(\sqrt{a}\tan(x)-\sqrt{a\tan(x)^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -tan(x)/sqrt(a*tan(x)^2 + a) + 2*sqrt(a)/((sqrt(a)*tan(x) - sqrt(a*tan(x)^2
 + a))^2 - a)
```

3.278 $\int \frac{\tan^3(x)}{(a+a\tan^2(x))^{3/2}}\,dx$

Optimal. Leaf size=30

$$\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}-\frac{1}{a\sqrt{a\sec^2(x)}}$$

```
[Out] 1/(3*(a*Sec[x]^2)^(3/2)) - 1/(a*Sqrt[a*Sec[x]^2])
```

Rubi [A] time = 0.0967719, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3657, 4124, 43}

$$\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}-\frac{1}{a\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3/(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] 1/(3*(a*Sec[x]^2)^(3/2)) - 1/(a*Sqrt[a*Sec[x]^2])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx &= \int \frac{\tan^3(x)}{\left(a\sec^2(x)\right)^{3/2}}\,dx\\
&= \frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{-1+x}{(ax)^{5/2}}\,dx, x, \sec^2(x)\right)\\
&= \frac{1}{2}a\,\mathrm{Subst}\left(\int \left(-\frac{1}{(ax)^{5/2}}+\frac{1}{a(ax)^{3/2}}\right)dx, x, \sec^2(x)\right)\\
&= \frac{1}{3\left(a\sec^2(x)\right)^{3/2}}-\frac{1}{a\sqrt{a\sec^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.0247385, size = 23, normalized size = 0.77

$$\frac{\cos(2x)-5}{6a\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3/(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] (-5 + Cos[2*x])/(6*a*Sqrt[a*Sec[x]^2])
```

Maple [A] time = 0.019, size = 29, normalized size = 1.

$$-\frac{1}{a}\frac{1}{\sqrt{a+a(\tan(x))^2}}+\frac{1}{3}\left(a+a(\tan(x))^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a+a*tan(x)^2)^(3/2),x)
```

```
[Out] -1/a/(a+a*tan(x)^2)^(1/2)+1/3/(a+a*tan(x)^2)^(3/2)
```

Maxima [A] time = 1.1809, size = 51, normalized size = 1.7

$$\frac{\left(\sin(x)^2+2\right)(\sin(x)+1)^{\frac{3}{2}}(-\sin(x)+1)^{\frac{3}{2}}}{3\left(a^{\frac{3}{2}}\sin(x)^2-a^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sin(x)^2 + 2)*(sin(x) + 1)^(3/2)*(-sin(x) + 1)^(3/2)/(a^(3/2)*sin(x)^2
 - a^(3/2))
```

Fricas [A] time = 1.29534, size = 111, normalized size = 3.7

$$-\frac{\sqrt{a\tan(x)^2+a}\left(3\tan(x)^2+2\right)}{3\left(a^2\tan(x)^4+2a^2\tan(x)^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(a*tan(x)^2 + a)*(3*tan(x)^2 + 2)/(a^2*tan(x)^4 + 2*a^2*tan(x)^2 +
 a^2)
```

Sympy [A] time = 3.49888, size = 36, normalized size = 1.2

$$\begin{cases} \frac{\frac{a}{3\left(a\tan^2(x)+a\right)^{\frac{3}{2}}}-\frac{1}{\sqrt{a\tan^2(x)+a}}}{a} & \text{for } a \neq 0 \\ \tilde{\infty}\tan^4(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3/(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] Piecewise(((a/(3*(a*tan(x)**2 + a)**(3/2)) - 1/sqrt(a*tan(x)**2 + a))/a, Ne
(a, 0)), (zoo*tan(x)**4, True))
```

Giac [A] time = 1.08396, size = 35, normalized size = 1.17

$$-\frac{3\,a\tan(x)^2+2\,a}{3\left(a\tan(x)^2+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*a*tan(x)^2 + 2*a)/((a*tan(x)^2 + a)^(3/2)*a)
```

3.279 $\int \frac{\tan^2(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=23

$$\frac{\sin^2(x)\tan(x)}{3a\sqrt{a\sec^2(x)}}$$

```
[Out] (Sin[x]^2*Tan[x])/(3*a*Sqrt[a*Sec[x]^2])
```

Rubi [A] time = 0.101071, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3657, 4125, 2564, 30}

$$\frac{\sin^2(x)\tan(x)}{3a\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2/(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] (Sin[x]^2*Tan[x])/(3*a*Sqrt[a*Sec[x]^2])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4125

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sec[e + f*x]^
n)^FracPart[p])/(Sec[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^2(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx=\int \frac{\tan^2(x)}{\left(a\sec^2(x)\right)^{3/2}}\,dx$$
$$=\frac{\sec(x)\int \cos(x)\sin^2(x)\,dx}{a\sqrt{a\sec^2(x)}}$$
$$=\frac{\sec(x)\,\text{Subst}\left(\int x^2\,dx,x,\sin(x)\right)}{a\sqrt{a\sec^2(x)}}$$
$$=\frac{\sin^2(x)\tan(x)}{3a\sqrt{a\sec^2(x)}}$$

Mathematica [A] time = 0.0179405, size = 18, normalized size = 0.78

$$\frac{\tan^3(x)}{3\left(a\sec^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2/(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] Tan[x]^3/(3*(a*Sec[x]^2)^(3/2))
```

Maple [B] time = 0.016, size = 56, normalized size = 2.4

$$\frac{\tan(x)}{a}\frac{1}{\sqrt{a+a(\tan(x))^2}}-a\left(\frac{\tan(x)}{3a}\left(a+a(\tan(x))^2\right)^{-\frac{3}{2}}+\frac{2\tan(x)}{3a^2}\frac{1}{\sqrt{a+a(\tan(x))^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a+a*tan(x)^2)^(3/2),x)
```

```
[Out] 1/a*tan(x)/(a+a*tan(x)^2)^(1/2)-a*(1/3/a*tan(x)/(a+a*tan(x)^2)^(3/2)+2/3/a^
2*tan(x)/(a+a*tan(x)^2)^(1/2))
```

Maxima [A] time = 1.92509, size = 19, normalized size = 0.83

$$-\frac{\sin(3x)-3\sin(x)}{12a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/12*(sin(3*x) - 3*sin(x))/a^(3/2)
```

Fricas [B] time = 1.34167, size = 99, normalized size = 4.3

$$\frac{\sqrt{a \tan\left(x\right)^{2} + a} \tan\left(x\right)^{3}}{3 \left(a^{2} \tan\left(x\right)^{4} + 2 a^{2} \tan\left(x\right)^{2} + a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(a*tan(x)^2 + a)*tan(x)^3/(a^2*tan(x)^4 + 2*a^2*tan(x)^2 + a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^{2}\left(x\right)}{\left(a \left(\tan^{2}\left(x\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2/(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] Integral(tan(x)**2/(a*(tan(x)**2 + 1))**(3/2), x)
```

Giac [A] time = 1.11288, size = 22, normalized size = 0.96

$$\frac{\tan\left(x\right)^{3}}{3 \left(a \tan\left(x\right)^{2} + a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*tan(x)^3/(a*tan(x)^2 + a)^(3/2)
```

3.280 $\int \frac{\tan(x)}{\left(a+a\tan^2(x)\right)^{3/2}} \, dx$

Optimal. Leaf size=14

$$-\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}$$

```
[Out] -1/(3*(a*Sec[x]^2)^(3/2))
```

Rubi [A] time = 0.0489939, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3657, 4124, 32}

$$-\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] -1/(3*(a*Sec[x]^2)^(3/2))
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\tan(x)}{\left(a+a\tan^2(x)\right)^{3/2}} \, dx &= \int \frac{\tan(x)}{\left(a\sec^2(x)\right)^{3/2}} \, dx \\ &= \frac{1}{2} a \operatorname{Subst}\left(\int \frac{1}{(ax)^{5/2}} \, dx, x, \sec^2(x)\right) \\ &= -\frac{1}{3\left(a\sec^2(x)\right)^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.0117573, size = 14, normalized size = 1.

$$-\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] -1/(3*(a*Sec[x]^2)^(3/2))
```

Maple [A] time = 0.011, size = 13, normalized size = 0.9

$$-\frac{1}{3}\left(a+a\left(\tan\left(x\right)\right)^{2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+a*tan(x)^2)^(3/2),x)
```

```
[Out] -1/3/(a+a*tan(x)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(x\right)}{\left(a\tan\left(x\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)/(a*tan(x)^2 + a)^(3/2), x)
```

Fricas [B] time = 1.35137, size = 88, normalized size = 6.29

$$-\frac{\sqrt{a\tan\left(x\right)^{2}+a}}{3\left(a^{2}\tan\left(x\right)^{4}+2\,a^{2}\tan\left(x\right)^{2}+a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(a*tan(x)^2 + a)/(a^2*tan(x)^4 + 2*a^2*tan(x)^2 + a^2)
```

Sympy [A] time = 2.25182, size = 15, normalized size = 1.07

$$-\frac{1}{3\left(a\tan^{2}\left(x\right)+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] -1/(3*(a*tan(x)**2 + a)**(3/2))
```

Giac [A] time = 1.08089, size = 16, normalized size = 1.14

$$-\frac{1}{3\left(a\tan\left(x\right)^{2}+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3/(a*tan(x)^2 + a)^(3/2)
```

3.281 $\int \frac{\cot(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=53

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{1}{a\sqrt{a\sec^2(x)}}+\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}$$

```
[Out] -(ArcTanh[Sqrt[a*Sec[x]^2]/Sqrt[a]]/a^(3/2)) + 1/(3*(a*Sec[x]^2)^(3/2)) + 1
/(a*Sqrt[a*Sec[x]^2])
```

Rubi [A] time = 0.0909338, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3657, 4124, 51, 63, 207}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{1}{a\sqrt{a\sec^2(x)}}+\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a*Sec[x]^2]/Sqrt[a]]/a^(3/2)) + 1/(3*(a*Sec[x]^2)^(3/2)) + 1
/(a*Sqrt[a*Sec[x]^2])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx &= \int \frac{\cot(x)}{\left(a\sec^2(x)\right)^{3/2}}\,dx\\
&= \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{(-1+x)(ax)^{5/2}}\,dx, x, \sec^2(x)\right)\\
&= \frac{1}{3\left(a\sec^2(x)\right)^{3/2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1+x)(ax)^{3/2}}\,dx, x, \sec^2(x)\right)\\
&= \frac{1}{3\left(a\sec^2(x)\right)^{3/2}} + \frac{1}{a\sqrt{a\sec^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{ax}}\,dx, x, \sec^2(x)\right)}{2a}\\
&= \frac{1}{3\left(a\sec^2(x)\right)^{3/2}} + \frac{1}{a\sqrt{a\sec^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a\sec^2(x)}\right)}{a^2}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{1}{3\left(a\sec^2(x)\right)^{3/2}} + \frac{1}{a\sqrt{a\sec^2(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.0584834, size = 47, normalized size = 0.89

$$\frac{\cos(3x)\sec(x) + 12\sec(x)\left(\log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right)\right) + 15}{12a\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] (15 + Cos[3*x]*Sec[x] + 12*(-Log[Cos[x/2]] + Log[Sin[x/2]])*Sec[x])/(12*a*S
qrt[a*Sec[x]^2])
```

Maple [A] time = 0.073, size = 38, normalized size = 0.7

$$\frac{1}{3\left(\cos(x)\right)^3}\left(\left(\cos(x)\right)^3 + 3\cos(x) + 3\ln\left(-\frac{\cos(x)-1}{\sin(x)}\right) + 4\right)\left(\frac{a}{\left(\cos(x)\right)^2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+a*tan(x)^2)^(3/2),x)
```

```
[Out] 1/3*(cos(x)^3+3*cos(x)+3*ln(-(cos(x)-1)/sin(x))+4)/cos(x)^3/(a/cos(x)^2)^(3
/2)
```

Maxima [A] time = 1.91696, size = 65, normalized size = 1.23

$$\frac{\cos(3x) + 15\cos(x) - 6\log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right) + 6\log\left(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1\right)}{12a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(cos(3*x) + 15*cos(x) - 6*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 6*
log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1))/a^(3/2)
```

Fricas [B] time = 1.60905, size = 258, normalized size = 4.87

$$\frac{3\left(\tan(x)^4 + 2\tan(x)^2 + 1\right)\sqrt{a}\log\left(\frac{a\tan(x)^2 - 2\sqrt{a\tan(x)^2 + a}\sqrt{a} + 2a}{\tan(x)^2}\right) + 2\sqrt{a\tan(x)^2 + a}\left(3\tan(x)^2 + 4\right)}{6\left(a^2\tan(x)^4 + 2a^2\tan(x)^2 + a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(tan(x)^4 + 2*tan(x)^2 + 1)*sqrt(a)*log((a*tan(x)^2 - 2*sqrt(a*tan(x
)^2 + a)*sqrt(a) + 2*a)/tan(x)^2) + 2*sqrt(a*tan(x)^2 + a)*(3*tan(x)^2 + 4)
)/(a^2*tan(x)^4 + 2*a^2*tan(x)^2 + a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{\left(a\left(\tan^2(x) + 1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] Integral(cot(x)/(a*(tan(x)**2 + 1))**(3/2), x)
```

Giac [A] time = 1.09109, size = 76, normalized size = 1.43

$$\frac{1}{3}a\left(\frac{3\arctan\left(\frac{\sqrt{a\tan(x)^2 + a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{3a\tan(x)^2 + 4a}{\left(a\tan(x)^2 + a\right)^{\frac{3}{2}}a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*a*(3*arctan(sqrt(a*tan(x)^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*a*tan(x)
^2 + 4*a)/((a*tan(x)^2 + a)^(3/2)*a^2))
```

3.282 $\int \frac{\cot^2(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=60

$$-\frac{\csc(x)\sec(x)}{a\sqrt{a\sec^2(x)}}-\frac{2\tan(x)}{a\sqrt{a\sec^2(x)}}+\frac{\sin^2(x)\tan(x)}{3a\sqrt{a\sec^2(x)}}$$

```
[Out] -((Csc[x]*Sec[x])/(a*Sqrt[a*Sec[x]^2])) - (2*Tan[x])/(a*Sqrt[a*Sec[x]^2]) +
 (Sin[x]^2*Tan[x])/(3*a*Sqrt[a*Sec[x]^2])
```

Rubi [A] time = 0.114722, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3657, 4125, 2590, 270}

$$-\frac{\csc(x)\sec(x)}{a\sqrt{a\sec^2(x)}}-\frac{2\tan(x)}{a\sqrt{a\sec^2(x)}}+\frac{\sin^2(x)\tan(x)}{3a\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2/(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] -((Csc[x]*Sec[x])/(a*Sqrt[a*Sec[x]^2])) - (2*Tan[x])/(a*Sqrt[a*Sec[x]^2]) +
 (Sin[x]^2*Tan[x])/(3*a*Sqrt[a*Sec[x]^2])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4125

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sec[e + f*x]^
n)^FracPart[p])/(Sec[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cot^2(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx = \int \frac{\cot^2(x)}{\left(a\sec^2(x)\right)^{3/2}}\,dx$$
$$= \frac{\sec(x)\int \cos^3(x)\cot^2(x)\,dx}{a\sqrt{a\sec^2(x)}}$$
$$= -\frac{\sec(x)\,\text{Subst}\left(\int \frac{(1-x^2)^2}{x^2}\,dx, x, -\sin(x)\right)}{a\sqrt{a\sec^2(x)}}$$
$$= -\frac{\sec(x)\,\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)dx, x, -\sin(x)\right)}{a\sqrt{a\sec^2(x)}}$$
$$= -\frac{\csc(x)\sec(x)}{a\sqrt{a\sec^2(x)}} - \frac{2\tan(x)}{a\sqrt{a\sec^2(x)}} + \frac{\sin^2(x)\tan(x)}{3a\sqrt{a\sec^2(x)}}$$

Mathematica [A] time = 0.056845, size = 31, normalized size = 0.52

$$\frac{\sec^3(x)\left(\sin^3(x)-6\sin(x)-3\csc(x)\right)}{3\left(a\sec^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2/(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] (Sec[x]^3*(-3*Csc[x] - 6*Sin[x] + Sin[x]^3))/(3*(a*Sec[x]^2)^(3/2))
```

Maple [A] time = 0.071, size = 31, normalized size = 0.5

$$\frac{(\cos(x))^4+4(\cos(x))^2-8}{3\sin(x)(\cos(x))^3}\left(\frac{a}{(\cos(x))^2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a+a*tan(x)^2)^(3/2),x)
```

```
[Out] 1/3*(cos(x)^4+4*cos(x)^2-8)/sin(x)/cos(x)^3/(a/cos(x)^2)^(3/2)
```

Maxima [B] time = 1.88868, size = 304, normalized size = 5.07

$$\frac{((\sin(5x)-\sin(3x))\cos(8x)+20(\sin(5x)-\sin(3x))\cos(6x)+10(9\sin(4x)-2\sin(2x))\cos(5x)-(\cos(5x)-c}{24\left(a^2\cos(5x)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*((sin(5*x) - sin(3*x))*cos(8*x) + 20*(sin(5*x) - sin(3*x))*cos(6*x) +
10*(9*sin(4*x) - 2*sin(2*x))*cos(5*x) - (cos(5*x) - cos(3*x))*sin(8*x) - 20
*(cos(5*x) - cos(3*x))*sin(6*x) - (90*cos(4*x) - 20*cos(2*x) - 1)*sin(5*x)
```

```
- 90*cos(3*x)*sin(4*x) - (20*cos(2*x) + 1)*sin(3*x) + 90*cos(4*x)*sin(3*x)
+ 20*cos(3*x)*sin(2*x))*sqrt(a)/(a^2*cos(5*x)^2 - 2*a^2*cos(5*x)*cos(3*x) +
 a^2*cos(3*x)^2 + a^2*sin(5*x)^2 - 2*a^2*sin(5*x)*sin(3*x) + a^2*sin(3*x)^2
)
```

Fricas [A] time = 1.59882, size = 139, normalized size = 2.32

$$-\frac{\left(8\,\tan\left(x\right)^{4}+12\,\tan\left(x\right)^{2}+3\right)\sqrt{a\tan\left(x\right)^{2}+a}}{3\left(a^{2}\tan\left(x\right)^{5}+2\,a^{2}\tan\left(x\right)^{3}+a^{2}\tan\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(8*tan(x)^4 + 12*tan(x)^2 + 3)*sqrt(a*tan(x)^2 + a)/(a^2*tan(x)^5 + 2*
a^2*tan(x)^3 + a^2*tan(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cot^{2}\left(x\right)}{\left(a\left(\tan^{2}\left(x\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2/(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] Integral(cot(x)**2/(a*(tan(x)**2 + 1))**(3/2), x)
```

Giac [A] time = 1.1077, size = 74, normalized size = 1.23

$$-\frac{\left(5\,\tan\left(x\right)^{2}+6\right)\tan\left(x\right)}{3\left(a\tan\left(x\right)^{2}+a\right)^{\frac{3}{2}}}+\frac{2}{\left(\left(\sqrt{a}\tan\left(x\right)-\sqrt{a\tan\left(x\right)^{2}+a}\right)^{2}-a\right)\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(5*tan(x)^2 + 6)*tan(x)/(a*tan(x)^2 + a)^(3/2) + 2/(((sqrt(a)*tan(x) -
 sqrt(a*tan(x)^2 + a))^2 - a)*sqrt(a))
```

3.283 $\int \frac{1}{\sqrt{a+a\tan^2(c+dx)}}\,dx$

Optimal. Leaf size=24

$$\frac{\tan(c+dx)}{d\sqrt{a\sec^2(c+dx)}}$$

```
[Out] Tan[c + d*x]/(d*Sqrt[a*Sec[c + d*x]^2])
```

Rubi [A] time = 0.0292873, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3657, 4122, 191}

$$\frac{\tan(c+dx)}{d\sqrt{a\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + a*Tan[c + d*x]^2],x]
```

```
[Out] Tan[c + d*x]/(d*Sqrt[a*Sec[c + d*x]^2])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+a\tan^2(c+dx)}}\,dx &= \int \frac{1}{\sqrt{a\sec^2(c+dx)}}\,dx \\
&= \frac{a\,\text{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{3/2}}\,dx, x, \tan(c+dx)\right)}{d} \\
&= \frac{\tan(c+dx)}{d\sqrt{a\sec^2(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0407648, size = 24, normalized size = 1.

$$\frac{\tan(c+dx)}{d\sqrt{a\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + a*Tan[c + d*x]^2],x]
```

```
[Out] Tan[c + d*x]/(d*Sqrt[a*Sec[c + d*x]^2])
```

Maple [A] time = 0.031, size = 25, normalized size = 1.

$$\frac{\tan(dx+c)}{d}\frac{1}{\sqrt{a+a(\tan(dx+c))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tan(d*x+c)^2)^(1/2),x)
```

```
[Out] 1/d*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(1/2)
```

Maxima [A] time = 1.80395, size = 18, normalized size = 0.75

$$\frac{\sin(dx+c)}{\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sin(d*x + c)/(sqrt(a)*d)
```

Fricas [A] time = 1.59211, size = 92, normalized size = 3.83

$$\frac{\sqrt{a\tan(dx+c)^2+a}\tan(dx+c)}{ad\tan(dx+c)^2+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a*tan(d*x + c)^2 + a)*tan(d*x + c)/(a*d*tan(d*x + c)^2 + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a\tan^2(c+dx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*tan(c + d*x)**2 + a), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\tan\left(dx+c\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*tan(d*x + c)^2 + a), x)
```

3.284 $\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{2\tan(c+dx)}{3ad\sqrt{a\sec^2(c+dx)}}+\frac{\tan(c+dx)}{3d\left(a\sec^2(c+dx)\right)^{3/2}}$$

```
[Out] Tan[c + d*x]/(3*d*(a*Sec[c + d*x]^2)^(3/2)) + (2*Tan[c + d*x])/(3*a*d*Sqrt[
a*Sec[c + d*x]^2])
```

Rubi [A] time = 0.0351514, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3657, 4122, 192, 191}

$$\frac{2\tan(c+dx)}{3ad\sqrt{a\sec^2(c+dx)}}+\frac{\tan(c+dx)}{3d\left(a\sec^2(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(-3/2),x]
```

```
[Out] Tan[c + d*x]/(3*d*(a*Sec[c + d*x]^2)^(3/2)) + (2*Tan[c + d*x])/(3*a*d*Sqrt[
a*Sec[c + d*x]^2])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{3/2}}\,dx=\int \frac{1}{\left(a\sec^2(c+dx)\right)^{3/2}}\,dx$$
$$=\frac{a\,\text{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{5/2}}\,dx,x,\tan(c+dx)\right)}{d}$$
$$=\frac{\tan(c+dx)}{3d\left(a\sec^2(c+dx)\right)^{3/2}}+\frac{2\,\text{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{3/2}}\,dx,x,\tan(c+dx)\right)}{3d}$$
$$=\frac{\tan(c+dx)}{3d\left(a\sec^2(c+dx)\right)^{3/2}}+\frac{2\tan(c+dx)}{3ad\sqrt{a\sec^2(c+dx)}}$$

Mathematica [A] time = 0.0606538, size = 40, normalized size = 0.69

$$-\frac{\left(\sin^2(c+dx)-3\right)\tan(c+dx)}{3ad\sqrt{a\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(-3/2),x]
```

```
[Out] -((-3 + Sin[c + d*x]^2)*Tan[c + d*x])/(3*a*d*Sqrt[a*Sec[c + d*x]^2])
```

Maple [A] time = 0.019, size = 57, normalized size = 1.

$$\frac{a}{d}\left(\frac{\tan(dx+c)}{3\,a}\left(a+a(\tan(dx+c))^2\right)^{-\frac{3}{2}}+\frac{2\,\tan(dx+c)}{3\,a^2}\frac{1}{\sqrt{a+a(\tan(dx+c))^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tan(d*x+c)^2)^(3/2),x)
```

```
[Out] 1/d*a*(1/3/a*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(3/2)+2/3/a^2*tan(d*x+c)/(a+a*ta
n(d*x+c)^2)^(1/2))
```

Maxima [A] time = 1.95516, size = 35, normalized size = 0.6

$$\frac{\sin(3\,dx+3\,c)+9\,\sin(dx+c)}{12\,a^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(sin(3*d*x + 3*c) + 9*sin(d*x + c))/(a^(3/2)*d)
```

Fricas [A] time = 1.54072, size = 167, normalized size = 2.88

$$\frac{\sqrt{a\tan\left(dx+c\right)^{2}+a}\left(2\,\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)}{3\left(a^{2}d\tan\left(dx+c\right)^{4}+2\,a^{2}d\tan\left(dx+c\right)^{2}+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(a*tan(d*x + c)^2 + a)*(2*tan(d*x + c)^3 + 3*tan(d*x + c))/(a^2*d*t
an(d*x + c)^4 + 2*a^2*d*tan(d*x + c)^2 + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a\tan^{2}\left(c+dx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((a*tan(c + d*x)**2 + a)**(-3/2), x)
```

Giac [A] time = 1.66812, size = 109, normalized size = 1.88

$$-\frac{2\left(3\sqrt{a}\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^{2}-4\sqrt{a}\right)}{3\,a^{2}d\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^{3}\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*sqrt(a)*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^2 - 4*sqrt(
a))/(a^2*d*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^3*sgn(tan(1/2*d*
x + 1/2*c)^4 - 1))
```

3.285 $\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{5/2}} dx$

Optimal. Leaf size=88

$$\frac{8\tan(c+dx)}{15a^2d\sqrt{a\sec^2(c+dx)}}+\frac{4\tan(c+dx)}{15ad\left(a\sec^2(c+dx)\right)^{3/2}}+\frac{\tan(c+dx)}{5d\left(a\sec^2(c+dx)\right)^{5/2}}$$

```
[Out] Tan[c + d*x]/(5*d*(a*Sec[c + d*x]^2)^(5/2)) + (4*Tan[c + d*x])/(15*a*d*(a*S
ec[c + d*x]^2)^(3/2)) + (8*Tan[c + d*x])/(15*a^2*d*Sqrt[a*Sec[c + d*x]^2])
```

Rubi [A] time = 0.0436301, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3657, 4122, 192, 191}

$$\frac{8\tan(c+dx)}{15a^2d\sqrt{a\sec^2(c+dx)}}+\frac{4\tan(c+dx)}{15ad\left(a\sec^2(c+dx)\right)^{3/2}}+\frac{\tan(c+dx)}{5d\left(a\sec^2(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(-5/2),x]
```

```
[Out] Tan[c + d*x]/(5*d*(a*Sec[c + d*x]^2)^(5/2)) + (4*Tan[c + d*x])/(15*a*d*(a*S
ec[c + d*x]^2)^(3/2)) + (8*Tan[c + d*x])/(15*a^2*d*Sqrt[a*Sec[c + d*x]^2])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{5/2}}\,dx=\int \frac{1}{\left(a\sec^2(c+dx)\right)^{5/2}}\,dx$$
$$=\frac{a\,\mathrm{Subst}\left(\int \frac{1}{(a+ax^2)^{7/2}}\,dx,x,\tan(c+dx)\right)}{d}$$
$$=\frac{\tan(c+dx)}{5d\left(a\sec^2(c+dx)\right)^{5/2}}+\frac{4\,\mathrm{Subst}\left(\int \frac{1}{(a+ax^2)^{5/2}}\,dx,x,\tan(c+dx)\right)}{5d}$$
$$=\frac{\tan(c+dx)}{5d\left(a\sec^2(c+dx)\right)^{5/2}}+\frac{4\tan(c+dx)}{15ad\left(a\sec^2(c+dx)\right)^{3/2}}+\frac{8\,\mathrm{Subst}\left(\int \frac{1}{(a+ax^2)^{3/2}}\,dx,x,\tan(c+\right.}{15ad}$$
$$=\frac{\tan(c+dx)}{5d\left(a\sec^2(c+dx)\right)^{5/2}}+\frac{4\tan(c+dx)}{15ad\left(a\sec^2(c+dx)\right)^{3/2}}+\frac{8\tan(c+dx)}{15a^2d\sqrt{a\sec^2(c+dx)}}$$

Mathematica [A] time = 0.0999582, size = 52, normalized size = 0.59

$$\frac{\left(3\sin^4(c+dx)-10\sin^2(c+dx)+15\right)\tan(c+dx)}{15a^2d\sqrt{a\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(-5/2),x]
```

```
[Out] ((15 - 10*Sin[c + d*x]^2 + 3*Sin[c + d*x]^4)*Tan[c + d*x])/(15*a^2*d*Sqrt[a
*Sec[c + d*x]^2])
```

Maple [A] time = 0.022, size = 88, normalized size = 1.

$$\frac{a}{d}\left(\frac{\tan(dx+c)}{5a}\left(a+a(\tan(dx+c))^2\right)^{-\frac{5}{2}}+\frac{4}{5a}\left(\frac{\tan(dx+c)}{3a}\left(a+a(\tan(dx+c))^2\right)^{-\frac{3}{2}}+\frac{2\tan(dx+c)}{3a^2}\frac{1}{\sqrt{a+a(\tan}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tan(d*x+c)^2)^(5/2),x)
```

```
[Out] 1/d*a*(1/5/a*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(5/2)+4/5/a*(1/3/a*tan(d*x+c)/(a
+a*tan(d*x+c)^2)^(3/2)+2/3/a^2*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(1/2)))
```

Maxima [A] time = 1.81389, size = 53, normalized size = 0.6

$$\frac{3\sin(5dx+5c)+25\sin(3dx+3c)+150\sin(dx+c)}{240a^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/240*(3*sin(5*d*x + 5*c) + 25*sin(3*d*x + 3*c) + 150*sin(d*x + c))/(a^(5/2
)*d)
```

Fricas [A] time = 1.57921, size = 231, normalized size = 2.62

$$\frac{\left(8 \tan\left(dx+c\right)^{5}+20 \tan\left(dx+c\right)^{3}+15 \tan\left(dx+c\right)\right)\sqrt{a \tan\left(dx+c\right)^{2}+a}}{15 \left(a^{3} d \tan\left(dx+c\right)^{6}+3 a^{3} d \tan\left(dx+c\right)^{4}+3 a^{3} d \tan\left(dx+c\right)^{2}+a^{3} d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*tan(d*x + c)^5 + 20*tan(d*x + c)^3 + 15*tan(d*x + c))*sqrt(a*tan(d*
x + c)^2 + a)/(a^3*d*tan(d*x + c)^6 + 3*a^3*d*tan(d*x + c)^4 + 3*a^3*d*tan(
d*x + c)^2 + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a \tan^{2}\left(c+dx\right)+a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((a*tan(c + d*x)**2 + a)**(-5/2), x)
```

Giac [A] time = 1.87904, size = 147, normalized size = 1.67

$$-\frac{2 \left(15 \sqrt{a}\left(\frac{1}{\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}+\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right)^{4}-40 \sqrt{a}\left(\frac{1}{\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}+\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right)^{2}+48 \sqrt{a}\right)}{15 a^{3} d\left(\frac{1}{\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}+\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right)^{5} \mathrm{sgn}\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*sqrt(a)*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^4 - 40*sq
rt(a)*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^2 + 48*sqrt(a))/(a^3*
d*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^5*sgn(tan(1/2*d*x + 1/2*c
)^4 - 1))
```

3.286 $\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=118

$$\frac{16\tan(c+dx)}{35a^3d\sqrt{a\sec^2(c+dx)}}+\frac{8\tan(c+dx)}{35a^2d\left(a\sec^2(c+dx)\right)^{3/2}}+\frac{6\tan(c+dx)}{35ad\left(a\sec^2(c+dx)\right)^{5/2}}+\frac{\tan(c+dx)}{7d\left(a\sec^2(c+dx)\right)^{7/2}}$$

```
[Out] Tan[c + d*x]/(7*d*(a*Sec[c + d*x]^2)^(7/2)) + (6*Tan[c + d*x])/(35*a*d*(a*S
ec[c + d*x]^2)^(5/2)) + (8*Tan[c + d*x])/(35*a^2*d*(a*Sec[c + d*x]^2)^(3/2)
) + (16*Tan[c + d*x])/(35*a^3*d*Sqrt[a*Sec[c + d*x]^2])
```

Rubi [A] time = 0.0532144, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3657, 4122, 192, 191}

$$\frac{16\tan(c+dx)}{35a^3d\sqrt{a\sec^2(c+dx)}}+\frac{8\tan(c+dx)}{35a^2d\left(a\sec^2(c+dx)\right)^{3/2}}+\frac{6\tan(c+dx)}{35ad\left(a\sec^2(c+dx)\right)^{5/2}}+\frac{\tan(c+dx)}{7d\left(a\sec^2(c+dx)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(-7/2),x]
```

```
[Out] Tan[c + d*x]/(7*d*(a*Sec[c + d*x]^2)^(7/2)) + (6*Tan[c + d*x])/(35*a*d*(a*S
ec[c + d*x]^2)^(5/2)) + (8*Tan[c + d*x])/(35*a^2*d*(a*Sec[c + d*x]^2)^(3/2)
) + (16*Tan[c + d*x])/(35*a^3*d*Sqrt[a*Sec[c + d*x]^2])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{7/2}}\,dx = \int \frac{1}{\left(a\sec^2(c+dx)\right)^{7/2}}\,dx$$
$$= \frac{a\,\text{Subst}\left(\int \frac{1}{(a+ax^2)^{9/2}}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\tan(c+dx)}{7d\left(a\sec^2(c+dx)\right)^{7/2}} + \frac{6\,\text{Subst}\left(\int \frac{1}{(a+ax^2)^{7/2}}\,dx, x, \tan(c+dx)\right)}{7d}$$
$$= \frac{\tan(c+dx)}{7d\left(a\sec^2(c+dx)\right)^{7/2}} + \frac{6\tan(c+dx)}{35ad\left(a\sec^2(c+dx)\right)^{5/2}} + \frac{24\,\text{Subst}\left(\int \frac{1}{(a+ax^2)^{5/2}}\,dx, x, \tan(c+\right.}{35ad}$$
$$= \frac{\tan(c+dx)}{7d\left(a\sec^2(c+dx)\right)^{7/2}} + \frac{6\tan(c+dx)}{35ad\left(a\sec^2(c+dx)\right)^{5/2}} + \frac{8\tan(c+dx)}{35a^2d\left(a\sec^2(c+dx)\right)^{3/2}} + \frac{16\,\text{Subst}}{}$$
$$= \frac{\tan(c+dx)}{7d\left(a\sec^2(c+dx)\right)^{7/2}} + \frac{6\tan(c+dx)}{35ad\left(a\sec^2(c+dx)\right)^{5/2}} + \frac{8\tan(c+dx)}{35a^2d\left(a\sec^2(c+dx)\right)^{3/2}} + \frac{16\,\text{ta}}{35a^3d\sqrt{a}}$$

Mathematica [A] time = 0.184002, size = 62, normalized size = 0.53

$$\frac{\left(-5\sin^6(c+dx)+21\sin^4(c+dx)-35\sin^2(c+dx)+35\right)\tan(c+dx)}{35a^3d\sqrt{a\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(-7/2),x]
```

```
[Out] ((35 - 35*Sin[c + d*x]^2 + 21*Sin[c + d*x]^4 - 5*Sin[c + d*x]^6)*Tan[c + d*
x])/(35*a^3*d*Sqrt[a*Sec[c + d*x]^2])
```

Maple [A] time = 0.021, size = 119, normalized size = 1.

$$\frac{a}{d}\left(\frac{\tan(dx+c)}{7a}\left(a+a(\tan(dx+c))^2\right)^{-\frac{7}{2}} + \frac{6}{7a}\left(\frac{\tan(dx+c)}{5a}\left(a+a(\tan(dx+c))^2\right)^{-\frac{5}{2}} + \frac{4}{5a}\left(\frac{\tan(dx+c)}{3a}\left(a+a(\tan(d\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tan(d*x+c)^2)^(7/2),x)
```

```
[Out] 1/d*a*(1/7/a*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(7/2)+6/7/a*(1/5/a*tan(d*x+c)/(a
+a*tan(d*x+c)^2)^(5/2)+4/5/a*(1/3/a*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(3/2)+2/3
/a^2*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(1/2))))
```

Maxima [A] time = 1.82782, size = 68, normalized size = 0.58

$$\frac{5\sin(7dx+7c)+49\sin(5dx+5c)+245\sin(3dx+3c)+1225\sin(dx+c)}{2240a^{\frac{7}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2240*(5*sin(7*d*x + 7*c) + 49*sin(5*d*x + 5*c) + 245*sin(3*d*x + 3*c) + 1
225*sin(d*x + c))/(a^(7/2)*d)
```

Fricas [A] time = 1.53594, size = 293, normalized size = 2.48

$$\frac{\left(16 \tan\left(dx+c\right)^{7}+56 \tan\left(dx+c\right)^{5}+70 \tan\left(dx+c\right)^{3}+35 \tan\left(dx+c\right)\right)\sqrt{a\tan\left(dx+c\right)^{2}+a}}{35\left(a^{4}d\tan\left(dx+c\right)^{8}+4\,a^{4}d\tan\left(dx+c\right)^{6}+6\,a^{4}d\tan\left(dx+c\right)^{4}+4\,a^{4}d\tan\left(dx+c\right)^{2}+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/35*(16*tan(d*x + c)^7 + 56*tan(d*x + c)^5 + 70*tan(d*x + c)^3 + 35*tan(d*
x + c))*sqrt(a*tan(d*x + c)^2 + a)/(a^4*d*tan(d*x + c)^8 + 4*a^4*d*tan(d*x
+ c)^6 + 6*a^4*d*tan(d*x + c)^4 + 4*a^4*d*tan(d*x + c)^2 + a^4*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\tan^{2}\left(c+dx\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)**2)**(7/2),x)
```

```
[Out] Integral((a*tan(c + d*x)**2 + a)**(-7/2), x)
```

Giac [A] time = 2.11193, size = 185, normalized size = 1.57

$$-\frac{2\left(35\sqrt{a}\left(\frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^{6}-140\sqrt{a}\left(\frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^{4}+336\sqrt{a}\left(\frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}+\tan\right.\right.}{35\,a^{4}d\left(\frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^{7}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -2/35*(35*sqrt(a)*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^6 - 140*s
qrt(a)*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^4 + 336*sqrt(a)*(1/t
an(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^2 - 320*sqrt(a))/(a^4*d*(1/tan(
1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^7*sgn(tan(1/2*d*x + 1/2*c)^4 - 1))
```

3.287 $\int \left(1+\tan^2(x)\right)^{3/2} \, dx$

Optimal. Leaf size=22

$$\frac{1}{2}\tan(x)\sqrt{\sec^2(x)}+\frac{1}{2}\sinh^{-1}(\tan(x))$$

```
[Out] ArcSinh[Tan[x]]/2 + (Sqrt[Sec[x]^2]*Tan[x])/2
```

Rubi [A] time = 0.0151697, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3657, 4122, 195, 215}

$$\frac{1}{2}\tan(x)\sqrt{\sec^2(x)}+\frac{1}{2}\sinh^{-1}(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tan[x]^2)^(3/2),x]
```

```
[Out] ArcSinh[Tan[x]]/2 + (Sqrt[Sec[x]^2]*Tan[x])/2
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \left(1+\tan^2(x)\right)^{3/2} dx &= \int \sec^2(x)^{3/2} dx \\
&= \text{Subst}\left(\int \sqrt{1+x^2}\,dx, x, \tan(x)\right) \\
&= \frac{1}{2}\sqrt{\sec^2(x)}\tan(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \tan(x)\right) \\
&= \frac{1}{2}\sinh^{-1}(\tan(x)) + \frac{1}{2}\sqrt{\sec^2(x)}\tan(x)
\end{aligned}
$$

Mathematica [B] time = 0.0649192, size = 52, normalized size = 2.36

$$
\frac{1}{2}\cos(x)\sqrt{\sec^2(x)}\left(\tan(x)\sec(x) - \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tan[x]^2)^(3/2),x]
```

```
[Out] (Cos[x]*Sqrt[Sec[x]^2]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]
] + Sec[x]*Tan[x]))/2
```

Maple [A] time = 0.023, size = 19, normalized size = 0.9

$$
\frac{\tan(x)}{2}\sqrt{1+(\tan(x))^2} + \frac{\text{Arcsinh}(\tan(x))}{2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tan(x)^2)^(3/2),x)
```

```
[Out] 1/2*tan(x)*(1+tan(x)^2)^(1/2)+1/2*arcsinh(tan(x))
```

Maxima [A] time = 1.60794, size = 24, normalized size = 1.09

$$
\frac{1}{2}\sqrt{\tan(x)^2+1}\tan(x) + \frac{1}{2}\text{arsinh}(\tan(x))
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(tan(x)^2 + 1)*tan(x) + 1/2*arcsinh(tan(x))
```

Fricas [B] time = 1.59911, size = 228, normalized size = 10.36

$$
\frac{1}{2}\sqrt{\tan(x)^2+1}\tan(x) + \frac{1}{4}\log\left(\frac{\tan(x)^2+\sqrt{\tan(x)^2+1}\tan(x)+1}{\tan(x)^2+1}\right) - \frac{1}{4}\log\left(\frac{\tan(x)^2-\sqrt{\tan(x)^2+1}\tan(x)+}{\tan(x)^2+1}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(tan(x)^2 + 1)*tan(x) + 1/4*log((tan(x)^2 + sqrt(tan(x)^2 + 1)*tan(
x) + 1)/(tan(x)^2 + 1)) - 1/4*log((tan(x)^2 - sqrt(tan(x)^2 + 1)*tan(x) + 1
)/(tan(x)^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\tan^2(x)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)**2)**(3/2),x)
```

```
[Out] Integral((tan(x)**2 + 1)**(3/2), x)
```

Giac [A] time = 1.07111, size = 39, normalized size = 1.77

$$\frac{1}{2}\sqrt{\tan(x)^2+1}\tan(x)-\frac{1}{2}\log\left(\sqrt{\tan(x)^2+1}-\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(tan(x)^2 + 1)*tan(x) - 1/2*log(sqrt(tan(x)^2 + 1) - tan(x))
```

3.288 $\int \sqrt{1+\tan^2(x)}\,dx$

Optimal. Leaf size=3

$$\sinh^{-1}(\tan(x))$$

```
[Out] ArcSinh[Tan[x]]
```

Rubi [A] time = 0.0112337, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3657, 4122, 215}

$$\sinh^{-1}(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Tan[x]^2],x]

[Out] ArcSinh[Tan[x]]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+\tan^2(x)}\,dx &= \int \sqrt{\sec^2(x)}\,dx\\ &= \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \tan(x)\right)\\ &= \sinh^{-1}(\tan(x))\end{aligned}$$

Mathematica [B] time = 0.0087386, size = 44, normalized size = 14.67

$$\cos(x)\sqrt{\sec^2(x)}\left(\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Tan[x]^2],x]
```

```
[Out] Cos[x]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])*Sqrt[Sec[x]^2
]
```

Maple [A] time = 0.021, size = 4, normalized size = 1.3

$$\text{Arcsinh}\left(\tan\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tan(x)^2)^(1/2),x)
```

```
[Out] arcsinh(tan(x))
```

Maxima [A] time = 1.61566, size = 4, normalized size = 1.33

$$\text{arsinh}\left(\tan\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(tan(x))
```

Fricas [B] time = 1.62911, size = 185, normalized size = 61.67

$$\frac{1}{2}\,\log\left(\frac{\tan\left(x\right)^{2}+\sqrt{\tan\left(x\right)^{2}+1}\,\tan\left(x\right)+1}{\tan\left(x\right)^{2}+1}\right)-\frac{1}{2}\,\log\left(\frac{\tan\left(x\right)^{2}-\sqrt{\tan\left(x\right)^{2}+1}\,\tan\left(x\right)+1}{\tan\left(x\right)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log((tan(x)^2 + sqrt(tan(x)^2 + 1)*tan(x) + 1)/(tan(x)^2 + 1)) - 1/2*lo
g((tan(x)^2 - sqrt(tan(x)^2 + 1)*tan(x) + 1)/(tan(x)^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\tan^{2}\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(tan(x)**2 + 1), x)
```

Giac [B] time = 1.13204, size = 22, normalized size = 7.33

$$-\log\left(\sqrt{\tan(x)^2+1}-\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(sqrt(tan(x)^2 + 1) - tan(x))
```

3.289 $\int \frac{1}{\sqrt{1+\tan^2(x)}} dx$

Optimal. Leaf size=11

$$\frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

[Out] Tan[x]/Sqrt[Sec[x]^2]

Rubi [A] time = 0.0128562, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3657, 4122, 191}

$$\frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 + Tan[x]^2],x]

[Out] Tan[x]/Sqrt[Sec[x]^2]

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1+\tan^2(x)}} dx &= \int \frac{1}{\sqrt{\sec^2(x)}} dx \\
&= \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{3/2}} dx, x, \tan(x)\right) \\
&= \frac{\tan(x)}{\sqrt{\sec^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.0067172, size = 11, normalized size = 1.

$$\frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Tan[x]^2],x]
```

```
[Out] Tan[x]/Sqrt[Sec[x]^2]
```

Maple [A] time = 0.013, size = 12, normalized size = 1.1

$$\tan(x)\frac{1}{\sqrt{1+(\tan(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+tan(x)^2)^(1/2),x)
```

```
[Out] 1/(1+tan(x)^2)^(1/2)*tan(x)
```

Maxima [A] time = 1.10782, size = 15, normalized size = 1.36

$$\frac{\tan(x)}{\sqrt{\tan(x)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] tan(x)/sqrt(tan(x)^2 + 1)
```

Fricas [A] time = 1.86436, size = 36, normalized size = 3.27

$$\frac{\tan(x)}{\sqrt{\tan(x)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] tan(x)/sqrt(tan(x)^2 + 1)
```

Sympy [A] time = 0.422575, size = 12, normalized size = 1.09

$$\frac{\tan(x)}{\sqrt{\tan^2(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)**2)**(1/2),x)
```

```
[Out] tan(x)/sqrt(tan(x)**2 + 1)
```

Giac [A] time = 1.09879, size = 15, normalized size = 1.36

$$\frac{\tan(x)}{\sqrt{\tan(x)^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] tan(x)/sqrt(tan(x)^2 + 1)
```

3.290 $\int \left(-1-\tan^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=35

$$\frac{1}{2}\tan^{-1}\left(\frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)-\frac{1}{2}\tan(x)\sqrt{-\sec^2(x)}$$

```
[Out] ArcTan[Tan[x]/Sqrt[-Sec[x]^2]]/2 - (Sqrt[-Sec[x]^2]*Tan[x])/2
```

Rubi [A] time = 0.0217918, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3657, 4122, 195, 217, 203}

$$\frac{1}{2}\tan^{-1}\left(\frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)-\frac{1}{2}\tan(x)\sqrt{-\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - Tan[x]^2)^(3/2),x]
```

```
[Out] ArcTan[Tan[x]/Sqrt[-Sec[x]^2]]/2 - (Sqrt[-Sec[x]^2]*Tan[x])/2
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \left(-1-\tan^2(x)\right)^{3/2}\,dx &= \int \left(-\sec^2(x)\right)^{3/2}\,dx\\ &= -\text{Subst}\left(\int \sqrt{-1-x^2}\,dx,x,\tan(x)\right)\\ &= -\frac{1}{2}\sqrt{-\sec^2(x)}\tan(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}}\,dx,x,\tan(x)\right)\\ &= -\frac{1}{2}\sqrt{-\sec^2(x)}\tan(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)\\ &= \frac{1}{2}\tan^{-1}\left(\frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)-\frac{1}{2}\sqrt{-\sec^2(x)}\tan(x)\end{aligned}$$

Mathematica [B] time = 0.0671557, size = 72, normalized size = 2.06

$$\frac{1}{4}\cos(x)\sqrt{-\sec^2(x)}\left(\frac{1}{\sin(x)-1}+\frac{1}{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2}+2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-2\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - Tan[x]^2)^(3/2),x]
```

```
[Out] (Cos[x]*Sqrt[-Sec[x]^2]*(2*Log[Cos[x/2] - Sin[x/2]] - 2*Log[Cos[x/2] + Sin[
x/2]] + (Cos[x/2] + Sin[x/2])^(-2) + (-1 + Sin[x])^(-1)))/4
```

Maple [A] time = 0.02, size = 32, normalized size = 0.9

$$-\frac{\tan(x)}{2}\sqrt{-1-(\tan(x))^2}+\frac{1}{2}\arctan\left(\tan(x)\frac{1}{\sqrt{-1-(\tan(x))^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-tan(x)^2)^(3/2),x)
```

```
[Out] -1/2*tan(x)*(-1-tan(x)^2)^(1/2)+1/2*arctan(tan(x)/(-1-tan(x)^2)^(1/2))
```

Maxima [C] time = 1.59439, size = 27, normalized size = 0.77

$$-\frac{1}{2}\sqrt{-\tan(x)^2-1}\tan(x)-\frac{1}{2}i\,\text{arsinh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-tan(x)^2 - 1)*tan(x) - 1/2*I*arcsinh(tan(x))
```

Fricas [C] time = 1.85355, size = 227, normalized size = 6.49

$$\frac{\left(-i e^{(4ix)} - 2i e^{(2ix)} - i\right)\log\left(e^{(ix)} + i\right) + \left(i e^{(4ix)} + 2i e^{(2ix)} + i\right)\log\left(e^{(ix)} - i\right) - 2 e^{(3ix)} + 2 e^{(ix)}}{2\left(e^{(4ix)} + 2 e^{(2ix)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((-I*e^(4*I*x) - 2*I*e^(2*I*x) - I)*log(e^(I*x) + I) + (I*e^(4*I*x) + 2
*I*e^(2*I*x) + I)*log(e^(I*x) - I) - 2*e^(3*I*x) + 2*e^(I*x))/(e^(4*I*x) +
2*e^(2*I*x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-\tan^2(x) - 1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)**2)**(3/2),x)
```

```
[Out] Integral((-tan(x)**2 - 1)**(3/2), x)
```

Giac [C] time = 1.09418, size = 27, normalized size = 0.77

$$-\frac{1}{2}i\sqrt{\tan(x)^2 + 1}\tan(x) - \frac{1}{2}i\arcsin(i\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*I*sqrt(tan(x)^2 + 1)*tan(x) - 1/2*I*arcsin(I*tan(x))
```

3.291 $\int \sqrt{-1-\tan^2(x)}\,dx$

Optimal. Leaf size=16

$$-\tan^{-1}\left(\frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)$$

```
[Out] -ArcTan[Tan[x]/Sqrt[-Sec[x]^2]]
```

Rubi [A] time = 0.0173641, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3657, 4122, 217, 203}

$$-\tan^{-1}\left(\frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 - Tan[x]^2],x]

[Out] -ArcTan[Tan[x]/Sqrt[-Sec[x]^2]]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{-1-\tan^2(x)}\,dx = \int \sqrt{-\sec^2(x)}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}}\,dx, x, \tan(x)\right)$$
$$= -\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)$$
$$= -\tan^{-1}\left(\frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)$$

Mathematica [B] time = 0.0069509, size = 46, normalized size = 2.88

$$\cos(x)\sqrt{-\sec^2(x)}\left(\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - Tan[x]^2],x]
```

```
[Out] Cos[x]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])*Sqrt[-Sec[x]^
2]
```

Maple [A] time = 0.029, size = 17, normalized size = 1.1

$$-\arctan\left(\tan(x)\frac{1}{\sqrt{-1-(\tan(x))^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-tan(x)^2)^(1/2),x)
```

```
[Out] -arctan(tan(x)/(-1-tan(x)^2)^(1/2))
```

Maxima [A] time = 1.84511, size = 23, normalized size = 1.44

$$\arctan(\cos(x), \sin(x)+1) + \arctan(\cos(x), -\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arctan2(cos(x), sin(x) + 1) + arctan2(cos(x), -sin(x) + 1)
```

Fricas [C] time = 1.87246, size = 55, normalized size = 3.44

$$i\log\left(e^{(ix)}+i\right)-i\log\left(e^{(ix)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] I*log(e^(I*x) + I) - I*log(e^(I*x) - I)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\tan^2(x) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-tan(x)**2 - 1), x)
```

Giac [C] time = 1.09306, size = 9, normalized size = 0.56

$$i \arcsin(i \tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] I*arcsin(I*tan(x))
```

3.292 $\int \frac{1}{\sqrt{-1-\tan^2(x)}} \, dx$

Optimal. Leaf size=13

$$\frac{\tan(x)}{\sqrt{-\sec^2(x)}}$$

```
[Out] Tan[x]/Sqrt[-Sec[x]^2]
```

Rubi [A] time = 0.0198659, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3657, 4122, 191}

$$\frac{\tan(x)}{\sqrt{-\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 - Tan[x]^2],x]
```

```
[Out] Tan[x]/Sqrt[-Sec[x]^2]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1-\tan^2(x)}} \, dx &= \int \frac{1}{\sqrt{-\sec^2(x)}} \, dx \\
&= -\text{Subst}\left(\int \frac{1}{\left(-1-x^2\right)^{3/2}} \, dx, x, \tan(x)\right) \\
&= \frac{\tan(x)}{\sqrt{-\sec^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.0070587, size = 13, normalized size = 1.

$$\frac{\tan(x)}{\sqrt{-\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 - Tan[x]^2],x]
```

```
[Out] Tan[x]/Sqrt[-Sec[x]^2]
```

Maple [A] time = 0.029, size = 14, normalized size = 1.1

$$\tan(x)\frac{1}{\sqrt{-1-(\tan(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1-tan(x)^2)^(1/2),x)
```

```
[Out] tan(x)/(-1-tan(x)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\tan(x)^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-tan(x)^2 - 1), x)
```

Fricas [C] time = 1.71258, size = 42, normalized size = 3.23

$$-\frac{1}{2}\left(e^{(2ix)}-1\right)e^{(-ix)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(e^(2*I*x) - 1)*e^(-I*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\tan^2(x)-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-tan(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-tan(x)**2 - 1), x)
```

Giac [C] time = 1.08866, size = 16, normalized size = 1.23

$$-\frac{i\tan(x)}{\sqrt{\tan(x)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -I*tan(x)/sqrt(tan(x)^2 + 1)
```

3.293 $\int \tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=117

$$\frac{\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2f}-\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{\sqrt{a+b\tan^2(e+fx)}}{f}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f) + Sqrt[a
 + b*Tan[e + f*x]^2]/f - ((a + b)*(a + b*Tan[e + f*x]^2)^(3/2))/(3*b^2*f) +
 (a + b*Tan[e + f*x]^2)^(5/2)/(5*b^2*f)
```

Rubi [A] time = 0.146467, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3670, 446, 88, 50, 63, 208}

$$\frac{\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2f}-\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{\sqrt{a+b\tan^2(e+fx)}}{f}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f) + Sqrt[a
 + b*Tan[e + f*x]^2]/f - ((a + b)*(a + b*Tan[e + f*x]^2)^(3/2))/(3*b^2*f) +
 (a + b*Tan[e + f*x]^2)^(5/2)/(5*b^2*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
```

```
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^5\sqrt{a+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx}}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(-a-b)\sqrt{a+bx}}{b} + \frac{\sqrt{a+bx}}{1+x} + \frac{(a+bx)^{3/2}}{b}\right)dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f} + \frac{\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2f} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+}}{1+x}\right.}{}$$

$$= \frac{\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f} + \frac{\left(a+b\tan^2(e+f\right.}{5b^2f}$$

$$= \frac{\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f} + \frac{\left(a+b\tan^2(e+f\right.}{5b^2f}$$

$$= -\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} + \frac{\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{(a+b)\left(a+b\tan\right.}{3b}$$

Mathematica [A] time = 1.35958, size = 109, normalized size = 0.93

$$\frac{\frac{\sqrt{a+b\tan^2(e+fx)}\left(-2a^2+b(a-5b)\tan^2(e+fx)-5ab+3b^2\tan^4(e+fx)+15b^2\right)}{b^2} - 15\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (-15*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + (Sqrt[a
+ b*Tan[e + f*x]^2]*(-2*a^2 - 5*a*b + 15*b^2 + (a - 5*b)*b*Tan[e + f*x]^2 +
```

```
3*b^2*Tan[e + f*x]^4))/b^2)/(15*f)
```

Maple [A] time = 0.046, size = 166, normalized size = 1.4

$$\frac{\left(\tan\left(fx+e\right)\right)^{2}}{5fb}\left(a+b\left(\tan\left(fx+e\right)\right)^{2}\right)^{\frac{3}{2}}-\frac{2a}{15fb^{2}}\left(a+b\left(\tan\left(fx+e\right)\right)^{2}\right)^{\frac{3}{2}}-\frac{1}{3fb}\left(a+b\left(\tan\left(fx+e\right)\right)^{2}\right)^{\frac{3}{2}}+\frac{1}{f}\sqrt{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x)
```

```
[Out] 1/5/f*tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2)/b-2/15/f*a/b^2*(a+b*tan(f*x+e)^
2)^(3/2)-1/3*(a+b*tan(f*x+e)^2)^(3/2)/b/f+(a+b*tan(f*x+e)^2)^(1/2)/f-1/f*b/
(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))+1/f*a/(-a+b)^(1/
2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\tan\left(fx+e\right)^{2}+a}\tan\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^5, x)
```

Fricas [A] time = 2.72676, size = 770, normalized size = 6.58

$$\left[\frac{15\sqrt{a-b}b^{2}\log\left(-\frac{b^{2}\tan\left(fx+e\right)^{4}+2\left(4ab-3b^{2}\right)\tan\left(fx+e\right)^{2}-4\left(b\tan\left(fx+e\right)^{2}+2a-b\right)\sqrt{b\tan\left(fx+e\right)^{2}+a}\sqrt{a-b}+8a^{2}-8ab+b^{2}}{\tan\left(fx+e\right)^{4}+2\tan\left(fx+e\right)^{2}+1}\right)+4\left(3b^{2}\tan\left(fx+\right.\right.}{60b^{2}f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [1/60*(15*sqrt(a - b)*b^2*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(
f*x + e)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt
(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4
*(3*b^2*tan(f*x + e)^4 + (a*b - 5*b^2)*tan(f*x + e)^2 - 2*a^2 - 5*a*b + 15*
b^2)*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), 1/30*(15*sqrt(-a + b)*b^2*arctan(
2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^2 + 2*a - b)) + 2
*(3*b^2*tan(f*x + e)^4 + (a*b - 5*b^2)*tan(f*x + e)^2 - 2*a^2 - 5*a*b + 15*
b^2)*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\tan^2\left(e+fx\right)}\tan^5\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2)*tan(f*x+e)**5,x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x)**5, x)
```

Giac [A] time = 1.20562, size = 193, normalized size = 1.65

$$\frac{(a-b)\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}f}+\frac{3\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}b^8f^4-5\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}ab^8f^4-5\left(b\tan\left(fx+e\right)^2\right.}{15\,b^{10}f^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] (a - b)*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-a + b)*f) +
1/15*(3*(b*tan(f*x + e)^2 + a)^(5/2)*b^8*f^4 - 5*(b*tan(f*x + e)^2 + a)^(3/
2)*a*b^8*f^4 - 5*(b*tan(f*x + e)^2 + a)^(3/2)*b^9*f^4 + 15*sqrt(b*tan(f*x +
 e)^2 + a)*b^10*f^4)/(b^10*f^5)
```

3.294 $\int \tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=88

$$\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3bf}-\frac{\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

```
[Out] (Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f - Sqrt[a +
b*Tan[e + f*x]^2]/f + (a + b*Tan[e + f*x]^2)^(3/2)/(3*b*f)
```

Rubi [A] time = 0.114322, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3670, 446, 80, 50, 63, 208}

$$\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3bf}-\frac{\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f - Sqrt[a +
b*Tan[e + f*x]^2]/f + (a + b*Tan[e + f*x]^2)^(3/2)/(3*b*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3\sqrt{a+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3bf} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= -\frac{\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3bf} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a}\ldots}\right.}{2\ldots}$$
$$= -\frac{\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3bf} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\ldots\right.}{\ldots}$$
$$= \frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} - \frac{\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\left(a+b\tan^2(e+\ldots\right.}{3bf}$$

Mathematica [A] time = 0.35181, size = 82, normalized size = 0.93

$$\frac{\sqrt{a+b\tan^2(e+fx)}\left(a+b\tan^2(e+fx)-3b\right)+3b\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{3bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (3*Sqrt[a - b]*b*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + Sqrt[a +
 b*Tan[e + f*x]^2]*(a - 3*b + b*Tan[e + f*x]^2))/(3*b*f)
```

Maple [A] time = 0.027, size = 114, normalized size = 1.3

$$\frac{1}{3fb}\left(a+b\left(\tan\left(fx+e\right)\right)^{2}\right)^{\frac{3}{2}}-\frac{1}{f}\sqrt{a+b\left(\tan\left(fx+e\right)\right)^{2}}+\frac{b}{f}\arctan\left(\sqrt{a+b\left(\tan\left(fx+e\right)\right)^{2}}\frac{1}{\sqrt{-a+b}}\right)\frac{1}{\sqrt{-a+b}}-\frac{a}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x)
```

```
[Out] 1/3*(a+b*tan(f*x+e)^2)^(3/2)/b/f-(a+b*tan(f*x+e)^2)^(1/2)/f+1/f*b/(-a+b)^(1
/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))-1/f*a/(-a+b)^(1/2)*arctan
((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx+e\right)^{2}+a}\tan\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^3, x)
```

Fricas [A] time = 2.47136, size = 621, normalized size = 7.06

$$\left[\frac{3\sqrt{a-b}b\log\left(-\frac{b^{2}\tan\left(fx+e\right)^{4}+2\left(4ab-3b^{2}\right)\tan\left(fx+e\right)^{2}+4\left(b\tan\left(fx+e\right)^{2}+2a-b\right)\sqrt{b\tan\left(fx+e\right)^{2}+a}\sqrt{a-b}+8a^{2}-8ab+b^{2}}{\tan\left(fx+e\right)^{4}+2\tan\left(fx+e\right)^{2}+1}\right)+4\left(b\tan\left(fx+e\right)^{2}\right.}{12bf}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [1/12*(3*sqrt(a - b)*b*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x
 + e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a
- b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4*(b
*tan(f*x + e)^2 + a - 3*b)*sqrt(b*tan(f*x + e)^2 + a))/(b*f), -1/6*(3*sqrt(
-a + b)*b*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^
2 + 2*a - b)) - 2*(b*tan(f*x + e)^2 + a - 3*b)*sqrt(b*tan(f*x + e)^2 + a))/
(b*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\tan^{2}\left(e+fx\right)}\tan^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2)*tan(f*x+e)**3,x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x)**3, x)
```

Giac [A] time = 1.17739, size = 131, normalized size = 1.49

$$-\frac{\frac{3\left(ab-b^{2}\right)\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}f}-\frac{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}f^{2}-3\sqrt{b\tan\left(fx+e\right)^{2}+a}bf^{2}}{f^{3}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] -1/3*(3*(a*b - b^2)*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-
a + b)*f) - ((b*tan(f*x + e)^2 + a)^(3/2)*f^2 - 3*sqrt(b*tan(f*x + e)^2 + a
)*b*f^2)/f^3)/b
```

3.295 $\int \tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=62

$$\frac{\sqrt{a+b\tan^2(e+fx)}}{f}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f) + Sqrt[a
 + b*Tan[e + f*x]^2]/f
```

Rubi [A] time = 0.0710242, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3670, 444, 50, 63, 208}

$$\frac{\sqrt{a+b\tan^2(e+fx)}}{f}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f) + Sqrt[a
 + b*Tan[e + f*x]^2]/f
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2f} \\
&= \frac{\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f} \\
&= \frac{\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right)}{bf} \\
&= -\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} + \frac{\sqrt{a+b\tan^2(e+fx)}}{f}
\end{aligned}$$

Mathematica [A] time = 0.0504613, size = 59, normalized size = 0.95

$$\frac{\sqrt{a+b\tan^2(e+fx)} - \sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (-(Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]) + Sqrt[a +
b*Tan[e + f*x]^2])/f
```

Maple [A] time = 0.032, size = 91, normalized size = 1.5

$$\frac{1}{f}\sqrt{a+b\left(\tan\left(fx+e\right)\right)^2} - \frac{b}{f}\arctan\left(\sqrt{a+b\left(\tan\left(fx+e\right)\right)^2}\frac{1}{\sqrt{-a+b}}\right)\frac{1}{\sqrt{-a+b}} + \frac{a}{f}\arctan\left(\sqrt{a+b\left(\tan\left(fx+e\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e),x)
```

```
[Out] (a+b*tan(f*x+e)^2)^(1/2)/f-1/f*b/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/
2)/(-a+b)^(1/2))+1/f*a/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^
```

```
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx+e\right)^2+a}\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e), x)
```

Fricas [A] time = 2.30909, size = 518, normalized size = 8.35

$$\left[\frac{\sqrt{a-b}\log\left(-\frac{b^2\tan\left(fx+e\right)^4+2\left(4ab-3b^2\right)\tan\left(fx+e\right)^2-4\left(b\tan\left(fx+e\right)^2+2a-b\right)\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{a-b}+8a^2-8ab+b^2}{\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1}\right)+4\sqrt{b\tan\left(fx+e\right)^2+}}{4f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e)
^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)
+ 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4*sqrt(b*
tan(f*x + e)^2 + a))/f, 1/2*(sqrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 +
a)*sqrt(-a + b)/(b*tan(f*x + e)^2 + 2*a - b)) + 2*sqrt(b*tan(f*x + e)^2 + a
))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\tan^2\left(e+fx\right)}\tan\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2)*tan(f*x+e),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x), x)
```

Giac [A] time = 1.15546, size = 81, normalized size = 1.31

$$\frac{(a-b)\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}f}+\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] (a - b)*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-a + b)*f) +
sqrt(b*tan(f*x + e)^2 + a)/f
```

3.296 $\int \cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/f) + (Sqrt[a - b]*A
rcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f
```

Rubi [A] time = 0.100548, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3670, 446, 83, 63, 208}

$$\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/f) + (Sqrt[a - b]*A
rcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f} - \frac{(a-b)\,\text{Subst}\Bigl(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \text{ta}\ldots}{2f}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right)}{bf} - \frac{(a-b)\,\text{Subst}\Bigl(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, \ldots}{bf}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Mathematica [A] time = 0.0612342, size = 72, normalized size = 0.97

$$\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right) - \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (-(Sqrt[a]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]) + Sqrt[a - b]*ArcTa
nh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f
```

Maple [B] time = 0.256, size = 615, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/4/f/(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)*4
^(1/2)*cos(f*x+e)*(cos(f*x+e)-1)*(ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+
e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2
)*a^(1/2)+b)/sin(f*x+e)^2)*a^(1/2)*(a-b)^(1/2)-a^(1/2)*ln(-4*(cos(f*x+e)*((
```

```
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)
*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*
a^(1/2)+b)/(cos(f*x+e)-1))*(a-b)^(1/2)+2*ln(4*cos(f*x+e)*(a-b)^(1/2)*((a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos
(f*x+e)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^
(1/2))*a-2*ln(4*cos(f*x+e)*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(
cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e)+4*(a-b)^(1/2)*((a*cos(
f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*b)/sin(f*x+e)^2/((a*cos
(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72163, size = 953, normalized size = 12.88

$$\left[\frac{\sqrt{a-b}\log\left(\frac{b\tan\left(fx+e\right)^2+2\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{a-b}+2a-b}{\tan\left(fx+e\right)^2+1}\right)+\sqrt{a}\log\left(\frac{b\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{a}+2a}{\tan\left(fx+e\right)^2}\right)}{2f}, \frac{2\sqrt{-a+b}\arctan\left(-\frac{\sqrt{b}\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a - b)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt
(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) + sqrt(a)*log((b*tan(f*x + e)^2 -
2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2))/f, 1/2*(2*sqrt
(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b)) + sqrt(a)
*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*
x + e)^2))/f, 1/2*(2*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a)
 + sqrt(a - b)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a
- b) + 2*a - b)/(tan(f*x + e)^2 + 1)))/f, (sqrt(-a)*arctan(sqrt(b*tan(f*x +
 e)^2 + a)*sqrt(-a)/a) + sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sq
rt(-a + b)/(a - b)))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\tan^2\left(e+fx\right)}\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx+e\right)^2+a}\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e), x)
```

3.297 $\int \cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=115

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

```
[Out] ((2*a - b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(2*Sqrt[a]*f) - (Sq
rt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f - (Cot[e + f*x
]^2*Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Rubi [A] time = 0.14774, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3670, 446, 99, 156, 63, 208}

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((2*a - b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(2*Sqrt[a]*f) - (Sq
rt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f - (Cot[e + f*x
]^2*Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^3\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-2a+b)-\frac{bx}{2}}{x(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+ \right.}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \tan\right.}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\right.}{bf}$$

$$= \frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f} - \frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} - \frac{\cot^2(}{}$$

Mathematica [A] time = 0.360486, size = 115, normalized size = 1.

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)-\sqrt{a}\left(2\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)}{2\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((2*a - b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]] - Sqrt[a]*(2*Sqrt[a
- b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + Cot[e + f*x]^2*Sqrt[
```

```
a + b*Tan[e + f*x]^2]))/(2*Sqrt[a]*f)
```

Maple [B] time = 0.319, size = 2135, normalized size = 18.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/8/f/a^(3/2)/(a-b)^(1/2)*(cos(f*x+e)-1)*(4*a^(5/2)*cos(f*x+e)^2*ln(4*cos(
f*x+e)*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/
2)+4*cos(f*x+e)*a-4*b*cos(f*x+e)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^
2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*4^(1/2)+2*a^(3/2)*(a-b)^(1/2)*cos(f*x+e)^2*
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*4^(1/2)-4*a^(3/2
)*cos(f*x+e)^2*ln(4*cos(f*x+e)*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e)+4*(a-b)^(1/2)*((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*4^(1/2)*b-4*a^(5/2)
*ln(4*cos(f*x+e)*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)
+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-c
os(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*4^(1/2)-2*a^(3/2)*(a-b)^(1/2)*cos
(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*4^(1/2)-
8*a^(1/2)*(a-b)^(1/2)*cos(f*x+e)^2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(
f*x+e)+1)^2)^(3/2)-4*a^(1/2)*(a-b)^(1/2)*cos(f*x+e)^2*((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*4^(1/2)*b+2*(a-b)^(1/2)*cos(f*x+e)^2*
ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)
/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*4^(1/2)
*a^2-(a-b)^(1/2)*cos(f*x+e)^2*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a
+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^
(1/2)+b)/sin(f*x+e)^2)*4^(1/2)*a*b-2*(a-b)^(1/2)*cos(f*x+e)^2*ln(-4*(cos(f*
x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos
(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)
^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*4^(1/2)*a^2+(a-b)^(1/2)*cos(f*x+e)^2*ln(-
4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^
(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x
+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*4^(1/2)*a*b+4*a^(3/2)*ln(4*cos(f
*x+e)*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2
)+4*cos(f*x+e)*a-4*b*cos(f*x+e)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/(cos(f*x+e)+1)^2)^(1/2))*4^(1/2)*b-16*a^(1/2)*(a-b)^(1/2)*cos(f*x+e)*
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)-8*a^(1/2)*(a-b)^
(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)+4*a^(1/2)*
(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*4^(1
/2)*b-2*(a-b)^(1/2)*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)
^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x
+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/s
in(f*x+e)^2)*4^(1/2)*a^2+(a-b)^(1/2)*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+
e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f
*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(
1/2)*a^(1/2)+b)/sin(f*x+e)^2)*4^(1/2)*a*b+2*(a-b)^(1/2)*ln(-4*(cos(f*x+e)*(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e
)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)
*a^(1/2)+b)/(cos(f*x+e)-1))*4^(1/2)*a^2-(a-b)^(1/2)*ln(-4*(cos(f*x+e)*((a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-
b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(
1/2)+b)/(cos(f*x+e)-1))*4^(1/2)*a*b)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)
```

```
^2*b+b)/cos(f*x+e)^2)^(1/2)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+
1)^2)^(1/2)/sin(f*x+e)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx+e\right)^2+a}\cot\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^3, x)
```

Fricas [A] time = 1.79282, size = 1467, normalized size = 12.76

$$\left[\frac{2\sqrt{a-b}a\log\left(\frac{b\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{a-b}+2a-b}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^2-\left(2a-b\right)\sqrt{a}\log\left(\frac{b\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{a}+\cdots}{\tan\left(fx+e\right)^2}\right.\cdots}{4af\tan\left(fx+e\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(a - b)*a*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*
sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 - (2*a - b)*sqr
t(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/ta
n(f*x + e)^2)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*a)/(a*f*tan(f*x
 + e)^2), -1/4*(4*a*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a
 + b)/(a - b))*tan(f*x + e)^2 + (2*a - b)*sqrt(a)*log((b*tan(f*x + e)^2 - 2
*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^2 +
 2*sqrt(b*tan(f*x + e)^2 + a)*a)/(a*f*tan(f*x + e)^2), -1/2*(sqrt(-a)*(2*a
- b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^2 - sqrt(a
- b)*a*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2
*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + sqrt(b*tan(f*x + e)^2 + a)*a
)/(a*f*tan(f*x + e)^2), -1/2*(sqrt(-a)*(2*a - b)*arctan(sqrt(b*tan(f*x + e)
^2 + a)*sqrt(-a)/a)*tan(f*x + e)^2 + 2*a*sqrt(-a + b)*arctan(-sqrt(b*tan(f*
x + e)^2 + a)*sqrt(-a + b)/(a - b))*tan(f*x + e)^2 + sqrt(b*tan(f*x + e)^2
+ a)*a)/(a*f*tan(f*x + e)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\tan^2\left(e+fx\right)}\cot^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \tan \left(fx+e\right)^{2}+a} \cot \left(fx+e\right)^{3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^3, x)
```

3.298 $\int \cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=163

$$-\frac{\left(8a^2-4ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f}+\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f}+($$

```
[Out] -((8*a^2 - 4*a*b - b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(8*a^(
3/2)*f) + (Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f +
 ((4*a - b)*Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2])/(8*a*f) - (Cot[e + f
*x]^4*Sqrt[a + b*Tan[e + f*x]^2])/(4*f)
```

Rubi [A] time = 0.211899, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3670, 446, 99, 151, 156, 63, 208}

$$-\frac{\left(8a^2-4ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f}+\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f}+($$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```


```
[Out] -((8*a^2 - 4*a*b - b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(8*a^(
3/2)*f) + (Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f +
 ((4*a - b)*Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2])/(8*a*f) - (Cot[e + f
*x]^4*Sqrt[a + b*Tan[e + f*x]^2])/(4*f)
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
```

```
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^5(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-4a+b)-\frac{3bx}{2}}{x^2(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+\right.}{4f}$$

$$= \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8af} - \frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e+f}{4f}$$

$$= \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8af} - \frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e+f}{4f}$$

$$= \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8af} - \frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e+f}{4f}$$

$$= -\frac{(8a^2-4ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f} + \frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx}}{\sqrt{a-b}}\right.}{f}$$

Mathematica [A] time = 1.31372, size = 138, normalized size = 0.85

$$\frac{(-8a^2+4ab+b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\left(8a\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)-\cot^2(e+fx)\sqrt{a+b\tan^2(e+f}\right.}{8a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((-8*a^2 + 4*a*b + b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]] + Sqrt[
a]*(8*a*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] - Cot[e
 + f*x]^2*(-4*a + b + 2*a*Cot[e + f*x]^2)*Sqrt[a + b*Tan[e + f*x]^2]))/(8*a
^(3/2)*f)
```

Maple [B] time = 0.31, size = 5676, normalized size = 34.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx+e\right)^{2}+a}\cot\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^5, x)
```

Fricas [A] time = 1.90633, size = 1756, normalized size = 10.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*sqrt(a - b)*a^2*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 +
a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 - (8*a^2 - 4
*a*b - b^2)*sqrt(a)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sq
rt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^4 + 2*((4*a^2 - a*b)*tan(f*x + e)
^2 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a^2*f*tan(f*x + e)^4), 1/16*(16*a^
2*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b))*tan
(f*x + e)^4 - (8*a^2 - 4*a*b - b^2)*sqrt(a)*log((b*tan(f*x + e)^2 + 2*sqrt(
b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^4 + 2*((4
*a^2 - a*b)*tan(f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a^2*f*tan(
f*x + e)^4), 1/8*(4*sqrt(a - b)*a^2*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*
x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 +
 (8*a^2 - 4*a*b - b^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/
a)*tan(f*x + e)^4 + ((4*a^2 - a*b)*tan(f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*x +
 e)^2 + a))/(a^2*f*tan(f*x + e)^4), 1/8*(8*a^2*sqrt(-a + b)*arctan(-sqrt(b*
tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b))*tan(f*x + e)^4 + (8*a^2 - 4*a*b -
 b^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^4
 + ((4*a^2 - a*b)*tan(f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a^2*
f*tan(f*x + e)^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\tan^{2}\left(e+fx\right)}\cot^{5}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x)**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx+e\right)^{2}+a}\cot\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^5, x)
```

3.299 $\int \tan^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=222

$$\frac{\left(2a^2b+a^3+8ab^2-16b^3\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{16b^{5/2}f}-\frac{(a-2b)(a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16b^2f}+\frac{\tan^5(e+fx)\sqrt{}}{}$$

```
[Out] -((Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f) + ((a^3 + 2*a^2*b + 8*a*b^2 - 16*b^3)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a + b*Tan[e + f*x]^2]])/(16*b^(5/2)*f) - ((a - 2*b)*(a + 4*b)*Tan[e +
f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(16*b^2*f) + ((a - 6*b)*Tan[e + f*x]^3*Sqr
t[a + b*Tan[e + f*x]^2])/(24*b*f) + (Tan[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]
^2])/(6*f)
```

Rubi [A] time = 0.337221, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3670, 478, 582, 523, 217, 206, 377, 203}

$$\frac{\left(2a^2b+a^3+8ab^2-16b^3\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{16b^{5/2}f}-\frac{(a-2b)(a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16b^2f}+\frac{\tan^5(e+fx)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]
```


```
[Out] -((Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f) + ((a^3 + 2*a^2*b + 8*a*b^2 - 16*b^3)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a + b*Tan[e + f*x]^2]])/(16*b^(5/2)*f) - ((a - 2*b)*(a + 4*b)*Tan[e +
f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(16*b^2*f) + ((a - 6*b)*Tan[e + f*x]^3*Sqr
t[a + b*Tan[e + f*x]^2])/(24*b*f) + (Tan[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]
^2])/(6*f)
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \tan^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6\sqrt{a+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{6f} - \frac{\text{Subst}\left(\int \frac{x^4(5a+(-a+6b)x^2)}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx\right.}{6f}$$

$$= \frac{(a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{24bf} + \frac{\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{6f}$$

$$= -\frac{(a-2b)(a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-6b)\tan^3(e+fx)\sqrt{a}}{24bf}$$

$$= -\frac{(a-2b)(a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-6b)\tan^3(e+fx)\sqrt{a}}{24bf}$$

$$= -\frac{(a-2b)(a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-6b)\tan^3(e+fx)\sqrt{a}}{24bf}$$

$$= -\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{(a^3+2a^2b+8ab^2-16b^3)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e}{\sqrt{a+b\tan^2}}\right.}{16b^{5/2}f}$$

Mathematica [C] time = 6.29949, size = 823, normalized size = 3.71

$$-\frac{b(a^3+2ba^2-8b^3)\sqrt{\frac{a+b+(a-b)\cos(2(e+fx))}{\cos(2(e+fx))+1}}\sqrt{-\frac{a\cot^2(e+fx)}{b}}\sqrt{-\frac{a(\cos(2(e+fx))+1)\csc^2(e+fx)}{b}}\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\csc(2(e+fx))\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{(a+b}{}}\right.\right.}{a(a+b+(a-b)\cos(2(e+fx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (-((b*(a^3 + 2*a^2*b - 8*b^3)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 +
Cos[2*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e +
f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e +
 f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*
(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a
- b)*Cos[2*(e + f*x)]))) - (4*b*(-8*a*b^2 + 8*b^3)*Sqrt[1 + Cos[2*(e + f*x)
]]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sqrt[-
((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)
]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*
x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^
2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(4*a*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a +
 b + (a - b)*Cos[2*(e + f*x)]]) - (Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*
(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e
+ f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticPi[-(b/(a - b)), ArcSi
n[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*
Sin[e + f*x]^4)/(2*(a - b)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*
Cos[2*(e + f*x)]])))/Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])/(8*b^2*f) + (S
```

```
qrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)]
)]*((Sec[e + f*x]^3*(a*Sin[e + f*x] - 14*b*Sin[e + f*x]))/(24*b) + (Sec[e +
 f*x]*(-3*a^2*Sin[e + f*x] - 8*a*b*Sin[e + f*x] + 44*b^2*Sin[e + f*x]))/(48
*b^2) + (Sec[e + f*x]^4*Tan[e + f*x])/6))/f
```

Maple [B] time = 0.035, size = 451, normalized size = 2.

$$\frac{\left(\tan\left(fx+e\right)\right)^{3}}{6fb}\left(a+b\left(\tan\left(fx+e\right)\right)^{2}\right)^{\frac{3}{2}}-\frac{a\tan\left(fx+e\right)}{8fb^{2}}\left(a+b\left(\tan\left(fx+e\right)\right)^{2}\right)^{\frac{3}{2}}+\frac{a^{2}\tan\left(fx+e\right)}{16fb^{2}}\sqrt{a+b\left(\tan\left(f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x)
```

```
[Out] 1/6/f*tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2)/b-1/8/f*a/b^2*tan(f*x+e)*(a+b*t
an(f*x+e)^2)^(3/2)+1/16/f*a^2/b^2*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)+1/16/
f*a^3/b^(5/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-1/4/f*tan(f*x
+e)*(a+b*tan(f*x+e)^2)^(3/2)/b+1/8/f*a/b*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e
)+1/8/f*a^2/b^(3/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))+1/2*(a+
b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+1/2/f*a/b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a
+b*tan(f*x+e)^2)^(1/2))-1/f*b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2
)^(1/2))+1/f*(b^4*(a-b))^(1/2)/b/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(
a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-1/f*a*(b^4*(a-b))^(1/2)/b^2/(a-b)*arcta
n(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\tan\left(fx+e\right)^{2}+a}\tan\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^6, x)
```

Fricas [A] time = 14.4093, size = 2030, normalized size = 9.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] [1/96*(48*sqrt(-a + b)*b^3*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*
x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - 3*(a^3
 + 2*a^2*b + 8*a*b^2 - 16*b^3)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*ta
n(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) + 2*(8*b^3*tan(f*x + e)^5 + 2*(
a*b^2 - 6*b^3)*tan(f*x + e)^3 - 3*(a^2*b + 2*a*b^2 - 8*b^3)*tan(f*x + e))*s
qrt(b*tan(f*x + e)^2 + a))/(b^3*f), -1/96*(96*sqrt(a - b)*b^3*arctan(-sqrt(
b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + 3*(a^3 + 2*a^2*b + 8*a*
```

```
b^2 - 16*b^3)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)
*sqrt(b)*tan(f*x + e) + a) - 2*(8*b^3*tan(f*x + e)^5 + 2*(a*b^2 - 6*b^3)*ta
n(f*x + e)^3 - 3*(a^2*b + 2*a*b^2 - 8*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e
)^2 + a))/(b^3*f), 1/48*(24*sqrt(-a + b)*b^3*log(-((a - 2*b)*tan(f*x + e)^2
 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e
)^2 + 1)) - 3*(a^3 + 2*a^2*b + 8*a*b^2 - 16*b^3)*sqrt(-b)*arctan(sqrt(b*tan
(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + (8*b^3*tan(f*x + e)^5 + 2*(a*
b^2 - 6*b^3)*tan(f*x + e)^3 - 3*(a^2*b + 2*a*b^2 - 8*b^3)*tan(f*x + e))*sqr
t(b*tan(f*x + e)^2 + a))/(b^3*f), -1/48*(48*sqrt(a - b)*b^3*arctan(-sqrt(b*
tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + 3*(a^3 + 2*a^2*b + 8*a*b^
2 - 16*b^3)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x
+ e))) - (8*b^3*tan(f*x + e)^5 + 2*(a*b^2 - 6*b^3)*tan(f*x + e)^3 - 3*(a^2*
b + 2*a*b^2 - 8*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^3*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\tan^2\left(e + fx\right)}\tan^6\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2)*tan(f*x+e)**6,x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x)**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx + e\right)^2 + a}\tan\left(fx + e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^6, x)
```

3.300 $\int \tan^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=169

$$-\frac{\left(a^2+4ab-8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8b^{3/2}f}+\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f}+\frac{(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e}{8bf}$$

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])
/f - ((a^2 + 4*a*b - 8*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e
 + f*x]^2]])/(8*b^(3/2)*f) + ((a - 4*b)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x
]^2])/(8*b*f) + (Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(4*f)
```

Rubi [A] time = 0.210355, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3670, 478, 582, 523, 217, 206, 377, 203}

$$-\frac{\left(a^2+4ab-8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8b^{3/2}f}+\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f}+\frac{(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])
/f - ((a^2 + 4*a*b - 8*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e
 + f*x]^2]])/(8*b^(3/2)*f) + ((a - 4*b)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x
]^2])/(8*b*f) + (Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(4*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
```

```
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \tan^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f} - \frac{\text{Subst}\left(\int \frac{x^2\left(3a+(-a+4b)x^2\right)}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx\right.}{4f}$$

$$= \frac{(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f}$$

$$= \frac{(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f}$$

$$= \frac{(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f}$$

$$= \frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\left(a^2+4ab-8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8b^{3/2}f} + \ldots$$

Mathematica [C] time = 6.20682, size = 767, normalized size = 4.54

$$\frac{\sqrt{\frac{a\cos(2(e+fx))+a-b\cos(2(e+fx))+b}{\cos(2(e+fx))+1}}\left(\frac{\sec(e+fx)(a\sin(e+fx)-6b\sin(e+fx))}{8b}+\frac{1}{4}\tan(e+fx)\sec^2(e+fx)\right)}{f}-\frac{b\left(a^2-4b^2\right)\sin^4(e+fx)\csc(2(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(-((b*(a^2 - 4*b^2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e
+ f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Cs
c[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/
b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)
])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a - b)*Cos[
2*(e + f*x)]))) - (4*b*(-4*a*b + 4*b^2)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a
+ b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot[e +
 f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a
+ b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*Ellipti
cF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[
2]], 1]*Sin[e + f*x]^4)/(4*a*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b
)*Cos[2*(e + f*x)]]) - (Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*
(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Cs
c[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a
+ b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x
]^4)/(2*(a - b)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e +
f*x)]])))/Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])/(4*b*f) + (Sqrt[(a + b +
a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sec[e +
f*x]*(a*Sin[e + f*x] - 6*b*Sin[e + f*x]))/(8*b) + (Sec[e + f*x]^2*Tan[e + f
*x])/4))/f
```

Maple [B] time = 0.03, size = 323, normalized size = 1.9

$$\frac{\tan\left(fx+e\right)}{4fb}\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^{\frac{3}{2}}-\frac{a\tan\left(fx+e\right)}{8fb}\sqrt{a+b\left(\tan\left(fx+e\right)\right)^2}-\frac{a^2}{8f}\ln\left(\sqrt{b}\tan\left(fx+e\right)+\sqrt{a+b(\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x)
```

```
[Out] 1/4/f*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)/b-1/8/f*a/b*(a+b*tan(f*x+e)^2)^(1
/2)*tan(f*x+e)-1/8/f*a^2/b^(3/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(
1/2))-1/2*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-1/2/f*a/b^(1/2)*ln(b^(1/2)*
tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))+1/f*b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b
*tan(f*x+e)^2)^(1/2))-1/f*(b^4*(a-b))^(1/2)/b/(a-b)*arctan(b^2*(a-b)/(b^4*(
a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))+1/f*a*(b^4*(a-b))^(1/2)/b^
2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x
+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx+e\right)^2+a}\tan\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

Fricas [A] time = 7.59838, size = 1675, normalized size = 9.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] [1/16*(8*sqrt(-a + b)*b^2*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x
 + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - (a^2 +
4*a*b - 8*b^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a
)*sqrt(b)*tan(f*x + e) + a) + 2*(2*b^2*tan(f*x + e)^3 + (a*b - 4*b^2)*tan(f
*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), 1/16*(16*sqrt(a - b)*b^2*arct
an(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (a^2 + 4*a*b -
 8*b^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
b)*tan(f*x + e) + a) + 2*(2*b^2*tan(f*x + e)^3 + (a*b - 4*b^2)*tan(f*x + e)
)*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), 1/8*(4*sqrt(-a + b)*b^2*log(-((a - 2
*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e)
 - a)/(tan(f*x + e)^2 + 1)) + (a^2 + 4*a*b - 8*b^2)*sqrt(-b)*arctan(sqrt(b*
tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + (2*b^2*tan(f*x + e)^3 + (a
*b - 4*b^2)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), 1/8*(8*sqrt(
a - b)*b^2*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) +
 (a^2 + 4*a*b - 8*b^2)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/
(b*tan(f*x + e))) + (2*b^2*tan(f*x + e)^3 + (a*b - 4*b^2)*tan(f*x + e))*sqr
t(b*tan(f*x + e)^2 + a))/(b^2*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\tan^2\left(e+fx\right)}\tan^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2)*tan(f*x+e)**4,x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx+e\right)^2+a}\tan\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

3.301 $\int \tan^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=123

$$-\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2\sqrt{b}f}$$

```
[Out] -((Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f) + ((a - 2*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2
]])/(2*Sqrt[b]*f) + (Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Rubi [A] time = 0.133592, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3670, 478, 523, 217, 206, 377, 203}

$$-\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f) + ((a - 2*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2
]])/(2*Sqrt[b]*f) + (Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \tan^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} - \frac{\text{Subst}\left(\int \frac{a+(-a+2b)x^2}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right.}{2f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tan(e+\right.}{2f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+f}{\sqrt{a+b\tan^2(}}\right.}{2f}$$

$$= -\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2\sqrt{b}f} + \tan($$

Mathematica [C] time = 5.86573, size = 251, normalized size = 2.04

$$\frac{\tan(e+fx)\left(-\sqrt{2}a\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}}{\sqrt{2}}\right),1\right)+\sec^2(e+fx)((a\right.}{2\sqrt{2}f\sqrt{\sec^2(e+fx)((a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((-(Sqrt[2]*a*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*E
llipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]
```

```
/Sqrt[2]], 1]) + 2*Sqrt[2]*a*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e
 + f*x]^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*
(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + (a + b + (a - b)*Cos[2*(e + f
*x)])*Sec[e + f*x]^2)*Tan[e + f*x])/(2*Sqrt[2]*f*Sqrt[(a + b + (a - b)*Cos[
2*(e + f*x)])*Sec[e + f*x]^2])
```

Maple [B] time = 0.028, size = 230, normalized size = 1.9

$$\frac{\tan\left(fx+e\right)}{2f}\sqrt{a+b\left(\tan\left(fx+e\right)\right)^{2}}+\frac{a}{2f}\ln\left(\sqrt{b}\tan\left(fx+e\right)+\sqrt{a+b\left(\tan\left(fx+e\right)\right)^{2}}\right)\frac{1}{\sqrt{b}}-\frac{1}{f}\sqrt{b}\ln\left(\sqrt{b}\tan\left(fx+e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x)
```

```
[Out] 1/2*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+1/2/f*a/b^(1/2)*ln(b^(1/2)*tan(f*
x+e)+(a+b*tan(f*x+e)^2)^(1/2))-1/f*b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f
*x+e)^2)^(1/2))+1/f*(b^4*(a-b))^(1/2)/b/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^
(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-1/f*a*(b^4*(a-b))^(1/2)/b^2/(a-b
)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\tan\left(fx+e\right)^{2}+a}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

Fricas [A] time = 3.45395, size = 1388, normalized size = 11.28

$$\left[-\frac{(a-2b)\sqrt{b}\log\left(2b\tan\left(fx+e\right)^{2}-2\sqrt{b\tan\left(fx+e\right)^{2}+a}\sqrt{b}\tan\left(fx+e\right)+a\right)-2\sqrt{-a+b}b\log\left(-\frac{(a-2b)\tan\left(fx+e\right)^{2}-2\cdots}{\cdots}\right.}{4bf}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((a - 2*b)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 +
 a)*sqrt(b)*tan(f*x + e) + a) - 2*sqrt(-a + b)*b*log(-((a - 2*b)*tan(f*x +
e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x
 + e)^2 + 1)) - 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/(b*f), -1/4*(4
*sqrt(a - b)*b*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e)
```

```
)) + (a - 2*b)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a
)*sqrt(b)*tan(f*x + e) + a) - 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/
(b*f), -1/2*((a - 2*b)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/
(b*tan(f*x + e))) - sqrt(-a + b)*b*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(
b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1))
- sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/(b*f), -1/2*(2*sqrt(a - b)*b*a
rctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + (a - 2*b)*s
qrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) - sqrt
(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/(b*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\tan^2\left(e + fx\right)}\tan^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2)*tan(f*x+e)**2,x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx+e\right)^2 + a}\tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

3.302 $\int \sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])
/f + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f
```

Rubi [A] time = 0.0525515, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3661, 402, 217, 206, 377, 203}

$$\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])
/f + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

$$= \frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

Mathematica [C] time = 0.798255, size = 203, normalized size = 2.39

$$\frac{-i\sqrt{a-b}\log\left(-\frac{4i\left(\sqrt{a-b}\sqrt{a+b\tan^2(e+fx)}+a-ib\tan(e+fx)\right)}{(a-b)^{3/2}(\tan(e+fx)+i)}\right)+i\sqrt{a-b}\log\left(\frac{4i\left(\sqrt{a-b}\sqrt{a+b\tan^2(e+fx)}+a+ib\tan(e+fx)\right)}{(a-b)^{3/2}(\tan(e+fx)-i)}\right)+2\sqrt{b}\log\left(\sqrt{b}\right.}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((-I)*Sqrt[a - b]*Log[((-4*I)*(a - I*b*Tan[e + f*x] + Sqrt[a - b]*Sqrt[a +
b*Tan[e + f*x]^2]))/((a - b)^(3/2)*(I + Tan[e + f*x]))] + I*Sqrt[a - b]*Log
[((4*I)*(a + I*b*Tan[e + f*x] + Sqrt[a - b]*Sqrt[a + b*Tan[e + f*x]^2]))/((
a - b)^(3/2)*(-I + Tan[e + f*x]))] + 2*Sqrt[b]*Log[b*Tan[e + f*x] + Sqrt[b]
*Sqrt[a + b*Tan[e + f*x]^2]])/(2*f)
```

Maple [B] time = 0., size = 169, normalized size = 2.

$$\frac{1}{f}\sqrt{b}\ln\left(\sqrt{b}\tan\left(fx+e\right)+\sqrt{a+b\left(\tan\left(fx+e\right)\right)^2}\right)-\frac{1}{fb(a-b)}\sqrt{b^4(a-b)}\arctan\left((a-b)b^2\tan\left(fx+e\right)\frac{1}{\sqrt{b^4(a-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f*b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-1/f*(b^4*(a-b))
^(1/2)/b/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*
tan(f*x+e))+1/f*a*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^
(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.16394, size = 1046, normalized size = 12.31

$$\left[\frac{\sqrt{b}\log\left(2\,b\tan\left(fx+e\right)^2+2\,\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{b}\tan\left(fx+e\right)+a\right)+\sqrt{-a+b}\log\left(-\frac{(a-2\,b)\tan\left(fx+e\right)^2+2\,\sqrt{b\tan\left(fx+e\right)^2}}{\tan\left(fx+e\right)^2+1}\right.}{2\,f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)
*tan(f*x + e) + a) + sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b
*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)))/
f, 1/2*(2*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f
*x + e))) + sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*s
qrt(b)*tan(f*x + e) + a))/f, -1/2*(2*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2
+ a)*sqrt(-b)/(b*tan(f*x + e))) - sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)
^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x +
 e)^2 + 1)))/f, (sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b
)*tan(f*x + e))) - sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*t
an(f*x + e))))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\tan^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \tan \left(fx+e\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a), x)
```

3.303 $\int \cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=75

$$-\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

```
[Out] -((Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f) - (Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/f
```

Rubi [A] time = 0.096368, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3670, 475, 12, 377, 203}

$$-\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f) - (Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/f
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\text{Subst}\left(\int \frac{-a+b}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+f\ldots\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan\ldots\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan\ldots}{\sqrt{a+b\,t\ldots}}\right.}{f}$$

$$= -\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

Mathematica [C] time = 0.25179, size = 64, normalized size = 0.85

$$-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{(a-b)\tan^2(e+fx)}{a+b\tan^2(e+fx)}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -(((a - b)*Tan[e + f*x]^2)/
(a + b*Tan[e + f*x]^2))]*Sqrt[a + b*Tan[e + f*x]^2])/f)
```

Maple [C] time = 0.349, size = 2233, normalized size = 29.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^
2*b+b)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)*(EllipticF((cos(f*x+e)-1)*((2*I*b^(1/
2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(
1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*2^(1/2)*cos(f*x+e)*sin(f*x+
```

```
e)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*
a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(
1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/
2)*a-2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+c
os(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/
2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+
e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(
1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a
*b+8*b^2)/a^2)^(1/2))*b*sin(f*x+e)*cos(f*x+e)-2*EllipticPi((cos(f*x+e)-1)*(
(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(
1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b
)^(1/2)+a-2*b)/a)^(1/2))*2^(1/2)*cos(f*x+e)*sin(f*x+e)*(1/a*(I*cos(f*x+e)*b
^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(
f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1
/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*a+2*EllipticPi((cos(
f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1
/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b
^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*2^(1/2)*cos(f*x+e)*sin(f*x+e)*(1/a*(I*c
os(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+
e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/
2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*b+2^(1/2)
*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-
b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/
2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)
*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x
+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^
2)^(1/2))*a*sin(f*x+e)-2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(
1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(
I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f
*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-
b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*b*sin(f*x+e)-2*2^(1/2)*(1/a*(I*cos(
f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+
b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*
(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((
cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*
b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2
*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*sin(f*x+e)+2*2^(1/2)*(1/a*(I*cos(
f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+
b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*
(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((
cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*
b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2
*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*b*sin(f*x+e)+cos(f*x+e)^2*((2*I*b^(
1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a-cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a
-2*b)/a)^(1/2)*b+((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b)/(a*cos(f*x+e)
^2-cos(f*x+e)^2*b+b)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \tan\left(fx+e\right)^2 + a} \cot\left(fx+e\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

Fricas [A] time = 2.40141, size = 640, normalized size = 8.53

$$\left[\frac{\sqrt{-a+b}\log\left(-\frac{\left(a^2-8ab+8b^2\right)\tan\left(fx+e\right)^4-2\left(3a^2-4ab\right)\tan\left(fx+e\right)^2+a^2-4\left(\left(a-2b\right)\tan\left(fx+e\right)^3-a\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{-a+b}}{\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1}\right)\tan\left(f\right.}{4f\tan\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 -
4*a*b)*tan(f*x + e)^2 + a^2 - 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))
*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^
2 + 1))*tan(f*x + e) - 4*sqrt(b*tan(f*x + e)^2 + a))/(f*tan(f*x + e)), -1/2
*(sqrt(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)
/((a - 2*b)*tan(f*x + e)^2 - a))*tan(f*x + e) + 2*sqrt(b*tan(f*x + e)^2 + a
))/(f*tan(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\tan^2\left(e+fx\right)}\cot^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx+e\right)^2+a}\cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

3.304 $\int \cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=117

$$\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f}+\frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af}$$

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])
/f + ((3*a - b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(3*a*f) - (Cot[e +
 f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(3*f)
```

Rubi [A] time = 0.154082, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3670, 475, 583, 12, 377, 203}

$$\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f}+\frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])
/f + ((3*a - b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(3*a*f) - (Cot[e +
 f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(3*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
```

```
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f} + \frac{\text{Subst}\left(\int \frac{-3a+b-2bx^2}{x^2(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+\right.}{3f}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx}}{3f}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx}}{3f}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx}}{3f}$$

$$= \frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af} -$$

Mathematica [C] time = 6.81588, size = 241, normalized size = 2.06

$$-\frac{\cos^2(e+fx)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\left(\frac{b\tan^2(e+fx)}{a}+1\right)\left(\frac{\sec^2(e+fx)\left(a-2b\tan^2(e+fx)\right)\left(\sqrt{\frac{(a-b)\sin^2(e+fx)}{a}}\sin^{-1}\left(\sqrt{\frac{(a-b)\sin^2(}{a}}\right.\right.}{\left(a+b\tan^2(e+fx)\right)\sqrt{\frac{b\sin^2(e+fx)}{a}+\cos^2}}\right.}{3f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(Cos[e + f*x]^2*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2]*(1 + (b*Tan[e +
f*x]^2)/a)*((Sec[e + f*x]^2*(ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sqrt[
```

```
((a - b)*Sin[e + f*x]^2)/a] + Sqrt[Cos[e + f*x]^2 + (b*Sin[e + f*x]^2)/a])*
(a - 2*b*Tan[e + f*x]^2))/(Sqrt[Cos[e + f*x]^2 + (b*Sin[e + f*x]^2)/a]*(a +
 b*Tan[e + f*x]^2)) - (4*(a - b)*Hypergeometric2F1[2, 2, 3/2, ((a - b)*Sin[
e + f*x]^2)/a]*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2))/(3*f)
```

Maple [C] time = 0.308, size = 4518, normalized size = 38.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3/f/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/a*(3*cos(f*x+e)^3*sin(f*x+
e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos
(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)
*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)
+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/
2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b
+8*b^2)/a^2)^(1/2))*a^2-3*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2
)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1
/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^3*sin(f*x+e)*(1/a*(I*cos(f*x+
e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(
cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b
)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*2^(1/2)*a*b-6*co
s(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1
/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I
*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*
x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2
*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^
(1/2))*a^2+6*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-
b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))
^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*
x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*
I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2
)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(
1/2)+a-2*b)/a)^(1/2))*a*b+3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x
+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/
(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-
b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(
f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2
)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^2-3*
EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+
e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2
)^(1/2))*cos(f*x+e)^2*sin(f*x+e)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b
^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a
*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos
(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*2^(1/2)*a*b-6*cos(f*x+e)^2*sin(f*x+e)*2^(1
/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)
*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^
(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1
/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin
(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2
*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a^2+6*cos(f*x+e)^2*
sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(
1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)
```

```
*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(co
s(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*
b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*
(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*b-
3*cos(f*x+e)*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^
(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*
(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(
f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b
)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a
-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^2+3*EllipticF((cos(f*x+e)-1)*((2
*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)
-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)*sin(f*x+
e)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*
a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(
1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/
2)*2^(1/2)*a*b+6*cos(f*x+e)*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(
a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1
))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(
f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((
2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1
/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)/a)^(1/2))*a^2-6*cos(f*x+e)*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x
+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/
(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-
b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos
(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(
1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*
b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*b+4*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)/a)^(1/2)*a^2-5*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/
a)^(1/2)*a*b+cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2-3*2
^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x
+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-
b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))
^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/s
in(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b
^2)/a^2)^(1/2))*a^2*sin(f*x+e)+3*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(
1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/
2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)
*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(
1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b
^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*b*sin(f*x+e)+6*2^(1/2)*
(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b
*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2
)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*
EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x
+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/
a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a^2*sin(f*x+e)-6*2^(1/2
)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a
-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1
/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2
)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f
*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b
)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*b*sin(f*x+e)-3*cos(
f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2+8*cos(f*x+e)^2*((2*I
*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b-2*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)/a)^(1/2)*b^2-3*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b+
((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2)*cos(f*x+e)*((a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/sin
(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx+e\right)^{2}+a}\cot\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

Fricas [A] time = 2.31175, size = 751, normalized size = 6.42

$$\left[\frac{3\,a\sqrt{-a+b}\log\left(-\frac{\left(a^{2}-8\,ab+8\,b^{2}\right)\tan\left(fx+e\right)^{4}-2\left(3\,a^{2}-4\,ab\right)\tan\left(fx+e\right)^{2}+a^{2}+4\left(\left(a-2\,b\right)\tan\left(fx+e\right)^{3}-a\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^{2}+a}\sqrt{-a+b}}{\tan\left(fx+e\right)^{4}+2\tan\left(fx+e\right)^{2}+1}\right)\tan\left(fx\right.}{12\,af\tan\left(fx+e\right)^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a
^2 - 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x
+ e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x
+ e)^2 + 1))*tan(f*x + e)^3 + 4*((3*a - b)*tan(f*x + e)^2 - a)*sqrt(b*tan(f
*x + e)^2 + a))/(a*f*tan(f*x + e)^3), 1/6*(3*sqrt(a - b)*a*arctan(-2*sqrt(b
*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a
))*tan(f*x + e)^3 + 2*((3*a - b)*tan(f*x + e)^2 - a)*sqrt(b*tan(f*x + e)^2
+ a))/(a*f*tan(f*x + e)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\tan^{2}\left(e+fx\right)}\cot^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx+e\right)^{2}+a}\cot\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

3.305 $\int \cot^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=167

$$-\frac{(15a^2-5ab-2b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f}-\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5f}$$

```
[Out] -((Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f) - ((15*a^2 - 5*a*b - 2*b^2)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/
(15*a^2*f) + ((5*a - b)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(15*a*f)
 - (Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(5*f)
```

Rubi [A] time = 0.230896, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3670, 475, 583, 12, 377, 203}

$$-\frac{(15a^2-5ab-2b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f}-\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f) - ((15*a^2 - 5*a*b - 2*b^2)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/
(15*a^2*f) + ((5*a - b)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(15*a*f)
 - (Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(5*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
```

```
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5f} + \frac{\text{Subst}\left(\int \frac{-5a+b-4bx^2}{x^4(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+\right.}{5f}$$

$$= \frac{(5a-b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15af} - \frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+f}}{5f}$$

$$= -\frac{(15a^2-5ab-2b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f} + \frac{(5a-b)\cot^3(e+f}{1}$$

$$= -\frac{(15a^2-5ab-2b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f} + \frac{(5a-b)\cot^3(e+f}{1}$$

$$= -\frac{(15a^2-5ab-2b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f} + \frac{(5a-b)\cot^3(e+f}{1}$$

$$= -\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{(15a^2-5ab-2b^2)\cot(e+fx)\sqrt{a+b\,\text{ta}}}{15a^2f}$$

Mathematica [C] time = 14.2186, size = 339, normalized size = 2.03

$$-\frac{\cos^4(e+fx)\cot^5(e+fx)\left(\frac{b\tan^2(e+fx)}{a}+1\right)\left(8(a-b)\tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^3 \text{HypergeometricPFQ}\left(\{2,\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(Cos[e + f*x]^4*Cot[e + f*x]^5*(1 + (b*Tan[e + f*x]^2)/a)*(8*(a - b)*Hyper
geometricPFQ[{2, 2, 2}, {1, 3/2}, ((a - b)*Sin[e + f*x]^2)/a]*Tan[e + f*x]^
2*(a + b*Tan[e + f*x]^2)^3 + 8*Hypergeometric2F1[2, 2, 3/2, ((a - b)*Sin[e
+ f*x]^2)/a]*Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2*(-2*a^2 - 3*b^2*Tan[e
+ f*x]^2 + a*b*(2 + 3*Tan[e + f*x]^2)) + (a^2*Sec[e + f*x]^4*(3*a^2 - 4*a*b
*Tan[e + f*x]^2 + 8*b^2*Tan[e + f*x]^4)*(ArcSin[Sqrt[((a - b)*Sin[e + f*x]^
2)/a]]*Sqrt[((a - b)*Sin[e + f*x]^2)/a] + Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e
 + f*x]^2))/a]))/Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]))/(15*a^3*
f*Sqrt[a + b*Tan[e + f*x]^2])
```

Maple [C] time = 0.401, size = 6894, normalized size = 41.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx+e\right)^2+a}\cot\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^6, x)
```

Fricas [A] time = 2.42358, size = 902, normalized size = 5.4

$$\left[\frac{15\,a^2\sqrt{-a+b}\log\left(-\frac{(a^2-8\,ab+8\,b^2)\tan\left(fx+e\right)^4-2\left(3\,a^2-4\,ab\right)\tan\left(fx+e\right)^2+a^2-4\left((a-2\,b)\tan\left(fx+e\right)^3-a\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{-a+b}}{\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1}\right)\tan\left(\right.}{60\,a^2f\tan\left(fx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*a^2*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(
3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 - 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f
*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f
```

```
*x + e)^2 + 1))*tan(f*x + e)^5 - 4*((15*a^2 - 5*a*b - 2*b^2)*tan(f*x + e)^4
 - (5*a^2 - a*b)*tan(f*x + e)^2 + 3*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a^2*f
*tan(f*x + e)^5), -1/30*(15*sqrt(a - b)*a^2*arctan(-2*sqrt(b*tan(f*x + e)^2
 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a))*tan(f*x + e)
^5 + 2*((15*a^2 - 5*a*b - 2*b^2)*tan(f*x + e)^4 - (5*a^2 - a*b)*tan(f*x + e
)^2 + 3*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a^2*f*tan(f*x + e)^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\tan^2\left(e + fx\right)}\cot^6\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x)**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx + e\right)^2 + a}\cot\left(fx + e\right)^6\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^6, x)
```

3.306 $\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=145

$$\frac{\left(a+b\tan^2(e+fx)\right)^{7/2}}{7b^2f}-\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}+\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}-\frac{(a-$$

```
[Out] -(((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f) + ((a
- b)*Sqrt[a + b*Tan[e + f*x]^2])/f + (a + b*Tan[e + f*x]^2)^(3/2)/(3*f) - (
(a + b)*(a + b*Tan[e + f*x]^2)^(5/2))/(5*b^2*f) + (a + b*Tan[e + f*x]^2)^(7
/2)/(7*b^2*f)
```

Rubi [A] time = 0.174963, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3670, 446, 88, 50, 63, 208}

$$\frac{\left(a+b\tan^2(e+fx)\right)^{7/2}}{7b^2f}-\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}+\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}-\frac{(a-$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```


```
[Out] -(((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f) + ((a
- b)*Sqrt[a + b*Tan[e + f*x]^2])/f + (a + b*Tan[e + f*x]^2)^(3/2)/(3*f) - (
(a + b)*(a + b*Tan[e + f*x]^2)^(5/2))/(5*b^2*f) + (a + b*Tan[e + f*x]^2)^(7
/2)/(7*b^2*f)
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^5\left(a+bx^2\right)^{3/2}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^{3/2}}{1+x}\,dx,x,\tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(-a-b)(a+bx)^{3/2}}{b}+\frac{(a+bx)^{3/2}}{1+x}+\frac{(a+bx)^{5/2}}{b}\right)dx,x,\tan^2(e+fx)\right)}{2f}$$

$$= -\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\left(a+b\tan^2(e+fx)\right)^{7/2}}{7b^2f}+\frac{\text{Subst}\left(\int \frac{(\ldots}{\ldots}\right.}{\ldots}$$

$$= \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\left(a+b\tan^2(\ldots\right.}{7b^2\ldots}$$

$$= \frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+b)\left(a+b\,\text{ta}\ldots\right.}{5b^{\ldots}}$$

$$= \frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+b)\left(a+b\,\text{ta}\ldots\right.}{5b^{\ldots}}$$

$$= -\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}+\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{(a\ldots}{\ldots}$$

Mathematica [A] time = 1.33256, size = 139, normalized size = 0.96

$$\frac{\frac{2\left(a+b\tan^2(e+fx)\right)^{7/2}}{7b^2}-\frac{2(a+b)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2}+\frac{2}{3}\left(a+b\tan^2(e+fx)\right)^{3/2}+2(a-b)\left(\sqrt{a+b\tan^2(e+fx)}-\sqrt{a-b}\tanh^{-\ldots}\right.}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*(a + b*Tan[e + f*x]^2)^(3/2))/3 - (2*(a + b)*(a + b*Tan[e + f*x]^2)^(5/
2))/(5*b^2) + (2*(a + b*Tan[e + f*x]^2)^(7/2))/(7*b^2) + 2*(a - b)*(-(Sqrt[
a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]) + Sqrt[a + b*Tan[e
+ f*x]^2]))/(2*f)
```

Maple [B] time = 0.026, size = 256, normalized size = 1.8

$$\frac{(\tan(fx+e))^2}{7fb}\left(a+b(\tan(fx+e))^2\right)^{\frac{5}{2}}-\frac{2a}{35fb^2}\left(a+b(\tan(fx+e))^2\right)^{\frac{5}{2}}-\frac{1}{5fb}\left(a+b(\tan(fx+e))^2\right)^{\frac{5}{2}}+\frac{b(\tan(f\ldots}{3\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/7/f*tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(5/2)/b-2/35/f*a/b^2*(a+b*tan(f*x+e)^
2)^(5/2)-1/5*(a+b*tan(f*x+e)^2)^(5/2)/b/f+1/3/f*b*tan(f*x+e)^2*(a+b*tan(f*x
+e)^2)^(1/2)+4/3/f*a*(a+b*tan(f*x+e)^2)^(1/2)-b*(a+b*tan(f*x+e)^2)^(1/2)/f+
1/f*b^2/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))-2/f*a*b/
(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))+1/f*a^2/(-a+b)^(
1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}\tan(fx+e)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^5, x)
```

Fricas [A] time = 2.29907, size = 992, normalized size = 6.84

$$\left[-\frac{105\left(ab^2-b^3\right)\sqrt{a-b}\log\left(-\frac{b^2\tan(fx+e)^4+2\left(4ab-3b^2\right)\tan(fx+e)^2+4\left(b\tan(fx+e)^2+2a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}+8a^2-8ab+b^2}{\tan(fx+e)^4+2\tan(fx+e)^2+1}\right)-4\left(15\ldots\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/420*(105*(a*b^2 - b^3)*sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b
- 3*b^2)*tan(f*x + e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e
)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e
```

```
)^2 + 1)) - 4*(15*b^3*tan(f*x + e)^6 + 3*(8*a*b^2 - 7*b^3)*tan(f*x + e)^4 -
 6*a^3 - 21*a^2*b + 140*a*b^2 - 105*b^3 + (3*a^2*b - 42*a*b^2 + 35*b^3)*tan
(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), 1/210*(105*(a*b^2 - b^3)*
sqrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x +
e)^2 + 2*a - b)) + 2*(15*b^3*tan(f*x + e)^6 + 3*(8*a*b^2 - 7*b^3)*tan(f*x +
 e)^4 - 6*a^3 - 21*a^2*b + 140*a*b^2 - 105*b^3 + (3*a^2*b - 42*a*b^2 + 35*b
^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\tan^5\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x)**5, x)
```

Giac [A] time = 1.28983, size = 265, normalized size = 1.83

$$\frac{\left(a^2-2\,ab+b^2\right)\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}f}+\frac{15\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{7}{2}}b^{12}f^6-21\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}ab^{12}f^6-21\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] (a^2 - 2*a*b + b^2)*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-
a + b)*f) + 1/105*(15*(b*tan(f*x + e)^2 + a)^(7/2)*b^12*f^6 - 21*(b*tan(f*x
 + e)^2 + a)^(5/2)*a*b^12*f^6 - 21*(b*tan(f*x + e)^2 + a)^(5/2)*b^13*f^6 +
35*(b*tan(f*x + e)^2 + a)^(3/2)*b^14*f^6 + 105*sqrt(b*tan(f*x + e)^2 + a)*a
*b^14*f^6 - 105*sqrt(b*tan(f*x + e)^2 + a)*b^15*f^6)/(b^14*f^7)
```

3.307 $\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{\left(a+b\tan^2(e+fx)\right)^{5/2}}{5bf}-\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}-\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

```
[Out] ((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f - ((a - b
)*Sqrt[a + b*Tan[e + f*x]^2])/f - (a + b*Tan[e + f*x]^2)^(3/2)/(3*f) + (a +
 b*Tan[e + f*x]^2)^(5/2)/(5*b*f)
```

Rubi [A] time = 0.14503, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3670, 446, 80, 50, 63, 208}

$$\frac{\left(a+b\tan^2(e+fx)\right)^{5/2}}{5bf}-\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}-\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f - ((a - b
)*Sqrt[a + b*Tan[e + f*x]^2])/f - (a + b*Tan[e + f*x]^2)^(3/2)/(3*f) + (a +
 b*Tan[e + f*x]^2)^(5/2)/(5*b*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
```

```
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{x^3\left(a+bx^2\right)^{3/2}}{1+x^2}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x(a+bx)^{3/2}}{1+x}dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\left(a+b\tan^2(e+fx)\right)^{5/2}}{5bf} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1+x}dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f} + \frac{\left(a+b\tan^2(e+fx)\right)^{5/2}}{5bf} - \frac{(a-b)\,\text{Subst}\left(\int \frac{\sqrt{}}{}\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f} + \frac{\left(a+b\tan^2(e\right.}{5bf}$$

$$= -\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f} + \frac{\left(a+b\tan^2(e\right.}{5bf}$$

$$= \frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} - \frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\left(a+\right.}{}$$

Mathematica [A] time = 0.906409, size = 112, normalized size = 0.97

$$\frac{\sqrt{a+b\tan^2(e+fx)}\left(3a^2+b(6a-5b)\tan^2(e+fx)-20ab+3b^2\tan^4(e+fx)+15b^2\right)+15b(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\text{t}}}{\sqrt{}}\right.}{15bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (15*(a - b)^(3/2)*b*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + Sqrt[
a + b*Tan[e + f*x]^2]*(3*a^2 - 20*a*b + 15*b^2 + (6*a - 5*b)*b*Tan[e + f*x]
```

```
^2 + 3*b^2*Tan[e + f*x]^4))/(15*b*f)
```

Maple [B] time = 0.017, size = 204, normalized size = 1.8

$$\frac{1}{5fb}\left(a+b\left(\tan\left(fx+e\right)\right)^{2}\right)^{\frac{5}{2}}-\frac{b\left(\tan\left(fx+e\right)\right)^{2}}{3f}\sqrt{a+b\left(\tan\left(fx+e\right)\right)^{2}}-\frac{4a}{3f}\sqrt{a+b\left(\tan\left(fx+e\right)\right)^{2}}+\frac{b}{f}\sqrt{a+b\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/5*(a+b*tan(f*x+e)^2)^(5/2)/b/f-1/3/f*b*tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1
/2)-4/3/f*a*(a+b*tan(f*x+e)^2)^(1/2)+b*(a+b*tan(f*x+e)^2)^(1/2)/f-1/f*b^2/(
-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))+2/f*a*b/(-a+b)^(1
/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))-1/f*a^2/(-a+b)^(1/2)*arct
an((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^3, x)
```

Fricas [A] time = 2.26934, size = 797, normalized size = 6.87

$$\left[-\frac{15\left(ab-b^{2}\right)\sqrt{a-b}\log\left(-\frac{b^{2}\tan\left(fx+e\right)^{4}+2\left(4ab-3b^{2}\right)\tan\left(fx+e\right)^{2}-4\left(b\tan\left(fx+e\right)^{2}+2a-b\right)\sqrt{b\tan\left(fx+e\right)^{2}+a}\sqrt{a-b}+8a^{2}-8ab+b^{2}}{\tan\left(fx+e\right)^{4}+2\tan\left(fx+e\right)^{2}+1}\right)-4\left(3b^{2}\mathrm{ta}\right.}{60bf}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(a*b - b^2)*sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*
b^2)*tan(f*x + e)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2
+ a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2
+ 1)) - 4*(3*b^2*tan(f*x + e)^4 + (6*a*b - 5*b^2)*tan(f*x + e)^2 + 3*a^2 -
20*a*b + 15*b^2)*sqrt(b*tan(f*x + e)^2 + a))/(b*f), -1/30*(15*(a*b - b^2)*s
qrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e
)^2 + 2*a - b)) - 2*(3*b^2*tan(f*x + e)^4 + (6*a*b - 5*b^2)*tan(f*x + e)^2
+ 3*a^2 - 20*a*b + 15*b^2)*sqrt(b*tan(f*x + e)^2 + a))/(b*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\tan^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x)**3, x)
```

Giac [A] time = 1.26429, size = 203, normalized size = 1.75

$$-\frac{\left(a^2-2\,ab+b^2\right)\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}f}+\frac{3\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}b^4f^4-5\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}b^5f^4-15\sqrt{b\,\mathrm{ta}}}{15\,b^5f^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(a^2 - 2*a*b + b^2)*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(
-a + b)*f) + 1/15*(3*(b*tan(f*x + e)^2 + a)^(5/2)*b^4*f^4 - 5*(b*tan(f*x +
e)^2 + a)^(3/2)*b^5*f^4 - 15*sqrt(b*tan(f*x + e)^2 + a)*a*b^5*f^4 + 15*sqrt
(b*tan(f*x + e)^2 + a)*b^6*f^4)/(b^5*f^5)
```

3.308 $\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=90

$$\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

```
[Out] -(((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f) + ((a
- b)*Sqrt[a + b*Tan[e + f*x]^2])/f + (a + b*Tan[e + f*x]^2)^(3/2)/(3*f)
```

Rubi [A] time = 0.0968939, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3670, 444, 50, 63, 208}

$$\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f) + ((a
- b)*Sqrt[a + b*Tan[e + f*x]^2])/f + (a + b*Tan[e + f*x]^2)^(3/2)/(3*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{x\left(a+bx^2\right)^{3/2}}{1+x^2}dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1+x}dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f} + \frac{(a-b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f} + \frac{(a-b)^2\,\text{Subst}\left(\right.}{}$$
$$= \frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f} + \frac{(a-b)^2\,\text{Subst}\left(\right.}{}$$
$$= -\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} + \frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{(a+}{}$$

Mathematica [A] time = 0.382881, size = 80, normalized size = 0.89

$$\frac{\sqrt{a+b\tan^2(e+fx)}\left(4a+b\tan^2(e+fx)-3b\right)-3(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*(a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + Sqrt[a
+ b*Tan[e + f*x]^2]*(4*a - 3*b + b*Tan[e + f*x]^2))/(3*f)
```

Maple [B] time = 0.016, size = 181, normalized size = 2.

$$\frac{b\left(\tan\left(fx+e\right)\right)^2}{3f}\sqrt{a+b\left(\tan\left(fx+e\right)\right)^2}+\frac{4a}{3f}\sqrt{a+b\left(\tan\left(fx+e\right)\right)^2}-\frac{b}{f}\sqrt{a+b\left(\tan\left(fx+e\right)\right)^2}+\frac{b^2}{f}\arctan\left(\sqrt{a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3/f*b*tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2)+4/3/f*a*(a+b*tan(f*x+e)^2)^(1
/2)-b*(a+b*tan(f*x+e)^2)^(1/2)/f+1/f*b^2/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e
)^2)^(1/2)/(-a+b)^(1/2))-2/f*a*b/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/
2)/(-a+b)^(1/2))+1/f*a^2/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e), x)
```

Fricas [A] time = 2.23965, size = 624, normalized size = 6.93

$$\left[-\frac{3\,(a-b)^{\frac{3}{2}}\log\left(-\frac{b^{2}\tan\left(fx+e\right)^{4}+2\left(4\,ab-3\,b^{2}\right)\tan\left(fx+e\right)^{2}+4\left(b\tan\left(fx+e\right)^{2}+2\,a-b\right)\sqrt{b\tan\left(fx+e\right)^{2}+a}\sqrt{a-b}+8\,a^{2}-8\,ab+b^{2}}{\tan\left(fx+e\right)^{4}+2\,\tan\left(fx+e\right)^{2}+1}\right)-4\left(b\tan\left(fx+e\right)^{2}\right.}{12\,f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(a - b)^(3/2)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*
x + e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a
 - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 4*(
b*tan(f*x + e)^2 + 4*a - 3*b)*sqrt(b*tan(f*x + e)^2 + a))/f, 1/6*(3*(a - b)
*sqrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x +
 e)^2 + 2*a - b)) + 2*(b*tan(f*x + e)^2 + 4*a - 3*b)*sqrt(b*tan(f*x + e)^2
+ a))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}\tan\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x), x)
```

Giac [A] time = 1.20663, size = 154, normalized size = 1.71

$$\frac{\left(a^2-2\,ab+b^2\right)\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}f}+\frac{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}f^2+3\sqrt{b\tan\left(fx+e\right)^2+a}af^2-3\sqrt{b\tan\left(fx+\cdots\right.}}{3\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] (a^2 - 2*a*b + b^2)*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-
a + b)*f) + 1/3*((b*tan(f*x + e)^2 + a)^(3/2)*f^2 + 3*sqrt(b*tan(f*x + e)^2
 + a)*a*f^2 - 3*sqrt(b*tan(f*x + e)^2 + a)*b*f^2)/f^3
```

3.309 $\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=95

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{b\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/f) + ((a - b)^(3/2)
*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f + (b*Sqrt[a + b*Tan[e +
 f*x]^2])/f
```

Rubi [A] time = 0.131187, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 446, 84, 156, 63, 208}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{b\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/f) + ((a - b)^(3/2)
*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f + (b*Sqrt[a + b*Tan[e +
 f*x]^2])/f
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{b\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\text{Subst}\left(\int \frac{a^2+(2a-b)bx}{x(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{b\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f} - \frac{(a-b}{}$$

$$= \frac{b\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right)}{bf} -$$

$$= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} + \frac{b\sqrt{}}{}$$

Mathematica [A] time = 0.250656, size = 90, normalized size = 0.95

$$\frac{a^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)\right)+b\sqrt{a+b\tan^2(e+fx)}+(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(a^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]) + (a - b)^(3/2)*Arc
Tanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + b*Sqrt[a + b*Tan[e + f*x]^2]
)/f
```

Maple [B] time = 0.186, size = 1765, normalized size = 18.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/4/f/a^(5/2)/(a-b)^(1/2)*(cos(f*x+e)-1)^3*(2*ln(4*cos(f*x+e)*(a-b)^(1/2)*
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4
*b*cos(f*x+e)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+
1)^2)^(1/2))*cos(f*x+e)*a^(9/2)-4*ln(4*cos(f*x+e)*(a-b)^(1/2)*((a*cos(f*x+e
)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e)
+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*
cos(f*x+e)*a^(7/2)*b+2*ln(4*cos(f*x+e)*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e)+4*(a-b)^(1
/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*cos(f*x+e)*
a^(5/2)*b^2+2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*co
s(f*x+e)*(a-b)^(1/2)*a^(5/2)*b+2*b*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(
f*x+e)+1)^2)^(1/2)*a^(5/2)*(a-b)^(1/2)+ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*
x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos
(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)
^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*cos(f*x+e)*(a-b)^(1/2)*a^4-3*ln(-2/a^(1/2)*
(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1
)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*cos(f*x+e)*(a-b)^(1/2
)*a^3*b+6*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^
2)*cos(f*x+e)*(a-b)^(1/2)*a^2*b^2-3*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e
)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*
x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1
/2)*a^(1/2)+b)/sin(f*x+e)^2)*cos(f*x+e)*(a-b)^(1/2)*a*b^3-ln(-4*(cos(f*x+e)
*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x
+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/
2)*a^(1/2)+b)/(cos(f*x+e)-1))*cos(f*x+e)*(a-b)^(1/2)*a^4+3*ln(-4/a^(1/2)*(c
os(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^
2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*cos(f*x+e)*(a-b)^(1/2)*
a^3*b-6*ln(-4/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e
)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(
f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)
*cos(f*x+e)*(a-b)^(1/2)*a^2*b^2+3*ln(-4/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+
e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2
)*a^(1/2)+b)/sin(f*x+e)^2)*cos(f*x+e)*(a-b)^(1/2)*a*b^3)*cos(f*x+e)^2*((a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)*4^(1/2)/sin(f*x+e)^6/((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e), x)
```

Fricas [A] time = 6.5598, size = 1481, normalized size = 15.59

$$\left[-\frac{(a-b)^{\frac{3}{2}}\log\left(-\frac{b^2\tan\left(fx+e\right)^4+2\left(4\,ab-3\,b^2\right)\tan\left(fx+e\right)^2-4\left(b\tan\left(fx+e\right)^2+2\,a-b\right)\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{a-b}+8\,a^2-8\,ab+b^2}{\tan\left(fx+e\right)^4+2\,\tan\left(fx+e\right)^2+1}\right)-2\,a^{\frac{3}{2}}\log\left(\frac{b\tan(f\cdots}{\cdots}\right.}{4\,f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((a - b)^(3/2)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x +
 e)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a -
b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 2*a^(3
/2)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan
(f*x + e)^2) - 4*sqrt(b*tan(f*x + e)^2 + a)*b)/f, 1/4*(4*sqrt(-a)*a*arctan(
sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) - (a - b)^(3/2)*log(-(b^2*tan(f*x +
e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sq
rt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4
 + 2*tan(f*x + e)^2 + 1)) + 4*sqrt(b*tan(f*x + e)^2 + a)*b)/f, 1/2*((-a + b
)^(3/2)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^2
+ 2*a - b)) + a^(3/2)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*
sqrt(a) + 2*a)/tan(f*x + e)^2) + 2*sqrt(b*tan(f*x + e)^2 + a)*b)/f, 1/2*(2*
sqrt(-a)*a*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) + (-a + b)^(3/2)*a
rctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^2 + 2*a - b
)) + 2*sqrt(b*tan(f*x + e)^2 + a)*b)/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*cot(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e), x)
```

3.310 $\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{\sqrt{a}(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{a\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

```
[Out] (Sqrt[a]*(2*a - 3*b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(2*f) - (
(a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f - (a*Cot[e
 + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Rubi [A] time = 0.173893, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3670, 446, 98, 156, 63, 208}

$$\frac{\sqrt{a}(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{a\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a]*(2*a - 3*b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(2*f) - (
(a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f - (a*Cot[e
 + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^3\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{a\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(2a-3b)+\frac{1}{2}(a-2b)bx}{x(1+x)\sqrt{a+bx}}\,dx, x\right.}{2f}$$

$$= -\frac{a\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} - \frac{(a(2a-3b))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,\right.}{4f}$$

$$= -\frac{a\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} - \frac{(a(2a-3b))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,\right.}{2bf}$$

$$= \frac{\sqrt{a}(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2f} - \frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right.}{f}$$

Mathematica [A] time = 0.328544, size = 109, normalized size = 0.94

$$\frac{\sqrt{a}(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)-2(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)-a\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a]*(2*a - 3*b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]] - 2*(a - b
)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] - a*Cot[e + f*x]^2*
Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Maple [B] time = 0.221, size = 2011, normalized size = 17.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/8/f/(a-b)^(1/2)*4^(1/2)*(cos(f*x+e)-1)^2*(2*cos(f*x+e)*ln(-2/a^(1/2)*(cos
(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)
^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)
/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*a^(3/2)*(a-b)^(1/2)-2*cos
(f*x+e)*ln(-4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)
^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b
+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*a^(3/2)*(a-b)^(1/2)-
3*cos(f*x+e)*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a
*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+
e)^2)*a^(1/2)*(a-b)^(1/2)*b+3*cos(f*x+e)*ln(-4*(cos(f*x+e)*((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e
)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(co
s(f*x+e)-1))*a^(1/2)*(a-b)^(1/2)*b-2*a^(3/2)*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(
cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/
2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)
+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*(a-b)^(1/2)+2*a^(3/2)*ln(-4*(cos(f*x+
e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f
*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(
1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*(a-b)^(1/2)-2*cos(f*x+e)*((a*cos(f*x+e)^2-c
os(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*(a-b)^(1/2)*a+4*cos(f*x+e)*ln(4*co
s(f*x+e)*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(
1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e
)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*a^2-8*cos(f*x+e)*ln(4*cos(f*x+e)*(a-b)^(1
/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)
*a-4*b*cos(f*x+e)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x
+e)+1)^2)^(1/2))*a*b+4*cos(f*x+e)*ln(4*cos(f*x+e)*(a-b)^(1/2)*((a*cos(f*x+e
)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e)
+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*
b^2+3*a^(1/2)*b*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-c
os(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f
*x+e)^2)*(a-b)^(1/2)-3*a^(1/2)*ln(-4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e
)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(
f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1
))*b*(a-b)^(1/2)-4*ln(4*cos(f*x+e)*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^
2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e)+4*(a-b)^(1/2)*
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*a^2+8*ln(4*cos(
f*x+e)*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/
2)+4*cos(f*x+e)*a-4*b*cos(f*x+e)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^
2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*a*b-4*ln(4*cos(f*x+e)*(a-b)^(1/2)*((a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*
x+e)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/
2))*b^2)*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2
)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)/sin(f*x+e)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^3, x)
```

Fricas [A] time = 1.8665, size = 1485, normalized size = 12.8

$$\left[-\frac{2\,(a-b)^{\frac{3}{2}}\log\left(\frac{b\tan\left(fx+e\right)^2+2\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{a-b}+2\,a-b}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^2+(2\,a-3\,b)\sqrt{a}\log\left(\frac{b\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}}{\tan\left(fx+e\right)^2}\right.}{4\,f\tan\left(fx+e\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a - b)^(3/2)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)
*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + (2*a - 3*b)*
sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)
/tan(f*x + e)^2)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*a)/(f*tan(f*
x + e)^2), -1/4*(4*(a - b)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*
sqrt(-a + b)/(a - b))*tan(f*x + e)^2 + (2*a - 3*b)*sqrt(a)*log((b*tan(f*x +
 e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x
 + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*a)/(f*tan(f*x + e)^2), -1/2*(sqrt(-a
)*(2*a - 3*b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^2
+ (a - b)^(3/2)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a
 - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + sqrt(b*tan(f*x + e)
^2 + a)*a)/(f*tan(f*x + e)^2), -1/2*(sqrt(-a)*(2*a - 3*b)*arctan(sqrt(b*tan
(f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^2 + 2*(a - b)*sqrt(-a + b)*arctan
(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b))*tan(f*x + e)^2 + sqrt(b*
tan(f*x + e)^2 + a)*a)/(f*tan(f*x + e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^3, x)
```

3.311 $\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=161

$$-\frac{\left(8a^2-12ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8\sqrt{a}f}+\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{a\cot^4(e+fx)\sqrt{a+b\tan^2(e+f}{4f}$$

```
[Out] -((8*a^2 - 12*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(8*
Sqrt[a]*f) + ((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]
)/f + ((4*a - 5*b)*Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2])/(8*f) - (a*Co
t[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2])/(4*f)
```

Rubi [A] time = 0.222276, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3670, 446, 98, 151, 156, 63, 208}

$$-\frac{\left(8a^2-12ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8\sqrt{a}f}+\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{a\cot^4(e+fx)\sqrt{a+b\tan^2(e+f}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```


```
[Out] -((8*a^2 - 12*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(8*
Sqrt[a]*f) + ((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]
)/f + ((4*a - 5*b)*Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2])/(8*f) - (a*Co
t[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2])/(4*f)
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^5\left(1+x^2\right)}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3(1+x)}dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{a\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(4a-5b)+\frac{1}{2}(3a-4b)bx}{x^2(1+x)\sqrt{a+bx}}dx, \cdots\right.}{4f}$$

$$= \frac{(4a-5b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} - \frac{a\cot^4(e+fx)\sqrt{a+b\tan^2 \cdots}}{4f}$$

$$= \frac{(4a-5b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} - \frac{a\cot^4(e+fx)\sqrt{a+b\tan^2 \cdots}}{4f}$$

$$= \frac{(4a-5b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} - \frac{a\cot^4(e+fx)\sqrt{a+b\tan^2 \cdots}}{4f}$$

$$= -\frac{\left(8a^2-12ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8\sqrt{a}f} + \frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan \cdots}}{\cdots}\right.}{f}$$

Mathematica [A] time = 1.36151, size = 140, normalized size = 0.87

$$\frac{\left(-8a^2+12ab-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\left(8(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\cot^2(e+fx)\sqrt{a+b\tan^2(e \cdots}\right.}{8\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-8*a^2 + 12*a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]] + Sq
rt[a]*(8*(a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + Co
t[e + f*x]^2*(4*a - 5*b - 2*a*Cot[e + f*x]^2)*Sqrt[a + b*Tan[e + f*x]^2]))/
(8*Sqrt[a]*f)
```

Maple [B] time = 0.197, size = 5224, normalized size = 32.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^5, x)
```

Fricas [A] time = 1.89837, size = 1817, normalized size = 11.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*(a^2 - a*b)*sqrt(a - b)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x
+ e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 - (
8*a^2 - 12*a*b + 3*b^2)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x +
e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^4 - 2*((4*a^2 - 5*a*b
)*tan(f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a*f*tan(f*x + e)^4),
 1/16*(16*(a^2 - a*b)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(
-a + b)/(a - b))*tan(f*x + e)^4 + (8*a^2 - 12*a*b + 3*b^2)*sqrt(a)*log((b*t
an(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)
*tan(f*x + e)^4 + 2*((4*a^2 - 5*a*b)*tan(f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*x
 + e)^2 + a))/(a*f*tan(f*x + e)^4), 1/8*((8*a^2 - 12*a*b + 3*b^2)*sqrt(-a)*
arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^4 - 4*(a^2 - a*b
)*sqrt(a - b)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a -
 b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 + ((4*a^2 - 5*a*b)*tan(
f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a*f*tan(f*x + e)^4), 1/8*(
(8*a^2 - 12*a*b + 3*b^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a
)/a)*tan(f*x + e)^4 + 8*(a^2 - a*b)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e
)^2 + a)*sqrt(-a + b)/(a - b))*tan(f*x + e)^4 + ((4*a^2 - 5*a*b)*tan(f*x +
e)^2 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a*f*tan(f*x + e)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^5, x)
```

3.312 $\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=294

$$\frac{\left(3a^2-56ab+48b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{192bf}-\frac{\left(8a^2b+3a^3-80ab^2+64b^3\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{128b^2f}+$$

```
[Out] -(((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^
2]])/f) + ((3*a^4 + 8*a^3*b + 48*a^2*b^2 - 192*a*b^3 + 128*b^4)*ArcTanh[(Sq
rt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(128*b^(5/2)*f) - ((3*a^3
+ 8*a^2*b - 80*a*b^2 + 64*b^3)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(12
8*b^2*f) + ((3*a^2 - 56*a*b + 48*b^2)*Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x
]^2])/(192*b*f) + ((9*a - 8*b)*Tan[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(
48*f) + (b*Tan[e + f*x]^7*Sqrt[a + b*Tan[e + f*x]^2])/(8*f)
```

Rubi [A] time = 0.447992, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3670, 477, 582, 523, 217, 206, 377, 203}

$$\frac{\left(3a^2-56ab+48b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{192bf}-\frac{\left(8a^2b+3a^3-80ab^2+64b^3\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{128b^2f}+$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^
2]])/f) + ((3*a^4 + 8*a^3*b + 48*a^2*b^2 - 192*a*b^3 + 128*b^4)*ArcTanh[(Sq
rt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(128*b^(5/2)*f) - ((3*a^3
+ 8*a^2*b - 80*a*b^2 + 64*b^3)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(12
8*b^2*f) + ((3*a^2 - 56*a*b + 48*b^2)*Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x
]^2])/(192*b*f) + ((9*a - 8*b)*Tan[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(
48*f) + (b*Tan[e + f*x]^7*Sqrt[a + b*Tan[e + f*x]^2])/(8*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^6\left(a+bx^2\right)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan^7(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} + \frac{\text{Subst}\left(\int \frac{x^6\left(a(8a-7b)+(9a-8b)bx^2\right)}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \mathrm{t}\right.}{8f}$$

$$= \frac{(9a-8b)\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{48f} + \frac{b\tan^7(e+fx)\sqrt{a+b\tan^2(e}{8f}$$

$$= \frac{\left(3a^2-56ab+48b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{192bf} + \frac{(9a-8b)\tan^5(e+}{}$$

$$= -\frac{\left(3a^3+8a^2b-80ab^2+64b^3\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{128b^2f} + \frac{\left(3a^2-5\right.}{}$$

$$= -\frac{\left(3a^3+8a^2b-80ab^2+64b^3\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{128b^2f} + \frac{\left(3a^2-5\right.}{}$$

$$= -\frac{\left(3a^3+8a^2b-80ab^2+64b^3\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{128b^2f} + \frac{\left(3a^2-5\right.}{}$$

$$= -\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{\left(3a^4+8a^3b+48a^2b^2-192ab^3+128b\right.}{128b^{5/2}f}$$

Mathematica [C] time = 6.48861, size = 908, normalized size = 3.09

$$-\frac{b\left(3a^4+8ba^3-16b^2a^2-64b^3a+64b^4\right)\sqrt{\frac{a+b+(a-b)\cos(2(e+fx))}{\cos(2(e+fx))+1}}\sqrt{-\frac{a\cot^2(e+fx)}{b}}\sqrt{-\frac{a(\cos(2(e+fx))+1)\csc^2(e+fx)}{b}}\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\csc(2(e+fx))\text{Ellip}}{a(a+b+(a-b)\cos(2(e+fx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-((b*(3*a^4 + 8*a^3*b - 16*a^2*b^2 - 64*a*b^3 + 64*b^4)*Sqrt[(a + b + (a -
 b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]
*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b
)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sq
rt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[
e + f*x]^4)/(a*(a + b + (a - b)*Cos[2*(e + f*x)]))) - (4*b*(-64*a^2*b^2 + 1
28*a*b^3 - 64*b^4)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b + (a - b)*Cos[2*(
e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((
a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(
e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a +
b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^
4)/(4*a*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])
- (Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f
```

```
*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[
2*(e + f*x)]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(
e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(2*(a - b)*Sqrt[
1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])))/Sqrt[a + b
+ (a - b)*Cos[2*(e + f*x)]])/(64*b^2*f) + (Sqrt[(a + b + a*Cos[2*(e + f*x)]
 - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sec[e + f*x]^5*(9*a*Sin[e
+ f*x] - 26*b*Sin[e + f*x]))/48 + (Sec[e + f*x]^3*(3*a^2*Sin[e + f*x] - 128
*a*b*Sin[e + f*x] + 184*b^2*Sin[e + f*x]))/(192*b) + (Sec[e + f*x]*(-9*a^3*
Sin[e + f*x] - 30*a^2*b*Sin[e + f*x] + 424*a*b^2*Sin[e + f*x] - 400*b^3*Sin
[e + f*x]))/(384*b^2) + (b*Sec[e + f*x]^6*Tan[e + f*x])/8))/f
```

Maple [B] time = 0.024, size = 669, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/8/f*tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(5/2)/b-1/16/f*a/b^2*tan(f*x+e)*(a+b*
tan(f*x+e)^2)^(5/2)+1/64/f*a^2/b^2*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)+3/12
8/f*a^3/b^2*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)+3/128/f*a^4/b^(5/2)*ln(b^(1
/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-1/6/f*tan(f*x+e)*(a+b*tan(f*x+e)^2
)^(5/2)/b+1/24/f*a/b*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)+1/16/f*a^2/b*(a+b*
tan(f*x+e)^2)^(1/2)*tan(f*x+e)+1/16/f*a^3/b^(3/2)*ln(b^(1/2)*tan(f*x+e)+(a+
b*tan(f*x+e)^2)^(1/2))+1/4/f*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)+3/8/f*a*(a
+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)+3/8/f*a^2/b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(
a+b*tan(f*x+e)^2)^(1/2))-1/2*b*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-3/2/f*
b^(1/2)*a*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))+1/f*b^(3/2)*ln(b^
(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-1/f*(b^4*(a-b))^(1/2)/(a-b)*arct
an(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))+2/f*a/b
*(b^4*(a-b))^(1/2)/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)
^2)^(1/2)*tan(f*x+e))-1/f*a^2*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/
(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 36.2956, size = 2572, normalized size = 8.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(3*a^4 + 8*a^3*b + 48*a^2*b^2 - 192*a*b^3 + 128*b^4)*sqrt(b)*log(
2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a)
 - 384*(a*b^3 - b^4)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b
*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) +
 2*(48*b^4*tan(f*x + e)^7 + 8*(9*a*b^3 - 8*b^4)*tan(f*x + e)^5 + 2*(3*a^2*b
^2 - 56*a*b^3 + 48*b^4)*tan(f*x + e)^3 - 3*(3*a^3*b + 8*a^2*b^2 - 80*a*b^3
+ 64*b^4)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^3*f), -1/384*(3*(3*a
^4 + 8*a^3*b + 48*a^2*b^2 - 192*a*b^3 + 128*b^4)*sqrt(-b)*arctan(sqrt(b*tan
(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + 192*(a*b^3 - b^4)*sqrt(-a + b
)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b
)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - (48*b^4*tan(f*x + e)^7 + 8*(9*a
*b^3 - 8*b^4)*tan(f*x + e)^5 + 2*(3*a^2*b^2 - 56*a*b^3 + 48*b^4)*tan(f*x +
e)^3 - 3*(3*a^3*b + 8*a^2*b^2 - 80*a*b^3 + 64*b^4)*tan(f*x + e))*sqrt(b*tan
(f*x + e)^2 + a))/(b^3*f), -1/768*(768*(a*b^3 - b^4)*sqrt(a - b)*arctan(-sq
rt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - 3*(3*a^4 + 8*a^3*b +
 48*a^2*b^2 - 192*a*b^3 + 128*b^4)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(
b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) - 2*(48*b^4*tan(f*x + e)^7
+ 8*(9*a*b^3 - 8*b^4)*tan(f*x + e)^5 + 2*(3*a^2*b^2 - 56*a*b^3 + 48*b^4)*ta
n(f*x + e)^3 - 3*(3*a^3*b + 8*a^2*b^2 - 80*a*b^3 + 64*b^4)*tan(f*x + e))*sq
rt(b*tan(f*x + e)^2 + a))/(b^3*f), -1/384*(384*(a*b^3 - b^4)*sqrt(a - b)*ar
ctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + 3*(3*a^4 + 8
*a^3*b + 48*a^2*b^2 - 192*a*b^3 + 128*b^4)*sqrt(-b)*arctan(sqrt(b*tan(f*x +
 e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) - (48*b^4*tan(f*x + e)^7 + 8*(9*a*b^3
 - 8*b^4)*tan(f*x + e)^5 + 2*(3*a^2*b^2 - 56*a*b^3 + 48*b^4)*tan(f*x + e)^3
 - 3*(3*a^3*b + 8*a^2*b^2 - 80*a*b^3 + 64*b^4)*tan(f*x + e))*sqrt(b*tan(f*x
 + e)^2 + a))/(b^3*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^{\frac{3}{2}} \tan^6\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x)**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx + e\right)^2 + a\right)^{\frac{3}{2}} \tan\left(fx + e\right)^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^6, x)
```

3.313 $\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=224

$$\frac{\left(a^2-10ab+8b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16bf}-\frac{\left(6a^2b+a^3-24ab^2+16b^3\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{16b^{3/2}f}+\frac{b\tan^5(e$$

```
[Out] ((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f - ((a^3 + 6*a^2*b - 24*a*b^2 + 16*b^3)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a + b*Tan[e + f*x]^2]])/(16*b^(3/2)*f) + ((a^2 - 10*a*b + 8*b^2)*Tan[e
 + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(16*b*f) + ((7*a - 6*b)*Tan[e + f*x]^3*
Sqrt[a + b*Tan[e + f*x]^2])/(24*f) + (b*Tan[e + f*x]^5*Sqrt[a + b*Tan[e + f
*x]^2])/(6*f)
```

Rubi [A] time = 0.354926, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3670, 477, 582, 523, 217, 206, 377, 203}

$$\frac{\left(a^2-10ab+8b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16bf}-\frac{\left(6a^2b+a^3-24ab^2+16b^3\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{16b^{3/2}f}+\frac{b\tan^5(e$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```


```
[Out] ((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f - ((a^3 + 6*a^2*b - 24*a*b^2 + 16*b^3)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a + b*Tan[e + f*x]^2]])/(16*b^(3/2)*f) + ((a^2 - 10*a*b + 8*b^2)*Tan[e
 + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(16*b*f) + ((7*a - 6*b)*Tan[e + f*x]^3*
Sqrt[a + b*Tan[e + f*x]^2])/(24*f) + (b*Tan[e + f*x]^5*Sqrt[a + b*Tan[e + f
*x]^2])/(6*f)
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^{3/2}}{1+x^2}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{6f} + \frac{\text{Subst}\left(\int \frac{x^4\left(a(6a-5b)+(7a-6b)bx^2\right)}{\left(1+x^2\right)\sqrt{a+bx^2}}dx,\right.}{6f}$$

$$= \frac{(7a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{24f} + \frac{b\tan^5(e+fx)\sqrt{a+b\tan}}{6f}$$

$$= \frac{\left(a^2-10ab+8b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16bf} + \frac{(7a-6b)\tan^3(e+}{}$$

$$= \frac{\left(a^2-10ab+8b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16bf} + \frac{(7a-6b)\tan^3(e+}{}$$

$$= \frac{\left(a^2-10ab+8b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16bf} + \frac{(7a-6b)\tan^3(e+}{}$$

$$= \frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\left(a^3+6a^2b-24ab^2+16b^3\right)\tanh^{-1}\left(\right.}{16b^{3/2}f}$$

Mathematica [C] time = 6.35628, size = 833, normalized size = 3.72

$$\frac{\sqrt{\frac{\cos(2(e+fx))a+a+b-b\cos(2(e+fx))}{\cos(2(e+fx))+1}}\left(\frac{1}{6}b\tan(e+fx)\sec^4(e+fx)+\frac{7}{24}(a\sin(e+fx)-2b\sin(e+fx))\sec^3(e+fx)+\frac{(3\sin(e}{}\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(-((b*(a^3 - 2*a^2*b - 8*a*b^2 + 8*b^3)*Sqrt[(a + b + (a - b)*Cos[2*(e + f
*x)])/(1 + Cos[2*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 +
Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x
)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a
- b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(
a + b + (a - b)*Cos[2*(e + f*x)]))) - (4*b*(-8*a^2*b + 16*a*b^2 - 8*b^3)*Sq
rt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2
*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x
)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*
x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e
+ f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(4*a*Sqrt[1 + Cos[
2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]) - (Sqrt[-((a*Cot[e +
f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a +
 b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*Elliptic
Pi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*
x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(2*(a - b)*Sqrt[1 + Cos[2*(e + f*x)]]
*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])))/Sqrt[a + b + (a - b)*Cos[2*(e +
```

```
f*x)]])/(8*b*f) + (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(
1 + Cos[2*(e + f*x)])]*((7*Sec[e + f*x]^3*(a*Sin[e + f*x] - 2*b*Sin[e + f*x
]))/24 + (Sec[e + f*x]*(3*a^2*Sin[e + f*x] - 44*a*b*Sin[e + f*x] + 44*b^2*S
in[e + f*x]))/(48*b) + (b*Sec[e + f*x]^4*Tan[e + f*x])/6))/f
```

Maple [B] time = 0.017, size = 510, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/6/f*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(5/2)/b-1/24/f*a/b*tan(f*x+e)*(a+b*tan(
f*x+e)^2)^(3/2)-1/16/f*a^2/b*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)-1/16/f*a^3
/b^(3/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-1/4/f*tan(f*x+e)*(
a+b*tan(f*x+e)^2)^(3/2)-3/8/f*a*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)-3/8/f*a
^2/b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))+1/2*b*(a+b*tan(f
*x+e)^2)^(1/2)*tan(f*x+e)/f+3/2/f*b^(1/2)*a*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(
f*x+e)^2)^(1/2))-1/f*b^(3/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2)
)+1/f*(b^4*(a-b))^(1/2)/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f
*x+e)^2)^(1/2)*tan(f*x+e))-2/f*a/b*(b^4*(a-b))^(1/2)/(a-b)*arctan(b^2*(a-b)
/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))+1/f*a^2*(b^4*(a-b))
^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2
)*tan(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan \left(fx+e\right)^2 + a\right)^{\frac{3}{2}} \tan \left(fx+e\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

Fricas [A] time = 21.1036, size = 2103, normalized size = 9.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(a^3 + 6*a^2*b - 24*a*b^2 + 16*b^3)*sqrt(b)*log(2*b*tan(f*x + e)^2
 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) - 48*(a*b^2 - b^3
)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a
)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + 2*(8*b^3*tan(f*x +
 e)^5 + 2*(7*a*b^2 - 6*b^3)*tan(f*x + e)^3 + 3*(a^2*b - 10*a*b^2 + 8*b^3)*t
an(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), 1/48*(3*(a^3 + 6*a^2*b -
24*a*b^2 + 16*b^3)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*t
```

```
an(f*x + e))) - 24*(a*b^2 - b^3)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^
2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x +
e)^2 + 1)) + (8*b^3*tan(f*x + e)^5 + 2*(7*a*b^2 - 6*b^3)*tan(f*x + e)^3 + 3
*(a^2*b - 10*a*b^2 + 8*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^2*
f), 1/96*(96*(a*b^2 - b^3)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(
sqrt(a - b)*tan(f*x + e))) + 3*(a^3 + 6*a^2*b - 24*a*b^2 + 16*b^3)*sqrt(b)*
log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e)
+ a) + 2*(8*b^3*tan(f*x + e)^5 + 2*(7*a*b^2 - 6*b^3)*tan(f*x + e)^3 + 3*(a^
2*b - 10*a*b^2 + 8*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f),
1/48*(48*(a*b^2 - b^3)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt
(a - b)*tan(f*x + e))) + 3*(a^3 + 6*a^2*b - 24*a*b^2 + 16*b^3)*sqrt(-b)*arc
tan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + (8*b^3*tan(f*x
+ e)^5 + 2*(7*a*b^2 - 6*b^3)*tan(f*x + e)^3 + 3*(a^2*b - 10*a*b^2 + 8*b^3)*
tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^{\frac{3}{2}} \tan^4\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx + e\right)^2 + a\right)^{\frac{3}{2}} \tan\left(fx + e\right)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

3.314 $\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=172

$$\frac{\left(3a^2-12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8\sqrt{b}f}+\frac{b\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f}+\frac{(5a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e}{8f}$$

```
[Out] -(((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^
2]])/f) + ((3*a^2 - 12*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
 b*Tan[e + f*x]^2]])/(8*Sqrt[b]*f) + ((5*a - 4*b)*Tan[e + f*x]*Sqrt[a + b*T
an[e + f*x]^2])/(8*f) + (b*Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(4*f)
```

Rubi [A] time = 0.248783, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3670, 477, 582, 523, 217, 206, 377, 203}

$$\frac{\left(3a^2-12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8\sqrt{b}f}+\frac{b\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f}+\frac{(5a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```


```
[Out] -(((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^
2]])/f) + ((3*a^2 - 12*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
 b*Tan[e + f*x]^2]])/(8*Sqrt[b]*f) + ((5*a - 4*b)*Tan[e + f*x]*Sqrt[a + b*T
an[e + f*x]^2])/(8*f) + (b*Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(4*f)
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
```

```
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2\left(a(4a-3b)+(5a-4b)bx^2\right)}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, t\right.}{4f}$$

$$= \frac{(5a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+fx)\sqrt{a+b\tan^2(e-}}{4f}$$

$$= \frac{(5a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+fx)\sqrt{a+b\tan^2(e-}}{4f}$$

$$= \frac{(5a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+fx)\sqrt{a+b\tan^2(e-}}{4f}$$

$$= -\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{\left(3a^2-12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+}{\sqrt{a+b\tan^2(e}}\right.}{8\sqrt{b}f}$$

Mathematica [C] time = 6.2563, size = 771, normalized size = 4.48

$$\frac{\frac{b\left(a^2+4ab-4b^2\right)\sin^4(e+fx)\csc(2(e+fx))\sqrt{\frac{(a-b)\cos(2(e+fx))+a+b}{\cos(2(e+fx))+1}}\sqrt{-\frac{a\cot^2(e+fx)}{b}}\sqrt{-\frac{a(\cos(2(e+fx))+1)\csc^2(e+fx)}{b}}\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}\text{EllipticF}\left(\sin^{-1}\right.}{a((a-b)\cos(2(e+fx))+a+b)}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((b*(a^2 + 4*a*b - 4*b^2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[
2*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)
])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x
]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e +
 f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a - b)
*Cos[2*(e + f*x)])) + (4*b*(4*a^2 - 8*a*b + 4*b^2)*Sqrt[1 + Cos[2*(e + f*x)
]]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sqrt[-
((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)
]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*
x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^
2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(4*a*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a +
 b + (a - b)*Cos[2*(e + f*x)]]) - (Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*
(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e
+ f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticPi[-(b/(a - b)), ArcSi
n[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*
Sin[e + f*x]^4)/(2*(a - b)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*
Cos[2*(e + f*x)]])))/Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])/(4*f) + (Sqrt[
(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(
(Sec[e + f*x]*(5*a*Sin[e + f*x] - 6*b*Sin[e + f*x]))/8 + (b*Sec[e + f*x]^2*
Tan[e + f*x])/4))/f
```

Maple [B] time = 0.016, size = 386, normalized size = 2.2

$$\frac{\tan\left(fx+e\right)}{4f}\left(a+b\left(\tan\left(fx+e\right)\right)^{2}\right)^{\frac{3}{2}}+\frac{3a\tan\left(fx+e\right)}{8f}\sqrt{a+b\left(\tan\left(fx+e\right)\right)^{2}}+\frac{3a^{2}}{8f}\ln\left(\sqrt{b}\tan\left(fx+e\right)+\sqrt{a+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4/f*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)+3/8/f*a*(a+b*tan(f*x+e)^2)^(1/2)*
tan(f*x+e)+3/8/f*a^2/b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2)
)-1/2*b*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-3/2/f*b^(1/2)*a*ln(b^(1/2)*ta
n(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))+1/f*b^(3/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*t
an(f*x+e)^2)^(1/2))-1/f*(b^4*(a-b))^(1/2)/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b)
)^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))+2/f*a/b*(b^4*(a-b))^(1/2)/(a-b
)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-1
/f*a^2*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*
tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

Fricas [A] time = 7.99549, size = 1739, normalized size = 10.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*a^2 - 12*a*b + 8*b^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*t
an(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) - 8*(a*b - b^2)*sqrt(-a + b)*l
og(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*t
an(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + 2*(2*b^2*tan(f*x + e)^3 + (5*a*b -
 4*b^2)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b*f), -1/8*((3*a^2 - 12*
a*b + 8*b^2)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x
 + e))) + 4*(a*b - b^2)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqr
t(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)
) - (2*b^2*tan(f*x + e)^3 + (5*a*b - 4*b^2)*tan(f*x + e))*sqrt(b*tan(f*x +
e)^2 + a))/(b*f), -1/16*(16*(a*b - b^2)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x
+ e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (3*a^2 - 12*a*b + 8*b^2)*sqrt(b)*
log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e)
+ a) - 2*(2*b^2*tan(f*x + e)^3 + (5*a*b - 4*b^2)*tan(f*x + e))*sqrt(b*tan(f
```

```
*x + e)^2 + a))/(b*f), -1/8*(8*(a*b - b^2)*sqrt(a - b)*arctan(-sqrt(b*tan(f
*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + (3*a^2 - 12*a*b + 8*b^2)*sqrt(
-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) - (2*b^2*t
an(f*x + e)^3 + (5*a*b - 4*b^2)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(
b*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^{\frac{3}{2}} \tan^2\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx + e\right)^2 + a\right)^{\frac{3}{2}} \tan\left(fx + e\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

3.315 $\int \left(a+b\tan^2(e+fx)\right)^{3/2} dx$

Optimal. Leaf size=125

$$\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{\sqrt{b}(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}$$

```
[Out] ((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f + ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e
 + f*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Rubi [A] time = 0.0949356, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3661, 416, 523, 217, 206, 377, 203}

$$\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{\sqrt{b}(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f + ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e
 + f*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{\text{Subst}\left(\int \frac{a(2a-b)+(3a-2b)bx^2}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx,x,\tan(e+fx)\right)}{2f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx,x,\tan(e+fx)\right)}{f}+$$

$$= \frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx,x,\frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+$$

$$= \frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{(3a-2b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}+\frac{b\tan(e+fx)}{}$$

Mathematica [C] time = 1.27094, size = 233, normalized size = 1.86

$$\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}-i(a-b)^{3/2}\log\left(-\frac{4i\left(\sqrt{a-b}\sqrt{a+b\tan^2(e+fx)}+a-ib\tan(e+fx)\right)}{(a-b)^{5/2}(\tan(e+fx)+i)}\right)+i(a-b)^{3/2}\log\left(\frac{4i\left(\sqrt{a-b}\sqrt{a+b\tan}\right.}{(a-b)^{5/2}}\right.}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-I)*(a - b)^(3/2)*Log[((-4*I)*(a - I*b*Tan[e + f*x] + Sqrt[a - b]*Sqrt[a
+ b*Tan[e + f*x]^2]))/((a - b)^(5/2)*(I + Tan[e + f*x]))] + I*(a - b)^(3/2)
*Log[((4*I)*(a + I*b*Tan[e + f*x] + Sqrt[a - b]*Sqrt[a + b*Tan[e + f*x]^2])
```

```
)/((a - b)^(5/2)*(-I + Tan[e + f*x]))] + (3*a - 2*b)*Sqrt[b]*Log[b*Tan[e +
f*x] + Sqrt[b]*Sqrt[a + b*Tan[e + f*x]^2]] + b*Tan[e + f*x]*Sqrt[a + b*Tan[
e + f*x]^2])/(2*f)
```

Maple [B] time = 0., size = 297, normalized size = 2.4

$$\frac{b\tan\left(fx+e\right)}{2f}\sqrt{a+b\left(\tan\left(fx+e\right)\right)^{2}}+\frac{3a}{2f}\sqrt{b}\ln\left(\sqrt{b}\tan\left(fx+e\right)+\sqrt{a+b\left(\tan\left(fx+e\right)\right)^{2}}\right)-\frac{1}{f}b^{\frac{3}{2}}\ln\left(\sqrt{b}\tan\left(fx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/2*b*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+3/2/f*b^(1/2)*a*ln(b^(1/2)*tan(
f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-1/f*b^(3/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan
(f*x+e)^2)^(1/2))+1/f*(b^4*(a-b))^(1/2)/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^
(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-2/f*a/b*(b^4*(a-b))^(1/2)/(a-b)*
arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))+1/f
*a^2*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*ta
n(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2), x)
```

Fricas [A] time = 4.83448, size = 1382, normalized size = 11.06

$$\left[-\frac{(3a-2b)\sqrt{b}\log\left(2b\tan\left(fx+e\right)^{2}-2\sqrt{b\tan\left(fx+e\right)^{2}+a}\sqrt{b}\tan\left(fx+e\right)+a\right)+2(a-b)\sqrt{-a+b}\log\left(-\frac{(a-2b)\tan\cdots}{\cdots}\right.}{4f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((3*a - 2*b)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2
 + a)*sqrt(b)*tan(f*x + e) + a) + 2*(a - b)*sqrt(-a + b)*log(-((a - 2*b)*ta
n(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/
(tan(f*x + e)^2 + 1)) - 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f, -1/
2*((3*a - 2*b)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f
```

```
*x + e))) - (-a + b)^(3/2)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*
x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - sqrt(b
*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f, 1/4*(4*(a - b)^(3/2)*arctan(-sqrt(b
*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (3*a - 2*b)*sqrt(b)*log(
2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a)
 + 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f, 1/2*(2*(a - b)^(3/2)*arc
tan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (3*a - 2*b)*s
qrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + sqrt
(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx + e\right)^2 + a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2), x)
```

3.316 $\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=114

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{a\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

```
[Out] -(((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^
2]])/f) + (b^(3/2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2
]])/f - (a*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/f
```

Rubi [A] time = 0.138248, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3670, 474, 523, 217, 206, 377, 203}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{a\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^
2]])/f) + (b^(3/2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2
]])/f - (a*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/f
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^2\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\text{Subst}\left(\int \frac{-a(a-2b)+b^2x^2}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \tan(e+\right.}{f}$$

$$= -\frac{a\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \text{t}\right.}{f}$$

$$= -\frac{a\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\ }{\sqrt{a-}}\right.}{f}$$

$$= -\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{a\cot(e}{}$$

Mathematica [C] time = 6.12369, size = 256, normalized size = 2.25

$$-\frac{a\tan(e+fx)\left(\sqrt{2}(a-2b)\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}}{\sqrt{2}}\right),1\right)+\csc^2(e+f\right.}{\sqrt{2}f\sqrt{\sec^2(e+fx)((a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a*((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2 + Sqrt[2]*(a - 2*b)
*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcS
in[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]
```

```
 + Sqrt[2]*(-a + b)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2
)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)
])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Tan[e + f*x])/(Sqrt[2]*f*Sqrt[(a + b +
(a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]))
```

Maple [C] time = 0.339, size = 3333, normalized size = 29.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*(cos(f*x+e)*sin(f*x+e)*2^(1/
2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*
a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(
1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/
2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f
*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/
a^2)^(1/2))*a^2-2*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)
/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^
2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)*sin(f*x+e)*(1/a*(I*cos(f*x+e)*b^(1/2)
*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)
+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-co
s(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*2^(1/2)*a*b+2*2^(1/2)*(1/a
*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos
(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*
b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*Elli
pticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),
1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1
/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*b^2*sin(f*x+e)*cos(f*x+e)-2*
cos(f*x+e)*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1
/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I
*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*
x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2
*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^
(1/2))*a^2+4*cos(f*x+e)*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)
^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(
1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+
e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*
b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+
a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/
2)+a-2*b)/a)^(1/2))*a*b-2*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*
b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/
a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*co
s(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(
a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,
(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)
/a)^(1/2))*b^2*sin(f*x+e)*cos(f*x+e)+2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-
b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))
^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*
x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I
*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4
*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^2*sin(f*x+e)-2*2^(1
/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)
*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^
```

```
(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1
/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(
f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)
/a^2)^(1/2))*a*b*sin(f*x+e)+2*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2
)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*
(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+
b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/
2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)
*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2
*b)/a)^(1/2))*b^2*sin(f*x+e)-2*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/
2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)
*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a
+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1
/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*
b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a
-2*b)/a)^(1/2))*a^2*sin(f*x+e)+4*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(
1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/
2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)
*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^
(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-
2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)
+a-2*b)/a)^(1/2))*a*b*sin(f*x+e)-2*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)
^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(
1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+
e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*
b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+
a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/
2)+a-2*b)/a)^(1/2))*b^2*sin(f*x+e)+cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a
-2*b)/a)^(1/2)*a^2-cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a
*b+((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b)*cos(f*x+e)^3*((a*cos(f*x+
e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)/sin(f*x+e)/(a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

Fricas [A] time = 6.95168, size = 1829, normalized size = 16.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b^(3/2)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
b)*tan(f*x + e) + a)*tan(f*x + e) - (a - b)*sqrt(-a + b)*log(-((a^2 - 8*a*b
 + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a -
```

```
 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a +
 b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))*tan(f*x + e) - 4*sqrt(b*tan(f
*x + e)^2 + a)*a)/(f*tan(f*x + e)), -1/4*(4*sqrt(-b)*b*arctan(sqrt(-b)*tan(
f*x + e)/sqrt(b*tan(f*x + e)^2 + a))*tan(f*x + e) + (a - b)*sqrt(-a + b)*lo
g(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2
 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(b*tan(f*x + e)^
2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))*tan(f*x + e)
+ 4*sqrt(b*tan(f*x + e)^2 + a)*a)/(f*tan(f*x + e)), -1/2*((a - b)^(3/2)*arc
tan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f
*x + e)^2 - a))*tan(f*x + e) - b^(3/2)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*ta
n(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a)*tan(f*x + e) + 2*sqrt(b*tan(f*x
 + e)^2 + a)*a)/(f*tan(f*x + e)), -1/2*((a - b)^(3/2)*arctan(-2*sqrt(b*tan(
f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a))*ta
n(f*x + e) + 2*sqrt(-b)*b*arctan(sqrt(-b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^
2 + a))*tan(f*x + e) + 2*sqrt(b*tan(f*x + e)^2 + a)*a)/(f*tan(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^{\frac{3}{2}}\cot^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*cot(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx + e\right)^2 + a\right)^{\frac{3}{2}}\cot\left(fx + e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

3.317 $\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=115

$$\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{a\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f}+\frac{(3a-4b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f}$$

```
[Out] ((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f + ((3*a - 4*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(3*f) - (a*Cot
[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(3*f)
```

Rubi [A] time = 0.173331, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3670, 474, 583, 12, 377, 203}

$$\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{a\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f}+\frac{(3a-4b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f + ((3*a - 4*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(3*f) - (a*Cot
[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(3*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
```

```
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^4\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f} + \frac{\text{Subst}\left(\int \frac{-a(3a-4b)-(2a-3b)bx^2}{x^2\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x\right.}{3f}$$

$$= \frac{(3a-4b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f} - \frac{a\cot^3(e+fx)\sqrt{a+b\tan^2(}}{3f}$$

$$= \frac{(3a-4b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f} - \frac{a\cot^3(e+fx)\sqrt{a+b\tan^2(}}{3f}$$

$$= \frac{(3a-4b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f} - \frac{a\cot^3(e+fx)\sqrt{a+b\tan^2(}}{3f}$$

$$= \frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a-4b)\cot(e+fx)\sqrt{a+b\tan^2(e+}}{3f}$$

Mathematica [C] time = 0.289824, size = 78, normalized size = 0.68

$$-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\left(a\cot^2(e+fx)+b\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\frac{(a-b)\tan^2(e+fx)}{a+b\tan^2(e+fx)}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cot[e + f*x]*(b + a*Cot[e + f*x]^2)*Hypergeometric2F1[-3/2, 1, -1/2, -(((
a - b)*Tan[e + f*x]^2)/(a + b*Tan[e + f*x]^2))]*Sqrt[a + b*Tan[e + f*x]^2])
/(3*f)
```

Maple [C] time = 0.319, size = 6591, normalized size = 57.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

Fricas [A] time = 2.36605, size = 760, normalized size = 6.61

$$\left[-\frac{3\,(a-b)\sqrt{-a+b}\log\left(-\frac{\left(a^2-8\,ab+8\,b^2\right)\tan\left(fx+e\right)^4-2\left(3\,a^2-4\,ab\right)\tan\left(fx+e\right)^2+a^2-4\left(\left(a-2\,b\right)\tan\left(fx+e\right)^3-a\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{-a+b}}{\tan\left(fx+e\right)^4+2\,\tan\left(fx+e\right)^2+1}\right)\mathrm{t}}{12\,f\tan\left(fx+e\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(a - b)*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 -
 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 - 4*((a - 2*b)*tan(f*x + e)^3 - a*t
an(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*t
an(f*x + e)^2 + 1))*tan(f*x + e)^3 - 4*((3*a - 4*b)*tan(f*x + e)^2 - a)*sqr
t(b*tan(f*x + e)^2 + a))/(f*tan(f*x + e)^3), 1/6*(3*(a - b)^(3/2)*arctan(-2
*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e
)^2 - a))*tan(f*x + e)^3 + 2*((3*a - 4*b)*tan(f*x + e)^2 - a)*sqrt(b*tan(f*
x + e)^2 + a))/(f*tan(f*x + e)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

3.318 $\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=165

$$-\frac{\left(15a^2-20ab+3b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15af}-\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{a\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5f}$$

```
[Out] -(((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^
2]])/f) - ((15*a^2 - 20*a*b + 3*b^2)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2
])/(15*a*f) + ((5*a - 6*b)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(15*f
) - (a*Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(5*f)
```

Rubi [A] time = 0.244786, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3670, 474, 583, 12, 377, 203}

$$-\frac{\left(15a^2-20ab+3b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15af}-\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{a\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]
```


```
[Out] -(((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^
2]])/f) - ((15*a^2 - 20*a*b + 3*b^2)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2
])/(15*a*f) + ((5*a - 6*b)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(15*f
) - (a*Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(5*f)
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
```

```
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+bx^2\right)^{3/2}}{x^6\left(1+x^2\right)}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{a\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5f}+\frac{\text{Subst}\left(\int\frac{-a(5a-6b)-(4a-5b)bx^2}{x^4\left(1+x^2\right)\sqrt{a+bx^2}}\,dx,x\right.}{5f}$$

$$=\frac{(5a-6b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15f}-\frac{a\cot^5(e+fx)\sqrt{a+b\tan^2}}{5f}$$

$$=-\frac{\left(15a^2-20ab+3b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15af}+\frac{(5a-6b)\cot^3(}{}$$

$$=-\frac{\left(15a^2-20ab+3b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15af}+\frac{(5a-6b)\cot^3(}{}$$

$$=-\frac{\left(15a^2-20ab+3b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15af}+\frac{(5a-6b)\cot^3(}{}$$

$$=-\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\left(15a^2-20ab+3b^2\right)\cot(e+fx)\sqrt{a}}{15af}$$

Mathematica [C] time = 8.50924, size = 140, normalized size = 0.85

$$-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)}\left(a\cot^2(e+fx)+b\right)^2\left(2(a-b)((a-b)\cos(2(e+fx))+a+b)\text{Hyperge}\right.}{15a^3f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cos[e + f*x]*(b + a*Cot[e + f*x]^2)^2*(a*(-2*b + 3*a*Cot[e + f*x]^2)*Hype
rgeometric2F1[1, 1, -1/2, ((a - b)*Sin[e + f*x]^2)/a] + 2*(a - b)*(a + b +
(a - b)*Cos[2*(e + f*x)])*Hypergeometric2F1[2, 2, 1/2, ((a - b)*Sin[e + f*x
]^2)/a])*Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(15*a^3*f)
```

Maple [C] time = 0.426, size = 10026, normalized size = 60.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^6, x)
```

Fricas [A] time = 2.47453, size = 927, normalized size = 5.62

$$\left[-\frac{15\left(a^2-ab\right)\sqrt{-a+b}\log\left(-\frac{\left(a^2-8ab+8b^2\right)\tan\left(fx+e\right)^4-2\left(3a^2-4ab\right)\tan\left(fx+e\right)^2+a^2+4\left((a-2b)\tan\left(fx+e\right)^3-a\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{-a-}}{\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1}\right.}{60af\tan\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(a^2 - a*b)*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e
)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3
- a*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4
+ 2*tan(f*x + e)^2 + 1))*tan(f*x + e)^5 + 4*((15*a^2 - 20*a*b + 3*b^2)*tan(
f*x + e)^4 - (5*a^2 - 6*a*b)*tan(f*x + e)^2 + 3*a^2)*sqrt(b*tan(f*x + e)^2
+ a))/(a*f*tan(f*x + e)^5), -1/30*(15*(a^2 - a*b)*sqrt(a - b)*arctan(-2*sqr
t(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2
- a))*tan(f*x + e)^5 + 2*((15*a^2 - 20*a*b + 3*b^2)*tan(f*x + e)^4 - (5*a^2
 - 6*a*b)*tan(f*x + e)^2 + 3*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a*f*tan(f*x
+ e)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^6, x)
```

3.319 $\int \left(a+b\tan^2(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=170

$$\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)}}\right)}{8d}+\frac{(a-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)}}\right)}{d}+\frac{b\tan(c+dx)\left(a+b\tan^2(c+dx)\right)}{4d}$$

```
[Out] ((a - b)^(5/2)*ArcTan[(Sqrt[a - b]*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]^2]
])/d + (Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tan[c + d*x])/Sq
rt[a + b*Tan[c + d*x]^2]])/(8*d) + ((7*a - 4*b)*b*Tan[c + d*x]*Sqrt[a + b*T
an[c + d*x]^2])/(8*d) + (b*Tan[c + d*x]*(a + b*Tan[c + d*x]^2)^(3/2))/(4*d)
```

Rubi [A] time = 0.178838, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3661, 416, 528, 523, 217, 206, 377, 203}

$$\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)}}\right)}{8d}+\frac{(a-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)}}\right)}{d}+\frac{b\tan(c+dx)\left(a+b\tan^2(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^2)^(5/2),x]
```

```
[Out] ((a - b)^(5/2)*ArcTan[(Sqrt[a - b]*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]^2]
])/d + (Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tan[c + d*x])/Sq
rt[a + b*Tan[c + d*x]^2]])/(8*d) + ((7*a - 4*b)*b*Tan[c + d*x]*Sqrt[a + b*T
an[c + d*x]^2])/(8*d) + (b*Tan[c + d*x]*(a + b*Tan[c + d*x]^2)^(3/2))/(4*d)
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\tan^2(c+dx)\right)^{5/2}\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{5/2}}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}$$

$$=\frac{b\tan(c+dx)\left(a+b\tan^2(c+dx)\right)^{3/2}}{4d}+\frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}\left(a(4a-b)+(7a-4b)bx^2\right)}{1+x^2}\,dx,x,\tan(c\right.}{4d}$$

$$=\frac{(7a-4b)b\tan(c+dx)\sqrt{a+b\tan^2(c+dx)}}{8d}+\frac{b\tan(c+dx)\left(a+b\tan^2(c+dx)\right)^{3/2}}{4d}+$$

$$=\frac{(7a-4b)b\tan(c+dx)\sqrt{a+b\tan^2(c+dx)}}{8d}+\frac{b\tan(c+dx)\left(a+b\tan^2(c+dx)\right)^{3/2}}{4d}+$$

$$=\frac{(7a-4b)b\tan(c+dx)\sqrt{a+b\tan^2(c+dx)}}{8d}+\frac{b\tan(c+dx)\left(a+b\tan^2(c+dx)\right)^{3/2}}{4d}+$$

$$=\frac{(a-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)}}\right)}{d}+\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)}}\right)}{8d}+$$

Mathematica [C] time = 1.24952, size = 259, normalized size = 1.52

$$\sqrt{b}\left(15a^2-20ab+8b^2\right)\log\left(\sqrt{b}\sqrt{a+b\tan^2(c+dx)}+b\tan(c+dx)\right)+b\tan(c+dx)\sqrt{a+b\tan^2(c+dx)}\left(9a+2b\tan^2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^2)^(5/2),x]
```

```
[Out] ((-4*I)*(a - b)^(5/2)*Log[((-4*I)*(a - I*b*Tan[c + d*x] + Sqrt[a - b]*Sqrt[
a + b*Tan[c + d*x]^2]))/((a - b)^(7/2)*(I + Tan[c + d*x]))] + (4*I)*(a - b)
^(5/2)*Log[((4*I)*(a + I*b*Tan[c + d*x] + Sqrt[a - b]*Sqrt[a + b*Tan[c + d*
x]^2]))/((a - b)^(7/2)*(-I + Tan[c + d*x]))] + Sqrt[b]*(15*a^2 - 20*a*b + 8
*b^2)*Log[b*Tan[c + d*x] + Sqrt[b]*Sqrt[a + b*Tan[c + d*x]^2]] + b*Tan[c +
d*x]*Sqrt[a + b*Tan[c + d*x]^2]*(9*a - 4*b + 2*b*Tan[c + d*x]^2))/(8*d)
```

Maple [B] time = 0.059, size = 461, normalized size = 2.7

$$\frac{b^2(\tan(dx+c))^3}{4d}\sqrt{a+b(\tan(dx+c))^2}+\frac{9ab\tan(dx+c)}{8d}\sqrt{a+b(\tan(dx+c))^2}+\frac{15a^2}{8d}\sqrt{b}\ln\left(\sqrt{b}\tan(dx+c)+\sqrt{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^2)^(5/2),x)
```

```
[Out] 1/4/d*b^2*tan(d*x+c)^3*(a+b*tan(d*x+c)^2)^(1/2)+9/8/d*b*a*tan(d*x+c)*(a+b*t
an(d*x+c)^2)^(1/2)+15/8/d*b^(1/2)*a^2*ln(b^(1/2)*tan(d*x+c)+(a+b*tan(d*x+c)
^2)^(1/2))-1/2/d*b^2*tan(d*x+c)*(a+b*tan(d*x+c)^2)^(1/2)-5/2/d*b^(3/2)*a*ln
(b^(1/2)*tan(d*x+c)+(a+b*tan(d*x+c)^2)^(1/2))+1/d*b^(5/2)*ln(b^(1/2)*tan(d*
x+c)+(a+b*tan(d*x+c)^2)^(1/2))-1/d*b*(b^4*(a-b))^(1/2)/(a-b)*arctan(b^2*(a-
b)/(b^4*(a-b))^(1/2)/(a+b*tan(d*x+c)^2)^(1/2)*tan(d*x+c))+3/d*a*(b^4*(a-b))
^(1/2)/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(d*x+c)^2)^(1/2)*ta
n(d*x+c))-3/d*a^2/b*(b^4*(a-b))^(1/2)/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1
/2)/(a+b*tan(d*x+c)^2)^(1/2)*tan(d*x+c))+1/d*a^3*(b^4*(a-b))^(1/2)/b^2/(a-b
)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(d*x+c)^2)^(1/2)*tan(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan(dx+c)^2+a\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c)^2 + a)^(5/2), x)
```

Fricas [A] time = 11.1738, size = 1763, normalized size = 10.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16*((15*a^2 - 20*a*b + 8*b^2)*sqrt(b)*log(2*b*tan(d*x + c)^2 + 2*sqrt(b*
tan(d*x + c)^2 + a)*sqrt(b)*tan(d*x + c) + a) + 8*(a^2 - 2*a*b + b^2)*sqrt(
-a + b)*log(-((a - 2*b)*tan(d*x + c)^2 + 2*sqrt(b*tan(d*x + c)^2 + a)*sqrt(
-a + b)*tan(d*x + c) - a)/(tan(d*x + c)^2 + 1)) + 2*(2*b^2*tan(d*x + c)^3 +
 (9*a*b - 4*b^2)*tan(d*x + c))*sqrt(b*tan(d*x + c)^2 + a))/d, 1/16*(16*(a^2
 - 2*a*b + b^2)*sqrt(a - b)*arctan(-sqrt(b*tan(d*x + c)^2 + a)/(sqrt(a - b)
*tan(d*x + c))) + (15*a^2 - 20*a*b + 8*b^2)*sqrt(b)*log(2*b*tan(d*x + c)^2
+ 2*sqrt(b*tan(d*x + c)^2 + a)*sqrt(b)*tan(d*x + c) + a) + 2*(2*b^2*tan(d*x
 + c)^3 + (9*a*b - 4*b^2)*tan(d*x + c))*sqrt(b*tan(d*x + c)^2 + a))/d, -1/8
*((15*a^2 - 20*a*b + 8*b^2)*sqrt(-b)*arctan(sqrt(b*tan(d*x + c)^2 + a)*sqrt
(-b)/(b*tan(d*x + c))) - 4*(a^2 - 2*a*b + b^2)*sqrt(-a + b)*log(-((a - 2*b)
*tan(d*x + c)^2 + 2*sqrt(b*tan(d*x + c)^2 + a)*sqrt(-a + b)*tan(d*x + c) -
a)/(tan(d*x + c)^2 + 1)) - (2*b^2*tan(d*x + c)^3 + (9*a*b - 4*b^2)*tan(d*x
+ c))*sqrt(b*tan(d*x + c)^2 + a))/d, 1/8*(8*(a^2 - 2*a*b + b^2)*sqrt(a - b)
*arctan(-sqrt(b*tan(d*x + c)^2 + a)/(sqrt(a - b)*tan(d*x + c))) - (15*a^2 -
 20*a*b + 8*b^2)*sqrt(-b)*arctan(sqrt(b*tan(d*x + c)^2 + a)*sqrt(-b)/(b*tan
(d*x + c))) + (2*b^2*tan(d*x + c)^3 + (9*a*b - 4*b^2)*tan(d*x + c))*sqrt(b*
tan(d*x + c)^2 + a))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tan^2\left(c + dx\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**(5/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.320 $\int \frac{\tan^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=95

$$\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f}-\frac{(a+b)\sqrt{a+b\tan^2(e+fx)}}{b^2f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f)) - ((a +
b)*Sqrt[a + b*Tan[e + f*x]^2])/(b^2*f) + (a + b*Tan[e + f*x]^2)^(3/2)/(3*b^
2*f)
```

Rubi [A] time = 0.140269, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3670, 446, 88, 63, 208}

$$\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f}-\frac{(a+b)\sqrt{a+b\tan^2(e+fx)}}{b^2f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f)) - ((a +
b)*Sqrt[a + b*Tan[e + f*x]^2])/(b^2*f) + (a + b*Tan[e + f*x]^2)^(3/2)/(3*b^
2*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{-a-b}{b\sqrt{a+bx}} + \frac{1}{(1+x)\sqrt{a+bx}} + \frac{\sqrt{a+bx}}{b}\right) dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= -\frac{(a+b)\sqrt{a+b\tan^2(e+fx)}}{b^2 f} + \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2 f} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \text{tan}\ldots\right.}{2f}$$
$$= -\frac{(a+b)\sqrt{a+b\tan^2(e+fx)}}{b^2 f} + \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2 f} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\ldots}\right.}{bf}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,f} - \frac{(a+b)\sqrt{a+b\tan^2(e+fx)}}{b^2 f} + \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2 f}$$

Mathematica [A] time = 2.34322, size = 87, normalized size = 0.92

$$-\frac{\frac{2\left(2a-b\tan^2(e+fx)+3b\right)\sqrt{a+b\tan^2(e+fx)}}{3b^2} + \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((2*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/Sqrt[a - b] + (2*(2*a
 + 3*b - b*Tan[e + f*x]^2)*Sqrt[a + b*Tan[e + f*x]^2])/(3*b^2))/(2*f)
```

Maple [A] time = 0.031, size = 111, normalized size = 1.2

$$\frac{\left(\tan(fx+e)\right)^2}{3fb}\sqrt{a+b\left(\tan(fx+e)\right)^2} - \frac{2a}{3fb^2}\sqrt{a+b\left(\tan(fx+e)\right)^2} - \frac{1}{fb}\sqrt{a+b\left(\tan(fx+e)\right)^2} + \frac{1}{f}\arctan\left(\sqrt{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3/f*tan(f*x+e)^2/b*(a+b*tan(f*x+e)^2)^(1/2)-2/3/f*a/b^2*(a+b*tan(f*x+e)^2
)^(1/2)-(a+b*tan(f*x+e)^2)^(1/2)/b/f+1/f/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e
)^2)^(1/2)/(-a+b)^(1/2))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.16573, size = 717, normalized size = 7.55

$$\left[\frac{3\sqrt{a-b}b^2\log\left(-\frac{b^2\tan\left(fx+e\right)^4+2\left(4ab-3b^2\right)\tan\left(fx+e\right)^2-4\left(b\tan\left(fx+e\right)^2+2a-b\right)\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{a-b}+8a^2-8ab+b^2}{\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1}\right)+4\left(\left(ab-b^2\right)\tan\left(\right.\right.}{12\left(ab^2-b^3\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*sqrt(a - b)*b^2*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f
*x + e)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4*
((a*b - b^2)*tan(f*x + e)^2 - 2*a^2 - a*b + 3*b^2)*sqrt(b*tan(f*x + e)^2 +
a))/((a*b^2 - b^3)*f), 1/6*(3*sqrt(-a + b)*b^2*arctan(2*sqrt(b*tan(f*x + e)
^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^2 + 2*a - b)) + 2*((a*b - b^2)*tan(f*x
 + e)^2 - 2*a^2 - a*b + 3*b^2)*sqrt(b*tan(f*x + e)^2 + a))/((a*b^2 - b^3)*f
)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^5\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [A] time = 1.63544, size = 154, normalized size = 1.62

$$\frac{\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}f}+\frac{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}b^4f^2-3\sqrt{b\tan\left(fx+e\right)^2+a}ab^4f^2-3\sqrt{b\tan\left(fx+e\right)^2+a}b^5f^2}{3b^6f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-a + b)*f) + 1/3*((b*
tan(f*x + e)^2 + a)^(3/2)*b^4*f^2 - 3*sqrt(b*tan(f*x + e)^2 + a)*a*b^4*f^2
- 3*sqrt(b*tan(f*x + e)^2 + a)*b^5*f^2)/(b^6*f^3)
```

3.321 $\int \frac{\tan^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{a+b\tan^2(e+fx)}}{bf}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

```
[Out] ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f) + Sqrt[a +
b*Tan[e + f*x]^2]/(b*f)
```

Rubi [A] time = 0.111124, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3670, 446, 80, 63, 208}

$$\frac{\sqrt{a+b\tan^2(e+fx)}}{bf}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f) + Sqrt[a +
b*Tan[e + f*x]^2]/(b*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{x}{(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f} \\
&= \frac{\sqrt{a+b\tan^2(e+fx)}}{bf} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f} \\
&= \frac{\sqrt{a+b\tan^2(e+fx)}}{bf} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right)}{bf} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}f} + \frac{\sqrt{a+b\tan^2(e+fx)}}{bf}
\end{aligned}$$

Mathematica [A] time = 0.265132, size = 62, normalized size = 0.97

$$\frac{\frac{\sqrt{a+b\tan^2(e+fx)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/Sqrt[a - b] + Sqrt[a + b*T
an[e + f*x]^2]/b)/f
```

Maple [A] time = 0.029, size = 58, normalized size = 0.9

$$\frac{1}{fb}\sqrt{a+b\left(\tan\left(fx+e\right)\right)^2}-\frac{1}{f}\arctan\left(\sqrt{a+b\left(\tan\left(fx+e\right)\right)^2}\frac{1}{\sqrt{-a+b}}\right)\frac{1}{\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] (a+b*tan(f*x+e)^2)^(1/2)/b/f-1/f/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/
2)/(-a+b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^{3}}{\sqrt{b\tan\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^3/sqrt(b*tan(f*x + e)^2 + a), x)
```

Fricas [A] time = 2.1055, size = 585, normalized size = 9.14

$$\left[\frac{\sqrt{a-b}b\log\left(-\frac{b^{2}\tan\left(fx+e\right)^{4}+2\left(4ab-3b^{2}\right)\tan\left(fx+e\right)^{2}+4\left(b\tan\left(fx+e\right)^{2}+2a-b\right)\sqrt{b\tan\left(fx+e\right)^{2}+a}\sqrt{a-b}+8a^{2}-8ab+b^{2}}{\tan\left(fx+e\right)^{4}+2\tan\left(fx+e\right)^{2}+1}\right)+4\sqrt{b\tan\left(fx+e\right)^{2}+ \cdots}{4\left(ab-b^{2}\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a - b)*b*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x +
e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b
) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4*sqrt(
b*tan(f*x + e)^2 + a)*(a - b))/((a*b - b^2)*f), -1/2*(sqrt(-a + b)*b*arctan
(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^2 + 2*a - b)) -
2*sqrt(b*tan(f*x + e)^2 + a)*(a - b))/((a*b - b^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^{3}\left(e+fx\right)}{\sqrt{a+b\tan^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/sqrt(a + b*tan(e + f*x)**2), x)
```

<u>Giac</u> [A] time = 1.46386, size = 84, normalized size = 1.31

$$-\frac{\frac{b\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}f}-\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}}{f}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(b*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-a + b)*f) - sqrt
(b*tan(f*x + e)^2 + a)/f)/b
```

3.322 $\int \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=41

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f))
```

Rubi [A] time = 0.063777, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3670, 444, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]

[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right)}{bf}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}f}$$

Mathematica [A] time = 0.0274421, size = 41, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f))
```

Maple [A] time = 0.03, size = 35, normalized size = 0.9

$$\frac{1}{f}\arctan\left(\sqrt{a+b\left(\tan\left(fx+e\right)\right)^2}\frac{1}{\sqrt{-a+b}}\right)\frac{1}{\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a), x)
```

Fricas [A] time = 2.05206, size = 446, normalized size = 10.88

$$\left[\frac{\log\left(-\frac{b^2\tan\left(fx+e\right)^4+2\left(4ab-3b^2\right)\tan\left(fx+e\right)^2-4\left(b\tan\left(fx+e\right)^2+2a-b\right)\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{a-b}+8a^2-8ab+b^2}{\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1}\right)}{4\sqrt{a-b}f}, \frac{\sqrt{-a+b}\arctan\left(\frac{2\sqrt{b\tan\left(fx+e\right)}}{b\tan\left(fx+e\right)}\right)}{2(a-b)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e)^2 - 4*(b*tan
(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a
*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/(sqrt(a - b)*f), 1/2*sqr
t(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^
2 + 2*a - b))/((a - b)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [A] time = 1.40035, size = 47, normalized size = 1.15

$$\frac{\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-a + b)*f)
```

3.323 $\int \frac{\cot(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=74

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f)) + ArcTanh[Sqrt[a
 + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f)
```

Rubi [A] time = 0.108394, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3670, 446, 86, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f)) + ArcTanh[Sqrt[a
 + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right)}{bf} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right)}{bf} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}f}
\end{aligned}$$

Mathematica [A] time = 0.079435, size = 72, normalized size = 0.97

$$\frac{\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (-(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]/Sqrt[a]) + ArcTanh[Sqrt[a +
b*Tan[e + f*x]^2]/Sqrt[a - b]]/Sqrt[a - b])/f
```

Maple [B] time = 0.24, size = 496, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2/f/a^(1/2)/(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1
)^2)^(1/2)*(ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*
```

```
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e
)^2)*(a-b)^(1/2)+2*ln(4*cos(f*x+e)*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^
2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e)+4*(a-b)^(1/2)*
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*a^(1/2)-ln(-4*(
cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/
2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)
+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*(a-b)^(1/2))*sin(f*x+e)^2/((a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/(cos(f*x+e)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)/sqrt(b*tan(f*x + e)^2 + a), x)
```

Fricas [A] time = 1.76104, size = 1083, normalized size = 14.64

$$\left[\frac{\sqrt{a-b}a\log\left(\frac{b\tan\left(fx+e\right)^2+2\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{a-b}+2a-b}{\tan\left(fx+e\right)^2+1}\right)+(a-b)\sqrt{a}\log\left(\frac{b\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{a}+2a}{\tan\left(fx+e\right)^2}\right)}{2\left(a^2-ab\right)f}, \frac{2a\sqrt{-a+b}\,\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a - b)*a*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sq
rt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) + (a - b)*sqrt(a)*log((b*tan(f*x
 + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2))/((a^
2 - a*b)*f), 1/2*(2*a*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(
-a + b)/(a - b)) + (a - b)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x
 + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2))/((a^2 - a*b)*f), 1/2*(2*sqrt(-
a)*(a - b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) + sqrt(a - b)*a*lo
g((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(
tan(f*x + e)^2 + 1)))/((a^2 - a*b)*f), (sqrt(-a)*(a - b)*arctan(sqrt(b*tan(
f*x + e)^2 + a)*sqrt(-a)/a) + a*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2
+ a)*sqrt(-a + b)/(a - b)))/((a^2 - a*b)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)/sqrt(b*tan(f*x + e)^2 + a), x)
```

3.324 $\int \frac{\cot^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=116

$$\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}-\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af}$$

```
[Out] ((2*a + b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(2*a^(3/2)*f) - Arc
Tanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f) - (Cot[e + f*x
]^2*Sqrt[a + b*Tan[e + f*x]^2])/(2*a*f)
```

Rubi [A] time = 0.162675, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3670, 446, 103, 156, 63, 208}

$$\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}-\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```


```
[Out] ((2*a + b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(2*a^(3/2)*f) - Arc
Tanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f) - (Cot[e + f*x
]^2*Sqrt[a + b*Tan[e + f*x]^2])/(2*a*f)
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2a+b)+\frac{bx}{2}}{x(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2af}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f} - \frac{(2a+b)\,\text{S}}{}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right)}{bf} - \frac{(2a}{}$$

$$= \frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}f} - \frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e}}{2af}$$

Mathematica [A] time = 0.733176, size = 135, normalized size = 1.16

$$\frac{\left(2a^2-ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\left((b-a)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}-2a\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+f}}{\sqrt{a-b}}\right.\right.}{2a^{3/2}f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((2*a^2 - a*b - b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]] + Sqrt[a]*
(-2*a*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + (-a + b
```

```
)*Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2]))/(2*a^(3/2)*(a - b)*f)
```

Maple [B] time = 0.276, size = 3601, normalized size = 31.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/4/f/a^(5/2)/(a-b)^(1/2)*(-4*a^(5/2)*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*
x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(4*cos(f*x+e)*(a-b)^(1/2)*((a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+
e)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)
)+2*a^(5/2)*(a-b)^(1/2)*cos(f*x+e)^3-4*a^(5/2)*cos(f*x+e)^2*((a*cos(f*x+e)^
2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(4*cos(f*x+e)*(a-b)^(1/2)*((a
*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*
cos(f*x+e)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^
2)^(1/2))+4*a^(5/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+
e)+1)^2)^(1/2)*ln(4*cos(f*x+e)*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e)+4*(a-b)^(1/2)*((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))-2*a^(3/2)*(a-b)^(1/
2)*cos(f*x+e)^3*b-2*(a-b)^(1/2)*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+
b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(
1/2)+b)/sin(f*x+e)^2)*a^2-(a-b)^(1/2)*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)
*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x
+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/
2)*a^(1/2)+b)/sin(f*x+e)^2)*a*b+2*(a-b)^(1/2)*cos(f*x+e)^3*((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*((a*cos(f*x+e)
^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x
+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(
cos(f*x+e)-1))*a^2+(a-b)^(1/2)*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b
+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*
a*b+4*a^(5/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln
(4*cos(f*x+e)*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)
^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))-2*(a-b)^(1/2)*cos(f*x+e)^2*((a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2/a^(1/2)*(cos(f*x+e)
-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*
a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f
*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*a^2-(a-b)^(1/2)*cos(f*x+e)^2*((a
*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2/a^(1/2)*(cos(
f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^
(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/
(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*a*b+2*(a-b)^(1/2)*cos(f*x+
e)^2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(
f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+c
os(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^
2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*a^2+(a-b)^(1/2)*cos(f*x+e)^2*((a*cos(f*
x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*((a*cos(
f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*c
os(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2
```

```
)+b)/(cos(f*x+e)-1))*a*b+2*a^(3/2)*(a-b)^(1/2)*cos(f*x+e)*b+2*(a-b)^(1/2)*c
os(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2/
a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(
f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*a^2+(a-b)^(1/
2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln
(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(
cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-c
os(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2)*a*b-2*(a-
b)^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1
/2)*ln(-4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^
(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/
(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*a^2-(a-b)^(1/2)*cos(f*x+
e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*
x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos
(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)
^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*a*b+2*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*
x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e
)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*
x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1
/2)*a^(1/2)+b)/sin(f*x+e)^2)*a^2+(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2/a^(1/2)*(cos(f*x+e)-1)*(cos(f*x+e)*((a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-cos(f*x+e)*a+
b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(
1/2)+b)/sin(f*x+e)^2)*a*b-2*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/
(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b
)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^
2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*a^2-
(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-
4*(cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^
(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x
+e)+1)^2)^(1/2)*a^(1/2)+b)/(cos(f*x+e)-1))*a*b)*sin(f*x+e)^2/(cos(f*x+e)-1)
^2/cos(f*x+e)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/(cos(f
*x+e)+1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(fx+e)^3}{\sqrt{b\tan(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^3/sqrt(b*tan(f*x + e)^2 + a), x)
```

Fricas [A] time = 1.83604, size = 1650, normalized size = 14.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(a - b)*a^2*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a
)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + (2*a^2 - a*
b - b^2)*sqrt(a)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*(a^
2 - a*b))/((a^3 - a^2*b)*f*tan(f*x + e)^2), -1/4*(4*a^2*sqrt(-a + b)*arctan
(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b))*tan(f*x + e)^2 - (2*a^2
- a*b - b^2)*sqrt(a)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*s
qrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)
*(a^2 - a*b))/((a^3 - a^2*b)*f*tan(f*x + e)^2), 1/2*(sqrt(a - b)*a^2*log((b
*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(
f*x + e)^2 + 1))*tan(f*x + e)^2 - (2*a^2 - a*b - b^2)*sqrt(-a)*arctan(sqrt(
b*tan(f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^2 - sqrt(b*tan(f*x + e)^2 +
a)*(a^2 - a*b))/((a^3 - a^2*b)*f*tan(f*x + e)^2), -1/2*(2*a^2*sqrt(-a + b)*
arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b))*tan(f*x + e)^2 + (
2*a^2 - a*b - b^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a)*t
an(f*x + e)^2 + sqrt(b*tan(f*x + e)^2 + a)*(a^2 - a*b))/((a^3 - a^2*b)*f*ta
n(f*x + e)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(fx+e)^3}{\sqrt{b\tan(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^3/sqrt(b*tan(f*x + e)^2 + a), x)
```

3.325 $\int \frac{\cot^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=166

$$-\frac{\left(8a^2+4ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f}+\frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8a^2f}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}-$$

```
[Out] -((8*a^2 + 4*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(8*a
^(5/2)*f) + ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f)
 + ((4*a + 3*b)*Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2])/(8*a^2*f) - (Cot
[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2])/(4*a*f)
```

Rubi [A] time = 0.213681, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3670, 446, 103, 151, 156, 63, 208}

$$-\frac{\left(8a^2+4ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f}+\frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8a^2f}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```


```
[Out] -((8*a^2 + 4*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(8*a
^(5/2)*f) + ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f)
 + ((4*a + 3*b)*Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2])/(8*a^2*f) - (Cot
[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2])/(4*a*f)
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
```

```
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^5(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= -\frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4af} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4a+3b)+\frac{3bx}{2}}{x^2(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{4af}$$
$$= \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8a^2f} - \frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4af} + \frac{\text{Subst}\left(\right.}{}$$
$$= \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8a^2f} - \frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4af} - \frac{\text{Subst}\left(\right.}{}$$
$$= \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8a^2f} - \frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4af} - \frac{\text{Subst}\left(\right.}{}$$
$$= -\frac{(8a^2+4ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}f} + \frac{(4a+3b)\cot^2(e}{}$$

Mathematica [A] time = 1.87309, size = 162, normalized size = 0.98

$$\frac{(4a^2b-8a^3+ab^2+3b^3)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\left(8a^2\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+(b-a)\cot^2(e+fx)\sqrt{a+b}\right.}{8a^{5/2}f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((-8*a^3 + 4*a^2*b + a*b^2 + 3*b^3)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt
[a]] + Sqrt[a]*(8*a^2*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a
 - b]] + (-a + b)*Cot[e + f*x]^2*(-4*a - 3*b + 2*a*Cot[e + f*x]^2)*Sqrt[a +
 b*Tan[e + f*x]^2]))/(8*a^(5/2)*(a - b)*f)
```

Maple [B] time = 0.266, size = 7641, normalized size = 46.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.94811, size = 1994, normalized size = 12.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*sqrt(a - b)*a^3*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 +
a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 + (8*a^3 - 4
*a^2*b - a*b^2 - 3*b^3)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x +
e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^4 - 2*(2*a^3 - 2*a^2*
b - (4*a^3 - a^2*b - 3*a*b^2)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/(
(a^4 - a^3*b)*f*tan(f*x + e)^4), 1/16*(16*a^3*sqrt(-a + b)*arctan(-sqrt(b*t
an(f*x + e)^2 + a)*sqrt(-a + b)/(a - b))*tan(f*x + e)^4 + (8*a^3 - 4*a^2*b
- a*b^2 - 3*b^3)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 +
a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^4 - 2*(2*a^3 - 2*a^2*b - (4*
a^3 - a^2*b - 3*a*b^2)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4 -
a^3*b)*f*tan(f*x + e)^4), 1/8*(4*sqrt(a - b)*a^3*log((b*tan(f*x + e)^2 + 2*
sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan
(f*x + e)^4 + (8*a^3 - 4*a^2*b - a*b^2 - 3*b^3)*sqrt(-a)*arctan(sqrt(b*tan(
f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^4 - (2*a^3 - 2*a^2*b - (4*a^3 - a^
2*b - 3*a*b^2)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4 - a^3*b)*f
*tan(f*x + e)^4), 1/8*(8*a^3*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a
)*sqrt(-a + b)/(a - b))*tan(f*x + e)^4 + (8*a^3 - 4*a^2*b - a*b^2 - 3*b^3)*
sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^4 - (2*
a^3 - 2*a^2*b - (4*a^3 - a^2*b - 3*a*b^2)*tan(f*x + e)^2)*sqrt(b*tan(f*x +
e)^2 + a))/((a^4 - a^3*b)*f*tan(f*x + e)^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^5\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^5}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^5/sqrt(b*tan(f*x + e)^2 + a), x)
```

3.326 $\int \frac{\tan^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=177

$$\frac{(3a^2+4ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8b^{5/2}f}-\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8b^2f}+\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4bf}$$

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b
]*f)) + ((3*a^2 + 4*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*
Tan[e + f*x]^2]])/(8*b^(5/2)*f) - ((3*a + 4*b)*Tan[e + f*x]*Sqrt[a + b*Tan[
e + f*x]^2])/(8*b^2*f) + (Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(4*b*f
)
```

Rubi [A] time = 0.222457, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3670, 479, 582, 523, 217, 206, 377, 203}

$$\frac{(3a^2+4ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8b^{5/2}f}-\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8b^2f}+\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```


```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b
]*f)) + ((3*a^2 + 4*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*
Tan[e + f*x]^2]])/(8*b^(5/2)*f) - ((3*a + 4*b)*Tan[e + f*x]*Sqrt[a + b*Tan[
e + f*x]^2])/(8*b^2*f) + (Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(4*b*f
)
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4bf} - \frac{\text{Subst}\left(\int \frac{x^2(3a+(3a+4b)x^2)}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{4bf}$$

$$= -\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8b^2f} + \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4bf} + \frac{\text{Sub}}{}$$

$$= -\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8b^2f} + \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4bf} - \frac{\text{Sub}}{}$$

$$= -\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8b^2f} + \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4bf} - \frac{\text{Sub}}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}f} + \frac{(3a^2+4ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8b^{5/2}f} - \frac{(3a+4b)\tan(}{}$$

Mathematica [C] time = 6.29817, size = 768, normalized size = 4.34

$$\frac{16b^3\sqrt{\cos(2(e+fx))+1}\sqrt{\frac{(a-b)\cos(2(e+fx))+a+b}{\cos(2(e+fx))+1}}\left(\frac{\sin^4(e+fx)\csc(2(e+fx))\sqrt{-\frac{a\cot^2(e+fx)}{b}}\sqrt{-\frac{a(\cos(2(e+fx))+1)\csc^2(e+fx)}{b}}\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}\text{EllipticF}\left(\right.}{4a\sqrt{\cos(2(e+fx))+1}\sqrt{(a-b)\cos(2(e+fx))+a+b}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (-((b*(3*a^2 + 4*a*b + 4*b^2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 +
Cos[2*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e +
f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e +
 f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*
(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a
- b)*Cos[2*(e + f*x)]))) + (16*b^3*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b +
 (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]
^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b +
 (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[Ar
cSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]],
1]*Sin[e + f*x]^4)/(4*a*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos
[2*(e + f*x)]]) - (Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e +
f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e +
 f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b +
 (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/
(2*(a - b)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]
])))/Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])/(4*b^2*f) + (Sqrt[(a + b + a*C
os[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((-3*Sec[e +
f*x]*(a*Sin[e + f*x] + 2*b*Sin[e + f*x]))/(8*b^2) + (Sec[e + f*x]^2*Tan[e +
```

```
f*x])/(4*b)))/f
```

Maple [A] time = 0.032, size = 261, normalized size = 1.5

$$\frac{\left(\tan\left(fx+e\right)\right)^{3}}{4fb}\sqrt{a+b\left(\tan\left(fx+e\right)\right)^{2}}-\frac{3a\tan\left(fx+e\right)}{8fb^{2}}\sqrt{a+b\left(\tan\left(fx+e\right)\right)^{2}}+\frac{3a^{2}}{8f}\ln\left(\sqrt{b}\tan\left(fx+e\right)+\sqrt{a+b\left(\tan\left(fx+e\right)\right)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/4*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3/b/f-3/8/f*a/b^2*tan(f*x+e)*(a+b*t
an(f*x+e)^2)^(1/2)+3/8/f*a^2/b^(5/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^
2)^(1/2))-1/2*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f+1/2/f*a/b^(3/2)*ln(b^
(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))+1/f*ln(b^(1/2)*tan(f*x+e)+(a+b*t
an(f*x+e)^2)^(1/2))/b^(1/2)-1/f*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b
)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 15.2032, size = 1918, normalized size = 10.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*sqrt(-a + b)*b^3*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*
x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - (3*a^3
 + a^2*b + 4*a*b^2 - 8*b^3)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f
*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) - 2*(2*(a*b^2 - b^3)*tan(f*x + e)^
3 - (3*a^2*b + a*b^2 - 4*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a
*b^3 - b^4)*f), -1/8*(4*sqrt(-a + b)*b^3*log(-((a - 2*b)*tan(f*x + e)^2 + 2
*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2
+ 1)) + (3*a^3 + a^2*b + 4*a*b^2 - 8*b^3)*sqrt(-b)*arctan(sqrt(b*tan(f*x +
e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) - (2*(a*b^2 - b^3)*tan(f*x + e)^3 - (3
*a^2*b + a*b^2 - 4*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a*b^3 -
 b^4)*f), -1/16*(16*sqrt(a - b)*b^3*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqr
t(a - b)*tan(f*x + e))) - (3*a^3 + a^2*b + 4*a*b^2 - 8*b^3)*sqrt(b)*log(2*b
*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) -
2*(2*(a*b^2 - b^3)*tan(f*x + e)^3 - (3*a^2*b + a*b^2 - 4*b^3)*tan(f*x + e))
*sqrt(b*tan(f*x + e)^2 + a))/((a*b^3 - b^4)*f), -1/8*(8*sqrt(a - b)*b^3*arc
```

```
tan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + (3*a^3 + a^2*
b + 4*a*b^2 - 8*b^3)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b
*tan(f*x + e))) - (2*(a*b^2 - b^3)*tan(f*x + e)^3 - (3*a^2*b + a*b^2 - 4*b^
3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a*b^3 - b^4)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**6/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(fx+e)^6}{\sqrt{b\tan(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^6/sqrt(b*tan(f*x + e)^2 + a), x)
```

3.327 $\int \frac{\tan^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=125

$$-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2b^{3/2}f}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2bf}$$

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]*
f) - ((a + 2*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])
/(2*b^(3/2)*f) + (Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*b*f)
```

Rubi [A] time = 0.139177, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3670, 479, 523, 217, 206, 377, 203}

$$-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2b^{3/2}f}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]*
f) - ((a + 2*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])
/(2*b^(3/2)*f) + (Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*b*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2bf} - \frac{\text{Subst}\left(\int \frac{a+(a+2b)x^2}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{2bf}$$

$$= \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2bf} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} - \frac{(a+2b}{}$$

$$= \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2bf} + \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{(a+2b)}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}f} - \frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\tan(e+fx)\sqrt{a+b\tan^2(e}{2bf}$$

Mathematica [C] time = 6.23047, size = 713, normalized size = 5.7

$$\frac{\tan(e+fx)\sqrt{\frac{a\cos(2(e+fx))+a-b\cos(2(e+fx))+b}{\cos(2(e+fx))+1}}}{2bf} - \frac{4b^2\sqrt{\cos(2(e+fx))+1}\sqrt{\frac{(a-b)\cos(2(e+fx))+a+b}{\cos(2(e+fx))+1}}\left(\frac{\sin^4(e+fx)\csc(2(e+fx))\sqrt{-\frac{a\cot^2(e+fx)}{b}}\sqrt{-\frac{a(\cos(2(e+}{}}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((-((b*(a + b)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x
)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e +
 f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Cs
c[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Cs
c[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a - b)*Cos[2*(e
+ f*x)]))) + (4*b^2*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b + (a - b)*Cos[2*
(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-(
(a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*
(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a +
 b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]
^4)/(4*a*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])
 - (Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e +
f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc
[2*(e + f*x)]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*
(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(2*(a - b)*Sqrt
[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])))/Sqrt[a + b
 + (a - b)*Cos[2*(e + f*x)]])/(b*f)) + (Sqrt[(a + b + a*Cos[2*(e + f*x)] -
b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*Tan[e + f*x])/(2*b*f)
```

Maple [A] time = 0.03, size = 165, normalized size = 1.3

$$\frac{\tan\left(fx+e\right)}{2fb}\sqrt{a+b\left(\tan\left(fx+e\right)\right)^{2}}-\frac{a}{2f}\ln\left(\sqrt{b}\tan\left(fx+e\right)+\sqrt{a+b\left(\tan\left(fx+e\right)\right)^{2}}\right)b^{-\frac{3}{2}}-\frac{1}{f}\ln\left(\sqrt{b}\tan\left(fx+e\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f-1/2/f*a/b^(3/2)*ln(b^(1/2)*tan(
f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-1/f*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^
2)^(1/2))/b^(1/2)+1/f*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-
b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^{4}}{\sqrt{b\tan\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)
```

Fricas [A] time = 8.43644, size = 1569, normalized size = 12.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-a + b)*b^2*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x
 + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - (a^2 +
a*b - 2*b^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*
sqrt(b)*tan(f*x + e) + a) - 2*sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan(f*
x + e))/((a*b^2 - b^3)*f), -1/2*(sqrt(-a + b)*b^2*log(-((a - 2*b)*tan(f*x +
 e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*
x + e)^2 + 1)) - (a^2 + a*b - 2*b^2)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2
+ a)*sqrt(-b)/(b*tan(f*x + e))) - sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*ta
n(f*x + e))/((a*b^2 - b^3)*f), 1/4*(4*sqrt(a - b)*b^2*arctan(-sqrt(b*tan(f*
x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + (a^2 + a*b - 2*b^2)*sqrt(b)*log
(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a
) + 2*sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan(f*x + e))/((a*b^2 - b^3)*f
), 1/2*(2*sqrt(a - b)*b^2*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*t
an(f*x + e))) + (a^2 + a*b - 2*b^2)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 +
 a)*sqrt(-b)/(b*tan(f*x + e))) + sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan
(f*x + e))/((a*b^2 - b^3)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^4}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)
```

3.328 $\int \frac{\tan^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=86

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}f}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b
]*f)) + ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[b]
*f)
```

Rubi [A] time = 0.104391, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3670, 483, 217, 206, 377, 203}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}f}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b
]*f)) + ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[b]
*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}f}
\end{aligned}$$

Mathematica [C] time = 0.730864, size = 149, normalized size = 1.73

$$\frac{a\sin(2(e+fx))\csc^2(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\,\Pi\left(-\frac{b}{a-b};\sin^{-1}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right.\right.}{2bf(a-b)\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (a*Csc[e + f*x]^2*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Co
s[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sqrt[(a + b + (a - b)*Cos[2
*(e + f*x)])*Sec[e + f*x]^2]*Sin[2*(e + f*x)])/(2*(a - b)*b*f*Sqrt[((a + b
+ (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b])
```

Maple [A] time = 0.027, size = 102, normalized size = 1.2

$$\frac{1}{f}\ln\left(\sqrt{b}\tan\left(fx+e\right)+\sqrt{a+b\left(\tan\left(fx+e\right)\right)^2}\right)\frac{1}{\sqrt{b}} - \frac{1}{fb^2(a-b)}\sqrt{b^4(a-b)}\arctan\left((a-b)b^2\tan\left(fx+e\right)\frac{1}{\sqrt{b^4(a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))/b^(1/2)-1/f*(b^4*(a-b))
^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2
)*tan(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^2}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

Fricas [A] time = 2.80032, size = 1179, normalized size = 13.71

$$\left[\frac{(a-b)\sqrt{b}\log\left(2\,b\tan\left(fx+e\right)^2+2\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{b}\tan\left(fx+e\right)+a\right)-\sqrt{-a+b}b\log\left(-\frac{(a-2\,b)\tan\left(fx+e\right)^2+2\sqrt{b\tan\ldots}}{\tan(\ldots}\right.}{2\left(ab-b^2\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a - b)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)
*sqrt(b)*tan(f*x + e) + a) - sqrt(-a + b)*b*log(-((a - 2*b)*tan(f*x + e)^2
+ 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)
^2 + 1)))/((a*b - b^2)*f), -1/2*(2*(a - b)*sqrt(-b)*arctan(sqrt(b*tan(f*x +
 e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + sqrt(-a + b)*b*log(-((a - 2*b)*tan(
f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(t
an(f*x + e)^2 + 1)))/((a*b - b^2)*f), -1/2*(2*sqrt(a - b)*b*arctan(-sqrt(b*
tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (a - b)*sqrt(b)*log(2*b*t
an(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a))/((a
*b - b^2)*f), -(sqrt(a - b)*b*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a -
b)*tan(f*x + e))) + (a - b)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt
(-b)/(b*tan(f*x + e))))/((a*b - b^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^2}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

3.329 $\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]*
f)
```

Rubi [A] time = 0.0307983, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3661, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]*
f)
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}f}$$

Mathematica [A] time = 0.0679673, size = 46, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]*
f)
```

Maple [A] time = 0., size = 67, normalized size = 1.5

$$\frac{1}{fb^2(a-b)}\sqrt{b^4(a-b)}\arctan\left((a-b)b^2\tan(fx+e)\frac{1}{\sqrt{b^4(a-b)}}\frac{1}{\sqrt{a+b\left(\tan(fx+e)\right)^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan
(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97741, size = 311, normalized size = 6.76

$$\left[-\frac{\sqrt{-a+b}\log\left(-\frac{(a-2\,b)\tan\left(fx+e\right)^2-2\,\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{-a+b}\tan\left(fx+e\right)-a}{\tan\left(fx+e\right)^2+1}\right)}{2\,(a-b)f},\ \frac{\arctan\left(-\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{a-b}\tan\left(fx+e\right)}\right)}{\sqrt{a-b}f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2
 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1))/((a - b)*f), arc
tan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e)))/(sqrt(a - b)*f)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*tan(f*x + e)^2 + a), x)
```

3.330 $\int \frac{\cot^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af}$$

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b
]*f)) - (Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(a*f)
```

Rubi [A] time = 0.119084, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3670, 480, 12, 377, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b
]*f)) - (Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(a*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af} - \frac{\text{Subst}\left(\int \frac{a}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{af} \\
&= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af} - \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}f} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af}
\end{aligned}
$$

Mathematica [C] time = 9.34255, size = 212, normalized size = 2.72

$$
-\frac{\cos^2(e+fx)\cot(e+fx)\left(\frac{b\tan^2(e+fx)}{a}+1\right)\left(\frac{4\sin^2(e+fx)\left(a^2+ab\left(\tan^2(e+fx)-1\right)-b^2\tan^2(e+fx)\right)\text{Hypergeometric2F1}\left(2,2,\frac{5}{2},\frac{(a-b)\sin^2(e+fx)}{a}\right)}{3a^2}+\right.}{f\sqrt{a+b\tan^2(e+fx)}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Cos[e + f*x]^2*Cot[e + f*x]*(1 + (b*Tan[e + f*x]^2)/a)*((4*Hypergeometri
c2F1[2, 2, 5/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*(a^2 - b^2*Tan[e
 + f*x]^2 + a*b*(-1 + Tan[e + f*x]^2)))/(3*a^2) + (ArcSin[Sqrt[((a - b)*Sin
[e + f*x]^2)/a]]*(a + 2*b*Tan[e + f*x]^2))/(a*Sqrt[(Cos[e + f*x]^2*Sin[e +
f*x]^2*(a^2 - b^2*Tan[e + f*x]^2 + a*b*(-1 + Tan[e + f*x]^2)))/a^2])))/(f*S
qrt[a + b*Tan[e + f*x]^2]))
```

Maple [C] time = 0.307, size = 1195, normalized size = 15.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/a*(EllipticF((cos(f*x+e)-1)*
((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1
/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*2^(1/2)*cos(f*x+
e)*sin(f*x+e)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+
cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1
/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x
+e)+1))^(1/2)*a-2*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b
)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(
a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*2^(1/
2)*cos(f*x+e)*sin(f*x+e)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(
a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(
f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-
b)/(cos(f*x+e)+1))^(1/2)*a+2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I
*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2
/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*c
os(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(
a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)
*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*sin(f*x+e)-2*2^(1/2)*(1/a*(I*
cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x
+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1
/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*Elliptic
Pi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(
2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)
/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*sin(f*x+e)+cos(f*x+e)^2*((2*I
*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a-cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/
2)+a-2*b)/a)^(1/2)*b+((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b)/cos(f*x+e
)/sin(f*x+e)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^2}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

Fricas [A] time = 2.80364, size = 701, normalized size = 8.99

$$\left[-\frac{a\sqrt{-a+b}\log\left(-\frac{\left(a^2-8\,ab+8\,b^2\right)\tan\left(fx+e\right)^4-2\left(3\,a^2-4\,ab\right)\tan\left(fx+e\right)^2+a^2+4\left(\left(a-2\,b\right)\tan\left(fx+e\right)^3-a\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{-a+b}}{\tan\left(fx+e\right)^4+2\,\tan\left(fx+e\right)^2+1}\right)\tan}{4\left(a^2-ab\right)f\tan\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(a*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2
 - 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x +
e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x +
e)^2 + 1))*tan(f*x + e) + 4*sqrt(b*tan(f*x + e)^2 + a)*(a - b))/((a^2 - a*b
)*f*tan(f*x + e)), -1/2*(sqrt(a - b)*a*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)
*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a))*tan(f*x + e) + 2*
sqrt(b*tan(f*x + e)^2 + a)*(a - b))/((a^2 - a*b)*f*tan(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^2}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

3.331 $\int \frac{\cot^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=120

$$\frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af}$$

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]*
f) + ((3*a + 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(3*a^2*f) - (Cot
[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(3*a*f)
```

Rubi [A] time = 0.164511, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3670, 480, 583, 12, 377, 203}

$$\frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]*
f) + ((3*a + 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(3*a^2*f) - (Cot
[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(3*a*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
```

```
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af} + \frac{\text{Subst}\left(\int \frac{-3a-2b-2bx^2}{x^2(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{3af}$$

$$= \frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af} - \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}f} + \frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f} - \frac{\cot^3(e+fx)\sqrt{a+b}}{3af}$$

Mathematica [C] time = 11.0362, size = 263, normalized size = 2.19

$$-\frac{\cos^2(e+fx)\cot^3(e+fx)\left(\frac{b\tan^2(e+fx)}{a}+1\right)\left(-8(a-b)\sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2 \text{HypergeometricPFQ}\left(\{2,2,\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(Cos[e + f*x]^2*Cot[e + f*x]^3*(1 + (b*Tan[e + f*x]^2)/a)*(-12*b*(-a + b)*
(-a - b + (-a + b)*Cos[2*(e + f*x)])*Hypergeometric2F1[2, 2, 5/2, ((a - b)*
Sin[e + f*x]^2)/a]*Tan[e + f*x]^4 - 8*(a - b)*HypergeometricPFQ[{2, 2, 2},
{1, 5/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)
^2 + (6*a*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*(a^2 - 4*a*b*Tan[e + f*x
]^2 - 8*b^2*Tan[e + f*x]^4))/Sqrt[((a - b)*Sin[2*(e + f*x)]^2*(a + b*Tan[e
+ f*x]^2))/a^2]))/(9*a^3*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Maple [C] time = 0.369, size = 2433, normalized size = 20.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3/f/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/a^2*(3*cos(f*x+e)^3*sin(f*
x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+c
os(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/
2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+
e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(
1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a
*b+8*b^2)/a^2)^(1/2))*a^2-6*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x
+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/
(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-
b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos
(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(
1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*
b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a^2+3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*
(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b
*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2
)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*
EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+
e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2
)^(1/2))*a^2-6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(
a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1
))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(
f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((
2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1
/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)/a)^(1/2))*a^2-3*cos(f*x+e)*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x
+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/
(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-
b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(
f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2
)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^2+6*
cos(f*x+e)*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1
/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I
*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*
x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2
*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^
(1/2))*a^2-3*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)
^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+
e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(
cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2
*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a
```

```
+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^2*sin(f*x+e)+6*2^(1/2)*(1/a*(I*cos(f*x+e)*b
^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(
f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1
/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+
e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*
(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/
2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a^2*sin(f*x+e)+4*cos(f*x+e)^4*((2*I*b^(1/2)
*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2-2*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a
-2*b)/a)^(1/2)*a*b-2*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)
*b^2-3*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2+5*cos(f*x
+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b+4*cos(f*x+e)^2*((2*I*b^
(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2-3*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)
^(1/2)*a*b-2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2)/cos(f*x+e)/sin(
f*x+e)^3/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.82968, size = 844, normalized size = 7.03

$$\left[-\frac{3\,a^2\sqrt{-a+b}\log\left(-\frac{\left(a^2-8\,ab+8\,b^2\right)\tan\left(fx+e\right)^4-2\left(3\,a^2-4\,ab\right)\tan\left(fx+e\right)^2+a^2-4\left(\left(a-2\,b\right)\tan\left(fx+e\right)^3-a\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{-a+b}}{\tan\left(fx+e\right)^4+2\,\tan\left(fx+e\right)^2+1}\right)\tan\left(\right.}{12\left(a^3-a^2b\right)f\tan\left(fx+e\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*a^2*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(
3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 - 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f
*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f
*x + e)^2 + 1))*tan(f*x + e)^3 - 4*((3*a^2 - a*b - 2*b^2)*tan(f*x + e)^2 -
a^2 + a*b)*sqrt(b*tan(f*x + e)^2 + a))/((a^3 - a^2*b)*f*tan(f*x + e)^3), 1/
6*(3*sqrt(a - b)*a^2*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f
*x + e)/((a - 2*b)*tan(f*x + e)^2 - a))*tan(f*x + e)^3 + 2*((3*a^2 - a*b -
2*b^2)*tan(f*x + e)^2 - a^2 + a*b)*sqrt(b*tan(f*x + e)^2 + a))/((a^3 - a^2*
b)*f*tan(f*x + e)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^4}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)
```

3.332 $\int \frac{\cot^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=170

$$-\frac{\left(15a^2+10ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f}+\frac{(5a+4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e}{\sqrt{a+b\tan^2(}}\right.}{f\sqrt{a-b}}$$

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b
]*f)) - ((15*a^2 + 10*a*b + 8*b^2)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])
/(15*a^3*f) + ((5*a + 4*b)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(15*a
^2*f) - (Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(5*a*f)
```

Rubi [A] time = 0.245944, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3670, 480, 583, 12, 377, 203}

$$-\frac{\left(15a^2+10ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f}+\frac{(5a+4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e}{\sqrt{a+b\tan^2(}}\right.}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b
]*f)) - ((15*a^2 + 10*a*b + 8*b^2)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])
/(15*a^3*f) + ((5*a + 4*b)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(15*a
^2*f) - (Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(5*a*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
```

```
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5af} + \frac{\text{Subst}\left(\int \frac{-5a-4b-4bx^2}{x^4(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{5af}$$

$$= \frac{(5a+4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f} - \frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5af} - \frac{\text{Subs}}{}$$

$$= -\frac{(15a^2+10ab+8b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f} + \frac{(5a+4b)\cot^3(e+fx)\sqrt{a+b}}{15a^2f}$$

$$= -\frac{(15a^2+10ab+8b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f} + \frac{(5a+4b)\cot^3(e+fx)\sqrt{a+b}}{15a^2f}$$

$$= -\frac{(15a^2+10ab+8b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f} + \frac{(5a+4b)\cot^3(e+fx)\sqrt{a+b}}{15a^2f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}f} - \frac{(15a^2+10ab+8b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f} + \frac{(5a}{}$$

Mathematica [C] time = 16.286, size = 794, normalized size = 4.67

$$\frac{b\sin^4(e+fx)\csc(2(e+fx))\sqrt{\frac{(a-b)\cos(2(e+fx))+a+b}{\cos(2(e+fx))+1}}\sqrt{-\frac{a\cot^2(e+fx)}{b}}\sqrt{-\frac{a(\cos(2(e+fx))+1)\csc^2(e+fx)}{b}}\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))}{b}}}{af((a-b)\cos(2(e+fx))+a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x
)])]*(((-23*a^2*Cos[e + f*x] - 14*a*b*Cos[e + f*x] - 8*b^2*Cos[e + f*x])*Cs
c[e + f*x])/(15*a^3) + ((11*a*Cos[e + f*x] + 4*b*Cos[e + f*x])*Csc[e + f*x]
^3)/(15*a^2) - (Cot[e + f*x]*Csc[e + f*x]^4)/(5*a)))/f + (b*Sqrt[(a + b + (
a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/
b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a
- b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin
[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*S
in[e + f*x]^4)/(a*f*(a + b + (a - b)*Cos[2*(e + f*x)])) + (4*b*Sqrt[1 + Cos
[2*(e + f*x)]]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)
])]*((Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e
+ f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*C
sc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*C
sc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(4*a*Sqrt[1 + Cos[2*(e + f*x
)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]) - (Sqrt[-((a*Cot[e + f*x]^2)/b)
]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a -
b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticPi[-(b/(a
- b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/S
qrt[2]], 1]*Sin[e + f*x]^4)/(2*(a - b)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a +
b + (a - b)*Cos[2*(e + f*x)]]))))/(f*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])
```

Maple [C] time = 0.346, size = 3741, normalized size = 22.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/15/f/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/a^3*(23*((2*I*b^(1/2)*(a-
b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^6*a^3-30*a^3*2^(1/2)*(1/a*(I*cos(f*x+e)
*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(co
s(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^
(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*
x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2
)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(
1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*sin(f*x+e)+15*2^(1/2)*(1/a*(I*cos(f*x+e)*
b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos
(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(
1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+
e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a
-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^3*sin(f*
x+e)+8*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^3+15*((2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a^3-8*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/
a)^(1/2)*cos(f*x+e)^6*b^3+24*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(
f*x+e)^4*b^3-24*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*b^3-
35*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^4*a^3+15*((2*I*b^(1
/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2*b+10*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a
)^(1/2)*a*b^2-9*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^6*a^2*
b-6*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^6*a*b^2+34*((2*I*b
^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^4*a^2*b+22*((2*I*b^(1/2)*(a-b
)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^4*a*b^2-40*((2*I*b^(1/2)*(a-b)^(1/2)+a-2
*b)/a)^(1/2)*cos(f*x+e)^2*a^2*b-26*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2
```

)*cos(f*x+e)^2*a*b^2+60*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^2*sin(f*x+e)*a^3+15*cos(f*x+e)*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^3-30*cos(f*x+e)*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a^3+15*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^5*sin(f*x+e)*a^3-30*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^5*sin(f*x+e)*a^3+15*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^4*sin(f*x+e)*a^3-30*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^4*sin(f*x+e)*a^3-30*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^3*sin(f*x+e)*a^3+60*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^3*sin(f*x+e)*a^3-30*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2

```
)^(1/2))*cos(f*x+e)^2*sin(f*x+e)*a^3)/cos(f*x+e)/sin(f*x+e)^5/((a*cos(f*x+e
)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.85725, size = 1022, normalized size = 6.01

$$\left[-\frac{15\,a^{3}\sqrt{-a+b}\log\left(-\frac{\left(a^{2}-8\,ab+8\,b^{2}\right)\tan\left(fx+e\right)^{4}-2\left(3\,a^{2}-4\,ab\right)\tan\left(fx+e\right)^{2}+a^{2}+4\left(\left(a-2\,b\right)\tan\left(fx+e\right)^{3}-a\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^{2}+a}\sqrt{-a+b}}{\tan\left(fx+e\right)^{4}+2\,\tan\left(fx+e\right)^{2}+1}\right)\tan}{60\left(a^{4}-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*a^3*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*
(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(
f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(
f*x + e)^2 + 1))*tan(f*x + e)^5 + 4*((15*a^3 - 5*a^2*b - 2*a*b^2 - 8*b^3)*t
an(f*x + e)^4 + 3*a^3 - 3*a^2*b - (5*a^3 - a^2*b - 4*a*b^2)*tan(f*x + e)^2)
*sqrt(b*tan(f*x + e)^2 + a))/((a^4 - a^3*b)*f*tan(f*x + e)^5), -1/30*(15*sq
rt(a - b)*a^3*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)
/((a - 2*b)*tan(f*x + e)^2 - a))*tan(f*x + e)^5 + 2*((15*a^3 - 5*a^2*b - 2*
a*b^2 - 8*b^3)*tan(f*x + e)^4 + 3*a^3 - 3*a^2*b - (5*a^3 - a^2*b - 4*a*b^2)
*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4 - a^3*b)*f*tan(f*x + e)^
5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^{6}\left(e+fx\right)}{\sqrt{a+b\tan^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**6/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^{6}}{\sqrt{b\tan\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^6/sqrt(b*tan(f*x + e)^2 + a), x)
```

3.333 $\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{a^2}{b^2f(a-b)\sqrt{a+b\tan^2(e+fx)}}+\frac{\sqrt{a+b\tan^2(e+fx)}}{b^2f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f)) + a^2/
((a - b)*b^2*f*Sqrt[a + b*Tan[e + f*x]^2]) + Sqrt[a + b*Tan[e + f*x]^2]/(b^
2*f)
```

Rubi [A] time = 0.169312, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3670, 446, 87, 63, 208}

$$\frac{a^2}{b^2f(a-b)\sqrt{a+b\tan^2(e+fx)}}+\frac{\sqrt{a+b\tan^2(e+fx)}}{b^2f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f)) + a^2/
((a - b)*b^2*f*Sqrt[a + b*Tan[e + f*x]^2]) + Sqrt[a + b*Tan[e + f*x]^2]/(b^
2*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx,x,\tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)(a+bx)^{3/2}}\,dx,x,\tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a^2}{(a-b)b(a+bx)^{3/2}}+\frac{1}{b\sqrt{a+bx}}+\frac{1}{(a-b)(1+x)\sqrt{a+bx}}\right)dx,x,\tan^2(e+fx)\right)}{2f}$$
$$= \frac{a^2}{(a-b)b^2f\sqrt{a+b\tan^2(e+fx)}}+\frac{\sqrt{a+b\tan^2(e+fx)}}{b^2f}+\frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx,x,\text{t}\right.}{2(a-b)f}$$
$$= \frac{a^2}{(a-b)b^2f\sqrt{a+b\tan^2(e+fx)}}+\frac{\sqrt{a+b\tan^2(e+fx)}}{b^2f}+\frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a}\right.}{(a-b)bf}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f}+\frac{a^2}{(a-b)b^2f\sqrt{a+b\tan^2(e+fx)}}+\frac{\sqrt{a+b\tan^2(e+fx)}}{b^2f}$$

Mathematica [C] time = 0.365129, size = 84, normalized size = 0.86

$$\frac{b^2\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\tan^2(e+fx)}{a-b}\right)+(a-b)\left(2a+b\tan^2(e+fx)+b\right)}{b^2f(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (b^2*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(a - b)] + (a -
 b)*(2*a + b + b*Tan[e + f*x]^2))/((a - b)*b^2*f*Sqrt[a + b*Tan[e + f*x]^2]
)
```

Maple [A] time = 0.026, size = 141, normalized size = 1.4

$$\frac{\left(\tan\left(fx+e\right)\right)^2}{fb}\frac{1}{\sqrt{a+b\left(\tan\left(fx+e\right)\right)^2}}+2\frac{a}{fb^2\sqrt{a+b\left(\tan\left(fx+e\right)\right)^2}}+\frac{1}{fb}\frac{1}{\sqrt{a+b\left(\tan\left(fx+e\right)\right)^2}}+\frac{1}{(a-b)f}\frac{}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/f*tan(f*x+e)^2/b/(a+b*tan(f*x+e)^2)^(1/2)+2/f*a/b^2/(a+b*tan(f*x+e)^2)^(1
/2)+1/f/b/(a+b*tan(f*x+e)^2)^(1/2)+1/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)+1/f/(
a-b)/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.75476, size = 987, normalized size = 10.07

$$\left[-\frac{\left(b^3\tan\left(fx+e\right)^2+ab^2\right)\sqrt{a-b}\log\left(-\frac{b^2\tan(fx+e)^4+2\left(4ab-3b^2\right)\tan(fx+e)^2+4\left(b\tan(fx+e)^2+2a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}+8a^2-8ab+b^2}{\tan(fx+e)^4+2\tan(fx+e)^2+1}\right.}{4\left(\left(a^2b^3-2ab^4+b^5\right)f\tan\left(fx+e\right)^2+\left(a^3b^2-2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^3*tan(f*x + e)^2 + a*b^2)*sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 +
2*(4*a*b - 3*b^2)*tan(f*x + e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*ta
n(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*ta
n(f*x + e)^2 + 1)) - 4*(2*a^3 - 3*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*t
an(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan
(f*x + e)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*f), 1/2*((b^3*tan(f*x + e)^2 +
a*b^2)*sqrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan
(f*x + e)^2 + 2*a - b)) + 2*(2*a^3 - 3*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b
^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^2*b^3 - 2*a*b^4 + b^5)*
f*tan(f*x + e)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^5\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [A] time = 1.40194, size = 134, normalized size = 1.37

$$\frac{a^2}{\left(ab^2f - b^3f\right)\sqrt{b\tan\left(fx+e\right)^2 + a}} + \frac{\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\left(af - bf\right)\sqrt{-a+b}} + \frac{\sqrt{b\tan\left(fx+e\right)^2 + a}}{b^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] a^2/((a*b^2*f - b^3*f)*sqrt(b*tan(f*x + e)^2 + a)) + arctan(sqrt(b*tan(f*x
+ e)^2 + a)/sqrt(-a + b))/((a*f - b*f)*sqrt(-a + b)) + sqrt(b*tan(f*x + e)^
2 + a)/(b^2*f)
```

3.334 $\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}-\frac{a}{bf(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f) - a/((a -
 b)*b*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.132861, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3670, 446, 78, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}-\frac{a}{bf(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f) - a/((a -
 b)*b*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{x}{(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}\\
&= -\frac{a}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2(a-b)f}\\
&= -\frac{a}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right)}{(a-b)bf}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} - \frac{a}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.362278, size = 75, normalized size = 1.03

$$\frac{\frac{a(b-a)}{b\sqrt{a+b\tan^2(e+fx)}} + \sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + (a*(-a + b))
/(b*Sqrt[a + b*Tan[e + f*x]^2]))/((a - b)^2*f)
```

Maple [A] time = 0.017, size = 92, normalized size = 1.3

$$-\frac{1}{fb}\frac{1}{\sqrt{a+b\left(\tan\left(fx+e\right)\right)^2}} - \frac{1}{(a-b)f}\frac{1}{\sqrt{a+b\left(\tan\left(fx+e\right)\right)^2}} - \frac{1}{(a-b)f}\arctan\left(\sqrt{a+b\left(\tan\left(fx+e\right)\right)^2}\frac{1}{\sqrt{-a+b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f/b/(a+b*tan(f*x+e)^2)^(1/2)-1/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)-1/f/(a-b
)/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.665, size = 829, normalized size = 11.36

$$\left[-\frac{\left(b^2\tan\left(fx+e\right)^2+ab\right)\sqrt{a-b}\log\left(-\frac{b^2\tan\left(fx+e\right)^4+2\left(4ab-3b^2\right)\tan\left(fx+e\right)^2-4\left(b\tan\left(fx+e\right)^2+2a-b\right)\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{a-b}+8a^2-8ab+b^2}{\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1}\right)}{4\left(\left(a^2b^2-2ab^3+b^4\right)f\tan\left(fx+e\right)^2+\left(a^3b-2a^2b^2+ab^3\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2*tan(f*x + e)^2 + a*b)*sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 + 2*
(4*a*b - 3*b^2)*tan(f*x + e)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(
f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(
f*x + e)^2 + 1)) + 4*sqrt(b*tan(f*x + e)^2 + a)*(a^2 - a*b))/((a^2*b^2 - 2*
a*b^3 + b^4)*f*tan(f*x + e)^2 + (a^3*b - 2*a^2*b^2 + a*b^3)*f), -1/2*((b^2*
tan(f*x + e)^2 + a*b)*sqrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt
(-a + b)/(b*tan(f*x + e)^2 + 2*a - b)) + 2*sqrt(b*tan(f*x + e)^2 + a)*(a^2
- a*b))/((a^2*b^2 - 2*a*b^3 + b^4)*f*tan(f*x + e)^2 + (a^3*b - 2*a^2*b^2 +
a*b^3)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [A] time = 1.37426, size = 103, normalized size = 1.41

$$-\frac{\frac{b\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}}{\sqrt{-a+b}}\right)}{\left(af-bf\right)\sqrt{-a+b}}+\frac{a}{\sqrt{b\tan\left(fx+e\right)^{2}+a}\left(af-bf\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(b*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/((a*f - b*f)*sqrt(-a +
b)) + a/(sqrt(b*tan(f*x + e)^2 + a)*(a*f - b*f)))/b
```

3.335 $\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{1}{f(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f)) + 1/((
a - b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.0884195, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3670, 444, 51, 63, 208}

$$\frac{1}{f(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f)) + 1/((
a - b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}\\
&= \frac{1}{(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2(a-b)f}\\
&= \frac{1}{(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right)}{(a-b)bf}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} + \frac{1}{(a-b)f\sqrt{a+b\tan^2(e+fx)}}
\end{aligned}
$$

Mathematica [C] time = 0.0734709, size = 56, normalized size = 0.81

$$
-\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\tan^2(e+fx)}{a-b}\right)}{f(b-a)\sqrt{a+b\tan^2(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(a - b)]/((-a + b)
*f*Sqrt[a + b*Tan[e + f*x]^2]))
```

Maple [A] time = 0.014, size = 68, normalized size = 1.

$$
\frac{1}{(a-b)\,f}\frac{1}{\sqrt{a+b\left(\tan\left(fx+e\right)\right)^2}}+\frac{1}{(a-b)\,f}\arctan\left(\sqrt{a+b\left(\tan\left(fx+e\right)\right)^2}\frac{1}{\sqrt{-a+b}}\right)\frac{1}{\sqrt{-a+b}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)+1/f/(a-b)/(-a+b)^(1/2)*arctan((a+b*tan(f
*x+e)^2)^(1/2)/(-a+b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 2.74776, size = 790, normalized size = 11.45

$$\left[-\frac{\left(b\tan\left(fx+e\right)^2+a\right)\sqrt{a-b}\log\left(-\frac{b^2\tan\left(fx+e\right)^4+2\left(4ab-3b^2\right)\tan\left(fx+e\right)^2+4\left(b\tan\left(fx+e\right)^2+2a-b\right)\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{a-b}+8a^2-8ab+b^2}{\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1}\right)-}{4\left(\left(a^2b-2ab^2+b^3\right)f\tan\left(fx+e\right)^2+\left(a^3-2a^2b+ab^2\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*tan(f*x + e)^2 + a)*sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a
*b - 3*b^2)*tan(f*x + e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x
+ e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x
+ e)^2 + 1)) - 4*sqrt(b*tan(f*x + e)^2 + a)*(a - b))/((a^2*b - 2*a*b^2 + b^
3)*f*tan(f*x + e)^2 + (a^3 - 2*a^2*b + a*b^2)*f), 1/2*((b*tan(f*x + e)^2 +
a)*sqrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x
 + e)^2 + 2*a - b)) + 2*sqrt(b*tan(f*x + e)^2 + a)*(a - b))/((a^2*b - 2*a*b
^2 + b^3)*f*tan(f*x + e)^2 + (a^3 - 2*a^2*b + a*b^2)*f)]
```

Sympy [A] time = 13.658, size = 56, normalized size = 0.81

$$\frac{1}{f\left(a-b\right)\sqrt{a+b\tan^2\left(e+fx\right)}}+\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\tan^2\left(e+fx\right)}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] 1/(f*(a - b)*sqrt(a + b*tan(e + f*x)**2)) + atan(sqrt(a + b*tan(e + f*x)**2
)/sqrt(-a + b))/(f*sqrt(-a + b)*(a - b))
```

Giac [A] time = 1.28866, size = 93, normalized size = 1.35

$$\frac{\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\left(af-bf\right)\sqrt{-a+b}}+\frac{1}{\sqrt{b\tan\left(fx+e\right)^2+a}\left(af-bf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/((a*f - b*f)*sqrt(-a + b))
+ 1/(sqrt(b*tan(f*x + e)^2 + a)*(a*f - b*f))
```

3.336 $\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{b}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + ArcTanh[Sqrt[a
 + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f) - b/(a*(a - b)*f*Sqrt[a
 + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.149207, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 446, 85, 156, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{b}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + ArcTanh[Sqrt[a
 + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f) - b/(a*(a - b)*f*Sqrt[a
 + b*Tan[e + f*x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x\left(1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{2f} \\
&= -\frac{b}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\operatorname{Subst}\left(\int \frac{a-b-bx}{x(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2a(a-b)f} \\
&= -\frac{b}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2af} - \frac{\operatorname{Subst}\left(\int \right.}{} \\
&= -\frac{b}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right)}{abf} - \frac{\mathrm{Sub}}{} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} - \frac{b}{a(a-b)f\sqrt{a+b\tan^2(e+fx}}
\end{aligned}
$$

Mathematica [C] time = 0.132571, size = 91, normalized size = 0.86

$$
\frac{(a-b)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{b\tan^2(e+fx)}{a}+1\right)-a\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\tan^2(e+fx)}{a-b}\right)}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(a*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(a - b)]) + (a
- b)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Tan[e + f*x]^2)/a])/(a*(a - b)*
```

```
f*Sqrt[a + b*Tan[e + f*x]^2])
```

Maple [B] time = 0.742, size = 32888, normalized size = 310.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x)

[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] Timed out
```

Fricas [B] time = 2.25042, size = 2129, normalized size = 20.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] [-1/2*((a^2*b*tan(f*x + e)^2 + a^3)*sqrt(a - b)*log((b*tan(f*x + e)^2 - 2*s
qrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) - (a
^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x + e)^2)*sqrt(a)*log(
(b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e
)^2) + 2*(a^2*b - a*b^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4*b - 2*a^3*b^2 +
a^2*b^3)*f*tan(f*x + e)^2 + (a^5 - 2*a^4*b + a^3*b^2)*f), 1/2*(2*(a^2*b*tan
(f*x + e)^2 + a^3)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a
+ b)/(a - b)) + (a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x +
e)^2)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a)
+ 2*a)/tan(f*x + e)^2) - 2*(a^2*b - a*b^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^
4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^2 + (a^5 - 2*a^4*b + a^3*b^2)*f),
 1/2*(2*(a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x + e)^2)*sq
rt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) - (a^2*b*tan(f*x + e)^
2 + a^3)*sqrt(a - b)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*s
qrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) - 2*(a^2*b - a*b^2)*sqrt(b*tan(
f*x + e)^2 + a))/((a^4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^2 + (a^5 - 2
*a^4*b + a^3*b^2)*f), ((a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan
(f*x + e)^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) + (a^2*
b*tan(f*x + e)^2 + a^3)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqr
t(-a + b)/(a - b)) - (a^2*b - a*b^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4*b -
2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^2 + (a^5 - 2*a^4*b + a^3*b^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

3.337 $\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=157

$$-\frac{b(a-3b)}{2a^2f(a-b)\sqrt{a+b\tan^2(e+fx)}}+\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}-\frac{\cot^2(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] ((2*a + 3*b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(2*a^(5/2)*f) - A
rcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f) - ((a - 3*
b)*b)/(2*a^2*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2]) - Cot[e + f*x]^2/(2*a*f*
Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.246375, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3670, 446, 103, 152, 156, 63, 208}

$$-\frac{b(a-3b)}{2a^2f(a-b)\sqrt{a+b\tan^2(e+fx)}}+\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}-\frac{\cot^2(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*a + 3*b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(2*a^(5/2)*f) - A
rcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f) - ((a - 3*
b)*b)/(2*a^2*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2]) - Cot[e + f*x]^2/(2*a*f*
Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= -\frac{\cot^2(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2a+3b)+\frac{3bx}{2}}{x(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{2af}$$
$$= -\frac{(a-3b)b}{2a^2(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^2(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{4}(a-b)(2a+3b)-}{x(1+x)\sqrt{a+}}\right.}{a^2(}$$
$$= -\frac{(a-3b)b}{2a^2(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^2(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx,\right.}{2(a-b)}$$
$$= -\frac{(a-3b)b}{2a^2(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^2(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x,\right.}{(a-b)}$$
$$= \frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} - \frac{(a-3b)b}{2a^2(a-b)f\sqrt{a+b\tan^2(}}$$

Mathematica [C] time = 0.426784, size = 115, normalized size = 0.73

$$\frac{(a-b)\left((2a+3b)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{b\tan^2(e+fx)}{a}+1\right)+a\cot^2(e+fx)\right)-2a^2\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2}\right.}{2a^2f(b-a)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*a^2*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(a - b)] + (
a - b)*(a*Cot[e + f*x]^2 + (2*a + 3*b)*Hypergeometric2F1[-1/2, 1, 1/2, 1 +
(b*Tan[e + f*x]^2)/a]))/(2*a^2*(-a + b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Maple [B] time = 1.523, size = 54353, normalized size = 346.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.35399, size = 2851, normalized size = 18.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a^3*b*tan(f*x + e)^4 + a^4*tan(f*x + e)^2)*sqrt(a - b)*log((b*tan
(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x
+ e)^2 + 1)) - ((2*a^3*b - a^2*b^2 - 4*a*b^3 + 3*b^4)*tan(f*x + e)^4 + (2*a
^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(a)*log((b*tan(f*x +
e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) + 2*(a^4
 - 2*a^3*b + a^2*b^2 + (a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(b
*tan(f*x + e)^2 + a))/((a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^4 + (a^
6 - 2*a^5*b + a^4*b^2)*f*tan(f*x + e)^2), -1/4*(4*(a^3*b*tan(f*x + e)^4 + a
^4*tan(f*x + e)^2)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a
+ b)/(a - b)) - ((2*a^3*b - a^2*b^2 - 4*a*b^3 + 3*b^4)*tan(f*x + e)^4 + (2*
a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(a)*log((b*tan(f*x +
 e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) + 2*(a^
4 - 2*a^3*b + a^2*b^2 + (a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(
b*tan(f*x + e)^2 + a))/((a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^4 + (a
^6 - 2*a^5*b + a^4*b^2)*f*tan(f*x + e)^2), -1/2*(((2*a^3*b - a^2*b^2 - 4*a*
b^3 + 3*b^4)*tan(f*x + e)^4 + (2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x
 + e)^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) + (a^3*b*ta
n(f*x + e)^4 + a^4*tan(f*x + e)^2)*sqrt(a - b)*log((b*tan(f*x + e)^2 + 2*sq
rt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) + (a^
4 - 2*a^3*b + a^2*b^2 + (a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(
b*tan(f*x + e)^2 + a))/((a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^4 + (a
^6 - 2*a^5*b + a^4*b^2)*f*tan(f*x + e)^2), -1/2*(((2*a^3*b - a^2*b^2 - 4*a*
b^3 + 3*b^4)*tan(f*x + e)^4 + (2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x
 + e)^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) + 2*(a^3*b*
tan(f*x + e)^4 + a^4*tan(f*x + e)^2)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x +
e)^2 + a)*sqrt(-a + b)/(a - b)) + (a^4 - 2*a^3*b + a^2*b^2 + (a^3*b - 4*a^2
*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^5*b - 2*a^4
*b^2 + a^3*b^3)*f*tan(f*x + e)^4 + (a^6 - 2*a^5*b + a^4*b^2)*f*tan(f*x + e)
^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^3}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^3/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

3.338 $\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=215

$$\frac{b\left(4a^2+3ab-15b^2\right)}{8a^3f(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{\left(8a^2+12ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f}+\frac{(4a+5b)\cot^2(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)}}+\tanh$$

```
[Out] -((8*a^2 + 12*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(8
*a^(7/2)*f) + ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2
)*f) + (b*(4*a^2 + 3*a*b - 15*b^2))/(8*a^3*(a - b)*f*Sqrt[a + b*Tan[e + f*x
]^2]) + ((4*a + 5*b)*Cot[e + f*x]^2)/(8*a^2*f*Sqrt[a + b*Tan[e + f*x]^2]) -
 Cot[e + f*x]^4/(4*a*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.34645, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3670, 446, 103, 151, 152, 156, 63, 208}

$$\frac{b\left(4a^2+3ab-15b^2\right)}{8a^3f(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{\left(8a^2+12ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f}+\frac{(4a+5b)\cot^2(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)}}+\tanh$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((8*a^2 + 12*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(8
*a^(7/2)*f) + ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2
)*f) + (b*(4*a^2 + 3*a*b - 15*b^2))/(8*a^3*(a - b)*f*Sqrt[a + b*Tan[e + f*x
]^2]) + ((4*a + 5*b)*Cot[e + f*x]^2)/(8*a^2*f*Sqrt[a + b*Tan[e + f*x]^2]) -
 Cot[e + f*x]^4/(4*a*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
```

```
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^5(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)}{4af\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4a+5b)+\frac{5bx}{2}}{x^2(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{4af}$$

$$= \frac{(4a+5b)\cot^2(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^4(e+fx)}{4af\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{4}\left(8a^2+12ab+15b^2\right)+\frac{3}{4}b(}{x(1+x)(a+bx)^{3/2}}\right.}{4a^2}$$

$$= \frac{b\left(4a^2+3ab-15b^2\right)}{8a^3(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{(4a+5b)\cot^2(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^4(e+fx)}{4af\sqrt{a+b\tan^2(e+}$$

$$= \frac{b\left(4a^2+3ab-15b^2\right)}{8a^3(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{(4a+5b)\cot^2(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^4(e+fx)}{4af\sqrt{a+b\tan^2(e+}$$

$$= \frac{b\left(4a^2+3ab-15b^2\right)}{8a^3(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{(4a+5b)\cot^2(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^4(e+fx)}{4af\sqrt{a+b\tan^2(e+}$$

$$= -\frac{\left(8a^2+12ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} + \frac{b\left(\right.}{8a^3(a-}$$

Mathematica [C] time = 1.19724, size = 142, normalized size = 0.66

$$\frac{(a-b)\left(a\cot^2(e+fx)\left(2a\cot^2(e+fx)-4a-5b\right)-\left(8a^2+12ab+15b^2\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{b\tan^2(e+fx)}{a}\right.\right.}{8a^3f(b-a)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (8*a^3*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(a - b)] + (a
 - b)*(a*Cot[e + f*x]^2*(-4*a - 5*b + 2*a*Cot[e + f*x]^2) - (8*a^2 + 12*a*b
 + 15*b^2)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Tan[e + f*x]^2)/a]))/(8*a
^3*(-a + b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Maple [B] time = 2.861, size = 79934, normalized size = 371.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x)

[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] Timed out
```

Fricas [A] time = 2.69519, size = 3421, normalized size = 15.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] [-1/16*(8*(a^4*b*tan(f*x + e)^6 + a^5*tan(f*x + e)^4)*sqrt(a - b)*log((b*ta
n(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x
 + e)^2 + 1)) - ((8*a^4*b - 4*a^3*b^2 - a^2*b^3 - 18*a*b^4 + 15*b^5)*tan(f*
x + e)^6 + (8*a^5 - 4*a^4*b - a^3*b^2 - 18*a^2*b^3 + 15*a*b^4)*tan(f*x + e)
^4)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) +
2*a)/tan(f*x + e)^2) + 2*(2*a^5 - 4*a^4*b + 2*a^3*b^2 - (4*a^4*b - a^3*b^2
- 18*a^2*b^3 + 15*a*b^4)*tan(f*x + e)^4 - (4*a^5 - 3*a^4*b - 6*a^3*b^2 + 5*
a^2*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^6*b - 2*a^5*b^2 +
a^4*b^3)*f*tan(f*x + e)^6 + (a^7 - 2*a^6*b + a^5*b^2)*f*tan(f*x + e)^4), 1/
16*(16*(a^4*b*tan(f*x + e)^6 + a^5*tan(f*x + e)^4)*sqrt(-a + b)*arctan(-sqr
t(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b)) + ((8*a^4*b - 4*a^3*b^2 - a^2
*b^3 - 18*a*b^4 + 15*b^5)*tan(f*x + e)^6 + (8*a^5 - 4*a^4*b - a^3*b^2 - 18*
a^2*b^3 + 15*a*b^4)*tan(f*x + e)^4)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(
b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) - 2*(2*a^5 - 4*a^4*b +
 2*a^3*b^2 - (4*a^4*b - a^3*b^2 - 18*a^2*b^3 + 15*a*b^4)*tan(f*x + e)^4 - (
4*a^5 - 3*a^4*b - 6*a^3*b^2 + 5*a^2*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e
)^2 + a))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*f*tan(f*x + e)^6 + (a^7 - 2*a^6*b
+ a^5*b^2)*f*tan(f*x + e)^4), 1/8*(((8*a^4*b - 4*a^3*b^2 - a^2*b^3 - 18*a*b
^4 + 15*b^5)*tan(f*x + e)^6 + (8*a^5 - 4*a^4*b - a^3*b^2 - 18*a^2*b^3 + 15*
a*b^4)*tan(f*x + e)^4)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/
a) - 4*(a^4*b*tan(f*x + e)^6 + a^5*tan(f*x + e)^4)*sqrt(a - b)*log((b*tan(f
*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x +
e)^2 + 1)) - (2*a^5 - 4*a^4*b + 2*a^3*b^2 - (4*a^4*b - a^3*b^2 - 18*a^2*b^3
 + 15*a*b^4)*tan(f*x + e)^4 - (4*a^5 - 3*a^4*b - 6*a^3*b^2 + 5*a^2*b^3)*tan
(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*f*t
an(f*x + e)^6 + (a^7 - 2*a^6*b + a^5*b^2)*f*tan(f*x + e)^4), 1/8*(((8*a^4*b
 - 4*a^3*b^2 - a^2*b^3 - 18*a*b^4 + 15*b^5)*tan(f*x + e)^6 + (8*a^5 - 4*a^4
*b - a^3*b^2 - 18*a^2*b^3 + 15*a*b^4)*tan(f*x + e)^4)*sqrt(-a)*arctan(sqrt(
b*tan(f*x + e)^2 + a)*sqrt(-a)/a) + 8*(a^4*b*tan(f*x + e)^6 + a^5*tan(f*x +
 e)^4)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b)
) - (2*a^5 - 4*a^4*b + 2*a^3*b^2 - (4*a^4*b - a^3*b^2 - 18*a^2*b^3 + 15*a*b
^4)*tan(f*x + e)^4 - (4*a^5 - 3*a^4*b - 6*a^3*b^2 + 5*a^2*b^3)*tan(f*x + e)
```

```
^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*f*tan(f*x +
e)^6 + (a^7 - 2*a^6*b + a^5*b^2)*f*tan(f*x + e)^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(fx+e)^5}{\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^5/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

3.339 $\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=182

$$\frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b^2f(a-b)}-\frac{(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2b^{5/2}f}-\frac{a\tan^3(e+fx)}{bf(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{\tan^{-1}\left(\ldots\right)}{f\ldots}$$

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3
/2)*f)) - ((3*a + 2*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*
x]^2]])/(2*b^(5/2)*f) - (a*Tan[e + f*x]^3)/((a - b)*b*f*Sqrt[a + b*Tan[e +
f*x]^2]) + ((3*a - b)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*(a - b)*b
^2*f)
```

Rubi [A] time = 0.250804, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3670, 470, 582, 523, 217, 206, 377, 203}

$$\frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b^2f(a-b)}-\frac{(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2b^{5/2}f}-\frac{a\tan^3(e+fx)}{bf(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{\tan^{-1}\left(\ldots\right)}{f\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3
/2)*f)) - ((3*a + 2*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*
x]^2]])/(2*b^(5/2)*f) - (a*Tan[e + f*x]^3)/((a - b)*b*f*Sqrt[a + b*Tan[e +
f*x]^2]) + ((3*a - b)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*(a - b)*b
^2*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan^3(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{x^2(3a+(3a-b)x^2)}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{(a-b)bf}$$

$$= -\frac{a\tan^3(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)b^2f} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{a\tan^3(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)b^2f} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{a\tan^3(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)b^2f} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2b^{5/2}f} - \frac{a\tan^3(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e\cdot}}$$

Mathematica [C] time = 6.39319, size = 787, normalized size = 4.32

$$\frac{\sqrt{\frac{a\cos(2(e+fx))+a-b\cos(2(e+fx))+b}{\cos(2(e+fx))+1}}\left(\frac{\tan(e+fx)}{2b^2} - \frac{a^2\sin(2(e+fx))}{b^2(a-b)(a(-\cos(2(e+fx)))-a+b\cos(2(e+fx))-b)}\right)}{f} - \frac{-\frac{b(3a^2-ab-b^2)\sin^4(e+fx)\csc(2(e+fx))\sqrt{\frac{(a-b)}{c}}}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((-((b*(3*a^2 - a*b - b^2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Co
s[2*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*
x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f
*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e
 + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a -
b)*Cos[2*(e + f*x)]))) - (4*b^3*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b + (a
 - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]^2)
/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a
 - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSi
n[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*
Sin[e + f*x]^4)/(4*a*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*
(e + f*x)]]) - (Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x
)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*
x]^2)/b]*Csc[2*(e + f*x)]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a
 - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(2*
(a - b)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]))
```

```
)/Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])/((a - b)*b^2*f)) + (Sqrt[(a + b +
 a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(-((a^2*S
in[2*(e + f*x)])/((a - b)*b^2*(-a - b - a*Cos[2*(e + f*x)] + b*Cos[2*(e + f
*x)]))) + Tan[e + f*x]/(2*b^2)))/f
```

Maple [A] time = 0.023, size = 286, normalized size = 1.6

$$\frac{(\tan(fx+e))^3}{2fb}\frac{1}{\sqrt{a+b(\tan(fx+e))^2}}+\frac{3a\tan(fx+e)}{2fb^2}\frac{1}{\sqrt{a+b(\tan(fx+e))^2}}-\frac{3a}{2f}\ln\left(\sqrt{b}\tan(fx+e)+\sqrt{a+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/2/f*tan(f*x+e)^3/b/(a+b*tan(f*x+e)^2)^(1/2)+3/2/f*a/b^2*tan(f*x+e)/(a+b*t
an(f*x+e)^2)^(1/2)-3/2/f*a/b^(5/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)
^(1/2))+1/f*tan(f*x+e)/b/(a+b*tan(f*x+e)^2)^(1/2)-1/f/b^(3/2)*ln(b^(1/2)*ta
n(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))+1/f*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2
)+b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)-1/f/(a-b)^2*(b^4*(a-b))^(
1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*
x+e))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 20.1148, size = 2768, normalized size = 15.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((3*a^4 - 4*a^3*b - a^2*b^2 + 2*a*b^3 + (3*a^3*b - 4*a^2*b^2 - a*b^3 +
 2*b^4)*tan(f*x + e)^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x +
 e)^2 + a)*sqrt(b)*tan(f*x + e) + a) + 2*(b^4*tan(f*x + e)^2 + a*b^3)*sqrt(
-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + 2*((a^2*b^2 - 2*a*b^3 + b
^4)*tan(f*x + e)^3 + (3*a^3*b - 4*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*tan
(f*x + e)^2 + a))/((a^2*b^4 - 2*a*b^5 + b^6)*f*tan(f*x + e)^2 + (a^3*b^3 -
2*a^2*b^4 + a*b^5)*f), 1/2*((3*a^4 - 4*a^3*b - a^2*b^2 + 2*a*b^3 + (3*a^3*b
 - 4*a^2*b^2 - a*b^3 + 2*b^4)*tan(f*x + e)^2)*sqrt(-b)*arctan(sqrt(b*tan(f*
x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + (b^4*tan(f*x + e)^2 + a*b^3)*sqr
t(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqr
```

```
t(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + ((a^2*b^2 - 2*a*b^3 + b
^4)*tan(f*x + e)^3 + (3*a^3*b - 4*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*tan
(f*x + e)^2 + a))/((a^2*b^4 - 2*a*b^5 + b^6)*f*tan(f*x + e)^2 + (a^3*b^3 -
2*a^2*b^4 + a*b^5)*f), -1/4*(4*(b^4*tan(f*x + e)^2 + a*b^3)*sqrt(a - b)*arc
tan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (3*a^4 - 4*a^
3*b - a^2*b^2 + 2*a*b^3 + (3*a^3*b - 4*a^2*b^2 - a*b^3 + 2*b^4)*tan(f*x + e
)^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*
tan(f*x + e) + a) - 2*((a^2*b^2 - 2*a*b^3 + b^4)*tan(f*x + e)^3 + (3*a^3*b
- 4*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a^2*b^4 -
2*a*b^5 + b^6)*f*tan(f*x + e)^2 + (a^3*b^3 - 2*a^2*b^4 + a*b^5)*f), -1/2*(2
*(b^4*tan(f*x + e)^2 + a*b^3)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a
)/(sqrt(a - b)*tan(f*x + e))) - (3*a^4 - 4*a^3*b - a^2*b^2 + 2*a*b^3 + (3*a
^3*b - 4*a^2*b^2 - a*b^3 + 2*b^4)*tan(f*x + e)^2)*sqrt(-b)*arctan(sqrt(b*ta
n(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) - ((a^2*b^2 - 2*a*b^3 + b^4)*t
an(f*x + e)^3 + (3*a^3*b - 4*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*tan(f*x
+ e)^2 + a))/((a^2*b^4 - 2*a*b^5 + b^6)*f*tan(f*x + e)^2 + (a^3*b^3 - 2*a^2
*b^4 + a*b^5)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^6\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**6/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^6}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^6/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

3.340 $\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=123

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{b^{3/2}f}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{a\tan(e+fx)}{bf(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3/2
)*f) + ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(b^(3/2)*
f) - (a*Tan[e + f*x])/((a - b)*b*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.158617, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3670, 470, 523, 217, 206, 377, 203}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{b^{3/2}f}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{a\tan(e+fx)}{bf(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```



```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3/2
)*f) + ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(b^(3/2)*
f) - (a*Tan[e + f*x])/((a - b)*b*f*Sqrt[a + b*Tan[e + f*x]^2])
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a+(a-b)x^2}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{(a-b)bf}$$

$$= -\frac{a\tan(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{(a-b)f} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{a\tan(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)f} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{b^{3/2}f} - \frac{a\tan(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}}$$

Mathematica [C] time = 2.93258, size = 250, normalized size = 2.03

$$\frac{a\sin(2(e+fx))\sec^2(e+fx)\left(\frac{(a-b)\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}}{\sqrt{2}}\right),1\right)}{\sqrt{2}} - \frac{b\sqrt{\frac{\csc^2(e+fx)((a-b)\,c}{b}}}{}\right.}{\sqrt{2}bf(a-b)^2\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*(-a + b + ((a - b)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]
^2)/b]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*
x]^2)/b]/Sqrt[2]], 1])/Sqrt[2] - (b*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)]
)*Csc[e + f*x]^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)
*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])/Sqrt[2])*Sec[e + f*x]^2
*Sin[2*(e + f*x)])/(Sqrt[2]*(a - b)^2*b*f*Sqrt[(a + b + (a - b)*Cos[2*(e +
f*x)])*Sec[e + f*x]^2])
```

Maple [A] time = 0.018, size = 193, normalized size = 1.6

$$-\frac{\tan\left(fx+e\right)}{fb}\frac{1}{\sqrt{a+b\left(\tan\left(fx+e\right)\right)^{2}}}+\frac{1}{f}\ln\left(\sqrt{b}\tan\left(fx+e\right)+\sqrt{a+b\left(\tan\left(fx+e\right)\right)^{2}}\right)b^{-\frac{3}{2}}-\frac{\tan\left(fx+e\right)}{fa}\frac{1}{\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f*tan(f*x+e)/b/(a+b*tan(f*x+e)^2)^(1/2)+1/f/b^(3/2)*ln(b^(1/2)*tan(f*x+e
)+(a+b*tan(f*x+e)^2)^(1/2))-1/f*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2)-b*tan
(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)+1/f/(a-b)^2*(b^4*(a-b))^(1/2)/b^
2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 10.8041, size = 2291, normalized size = 18.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x + e)^2)*sqrt
(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x +
 e) + a) + (b^3*tan(f*x + e)^2 + a*b^2)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*
x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan
(f*x + e)^2 + 1)) - 2*(a^2*b - a*b^2)*sqrt(b*tan(f*x + e)^2 + a)*tan(f*x +
e))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan(f*x + e)^2 + (a^3*b^2 - 2*a^2*b^3 + a*
```

```
b^4)*f), -1/2*(2*(a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x +
 e)^2)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))
) - (b^3*tan(f*x + e)^2 + a*b^2)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^
2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x +
e)^2 + 1)) + 2*(a^2*b - a*b^2)*sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e))/((a
^2*b^3 - 2*a*b^4 + b^5)*f*tan(f*x + e)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*f)
, 1/2*(2*(b^3*tan(f*x + e)^2 + a*b^2)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x +
e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + (a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2
*a*b^2 + b^3)*tan(f*x + e)^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan
(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) - 2*(a^2*b - a*b^2)*sqrt(b*tan(f
*x + e)^2 + a)*tan(f*x + e))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan(f*x + e)^2 +
(a^3*b^2 - 2*a^2*b^3 + a*b^4)*f), ((b^3*tan(f*x + e)^2 + a*b^2)*sqrt(a - b)
*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (a^3 - 2*
a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x + e)^2)*sqrt(-b)*arctan(sqr
t(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) - (a^2*b - a*b^2)*sqrt(b
*tan(f*x + e)^2 + a)*tan(f*x + e))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan(f*x + e
)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^4}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

3.341 $\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=81

$$\frac{\tan(e+fx)}{f(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}$$

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3
/2)*f)) + Tan[e + f*x]/((a - b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.1111, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3670, 471, 377, 203}

$$\frac{\tan(e+fx)}{f(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3
/2)*f)) + Tan[e + f*x]/((a - b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\tan(e+fx)}{(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{(a-b)f}$$
$$= \frac{\tan(e+fx)}{(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)f}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} + \frac{\tan(e+fx)}{(a-b)f\sqrt{a+b\tan^2(e+fx)}}$$

Mathematica [A] time = 3.18117, size = 154, normalized size = 1.9

$$\frac{\tan(e+fx)\left((a-b)\sqrt{\frac{b\tan^2(e+fx)}{a}+1}+\sqrt{\frac{(b-a)\tan^2(e+fx)}{a}}\left(a\cot^2(e+fx)+b\right)\tanh^{-1}\left(\frac{\sqrt{\frac{(b-a)\tan^2(e+fx)}{a}}}{\sqrt{\frac{b\tan^2(e+fx)}{a}+1}}\right)\right)}{f(a-b)^2\sqrt{a+b\tan^2(e+fx)}\sqrt{\frac{b\tan^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Tan[e + f*x]*(ArcTanh[Sqrt[((-a + b)*Tan[e + f*x]^2)/a]/Sqrt[1 + (b*Tan[e
 + f*x]^2)/a]]*(b + a*Cot[e + f*x]^2)*Sqrt[((-a + b)*Tan[e + f*x]^2)/a] + (a
 - b)*Sqrt[1 + (b*Tan[e + f*x]^2)/a]))/((a - b)^2*f*Sqrt[a + b*Tan[e + f*x]
^2]*Sqrt[1 + (b*Tan[e + f*x]^2)/a])
```

Maple [A] time = 0.016, size = 131, normalized size = 1.6

$$\frac{\tan(fx+e)}{fa}\frac{1}{\sqrt{a+b\left(\tan(fx+e)\right)^2}}+\frac{b\tan(fx+e)}{a(a-b)f}\frac{1}{\sqrt{a+b\left(\tan(fx+e)\right)^2}}-\frac{1}{f(a-b)^2b^2}\sqrt{b^4(a-b)}\arctan\left((a-b)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/f*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2)+b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f
*x+e)^2)^(1/2)-1/f/(a-b)^2*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b
```

```
))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.13058, size = 686, normalized size = 8.47

$$\left[\frac{\left(b\tan\left(fx+e\right)^{2}+a\right)\sqrt{-a+b}\log\left(-\frac{(a-2b)\tan\left(fx+e\right)^{2}-2\sqrt{b\tan\left(fx+e\right)^{2}+a}\sqrt{-a+b}\tan\left(fx+e\right)-a}{\tan\left(fx+e\right)^{2}+1}\right)+2\sqrt{b\tan\left(fx+e\right)^{2}+a}(a-b}{2\left(\left(a^{2}b-2ab^{2}+b^{3}\right)f\tan\left(fx+e\right)^{2}+\left(a^{3}-2a^{2}b+ab^{2}\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*tan(f*x + e)^2 + a)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 -
2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2
 + 1)) + 2*sqrt(b*tan(f*x + e)^2 + a)*(a - b)*tan(f*x + e))/((a^2*b - 2*a*b
^2 + b^3)*f*tan(f*x + e)^2 + (a^3 - 2*a^2*b + a*b^2)*f), -((b*tan(f*x + e)^
2 + a)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x
+ e))) - sqrt(b*tan(f*x + e)^2 + a)*(a - b)*tan(f*x + e))/((a^2*b - 2*a*b^2
 + b^3)*f*tan(f*x + e)^2 + (a^3 - 2*a^2*b + a*b^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^{2}\left(e+fx\right)}{\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^{2}}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

3.342 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{b\tan(e+fx)}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3/2
)*f) - (b*Tan[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.0651576, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3661, 382, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{b\tan(e+fx)}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-3/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3/2
)*f) - (b*Tan[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}+\frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx,x,\tan(e+fx)\right)}{(a-b)f}$$

$$= -\frac{b\tan(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}+\frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx,x,\frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f}-\frac{b\tan(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}$$

Mathematica [C] time = 6.12497, size = 214, normalized size = 2.52

$$\frac{4\sin(e+fx)\cos^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\left(a(a-b)\tan^2(e+fx)\text{Hypergeometric2F1}\left(2,2,\frac{7}{2},\frac{(a-b)\sin^2(e+fx)}{a}\right)+\frac{15(3\cdots}{\cdots}\right.}{15a^4f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-3/2),x]
```

```
[Out] (4*Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2]*(a*(a - b)*Hyperg
eometric2F1[2, 2, 7/2, ((a - b)*Sin[e + f*x]^2)/a]*Tan[e + f*x]^2 + (15*(3*
a + 2*b*Tan[e + f*x]^2)*(-2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*(a*Cos
[e + f*x]^2 + b*Sin[e + f*x]^2) + a*Sqrt[((a - b)*Sin[2*(e + f*x)]^2*(a + b
*Tan[e + f*x]^2))/a^2]))/(((a - b)*Sin[2*(e + f*x)]^2*(a + b*Tan[e + f*x]^2
))/a^2)^(3/2)))/(15*a^4*f)
```

Maple [A] time = 0., size = 104, normalized size = 1.2

$$-\frac{b\tan(fx+e)}{a(a-b)f}\frac{1}{\sqrt{a+b\left(\tan(fx+e)\right)^2}}+\frac{1}{f(a-b)^2b^2}\sqrt{b^4(a-b)}\arctan\left((a-b)b^2\tan(fx+e)\frac{1}{\sqrt{b^4(a-b)}}\frac{1}{\sqrt{a+b\left(\tan\cdots\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)+1/f/(a-b)^2*(b^4*(a-b))^(1
/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x
```

```
+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.27543, size = 722, normalized size = 8.49

$$\left[\frac{\left(ab\tan\left(fx+e\right)^2+a^2\right)\sqrt{-a+b}\log\left(-\frac{(a-2\,b)\tan\left(fx+e\right)^2+2\,\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{-a+b}\tan\left(fx+e\right)-a}{\tan\left(fx+e\right)^2+1}\right)-2\,\sqrt{b\tan\left(fx+e\right)^2+a}\left(ab\right.}{2\left(\left(a^3b-2\,a^2b^2+ab^3\right)f\tan\left(fx+e\right)^2+\left(a^4-2\,a^3b+a^2b^2\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b*tan(f*x + e)^2 + a^2)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^
2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x +
e)^2 + 1)) - 2*sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan(f*x + e))/((a^3*b
 - 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f), ((a*
b*tan(f*x + e)^2 + a^2)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqr
t(a - b)*tan(f*x + e))) - sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan(f*x +
e))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^
2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(-3/2), x)
```

3.343 $\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{(a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a^2f(a-b)}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{b\cot(e+fx)}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3
/2)*f)) - (b*Cot[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2]) - ((a -
 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(a^2*(a - b)*f)
```

Rubi [A] time = 0.184107, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3670, 472, 583, 12, 377, 203}

$$-\frac{(a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a^2f(a-b)}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{b\cot(e+fx)}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```



```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3
/2)*f)) - (b*Cot[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2]) - ((a -
 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(a^2*(a - b)*f)
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
```

```
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx=\frac{\text{Subst}\left(\int\frac{1}{x^2(1+x^2)(a+bx^2)^{3/2}}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{b\cot(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}+\frac{\text{Subst}\left(\int\frac{a-2b-2bx^2}{x^2(1+x^2)\sqrt{a+bx^2}}\,dx,x,\tan(e+fx)\right)}{a(a-b)f}$$

$$=-\frac{b\cot(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}-\frac{(a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a^2(a-b)f}-\frac{\text{Subst}\left(\int\right.}{}$$

$$=-\frac{b\cot(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}-\frac{(a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a^2(a-b)f}-\frac{\text{Subst}\left(\int\right.}{}$$

$$=-\frac{b\cot(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}-\frac{(a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a^2(a-b)f}-\frac{\text{Subst}\left(\int\right.}{}$$

$$=-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f}-\frac{b\cot(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}-\frac{(a-2b)\cot(e+fx)\sqrt{a+b\,\text{t}}}{a^2(a-b)f}$$

Mathematica [C] time = 13.503, size = 882, normalized size = 6.89

$$-\frac{\cos^2(e+fx)\cot(e+fx)\left(\frac{8(a-b)b^2\text{Hypergeometric2F1}\left(2,2,\frac{7}{2},\frac{(a-b)\sin^2(e+fx)}{a}\right)\sin^2(e+fx)\tan^4(e+fx)}{5a^3}+\frac{8(a-b)b^2\text{HypergeometricPFQ}\left(\{2,2,2\},\{\right.}{1}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((Cos[e + f*x]^2*Cot[e + f*x]*((3*a*Csc[e + f*x]^2)/(a - b) + (12*b*Sec[e
+ f*x]^2)/(a - b) + (16*(a - b)*Hypergeometric2F1[2, 2, 7/2, ((a - b)*Sin[e
 + f*x]^2)/a]*Sin[e + f*x]^2)/(15*a) + (8*(a - b)*HypergeometricPFQ[{2, 2,
2}, {1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2)/(15*a) + (8*b^2*S
ec[e + f*x]^2*Tan[e + f*x]^2)/(a*(a - b)) + (8*(a - b)*b*Hypergeometric2F1[
2, 2, 7/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(3*a^
2) + (16*(a - b)*b*HypergeometricPFQ[{2, 2, 2}, {1, 7/2}, ((a - b)*Sin[e +
f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(15*a^2) + (8*(a - b)*b^2*Hyperge
ometric2F1[2, 2, 7/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*
x]^4)/(5*a^3) + (8*(a - b)*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 7/2}, ((a -
 b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^4)/(15*a^3) - (3*ArcSin[
Sqrt[((a - b)*Sin[e + f*x]^2)/a]])/((((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Sqrt
[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]) - (12*b*ArcSin[Sqrt[((a - b)*S
in[e + f*x]^2)/a]]*Tan[e + f*x]^2)/(a*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Sq
rt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]) - (8*b^2*ArcSin[Sqrt[((a - b
)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^4)/(a^2*(((a - b)*Sin[e + f*x]^2)/a)^(3/
2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]) + (3*ArcSin[Sqrt[((a -
b)*Sin[e + f*x]^2)/a]])/Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*
Tan[e + f*x]^2))/a^2] + (12*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[
e + f*x]^2)/(a*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f
*x]^2))/a^2]) + (8*b^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x
]^4)/(a^2*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2
))/a^2])))/(a*f*Sqrt[a + b*Tan[e + f*x]^2]))
```

Maple [C] time = 0.304, size = 1305, normalized size = 10.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f/a^2/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/(a-b)/(a*cos(f*x+e)^2-co
s(f*x+e)^2*b+b)^2*(cos(f*x+e)*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)
*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)
+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-co
s(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*(
(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/
2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^2-2*cos(f*x+e)*
sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(
1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)
*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(co
s(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*
b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*
(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a^2+
2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*
x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a
-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)
)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/
sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*
b^2)/a^2)^(1/2))*a^2*sin(f*x+e)-2*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^
(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1
/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e
```

```
)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b
^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a
-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2
)+a-2*b)/a)^(1/2))*a^2*sin(f*x+e)+cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-
2*b)/a)^(1/2)*a^2-2*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*
a*b+2*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2+((2*I*b^(1
/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b-2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(
1/2)*b^2)*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/
2)/sin(f*x+e)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.99376, size = 1102, normalized size = 8.61

$$\left[\frac{\left(a^{2} b \tan\left(f x + e\right)^{3} + a^{3} \tan\left(f x + e\right)\right) \sqrt{-a + b} \log\left(-\frac{\left(a^{2} - 8 \, a b + 8 \, b^{2}\right) \tan\left(f x + e\right)^{4} - 2 \, \left(3 \, a^{2} - 4 \, a b\right) \tan\left(f x + e\right)^{2} + a^{2} - 4 \, \left(\left(a - 2 \, b\right) \tan\left(f x + e\right)^{3} - a \tan \cdots}{\tan\left(f x + e\right)^{4} + 2 \, \tan\left(f x + e\right)^{2} + 1}\cdots\right.}{4 \, \left(\left(a^{4} b - 2 \, a^{3} b^{2} + a^{2} b^{3}\right) f \tan\left(f x + e\right)^{3} + \left(a^{5} \cdots\right.\right.}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a^2*b*tan(f*x + e)^3 + a^3*tan(f*x + e))*sqrt(-a + b)*log(-((a^2 - 8
*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 - 4*(
(a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 4*(a^3 - 2*a^2*b + a*b^
2 + (a^2*b - 3*a*b^2 + 2*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/(
(a^4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^3 + (a^5 - 2*a^4*b + a^3*b^2)*
f*tan(f*x + e)), -1/2*((a^2*b*tan(f*x + e)^3 + a^3*tan(f*x + e))*sqrt(a - b
)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*
tan(f*x + e)^2 - a)) + 2*(a^3 - 2*a^2*b + a*b^2 + (a^2*b - 3*a*b^2 + 2*b^3)
*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4*b - 2*a^3*b^2 + a^2*b^3)
*f*tan(f*x + e)^3 + (a^5 - 2*a^4*b + a^3*b^2)*f*tan(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^{2}\left(e + f x\right)}{\left(a + b \tan^{2}\left(e + f x\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^{2}}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

3.344 $\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{(a-4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f(a-b)}+\frac{(3a-4b)(a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^3f(a-b)}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}$$

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3/2
)*f) - (b*Cot[e + f*x]^3)/(a*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2]) + ((3*a
- 4*b)*(a + 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(3*a^3*(a - b)*f)
 - ((a - 4*b)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(3*a^2*(a - b)*f)
```

Rubi [A] time = 0.259454, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3670, 472, 583, 12, 377, 203}

$$-\frac{(a-4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f(a-b)}+\frac{(3a-4b)(a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^3f(a-b)}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3/2
)*f) - (b*Cot[e + f*x]^3)/(a*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2]) + ((3*a
- 4*b)*(a + 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(3*a^3*(a - b)*f)
 - ((a - 4*b)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(3*a^2*(a - b)*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-4b-4bx^2}{x^4(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{a(a-b)f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{(a-4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2(a-b)f} - \frac{\text{Subst}}{}$$

$$= -\frac{b\cot^3(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-4b)(a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^3(a-b)f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-4b)(a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^3(a-b)f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-4b)(a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^3(a-b)f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{b\cot^3(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-4b)(a+2b)\cot(e+f}{3a^3(a-}$$

Mathematica [C] time = 16.3827, size = 802, normalized size = 4.36

$$-\frac{b\sqrt{\frac{a+b+(a-b)\cos(2(e+fx))}{\cos(2(e+fx))+1}}\sqrt{-\frac{a\cot^2(e+fx)}{b}}\sqrt{-\frac{a(\cos(2(e+fx))+1)\csc^2(e+fx)}{b}}\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\csc(2(e+fx))\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+f}{b}}}{\sqrt{2}}\right.\right.}{a(a+b+(a-b)\cos(2(e+fx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-((b*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*Sqrt[
-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b
)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f
*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]
^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a - b)*Cos[2*(e + f*x)])))
 - (4*b*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/
(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[
2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*
Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)
*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(4*a*Sqr
t[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]) - (Sqrt[-((
a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*
Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)
]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*
Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(2*(a - b)*Sqrt[1 + Cos[2*(
e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])))/Sqrt[a + b + (a - b)*C
os[2*(e + f*x)]])/((a - b)*f) + (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2
*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(((4*a*Cos[e + f*x] + 5*b*Cos[e + f*x]
)*Csc[e + f*x])/(3*a^3) - (Cot[e + f*x]*Csc[e + f*x]^2)/(3*a^2) - (b^3*Sin[
2*(e + f*x)])/(a^3*(a - b)*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)]
))))/f
```

Maple [C] time = 0.266, size = 2577, normalized size = 14.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3/f/a^3/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/(a-b)/(a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)^2*(-6*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^
(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*
(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(
f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-
b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-
(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a
)^(1/2))*cos(f*x+e)^3*sin(f*x+e)*a^3+3*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(
a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1
))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(
f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2
*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)
```

```
-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^3*sin(f*
x+e)*a^3-6*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(
1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)
*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(co
s(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*
b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*
(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*cos(
f*x+e)^2*sin(f*x+e)*a^3+3*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*
b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/
a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*co
s(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a
-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*
(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^2*sin(f*x+e)*a^3+6*co
s(f*x+e)*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2
)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*c
os(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+
e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(
1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I
*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1
/2))*a^3-3*cos(f*x+e)*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(
1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/
2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)
*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(
1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b
^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^3+4*((2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^4*a^3-3*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/
a)^(1/2)*cos(f*x+e)^4*a^2*b-6*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos
(f*x+e)^4*a*b^2+8*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^4*b^
3+6*a^3*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2
)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^
(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f
*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/
a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-
b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*sin(f*x
+e)-3*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+
cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1
/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x
+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^
(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*
a*b+8*b^2)/a^2)^(1/2))*a^3*sin(f*x+e)-3*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)
^(1/2)*cos(f*x+e)^2*a^3+5*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x
+e)^2*a^2*b+8*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a*b^2-
16*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*b^3-3*((2*I*b^(1/
2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2*b-2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^
(1/2)*a*b^2+8*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^3)*cos(f*x+e)^3*(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)/sin(f*x+e)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 3.25641, size = 1312, normalized size = 7.13

$$\left[\frac{3\left(a^3b\tan\left(fx+e\right)^5+a^4\tan\left(fx+e\right)^3\right)\sqrt{-a+b}\log\left(-\frac{\left(a^2-8ab+8b^2\right)\tan\left(fx+e\right)^4-2\left(3a^2-4ab\right)\tan\left(fx+e\right)^2+a^2+4\left((a-2b)\tan\left(fx+e\right)^3-a\right.}{\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1}\right.}{12\left(\left(a^5b-2a^4b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^3*b*tan(f*x + e)^5 + a^4*tan(f*x + e)^3)*sqrt(-a + b)*log(-((a^
2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2
+ 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*
sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4*((3*a^3*b - a^2*
b^2 - 10*a*b^3 + 8*b^4)*tan(f*x + e)^4 - a^4 + 2*a^3*b - a^2*b^2 + (3*a^4 -
 2*a^3*b - 5*a^2*b^2 + 4*a*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))
/((a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^5 + (a^6 - 2*a^5*b + a^4*b^2
)*f*tan(f*x + e)^3), 1/6*(3*(a^3*b*tan(f*x + e)^5 + a^4*tan(f*x + e)^3)*sqr
t(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a
- 2*b)*tan(f*x + e)^2 - a)) + 2*((3*a^3*b - a^2*b^2 - 10*a*b^3 + 8*b^4)*tan
(f*x + e)^4 - a^4 + 2*a^3*b - a^2*b^2 + (3*a^4 - 2*a^3*b - 5*a^2*b^2 + 4*a*
b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^5*b - 2*a^4*b^2 + a^3*
b^3)*f*tan(f*x + e)^5 + (a^6 - 2*a^5*b + a^4*b^2)*f*tan(f*x + e)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^4}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

3.345 $\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=252

$$\frac{\left(5a^2+4ab-24b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f(a-b)}-\frac{\left(10a^2b+15a^3+8ab^2-48b^3\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx}}{15a^4f(a-b)}$$

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3
/2)*f)) - (b*Cot[e + f*x]^5)/(a*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2]) - ((1
5*a^3 + 10*a^2*b + 8*a*b^2 - 48*b^3)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2
])/(15*a^4*(a - b)*f) + ((5*a^2 + 4*a*b - 24*b^2)*Cot[e + f*x]^3*Sqrt[a + b
*Tan[e + f*x]^2])/(15*a^3*(a - b)*f) - ((a - 6*b)*Cot[e + f*x]^5*Sqrt[a + b
*Tan[e + f*x]^2])/(5*a^2*(a - b)*f)
```

Rubi [A] time = 0.368314, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3670, 472, 583, 12, 377, 203}

$$\frac{\left(5a^2+4ab-24b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f(a-b)}-\frac{\left(10a^2b+15a^3+8ab^2-48b^3\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx}}{15a^4f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```


```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3
/2)*f)) - (b*Cot[e + f*x]^5)/(a*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2]) - ((1
5*a^3 + 10*a^2*b + 8*a*b^2 - 48*b^3)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2
])/(15*a^4*(a - b)*f) + ((5*a^2 + 4*a*b - 24*b^2)*Cot[e + f*x]^3*Sqrt[a + b
*Tan[e + f*x]^2])/(15*a^3*(a - b)*f) - ((a - 6*b)*Cot[e + f*x]^5*Sqrt[a + b
*Tan[e + f*x]^2])/(5*a^2*(a - b)*f)
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-6b-6bx^2}{x^6(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{a(a-b)f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{(a-6b)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5a^2(a-b)f} - \text{Subst}$$

$$= -\frac{b\cot^5(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\left(5a^2+4ab-24b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+f}}{15a^3(a-b)f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\left(15a^3+10a^2b+8ab^2-48b^3\right)\cot(e+fx)\sqrt{a+b\,\text{t}}}{15a^4(a-b)f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\left(15a^3+10a^2b+8ab^2-48b^3\right)\cot(e+fx)\sqrt{a+b\,\text{t}}}{15a^4(a-b)f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\left(15a^3+10a^2b+8ab^2-48b^3\right)\cot(e+fx)\sqrt{a+b\,\text{t}}}{15a^4(a-b)f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{b\cot^5(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\left(15a^3+10a^2b+8ab^2-4\right.}{}$$

Mathematica [C] time = 16.4838, size = 850, normalized size = 3.37

$$\frac{\sqrt{\frac{\cos(2(e+fx))a+a+b-b\cos(2(e+fx))}{\cos(2(e+fx))+1}}\left(\frac{\sin(2(e+fx))b^4}{a^4(a-b)(\cos(2(e+fx))a+a+b-b\cos(2(e+fx)))} - \frac{\cot(e+fx)\csc^4(e+fx)}{5a^2} + \frac{(11a\cos(e+fx)+9b\cos(e+fx))\csc^3(e-}{15a^3}\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((-((b*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*Sqr
t[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)
/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e +
 f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*
x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a - b)*Cos[2*(e + f*x)])
)) - (4*b*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)]
)/(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Co
s[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)]
```

```
)*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a -
b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(4*a*S
qrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]) - (Sqrt[-
((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)
]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*
x)]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)]
)*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(2*(a - b)*Sqrt[1 + Cos[2
*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])))/Sqrt[a + b + (a - b)
*Cos[2*(e + f*x)]])/((a - b)*f)) + (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Co
s[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(((-23*a^2*Cos[e + f*x] - 34*a*b*Co
s[e + f*x] - 33*b^2*Cos[e + f*x])*Csc[e + f*x])/(15*a^4) + ((11*a*Cos[e + f
*x] + 9*b*Cos[e + f*x])*Csc[e + f*x]^3)/(15*a^3) - (Cot[e + f*x]*Csc[e + f*
x]^4)/(5*a^2) + (b^4*Sin[2*(e + f*x)])/(a^4*(a - b)*(a + b + a*Cos[2*(e + f
*x)] - b*Cos[2*(e + f*x)]))))/f
```

Maple [C] time = 0.355, size = 3925, normalized size = 15.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/15/f/a^4/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/(a-b)/(a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)^2*(-30*cos(f*x+e)^5*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e
)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(c
os(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)
^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f
*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/
2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^
(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a^4+15*cos(f*x+e)^5*sin(f*x+e)*2^(1/2)*(
1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*
cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)
-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*E
llipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e
),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)
^(1/2))*a^4-30*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(
a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1
))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(
f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((
2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1
/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)/a)^(1/2))*a^4+15*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(
f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+
b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*
(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((c
os(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(
3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^4
+60*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-
I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-
2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*
cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)
*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*
a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*
b)/a)^(1/2))*a^4-30*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1
/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x
+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)
```

```
-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1
)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^
(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^4+60*cos(f*x
+e)^2*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(
a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(
f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-
b)/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2
)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^
(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)
)*a^4-30*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(
1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/
2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)
*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(
1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b
^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^4-30*cos(f*x+e)*sin(f*x
+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+co
s(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2
)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e
)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(
1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(
1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a^4+15*cos(
f*x+e)*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*
(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos
(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)
-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2
)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1
/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^4+10*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/
a)^(1/2)*a^2*b^2+8*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b^3-48*((2*I
*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^4+15*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(
f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+
b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*
(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1))^(1/2)*EllipticF((c
os(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(
3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^4
-30*sin(f*x+e)*2^(1/2)*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-
b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1))^(1/2)*(-2/a*(I*cos(f*
x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)
/(cos(f*x+e)+1))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+
a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1
/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*
a^4-12*cos(f*x+e)^6*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^3*b+34*cos(
f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^3*b-40*cos(f*x+e)^2*((
2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^3*b-26*cos(f*x+e)^2*((2*I*b^(1/2)
*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2*b^2-48*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1
/2)+a-2*b)/a)^(1/2)*a*b^3-12*cos(f*x+e)^6*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/
a)^(1/2)*a^2*b^2-32*cos(f*x+e)^6*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*
a*b^3+28*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2*b^2+72*
cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b^3+23*cos(f*x+e)^
6*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^4-144*cos(f*x+e)^4*((2*I*b^(1
/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^4+48*cos(f*x+e)^6*((2*I*b^(1/2)*(a-b)^(1/
2)+a-2*b)/a)^(1/2)*b^4-35*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^
(1/2)*a^4+15*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^4+15*
((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^3*b+144*cos(f*x+e)^2*((2*I*b^(1
/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^4)*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+
e)^2*b+b)/cos(f*x+e)^2)^(3/2)/sin(f*x+e)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.85181, size = 1569, normalized size = 6.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*(a^4*b*tan(f*x + e)^7 + a^5*tan(f*x + e)^5)*sqrt(-a + b)*log(-((a
^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2
 - 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)
*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 4*((15*a^4*b - 5*
a^3*b^2 - 2*a^2*b^3 - 56*a*b^4 + 48*b^5)*tan(f*x + e)^6 + 3*a^5 - 6*a^4*b +
 3*a^3*b^2 + (15*a^5 - 10*a^4*b - a^3*b^2 - 28*a^2*b^3 + 24*a*b^4)*tan(f*x
+ e)^4 - (5*a^5 - 4*a^4*b - 7*a^3*b^2 + 6*a^2*b^3)*tan(f*x + e)^2)*sqrt(b*t
an(f*x + e)^2 + a))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*f*tan(f*x + e)^7 + (a^7
- 2*a^6*b + a^5*b^2)*f*tan(f*x + e)^5), -1/30*(15*(a^4*b*tan(f*x + e)^7 + a
^5*tan(f*x + e)^5)*sqrt(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a
- b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a)) + 2*((15*a^4*b - 5*a^3*b^
2 - 2*a^2*b^3 - 56*a*b^4 + 48*b^5)*tan(f*x + e)^6 + 3*a^5 - 6*a^4*b + 3*a^3
*b^2 + (15*a^5 - 10*a^4*b - a^3*b^2 - 28*a^2*b^3 + 24*a*b^4)*tan(f*x + e)^4
 - (5*a^5 - 4*a^4*b - 7*a^3*b^2 + 6*a^2*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x
 + e)^2 + a))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*f*tan(f*x + e)^7 + (a^7 - 2*a^
6*b + a^5*b^2)*f*tan(f*x + e)^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^6\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**6/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^6}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^6/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

3.346 $\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=115

$$\frac{a^2}{3b^2f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{a(a-2b)}{b^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f)) + a^2/
(3*(a - b)*b^2*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (a*(a - 2*b))/((a - b)^2*b
^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.203918, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3670, 446, 87, 63, 208}

$$\frac{a^2}{3b^2f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{a(a-2b)}{b^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f)) + a^2/
(3*(a - b)*b^2*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (a*(a - 2*b))/((a - b)^2*b
^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)(a+bx)^{5/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{a^2}{(a-b)b(a+bx)^{5/2}} + \frac{a(a-2b)}{(a-b)^2b(a+bx)^{3/2}} + \frac{1}{(a-b)^2(1+x)\sqrt{a+bx}}\right)dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{a^2}{3(a-b)b^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(a-2b)}{(a-b)^2b^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{}{(1\cdots}\right.}{}$$

$$= \frac{a^2}{3(a-b)b^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(a-2b)}{(a-b)^2b^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{}{1-\cdots}\right.}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} + \frac{a^2}{3(a-b)b^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(a-2\cdots}{(a-b)^2b^2f\sqrt{a+b\cdots}}$$

Mathematica [C] time = 0.452866, size = 91, normalized size = 0.79

$$\frac{b^2\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\tan^2(e+fx)}{a-b}\right) - (a-b)\left(2a+3b\tan^2(e+fx)-b\right)}{3b^2f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (b^2*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[e + f*x]^2)/(a - b)] - (a
- b)*(2*a - b + 3*b*Tan[e + f*x]^2))/(3*(a - b)*b^2*f*(a + b*Tan[e + f*x]^2
)^(3/2))
```

Maple [A] time = 0.027, size = 169, normalized size = 1.5

$$-\frac{\left(\tan\left(fx+e\right)\right)^2}{fb}\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^{-\frac{3}{2}} - \frac{2a}{3fb^2}\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^{-\frac{3}{2}} + \frac{1}{3fb}\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^{-\frac{3}{2}} + \frac{}{(a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/f*tan(f*x+e)^2/b/(a+b*tan(f*x+e)^2)^(3/2)-2/3/f*a/b^2/(a+b*tan(f*x+e)^2)
^(3/2)+1/3/f/b/(a+b*tan(f*x+e)^2)^(3/2)+1/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2
)+1/f/(a-b)^2/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))+1/
3/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.50223, size = 1338, normalized size = 11.63

$$\left[\frac{3\left(b^4\tan\left(fx+e\right)^4+2\,ab^3\tan\left(fx+e\right)^2+a^2b^2\right)\sqrt{a-b}\log\left(-\frac{b^2\tan(fx+e)^4+2\,(4\,ab-3\,b^2)\tan(fx+e)^2-4\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan(\cdots}}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}\cdots\right.}{12\left(\left(a^3b^4-3\,a^2b^5+3\,ab^6-b^7\right)f\tan\left(fx+e\right)^4+2\left(a^4b^3-3\,a^3b^4+3\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^4*tan(f*x + e)^4 + 2*a*b^3*tan(f*x + e)^2 + a^2*b^2)*sqrt(a - b
)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e)^2 - 4*(b*tan(f*
x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b
+ b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 4*(2*a^4 - 7*a^3*b + 5*a^
2*b^2 + 3*(a^3*b - 3*a^2*b^2 + 2*a*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)
^2 + a))/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*f*tan(f*x + e)^4 + 2*(a^4*b
^3 - 3*a^3*b^4 + 3*a^2*b^5 - a*b^6)*f*tan(f*x + e)^2 + (a^5*b^2 - 3*a^4*b^3
 + 3*a^3*b^4 - a^2*b^5)*f), 1/6*(3*(b^4*tan(f*x + e)^4 + 2*a*b^3*tan(f*x +
e)^2 + a^2*b^2)*sqrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a +
b)/(b*tan(f*x + e)^2 + 2*a - b)) - 2*(2*a^4 - 7*a^3*b + 5*a^2*b^2 + 3*(a^3*
b - 3*a^2*b^2 + 2*a*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^3*
b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*f*tan(f*x + e)^4 + 2*(a^4*b^3 - 3*a^3*b^4
+ 3*a^2*b^5 - a*b^6)*f*tan(f*x + e)^2 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 -
a^2*b^5)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^5\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [A] time = 1.49219, size = 185, normalized size = 1.61

$$\frac{\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\left(a^2f-2\,abf+b^2f\right)\sqrt{-a+b}}-\frac{3\left(b\tan\left(fx+e\right)^2+a\right)a^2-a^3-6\left(b\tan\left(fx+e\right)^2+a\right)ab+a^2b}{3\left(a^2b^2f-2\,ab^3f+b^4f\right)\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/((a^2*f - 2*a*b*f + b^2*f)*
sqrt(-a + b)) - 1/3*(3*(b*tan(f*x + e)^2 + a)*a^2 - a^3 - 6*(b*tan(f*x + e)
^2 + a)*a*b + a^2*b)/((a^2*b^2*f - 2*a*b^3*f + b^4*f)*(b*tan(f*x + e)^2 + a
)^(3/2))
```

3.347 $\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{a}{3bf(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{1}{f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

```
[Out] ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f) - a/(3*(a
 - b)*b*f*(a + b*Tan[e + f*x]^2)^(3/2)) - 1/((a - b)^2*f*Sqrt[a + b*Tan[e +
 f*x]^2])
```

Rubi [A] time = 0.155672, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3670, 446, 78, 51, 63, 208}

$$-\frac{a}{3bf(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{1}{f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```


```
[Out] ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f) - a/(3*(a
 - b)*b*f*(a + b*Tan[e + f*x]^2)^(3/2)) - 1/((a - b)^2*f*Sqrt[a + b*Tan[e +
 f*x]^2])
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{(1+x)(a+bx)^{5/2}}\,dx,x,\tan^2(e+fx)\right)}{2f}$$

$$= -\frac{a}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx,x,\tan^2(e+fx)\right)}{2(a-b)f}$$

$$= -\frac{a}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{1}{(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{}{(1+x}\right.}{}$$

$$= -\frac{a}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{1}{(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}}\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} - \frac{a}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{1}{(a-b)^2f\sqrt{a+b\tan^2}}$$

Mathematica [C] time = 0.297833, size = 84, normalized size = 0.82

$$\frac{a(b-a)-3b\left(a+b\tan^2(e+fx)\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\tan^2(e+fx)}{a-b}\right)}{3bf(a-b)^2\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (a*(-a + b) - 3*b*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(a
 - b)]*(a + b*Tan[e + f*x]^2))/(3*(a - b)^2*b*f*(a + b*Tan[e + f*x]^2)^(3/2
))
```

Maple [A] time = 0.018, size = 118, normalized size = 1.2

$$-\frac{1}{3fb}\left(a+b\left(\tan\left(fx+e\right)\right)^{2}\right)^{-\frac{3}{2}}-\frac{1}{\left(a-b\right)^{2}f}\frac{1}{\sqrt{a+b\left(\tan\left(fx+e\right)\right)^{2}}}-\frac{1}{\left(a-b\right)^{2}f}\arctan\left(\sqrt{a+b\left(\tan\left(fx+e\right)\right)^{2}}\frac{1}{\sqrt{-a+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3/f/b/(a+b*tan(f*x+e)^2)^(3/2)-1/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)-1/f/
(a-b)^2/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))-1/3/(a-b
)/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.50577, size = 1269, normalized size = 12.32

$$\left[\frac{3\left(b^{3}\tan\left(fx+e\right)^{4}+2\,ab^{2}\tan\left(fx+e\right)^{2}+a^{2}b\right)\sqrt{a-b}\log\left(-\frac{b^{2}\tan\left(fx+e\right)^{4}+2\left(4\,ab-3\,b^{2}\right)\tan\left(fx+e\right)^{2}+4\left(b\tan\left(fx+e\right)^{2}+2\,a-b\right)\sqrt{b\tan\left(f\right.}}{\tan\left(fx+e\right)^{4}+2\,\tan\left(fx+e\right)^{2}+1}\right.}{12\left(\left(a^{3}b^{3}-3\,a^{2}b^{4}+3\,ab^{5}-b^{6}\right)f\tan\left(fx+e\right)^{4}+2\left(a^{4}b^{2}-3\,a^{3}b^{3}+3\,a^{2}b^{4}-a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^3*tan(f*x + e)^4 + 2*a*b^2*tan(f*x + e)^2 + a^2*b)*sqrt(a - b)*
log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e)^2 + 4*(b*tan(f*x
+ e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b +
b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 4*(a^3 + a^2*b - 2*a*b^2 +
3*(a*b^2 - b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^3*b^3 - 3*a
^2*b^4 + 3*a*b^5 - b^6)*f*tan(f*x + e)^4 + 2*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b
^4 - a*b^5)*f*tan(f*x + e)^2 + (a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*f)
, -1/6*(3*(b^3*tan(f*x + e)^4 + 2*a*b^2*tan(f*x + e)^2 + a^2*b)*sqrt(-a + b
)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^2 + 2*a
- b)) + 2*(a^3 + a^2*b - 2*a*b^2 + 3*(a*b^2 - b^3)*tan(f*x + e)^2)*sqrt(b*t
an(f*x + e)^2 + a))/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*f*tan(f*x + e)^4
```

```
 + 2*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*f*tan(f*x + e)^2 + (a^5*b -
3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [A] time = 1.44239, size = 157, normalized size = 1.52

$$-\frac{\frac{3\,b\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\left(a^2f-2\,abf+b^2f\right)\sqrt{-a+b}}+\frac{a^2+3\left(b\tan\left(fx+e\right)^2+a\right)b-ab}{\left(a^2f-2\,abf+b^2f\right)\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/((a^2*f - 2*a*b*f
 + b^2*f)*sqrt(-a + b)) + (a^2 + 3*(b*tan(f*x + e)^2 + a)*b - a*b)/((a^2*f
- 2*a*b*f + b^2*f)*(b*tan(f*x + e)^2 + a)^(3/2)))/b
```

3.348 $\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=99

$$\frac{1}{f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{1}{3f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f)) + 1/(3
*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) + 1/((a - b)^2*f*Sqrt[a + b*Tan[e
+ f*x]^2])
```

Rubi [A] time = 0.107685, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3670, 444, 51, 63, 208}

$$\frac{1}{f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{1}{3f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f)) + 1/(3
*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) + 1/((a - b)^2*f*Sqrt[a + b*Tan[e
+ f*x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{5/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{1}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{2(a-b)f}$$

$$= \frac{1}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{1}{(a-b)^2 f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a}}\right.}{2}$$

$$= \frac{1}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{1}{(a-b)^2 f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\right.}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} + \frac{1}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{1}{(a-b)^2 f\sqrt{a+b\tan}}$$

Mathematica [C] time = 0.16327, size = 58, normalized size = 0.59

$$\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\tan^2(e+fx)}{a-b}\right)}{3f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[e + f*x]^2)/(a - b)]/(3*(a - b)
*f*(a + b*Tan[e + f*x]^2)^(3/2))
```

Maple [A] time = 0.016, size = 94, normalized size = 1.

$$\frac{1}{(a-b)^2 f}\frac{1}{\sqrt{a+b\left(\tan\left(fx+e\right)\right)^2}} + \frac{1}{(a-b)^2 f}\arctan\left(\sqrt{a+b\left(\tan\left(fx+e\right)\right)^2}\frac{1}{\sqrt{-a+b}}\right)\frac{1}{\sqrt{-a+b}} + \frac{1}{(3a-3b)f}\left(a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)+1/f/(a-b)^2/(-a+b)^(1/2)*arctan((a+b*t
an(f*x+e)^2)^(1/2)/(-a+b)^(1/2))+1/3/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] time = 2.51089, size = 1230, normalized size = 12.42

$$\left[\frac{3\left(b^{2}\tan\left(fx+e\right)^{4}+2\,ab\tan\left(fx+e\right)^{2}+a^{2}\right)\sqrt{a-b}\log\left(-\frac{b^{2}\tan\left(fx+e\right)^{4}+2\left(4\,ab-3\,b^{2}\right)\tan\left(fx+e\right)^{2}-4\left(b\tan\left(fx+e\right)^{2}+2\,a-b\right)\sqrt{b\tan\left(fx+\cdots\right.}}{\tan\left(fx+e\right)^{4}+2\tan\left(fx+e\right)^{2}+1}\cdots\right.}{12\left(\left(a^{3}b^{2}-3\,a^{2}b^{3}+3\,ab^{4}-b^{5}\right)f\tan\left(fx+e\right)^{4}+2\left(a^{4}b-3\,a^{3}b^{2}+3\,a^{2}b^{3}-ab\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*sqrt(a - b)*log(
-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e)^2 - 4*(b*tan(f*x + e)
^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)
/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4*(3*(a*b - b^2)*tan(f*x + e)^2
 + 4*a^2 - 5*a*b + b^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^3*b^2 - 3*a^2*b^3 +
 3*a*b^4 - b^5)*f*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4
)*f*tan(f*x + e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f), 1/6*(3*(b^2*
tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*sqrt(-a + b)*arctan(2*sqrt(b*t
an(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^2 + 2*a - b)) + 2*(3*(a*b -
 b^2)*tan(f*x + e)^2 + 4*a^2 - 5*a*b + b^2)*sqrt(b*tan(f*x + e)^2 + a))/((a
^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2
 + 3*a^2*b^3 - a*b^4)*f*tan(f*x + e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b
^3)*f)]
```

Sympy [A] time = 19.7342, size = 83, normalized size = 0.84

$$\frac{1}{3f\left(a-b\right)\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}+\frac{1}{f\left(a-b\right)^{2}\sqrt{a+b\tan^{2}\left(e+fx\right)}}+\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\tan^{2}\left(e+fx\right)}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}\left(a-b\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] 1/(3*f*(a - b)*(a + b*tan(e + f*x)**2)**(3/2)) + 1/(f*(a - b)**2*sqrt(a + b
*tan(e + f*x)**2)) + atan(sqrt(a + b*tan(e + f*x)**2)/sqrt(-a + b))/(f*sqrt
(-a + b)*(a - b)**2)
```

Giac [A] time = 1.34103, size = 142, normalized size = 1.43

$$\frac{\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\left(a^2f-2\,abf+b^2f\right)\sqrt{-a+b}}+\frac{3\,b\tan\left(fx+e\right)^2+4\,a-b}{3\left(a^2f-2\,abf+b^2f\right)\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/((a^2*f - 2*a*b*f + b^2*f)*
sqrt(-a + b)) + 1/3*(3*b*tan(f*x + e)^2 + 4*a - b)/((a^2*f - 2*a*b*f + b^2*
f)*(b*tan(f*x + e)^2 + a)^(3/2))
```

3.349 $\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{b(2a-b)}{a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{b}{3af(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + ArcTanh[Sqrt[a
 + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f) - b/(3*a*(a - b)*f*(a +
 b*Tan[e + f*x]^2)^(3/2)) - ((2*a - b)*b)/(a^2*(a - b)^2*f*Sqrt[a + b*Tan[e
 + f*x]^2])
```

Rubi [A] time = 0.213703, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3670, 446, 85, 152, 156, 63, 208}

$$-\frac{b(2a-b)}{a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{b}{3af(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + ArcTanh[Sqrt[a
 + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f) - b/(3*a*(a - b)*f*(a +
 b*Tan[e + f*x]^2)^(3/2)) - ((2*a - b)*b)/(a^2*(a - b)^2*f*Sqrt[a + b*Tan[e
 + f*x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(1+x^2)(a+bx^2)^{5/2}}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(1+x)(a+bx)^{5/2}}\,dx,x,\tan^2(e+fx)\right)}{2f}$$

$$= -\frac{b}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a-b-bx}{x(1+x)(a+bx)^{3/2}}\,dx,x,\tan^2(e+fx)\right)}{2a(a-b)f}$$

$$= -\frac{b}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(2a-b)b}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(a-}{x(}\right.}{}$$

$$= -\frac{b}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(2a-b)b}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+}}\right.}{}$$

$$= -\frac{b}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(2a-b)b}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x}{}}\right.}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} - \frac{b}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3}}$$

Mathematica [C] time = 0.370316, size = 94, normalized size = 0.64

$$\frac{(a-b)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{b\tan^2(e+fx)}{a}+1\right)-a\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\tan^2(e+fx)}{a-b}\right)}{3af(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (-(a*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[e + f*x]^2)/(a - b)]) + (a
 - b)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Tan[e + f*x]^2)/a])/(3*a*(a -
 b)*f*(a + b*Tan[e + f*x]^2)^(3/2))
```

Maple [B] time = 15.647, size = 331597, normalized size = 2255.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.18106, size = 3623, normalized size = 24.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^3*b^2*tan(f*x + e)^4 + 2*a^4*b*tan(f*x + e)^2 + a^5)*sqrt(a - b)
*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b
)/(tan(f*x + e)^2 + 1)) + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^3*b^2
 - 3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2
*b^3 - a*b^4)*tan(f*x + e)^2)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(
f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) - 2*(7*a^4*b - 11*a^3*b^2 +
4*a^2*b^3 + 3*(2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*
x + e)^2 + a))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^
4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x + e)^2 + (a^8 - 3
*a^7*b + 3*a^6*b^2 - a^5*b^3)*f), 1/6*(6*(a^3*b^2*tan(f*x + e)^4 + 2*a^4*b*
tan(f*x + e)^2 + a^5)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(
-a + b)/(a - b)) + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^3*b^2 - 3*a^
2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 -
a*b^4)*tan(f*x + e)^2)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e
)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) - 2*(7*a^4*b - 11*a^3*b^2 + 4*a^2*b
^3 + 3*(2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^
2 + a))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^4 + 2*(
a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x + e)^2 + (a^8 - 3*a^7*b
+ 3*a^6*b^2 - a^5*b^3)*f), 1/6*(6*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a
^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 +
 3*a^2*b^3 - a*b^4)*tan(f*x + e)^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 +
 a)*sqrt(-a)/a) + 3*(a^3*b^2*tan(f*x + e)^4 + 2*a^4*b*tan(f*x + e)^2 + a^5)
*sqrt(a - b)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a -
b) + 2*a - b)/(tan(f*x + e)^2 + 1)) - 2*(7*a^4*b - 11*a^3*b^2 + 4*a^2*b^3 +
 3*(2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 +
a))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^4 + 2*(a^7*
b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x + e)^2 + (a^8 - 3*a^7*b + 3*
a^6*b^2 - a^5*b^3)*f), 1/3*(3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^3*b
^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a
^2*b^3 - a*b^4)*tan(f*x + e)^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*
sqrt(-a)/a) + 3*(a^3*b^2*tan(f*x + e)^4 + 2*a^4*b*tan(f*x + e)^2 + a^5)*sqr
t(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b)) - (7*a^4
*b - 11*a^3*b^2 + 4*a^2*b^3 + 3*(2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(f*x + e
)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^
5)*f*tan(f*x + e)^4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x
 + e)^2 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

3.350 $\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=206

$$-\frac{b\left(a^2-8ab+5b^2\right)}{2a^3f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{b(3a-5b)}{6a^2f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}+\frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f}-\tanh$$

```
[Out] ((2*a + 5*b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(2*a^(7/2)*f) - A
rcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f) - ((3*a -
5*b)*b)/(6*a^2*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - Cot[e + f*x]^2/(2*
a*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (b*(a^2 - 8*a*b + 5*b^2))/(2*a^3*(a - b
)^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.350195, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3670, 446, 103, 152, 156, 63, 208}

$$-\frac{b\left(a^2-8ab+5b^2\right)}{2a^3f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{b(3a-5b)}{6a^2f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}+\frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f}-\tanh$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*a + 5*b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(2*a^(7/2)*f) - A
rcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f) - ((3*a -
5*b)*b)/(6*a^2*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - Cot[e + f*x]^2/(2*
a*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (b*(a^2 - 8*a*b + 5*b^2))/(2*a^3*(a - b
)^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
```

```
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x)(a+bx)^{5/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2a+5b)+\frac{5bx}{2}}{x(1+x)(a+bx)^{5/2}}\,dx, x, \tan^2(e+fx)\right)}{2af}$$

$$= -\frac{(3a-5b)b}{6a^2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^2(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{4}(a-}{}\right.}{}$$

$$= -\frac{(3a-5b)b}{6a^2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^2(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{b\left(a^2-8\right.}{2a^3(a-b)^2f\sqrt{a}}$$

$$= -\frac{(3a-5b)b}{6a^2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^2(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{b\left(a^2-8\right.}{2a^3(a-b)^2f\sqrt{a}}$$

$$= -\frac{(3a-5b)b}{6a^2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^2(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{b\left(a^2-8\right.}{2a^3(a-b)^2f\sqrt{a}}$$

$$= \frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} - \frac{(3a-5b)}{6a^2(a-b)f\left(a+b\tan\right.}$$

Mathematica [C] time = 0.583185, size = 138, normalized size = 0.67

$$\frac{\cot^2(e+fx)\left((a-b)\left((2a+5b)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{b\tan^2(e+fx)}{a}+1\right)+3a\cot^2(e+fx)\right)-2a^2\text{Hypergeo}\right.}{6a^2f(b-a)\sqrt{a+b\tan^2(e+fx)}\left(a\cot^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cot[e + f*x]^2*(-2*a^2*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[e + f*x
]^2)/(a - b)] + (a - b)*(3*a*Cot[e + f*x]^2 + (2*a + 5*b)*Hypergeometric2F1
[-3/2, 1, -1/2, 1 + (b*Tan[e + f*x]^2)/a])))/(6*a^2*(-a + b)*f*(b + a*Cot[e
 + f*x]^2)*Sqrt[a + b*Tan[e + f*x]^2])
```

Maple [B] time = 34.855, size = 531573, normalized size = 2580.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.66252, size = 4601, normalized size = 22.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(6*(a^4*b^2*tan(f*x + e)^6 + 2*a^5*b*tan(f*x + e)^4 + a^6*tan(f*x + e
)^2)*sqrt(a - b)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) + 3*((2*a^4*b^2 - a^3*b^3 - 9*a^2*b
^4 + 13*a*b^5 - 5*b^6)*tan(f*x + e)^6 + 2*(2*a^5*b - a^4*b^2 - 9*a^3*b^3 +
13*a^2*b^4 - 5*a*b^5)*tan(f*x + e)^4 + (2*a^6 - a^5*b - 9*a^4*b^2 + 13*a^3*
b^3 - 5*a^2*b^4)*tan(f*x + e)^2)*sqrt(a)*log((b*tan(f*x + e)^2 + 2*sqrt(b*t
an(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) - 2*(3*a^6 - 9*a^5*b + 9*
a^4*b^2 - 3*a^3*b^3 + 3*(a^4*b^2 - 9*a^3*b^3 + 13*a^2*b^4 - 5*a*b^5)*tan(f*
x + e)^4 + 2*(3*a^5*b - 19*a^4*b^2 + 26*a^3*b^3 - 10*a^2*b^4)*tan(f*x + e)^
2)*sqrt(b*tan(f*x + e)^2 + a))/((a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)
*f*tan(f*x + e)^6 + 2*(a^8*b - 3*a^7*b^2 + 3*a^6*b^3 - a^5*b^4)*f*tan(f*x +
 e)^4 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^6*b^3)*f*tan(f*x + e)^2), -1/12*(12*
(a^4*b^2*tan(f*x + e)^6 + 2*a^5*b*tan(f*x + e)^4 + a^6*tan(f*x + e)^2)*sqrt
(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b)) - 3*((2*a
^4*b^2 - a^3*b^3 - 9*a^2*b^4 + 13*a*b^5 - 5*b^6)*tan(f*x + e)^6 + 2*(2*a^5*
b - a^4*b^2 - 9*a^3*b^3 + 13*a^2*b^4 - 5*a*b^5)*tan(f*x + e)^4 + (2*a^6 - a
^5*b - 9*a^4*b^2 + 13*a^3*b^3 - 5*a^2*b^4)*tan(f*x + e)^2)*sqrt(a)*log((b*t
an(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)
 + 2*(3*a^6 - 9*a^5*b + 9*a^4*b^2 - 3*a^3*b^3 + 3*(a^4*b^2 - 9*a^3*b^3 + 13
*a^2*b^4 - 5*a*b^5)*tan(f*x + e)^4 + 2*(3*a^5*b - 19*a^4*b^2 + 26*a^3*b^3 -
 10*a^2*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^7*b^2 - 3*a^6*
b^3 + 3*a^5*b^4 - a^4*b^5)*f*tan(f*x + e)^6 + 2*(a^8*b - 3*a^7*b^2 + 3*a^6*
b^3 - a^5*b^4)*f*tan(f*x + e)^4 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^6*b^3)*f*t
an(f*x + e)^2), -1/6*(3*((2*a^4*b^2 - a^3*b^3 - 9*a^2*b^4 + 13*a*b^5 - 5*b^
6)*tan(f*x + e)^6 + 2*(2*a^5*b - a^4*b^2 - 9*a^3*b^3 + 13*a^2*b^4 - 5*a*b^5
)*tan(f*x + e)^4 + (2*a^6 - a^5*b - 9*a^4*b^2 + 13*a^3*b^3 - 5*a^2*b^4)*tan
(f*x + e)^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) - 3*(a^
4*b^2*tan(f*x + e)^6 + 2*a^5*b*tan(f*x + e)^4 + a^6*tan(f*x + e)^2)*sqrt(a
- b)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a
 - b)/(tan(f*x + e)^2 + 1)) + (3*a^6 - 9*a^5*b + 9*a^4*b^2 - 3*a^3*b^3 + 3*
(a^4*b^2 - 9*a^3*b^3 + 13*a^2*b^4 - 5*a*b^5)*tan(f*x + e)^4 + 2*(3*a^5*b -
19*a^4*b^2 + 26*a^3*b^3 - 10*a^2*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2
```

```
 + a))/((a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*f*tan(f*x + e)^6 + 2*(a
^8*b - 3*a^7*b^2 + 3*a^6*b^3 - a^5*b^4)*f*tan(f*x + e)^4 + (a^9 - 3*a^8*b +
 3*a^7*b^2 - a^6*b^3)*f*tan(f*x + e)^2), -1/6*(3*((2*a^4*b^2 - a^3*b^3 - 9*
a^2*b^4 + 13*a*b^5 - 5*b^6)*tan(f*x + e)^6 + 2*(2*a^5*b - a^4*b^2 - 9*a^3*b
^3 + 13*a^2*b^4 - 5*a*b^5)*tan(f*x + e)^4 + (2*a^6 - a^5*b - 9*a^4*b^2 + 13
*a^3*b^3 - 5*a^2*b^4)*tan(f*x + e)^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2
 + a)*sqrt(-a)/a) + 6*(a^4*b^2*tan(f*x + e)^6 + 2*a^5*b*tan(f*x + e)^4 + a^
6*tan(f*x + e)^2)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a +
 b)/(a - b)) + (3*a^6 - 9*a^5*b + 9*a^4*b^2 - 3*a^3*b^3 + 3*(a^4*b^2 - 9*a^
3*b^3 + 13*a^2*b^4 - 5*a*b^5)*tan(f*x + e)^4 + 2*(3*a^5*b - 19*a^4*b^2 + 26
*a^3*b^3 - 10*a^2*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^7*b^
2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*f*tan(f*x + e)^6 + 2*(a^8*b - 3*a^7*b^
2 + 3*a^6*b^3 - a^5*b^4)*f*tan(f*x + e)^4 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^
6*b^3)*f*tan(f*x + e)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^3}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^3/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

3.351 $\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=272

$$\frac{b\left(3a^2b+4a^3-50ab^2+35b^3\right)}{8a^4f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{b\left(12a^2+15ab-35b^2\right)}{24a^3f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2+20ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f}$$

```
[Out] -((8*a^2 + 20*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(8
*a^(9/2)*f) + ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2
)*f) + (b*(12*a^2 + 15*a*b - 35*b^2))/(24*a^3*(a - b)*f*(a + b*Tan[e + f*x]
^2)^(3/2)) + ((4*a + 7*b)*Cot[e + f*x]^2)/(8*a^2*f*(a + b*Tan[e + f*x]^2)^(
3/2)) - Cot[e + f*x]^4/(4*a*f*(a + b*Tan[e + f*x]^2)^(3/2)) + (b*(4*a^3 + 3
*a^2*b - 50*a*b^2 + 35*b^3))/(8*a^4*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.443041, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3670, 446, 103, 151, 152, 156, 63, 208}

$$\frac{b\left(3a^2b+4a^3-50ab^2+35b^3\right)}{8a^4f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{b\left(12a^2+15ab-35b^2\right)}{24a^3f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2+20ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```


```
[Out] -((8*a^2 + 20*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(8
*a^(9/2)*f) + ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2
)*f) + (b*(12*a^2 + 15*a*b - 35*b^2))/(24*a^3*(a - b)*f*(a + b*Tan[e + f*x]
^2)^(3/2)) + ((4*a + 7*b)*Cot[e + f*x]^2)/(8*a^2*f*(a + b*Tan[e + f*x]^2)^(
3/2)) - Cot[e + f*x]^4/(4*a*f*(a + b*Tan[e + f*x]^2)^(3/2)) + (b*(4*a^3 + 3
*a^2*b - 50*a*b^2 + 35*b^3))/(8*a^4*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^5(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x)(a+bx)^{5/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4a+7b)+\frac{7bx}{2}}{x^2(1+x)(a+bx)^{5/2}}\,dx, x, \tan^2(e+fx)\right)}{4af}$$

$$= \frac{(4a+7b)\cot^2(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^4(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{4}(8a^2+20ab+35b^2)+ \ldots}{x(1+x)(a+bx) \ldots}\right.}{4 \ldots}$$

$$= \frac{b\left(12a^2+15ab-35b^2\right)}{24a^3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(4a+7b)\cot^2(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^4(e+f \ldots}{4af\left(a+b\tan^2(e \ldots\right.}$$

$$= \frac{b\left(12a^2+15ab-35b^2\right)}{24a^3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(4a+7b)\cot^2(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^4(e+f \ldots}{4af\left(a+b\tan^2(e \ldots\right.}$$

$$= \frac{b\left(12a^2+15ab-35b^2\right)}{24a^3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(4a+7b)\cot^2(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^4(e+f \ldots}{4af\left(a+b\tan^2(e \ldots\right.}$$

$$= \frac{b\left(12a^2+15ab-35b^2\right)}{24a^3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(4a+7b)\cot^2(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^4(e+f \ldots}{4af\left(a+b\tan^2(e \ldots\right.}$$

$$= -\frac{\left(8a^2+20ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} + \frac{b\,(12 \ldots}{24a^3(a-b) \ldots}$$

Mathematica [C] time = 1.99222, size = 165, normalized size = 0.61

$$\frac{\cot^2(e+fx)\left((a-b)\left(3a\cot^2(e+fx)\left(2a\cot^2(e+fx)-4a-7b\right)-\left(8a^2+20ab+35b^2\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,- \ldots\right.\right.\right.}{24a^3f(b-a)\sqrt{a+b\tan^2(e+fx)}\left(a\cot^2(e+fx) \ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cot[e + f*x]^2*(8*a^3*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[e + f*x]
^2)/(a - b)] + (a - b)*(3*a*Cot[e + f*x]^2*(-4*a - 7*b + 2*a*Cot[e + f*x]^2
) - (8*a^2 + 20*a*b + 35*b^2)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Tan[e
 + f*x]^2)/a])))/(24*a^3*(-a + b)*f*(b + a*Cot[e + f*x]^2)*Sqrt[a + b*Tan[e
 + f*x]^2])
```

Maple [B] time = 58.722, size = 790286, normalized size = 2905.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.66564, size = 5387, normalized size = 19.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(24*(a^5*b^2*tan(f*x + e)^8 + 2*a^6*b*tan(f*x + e)^6 + a^7*tan(f*x +
e)^4)*sqrt(a - b)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt
(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) + 3*((8*a^5*b^2 - 4*a^4*b^3 - a^3*
b^4 - 53*a^2*b^5 + 85*a*b^6 - 35*b^7)*tan(f*x + e)^8 + 2*(8*a^6*b - 4*a^5*b
^2 - a^4*b^3 - 53*a^3*b^4 + 85*a^2*b^5 - 35*a*b^6)*tan(f*x + e)^6 + (8*a^7
- 4*a^6*b - a^5*b^2 - 53*a^4*b^3 + 85*a^3*b^4 - 35*a^2*b^5)*tan(f*x + e)^4)
*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a
)/tan(f*x + e)^2) - 2*(6*a^7 - 18*a^6*b + 18*a^5*b^2 - 6*a^4*b^3 - 3*(4*a^5
*b^2 - a^4*b^3 - 53*a^3*b^4 + 85*a^2*b^5 - 35*a*b^6)*tan(f*x + e)^6 - 4*(6*
a^6*b - 3*a^5*b^2 - 53*a^4*b^3 + 85*a^3*b^4 - 35*a^2*b^5)*tan(f*x + e)^4 -
3*(4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*tan(f*x + e)^2)*sq
rt(b*tan(f*x + e)^2 + a))/((a^8*b^2 - 3*a^7*b^3 + 3*a^6*b^4 - a^5*b^5)*f*ta
n(f*x + e)^8 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*b^3 - a^6*b^4)*f*tan(f*x + e)^6
 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f*tan(f*x + e)^4), 1/48*(48*(a^5*
b^2*tan(f*x + e)^8 + 2*a^6*b*tan(f*x + e)^6 + a^7*tan(f*x + e)^4)*sqrt(-a +
 b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b)) + 3*((8*a^5*b^
2 - 4*a^4*b^3 - a^3*b^4 - 53*a^2*b^5 + 85*a*b^6 - 35*b^7)*tan(f*x + e)^8 +
2*(8*a^6*b - 4*a^5*b^2 - a^4*b^3 - 53*a^3*b^4 + 85*a^2*b^5 - 35*a*b^6)*tan(
f*x + e)^6 + (8*a^7 - 4*a^6*b - a^5*b^2 - 53*a^4*b^3 + 85*a^3*b^4 - 35*a^2*
b^5)*tan(f*x + e)^4)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^
2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) - 2*(6*a^7 - 18*a^6*b + 18*a^5*b^2 -
6*a^4*b^3 - 3*(4*a^5*b^2 - a^4*b^3 - 53*a^3*b^4 + 85*a^2*b^5 - 35*a*b^6)*ta
n(f*x + e)^6 - 4*(6*a^6*b - 3*a^5*b^2 - 53*a^4*b^3 + 85*a^3*b^4 - 35*a^2*b^
5)*tan(f*x + e)^4 - 3*(4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4
)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^8*b^2 - 3*a^7*b^3 + 3*a^6
*b^4 - a^5*b^5)*f*tan(f*x + e)^8 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*b^3 - a^6*b
^4)*f*tan(f*x + e)^6 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f*tan(f*x + e
)^4), 1/24*(3*((8*a^5*b^2 - 4*a^4*b^3 - a^3*b^4 - 53*a^2*b^5 + 85*a*b^6 - 3
5*b^7)*tan(f*x + e)^8 + 2*(8*a^6*b - 4*a^5*b^2 - a^4*b^3 - 53*a^3*b^4 + 85*
a^2*b^5 - 35*a*b^6)*tan(f*x + e)^6 + (8*a^7 - 4*a^6*b - a^5*b^2 - 53*a^4*b^
```

```
3 + 85*a^3*b^4 - 35*a^2*b^5)*tan(f*x + e)^4)*sqrt(-a)*arctan(sqrt(b*tan(f*x
 + e)^2 + a)*sqrt(-a)/a) + 12*(a^5*b^2*tan(f*x + e)^8 + 2*a^6*b*tan(f*x + e
)^6 + a^7*tan(f*x + e)^4)*sqrt(a - b)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(
f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) - (6*a^7 - 18*
a^6*b + 18*a^5*b^2 - 6*a^4*b^3 - 3*(4*a^5*b^2 - a^4*b^3 - 53*a^3*b^4 + 85*a
^2*b^5 - 35*a*b^6)*tan(f*x + e)^6 - 4*(6*a^6*b - 3*a^5*b^2 - 53*a^4*b^3 + 8
5*a^3*b^4 - 35*a^2*b^5)*tan(f*x + e)^4 - 3*(4*a^7 - 5*a^6*b - 9*a^5*b^2 + 1
7*a^4*b^3 - 7*a^3*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^8*b^
2 - 3*a^7*b^3 + 3*a^6*b^4 - a^5*b^5)*f*tan(f*x + e)^8 + 2*(a^9*b - 3*a^8*b^
2 + 3*a^7*b^3 - a^6*b^4)*f*tan(f*x + e)^6 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a
^7*b^3)*f*tan(f*x + e)^4), 1/24*(3*((8*a^5*b^2 - 4*a^4*b^3 - a^3*b^4 - 53*a
^2*b^5 + 85*a*b^6 - 35*b^7)*tan(f*x + e)^8 + 2*(8*a^6*b - 4*a^5*b^2 - a^4*b
^3 - 53*a^3*b^4 + 85*a^2*b^5 - 35*a*b^6)*tan(f*x + e)^6 + (8*a^7 - 4*a^6*b
- a^5*b^2 - 53*a^4*b^3 + 85*a^3*b^4 - 35*a^2*b^5)*tan(f*x + e)^4)*sqrt(-a)*
arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) + 24*(a^5*b^2*tan(f*x + e)^8
+ 2*a^6*b*tan(f*x + e)^6 + a^7*tan(f*x + e)^4)*sqrt(-a + b)*arctan(-sqrt(b*
tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b)) - (6*a^7 - 18*a^6*b + 18*a^5*b^2
- 6*a^4*b^3 - 3*(4*a^5*b^2 - a^4*b^3 - 53*a^3*b^4 + 85*a^2*b^5 - 35*a*b^6)*
tan(f*x + e)^6 - 4*(6*a^6*b - 3*a^5*b^2 - 53*a^4*b^3 + 85*a^3*b^4 - 35*a^2*
b^5)*tan(f*x + e)^4 - 3*(4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b
^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^8*b^2 - 3*a^7*b^3 + 3*a
^6*b^4 - a^5*b^5)*f*tan(f*x + e)^8 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*b^3 - a^6
*b^4)*f*tan(f*x + e)^6 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f*tan(f*x +
 e)^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(fx+e)^5}{\left(b\tan(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^5/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

3.352 $\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{a(a-2b)\tan(e+fx)}{b^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{b^{5/2}f}-\frac{a\tan^3(e+fx)}{3bf(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}$$

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5
/2)*f)) + ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(b^(5/
2)*f) - (a*Tan[e + f*x]^3)/(3*(a - b)*b*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (
a*(a - 2*b)*Tan[e + f*x])/((a - b)^2*b^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.266733, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3670, 470, 578, 523, 217, 206, 377, 203}

$$-\frac{a(a-2b)\tan(e+fx)}{b^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{b^{5/2}f}-\frac{a\tan^3(e+fx)}{3bf(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5
/2)*f)) + ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(b^(5/
2)*f) - (a*Tan[e + f*x]^3)/(3*(a - b)*b*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (
a*(a - 2*b)*Tan[e + f*x])/((a - b)^2*b^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
```

```
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan^3(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{x^2\left(3a+3(a-b)x^2\right)}{(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3(a-b)bf}$$

$$= -\frac{a\tan^3(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(a-2b)\tan(e+fx)}{(a-b)^2b^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-3}{\ldots}\right.}{\ldots}$$

$$= -\frac{a\tan^3(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(a-2b)\tan(e+fx)}{(a-b)^2b^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\ldots}{(1\ldots}\right.}{\ldots}$$

$$= -\frac{a\tan^3(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(a-2b)\tan(e+fx)}{(a-b)^2b^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\ldots}{1-\ldots}\right.}{\ldots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{b^{5/2}f} - \frac{a\tan^3(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)\ldots}$$

Mathematica [C] time = 4.44009, size = 295, normalized size = 1.73

$$-\frac{\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(a^2(a-b)\sin(2(e+fx))((3a-7b)((a-b)\cos(2(e+fx))+a+b)+2a\ldots\right.}{3\sqrt{2}a\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(a^2*(a - b)*(2*a
*b + (3*a - 7*b)*(a + b + (a - b)*Cos[2*(e + f*x)]))*Sin[2*(e + f*x)] - (3*
a^2*b*(((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)^(3/2)*((a^2 -
 3*a*b + 2*b^2)*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*C
sc[e + f*x]^2)/b]/Sqrt[2]], 1] + b^2*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[(
(a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sin[e +
 f*x]^2*Sin[2*(e + f*x)])/Sqrt[2]))/(3*Sqrt[2]*a*(a - b)^3*b^2*f*(a + b + (
a - b)*Cos[2*(e + f*x)])^2)
```

Maple [B] time = 0.047, size = 382, normalized size = 2.2

$$-\frac{\left(\tan\left(fx+e\right)\right)^3}{3fb}\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^{-\frac{3}{2}} - \frac{\tan\left(fx+e\right)}{fb^2}\frac{1}{\sqrt{a+b\left(\tan\left(fx+e\right)\right)^2}} + \frac{1}{f}\ln\left(\sqrt{b}\tan\left(fx+e\right)+\sqrt{a+\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3/f*tan(f*x+e)^3/b/(a+b*tan(f*x+e)^2)^(3/2)-1/f/b^2*tan(f*x+e)/(a+b*tan(
f*x+e)^2)^(1/2)+1/f/b^(5/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))
+1/3/f/b*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2)-1/3/f/a/b*tan(f*x+e)/(a+b*tan(
f*x+e)^2)^(1/2)+1/3/f*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(3/2)+2/3/f/a^2*tan(f
*x+e)/(a+b*tan(f*x+e)^2)^(1/2)+1/f/(a-b)^2*b*tan(f*x+e)/a/(a+b*tan(f*x+e)^2
)^(1/2)-1/f/(a-b)^3*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2
)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))+1/3*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(
f*x+e)^2)^(3/2)+2/3/f/(a-b)*b/a^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 18.9618, size = 3800, normalized size = 22.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b
^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*tan(f*
x + e)^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqr
t(b)*tan(f*x + e) + a) - 3*(b^5*tan(f*x + e)^4 + 2*a*b^4*tan(f*x + e)^2 + a
^2*b^3)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)
^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - 2*((4*a^3*b^
2 - 11*a^2*b^3 + 7*a*b^4)*tan(f*x + e)^3 + 3*(a^4*b - 3*a^3*b^2 + 2*a^2*b^3
)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a^3*b^5 - 3*a^2*b^6 + 3*a*b^7
 - b^8)*f*tan(f*x + e)^4 + 2*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*f*ta
n(f*x + e)^2 + (a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*f), -1/6*(6*(a^5
 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*ta
n(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*tan(f*x + e)^2)*sq
rt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + 3*(b^
5*tan(f*x + e)^4 + 2*a*b^4*tan(f*x + e)^2 + a^2*b^3)*sqrt(-a + b)*log(-((a
- 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x +
 e) - a)/(tan(f*x + e)^2 + 1)) + 2*((4*a^3*b^2 - 11*a^2*b^3 + 7*a*b^4)*tan(
f*x + e)^3 + 3*(a^4*b - 3*a^3*b^2 + 2*a^2*b^3)*tan(f*x + e))*sqrt(b*tan(f*x
 + e)^2 + a))/((a^3*b^5 - 3*a^2*b^6 + 3*a*b^7 - b^8)*f*tan(f*x + e)^4 + 2*(
a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*f*tan(f*x + e)^2 + (a^5*b^3 - 3*a^
4*b^4 + 3*a^3*b^5 - a^2*b^6)*f), -1/6*(6*(b^5*tan(f*x + e)^4 + 2*a*b^4*tan(
f*x + e)^2 + a^2*b^3)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(
a - b)*tan(f*x + e))) - 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^3*b^2 -
 3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b
^3 - a*b^4)*tan(f*x + e)^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f
*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) + 2*((4*a^3*b^2 - 11*a^2*b^3 + 7*a
```

```
*b^4)*tan(f*x + e)^3 + 3*(a^4*b - 3*a^3*b^2 + 2*a^2*b^3)*tan(f*x + e))*sqrt
(b*tan(f*x + e)^2 + a))/((a^3*b^5 - 3*a^2*b^6 + 3*a*b^7 - b^8)*f*tan(f*x +
e)^4 + 2*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*f*tan(f*x + e)^2 + (a^5*
b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*f), -1/3*(3*(b^5*tan(f*x + e)^4 + 2*
a*b^4*tan(f*x + e)^2 + a^2*b^3)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 +
 a)/(sqrt(a - b)*tan(f*x + e))) + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 +
(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2
 + 3*a^2*b^3 - a*b^4)*tan(f*x + e)^2)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2
 + a)*sqrt(-b)/(b*tan(f*x + e))) + ((4*a^3*b^2 - 11*a^2*b^3 + 7*a*b^4)*tan(
f*x + e)^3 + 3*(a^4*b - 3*a^3*b^2 + 2*a^2*b^3)*tan(f*x + e))*sqrt(b*tan(f*x
 + e)^2 + a))/((a^3*b^5 - 3*a^2*b^6 + 3*a*b^7 - b^8)*f*tan(f*x + e)^4 + 2*(
a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*f*tan(f*x + e)^2 + (a^5*b^3 - 3*a^
4*b^4 + 3*a^3*b^5 - a^2*b^6)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^6\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**6/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^6}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^6/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

3.353 $\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=131

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}+\frac{(a-4b)\tan(e+fx)}{3bf(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{a\tan(e+fx)}{3bf(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5/2
)*f) - (a*Tan[e + f*x])/(3*(a - b)*b*f*(a + b*Tan[e + f*x]^2)^(3/2)) + ((a
- 4*b)*Tan[e + f*x])/(3*(a - b)^2*b*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.158553, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3670, 470, 527, 12, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}+\frac{(a-4b)\tan(e+fx)}{3bf(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{a\tan(e+fx)}{3bf(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5/2
)*f) - (a*Tan[e + f*x])/(3*(a - b)*b*f*(a + b*Tan[e + f*x]^2)^(3/2)) + ((a
- 4*b)*Tan[e + f*x])/(3*(a - b)^2*b*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
```

```
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+(a-3b)x^2}{\left(1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3(a-b)bf}$$

$$= -\frac{a\tan(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-4b)\tan(e+fx)}{3(a-b)^2bf\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{a\tan(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-4b)\tan(e+fx)}{3(a-b)^2bf\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{a\tan(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-4b)\tan(e+fx)}{3(a-b)^2bf\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{a\tan(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-4b)\tan(e+}{3(a-b)^2bf\sqrt{a+b\tan}}$$

Mathematica [A] time = 6.03819, size = 260, normalized size = 1.98

$$\frac{\tan^5(e+fx)\left(\frac{b\tan^2(e+fx)}{a}+1\right)\left(-\frac{\left(\frac{b\tan^2(e+fx)}{a}-\tan^2(e+fx)\right)^2}{3\left(\frac{b\tan^2(e+fx)}{a}+1\right)^2} - \frac{\frac{b\tan^2(e+fx)}{a}-\tan^2(e+fx)}{\frac{b\tan^2(e+fx)}{a}+1} + \frac{\sqrt{\frac{b\tan^2(e+fx)}{a}-\tan^2(e+fx)}\tanh^{-1}\left(\frac{\sqrt{\frac{b\tan^2(e+f}{a}}}{\sqrt{\frac{b\tan^2}{}}}\right.}{\sqrt{\frac{b\tan^2(e+fx)}{a}+1}}\right.}{a^2f\sqrt{a+b\tan^2(e+fx)}\left(\frac{b\tan^2(e+fx)}{a}-\tan^2(e+fx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Tan[e + f*x]^5*(1 + (b*Tan[e + f*x]^2)/a)*((ArcTanh[Sqrt[-Tan[e + f*x]^2 +
 (b*Tan[e + f*x]^2)/a]/Sqrt[1 + (b*Tan[e + f*x]^2)/a]]*Sqrt[-Tan[e + f*x]^2
 + (b*Tan[e + f*x]^2)/a])/Sqrt[1 + (b*Tan[e + f*x]^2)/a] - (-Tan[e + f*x]^2
 + (b*Tan[e + f*x]^2)/a)/(1 + (b*Tan[e + f*x]^2)/a) - (-Tan[e + f*x]^2 + (b
*Tan[e + f*x]^2)/a)^2/(3*(1 + (b*Tan[e + f*x]^2)/a)^2)))/(a^2*f*Sqrt[a + b*
Tan[e + f*x]^2]*(-Tan[e + f*x]^2 + (b*Tan[e + f*x]^2)/a)^3)
```

Maple [B] time = 0.046, size = 291, normalized size = 2.2

$$-\frac{\tan(fx+e)}{3fb}\left(a+b\left(\tan(fx+e)\right)^2\right)^{-\frac{3}{2}}+\frac{\tan(fx+e)}{3fab}\frac{1}{\sqrt{a+b\left(\tan(fx+e)\right)^2}}-\frac{\tan(fx+e)}{3fa}\left(a+b\left(\tan(fx+e)\right)^2\right)^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3/f/b*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2)+1/3/f/a/b*tan(f*x+e)/(a+b*tan(
f*x+e)^2)^(1/2)-1/3/f*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(3/2)-2/3/f/a^2*tan(f
*x+e)/(a+b*tan(f*x+e)^2)^(1/2)-1/f/(a-b)^2*b*tan(f*x+e)/a/(a+b*tan(f*x+e)^2
)^(1/2)+1/f/(a-b)^3*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2
)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-1/3*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(
f*x+e)^2)^(3/2)-2/3/f/(a-b)*b/a^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.19079, size = 1137, normalized size = 8.68

$$\left[-\frac{3\left(b^2\tan(fx+e)^4+2\,ab\tan(fx+e)^2+a^2\right)\sqrt{-a+b}\log\left(-\frac{(a-2\,b)\tan(fx+e)^2-2\,\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}\tan(fx+e)-a}{\tan(fx+e)^2+1}\right)-2\left(\left(a^2\right.\right.}{6\left(\left(a^3b^2-3\,a^2b^3+3\,ab^4-b^5\right)f\tan(fx+e)^4+2\left(a^4b-3\,a^3b^2+3\,a^2b^3-ab^4\right)f\tan\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*sqrt(-a + b)*log
(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan
(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - 2*((a^2 - 5*a*b + 4*b^2)*tan(f*x + e
)^3 - 3*(a^2 - a*b)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a^3*b^2 - 3
*a^2*b^3 + 3*a*b^4 - b^5)*f*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b
^3 - a*b^4)*f*tan(f*x + e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f), 1/
3*(3*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*sqrt(a - b)*arctan(-
sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + ((a^2 - 5*a*b + 4*
b^2)*tan(f*x + e)^3 - 3*(a^2 - a*b)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a
))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*tan(f*x + e)^4 + 2*(a^4*b - 3*a
^3*b^2 + 3*a^2*b^3 - a*b^4)*f*tan(f*x + e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 -
 a^2*b^3)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^4}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

3.354 $\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}+\frac{(2a+b)\tan(e+fx)}{3af(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\tan(e+fx)}{3f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5
/2)*f)) + Tan[e + f*x]/(3*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) + ((2*a +
 b)*Tan[e + f*x])/(3*a*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.152694, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3670, 471, 527, 12, 377, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}+\frac{(2a+b)\tan(e+fx)}{3af(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\tan(e+fx)}{3f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```


```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5
/2)*f)) + Tan[e + f*x]/(3*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) + ((2*a +
 b)*Tan[e + f*x])/(3*a*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
```

```
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1-2x^2}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3(a-b)f}$$

$$= \frac{\tan(e+fx)}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+b)\tan(e+fx)}{3a(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\cdots}{(1+x\cdots} \cdots\right.}{\cdots}$$

$$= \frac{\tan(e+fx)}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+b)\tan(e+fx)}{3a(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\cdots}{(1+x\cdots} \cdots\right.}{\cdots}$$

$$= \frac{\tan(e+fx)}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+b)\tan(e+fx)}{3a(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\cdots}{1-(-\cdots} \cdots\right.}{\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} + \frac{\tan(e+fx)}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+b)\tan(e+\cdots}{3a(a-b)^2f\sqrt{a+b\tan\cdots}}$$

Mathematica [C] time = 7.73209, size = 365, normalized size = 2.85

$$\frac{\cos^4(e+fx)\cot(e+fx)\left(12(a-b)^3\tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)\text{Hypergeometric2F1}\left(2,2,\frac{9}{2},\frac{(a-b)\sin^2(e+fx)}{a}\right)\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cos[e + f*x]^4*Cot[e + f*x]*(12*(a - b)^3*Hypergeometric2F1[2, 2, 9/2, ((a
 - b)*Sin[e + f*x]^2)/a]*Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)*Sqrt[(Cos[e
+ f*x]^2*Sin[e + f*x]^2*(a^2 - b^2*Tan[e + f*x]^2 + a*b*(-1 + Tan[e + f*x]^
2)))/a^2] + 35*a*Sec[e + f*x]^2*(5*a + 2*b*Tan[e + f*x]^2)*(3*ArcSin[Sqrt[(
(a - b)*Sin[e + f*x]^2)/a]]*(a + b*Tan[e + f*x]^2)^2 + a*Sec[e + f*x]^2*(-4
*b*Tan[e + f*x]^2 + a*(-3 + Tan[e + f*x]^2))*Sqrt[(Cos[e + f*x]^2*Sin[e + f
*x]^2*(a^2 - b^2*Tan[e + f*x]^2 + a*b*(-1 + Tan[e + f*x]^2)))/a^2])))/(315*
a^4*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2]*Sqrt[((a - b)*Cos[e + f*x]^2*Sin
[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2]*(1 + (b*Tan[e + f*x]^2)/a))
```

Maple [B] time = 0.044, size = 232, normalized size = 1.8

$$\frac{\tan\left(fx+e\right)}{3\,fa}\left(a+b\left(\tan\left(fx+e\right)\right)^{2}\right)^{-\frac{3}{2}}+\frac{2\,\tan\left(fx+e\right)}{3\,fa^{2}}\frac{1}{\sqrt{a+b\left(\tan\left(fx+e\right)\right)^{2}}}+\frac{b\tan\left(fx+e\right)}{f\left(a-b\right)^{2}a}\frac{1}{\sqrt{a+b\left(\tan\left(fx+e\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/f*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(3/2)+2/3/f/a^2*tan(f*x+e)/(a+b*tan(f
*x+e)^2)^(1/2)+1/f/(a-b)^2*b*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2)-1/f/(a-b
)^3*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e
)^2)^(1/2)*tan(f*x+e))+1/3*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)+
2/3/f/(a-b)*b/a^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.14815, size = 1176, normalized size = 9.19

$$\left[-\frac{3\left(ab^{2}\tan\left(fx+e\right)^{4}+2\,a^{2}b\tan\left(fx+e\right)^{2}+a^{3}\right)\sqrt{-a+b}\log\left(-\frac{(a-2\,b)\tan\left(fx+e\right)^{2}+2\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\sqrt{-a+b}\tan\left(fx+e\right)-a}{\tan\left(fx+e\right)^{2}+1}\right)-2\,\left(\left(\right.\right.}{6\left(\left(a^{4}b^{2}-3\,a^{3}b^{3}+3\,a^{2}b^{4}-ab^{5}\right)f\tan\left(fx+e\right)^{4}+2\left(a^{5}b-3\,a^{4}b^{2}+3\,a^{3}b^{3}-a^{2}b^{4}\right)f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(a*b^2*tan(f*x + e)^4 + 2*a^2*b*tan(f*x + e)^2 + a^3)*sqrt(-a + b)
*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)
*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - 2*((2*a^2*b - a*b^2 - b^3)*tan(f
*x + e)^3 + 3*(a^3 - a^2*b)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a^4
*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*f*tan(f*x + e)^4 + 2*(a^5*b - 3*a^4*b
^2 + 3*a^3*b^3 - a^2*b^4)*f*tan(f*x + e)^2 + (a^6 - 3*a^5*b + 3*a^4*b^2 - a
^3*b^3)*f), -1/3*(3*(a*b^2*tan(f*x + e)^4 + 2*a^2*b*tan(f*x + e)^2 + a^3)*s
qrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) -
 ((2*a^2*b - a*b^2 - b^3)*tan(f*x + e)^3 + 3*(a^3 - a^2*b)*tan(f*x + e))*sq
rt(b*tan(f*x + e)^2 + a))/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*f*tan(
f*x + e)^4 + 2*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*f*tan(f*x + e)^2 +
 (a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^2}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

3.355 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{b(5a-2b)\tan(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}-\frac{b\tan(e+fx)}{3af(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5/2
)*f) - (b*Tan[e + f*x])/(3*a*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - ((5*
a - 2*b)*b*Tan[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.100929, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3661, 414, 527, 12, 377, 203}

$$-\frac{b(5a-2b)\tan(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}-\frac{b\tan(e+fx)}{3af(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-5/2),x]
```


```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5/2
)*f) - (b*Tan[e + f*x])/(3*a*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - ((5*
a - 2*b)*b*Tan[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```


Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-2b-2bx^2}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3a(a-b)f}$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\tan(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\tan(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\tan(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\tan(e+\cdots}{3a^2(a-b)^2f\sqrt{a+b\,\text{ta}\cdots}}$$

Mathematica [C] time = 7.52667, size = 1331, normalized size = 9.93

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-5/2),x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*(1575*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]] -
 (3150*(a - b)*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2)/a +
```

```
 (1575*(a - b)^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^4)/a
^2 + (2100*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^2)/a - (
4200*(a - b)*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2*Tan[
e + f*x]^2)/a^2 + (2100*(a - b)^2*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]
]*Sin[e + f*x]^4*Tan[e + f*x]^2)/a^3 + (840*b^2*ArcSin[Sqrt[((a - b)*Sin[e
+ f*x]^2)/a]]*Tan[e + f*x]^4)/a^2 - (1680*(a - b)*b^2*ArcSin[Sqrt[((a - b)*
Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2*Tan[e + f*x]^4)/a^3 + (840*(a - b)^2*b^2
*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^4*Tan[e + f*x]^4)/a^
4 + 2100*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan
[e + f*x]^2))/a] + 96*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)
/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e +
f*x]^2))/a] + 24*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*
x]^2)/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan
[e + f*x]^2))/a] + (2800*b*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Tan[e + f*x]^
2*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a])/a + (168*b*Hypergeometri
c2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7
/2)*Tan[e + f*x]^2*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a])/a + (48
*b*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*x]^2)/a]*(((a
- b)*Sin[e + f*x]^2)/a)^(7/2)*Tan[e + f*x]^2*Sqrt[(Cos[e + f*x]^2*(a + b*Ta
n[e + f*x]^2))/a])/a + (1120*b^2*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Tan[e +
 f*x]^4*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a])/a^2 + (72*b^2*Hype
rgeometric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*(((a - b)*Sin[e + f*x]
^2)/a)^(7/2)*Tan[e + f*x]^4*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]
)/a^2 + (24*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*x
]^2)/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7/2)*Tan[e + f*x]^4*Sqrt[(Cos[e + f*x
]^2*(a + b*Tan[e + f*x]^2))/a])/a^2 - 1575*Sqrt[((a - b)*Cos[e + f*x]^2*Sin
[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2] - (2100*b*Tan[e + f*x]^2*Sqrt[((a
- b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2])/a - (840*b
^2*Tan[e + f*x]^4*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e
+ f*x]^2))/a^2])/a^2))/(315*a^2*f*(((a - b)*Sin[e + f*x]^2)/a)^(5/2)*Sqrt[a
 + b*Tan[e + f*x]^2]*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]*(1 + (
b*Tan[e + f*x]^2)/a))
```

Maple [A] time = 0.046, size = 176, normalized size = 1.3

$$-\frac{b\tan\left(fx+e\right)}{f\left(a-b\right)^{2}a}\frac{1}{\sqrt{a+b\left(\tan\left(fx+e\right)\right)^{2}}}+\frac{1}{f\left(a-b\right)^{3}b^{2}}\sqrt{b^{4}\left(a-b\right)}\arctan\left(\left(a-b\right)b^{2}\tan\left(fx+e\right)\frac{1}{\sqrt{b^{4}\left(a-b\right)}}\frac{1}{\sqrt{a+b\left(\text{ta}\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/f/(a-b)^2*b*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2)+1/f/(a-b)^3*(b^4*(a-b)
)^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan
(f*x+e))-1/3*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)-2/3/f/(a-b)*b/
a^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.13218, size = 1245, normalized size = 9.29

$$\left[-\frac{3\left(a^2b^2\tan\left(fx+e\right)^4 + 2\,a^3b\tan\left(fx+e\right)^2 + a^4\right)\sqrt{-a+b}\log\left(-\frac{(a-2\,b)\tan\left(fx+e\right)^2 - 2\,\sqrt{b\tan\left(fx+e\right)^2 + a}\sqrt{-a+b}\tan\left(fx+e\right) - a}{\tan\left(fx+e\right)^2 + 1}\right) + \cdots}{6\left(\left(a^5b^2 - 3\,a^4b^3 + 3\,a^3b^4 - a^2b^5\right)f\tan\left(fx+e\right)^4 + 2\left(a^6b - 3\,a^5b^2 + 3\,a^4 \cdots\right.\right.} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(a^2*b^2*tan(f*x + e)^4 + 2*a^3*b*tan(f*x + e)^2 + a^4)*sqrt(-a +
b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a +
b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + 2*((5*a^2*b^2 - 7*a*b^3 + 2*b^
4)*tan(f*x + e)^3 + 3*(2*a^3*b - 3*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*ta
n(f*x + e)^2 + a))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(f*x +
 e)^4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2 + (a^7
 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f), 1/3*(3*(a^2*b^2*tan(f*x + e)^4 + 2*a^
3*b*tan(f*x + e)^2 + a^4)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(s
qrt(a - b)*tan(f*x + e))) - ((5*a^2*b^2 - 7*a*b^3 + 2*b^4)*tan(f*x + e)^3 +
 3*(2*a^3*b - 3*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/
((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(f*x + e)^4 + 2*(a^6*b -
3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2 + (a^7 - 3*a^6*b + 3*a^5*
b^2 - a^4*b^3)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + b\tan^2\left(e + fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan\left(fx+e\right)^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(-5/2), x)
```

3.356 $\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=186

$$-\frac{(a-4b)(3a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^3f(a-b)^2}-\frac{b(7a-4b)\cot(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}-\frac{}{3af(\cdots}$$

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5
/2)*f)) - (b*Cot[e + f*x])/(3*a*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (
(7*a - 4*b)*b*Cot[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2])
 - ((a - 4*b)*(3*a - 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(3*a^3*(a
 - b)^2*f)
```

Rubi [A] time = 0.278731, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3670, 472, 579, 583, 12, 377, 203}

$$-\frac{(a-4b)(3a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^3f(a-b)^2}-\frac{b(7a-4b)\cot(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}-\frac{}{3af(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5
/2)*f)) - (b*Cot[e + f*x])/(3*a*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (
(7*a - 4*b)*b*Cot[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2])
 - ((a - 4*b)*(3*a - 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(3*a^3*(a
 - b)^2*f)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-4b-4bx^2}{x^2(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3a(a-b)f}$$

$$= -\frac{b\cot(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(7a-4b)b\cot(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{(a-4}{}\right.}{}$$

$$= -\frac{b\cot(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(7a-4b)b\cot(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{(a-4b)(3a-}{}$$

$$= -\frac{b\cot(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(7a-4b)b\cot(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{(a-4b)(3a-}{}$$

$$= -\frac{b\cot(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(7a-4b)b\cot(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{(a-4b)(3a-}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\cot(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(7a-4b)b\cot(e+}{3a^2(a-b)^2f\sqrt{a+b\tan^2}}$$

Mathematica [C] time = 15.9142, size = 1890, normalized size = 10.16

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -((Cos[e + f*x]^2*Cot[e + f*x]*((20*a*Csc[e + f*x]^2)/(3*(a - b)) - (5*a^2*
Csc[e + f*x]^4)/(a - b)^2 + (40*b*Sec[e + f*x]^2)/(a - b) - (30*a*b*Csc[e +
 f*x]^2*Sec[e + f*x]^2)/(a - b)^2 - (40*b^2*Sec[e + f*x]^4)/(a - b)^2 + (92
*(a - b)*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f
*x]^2)/(105*a) + (24*(a - b)*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b
)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2)/(35*a) + (16*(a - b)*HypergeometricPFQ
[{2, 2, 2, 2}, {1, 1, 9/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2)/(10
5*a) + (160*b^2*Sec[e + f*x]^2*Tan[e + f*x]^2)/(3*a*(a - b)) + (124*(a - b)
*b*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*
Tan[e + f*x]^2)/(35*a^2) + (16*(a - b)*b*HypergeometricPFQ[{2, 2, 2}, {1, 9
/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(7*a^2) + (
16*(a - b)*b*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, ((a - b)*Sin[e +
f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(35*a^2) + (64*b^3*Sec[e + f*x]^2
*Tan[e + f*x]^4)/(3*a^2*(a - b)) + (152*(a - b)*b^2*Hypergeometric2F1[2, 2,
 9/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^4)/(35*a^3) +
 (88*(a - b)*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*
x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^4)/(35*a^3) + (16*(a - b)*b^2*Hypergeo
metricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*
```

```
x]^2*Tan[e + f*x]^4)/(35*a^3) + (176*(a - b)*b^3*Hypergeometric2F1[2, 2, 9/
2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^6)/(105*a^4) + (
32*(a - b)*b^3*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*x]
^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^6)/(35*a^4) + (16*(a - b)*b^3*Hypergeome
tricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]
^2*Tan[e + f*x]^6)/(105*a^4) + (5*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]])
/((((a - b)*Sin[e + f*x]^2)/a)^(5/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*
x]^2))/a]) + (30*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^2)
/(a*(((a - b)*Sin[e + f*x]^2)/a)^(5/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e +
f*x]^2))/a]) + (40*b^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x
]^4)/(a^2*(((a - b)*Sin[e + f*x]^2)/a)^(5/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Ta
n[e + f*x]^2))/a]) + (16*b^3*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e
 + f*x]^6)/(a^3*(((a - b)*Sin[e + f*x]^2)/a)^(5/2)*Sqrt[(Cos[e + f*x]^2*(a
+ b*Tan[e + f*x]^2))/a]) + (5*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]])/Sqr
t[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2] - (10
*a*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Csc[e + f*x]^2)/((a - b)*Sqrt[(
(a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2]) - (60*b
*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sec[e + f*x]^2)/((a - b)*Sqrt[((a
 - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2]) + (30*b*A
rcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^2)/(a*Sqrt[((a - b)*Co
s[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2]) - (80*b^2*ArcSin[
Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sec[e + f*x]^2*Tan[e + f*x]^2)/(a*(a - b)
*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2])
+ (40*b^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^4)/(a^2*Sqr
t[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2]) - (3
2*b^3*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sec[e + f*x]^2*Tan[e + f*x]^
4)/(a^2*(a - b)*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e +
f*x]^2))/a^2]) + (16*b^3*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f
*x]^6)/(a^3*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]
^2))/a^2]) - (16*b^3*(Tan[e + f*x] + Tan[e + f*x]^3)^2)/(a*(a - b)^2)))/(a^
2*f*Sqrt[a + b*Tan[e + f*x]^2]*(1 + (b*Tan[e + f*x]^2)/a)))
```

Maple [F] time = 0.219, size = 0, normalized size = 0.

$$\int \left(\cot \left(f x+e\right)\right)^{2} \left(a+b \left(\tan \left(f x+e\right)\right)^{2}\right)^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x)

[Out] int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] Timed out
```

Fricas [B] time = 3.00555, size = 1693, normalized size = 9.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(a^3*b^2*tan(f*x + e)^5 + 2*a^4*b*tan(f*x + e)^3 + a^5*tan(f*x +
e))*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*
a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*s
qrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2
+ 1)) + 4*(3*a^5 - 9*a^4*b + 9*a^3*b^2 - 3*a^2*b^3 + (3*a^3*b^2 - 17*a^2*b^
3 + 22*a*b^4 - 8*b^5)*tan(f*x + e)^4 + 3*(2*a^4*b - 9*a^3*b^2 + 11*a^2*b^3
- 4*a*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^6*b^2 - 3*a^5*b^
3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^5 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^
3 - a^4*b^4)*f*tan(f*x + e)^3 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*f*tan
(f*x + e)), -1/6*(3*(a^3*b^2*tan(f*x + e)^5 + 2*a^4*b*tan(f*x + e)^3 + a^5*
tan(f*x + e))*sqrt(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*
tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a)) + 2*(3*a^5 - 9*a^4*b + 9*a^3*b
^2 - 3*a^2*b^3 + (3*a^3*b^2 - 17*a^2*b^3 + 22*a*b^4 - 8*b^5)*tan(f*x + e)^4
 + 3*(2*a^4*b - 9*a^3*b^2 + 11*a^2*b^3 - 4*a*b^4)*tan(f*x + e)^2)*sqrt(b*ta
n(f*x + e)^2 + a))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x +
 e)^5 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x + e)^3 + (a^8
 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*f*tan(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^2}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

3.357 $\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=249

$$-\frac{\left(a^2-12ab+8b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^3f(a-b)^2}+\frac{(a-2b)\left(3a^2+8ab-8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^4f(a-b)^2}-$$

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5/2
)*f) - (b*Cot[e + f*x]^3)/(3*a*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - ((
3*a - 2*b)*b*Cot[e + f*x]^3)/(a^2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2]) +
 ((a - 2*b)*(3*a^2 + 8*a*b - 8*b^2)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2]
)/(3*a^4*(a - b)^2*f) - ((a^2 - 12*a*b + 8*b^2)*Cot[e + f*x]^3*Sqrt[a + b*T
an[e + f*x]^2])/(3*a^3*(a - b)^2*f)
```

Rubi [A] time = 0.377712, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3670, 472, 579, 583, 12, 377, 203}

$$-\frac{\left(a^2-12ab+8b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^3f(a-b)^2}+\frac{(a-2b)\left(3a^2+8ab-8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^4f(a-b)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```


```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5/2
)*f) - (b*Cot[e + f*x]^3)/(3*a*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - ((
3*a - 2*b)*b*Cot[e + f*x]^3)/(a^2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2]) +
 ((a - 2*b)*(3*a^2 + 8*a*b - 8*b^2)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2]
)/(3*a^4*(a - b)^2*f) - ((a^2 - 12*a*b + 8*b^2)*Cot[e + f*x]^3*Sqrt[a + b*T
an[e + f*x]^2])/(3*a^3*(a - b)^2*f)
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3(a-2b)-6bx^2}{x^4(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3a(a-b)f}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(3a-2b)b\cot^3(e+fx)}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{3(}{}\right.}{}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(3a-2b)b\cot^3(e+fx)}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\left(a^2-12ab\right.}{}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(3a-2b)b\cot^3(e+fx)}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{(a-2b)\left(3a\right.}{}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(3a-2b)b\cot^3(e+fx)}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{(a-2b)\left(3a\right.}{}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(3a-2b)b\cot^3(e+fx)}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{(a-2b)\left(3a\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\cot^3(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(3a-2b)b\cot^3(e-}{a^2(a-b)^2f\sqrt{a+b\tan}}$$

Mathematica [C] time = 16.506, size = 871, normalized size = 3.5

$$-\frac{b\sqrt{\frac{a+b+(a-b)\cos(2(e+fx))}{\cos(2(e+fx))+1}}\sqrt{-\frac{a\cot^2(e+fx)}{b}}\sqrt{-\frac{a(\cos(2(e+fx))+1)\csc^2(e+fx)}{b}}\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\csc(2(e+fx))\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{(a+b+(a-b)\cos(2}}{\sqrt{}}\right.\right.}{a(a+b+(a-b)\cos(2(e+fx)))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (-((b*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*Sqrt[
-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b
)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f
*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]
^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a - b)*Cos[2*(e + f*x)])))
 - (4*b*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/
(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[
2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*
```

```
Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)
*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(4*a*Sqr
t[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]) - (Sqrt[-((
a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*
Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)
]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*
Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(2*(a - b)*Sqrt[1 + Cos[2*(
e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])))/Sqrt[a + b + (a - b)*C
os[2*(e + f*x)]])/((a - b)^2*f) + (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos
[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((4*(a*Cos[e + f*x] + 2*b*Cos[e + f*
x])*Csc[e + f*x])/(3*a^4) - (Cot[e + f*x]*Csc[e + f*x]^2)/(3*a^3) + (2*b^4*
Sin[2*(e + f*x)])/(3*a^3*(a - b)^2*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e
 + f*x)])^2) - (4*(3*a*b^3*Sin[2*(e + f*x)] - 2*b^4*Sin[2*(e + f*x)]))/(3*a
^4*(a - b)^2*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)]))))/f
```

Maple [F] time = 0.23, size = 0, normalized size = 0.

$$\int \left(\cot \left(fx+e\right)\right)^{4} \left(a+b \left(\tan \left(fx+e\right)\right)^{2}\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 3.21615, size = 1937, normalized size = 7.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(a^4*b^2*tan(f*x + e)^7 + 2*a^5*b*tan(f*x + e)^5 + a^6*tan(f*x +
e)^3)*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 -
4*a*b)*tan(f*x + e)^2 + a^2 - 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))
*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^
2 + 1)) - 4*((3*a^4*b^2 - a^3*b^3 - 26*a^2*b^4 + 40*a*b^5 - 16*b^6)*tan(f*x
 + e)^6 - a^6 + 3*a^5*b - 3*a^4*b^2 + a^3*b^3 + 3*(2*a^5*b - a^4*b^2 - 13*a
^3*b^3 + 20*a^2*b^4 - 8*a*b^5)*tan(f*x + e)^4 + 3*(a^6 - a^5*b - 3*a^4*b^2
+ 5*a^3*b^3 - 2*a^2*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^7*
```

```
b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*f*tan(f*x + e)^7 + 2*(a^8*b - 3*a^7*
b^2 + 3*a^6*b^3 - a^5*b^4)*f*tan(f*x + e)^5 + (a^9 - 3*a^8*b + 3*a^7*b^2 -
a^6*b^3)*f*tan(f*x + e)^3), 1/6*(3*(a^4*b^2*tan(f*x + e)^7 + 2*a^5*b*tan(f*
x + e)^5 + a^6*tan(f*x + e)^3)*sqrt(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2
+ a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a)) + 2*((3*a^4*b
^2 - a^3*b^3 - 26*a^2*b^4 + 40*a*b^5 - 16*b^6)*tan(f*x + e)^6 - a^6 + 3*a^5
*b - 3*a^4*b^2 + a^3*b^3 + 3*(2*a^5*b - a^4*b^2 - 13*a^3*b^3 + 20*a^2*b^4 -
 8*a*b^5)*tan(f*x + e)^4 + 3*(a^6 - a^5*b - 3*a^4*b^2 + 5*a^3*b^3 - 2*a^2*b
^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^7*b^2 - 3*a^6*b^3 + 3*a
^5*b^4 - a^4*b^5)*f*tan(f*x + e)^7 + 2*(a^8*b - 3*a^7*b^2 + 3*a^6*b^3 - a^5
*b^4)*f*tan(f*x + e)^5 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^6*b^3)*f*tan(f*x +
e)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^4}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

3.358 $\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=327

$$-\frac{\left(a^2-22ab+16b^2\right)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5a^3f(a-b)^2}+\frac{\left(4a^2b+5a^3-88ab^2+64b^3\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^4f(a-b)^2}$$

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5
/2)*f)) - (b*Cot[e + f*x]^5)/(3*a*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) -
 ((11*a - 8*b)*b*Cot[e + f*x]^5)/(3*a^2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]
^2]) - ((15*a^4 + 10*a^3*b + 8*a^2*b^2 - 176*a*b^3 + 128*b^4)*Cot[e + f*x]*
Sqrt[a + b*Tan[e + f*x]^2])/(15*a^5*(a - b)^2*f) + ((5*a^3 + 4*a^2*b - 88*a
*b^2 + 64*b^3)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(15*a^4*(a - b)^2
*f) - ((a^2 - 22*a*b + 16*b^2)*Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(
5*a^3*(a - b)^2*f)
```

Rubi [A] time = 0.49624, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3670, 472, 579, 583, 12, 377, 203}

$$-\frac{\left(a^2-22ab+16b^2\right)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5a^3f(a-b)^2}+\frac{\left(4a^2b+5a^3-88ab^2+64b^3\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^4f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]
```


```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5
/2)*f)) - (b*Cot[e + f*x]^5)/(3*a*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) -
 ((11*a - 8*b)*b*Cot[e + f*x]^5)/(3*a^2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]
^2]) - ((15*a^4 + 10*a^3*b + 8*a^2*b^2 - 176*a*b^3 + 128*b^4)*Cot[e + f*x]*
Sqrt[a + b*Tan[e + f*x]^2])/(15*a^5*(a - b)^2*f) + ((5*a^3 + 4*a^2*b - 88*a
*b^2 + 64*b^3)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(15*a^4*(a - b)^2
*f) - ((a^2 - 22*a*b + 16*b^2)*Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(
5*a^3*(a - b)^2*f)
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
```

```
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-8b-8bx^2}{x^6(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3a(a-b)f}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(11a-8b)b\cot^5(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{3(a^2}{\ldots}\right.}{\ldots}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(11a-8b)b\cot^5(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\left(a^2-22ab+\ldots\right.}{\ldots}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(11a-8b)b\cot^5(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\left(5a^3+4a^2b-\ldots\right.}{\ldots}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(11a-8b)b\cot^5(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\left(15a^4+10a^3b\ldots\right.}{\ldots}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(11a-8b)b\cot^5(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\left(15a^4+10a^3b\ldots\right.}{\ldots}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(11a-8b)b\cot^5(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\left(15a^4+10a^3b\ldots\right.}{\ldots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(11a-8b)b\cot^5(e+\ldots}{3a^2(a-b)^2f\sqrt{a+b\tan^2\ldots}}$$

Mathematica [C] time = 16.3189, size = 441, normalized size = 1.35

$$\frac{\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(-\frac{15a^5b\sin^2(e+fx)\sin(2(e+fx))\left(\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}\right)^{3/2}\left(2(a-b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\ldots}\right.\right.\right.}{2\sqrt{\ldots}}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*((-15*a^5*b*(((a +
 b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)^(3/2)*(2*(a - b)*Elliptic
F[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2
]], 1] - 2*a*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(
e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sin[e + f*x]^2*Sin[2*(e + f*x)]
```

```
)/(2*Sqrt[2]) - (a - b)*((a - b)^2*(23*a^2 + 54*a*b + 73*b^2)*(a + b + (a -
 b)*Cos[2*(e + f*x)])^2*Cot[e + f*x] - a*(a - b)^2*(11*a + 14*b)*(a + b + (
a - b)*Cos[2*(e + f*x)])^2*Cot[e + f*x]*Csc[e + f*x]^2 + 3*a^2*(a - b)^2*(a
 + b + (a - b)*Cos[2*(e + f*x)])^2*Cot[e + f*x]*Csc[e + f*x]^4 + 10*a*b^5*S
in[2*(e + f*x)] - 5*(15*a - 11*b)*b^4*(a + b + (a - b)*Cos[2*(e + f*x)])*Si
n[2*(e + f*x)])))/(15*Sqrt[2]*a^5*(a - b)^3*f*(a + b + (a - b)*Cos[2*(e + f
*x)])^2)
```

Maple [F] time = 0.255, size = 0, normalized size = 0.

$$\int \left(\cot \left(fx+e\right)\right)^{6} \left(a+b\left(\tan \left(fx+e\right)\right)^{2}\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x)

[Out] int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] Timed out
```

Fricas [A] time = 3.35924, size = 2286, normalized size = 6.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] [-1/60*(15*(a^5*b^2*tan(f*x + e)^9 + 2*a^6*b*tan(f*x + e)^7 + a^7*tan(f*x +
 e)^5)*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 -
 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e)
)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)
^2 + 1)) + 4*((15*a^5*b^2 - 5*a^4*b^3 - 2*a^3*b^4 - 184*a^2*b^5 + 304*a*b^6
 - 128*b^7)*tan(f*x + e)^8 + 3*a^7 - 9*a^6*b + 9*a^5*b^2 - 3*a^4*b^3 + 3*(1
0*a^6*b - 5*a^5*b^2 - a^4*b^3 - 92*a^3*b^4 + 152*a^2*b^5 - 64*a*b^6)*tan(f*
x + e)^6 + 3*(5*a^7 - 5*a^6*b + a^5*b^2 - 23*a^4*b^3 + 38*a^3*b^4 - 16*a^2*
b^5)*tan(f*x + e)^4 - (5*a^7 - 7*a^6*b - 9*a^5*b^2 + 19*a^4*b^3 - 8*a^3*b^4
)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^8*b^2 - 3*a^7*b^3 + 3*a^6
*b^4 - a^5*b^5)*f*tan(f*x + e)^9 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*b^3 - a^6*b
^4)*f*tan(f*x + e)^7 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f*tan(f*x + e
)^5), -1/30*(15*(a^5*b^2*tan(f*x + e)^9 + 2*a^6*b*tan(f*x + e)^7 + a^7*tan(
f*x + e)^5)*sqrt(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*ta
n(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a)) + 2*((15*a^5*b^2 - 5*a^4*b^3 - 2
```

```
*a^3*b^4 - 184*a^2*b^5 + 304*a*b^6 - 128*b^7)*tan(f*x + e)^8 + 3*a^7 - 9*a^
6*b + 9*a^5*b^2 - 3*a^4*b^3 + 3*(10*a^6*b - 5*a^5*b^2 - a^4*b^3 - 92*a^3*b^
4 + 152*a^2*b^5 - 64*a*b^6)*tan(f*x + e)^6 + 3*(5*a^7 - 5*a^6*b + a^5*b^2 -
 23*a^4*b^3 + 38*a^3*b^4 - 16*a^2*b^5)*tan(f*x + e)^4 - (5*a^7 - 7*a^6*b -
9*a^5*b^2 + 19*a^4*b^3 - 8*a^3*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 +
 a))/((a^8*b^2 - 3*a^7*b^3 + 3*a^6*b^4 - a^5*b^5)*f*tan(f*x + e)^9 + 2*(a^9
*b - 3*a^8*b^2 + 3*a^7*b^3 - a^6*b^4)*f*tan(f*x + e)^7 + (a^10 - 3*a^9*b +
3*a^8*b^2 - a^7*b^3)*f*tan(f*x + e)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^6}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^6/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

3.359 $\int (d\tan(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=72

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p (d\tan(e+fx))^m \text{Hypergeometric2F1}\left(1, \frac{1}{2}(m+2p+1), \frac{1}{2}(m+2p+3), -\tan^2(e+fx\right.}{f(m+2p+1)}$$

```
[Out] (Hypergeometric2F1[1, (1 + m + 2*p)/2, (3 + m + 2*p)/2, -Tan[e + f*x]^2]*Ta
n[e + f*x]*(d*Tan[e + f*x])^m*(b*Tan[e + f*x]^2)^p)/(f*(1 + m + 2*p))
```

Rubi [A] time = 0.0826926, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3578, 20, 3476, 364}

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p (d\tan(e+fx))^m {}_2F_1\left(1, \frac{1}{2}(m+2p+1); \frac{1}{2}(m+2p+3); -\tan^2(e+fx)\right)}{f(m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + m + 2*p)/2, (3 + m + 2*p)/2, -Tan[e + f*x]^2]*Ta
n[e + f*x]*(d*Tan[e + f*x])^m*(b*Tan[e + f*x]^2)^p)/(f*(1 + m + 2*p))
```

Rule 3578

```
Int[((c_.)*((d_.)*tan[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Tan[e + f*x
])^p)^FracPart[n])/(d*Tan[e + f*x])^(p*FracPart[n]), Int[(a + b*Tan[e + f*x
])^m*(d*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && !IntegerQ[n] && !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (d\tan(e+fx))^m \left(b\tan^2(e+fx)\right)^p dx = \left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int \tan^{2p}(e+fx)(d\tan(e+fx))^m dx$$
$$= \left(\tan^{-m-2p}(e+fx)(d\tan(e+fx))^m \left(b\tan^2(e+fx)\right)^p\right)\int \tan^{m+2p}(e+fx)\,dx$$
$$= \frac{\left(\tan^{-m-2p}(e+fx)(d\tan(e+fx))^m \left(b\tan^2(e+fx)\right)^p\right)\text{Subst}\left(\int \frac{x^{m+2p}}{1+x^2}dx, x, \text{t}$$
$$= \frac{{}_2F_1\left(1, \frac{1}{2}(1+m+2p); \frac{1}{2}(3+m+2p); -\tan^2(e+fx)\right)\tan(e+fx)(d\tan(e+}{f(1+m+2p)}$$

Mathematica [A] time = 0.0795028, size = 74, normalized size = 1.03

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p(d\tan(e+fx))^m\text{Hypergeometric2F1}\left(1,\frac{1}{2}(m+2p+1),\frac{1}{2}(m+2p+1)+1,-\tan^2(e+f}{f(m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + m + 2*p)/2, 1 + (1 + m + 2*p)/2, -Tan[e + f*x]^2
]*Tan[e + f*x]*(d*Tan[e + f*x])^m*(b*Tan[e + f*x]^2)^p)/(f*(1 + m + 2*p))
```

Maple [F] time = 0.786, size = 0, normalized size = 0.

$$\int (d\tan(fx+e))^m \left(b(\tan(fx+e))^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*tan(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan(fx+e)^2\right)^p (d\tan(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*tan(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan(fx+e)^2\right)^p (d\tan(fx+e))^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2)^p*(d*tan(f*x + e))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan^2\left(e+fx\right)\right)^p \left(d\tan\left(e+fx\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**p*(d*tan(e + f*x))**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2\right)^p \left(d\tan\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*tan(f*x + e))^m, x)
```

3.360 $\int (d\tan(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=100

$$\frac{(d\tan(e+fx))^{m+1}\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};1,-p;\frac{m+3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{df(m+1)}$$

```
[Out] (AppellF1[(1 + m)/2, 1, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/a)]*(d*Tan[e + f*x])^(1 + m)*(a + b*Tan[e + f*x]^2)^p)/(d*f*(1 + m)*(1 +
(b*Tan[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.119082, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3670, 511, 510}

$$\frac{(d\tan(e+fx))^{m+1}\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};1,-p;\frac{m+3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{df(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[(1 + m)/2, 1, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/a)]*(d*Tan[e + f*x])^(1 + m)*(a + b*Tan[e + f*x]^2)^p)/(d*f*(1 + m)*(1 +
(b*Tan[e + f*x]^2)/a)^p)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (d\tan(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx = \frac{\text{Subst}\left(\int \frac{(dx)^m\left(a+bx^2\right)^p}{1+x^2}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{(dx)^m\left(1+\frac{bx^2}{a}\right)^p}{1+x^2}dx, x\right.}{f}$$

$$= \frac{F_1\left(\frac{1+m}{2};1,-p;\frac{3+m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)(d\tan(e+fx))^{1+m}\left(\right.}{df(1+m)}$$

Mathematica [A] time = 0.230252, size = 101, normalized size = 1.01

$$\frac{\tan(e+fx)(d\tan(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};-p,1;\frac{m+3}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx\right.}{f(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Tan[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*
x]^2]*Tan[e + f*x]*(d*Tan[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + m)*
(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Maple [F] time = 0.589, size = 0, normalized size = 0.

$$\int \left(d\tan\left(fx+e\right)\right)^m \left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*tan(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p \left(d\tan\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\left(d\tan\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p \left(d\tan\left(fx+e\right)\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

3.361 $\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=129

$$-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{a+b\tan^2(e+fx)}{a-b}\right)}{2f(p+1)(a-b)}-\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{p+1}}{2b^2f(p+1)}+\frac{(a$$

```
[Out] -((a + b)*(a + b*Tan[e + f*x]^2)^(1 + p))/(2*b^2*f*(1 + p)) - (Hypergeometr
ic2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*Tan[e + f*x]^
2)^(1 + p))/(2*(a - b)*f*(1 + p)) + (a + b*Tan[e + f*x]^2)^(2 + p)/(2*b^2*f
*(2 + p))
```

Rubi [A] time = 0.171291, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3670, 446, 88, 68}

$$-\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{p+1}}{2b^2f(p+1)}+\frac{\left(a+b\tan^2(e+fx)\right)^{p+2}}{2b^2f(p+2)}-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+}{a-b}\right.}{2f(p+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((a + b)*(a + b*Tan[e + f*x]^2)^(1 + p))/(2*b^2*f*(1 + p)) - (Hypergeometr
ic2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*Tan[e + f*x]^
2)^(1 + p))/(2*(a - b)*f*(1 + p)) + (a + b*Tan[e + f*x]^2)^(2 + p)/(2*b^2*f
*(2 + p))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
```

```
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{x^5\left(a+bx^2\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^p}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(-a-b)(a+bx)^p}{b} + \frac{(a+bx)^p}{1+x} + \frac{(a+bx)^{1+p}}{b}\right)dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= -\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2b^2f(1+p)} + \frac{\left(a+b\tan^2(e+fx)\right)^{2+p}}{2b^2f(2+p)} + \frac{\text{Subst}\left(\int \frac{(a+b\ldots}{1+\ldots}\right.}{\ldots}$$
$$= -\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2b^2f(1+p)} - \frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2(e+fx)}{a-b}\right)\left(a+b\,\mathrm{t}\ldots\right.}{2(a-b)f(1+p)}$$

Mathematica [A] time = 0.735409, size = 106, normalized size = 0.82

$$\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\left(b^2(p+2)\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{a+b\tan^2(e+fx)}{a-b}\right)+(a-b)\left(a-b(p+1)\tan^2(e+f\ldots\right.\right.}{2b^2f(p+1)(p+2)(b-a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((a + b*Tan[e + f*x]^2)^(1 + p)*(b^2*(2 + p)*Hypergeometric2F1[1, 1 + p, 2
+ p, (a + b*Tan[e + f*x]^2)/(a - b)] + (a - b)*(a + b*(2 + p) - b*(1 + p)*T
an[e + f*x]^2)))/(2*b^2*(-a + b)*f*(1 + p)*(2 + p))
```

Maple [F] time = 0.398, size = 0, normalized size = 0.

$$\int \left(\tan\left(fx+e\right)\right)^5\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\tan\left(fx+e\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\tan\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

3.362 $\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=95

$$\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{a+b\tan^2(e+fx)}{a-b}\right)}{2f(p+1)(a-b)}+\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}}{2bf(p+1)}$$

```
[Out] (a + b*Tan[e + f*x]^2)^(1 + p)/(2*b*f*(1 + p)) + (Hypergeometric2F1[1, 1 +
p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*Tan[e + f*x]^2)^(1 + p))/(
2*(a - b)*f*(1 + p))
```

Rubi [A] time = 0.10289, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3670, 446, 80, 68}

$$\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a}{a-b}\right)}{2f(p+1)(a-b)}+\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}}{2bf(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (a + b*Tan[e + f*x]^2)^(1 + p)/(2*b*f*(1 + p)) + (Hypergeometric2F1[1, 1 +
p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*Tan[e + f*x]^2)^(1 + p))/(
2*(a - b)*f*(1 + p))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
```

```
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p dx = \frac{\text{Subst}\left(\int \frac{x^3\left(a+bx^2\right)^p}{1+x^2}dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x(a+bx)^p}{1+x}dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\left(a+b\tan^2(e+fx)\right)^{1+p}}{2bf(1+p)} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{1+x}dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\left(a+b\tan^2(e+fx)\right)^{1+p}}{2bf(1+p)} + \frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2(e+fx)}{a-b}\right)\left(a+b\tan^2(e\right.}{2(a-b)f(1+p)}$$

Mathematica [A] time = 0.125985, size = 73, normalized size = 0.77

$$-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\left(b\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{a+b\tan^2(e+fx)}{a-b}\right)+a-b\right)}{2bf(p+1)(b-a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((a - b + b*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a -
 b)])*(a + b*Tan[e + f*x]^2)^(1 + p))/(2*b*(-a + b)*f*(1 + p))
```

Maple [F] time = 0.316, size = 0, normalized size = 0.

$$\int \left(\tan\left(fx+e\right)\right)^3\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p \tan\left(fx+e\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

3.363 $\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=63

$$-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{a+b\tan^2(e+fx)}{a-b}\right)}{2f(p+1)(a-b)}$$

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b
*Tan[e + f*x]^2)^(1 + p))/(2*(a - b)*f*(1 + p))
```

Rubi [A] time = 0.0669837, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3670, 444, 68}

$$-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a}{a-b}\right)}{2f(p+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b
*Tan[e + f*x]^2)^(1 + p))/(2*(a - b)*f*(1 + p))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx^2)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= -\frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2(e+fx)}{a-b}\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2(a-b)f(1+p)}$$

Mathematica [A] time = 0.0808765, size = 63, normalized size = 1.

$$-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{a+b\tan^2(e+fx)}{a-b}\right)}{2f(p+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b
*Tan[e + f*x]^2)^(1 + p))/(2*(a - b)*f*(1 + p))
```

Maple [F] time = 0.294, size = 0, normalized size = 0.

$$\int \tan\left(fx+e\right)\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(tan(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tan^{2}\left(e + fx\right)\right)^{p}\tan\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**p*tan(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx + e\right)^{2} + a\right)^{p}\tan\left(fx + e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

3.364 $\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=118

$$\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{a+b\tan^2(e+fx)}{a-b}\right)}{2f(p+1)(a-b)}-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\text{Hypergeometri}}{2af(p+}$$

```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*
Tan[e + f*x]^2)^(1 + p))/(2*(a - b)*f*(1 + p)) - (Hypergeometric2F1[1, 1 +
p, 2 + p, 1 + (b*Tan[e + f*x]^2)/a]*(a + b*Tan[e + f*x]^2)^(1 + p))/(2*a*f*
(1 + p))
```

Rubi [A] time = 0.11478, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3670, 446, 86, 65, 68}

$$\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a}{a-b}\right)}{2f(p+1)(a-b)}-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)}{a}+1\right)}{2af(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*
Tan[e + f*x]^2)^(1 + p))/(2*(a - b)*f*(1 + p)) - (Hypergeometric2F1[1, 1 +
p, 2 + p, 1 + (b*Tan[e + f*x]^2)/a]*(a + b*Tan[e + f*x]^2)^(1 + p))/(2*a*f*
(1 + p))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
```

```
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{x\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \tan^2(e+fx)\right)}{2f} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{1+x}\,dx, x, \tan^2(e+fx\right.}{2f}$$

$$= \frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2(e+fx)}{a-b}\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2(a-b)f(1+p)} - \frac{{}_2F_1\left(1,1+p;\right.}{}$$

Mathematica [A] time = 0.164678, size = 98, normalized size = 0.83

$$\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\left(a\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{a+b\tan^2(e+fx)}{a-b}\right)+(b-a)\text{Hypergeometric2F1}\left(1,p+\right.\right.}{2af(p+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((a*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)] + (-
a + b)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Tan[e + f*x]^2)/a])*(a + b
*Tan[e + f*x]^2)^(1 + p))/(2*a*(a - b)*f*(1 + p))
```

Maple [F] time = 0.314, size = 0, normalized size = 0.

$$\int \cot\left(fx+e\right)\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

3.365 $\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=158

$$\frac{(a-bp)\left(a+b\tan^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\tan^2(e+fx)}{a}+1\right)}{2a^2f(p+1)}-\left(a+b\tan^2(e+fx)\right)^{p+1}\text{Hy}$$

```
[Out] -(Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(1 + p))/(2*a*f) - (Hypergeometric2
F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*Tan[e + f*x]^2)^
(1 + p))/(2*(a - b)*f*(1 + p)) + ((a - b*p)*Hypergeometric2F1[1, 1 + p, 2 +
 p, 1 + (b*Tan[e + f*x]^2)/a]*(a + b*Tan[e + f*x]^2)^(1 + p))/(2*a^2*f*(1 +
 p))
```

Rubi [A] time = 0.167758, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 446, 103, 156, 65, 68}

$$\frac{(a-bp)\left(a+b\tan^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)}{a}+1\right)}{2a^2f(p+1)}-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\tan}{}\right.}{2f(p+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```


```
[Out] -(Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(1 + p))/(2*a*f) - (Hypergeometric2
F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*Tan[e + f*x]^2)^
(1 + p))/(2*(a - b)*f*(1 + p)) + ((a - b*p)*Hypergeometric2F1[1, 1 + p, 2 +
 p, 1 + (b*Tan[e + f*x]^2)/a]*(a + b*Tan[e + f*x]^2)^(1 + p))/(2*a^2*f*(1 +
 p))
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
```

```
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{x^3\left(1+x^2\right)}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x^2(1+x)}\,dx,x,\tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2af} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p(a-bp-bpx)}{x(1+x)}\,dx,x,\tan^2\right.}{2af}$$

$$= -\frac{\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2af} + \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{1+x}\,dx,x,\tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2af} - \frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2(e+fx)}{a-b}\right)(a}{2(a-b)f(1+p}$$

Mathematica [A] time = 0.644604, size = 142, normalized size = 0.9

$$\frac{\tan^2(e+fx)\left(a\cot^2(e+fx)+b\right)\left(a+b\tan^2(e+fx)\right)^p\left(a^2\left(-\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{a+b\tan^2(e+fx)}{a-b}\right)\right)-(a\right.}{2a^2f(p+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((b + a*Cot[e + f*x]^2)*(-(a^2*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Ta
n[e + f*x]^2)/(a - b)]) - (a - b)*(a*(1 + p)*Cot[e + f*x]^2 + (-a + b*p)*Hy
pergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Tan[e + f*x]^2)/a]))*Tan[e + f*x]^2
*(a + b*Tan[e + f*x]^2)^p)/(2*a^2*(a - b)*f*(1 + p))
```

Maple [F] time = 0.296, size = 0, normalized size = 0.

$$\int \left(\cot \left(f x+e\right)\right)^{3} \left(a+b \left(\tan \left(f x+e\right)\right)^{2}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan \left(f x+e\right)^{2}+a\right)^{p} \cot \left(f x+e\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \tan \left(f x+e\right)^{2}+a\right)^{p} \cot \left(f x+e\right)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan \left(f x+e\right)^{2}+a\right)^{p} \cot \left(f x+e\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

3.366 $\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=217

$$-\frac{\left(2a^2-2abp-b^2(1-p)p\right)\left(a+b\tan^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\tan^2(e+fx)}{a}+1\right)}{4a^3f(p+1)}+\frac{\left(a+b\,\text{ta}\right.}{}$$

```
[Out] ((2*a + b - b*p)*Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(1 + p))/(4*a^2*f) -
 (Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(1 + p))/(4*a*f) + (Hypergeometric2
F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*Tan[e + f*x]^2)^
(1 + p))/(2*(a - b)*f*(1 + p)) - ((2*a^2 - 2*a*b*p - b^2*(1 - p)*p)*Hyperge
ometric2F1[1, 1 + p, 2 + p, 1 + (b*Tan[e + f*x]^2)/a]*(a + b*Tan[e + f*x]^2
)^(1 + p))/(4*a^3*f*(1 + p))
```

Rubi [A] time = 0.253299, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3670, 446, 103, 151, 156, 65, 68}

$$-\frac{\left(2a^2-2abp-b^2(1-p)p\right)\left(a+b\tan^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)}{a}+1\right)}{4a^3f(p+1)}+\frac{(2a-bp+b)\cot^2(e+fx)\left(\right.}{4a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((2*a + b - b*p)*Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(1 + p))/(4*a^2*f) -
 (Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(1 + p))/(4*a*f) + (Hypergeometric2
F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*Tan[e + f*x]^2)^
(1 + p))/(2*(a - b)*f*(1 + p)) - ((2*a^2 - 2*a*b*p - b^2*(1 - p)*p)*Hyperge
ometric2F1[1, 1 + p, 2 + p, 1 + (b*Tan[e + f*x]^2)/a]*(a + b*Tan[e + f*x]^2
)^(1 + p))/(4*a^3*f*(1 + p))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{x^5\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x^3(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{4af} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p(2a+b-bp+b(1-p)x)}{x^2(1+x)}\,dx, \ldots\right.}{4af}$$

$$= \frac{(2a+b-bp)\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{4a^2f} - \frac{\cot^4(e+fx)\left(a+b\tan^2 \ldots\right.}{4af}$$

$$= \frac{(2a+b-bp)\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{4a^2f} - \frac{\cot^4(e+fx)\left(a+b\tan^2 \ldots\right.}{4af}$$

$$= \frac{(2a+b-bp)\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{4a^2f} - \frac{\cot^4(e+fx)\left(a+b\tan^2 \ldots\right.}{4af}$$

Mathematica [A] time = 2.54215, size = 172, normalized size = 0.79

$$-\frac{\tan^2(e+fx)\left(a\cot^2(e+fx)+b\right)\left(a+b\tan^2(e+fx)\right)^p\left((a-b)\left(\left(2a^2-2abp+b^2(p-1)p\right)\text{Hypergeometric2F1}\left(1\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((b + a*Cot[e + f*x]^2)*(-2*a^3*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*
Tan[e + f*x]^2)/(a - b)] + (a - b)*(a*(1 + p)*Cot[e + f*x]^2*(-2*a + b*(-1
+ p) + a*Cot[e + f*x]^2) + (2*a^2 - 2*a*b*p + b^2*(-1 + p)*p)*Hypergeometri
c2F1[1, 1 + p, 2 + p, 1 + (b*Tan[e + f*x]^2)/a]))*Tan[e + f*x]^2*(a + b*Tan
[e + f*x]^2)^p)/(4*a^3*(a - b)*f*(1 + p))
```

Maple [F] time = 0.301, size = 0, normalized size = 0.

$$\int \left(\cot\left(fx+e\right)\right)^5\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^5,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^5, x)
```

3.367 $\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{\tan^7(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{7}{2};1,-p;\frac{9}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{7f}$$

```
[Out] (AppellF1[7/2, 1, -p, 9/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
+ f*x]^7*(a + b*Tan[e + f*x]^2)^p)/(7*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.100262, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 511, 510}

$$\frac{\tan^7(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{7}{2};1,-p;\frac{9}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[7/2, 1, -p, 9/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
+ f*x]^7*(a + b*Tan[e + f*x]^2)^p)/(7*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\frac{x^6(a+bx^2)^p}{1+x^2}dx,x,\tan(e+fx)\right)}{f}$$
$$=\frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\frac{x^6\left(1+\frac{bx^2}{a}\right)^p}{1+x^2}dx,x,\tan(e+\right.}{f}$$
$$=\frac{F_1\left(\frac{7}{2};1,-p;\frac{9}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan^7(e+fx)\left(a+b\tan^2(e+fx)\right.}{7f}$$

Mathematica [F] time = 2.87589, size = 0, normalized size = 0.

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p, x]
```

Maple [F] time = 0.418, size = 0, normalized size = 0.

$$\int\left(\tan\left(fx+e\right)\right)^6\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^6,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\tan\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^6, x)
```

3.368 $\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{\tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};1,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{5f}$$

```
[Out] (AppellF1[5/2, 1, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
+ f*x]^5*(a + b*Tan[e + f*x]^2)^p)/(5*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.0989361, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 511, 510}

$$\frac{\tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};1,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[5/2, 1, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
+ f*x]^5*(a + b*Tan[e + f*x]^2)^p)/(5*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p dx = \frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)^p}{1+x^2} dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^4\left(1+\frac{bx^2}{a}\right)^p}{1+x^2} dx, x, \tan(\right.}{f}$$

$$= \frac{F_1\left(\frac{5}{2}; 1, -p; \frac{7}{2}; -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a}\right)\tan^5(e+fx)\left(a+b\tan^2(e+\right.}{5f}$$

Mathematica [B] time = 14.8471, size = 2250, normalized size = 27.11

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(2*p)*((9*a*AppellF1[1/2, -p, 1, 3/2
, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*a*AppellF1[1
/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1
[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1
[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2
) + (-3*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/a)] + Hypergeo
metric2F1[3/2, -p, 5/2, -((b*Tan[e + f*x]^2)/a)]*Tan[e + f*x]^2)/(1 + (b*Ta
n[e + f*x]^2)/a)^p))/(3*f*((2*b*p*Sec[e + f*x]^2*Tan[e + f*x]^2*(a + b*Tan[
e + f*x]^2)^(-1 + p)*((9*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a
), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan
[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((
b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*T
an[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (-3*Hypergeometric2F
1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/a)] + Hypergeometric2F1[3/2, -p, 5/2,
-((b*Tan[e + f*x]^2)/a)]*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/a)^p))/3 +
 (Sec[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p*((9*a*AppellF1[1/2, -p, 1, 3/2, -
((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*a*AppellF1[1/2,
 -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/
2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/
2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) +
 (-3*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/a)] + Hypergeomet
ric2F1[3/2, -p, 5/2, -((b*Tan[e + f*x]^2)/a)]*Tan[e + f*x]^2)/(1 + (b*Tan[e
 + f*x]^2)/a)^p))/3 + (Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p*((-18*a*Appell
F1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]*
Sin[e + f*x])/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[
e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a),
-Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Ta
n[e + f*x]^2])*Tan[e + f*x]^2) + (9*a*Cos[e + f*x]^2*((2*b*p*AppellF1[3/2,
1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan
[e + f*x])/(3*a) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -T
an[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*a*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p,
 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2,
 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (2*b*p*S
ec[e + f*x]^2*Tan[e + f*x]*(1 + (b*Tan[e + f*x]^2)/a)^(-1 - p)*(-3*Hypergeo
metric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/a)] + Hypergeometric2F1[3/2, -
p, 5/2, -((b*Tan[e + f*x]^2)/a)]*Tan[e + f*x]^2))/a - (9*a*AppellF1[1/2, -p
```

```
, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]^2*(4*(b*p*
AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*
AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Sec[e
+ f*x]^2*Tan[e + f*x] + 3*a*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e
 + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x]))/(3*a) - (2*App
ellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*
x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*AppellF1[5/2, 1 - p, 2,
7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])
/5 - (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -((b*Tan[e + f*x]^2)/a), -Ta
n[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*a)) - a*((6*b*p*AppellF1[5/2,
 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Ta
n[e + f*x])/(5*a) - (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/a),
-Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*a*AppellF1[1/2, -p,
1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1
- p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p
, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)^2 + (2
*Hypergeometric2F1[3/2, -p, 5/2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Ta
n[e + f*x] - 3*Csc[e + f*x]*Sec[e + f*x]*(-Hypergeometric2F1[1/2, -p, 3/2,
-((b*Tan[e + f*x]^2)/a)] + (1 + (b*Tan[e + f*x]^2)/a)^p) + 3*Sec[e + f*x]^2
*Tan[e + f*x]*(-Hypergeometric2F1[3/2, -p, 5/2, -((b*Tan[e + f*x]^2)/a)] +
(1 + (b*Tan[e + f*x]^2)/a)^p))/(1 + (b*Tan[e + f*x]^2)/a)^p))/3))
```

Maple [F] time = 0.332, size = 0, normalized size = 0.

$$\int \left(\tan \left(fx+e\right)\right)^{4} \left(a+b \left(\tan \left(fx+e\right)\right)^{2}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan \left(fx+e\right)^{2}+a\right)^{p} \tan \left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \tan \left(fx+e\right)^{2}+a\right)^{p} \tan \left(fx+e\right)^{4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

3.369 $\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};1,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{3f}$$

```
[Out] (AppellF1[3/2, 1, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
+ f*x]^3*(a + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.101501, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 511, 510}

$$\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};1,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[3/2, 1, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
+ f*x]^3*(a + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^2\left(1+\frac{bx^2}{a}\right)^p}{1+x^2}\,dx, x, \tan(\right.}{f}$$

$$= \frac{F_1\left(\frac{3}{2};1,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan^3(e+fx)\left(a+b\tan^2(e+\right.}{3f}$$

Mathematica [B] time = 14.5488, size = 1992, normalized size = 24.

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(2*p)*(Hypergeometric2F1[1/2, -p, 3/
2, -((b*Tan[e + f*x]^2)/a)]/(1 + (b*Tan[e + f*x]^2)/a)^p + (3*a*AppellF1[1/
2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(-
3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2
*(-(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]
^2]) + a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2
])*Tan[e + f*x]^2)))/(f*(2*b*p*Sec[e + f*x]^2*Tan[e + f*x]^2*(a + b*Tan[e +
 f*x]^2)^(-1 + p)*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/a)]
/(1 + (b*Tan[e + f*x]^2)/a)^p + (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e +
 f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(-3*a*AppellF1[1/2, -p, 1, 3/
2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*AppellF1[3/2, 1 - p
, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*AppellF1[3/2, -p,
2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + Sec[e
 + f*x]^2*(a + b*Tan[e + f*x]^2)^p*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Ta
n[e + f*x]^2)/a)]/(1 + (b*Tan[e + f*x]^2)/a)^p + (3*a*AppellF1[1/2, -p, 1,
3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(-3*a*Appell
F1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*Ap
pellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*A
ppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e +
 f*x]^2)) + Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p*((-2*b*p*Hypergeometric2F
1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x]*(1 + (
b*Tan[e + f*x]^2)/a)^(-1 - p))/a - (6*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[
e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]*Sin[e + f*x])/(-3*a*AppellF1[
1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*Appel
lF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*Appe
llF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*
x]^2) + (3*a*Cos[e + f*x]^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e
 + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*App
ellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*
x]^2*Tan[e + f*x])/3))/(-3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)
/a), -Tan[e + f*x]^2] + 2*(-(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e +
f*x]^2)/a), -Tan[e + f*x]^2]) + a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*
x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (Csc[e + f*x]*Sec[e + f*x]*(-
Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/a)] + (1 + (b*Tan[e +
f*x]^2)/a)^p))/(1 + (b*Tan[e + f*x]^2)/a)^p - (3*a*AppellF1[1/2, -p, 1, 3/2
, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]^2*(4*(-(b*p*Appell
F1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*Appel
```

```
lF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Sec[e + f*x
]^2*Tan[e + f*x] - 3*a*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*
x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*AppellF1
[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*
Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(-(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2
, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5
- (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e
 + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*a))) + a*((6*b*p*AppellF1[5/2, 1
 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[
e + f*x])/(5*a) - (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/a), -T
an[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(-3*a*AppellF1[1/2, -p, 1
, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*AppellF1[3/2, 1
 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*AppellF1[3/2,
-p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)^2)))
```

Maple [F] time = 0.283, size = 0, normalized size = 0.

$$\int \left(\tan \left(fx+e\right)\right)^{2} \left(a+b \left(\tan \left(fx+e\right)\right)^{2}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan \left(fx+e\right)^{2}+a\right)^{p} \tan \left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \tan \left(fx+e\right)^{2}+a\right)^{p} \tan \left(fx+e\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\tan^{2}\left(e+fx\right)\right)^{p}\tan^{2}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**p*tan(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

3.370 $\int \left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=78

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
+ f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.0507798, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3661, 430, 429}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
+ f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+b\tan^2(e+fx)\right)^p dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{1+x^2} dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p \left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{1+x^2} dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p \left(1+\frac{b\tan}{}\right.}{f}$$

Mathematica [B] time = 0.378287, size = 192, normalized size = 2.46

$$\frac{3a\sin(2(e+fx))\left(a+b\tan^2(e+fx)\right)^p F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e\right.}{4f\tan^2(e+fx)\left(bpF_1\left(\frac{3}{2};1-p,1;\frac{5}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx)\right)-aF_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx)\right)\right)+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Si
n[2*(e + f*x)]*(a + b*Tan[e + f*x]^2)^p)/(6*a*f*AppellF1[1/2, -p, 1, 3/2, -
((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 4*f*(b*p*AppellF1[3/2, 1 - p, 1,
 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/
2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)
```

Maple [F] time = 0.001, size = 0, normalized size = 0.

$$\int \left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan^2\left(e+fx\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(fx+e\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p, x)
```

3.371 $\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=79

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};1,-p;\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

```
[Out] -((AppellF1[-1/2, 1, -p, 1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Cot
[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p))
```

Rubi [A] time = 0.0984764, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 511, 510}

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};1,-p;\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[-1/2, 1, -p, 1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Cot
[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\mathrm{Subst}\left(\int \frac{(a+bx^2)^p}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\mathrm{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2(1+x^2)}\,dx, x, \tan(e+f\right.}{f}$$

$$= -\frac{F_1\left(-\frac{1}{2};1,-p;\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\cot(e+fx)\left(a+b\tan^2(e+f\right.}{f}$$

Mathematica [B] time = 14.5048, size = 1989, normalized size = 25.18

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(2*p)*(-(Hypergeometric2F1[-1/2, -p,
 1/2, -((b*Tan[e + f*x]^2)/a)]/(1 + (b*Tan[e + f*x]^2)/a)^p) + (3*a*AppellF
1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sin[e + f*x]^2
)/(-3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]
 + 2*(-(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e +
f*x]^2]) + a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*
x]^2])*Tan[e + f*x]^2)))/(f*(2*b*p*Sec[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(-
1 + p)*(-(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]/(1 + (b
*Tan[e + f*x]^2)/a)^p) + (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/a), -Tan[e + f*x]^2]*Sin[e + f*x]^2)/(-3*a*AppellF1[1/2, -p, 1, 3/2, -((b
*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*AppellF1[3/2, 1 - p, 1, 5/
2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*AppellF1[3/2, -p, 2, 5/2,
 -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) - Csc[e + f*x]
^2*(a + b*Tan[e + f*x]^2)^p*(-(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e
+ f*x]^2)/a)]/(1 + (b*Tan[e + f*x]^2)/a)^p) + (3*a*AppellF1[1/2, -p, 1, 3/2
, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sin[e + f*x]^2)/(-3*a*AppellF1[
1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*Appel
lF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*Appe
llF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*
x]^2)) + Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^p*((2*b*p*Hypergeometric2F1[-1
/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x]*(1 + (b*T
an[e + f*x]^2)/a)^(-1 - p))/a + (6*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e +
 f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]*Sin[e + f*x])/(-3*a*AppellF1[1/2
, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*AppellF1
[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*AppellF
1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^
2) + (3*a*Sin[e + f*x]^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e +
f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*Appell
F1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^
2*Tan[e + f*x])/3))/(-3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a)
, -Tan[e + f*x]^2] + 2*(-(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x
]^2)/a), -Tan[e + f*x]^2]) + a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^
2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (Csc[e + f*x]*Sec[e + f*x]*(Hype
rgeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)] - (1 + (b*Tan[e + f*x
]^2)/a)^p))/(1 + (b*Tan[e + f*x]^2)/a)^p - (3*a*AppellF1[1/2, -p, 1, 3/2, -
((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sin[e + f*x]^2*(4*(-(b*p*AppellF1[
3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*AppellF1
```

```
[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Sec[e + f*x]^2
*Tan[e + f*x] - 3*a*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^
2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*AppellF1[3/
2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan
[e + f*x])/3) + 2*Tan[e + f*x]^2*(-(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2, -
((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5 - (
6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e +
f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*a))) + a*((6*b*p*AppellF1[5/2, 1 -
p, 2, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e +
 f*x])/(5*a) - (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[
e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(-3*a*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*AppellF1[3/2, 1 -
p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*AppellF1[3/2, -p,
 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)^2)))
```

Maple [F] time = 0.281, size = 0, normalized size = 0.

$$\int \left(\cot \left(fx+e\right)\right)^{2}\left(a+b\left(\tan \left(fx+e\right)\right)^{2}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan \left(fx+e\right)^{2}+a\right)^{p}\cot \left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan \left(fx+e\right)^{2}+a\right)^{p}\cot \left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

3.372 $\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};1,-p;-\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{3f}$$

```
[Out] -(AppellF1[-3/2, 1, -p, -1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Cot
[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.0992018, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 511, 510}

$$-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};1,-p;-\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -(AppellF1[-3/2, 1, -p, -1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Cot
[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^4(1+x^2)}\,dx, x, \tan(e+f\right.}{f}$$

$$= -\frac{F_1\left(-\frac{3}{2};1,-p;-\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\cot^3(e+fx)\left(a+b\tan^2(e+\right.}{3f}$$

Mathematica [B] time = 14.5561, size = 2468, normalized size = 29.73

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Cot[e + f*x]^7*(a + b*Tan[e + f*x]^2)^(2*p)*((9*a*AppellF1[1/2, -p, 1, 3/2
, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sin[e + f*x]^2*Tan[e + f*x]^2)/
(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] +
2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^
2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])
*Tan[e + f*x]^2) - (Hypergeometric2F1[-3/2, -p, -1/2, -((b*Tan[e + f*x]^2)/
a)] - 3*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*Tan[e + f
*x]^2)/(1 + (b*Tan[e + f*x]^2)/a)^p))/(3*f*((2*b*p*Csc[e + f*x]^2*(a + b*Ta
n[e + f*x]^2)^(-1 + p)*((9*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)
/a), -Tan[e + f*x]^2]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(3*a*AppellF1[1/2, -p,
 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1
 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -
p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (Hy
pergeometric2F1[-3/2, -p, -1/2, -((b*Tan[e + f*x]^2)/a)] - 3*Hypergeometric
2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*Tan[e + f*x]^2)/(1 + (b*Tan[e +
 f*x]^2)/a)^p))/3 - Cot[e + f*x]^2*Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p*
((9*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*S
in[e + f*x]^2*Tan[e + f*x]^2)/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f
*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[
e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e +
 f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (Hypergeometric2F1[-3/2, -
p, -1/2, -((b*Tan[e + f*x]^2)/a)] - 3*Hypergeometric2F1[-1/2, -p, 1/2, -((b
*Tan[e + f*x]^2)/a)]*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/a)^p) + (Cot[e
 + f*x]^3*(a + b*Tan[e + f*x]^2)^p*((18*a*AppellF1[1/2, -p, 1, 3/2, -((b*Ta
n[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sin[e + f*x]^2*Tan[e + f*x])/(3*a*Appell
F1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*Appe
llF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*Appe
llF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*
x]^2) + (18*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f
*x]^2]*Tan[e + f*x]^3)/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/
a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x
]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2
)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (9*a*Sin[e + f*x]^2*Tan[e + f*x]^
2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*
x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*AppellF1[3/2, -p, 2, 5/2, -((
b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*
a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(
```

```
b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]
- a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Ta
n[e + f*x]^2) + (2*b*p*Sec[e + f*x]^2*Tan[e + f*x]*(1 + (b*Tan[e + f*x]^2)/
a)^(-1 - p)*(Hypergeometric2F1[-3/2, -p, -1/2, -((b*Tan[e + f*x]^2)/a)] - 3
*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*Tan[e + f*x]^2))
/a - (9*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^
2]*Sin[e + f*x]^2*Tan[e + f*x]^2*(4*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*
Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan
[e + f*x]^2)/a), -Tan[e + f*x]^2])*Sec[e + f*x]^2*Tan[e + f*x] + 3*a*((2*b*
p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Se
c[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e
+ f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e +
f*x]^2*(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan
[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2, 2
- p, 1, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e
 + f*x])/(5*a)) - a*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^
2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*a) - (12*AppellF1[5
/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Ta
n[e + f*x])/5))))/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -
Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/
a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a),
 -Tan[e + f*x]^2])*Tan[e + f*x]^2)^2 - (-6*Hypergeometric2F1[-1/2, -p, 1/2,
 -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x] - 3*Sec[e + f*x]^2*Ta
n[e + f*x]*(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)] - (1
+ (b*Tan[e + f*x]^2)/a)^p) - 3*Csc[e + f*x]*Sec[e + f*x]*(-Hypergeometric2F
1[-3/2, -p, -1/2, -((b*Tan[e + f*x]^2)/a)] + (1 + (b*Tan[e + f*x]^2)/a)^p))
/(1 + (b*Tan[e + f*x]^2)/a)^p))/3))
```

Maple [F] time = 0.283, size = 0, normalized size = 0.

$$\int \left(\cot \left(fx+e\right)\right)^{4} \left(a+b \left(\tan \left(fx+e\right)\right)^{2}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right)^{4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

3.373 $\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{5}{2};1,-p;-\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{5f}$$

```
[Out] -(AppellF1[-5/2, 1, -p, -3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Cot
[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p)/(5*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.0994013, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 511, 510}

$$-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{5}{2};1,-p;-\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -(AppellF1[-5/2, 1, -p, -3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Cot
[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p)/(5*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^6(1+x^2)}\,dx, x, \tan(e+f}{f}$$
$$= -\frac{F_1\left(-\frac{5}{2};1,-p;-\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\cot^5(e+fx)\left(a+b\tan^2(e+\right.}{5f}$$

Mathematica [F] time = 3.614, size = 0, normalized size = 0.

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p, x]
```

Maple [F] time = 0.316, size = 0, normalized size = 0.

$$\int \left(\cot\left(fx+e\right)\right)^6\left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^6, x)
```

3.374 $\int \left(a+b\tan^3(c+dx)\right)^4\,dx$

Optimal. Leaf size=255

$$\frac{b^2\left(6a^2-b^2\right)\tan^5(c+dx)}{5d}-\frac{b^2\left(6a^2-b^2\right)\tan^3(c+dx)}{3d}+\frac{2ab\left(a^2-b^2\right)\tan^2(c+dx)}{d}+\frac{b^2\left(6a^2-b^2\right)\tan(c+dx)}{d}+\frac{4ab(}{}$$

```
[Out] (a^4 - 6*a^2*b^2 + b^4)*x + (4*a*b*(a^2 - b^2)*Log[Cos[c + d*x]])/d + (b^2*
(6*a^2 - b^2)*Tan[c + d*x])/d + (2*a*b*(a^2 - b^2)*Tan[c + d*x]^2)/d - (b^2
*(6*a^2 - b^2)*Tan[c + d*x]^3)/(3*d) + (a*b^3*Tan[c + d*x]^4)/d + (b^2*(6*a
^2 - b^2)*Tan[c + d*x]^5)/(5*d) - (2*a*b^3*Tan[c + d*x]^6)/(3*d) + (b^4*Tan
[c + d*x]^7)/(7*d) + (a*b^3*Tan[c + d*x]^8)/(2*d) - (b^4*Tan[c + d*x]^9)/(9
*d) + (b^4*Tan[c + d*x]^11)/(11*d)
```

Rubi [A] time = 0.147243, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3661, 1810, 635, 203, 260}

$$\frac{b^2\left(6a^2-b^2\right)\tan^5(c+dx)}{5d}-\frac{b^2\left(6a^2-b^2\right)\tan^3(c+dx)}{3d}+\frac{2ab\left(a^2-b^2\right)\tan^2(c+dx)}{d}+\frac{b^2\left(6a^2-b^2\right)\tan(c+dx)}{d}+\frac{4ab(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^3)^4,x]
```

```
[Out] (a^4 - 6*a^2*b^2 + b^4)*x + (4*a*b*(a^2 - b^2)*Log[Cos[c + d*x]])/d + (b^2*
(6*a^2 - b^2)*Tan[c + d*x])/d + (2*a*b*(a^2 - b^2)*Tan[c + d*x]^2)/d - (b^2
*(6*a^2 - b^2)*Tan[c + d*x]^3)/(3*d) + (a*b^3*Tan[c + d*x]^4)/d + (b^2*(6*a
^2 - b^2)*Tan[c + d*x]^5)/(5*d) - (2*a*b^3*Tan[c + d*x]^6)/(3*d) + (b^4*Tan
[c + d*x]^7)/(7*d) + (a*b^3*Tan[c + d*x]^8)/(2*d) - (b^4*Tan[c + d*x]^9)/(9
*d) + (b^4*Tan[c + d*x]^11)/(11*d)
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \left(a+b\tan^3(c+dx)\right)^4\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^3\right)^4}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(6a^2b^2-b^4+4ab\left(a^2-b^2\right)x-b^2\left(6a^2-b^2\right)x^2+4ab^3x^3+b^2\left(6a^2-b^2\right)x^4-\right.\right.}{d}$$

$$= \frac{b^2\left(6a^2-b^2\right)\tan(c+dx)}{d}+\frac{2ab\left(a^2-b^2\right)\tan^2(c+dx)}{d}-\frac{b^2\left(6a^2-b^2\right)\tan^3(c+dx)}{3d}+$$

$$= \frac{b^2\left(6a^2-b^2\right)\tan(c+dx)}{d}+\frac{2ab\left(a^2-b^2\right)\tan^2(c+dx)}{d}-\frac{b^2\left(6a^2-b^2\right)\tan^3(c+dx)}{3d}+$$

$$= \left(a^4-6a^2b^2+b^4\right)x+\frac{4ab\left(a^2-b^2\right)\log(\cos(c+dx))}{d}+\frac{b^2\left(6a^2-b^2\right)\tan(c+dx)}{d}+\frac{2ab}{}$$

Mathematica [C] time = 0.904277, size = 224, normalized size = 0.88

$$\frac{-1386b^2\left(b^2-6a^2\right)\tan^5(c+dx)+2310b^2\left(b^2-6a^2\right)\tan^3(c+dx)+13860ab\left(a^2-b^2\right)\tan^2(c+dx)-6930b^2\left(b^2-6a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^3)^4,x]
```

```
[Out] ((-3465*I)*((a - I*b)^4*Log[I - Tan[c + d*x]] - (a + I*b)^4*Log[I + Tan[c +
 d*x]]) - 6930*b^2*(-6*a^2 + b^2)*Tan[c + d*x] + 13860*a*b*(a^2 - b^2)*Tan[
c + d*x]^2 + 2310*b^2*(-6*a^2 + b^2)*Tan[c + d*x]^3 + 6930*a*b^3*Tan[c + d*
x]^4 - 1386*b^2*(-6*a^2 + b^2)*Tan[c + d*x]^5 - 4620*a*b^3*Tan[c + d*x]^6 +
 990*b^4*Tan[c + d*x]^7 + 3465*a*b^3*Tan[c + d*x]^8 - 770*b^4*Tan[c + d*x]^
9 + 630*b^4*Tan[c + d*x]^11)/(6930*d)
```

Maple [A] time = 0.008, size = 321, normalized size = 1.3

$$\frac{b^4\left(\tan(dx+c)\right)^{11}}{11\,d}-\frac{b^4\left(\tan(dx+c)\right)^{9}}{9\,d}+\frac{ab^3\left(\tan(dx+c)\right)^{8}}{2\,d}+\frac{b^4\left(\tan(dx+c)\right)^{7}}{7\,d}-\frac{2\,ab^3\left(\tan(dx+c)\right)^{6}}{3\,d}+\frac{6\,(\tan(d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^3)^4,x)
```

```
[Out] 1/11*b^4*tan(d*x+c)^11/d-1/9*b^4*tan(d*x+c)^9/d+1/2*a*b^3*tan(d*x+c)^8/d+1/
7*b^4*tan(d*x+c)^7/d-2/3*a*b^3*tan(d*x+c)^6/d+6/5/d*tan(d*x+c)^5*a^2*b^2-1/
5/d*tan(d*x+c)^5*b^4+a*b^3*tan(d*x+c)^4/d-2/d*tan(d*x+c)^3*a^2*b^2+1/3/d*ta
n(d*x+c)^3*b^4+2/d*tan(d*x+c)^2*a^3*b-2/d*a*b^3*tan(d*x+c)^2+6/d*a^2*b^2*ta
```

```
n(d*x+c)-1/d*b^4*tan(d*x+c)-2/d*ln(tan(d*x+c)^2+1)*a^3*b+2/d*ln(tan(d*x+c)^
2+1)*a*b^3+1/d*arctan(tan(d*x+c))*a^4-6/d*arctan(tan(d*x+c))*a^2*b^2+1/d*ar
ctan(tan(d*x+c))*b^4
```

Maxima [A] time = 1.51901, size = 351, normalized size = 1.38

$$a^4x + \frac{2\left(3\,\tan\left(dx+c\right)^5 - 5\,\tan\left(dx+c\right)^3 - 15\,dx - 15\,c + 15\,\tan\left(dx+c\right)\right)a^2b^2}{5\,d} + \frac{\left(315\,\tan\left(dx+c\right)^{11} - 385\,\tan\left(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 2/5*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d
*x + c))*a^2*b^2/d + 1/3465*(315*tan(d*x + c)^11 - 385*tan(d*x + c)^9 + 495
*tan(d*x + c)^7 - 693*tan(d*x + c)^5 + 1155*tan(d*x + c)^3 + 3465*d*x + 346
5*c - 3465*tan(d*x + c))*b^4/d + 1/6*a*b^3*((48*sin(d*x + c)^6 - 108*sin(d*
x + c)^4 + 88*sin(d*x + c)^2 - 25)/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*s
in(d*x + c)^4 - 4*sin(d*x + c)^2 + 1) - 12*log(sin(d*x + c)^2 - 1))/d - 2*a
^3*b*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d
```

Fricas [A] time = 1.77588, size = 559, normalized size = 2.19

$$\frac{630\,b^4\tan\left(dx+c\right)^{11} - 770\,b^4\tan\left(dx+c\right)^9 + 3465\,ab^3\tan\left(dx+c\right)^8 + 990\,b^4\tan\left(dx+c\right)^7 - 4620\,ab^3\tan\left(dx+c\right)^6 + 69}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^4,x, algorithm="fricas")
```

```
[Out] 1/6930*(630*b^4*tan(d*x + c)^11 - 770*b^4*tan(d*x + c)^9 + 3465*a*b^3*tan(d
*x + c)^8 + 990*b^4*tan(d*x + c)^7 - 4620*a*b^3*tan(d*x + c)^6 + 6930*a*b^3
*tan(d*x + c)^4 + 1386*(6*a^2*b^2 - b^4)*tan(d*x + c)^5 - 2310*(6*a^2*b^2 -
 b^4)*tan(d*x + c)^3 + 6930*(a^4 - 6*a^2*b^2 + b^4)*d*x + 13860*(a^3*b - a*
b^3)*tan(d*x + c)^2 + 13860*(a^3*b - a*b^3)*log(1/(tan(d*x + c)^2 + 1)) + 6
930*(6*a^2*b^2 - b^4)*tan(d*x + c))/d
```

Sympy [A] time = 4.69613, size = 301, normalized size = 1.18

$$\begin{cases} a^4x - \frac{2a^3b\log\left(\tan^2\left(c+dx\right)+1\right)}{d} + \frac{2a^3b\tan^2\left(c+dx\right)}{d} - 6a^2b^2x + \frac{6a^2b^2\tan^5\left(c+dx\right)}{5d} - \frac{2a^2b^2\tan^3\left(c+dx\right)}{d} + \frac{6a^2b^2\tan\left(c+dx\right)}{d} + \frac{2ab^3\log\left(\tan^2\left(c+dx\right.\right.}{d} \\ x\left(a+b\tan^3\left(c\right)\right)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)**3)**4,x)
```

```
[Out] Piecewise((a**4*x - 2*a**3*b*log(tan(c + d*x)**2 + 1)/d + 2*a**3*b*tan(c +
d*x)**2/d - 6*a**2*b**2*x + 6*a**2*b**2*tan(c + d*x)**5/(5*d) - 2*a**2*b**2
*tan(c + d*x)**3/d + 6*a**2*b**2*tan(c + d*x)/d + 2*a*b**3*log(tan(c + d*x)
**2 + 1)/d + a*b**3*tan(c + d*x)**8/(2*d) - 2*a*b**3*tan(c + d*x)**6/(3*d)
```

```
+ a*b**3*tan(c + d*x)**4/d - 2*a*b**3*tan(c + d*x)**2/d + b**4*x + b**4*tan
(c + d*x)**11/(11*d) - b**4*tan(c + d*x)**9/(9*d) + b**4*tan(c + d*x)**7/(7
*d) - b**4*tan(c + d*x)**5/(5*d) + b**4*tan(c + d*x)**3/(3*d) - b**4*tan(c
+ d*x)/d, Ne(d, 0)), (x*(a + b*tan(c)**3)**4, True))
```

Giac [B] time = 75.9038, size = 8031, normalized size = 31.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^4,x, algorithm="giac")
```

```
[Out] 1/6930*(6930*a^4*d*x*tan(d*x)^11*tan(c)^11 - 41580*a^2*b^2*d*x*tan(d*x)^11*
tan(c)^11 + 6930*b^4*d*x*tan(d*x)^11*tan(c)^11 + 13860*a^3*b*log(4*(tan(c)^
2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + t
an(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^11*tan(c)^11 - 13860*a*b^3*log
(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*t
an(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^11*tan(c)^11 - 7623
0*a^4*d*x*tan(d*x)^10*tan(c)^10 + 457380*a^2*b^2*d*x*tan(d*x)^10*tan(c)^10
- 76230*b^4*d*x*tan(d*x)^10*tan(c)^10 + 13860*a^3*b*tan(d*x)^11*tan(c)^11 -
 28875*a*b^3*tan(d*x)^11*tan(c)^11 - 152460*a^3*b*log(4*(tan(c)^2 + 1)/(tan
(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 -
 2*tan(d*x)*tan(c) + 1))*tan(d*x)^10*tan(c)^10 + 152460*a*b^3*log(4*(tan(c)
^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 +
tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^10*tan(c)^10 - 41580*a^2*b^2*
tan(d*x)^11*tan(c)^10 + 6930*b^4*tan(d*x)^11*tan(c)^10 - 41580*a^2*b^2*tan(
d*x)^10*tan(c)^11 + 6930*b^4*tan(d*x)^10*tan(c)^11 + 381150*a^4*d*x*tan(d*x
)^9*tan(c)^9 - 2286900*a^2*b^2*d*x*tan(d*x)^9*tan(c)^9 + 381150*b^4*d*x*tan
(d*x)^9*tan(c)^9 + 13860*a^3*b*tan(d*x)^11*tan(c)^9 - 13860*a*b^3*tan(d*x)^
11*tan(c)^9 - 124740*a^3*b*tan(d*x)^10*tan(c)^10 + 289905*a*b^3*tan(d*x)^10
*tan(c)^10 + 13860*a^3*b*tan(d*x)^9*tan(c)^11 - 13860*a*b^3*tan(d*x)^9*tan(
c)^11 + 13860*a^2*b^2*tan(d*x)^11*tan(c)^8 - 2310*b^4*tan(d*x)^11*tan(c)^8
+ 762300*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan
(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^9
*tan(c)^9 - 762300*a*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(
d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*
tan(d*x)^9*tan(c)^9 + 457380*a^2*b^2*tan(d*x)^10*tan(c)^9 - 76230*b^4*tan(d
*x)^10*tan(c)^9 + 457380*a^2*b^2*tan(d*x)^9*tan(c)^10 - 76230*b^4*tan(d*x)^
9*tan(c)^10 + 13860*a^2*b^2*tan(d*x)^8*tan(c)^11 - 2310*b^4*tan(d*x)^8*tan(
c)^11 + 6930*a*b^3*tan(d*x)^11*tan(c)^7 - 1143450*a^4*d*x*tan(d*x)^8*tan(c)
^8 + 6860700*a^2*b^2*d*x*tan(d*x)^8*tan(c)^8 - 1143450*b^4*d*x*tan(d*x)^8*t
an(c)^8 - 124740*a^3*b*tan(d*x)^10*tan(c)^8 + 152460*a*b^3*tan(d*x)^10*tan(
c)^8 + 512820*a^3*b*tan(d*x)^9*tan(c)^9 - 1297065*a*b^3*tan(d*x)^9*tan(c)^9
 - 124740*a^3*b*tan(d*x)^8*tan(c)^10 + 152460*a*b^3*tan(d*x)^8*tan(c)^10 +
6930*a*b^3*tan(d*x)^7*tan(c)^11 - 8316*a^2*b^2*tan(d*x)^11*tan(c)^6 + 1386*
b^4*tan(d*x)^11*tan(c)^6 - 152460*a^2*b^2*tan(d*x)^10*tan(c)^7 + 25410*b^4*
tan(d*x)^10*tan(c)^7 - 2286900*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c
)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*t
an(c) + 1))*tan(d*x)^8*tan(c)^8 + 2286900*a*b^3*log(4*(tan(c)^2 + 1)/(tan(d
*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2
*tan(d*x)*tan(c) + 1))*tan(d*x)^8*tan(c)^8 - 2286900*a^2*b^2*tan(d*x)^9*tan
(c)^8 + 381150*b^4*tan(d*x)^9*tan(c)^8 - 2286900*a^2*b^2*tan(d*x)^8*tan(c)^
9 + 381150*b^4*tan(d*x)^8*tan(c)^9 - 152460*a^2*b^2*tan(d*x)^7*tan(c)^10 +
25410*b^4*tan(d*x)^7*tan(c)^10 - 8316*a^2*b^2*tan(d*x)^6*tan(c)^11 + 1386*b
^4*tan(d*x)^6*tan(c)^11 - 4620*a*b^3*tan(d*x)^11*tan(c)^5 - 76230*a*b^3*tan
(d*x)^10*tan(c)^6 + 2286900*a^4*d*x*tan(d*x)^7*tan(c)^7 - 13721400*a^2*b^2*
```

d*x*tan(d*x)^7*tan(c)^7 + 2286900*b^4*d*x*tan(d*x)^7*tan(c)^7 + 498960*a^3*
b*tan(d*x)^9*tan(c)^7 - 762300*a*b^3*tan(d*x)^9*tan(c)^7 - 1288980*a^3*b*ta
n(d*x)^8*tan(c)^8 + 3382995*a*b^3*tan(d*x)^8*tan(c)^8 + 498960*a^3*b*tan(d*
x)^7*tan(c)^9 - 762300*a*b^3*tan(d*x)^7*tan(c)^9 - 76230*a*b^3*tan(d*x)^6*t
an(c)^10 - 4620*a*b^3*tan(d*x)^5*tan(c)^11 - 990*b^4*tan(d*x)^11*tan(c)^4 +
49896*a^2*b^2*tan(d*x)^10*tan(c)^5 - 15246*b^4*tan(d*x)^10*tan(c)^5 + 6375
60*a^2*b^2*tan(d*x)^9*tan(c)^6 - 127050*b^4*tan(d*x)^9*tan(c)^6 + 4573800*a
^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(
d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^7*tan(c)^7
- 4573800*a*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*ta
n(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^
7*tan(c)^7 + 6652800*a^2*b^2*tan(d*x)^8*tan(c)^7 - 1143450*b^4*tan(d*x)^8*t
an(c)^7 + 6652800*a^2*b^2*tan(d*x)^7*tan(c)^8 - 1143450*b^4*tan(d*x)^7*tan(
c)^8 + 637560*a^2*b^2*tan(d*x)^6*tan(c)^9 - 127050*b^4*tan(d*x)^6*tan(c)^9
+ 49896*a^2*b^2*tan(d*x)^5*tan(c)^10 - 15246*b^4*tan(d*x)^5*tan(c)^10 - 990
*b^4*tan(d*x)^4*tan(c)^11 + 3465*a*b^3*tan(d*x)^11*tan(c)^3 + 50820*a*b^3*t
an(d*x)^10*tan(c)^4 + 381150*a*b^3*tan(d*x)^9*tan(c)^5 - 3201660*a^4*d*x*ta
n(d*x)^6*tan(c)^6 + 19209960*a^2*b^2*d*x*tan(d*x)^6*tan(c)^6 - 3201660*b^4*
d*x*tan(d*x)^6*tan(c)^6 - 1164240*a^3*b*tan(d*x)^8*tan(c)^6 + 2286900*a*b^3
*tan(d*x)^8*tan(c)^6 + 2245320*a^3*b*tan(d*x)^7*tan(c)^7 - 5622540*a*b^3*ta
n(d*x)^7*tan(c)^7 - 1164240*a^3*b*tan(d*x)^6*tan(c)^8 + 2286900*a*b^3*tan(d
*x)^6*tan(c)^8 + 381150*a*b^3*tan(d*x)^5*tan(c)^9 + 50820*a*b^3*tan(d*x)^4*
tan(c)^10 + 3465*a*b^3*tan(d*x)^3*tan(c)^11 + 770*b^4*tan(d*x)^11*tan(c)^2
+ 10890*b^4*tan(d*x)^10*tan(c)^3 - 124740*a^2*b^2*tan(d*x)^9*tan(c)^4 + 762
30*b^4*tan(d*x)^9*tan(c)^4 - 1399860*a^2*b^2*tan(d*x)^8*tan(c)^5 + 381150*b
^4*tan(d*x)^8*tan(c)^5 - 6403320*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan
(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)
*tan(c) + 1))*tan(d*x)^6*tan(c)^6 + 6403320*a*b^3*log(4*(tan(c)^2 + 1)/(tan
(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 -
2*tan(d*x)*tan(c) + 1))*tan(d*x)^6*tan(c)^6 - 12307680*a^2*b^2*tan(d*x)^7*
tan(c)^6 + 2286900*b^4*tan(d*x)^7*tan(c)^6 - 12307680*a^2*b^2*tan(d*x)^6*ta
n(c)^7 + 2286900*b^4*tan(d*x)^6*tan(c)^7 - 1399860*a^2*b^2*tan(d*x)^5*tan(c
)^8 + 381150*b^4*tan(d*x)^5*tan(c)^8 - 124740*a^2*b^2*tan(d*x)^4*tan(c)^9 +
76230*b^4*tan(d*x)^4*tan(c)^9 + 10890*b^4*tan(d*x)^3*tan(c)^10 + 770*b^4*t
an(d*x)^2*tan(c)^11 - 10395*a*b^3*tan(d*x)^10*tan(c)^2 - 129360*a*b^3*tan(d
*x)^9*tan(c)^3 - 810810*a*b^3*tan(d*x)^8*tan(c)^4 + 3201660*a^4*d*x*tan(d*x
)^5*tan(c)^5 - 19209960*a^2*b^2*d*x*tan(d*x)^5*tan(c)^5 + 3201660*b^4*d*x*t
an(d*x)^5*tan(c)^5 + 1746360*a^3*b*tan(d*x)^7*tan(c)^5 - 3991680*a*b^3*tan(
d*x)^7*tan(c)^5 - 2910600*a^3*b*tan(d*x)^6*tan(c)^6 + 6740580*a*b^3*tan(d*x
)^6*tan(c)^6 + 1746360*a^3*b*tan(d*x)^5*tan(c)^7 - 3991680*a*b^3*tan(d*x)^5
*tan(c)^7 - 810810*a*b^3*tan(d*x)^4*tan(c)^8 - 129360*a*b^3*tan(d*x)^3*tan(
c)^9 - 10395*a*b^3*tan(d*x)^2*tan(c)^10 - 630*b^4*tan(d*x)^11 - 8470*b^4*ta
n(d*x)^10*tan(c) - 54450*b^4*tan(d*x)^9*tan(c)^2 + 166320*a^2*b^2*tan(d*x)^
8*tan(c)^3 - 228690*b^4*tan(d*x)^8*tan(c)^3 + 1801800*a^2*b^2*tan(d*x)^7*ta
n(c)^4 - 762300*b^4*tan(d*x)^7*tan(c)^4 + 6403320*a^3*b*log(4*(tan(c)^2 + 1
)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*
x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^5*tan(c)^5 - 6403320*a*b^3*log(4*(t
an(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)
^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^5*tan(c)^5 + 15051960*a^
2*b^2*tan(d*x)^6*tan(c)^5 - 3201660*b^4*tan(d*x)^6*tan(c)^5 + 15051960*a^2*
b^2*tan(d*x)^5*tan(c)^6 - 3201660*b^4*tan(d*x)^5*tan(c)^6 + 1801800*a^2*b^2
*tan(d*x)^4*tan(c)^7 - 762300*b^4*tan(d*x)^4*tan(c)^7 + 166320*a^2*b^2*tan(
d*x)^3*tan(c)^8 - 228690*b^4*tan(d*x)^3*tan(c)^8 - 54450*b^4*tan(d*x)^2*tan
(c)^9 - 8470*b^4*tan(d*x)*tan(c)^10 - 630*b^4*tan(c)^11 + 10395*a*b^3*tan(d
*x)^9*tan(c) + 129360*a*b^3*tan(d*x)^8*tan(c)^2 + 810810*a*b^3*tan(d*x)^7*t
an(c)^3 - 2286900*a^4*d*x*tan(d*x)^4*tan(c)^4 + 13721400*a^2*b^2*d*x*tan(d*
x)^4*tan(c)^4 - 2286900*b^4*d*x*tan(d*x)^4*tan(c)^4 - 1746360*a^3*b*tan(d*x
)^6*tan(c)^4 + 3991680*a*b^3*tan(d*x)^6*tan(c)^4 + 2910600*a^3*b*tan(d*x)^5
*tan(c)^5 - 6740580*a*b^3*tan(d*x)^5*tan(c)^5 - 1746360*a^3*b*tan(d*x)^4*ta

n(c)^6 + 3991680*a*b^3*tan(d*x)^4*tan(c)^6 + 810810*a*b^3*tan(d*x)^3*tan(c)
^7 + 129360*a*b^3*tan(d*x)^2*tan(c)^8 + 10395*a*b^3*tan(d*x)*tan(c)^9 + 770
*b^4*tan(d*x)^9 + 10890*b^4*tan(d*x)^8*tan(c) - 124740*a^2*b^2*tan(d*x)^7*t
an(c)^2 + 76230*b^4*tan(d*x)^7*tan(c)^2 - 1399860*a^2*b^2*tan(d*x)^6*tan(c)
^3 + 381150*b^4*tan(d*x)^6*tan(c)^3 - 4573800*a^3*b*log(4*(tan(c)^2 + 1)/(t
an(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2
 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4 + 4573800*a*b^3*log(4*(tan(c
)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 +
 tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4 - 12307680*a^2*b^
2*tan(d*x)^5*tan(c)^4 + 2286900*b^4*tan(d*x)^5*tan(c)^4 - 12307680*a^2*b^2*
tan(d*x)^4*tan(c)^5 + 2286900*b^4*tan(d*x)^4*tan(c)^5 - 1399860*a^2*b^2*tan
(d*x)^3*tan(c)^6 + 381150*b^4*tan(d*x)^3*tan(c)^6 - 124740*a^2*b^2*tan(d*x)
^2*tan(c)^7 + 76230*b^4*tan(d*x)^2*tan(c)^7 + 10890*b^4*tan(d*x)*tan(c)^8 +
 770*b^4*tan(c)^9 - 3465*a*b^3*tan(d*x)^8 - 50820*a*b^3*tan(d*x)^7*tan(c) -
 381150*a*b^3*tan(d*x)^6*tan(c)^2 + 1143450*a^4*d*x*tan(d*x)^3*tan(c)^3 - 6
860700*a^2*b^2*d*x*tan(d*x)^3*tan(c)^3 + 1143450*b^4*d*x*tan(d*x)^3*tan(c)^
3 + 1164240*a^3*b*tan(d*x)^5*tan(c)^3 - 2286900*a*b^3*tan(d*x)^5*tan(c)^3 -
 2245320*a^3*b*tan(d*x)^4*tan(c)^4 + 5622540*a*b^3*tan(d*x)^4*tan(c)^4 + 11
64240*a^3*b*tan(d*x)^3*tan(c)^5 - 2286900*a*b^3*tan(d*x)^3*tan(c)^5 - 38115
0*a*b^3*tan(d*x)^2*tan(c)^6 - 50820*a*b^3*tan(d*x)*tan(c)^7 - 3465*a*b^3*ta
n(c)^8 - 990*b^4*tan(d*x)^7 + 49896*a^2*b^2*tan(d*x)^6*tan(c) - 15246*b^4*t
an(d*x)^6*tan(c) + 637560*a^2*b^2*tan(d*x)^5*tan(c)^2 - 127050*b^4*tan(d*x)
^5*tan(c)^2 + 2286900*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*t
an(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1
))*tan(d*x)^3*tan(c)^3 - 2286900*a*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan
(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)
*tan(c) + 1))*tan(d*x)^3*tan(c)^3 + 6652800*a^2*b^2*tan(d*x)^4*tan(c)^3 - 1
143450*b^4*tan(d*x)^4*tan(c)^3 + 6652800*a^2*b^2*tan(d*x)^3*tan(c)^4 - 1143
450*b^4*tan(d*x)^3*tan(c)^4 + 637560*a^2*b^2*tan(d*x)^2*tan(c)^5 - 127050*b
^4*tan(d*x)^2*tan(c)^5 + 49896*a^2*b^2*tan(d*x)*tan(c)^6 - 15246*b^4*tan(d*
x)*tan(c)^6 - 990*b^4*tan(c)^7 + 4620*a*b^3*tan(d*x)^6 + 76230*a*b^3*tan(d*
x)^5*tan(c) - 381150*a^4*d*x*tan(d*x)^2*tan(c)^2 + 2286900*a^2*b^2*d*x*tan(
d*x)^2*tan(c)^2 - 381150*b^4*d*x*tan(d*x)^2*tan(c)^2 - 498960*a^3*b*tan(d*x
)^4*tan(c)^2 + 762300*a*b^3*tan(d*x)^4*tan(c)^2 + 1288980*a^3*b*tan(d*x)^3*
tan(c)^3 - 3382995*a*b^3*tan(d*x)^3*tan(c)^3 - 498960*a^3*b*tan(d*x)^2*tan(
c)^4 + 762300*a*b^3*tan(d*x)^2*tan(c)^4 + 76230*a*b^3*tan(d*x)*tan(c)^5 + 4
620*a*b^3*tan(c)^6 - 8316*a^2*b^2*tan(d*x)^5 + 1386*b^4*tan(d*x)^5 - 152460
*a^2*b^2*tan(d*x)^4*tan(c) + 25410*b^4*tan(d*x)^4*tan(c) - 762300*a^3*b*log
(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*t
an(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 + 762300
*a*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + ta
n(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^
2 - 2286900*a^2*b^2*tan(d*x)^3*tan(c)^2 + 381150*b^4*tan(d*x)^3*tan(c)^2 -
2286900*a^2*b^2*tan(d*x)^2*tan(c)^3 + 381150*b^4*tan(d*x)^2*tan(c)^3 - 1524
60*a^2*b^2*tan(d*x)*tan(c)^4 + 25410*b^4*tan(d*x)*tan(c)^4 - 8316*a^2*b^2*t
an(c)^5 + 1386*b^4*tan(c)^5 - 6930*a*b^3*tan(d*x)^4 + 76230*a^4*d*x*tan(d*x
)*tan(c) - 457380*a^2*b^2*d*x*tan(d*x)*tan(c) + 76230*b^4*d*x*tan(d*x)*tan(
c) + 124740*a^3*b*tan(d*x)^3*tan(c) - 152460*a*b^3*tan(d*x)^3*tan(c) - 5128
20*a^3*b*tan(d*x)^2*tan(c)^2 + 1297065*a*b^3*tan(d*x)^2*tan(c)^2 + 124740*a
^3*b*tan(d*x)*tan(c)^3 - 152460*a*b^3*tan(d*x)*tan(c)^3 - 6930*a*b^3*tan(c)
^4 + 13860*a^2*b^2*tan(d*x)^3 - 2310*b^4*tan(d*x)^3 + 152460*a^3*b*log(4*(t
an(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)
^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)*tan(c) - 152460*a*b^3*lo
g(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*
tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)*tan(c) + 457380*a^
2*b^2*tan(d*x)^2*tan(c) - 76230*b^4*tan(d*x)^2*tan(c) + 457380*a^2*b^2*tan(
d*x)*tan(c)^2 - 76230*b^4*tan(d*x)*tan(c)^2 + 13860*a^2*b^2*tan(c)^3 - 2310
*b^4*tan(c)^3 - 6930*a^4*d*x + 41580*a^2*b^2*d*x - 6930*b^4*d*x - 13860*a^3
*b*tan(d*x)^2 + 13860*a*b^3*tan(d*x)^2 + 124740*a^3*b*tan(d*x)*tan(c) - 289

```
905*a*b^3*tan(d*x)*tan(c) - 13860*a^3*b*tan(c)^2 + 13860*a*b^3*tan(c)^2 - 1
3860*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c)
+ tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)) + 13860*a*b^3*
log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^
2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)) - 41580*a^2*b^2*tan(d*x)
+ 6930*b^4*tan(d*x) - 41580*a^2*b^2*tan(c) + 6930*b^4*tan(c) - 13860*a^3*b
+ 28875*a*b^3)/(d*tan(d*x)^11*tan(c)^11 - 11*d*tan(d*x)^10*tan(c)^10 + 55*d
*tan(d*x)^9*tan(c)^9 - 165*d*tan(d*x)^8*tan(c)^8 + 330*d*tan(d*x)^7*tan(c)^
7 - 462*d*tan(d*x)^6*tan(c)^6 + 462*d*tan(d*x)^5*tan(c)^5 - 330*d*tan(d*x)^
4*tan(c)^4 + 165*d*tan(d*x)^3*tan(c)^3 - 55*d*tan(d*x)^2*tan(c)^2 + 11*d*ta
n(d*x)*tan(c) - d)
```

3.375 $\int \left(a+b\tan^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=168

$$\frac{b\left(3a^2-b^2\right)\tan^2(c+dx)}{2d}+\frac{b\left(3a^2-b^2\right)\log(\cos(c+dx))}{d}+ax\left(a^2-3b^2\right)+\frac{3ab^2\tan^5(c+dx)}{5d}-\frac{ab^2\tan^3(c+dx)}{d}+$$

```
[Out] a*(a^2 - 3*b^2)*x + (b*(3*a^2 - b^2)*Log[Cos[c + d*x]])/d + (3*a*b^2*Tan[c
+ d*x])/d + (b*(3*a^2 - b^2)*Tan[c + d*x]^2)/(2*d) - (a*b^2*Tan[c + d*x]^3)
/d + (b^3*Tan[c + d*x]^4)/(4*d) + (3*a*b^2*Tan[c + d*x]^5)/(5*d) - (b^3*Tan
[c + d*x]^6)/(6*d) + (b^3*Tan[c + d*x]^8)/(8*d)
```

Rubi [A] time = 0.0977967, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3661, 1810, 635, 203, 260}

$$\frac{b\left(3a^2-b^2\right)\tan^2(c+dx)}{2d}+\frac{b\left(3a^2-b^2\right)\log(\cos(c+dx))}{d}+ax\left(a^2-3b^2\right)+\frac{3ab^2\tan^5(c+dx)}{5d}-\frac{ab^2\tan^3(c+dx)}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^3)^3,x]
```


```
[Out] a*(a^2 - 3*b^2)*x + (b*(3*a^2 - b^2)*Log[Cos[c + d*x]])/d + (3*a*b^2*Tan[c
+ d*x])/d + (b*(3*a^2 - b^2)*Tan[c + d*x]^2)/(2*d) - (a*b^2*Tan[c + d*x]^3)
/d + (b^3*Tan[c + d*x]^4)/(4*d) + (3*a*b^2*Tan[c + d*x]^5)/(5*d) - (b^3*Tan
[c + d*x]^6)/(6*d) + (b^3*Tan[c + d*x]^8)/(8*d)
```


Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \left(a+b\tan^3(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int\frac{(a+bx^3)^3}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}$$
$$=\frac{\text{Subst}\left(\int\left(3ab^2+b\left(3a^2-b^2\right)x-3ab^2x^2+b^3x^3+3ab^2x^4-b^3x^5+b^3x^7+\frac{a^3-3ab^2-b\left(3a^2-b^2\right.}{1+x^2}\right.\right.}{d}$$
$$=\frac{3ab^2\tan(c+dx)}{d}+\frac{b\left(3a^2-b^2\right)\tan^2(c+dx)}{2d}-\frac{ab^2\tan^3(c+dx)}{d}+\frac{b^3\tan^4(c+dx)}{4d}+\frac{3ab}{}$$
$$=\frac{3ab^2\tan(c+dx)}{d}+\frac{b\left(3a^2-b^2\right)\tan^2(c+dx)}{2d}-\frac{ab^2\tan^3(c+dx)}{d}+\frac{b^3\tan^4(c+dx)}{4d}+\frac{3ab}{}$$
$$=a\left(a^2-3b^2\right)x+\frac{b\left(3a^2-b^2\right)\log(\cos(c+dx))}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b\left(3a^2-b^2\right)\tan^2(c+}{2d}$$

Mathematica [C] time = 0.428701, size = 160, normalized size = 0.95

$$\frac{-60b\left(b^2-3a^2\right)\tan^2(c+dx)+72ab^2\tan^5(c+dx)-120ab^2\tan^3(c+dx)+360ab^2\tan(c+dx)+60\left(i(a+ib)^3\log(\tan(\right.}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^3)^3,x]
```

```
[Out] (60*((-I)*(a - I*b)^3*Log[I - Tan[c + d*x]] + I*(a + I*b)^3*Log[I + Tan[c +
 d*x]]) + 360*a*b^2*Tan[c + d*x] - 60*b*(-3*a^2 + b^2)*Tan[c + d*x]^2 - 120
*a*b^2*Tan[c + d*x]^3 + 30*b^3*Tan[c + d*x]^4 + 72*a*b^2*Tan[c + d*x]^5 - 2
0*b^3*Tan[c + d*x]^6 + 15*b^3*Tan[c + d*x]^8)/(120*d)
```

Maple [A] time = 0.007, size = 201, normalized size = 1.2

$$\frac{b^3(\tan(dx+c))^8}{8d}-\frac{b^3(\tan(dx+c))^6}{6d}+\frac{3ab^2(\tan(dx+c))^5}{5d}+\frac{b^3(\tan(dx+c))^4}{4d}-\frac{ab^2(\tan(dx+c))^3}{d}+\frac{3(\tan(dx+}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^3)^3,x)
```

```
[Out] 1/8*b^3*tan(d*x+c)^8/d-1/6*b^3*tan(d*x+c)^6/d+3/5*a*b^2*tan(d*x+c)^5/d+1/4*
b^3*tan(d*x+c)^4/d-a*b^2*tan(d*x+c)^3/d+3/2/d*tan(d*x+c)^2*a^2*b-1/2/d*tan(
d*x+c)^2*b^3+3*a*b^2*tan(d*x+c)/d-3/2/d*ln(tan(d*x+c)^2+1)*a^2*b+1/2/d*ln(t
an(d*x+c)^2+1)*b^3+1/d*arctan(tan(d*x+c))*a^3-3/d*arctan(tan(d*x+c))*a*b^2
```

Maxima [A] time = 1.52337, size = 247, normalized size = 1.47

$$a^3x+\frac{\left(3\tan(dx+c)^5-5\tan(dx+c)^3-15dx-15c+15\tan(dx+c)\right)ab^2}{5d}+\frac{b^3\left(\frac{48\sin(dx+c)^6-108\sin(dx+c)^4+88\sin(dx+c)}{\sin(dx+c)^8-4\sin(dx+c)^6+6\sin(dx+c)^4-4\sin(d}\right.}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 1/5*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d
*x + c))*a*b^2/d + 1/24*b^3*((48*sin(d*x + c)^6 - 108*sin(d*x + c)^4 + 88*s
in(d*x + c)^2 - 25)/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 -
 4*sin(d*x + c)^2 + 1) - 12*log(sin(d*x + c)^2 - 1))/d - 3/2*a^2*b*(1/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d
```

Fricas [A] time = 1.672, size = 365, normalized size = 2.17

$$\frac{15\,b^3\tan(dx+c)^8 - 20\,b^3\tan(dx+c)^6 + 72\,ab^2\tan(dx+c)^5 + 30\,b^3\tan(dx+c)^4 - 120\,ab^2\tan(dx+c)^3 + 360\,ab^2}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(15*b^3*tan(d*x + c)^8 - 20*b^3*tan(d*x + c)^6 + 72*a*b^2*tan(d*x + c
)^5 + 30*b^3*tan(d*x + c)^4 - 120*a*b^2*tan(d*x + c)^3 + 360*a*b^2*tan(d*x
+ c) + 120*(a^3 - 3*a*b^2)*d*x + 60*(3*a^2*b - b^3)*tan(d*x + c)^2 + 60*(3*
a^2*b - b^3)*log(1/(tan(d*x + c)^2 + 1)))/d
```

Sympy [A] time = 1.90791, size = 194, normalized size = 1.15

$$\begin{cases} a^3x - \frac{3a^2b\log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{3a^2b\tan^2(c+dx)}{2d} - 3ab^2x + \frac{3ab^2\tan^5(c+dx)}{5d} - \frac{ab^2\tan^3(c+dx)}{d} + \frac{3ab^2\tan(c+dx)}{d} + \frac{b^3\log\left(\tan^2(c+dx)+1\right)}{2d} \\ x\left(a+b\tan^3(c)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)**3)**3,x)
```

```
[Out] Piecewise((a**3*x - 3*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d) + 3*a**2*b*tan(
c + d*x)**2/(2*d) - 3*a*b**2*x + 3*a*b**2*tan(c + d*x)**5/(5*d) - a*b**2*ta
n(c + d*x)**3/d + 3*a*b**2*tan(c + d*x)/d + b**3*log(tan(c + d*x)**2 + 1)/(
2*d) + b**3*tan(c + d*x)**8/(8*d) - b**3*tan(c + d*x)**6/(6*d) + b**3*tan(c
 + d*x)**4/(4*d) - b**3*tan(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tan(c)*
*3)**3, True))
```

Giac [B] time = 17.7387, size = 4219, normalized size = 25.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/120*(120*a^3*d*x*tan(d*x)^8*tan(c)^8 - 360*a*b^2*d*x*tan(d*x)^8*tan(c)^8
+ 180*a^2*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c)
```

+ tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^8*ta
n(c)^8 - 60*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*ta
n(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^
8*tan(c)^8 - 960*a^3*d*x*tan(d*x)^7*tan(c)^7 + 2880*a*b^2*d*x*tan(d*x)^7*ta
n(c)^7 + 180*a^2*b*tan(d*x)^8*tan(c)^8 - 125*b^3*tan(d*x)^8*tan(c)^8 - 1440
*a^2*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + ta
n(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^7*tan(c)^
7 + 480*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c)
 + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^7*ta
n(c)^7 - 360*a*b^2*tan(d*x)^8*tan(c)^7 - 360*a*b^2*tan(d*x)^7*tan(c)^8 + 33
60*a^3*d*x*tan(d*x)^6*tan(c)^6 - 10080*a*b^2*d*x*tan(d*x)^6*tan(c)^6 + 180*
a^2*b*tan(d*x)^8*tan(c)^6 - 60*b^3*tan(d*x)^8*tan(c)^6 - 1080*a^2*b*tan(d*x
)^7*tan(c)^7 + 880*b^3*tan(d*x)^7*tan(c)^7 + 180*a^2*b*tan(d*x)^6*tan(c)^8
- 60*b^3*tan(d*x)^6*tan(c)^8 + 120*a*b^2*tan(d*x)^8*tan(c)^5 + 5040*a^2*b*l
og(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2
*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^6*tan(c)^6 - 1680
*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(
d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^6*tan(c)^6
+ 2880*a*b^2*tan(d*x)^7*tan(c)^6 + 2880*a*b^2*tan(d*x)^6*tan(c)^7 + 120*a*b
^2*tan(d*x)^5*tan(c)^8 + 30*b^3*tan(d*x)^8*tan(c)^4 - 6720*a^3*d*x*tan(d*x)
^5*tan(c)^5 + 20160*a*b^2*d*x*tan(d*x)^5*tan(c)^5 - 1080*a^2*b*tan(d*x)^7*t
an(c)^5 + 480*b^3*tan(d*x)^7*tan(c)^5 + 2880*a^2*b*tan(d*x)^6*tan(c)^6 - 26
00*b^3*tan(d*x)^6*tan(c)^6 - 1080*a^2*b*tan(d*x)^5*tan(c)^7 + 480*b^3*tan(d
*x)^5*tan(c)^7 + 30*b^3*tan(d*x)^4*tan(c)^8 - 72*a*b^2*tan(d*x)^8*tan(c)^3
- 960*a*b^2*tan(d*x)^7*tan(c)^4 - 10080*a^2*b*log(4*(tan(c)^2 + 1)/(tan(d*x
)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*t
an(d*x)*tan(c) + 1))*tan(d*x)^5*tan(c)^5 + 3360*b^3*log(4*(tan(c)^2 + 1)/(t
an(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2
 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^5*tan(c)^5 - 10080*a*b^2*tan(d*x)^6*tan
(c)^5 - 10080*a*b^2*tan(d*x)^5*tan(c)^6 - 960*a*b^2*tan(d*x)^4*tan(c)^7 - 7
2*a*b^2*tan(d*x)^3*tan(c)^8 - 20*b^3*tan(d*x)^8*tan(c)^2 - 240*b^3*tan(d*x)
^7*tan(c)^3 + 8400*a^3*d*x*tan(d*x)^4*tan(c)^4 - 25200*a*b^2*d*x*tan(d*x)^4
*tan(c)^4 + 2700*a^2*b*tan(d*x)^6*tan(c)^4 - 1680*b^3*tan(d*x)^6*tan(c)^4 -
 4680*a^2*b*tan(d*x)^5*tan(c)^5 + 4080*b^3*tan(d*x)^5*tan(c)^5 + 2700*a^2*b
*tan(d*x)^4*tan(c)^6 - 1680*b^3*tan(d*x)^4*tan(c)^6 - 240*b^3*tan(d*x)^3*ta
n(c)^7 - 20*b^3*tan(d*x)^2*tan(c)^8 + 216*a*b^2*tan(d*x)^7*tan(c)^2 + 2280*
a*b^2*tan(d*x)^6*tan(c)^3 + 12600*a^2*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*ta
n(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x
)*tan(c) + 1))*tan(d*x)^4*tan(c)^4 - 4200*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x
)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*t
an(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4 + 18360*a*b^2*tan(d*x)^5*tan(c)^4
+ 18360*a*b^2*tan(d*x)^4*tan(c)^5 + 2280*a*b^2*tan(d*x)^3*tan(c)^6 + 216*a*
b^2*tan(d*x)^2*tan(c)^7 + 15*b^3*tan(d*x)^8 + 160*b^3*tan(d*x)^7*tan(c) + 8
40*b^3*tan(d*x)^6*tan(c)^2 - 6720*a^3*d*x*tan(d*x)^3*tan(c)^3 + 20160*a*b^2
*d*x*tan(d*x)^3*tan(c)^3 - 3600*a^2*b*tan(d*x)^5*tan(c)^3 + 3360*b^3*tan(d*
x)^5*tan(c)^3 + 5400*a^2*b*tan(d*x)^4*tan(c)^4 - 3420*b^3*tan(d*x)^4*tan(c)
^4 - 3600*a^2*b*tan(d*x)^3*tan(c)^5 + 3360*b^3*tan(d*x)^3*tan(c)^5 + 840*b^
3*tan(d*x)^2*tan(c)^6 + 160*b^3*tan(d*x)*tan(c)^7 + 15*b^3*tan(c)^8 - 216*a
*b^2*tan(d*x)^6*tan(c) - 2280*a*b^2*tan(d*x)^5*tan(c)^2 - 10080*a^2*b*log(4
*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan
(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^3 + 3360*b^3
log(4(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)
^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^3 - 18
360*a*b^2*tan(d*x)^4*tan(c)^3 - 18360*a*b^2*tan(d*x)^3*tan(c)^4 - 2280*a*b^
2*tan(d*x)^2*tan(c)^5 - 216*a*b^2*tan(d*x)*tan(c)^6 - 20*b^3*tan(d*x)^6 - 2
40*b^3*tan(d*x)^5*tan(c) + 3360*a^3*d*x*tan(d*x)^2*tan(c)^2 - 10080*a*b^2*d
*x*tan(d*x)^2*tan(c)^2 + 2700*a^2*b*tan(d*x)^4*tan(c)^2 - 1680*b^3*tan(d*x)
^4*tan(c)^2 - 4680*a^2*b*tan(d*x)^3*tan(c)^3 + 4080*b^3*tan(d*x)^3*tan(c)^3
 + 2700*a^2*b*tan(d*x)^2*tan(c)^4 - 1680*b^3*tan(d*x)^2*tan(c)^4 - 240*b^3*

```
tan(d*x)*tan(c)^5 - 20*b^3*tan(c)^6 + 72*a*b^2*tan(d*x)^5 + 960*a*b^2*tan(d
*x)^4*tan(c) + 5040*a^2*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan
(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))
*tan(d*x)^2*tan(c)^2 - 1680*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 -
 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
 + 1))*tan(d*x)^2*tan(c)^2 + 10080*a*b^2*tan(d*x)^3*tan(c)^2 + 10080*a*b^2*
tan(d*x)^2*tan(c)^3 + 960*a*b^2*tan(d*x)*tan(c)^4 + 72*a*b^2*tan(c)^5 + 30*
b^3*tan(d*x)^4 - 960*a^3*d*x*tan(d*x)*tan(c) + 2880*a*b^2*d*x*tan(d*x)*tan(
c) - 1080*a^2*b*tan(d*x)^3*tan(c) + 480*b^3*tan(d*x)^3*tan(c) + 2880*a^2*b*
tan(d*x)^2*tan(c)^2 - 2600*b^3*tan(d*x)^2*tan(c)^2 - 1080*a^2*b*tan(d*x)*ta
n(c)^3 + 480*b^3*tan(d*x)*tan(c)^3 + 30*b^3*tan(c)^4 - 120*a*b^2*tan(d*x)^3
 - 1440*a^2*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(
c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)*ta
n(c) + 480*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan
(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)*t
an(c) - 2880*a*b^2*tan(d*x)^2*tan(c) - 2880*a*b^2*tan(d*x)*tan(c)^2 - 120*a
*b^2*tan(c)^3 + 120*a^3*d*x - 360*a*b^2*d*x + 180*a^2*b*tan(d*x)^2 - 60*b^3
*tan(d*x)^2 - 1080*a^2*b*tan(d*x)*tan(c) + 880*b^3*tan(d*x)*tan(c) + 180*a^
2*b*tan(c)^2 - 60*b^3*tan(c)^2 + 180*a^2*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4
*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(
d*x)*tan(c) + 1)) - 60*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*ta
n(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)
) + 360*a*b^2*tan(d*x) + 360*a*b^2*tan(c) + 180*a^2*b - 125*b^3)/(d*tan(d*x
)^8*tan(c)^8 - 8*d*tan(d*x)^7*tan(c)^7 + 28*d*tan(d*x)^6*tan(c)^6 - 56*d*ta
n(d*x)^5*tan(c)^5 + 70*d*tan(d*x)^4*tan(c)^4 - 56*d*tan(d*x)^3*tan(c)^3 + 2
8*d*tan(d*x)^2*tan(c)^2 - 8*d*tan(d*x)*tan(c) + d)
```

3.376 $\int \left(a+b\tan^3(c+dx)\right)^2 dx$

Optimal. Leaf size=89

$$x\left(a^2-b^2\right)+\frac{ab\tan^2(c+dx)}{d}+\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan^5(c+dx)}{5d}-\frac{b^2\tan^3(c+dx)}{3d}+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] (a^2 - b^2)*x + (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Tan[c + d*x])/d + (a*b*T
an[c + d*x]^2)/d - (b^2*Tan[c + d*x]^3)/(3*d) + (b^2*Tan[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.059836, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3661, 1810, 635, 203, 260}

$$x\left(a^2-b^2\right)+\frac{ab\tan^2(c+dx)}{d}+\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan^5(c+dx)}{5d}-\frac{b^2\tan^3(c+dx)}{3d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^3)^2,x]
```

```
[Out] (a^2 - b^2)*x + (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Tan[c + d*x])/d + (a*b*T
an[c + d*x]^2)/d - (b^2*Tan[c + d*x]^3)/(3*d) + (b^2*Tan[c + d*x]^5)/(5*d)
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \left(a+b\tan^3(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^3\right)^2}{1+x^2}dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(b^2+2abx-b^2x^2+b^2x^4+\frac{a^2-b^2-2abx}{1+x^2}\right)dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{b^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}-\frac{b^2\tan^3(c+dx)}{3d}+\frac{b^2\tan^5(c+dx)}{5d}+\frac{\text{Subst}\left(\int \frac{a^2-}{}\right.}{}$$
$$= \frac{b^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}-\frac{b^2\tan^3(c+dx)}{3d}+\frac{b^2\tan^5(c+dx)}{5d}-\frac{(2ab)\,\text{Subst}\left(\right.}{}$$
$$= \left(a^2-b^2\right)x+\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}-\frac{b^2\tan^3(c+d}{3d}$$

Mathematica [C] time = 0.473047, size = 107, normalized size = 1.2

$$\frac{30ab\tan^2(c+dx)-15i\left((a-ib)^2\log(-\tan(c+dx)+i)-(a+ib)^2\log(\tan(c+dx)+i)\right)+6b^2\tan^5(c+dx)-10b^2\text{ta}}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^3)^2,x]
```

```
[Out] ((-15*I)*((a - I*b)^2*Log[I - Tan[c + d*x]] - (a + I*b)^2*Log[I + Tan[c + d
*x]]) + 30*b^2*Tan[c + d*x] + 30*a*b*Tan[c + d*x]^2 - 10*b^2*Tan[c + d*x]^3
 + 6*b^2*Tan[c + d*x]^5)/(30*d)
```

Maple [A] time = 0.007, size = 108, normalized size = 1.2

$$\frac{b^2(\tan(dx+c))^5}{5d}-\frac{b^2(\tan(dx+c))^3}{3d}+\frac{ab(\tan(dx+c))^2}{d}+\frac{b^2\tan(dx+c)}{d}-\frac{ab\ln\left((\tan(dx+c))^2+1\right)}{d}+\frac{\arctan(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^3)^2,x)
```

```
[Out] 1/5*b^2*tan(d*x+c)^5/d-1/3*b^2*tan(d*x+c)^3/d+a*b*tan(d*x+c)^2/d+b^2*tan(d*
x+c)/d-1/d*a*b*ln(tan(d*x+c)^2+1)+1/d*arctan(tan(d*x+c))*a^2-1/d*arctan(tan
(d*x+c))*b^2
```

Maxima [A] time = 1.50395, size = 112, normalized size = 1.26

$$a^2x+\frac{\left(3\tan(dx+c)^5-5\tan(dx+c)^3-15\,dx-15\,c+15\tan(dx+c)\right)b^2}{15\,d}-\frac{ab\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin(dx+c)^2-1\right)\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(
d*x + c))*b^2/d - a*b*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d
```

Fricas [A] time = 1.65648, size = 213, normalized size = 2.39

$$\frac{3 b^2 \tan (dx+c)^5 - 5 b^2 \tan (dx+c)^3 + 15 ab \tan (dx+c)^2 + 15 \left(a^2-b^2\right)dx + 15 ab \log \left(\frac{1}{\tan(dx+c)^2+1}\right) + 15 b^2 \tan (dx+c}{15 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*tan(d*x + c)^5 - 5*b^2*tan(d*x + c)^3 + 15*a*b*tan(d*x + c)^2 +
 15*(a^2 - b^2)*d*x + 15*a*b*log(1/(tan(d*x + c)^2 + 1)) + 15*b^2*tan(d*x +
 c))/d
```

Sympy [A] time = 0.760173, size = 94, normalized size = 1.06

$$\begin{cases} a^2x - \frac{ab \log \left(\tan^2 (c+dx)+1\right)}{d} + \frac{ab \tan^2 (c+dx)}{d} - b^2x + \frac{b^2 \tan^5 (c+dx)}{5d} - \frac{b^2 \tan^3 (c+dx)}{3d} + \frac{b^2 \tan (c+dx)}{d} & \text{for } d \neq 0 \\ x \left(a + b \tan^3 (c)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((a**2*x - a*b*log(tan(c + d*x)**2 + 1)/d + a*b*tan(c + d*x)**2/d
- b**2*x + b**2*tan(c + d*x)**5/(5*d) - b**2*tan(c + d*x)**3/(3*d) + b**2*t
an(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c)**3)**2, True))
```

Giac [B] time = 3.70932, size = 1438, normalized size = 16.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/15*(15*a^2*d*x*tan(d*x)^5*tan(c)^5 - 15*b^2*d*x*tan(d*x)^5*tan(c)^5 + 15*
a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d
*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^5*tan(c)^5 -
 75*a^2*d*x*tan(d*x)^4*tan(c)^4 + 75*b^2*d*x*tan(d*x)^4*tan(c)^4 + 15*a*b*t
an(d*x)^5*tan(c)^5 - 75*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*t
an(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1
))*tan(d*x)^4*tan(c)^4 - 15*b^2*tan(d*x)^5*tan(c)^4 - 15*b^2*tan(d*x)^4*tan
(c)^5 + 150*a^2*d*x*tan(d*x)^3*tan(c)^3 - 150*b^2*d*x*tan(d*x)^3*tan(c)^3 +
 15*a*b*tan(d*x)^5*tan(c)^3 - 45*a*b*tan(d*x)^4*tan(c)^4 + 15*a*b*tan(d*x)^
3*tan(c)^5 + 5*b^2*tan(d*x)^5*tan(c)^2 + 150*a*b*log(4*(tan(c)^2 + 1)/(tan(
d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 -
2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^3 + 75*b^2*tan(d*x)^4*tan(c)^3 +
75*b^2*tan(d*x)^3*tan(c)^4 + 5*b^2*tan(d*x)^2*tan(c)^5 - 150*a^2*d*x*tan(d*
```

```
x)^2*tan(c)^2 + 150*b^2*d*x*tan(d*x)^2*tan(c)^2 - 45*a*b*tan(d*x)^4*tan(c)^
2 + 60*a*b*tan(d*x)^3*tan(c)^3 - 45*a*b*tan(d*x)^2*tan(c)^4 - 3*b^2*tan(d*x
)^5 - 25*b^2*tan(d*x)^4*tan(c) - 150*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*t
an(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*
x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 - 150*b^2*tan(d*x)^3*tan(c)^2 - 150*b^2
*tan(d*x)^2*tan(c)^3 - 25*b^2*tan(d*x)*tan(c)^4 - 3*b^2*tan(c)^5 + 75*a^2*d
*x*tan(d*x)*tan(c) - 75*b^2*d*x*tan(d*x)*tan(c) + 45*a*b*tan(d*x)^3*tan(c)
- 60*a*b*tan(d*x)^2*tan(c)^2 + 45*a*b*tan(d*x)*tan(c)^3 + 5*b^2*tan(d*x)^3
+ 75*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) +
tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)*tan(c)
+ 75*b^2*tan(d*x)^2*tan(c) + 75*b^2*tan(d*x)*tan(c)^2 + 5*b^2*tan(c)^3 - 15
*a^2*d*x + 15*b^2*d*x - 15*a*b*tan(d*x)^2 + 45*a*b*tan(d*x)*tan(c) - 15*a*b
*tan(c)^2 - 15*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3
*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)) - 15*b
^2*tan(d*x) - 15*b^2*tan(c) - 15*a*b)/(d*tan(d*x)^5*tan(c)^5 - 5*d*tan(d*x)
^4*tan(c)^4 + 10*d*tan(d*x)^3*tan(c)^3 - 10*d*tan(d*x)^2*tan(c)^2 + 5*d*tan
(d*x)*tan(c) - d)
```

3.377 $\int \left(a+b\tan^3(c+dx)\right)\,dx$

Optimal. Leaf size=32

$$ax+\frac{b\tan^2(c+dx)}{2d}+\frac{b\log(\cos(c+dx))}{d}$$

```
[Out] a*x + (b*Log[Cos[c + d*x]])/d + (b*Tan[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0194858, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3473, 3475}

$$ax+\frac{b\tan^2(c+dx)}{2d}+\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Tan[c + d*x]^3,x]
```

```
[Out] a*x + (b*Log[Cos[c + d*x]])/d + (b*Tan[c + d*x]^2)/(2*d)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tan^3(c+dx)\right)\,dx &= ax+b\int \tan^3(c+dx)\,dx\\ &= ax+\frac{b\tan^2(c+dx)}{2d}-b\int \tan(c+dx)\,dx\\ &= ax+\frac{b\log(\cos(c+dx))}{d}+\frac{b\tan^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0768237, size = 30, normalized size = 0.94

$$ax+\frac{b\left(\tan^2(c+dx)+2\log(\cos(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Tan[c + d*x]^3,x]
```

```
[Out] a*x + (b*(2*Log[Cos[c + d*x]] + Tan[c + d*x]^2))/(2*d)
```

Maple [A] time = 0.005, size = 36, normalized size = 1.1

$$ax+\frac{b\left(\tan\left(dx+c\right)\right)^{2}}{2d}-\frac{b\ln\left(\left(\tan\left(dx+c\right)\right)^{2}+1\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*tan(d*x+c)^3,x)
```

```
[Out] a*x+1/2*b*tan(d*x+c)^2/d-1/2*b/d*ln(tan(d*x+c)^2+1)
```

Maxima [A] time = 0.998025, size = 49, normalized size = 1.53

$$ax-\frac{b\left(\frac{1}{\sin\left(dx+c\right)^{2}-1}-\log\left(\sin\left(dx+c\right)^{2}-1\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] a*x - 1/2*b*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d
```

Fricas [A] time = 1.61172, size = 92, normalized size = 2.88

$$\frac{2\,adx+b\tan\left(dx+c\right)^{2}+b\log\left(\frac{1}{\tan\left(dx+c\right)^{2}+1}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d*x + b*tan(d*x + c)^2 + b*log(1/(tan(d*x + c)^2 + 1)))/d
```

Sympy [A] time = 0.209, size = 37, normalized size = 1.16

$$ax+b\left(\begin{cases}-\frac{\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{\tan^{2}\left(c+dx\right)}{2d} & \text{for}\ d\neq 0\\ x\tan^{3}\left(c\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)**3,x)
```

```
[Out] a*x + b*Piecewise((-log(tan(c + d*x)**2 + 1)/(2*d) + tan(c + d*x)**2/(2*d),
 Ne(d, 0)), (x*tan(c)**3, True))
```

Giac [B] time = 1.50465, size = 339, normalized size = 10.59

$$ax + \frac{\left(\log\left(\frac{4\left(\tan(c)^2+1\right)}{\tan(dx)^4\tan(c)^2-2\tan(dx)^3\tan(c)+\tan(dx)^2\tan(c)^2+\tan(dx)^2-2\tan(dx)\tan(c)+1}\right)\tan(dx)^2\tan(c)^2 + \tan(dx)^2\tan(c)^2 - 2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] a*x + 1/2*(log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c)
+ tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan
(c)^2 + tan(d*x)^2*tan(c)^2 - 2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 -
 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
 + 1))*tan(d*x)*tan(c) + tan(d*x)^2 + tan(c)^2 + log(4*(tan(c)^2 + 1)/(tan(
d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 -
2*tan(d*x)*tan(c) + 1)) + 1)*b/(d*tan(d*x)^2*tan(c)^2 - 2*d*tan(d*x)*tan(c)
 + d)
```

3.378 $\int \frac{1}{a+b\tan^3(c+dx)}\,dx$

Optimal. Leaf size=256

$$\frac{\sqrt[3]{b}\left(a^{4/3}-b^{4/3}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}d\left(a^2+b^2\right)}-\frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tan(c+dx)+b^{2/3}\tan^2(c+dx)\right)}{6a^{2/3}d\left(a^2+b^2\right)}+\frac{\sqrt[3]{b}\,(}{}$$

```
[Out] (a*x)/(a^2 + b^2) + (b^(1/3)*(a^(4/3) - b^(4/3))*ArcTan[(a^(1/3) - 2*b^(1/3
)*Tan[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a^2 + b^2)*d) - (b*Lo
g[a*Cos[c + d*x]^3 + b*Sin[c + d*x]^3])/(3*(a^2 + b^2)*d) + (b^(1/3)*(a^(4/
3) + b^(4/3))*Log[a^(1/3) + b^(1/3)*Tan[c + d*x]])/(3*a^(2/3)*(a^2 + b^2)*d
) - (b^(1/3)*(a^(4/3) + b^(4/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tan[c + d*x]
 + b^(2/3)*Tan[c + d*x]^2])/(6*a^(2/3)*(a^2 + b^2)*d)
```

Rubi [A] time = 0.380429, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3661, 6725, 635, 203, 260, 1871, 1860, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}\left(a^{4/3}-b^{4/3}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}d\left(a^2+b^2\right)}-\frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tan(c+dx)+b^{2/3}\tan^2(c+dx)\right)}{6a^{2/3}d\left(a^2+b^2\right)}+\frac{\sqrt[3]{b}\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^3)^(-1),x]
```


```
[Out] (a*x)/(a^2 + b^2) + (b^(1/3)*(a^(4/3) - b^(4/3))*ArcTan[(a^(1/3) - 2*b^(1/3
)*Tan[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a^2 + b^2)*d) - (b*Lo
g[a*Cos[c + d*x]^3 + b*Sin[c + d*x]^3])/(3*(a^2 + b^2)*d) + (b^(1/3)*(a^(4/
3) + b^(4/3))*Log[a^(1/3) + b^(1/3)*Tan[c + d*x]])/(3*a^(2/3)*(a^2 + b^2)*d
) - (b^(1/3)*(a^(4/3) + b^(4/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tan[c + d*x]
 + b^(2/3)*Tan[c + d*x]^2])/(6*a^(2/3)*(a^2 + b^2)*d)
```


Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{a+b\tan^3(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^3)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a+bx}{(a^2+b^2)(1+x^2)} - \frac{b(-b+ax+bx^2)}{(a^2+b^2)(a+bx^3)}\right) dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{a+bx}{1+x^2}\,dx, x, \tan(c+dx)\right)}{(a^2+b^2)\,d} - \frac{b\,\text{Subst}\left(\int \frac{-b+ax+bx^2}{a+bx^3}\,dx, x, \tan(c+dx)\right)}{(a^2+b^2)\,d}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{(a^2+b^2)\,d} + \frac{b\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \tan(c+dx)\right)}{(a^2+b^2)\,d} - \frac{b\,\text{Subst}\left(\int \frac{-}{a}\right.}{(}$$

$$= \frac{ax}{a^2+b^2} - \frac{b\log(\cos(c+dx))}{(a^2+b^2)\,d} - \frac{b\log\left(a+b\tan^3(c+dx)\right)}{3(a^2+b^2)\,d} - \frac{b^{2/3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(a^{4/3}-2b^{4/3}\right)+\sqrt[3]{b}}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+}\right.}{3a^{2/3}\left(a^2\right.}$$

$$= \frac{ax}{a^2+b^2} - \frac{b\log(\cos(c+dx))}{(a^2+b^2)\,d} + \frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tan(c+dx)\right)}{3a^{2/3}(a^2+b^2)\,d} - \frac{b\log\left(a+b\right.}{3\left(a^2\right.}$$

$$= \frac{ax}{a^2+b^2} - \frac{b\log(\cos(c+dx))}{(a^2+b^2)\,d} + \frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tan(c+dx)\right)}{3a^{2/3}(a^2+b^2)\,d} - \frac{\sqrt[3]{b}\left(a^{4/3}+b\right.}{}$$

$$= \frac{ax}{a^2+b^2} + \frac{\sqrt[3]{b}\left(a^{4/3}-b^{4/3}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\tan(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}(a^2+b^2)\,d} - \frac{b\log(\cos(c+dx))}{(a^2+b^2)\,d} + \frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log}{3a^{2/3}}$$

Mathematica [C] time = 0.575201, size = 278, normalized size = 1.09

$$-3a^{2/3}b\tan^2(c+dx)\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{b\tan^3(c+dx)}{a}\right) - b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tan(c+dx)+b^{2/3}\tan^2(c+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^3)^(-1),x]
```

```
[Out] (-2*Sqrt[3]*b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Tan[c + d*x])/(Sqrt[3]*a^(1
/3))] - (3*I)*a^(5/3)*Log[I - Tan[c + d*x]] + 3*a^(2/3)*b*Log[I - Tan[c + d
*x]] + (3*I)*a^(5/3)*Log[I + Tan[c + d*x]] + 3*a^(2/3)*b*Log[I + Tan[c + d*
x]] + 2*b^(5/3)*Log[a^(1/3) + b^(1/3)*Tan[c + d*x]] - b^(5/3)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*Tan[c + d*x] + b^(2/3)*Tan[c + d*x]^2] - 2*a^(2/3)*b*Log[a
 + b*Tan[c + d*x]^3] - 3*a^(2/3)*b*Hypergeometric2F1[2/3, 1, 5/3, -((b*Tan[
c + d*x]^3)/a)]*Tan[c + d*x]^2)/(6*a^(2/3)*(a^2 + b^2)*d)
```

Maple [A] time = 0.027, size = 355, normalized size = 1.4

$$\frac{b}{3d(a^2+b^2)}\ln\left(\tan(dx+c)+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{b}{6d(a^2+b^2)}\ln\left((\tan(dx+c))^2-\sqrt[3]{\frac{a}{b}}\tan(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^3),x)
```

```
[Out] 1/3/d/(a^2+b^2)*b/(a/b)^(2/3)*ln(tan(d*x+c)+(a/b)^(1/3))-1/6/d/(a^2+b^2)*b/
(a/b)^(2/3)*ln(tan(d*x+c)^2-(a/b)^(1/3)*tan(d*x+c)+(a/b)^(2/3))+1/3/d/(a^2+
b^2)*b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*tan(d*x+c)-1))
+1/3/d/(a^2+b^2)*a/(a/b)^(1/3)*ln(tan(d*x+c)+(a/b)^(1/3))-1/6/d/(a^2+b^2)*a
/(a/b)^(1/3)*ln(tan(d*x+c)^2-(a/b)^(1/3)*tan(d*x+c)+(a/b)^(2/3))-1/3/d/(a^2
+b^2)*a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*tan(d*x+c)-1)
)-1/3/d/(a^2+b^2)*b*ln(a+b*tan(d*x+c)^3)+1/2/d/(a^2+b^2)*b*ln(tan(d*x+c)^2+
1)+1/d/(a^2+b^2)*a*arctan(tan(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 10.1303, size = 10283, normalized size = 40.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/24*(2*(a^2 + b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3
) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3)
+ 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b
^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^
(1/3)) + 2*b/(a^2*d + b^2*d))*d*log(-1/4*(4*b^2*tan(d*x + c)^2 - ((a^4 + a^
2*b^2)*d^2*tan(d*x + c)^2 - (a^4 + a^2*b^2)*d^2)*((1/2)^(1/3)*(I*sqrt(3) +
1)*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a
^2 + b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d +
b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)
*b/((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d))^2 + 2*(a^2*b*d*ta
n(d*x + c)^2 - a^2*b*d + 2*(a^3 - a*b^2)*d*tan(d*x + c))*((1/2)^(1/3)*(I*sq
rt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^
2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((
a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^
2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d)) - 4*a^2)/
(tan(d*x + c)^2 + 1)) - 24*a*d*x - ((a^2 + b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1
)*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^
2 + b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b
^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*
b/((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d))*d - 3*sqrt(1/3)*(a
^2 + b^2)*d*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/
(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b
^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)
^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a
^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d))^2*d^2 - 4*(a^2*b + b^3)
```

```
*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b
^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*
(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2
*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d
+ b^2*d))*d - 12*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 6*b)*log(1/4*(8*a^4
- 16*a^2*b^2 - ((a^6 + 2*a^4*b^2 + a^2*b^4)*d^2*tan(d*x + c)^2 - (a^6 + 2*a
^4*b^2 + a^2*b^4)*d^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d
^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3
) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2
*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3)
)^(1/3)) + 2*b/(a^2*d + b^2*d))^2 + 8*(2*a^2*b^2 - b^4)*tan(d*x + c)^2 + 2*
((a^4*b + a^2*b^3)*d*tan(d*x + c)^2 + 2*(a^5 - a*b^4)*d*tan(d*x + c) - (a^4
*b + a^2*b^3)*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3) -
2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) + 2*
(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d
^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3
)) + 2*b/(a^2*d + b^2*d)) + 3*sqrt(1/3)*(4*(a^4*b + a^2*b^3)*d*tan(d*x + c)
^2 - 4*(a^5 - a*b^4)*d*tan(d*x + c) - ((a^6 + 2*a^4*b^2 + a^2*b^4)*d^2*tan(
d*x + c)^2 - (a^6 + 2*a^4*b^2 + a^2*b^4)*d^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*
(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2
+ b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2
*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/
((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d)) - 4*(a^4*b + a^2*b^3
)*d)*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^
3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a
^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(
a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^
2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d))^2*d^2 - 4*(a^2*b + b^3)*((1/2)
^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3
 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqr
t(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^
2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d
))*d - 12*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 24*(a^3*b - a*b^3)*tan(d*x
+ c))/(tan(d*x + c)^2 + 1)) - ((a^2 + b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/
(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b
^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)
^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a
^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d))*d + 3*sqrt(1/3)*(a^2 +
b^2)*d*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*
d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2
*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b
/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 +
b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d))^2*d^2 - 4*(a^2*b + b^3)*((1/
2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)
^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*s
qrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d +
b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2
*d))*d - 12*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 6*b)*log(-1/4*(8*a^4 - 16
*a^2*b^2 - ((a^6 + 2*a^4*b^2 + a^2*b^4)*d^2*tan(d*x + c)^2 - (a^6 + 2*a^4*b
^2 + a^2*b^4)*d^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3)
- 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) +
2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2
*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1
/3)) + 2*b/(a^2*d + b^2*d))^2 + 8*(2*a^2*b^2 - b^4)*tan(d*x + c)^2 + 2*((a^
4*b + a^2*b^3)*d*tan(d*x + c)^2 + 2*(a^5 - a*b^4)*d*tan(d*x + c) - (a^4*b +
 a^2*b^3)*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^
3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) + 2*(1/2
)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3)
- 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3)) +
```

```
2*b/(a^2*d + b^2*d)) - 3*sqrt(1/3)*(4*(a^4*b + a^2*b^3)*d*tan(d*x + c)^2 -
4*(a^5 - a*b^4)*d*tan(d*x + c) - ((a^6 + 2*a^4*b^2 + a^2*b^4)*d^2*tan(d*x
+ c)^2 - (a^6 + 2*a^4*b^2 + a^2*b^4)*d^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(
a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^
2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^
2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^
2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d)) - 4*(a^4*b + a^2*b^3)*d)
*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 +
a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d
^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*
d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2
*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d))^2*d^2 - 4*(a^2*b + b^3)*((1/2)^(1/
3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (
a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3)
 + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)
^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d))*d
 - 12*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 24*(a^3*b - a*b^3)*tan(d*x + c)
)/(tan(d*x + c)^2 + 1)))/((a^2 + b^2)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a + b\tan^3(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)**3),x)
```

```
[Out] Integral(1/(a + b*tan(c + d*x)**3), x)
```

Giac [A] time = 1.58741, size = 451, normalized size = 1.76

$$\frac{\frac{2\left(a^3b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^3-b^5\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\tan(dx+c)\right|\right)}{a^5b+2a^3b^3+ab^5}+\frac{6\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\tan(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\right)\left(\left(-ab^2\right)^{\frac{1}{3}}b^2+\left(-ab^2\right)^{\frac{2}{3}}\ldots}{\sqrt{3}a^3b+\sqrt{3}ab^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/6*(2*(a^3*b^2*(-a/b)^(1/3) + a*b^4*(-a/b)^(1/3) - a^2*b^3 - b^5)*(-a/b)^(
1/3)*log(abs(-(-a/b)^(1/3) + tan(d*x + c)))/(a^5*b + 2*a^3*b^3 + a*b^5) + 6
*(pi*floor((d*x + c)/pi + 1/2)*sgn((-a/b)^(1/3)) + arctan(1/3*sqrt(3)*((-a/
b)^(1/3) + 2*tan(d*x + c))/(-a/b)^(1/3)))*((-a*b^2)^(1/3)*b^2 + (-a*b^2)^(2
/3)*a)/(sqrt(3)*a^3*b + sqrt(3)*a*b^3) + 6*(d*x + c)*a/(a^2 + b^2) + ((-a*b
^2)^(1/3)*b^2 - (-a*b^2)^(2/3)*a)*log(tan(d*x + c)^2 + (-a/b)^(1/3)*tan(d*x
 + c) + (-a/b)^(2/3))/(a^3*b + a*b^3) + 3*b*log(tan(d*x + c)^2 + 1)/(a^2 +
b^2) - 2*b*log(abs(b*tan(d*x + c)^3 + a))/(a^2 + b^2))/d
```

3.379 $\int \frac{1}{\left(a+b\tan^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=558

$$\frac{\sqrt[3]{b}\left(a^{4/3}-2b^{4/3}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}d\left(a^2+b^2\right)}+\frac{\sqrt[3]{b}\left(-2a^{2/3}b^{4/3}+a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}d\left(a^2+b^2\right)^2}+\frac{b(\tan(c+dx)(b-a}{3ad\left(a^2+b^2\right)\left(a+\right.}$$

```
[Out] ((a^2 - b^2)*x)/(a^2 + b^2)^2 + (b^(1/3)*(a^2 - 2*a^(2/3)*b^(4/3) - b^2)*Ar
cTan[(a^(1/3) - 2*b^(1/3)*Tan[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3
)*(a^2 + b^2)^2*d) + (b^(1/3)*(a^(4/3) - 2*b^(4/3))*ArcTan[(a^(1/3) - 2*b^(
1/3)*Tan[c + d*x])/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*(a^2 + b^2)*d) -
(2*a*b*Log[a*Cos[c + d*x]^3 + b*Sin[c + d*x]^3])/(3*(a^2 + b^2)^2*d) + (b^(
1/3)*(a^2 + 2*a^(2/3)*b^(4/3) - b^2)*Log[a^(1/3) + b^(1/3)*Tan[c + d*x]])/(
3*a^(1/3)*(a^2 + b^2)^2*d) + (b^(1/3)*(a^(4/3) + 2*b^(4/3))*Log[a^(1/3) + b
^(1/3)*Tan[c + d*x]])/(9*a^(5/3)*(a^2 + b^2)*d) - (b^(1/3)*(a^2 + 2*a^(2/3)
*b^(4/3) - b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tan[c + d*x] + b^(2/3)*Tan[c
+ d*x]^2])/(6*a^(1/3)*(a^2 + b^2)^2*d) - (b^(1/3)*(a^(4/3) + 2*b^(4/3))*Log
[a^(2/3) - a^(1/3)*b^(1/3)*Tan[c + d*x] + b^(2/3)*Tan[c + d*x]^2])/(18*a^(5
/3)*(a^2 + b^2)*d) + (b*(a + Tan[c + d*x]*(b - a*Tan[c + d*x])))/(3*a*(a^2
+ b^2)*d*(a + b*Tan[c + d*x]^3))
```

Rubi [A] time = 0.727999, antiderivative size = 558, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 13, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {3661, 6725, 635, 203, 260, 1854, 1860, 31, 634, 617, 204, 628, 1871}

$$\frac{\sqrt[3]{b}\left(a^{4/3}-2b^{4/3}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}d\left(a^2+b^2\right)}+\frac{\sqrt[3]{b}\left(-2a^{2/3}b^{4/3}+a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}d\left(a^2+b^2\right)^2}+\frac{b(\tan(c+dx)(b-a}{3ad\left(a^2+b^2\right)\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^3)^(-2),x]
```


```
[Out] ((a^2 - b^2)*x)/(a^2 + b^2)^2 + (b^(1/3)*(a^2 - 2*a^(2/3)*b^(4/3) - b^2)*Ar
cTan[(a^(1/3) - 2*b^(1/3)*Tan[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3
)*(a^2 + b^2)^2*d) + (b^(1/3)*(a^(4/3) - 2*b^(4/3))*ArcTan[(a^(1/3) - 2*b^(
1/3)*Tan[c + d*x])/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*(a^2 + b^2)*d) -
(2*a*b*Log[a*Cos[c + d*x]^3 + b*Sin[c + d*x]^3])/(3*(a^2 + b^2)^2*d) + (b^(
1/3)*(a^2 + 2*a^(2/3)*b^(4/3) - b^2)*Log[a^(1/3) + b^(1/3)*Tan[c + d*x]])/(
3*a^(1/3)*(a^2 + b^2)^2*d) + (b^(1/3)*(a^(4/3) + 2*b^(4/3))*Log[a^(1/3) + b
^(1/3)*Tan[c + d*x]])/(9*a^(5/3)*(a^2 + b^2)*d) - (b^(1/3)*(a^2 + 2*a^(2/3)
*b^(4/3) - b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tan[c + d*x] + b^(2/3)*Tan[c
+ d*x]^2])/(6*a^(1/3)*(a^2 + b^2)^2*d) - (b^(1/3)*(a^(4/3) + 2*b^(4/3))*Log
[a^(2/3) - a^(1/3)*b^(1/3)*Tan[c + d*x] + b^(2/3)*Tan[c + d*x]^2])/(18*a^(5
/3)*(a^2 + b^2)*d) + (b*(a + Tan[c + d*x]*(b - a*Tan[c + d*x])))/(3*a*(a^2
+ b^2)*d*(a + b*Tan[c + d*x]^3))
```


Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
```

```
qQ[n^2, 16])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^3(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\left(a+bx^3\right)^2}\,dx,x,\tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2-b^2+2abx}{\left(a^2+b^2\right)^2\left(1+x^2\right)}-\frac{b\left(-b+ax+bx^2\right)}{\left(a^2+b^2\right)\left(a+bx^3\right)^2}+\frac{b\left(2ab-\left(a^2-b^2\right)x-2abx^2\right)}{\left(a^2+b^2\right)^2\left(a+bx^3\right)}\right)dx,x,\tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{a^2-b^2+2abx}{1+x^2}\,dx,x,\tan(c+dx)\right)}{\left(a^2+b^2\right)^2 d}+\frac{b\,\text{Subst}\left(\int \frac{2ab-\left(a^2-b^2\right)x-2abx^2}{a+bx^3}\,dx,x,\tan(c+d\right.}{\left(a^2+b^2\right)^2 d}$$

$$= \frac{b(a+\tan(c+dx)(b-a\tan(c+dx)))}{3a\left(a^2+b^2\right)d\left(a+b\tan^3(c+dx)\right)}+\frac{b\,\text{Subst}\left(\int \frac{2ab+\left(-a^2+b^2\right)x}{a+bx^3}\,dx,x,\tan(c+dx)\right)}{\left(a^2+b^2\right)^2 d}+$$

$$= \frac{\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^2}-\frac{2ab\log(\cos(c+dx))}{\left(a^2+b^2\right)^2 d}-\frac{2ab\log\left(a+b\tan^3(c+dx)\right)}{3\left(a^2+b^2\right)^2 d}+\frac{b(a+\tan(c+dx)}{3a\left(a^2+b^2\right)d\,(a}$$

$$= \frac{\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^2}-\frac{2ab\log(\cos(c+dx))}{\left(a^2+b^2\right)^2 d}+\frac{\sqrt[3]{b}\left(a^2+2a^{2/3}b^{4/3}-b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tan(c+dx\right.}{3\sqrt[3]{a}\left(a^2+b^2\right)^2 d}$$

$$= \frac{\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^2}-\frac{2ab\log(\cos(c+dx))}{\left(a^2+b^2\right)^2 d}+\frac{\sqrt[3]{b}\left(a^2+2a^{2/3}b^{4/3}-b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tan(c+dx\right.}{3\sqrt[3]{a}\left(a^2+b^2\right)^2 d}$$

$$= \frac{\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^2}+\frac{\sqrt[3]{b}\left(a^2-2a^{2/3}b^{4/3}-b^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\tan(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\left(a^2+b^2\right)^2 d}+\frac{\sqrt[3]{b}\left(a^{4/3}-2b^{4/3}\right)\tan^{-1}\left(\right.}{3\sqrt{3}a^{5/3}\left(a^2+\right.}$$

Mathematica [C] time = 6.27911, size = 575, normalized size = 1.03

$$-\frac{b(a-b)(a+b)\tan^2(c+dx)\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{b\tan^3(c+dx)}{a}\right)}{2ad\left(a^2+b^2\right)^2}-\frac{b\tan^2(c+dx)\text{Hypergeometric2F1}\left(\frac{2}{3},2,\frac{5}{3}\right.}{2ad\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^3)^(-2),x]
```

```
[Out] ((-I/2)*Log[I - Tan[c + d*x]])/((a - I*b)^2*d) + ((I/2)*Log[I + Tan[c + d*x
]])/((a + I*b)^2*d) + (2*a^(1/3)*b^(5/3)*Log[a^(1/3) + b^(1/3)*Tan[c + d*x]
])/(3*(a^2 + b^2)^2*d) - (a^(1/3)*(2*Sqrt[3]*b^(5/3)*ArcTan[(a^(1/3) - 2*b^
(1/3)*Tan[c + d*x])/(Sqrt[3]*a^(1/3))] + b^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*Tan[c + d*x] + b^(2/3)*Tan[c + d*x]^2]))/(3*(a^2 + b^2)^2*d) + ((2*b^(5
/3)*Log[a^(1/3) + b^(1/3)*Tan[c + d*x]])/a^(2/3) - (2*Sqrt[3]*b^(5/3)*ArcTa
n[(a^(1/3) - 2*b^(1/3)*Tan[c + d*x])/(Sqrt[3]*a^(1/3))] + b^(5/3)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*Tan[c + d*x] + b^(2/3)*Tan[c + d*x]^2])/a^(2/3))/(9*a*
(a^2 + b^2)*d) - (2*a*b*Log[a + b*Tan[c + d*x]^3])/(3*(a^2 + b^2)^2*d) - ((
a - b)*b*(a + b)*Hypergeometric2F1[2/3, 1, 5/3, -((b*Tan[c + d*x]^3)/a)]*Ta
n[c + d*x]^2)/(2*a*(a^2 + b^2)^2*d) - (b*Hypergeometric2F1[2/3, 2, 5/3, -((
b*Tan[c + d*x]^3)/a)]*Tan[c + d*x]^2)/(2*a*(a^2 + b^2)*d) + b/(3*(a^2 + b^2
)*d*(a + b*Tan[c + d*x]^3)) + (b^2*Tan[c + d*x])/(3*a*(a^2 + b^2)*d*(a + b*
Tan[c + d*x]^3))
```

Maple [B] time = 0.036, size = 1086, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^3)^2,x)
```

```
[Out] -1/3/d*b/(a^4+2*a^2*b^2+b^4)/(a+b*tan(d*x+c)^3)*tan(d*x+c)^2*a^2-1/3/d*b^3/
(a^4+2*a^2*b^2+b^4)/(a+b*tan(d*x+c)^3)*tan(d*x+c)^2+1/3/d*b^2/(a^4+2*a^2*b^
2+b^4)/(a+b*tan(d*x+c)^3)*a*tan(d*x+c)+1/3/d*b^4/(a^4+2*a^2*b^2+b^4)/(a+b*t
an(d*x+c)^3)/a*tan(d*x+c)+1/3/d*b/(a^4+2*a^2*b^2+b^4)/(a+b*tan(d*x+c)^3)*a^
2+1/3/d*b^3/(a^4+2*a^2*b^2+b^4)/(a+b*tan(d*x+c)^3)+8/9/d*b/(a^4+2*a^2*b^2+b
^4)*a/(a/b)^(2/3)*ln(tan(d*x+c)+(a/b)^(1/3))+2/9/d*b^3/(a^4+2*a^2*b^2+b^4)/
a/(a/b)^(2/3)*ln(tan(d*x+c)+(a/b)^(1/3))-4/9/d*b/(a^4+2*a^2*b^2+b^4)*a/(a/b
)^(2/3)*ln(tan(d*x+c)^2-(a/b)^(1/3)*tan(d*x+c)+(a/b)^(2/3))-1/9/d*b^3/(a^4+
2*a^2*b^2+b^4)/a/(a/b)^(2/3)*ln(tan(d*x+c)^2-(a/b)^(1/3)*tan(d*x+c)+(a/b)^(
2/3))+8/9/d*b/(a^4+2*a^2*b^2+b^4)*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*
(2/(a/b)^(1/3)*tan(d*x+c)-1))+2/9/d*b^3/(a^4+2*a^2*b^2+b^4)/a/(a/b)^(2/3)*3
^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*tan(d*x+c)-1))+4/9/d/(a^4+2*a^2*b^
2+b^4)*a^2/(a/b)^(1/3)*ln(tan(d*x+c)+(a/b)^(1/3))-2/9/d*b^2/(a^4+2*a^2*b^2+
b^4)/(a/b)^(1/3)*ln(tan(d*x+c)+(a/b)^(1/3))-2/9/d/(a^4+2*a^2*b^2+b^4)*a^2/(
a/b)^(1/3)*ln(tan(d*x+c)^2-(a/b)^(1/3)*tan(d*x+c)+(a/b)^(2/3))+1/9/d*b^2/(a
^4+2*a^2*b^2+b^4)/(a/b)^(1/3)*ln(tan(d*x+c)^2-(a/b)^(1/3)*tan(d*x+c)+(a/b)^
(2/3))-4/9/d/(a^4+2*a^2*b^2+b^4)*a^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*tan(d*x+c)-1))+2/9/d*b^2/(a^4+2*a^2*b^2+b^4)*3^(1/2)/(a/b)^
(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*tan(d*x+c)-1))-2/3/d*b/(a^4+2*a^2*b
^2+b^4)*a*ln(a+b*tan(d*x+c)^3)+1/d/(a^4+2*a^2*b^2+b^4)*a*b*ln(tan(d*x+c)^2+
1)+1/d/(a^4+2*a^2*b^2+b^4)*arctan(tan(d*x+c))*a^2-1/d/(a^4+2*a^2*b^2+b^4)*a
rctan(tan(d*x+c))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^3)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [C] time = 13.9013, size = 24260, normalized size = 43.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^3)^2,x, algorithm="fricas")

[Out] -1/648*(216*a^3*b - 432*a*b^3 + 216*(2*a^2*b^2 - b^4 - 3*(a^3*b - a*b^3)*d*
x)*tan(d*x + c)^3 - 648*(a^4 - a^2*b^2)*d*x + 2*((a^5*b + 2*a^3*b^3 + a*b^5
)*d*tan(d*x + c)^3 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d)*(4*(9*a^2*b^2/(a^4*d +
2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*s
qrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a
^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/72
9*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 -
28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27
*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2
*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/
(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^
2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^
4*d + 2*a^2*b^2*d + b^4*d))*log(1/324*(10368*a^6 - 12960*a^4*b^2 - 3888*a^2
*b^4 + ((2*a^10 + 5*a^8*b^2 + 4*a^6*b^4 + a^4*b^6)*d^2*tan(d*x + c)^2 - 4*(
a^9*b + 2*a^7*b^3 + a^5*b^5)*d^2*tan(d*x + c) - (2*a^10 + 5*a^8*b^2 + 4*a^6
*b^4 + a^4*b^6)*d^2)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a
^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4
*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^
4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*
a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*
b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d +
 b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*
a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*
b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a
^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d))^2 -
 1296*(18*a^4*b^2 + 7*a^2*b^4 + b^6)*tan(d*x + c)^2 - 36*((8*a^7*b - 2*a^5*
b^3 - a^3*b^5)*d*tan(d*x + c)^2 + 2*(4*a^8 - 20*a^6*b^2 - 7*a^4*b^4 - a^2*b
^6)*d*tan(d*x + c) - (8*a^7*b - 2*a^5*b^3 - a^3*b^5)*d)*(4*(9*a^2*b^2/(a^4*
d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(
-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3
/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) +
4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^
6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-
8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4
*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b
^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 1
0*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b
```

/(a^4*d + 2*a^2*b^2*d + b^4*d)) + 2592*(4*a^5*b + 2*a^3*b^3 + a*b^5)*tan(d*
x + c))/(tan(d*x + c)^2 + 1)) + 216*(a^3*b + a*b^3)*tan(d*x + c)^2 + (324*a
^2*b^2*tan(d*x + c)^3 + 324*a^3*b - ((a^5*b + 2*a^3*b^3 + a*b^5)*d*tan(d*x
+ c)^3 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d
+ b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/
(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a
^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b +
b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 -
10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^
4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b
^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2
*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)
*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*
b^2*d + b^4*d)) + 3*sqrt(1/3)*((a^5*b + 2*a^3*b^3 + a*b^5)*d*tan(d*x + c)^3
+ (a^6 + 2*a^4*b^2 + a^2*b^4)*d)*sqrt((29808*a^4*b^2 - 10368*a^2*b^4 - 518
4*b^6 - (a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*b^8)*(4*(9*a^2*b^2/
(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^
2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*
a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d
)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*
(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) +
81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 +
2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*
b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^
2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 10
8*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d))^2*d^2 + 216*(a^7*b + 2*a^5*b^3 + a^3*b
^5)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^
2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d
+ b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2
*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5
*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*
a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*
a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d
)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*
(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I
*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d))*d)/((a^10 + 4*a^8*b^
2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*b^8)*d^2)))*log(1/324*(20736*a^8 - 106272*a
^6*b^2 - 22032*a^4*b^4 - ((2*a^12 + 7*a^10*b^2 + 9*a^8*b^4 + 5*a^6*b^6 + a^
4*b^8)*d^2*tan(d*x + c)^2 - 4*(a^11*b + 3*a^9*b^3 + 3*a^7*b^5 + a^5*b^7)*d^
2*tan(d*x + c) - (2*a^12 + 7*a^10*b^2 + 9*a^8*b^4 + 5*a^6*b^6 + a^4*b^8)*d^
2)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2
*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d +
b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*
a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*
b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a
^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a
*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)
) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(
8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*
sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d))^2 + 1296*(42*a^6*b^2
- 59*a^4*b^4 - 22*a^2*b^6 - 2*b^8)*tan(d*x + c)^2 + 36*((8*a^9*b + 6*a^7*b^
3 - 3*a^5*b^5 - a^3*b^7)*d*tan(d*x + c)^2 + 2*(4*a^10 - 16*a^8*b^2 - 27*a^6
*b^4 - 8*a^4*b^6 - a^2*b^8)*d*tan(d*x + c) - (8*a^9*b + 6*a^7*b^3 - 3*a^5*b
^5 - a^3*b^7)*d)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d
^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d +
2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^
2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*
b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((
a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4

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*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*
b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*
d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d
^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d)) + 3*sqr
t(1/3)*(36*(10*a^9*b + 21*a^7*b^3 + 12*a^5*b^5 + a^3*b^7)*d*tan(d*x + c)^2
- 72*(4*a^10 + 2*a^8*b^2 - 9*a^6*b^4 - 8*a^4*b^6 - a^2*b^8)*d*tan(d*x + c)
- ((2*a^12 + 7*a^10*b^2 + 9*a^8*b^4 + 5*a^6*b^6 + a^4*b^8)*d^2*tan(d*x + c)
^2 - 4*(a^11*b + 3*a^9*b^3 + 3*a^7*b^5 + a^5*b^7)*d^2*tan(d*x + c) - (2*a^1
2 + 7*a^10*b^2 + 9*a^8*b^4 + 5*a^6*b^6 + a^4*b^8)*d^2)*(4*(9*a^2*b^2/(a^4*d
 + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-
I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/
((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4
/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6
 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8
/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*
b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^
3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10
*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/
(a^4*d + 2*a^2*b^2*d + b^4*d)) - 36*(10*a^9*b + 21*a^7*b^3 + 12*a^5*b^5 + a
^3*b^7)*d)*sqrt((29808*a^4*b^2 - 10368*a^2*b^4 - 5184*b^6 - (a^10 + 4*a^8*b
^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*b^8)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d +
b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-
8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4
*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b
^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 1
0*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*
d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4
*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a
^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b
/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*b^
2*d + b^4*d))^2*d^2 + 216*(a^7*b + 2*a^5*b^3 + a^3*b^5)*(4*(9*a^2*b^2/(a^4*
d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(
-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3
/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) +
4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^
6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-
8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4
*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b
^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 1
0*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b
/(a^4*d + 2*a^2*b^2*d + b^4*d))*d)/((a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b
^6 + a^2*b^8)*d^2)) - 2592*(28*a^7*b - 78*a^5*b^3 - 27*a^3*b^5 - 2*a*b^7)*t
an(d*x + c))/(tan(d*x + c)^2 + 1)) + (324*a^2*b^2*tan(d*x + c)^3 + 324*a^3*
b - ((a^5*b + 2*a^3*b^3 + a*b^5)*d*tan(d*x + c)^3 + (a^6 + 2*a^4*b^2 + a^2*
b^4)*d)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^
4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^
2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d
 + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 +
 a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2
)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4
/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b
^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/
729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3
)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d)) - 3*sqrt(1/3)*((
a^5*b + 2*a^3*b^3 + a*b^5)*d*tan(d*x + c)^3 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d
)*sqrt((29808*a^4*b^2 - 10368*a^2*b^4 - 5184*b^6 - (a^10 + 4*a^8*b^2 + 6*a^
6*b^4 + 4*a^4*b^6 + a^2*b^8)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d + b^4*d)^2
- b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*
b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2
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+ a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*
d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4
 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d + 2*a^2
*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^
4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^
3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 +
b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*b^2*d + b^4
*d))^2*d^2 + 216*(a^7*b + 2*a^5*b^3 + a^3*b^5)*(4*(9*a^2*b^2/(a^4*d + 2*a^2
*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3
) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^
2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*
a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^
4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*
b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2
+ a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*
d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4
 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d +
 2*a^2*b^2*d + b^4*d))*d)/((a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*
b^8)*d^2)))*log(-1/324*(20736*a^8 - 106272*a^6*b^2 - 22032*a^4*b^4 - ((2*a^
12 + 7*a^10*b^2 + 9*a^8*b^4 + 5*a^6*b^6 + a^4*b^8)*d^2*tan(d*x + c)^2 - 4*(
a^11*b + 3*a^9*b^3 + 3*a^7*b^5 + a^5*b^7)*d^2*tan(d*x + c) - (2*a^12 + 7*a^
10*b^2 + 9*a^8*b^4 + 5*a^6*b^6 + a^4*b^8)*d^2)*(4*(9*a^2*b^2/(a^4*d + 2*a^2
*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3
) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^
2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*
a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^
4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*
b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2
+ a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*
d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4
 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d +
 2*a^2*b^2*d + b^4*d))^2 + 1296*(42*a^6*b^2 - 59*a^4*b^4 - 22*a^2*b^6 - 2*b
^8)*tan(d*x + c)^2 + 36*((8*a^9*b + 6*a^7*b^3 - 3*a^5*b^5 - a^3*b^7)*d*tan(
d*x + c)^2 + 2*(4*a^10 - 16*a^8*b^2 - 27*a^6*b^4 - 8*a^4*b^6 - a^2*b^8)*d*t
an(d*x + c) - (8*a^9*b + 6*a^7*b^3 - 3*a^5*b^5 - a^3*b^7)*d)*(4*(9*a^2*b^2/
(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^
2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*
a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d
)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*
(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) +
81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 +
2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*
b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^
2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 10
8*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d)) - 3*sqrt(1/3)*(36*(10*a^9*b + 21*a^7*b
^3 + 12*a^5*b^5 + a^3*b^7)*d*tan(d*x + c)^2 - 72*(4*a^10 + 2*a^8*b^2 - 9*a^
6*b^4 - 8*a^4*b^6 - a^2*b^8)*d*tan(d*x + c) - ((2*a^12 + 7*a^10*b^2 + 9*a^8
*b^4 + 5*a^6*b^6 + a^4*b^8)*d^2*tan(d*x + c)^2 - 4*(a^11*b + 3*a^9*b^3 + 3*
a^7*b^5 + a^5*b^7)*d^2*tan(d*x + c) - (2*a^12 + 7*a^10*b^2 + 9*a^8*b^4 + 5*
a^6*b^6 + a^4*b^8)*d^2)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2
/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(
a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2
*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 +
 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^
6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*
d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d +
 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a
^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^
4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d))
```

```
- 36*(10*a^9*b + 21*a^7*b^3 + 12*a^5*b^5 + a^3*b^7)*d)*sqrt((29808*a^4*b^2
- 10368*a^2*b^4 - 5184*b^6 - (a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^
2*b^8)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4
*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2
*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d
+ 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 +
a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)
^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/
81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^
4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/7
29*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)
*(I*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d))^2*d^2 + 216*(a^7*
b + 2*a^5*b^3 + a^3*b^5)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^
2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/
(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^
2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3
+ 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b
^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2
*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d
+ 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 +
a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)
^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d))
*d)/((a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*b^8)*d^2)) - 2592*(28*
a^7*b - 78*a^5*b^3 - 27*a^3*b^5 - 2*a*b^7)*tan(d*x + c))/(tan(d*x + c)^2 +
1)) - 216*(a^2*b^2 + b^4)*tan(d*x + c))/((a^5*b + 2*a^3*b^3 + a*b^5)*d*tan(
d*x + c)^3 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.6547, size = 811, normalized size = 1.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/9*(9*a*b*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 6*a*b*log(abs(
b*tan(d*x + c)^3 + a))/(a^4 + 2*a^2*b^2 + b^4) + 2*(2*a^8*b^2*(-a/b)^(1/3)
+ 3*a^6*b^4*(-a/b)^(1/3) - a^2*b^8*(-a/b)^(1/3) - 4*a^7*b^3 - 9*a^5*b^5 - 6
*a^3*b^7 - a*b^9)*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + tan(d*x + c)))/(a^11
*b + 4*a^9*b^3 + 6*a^7*b^5 + 4*a^5*b^7 + a^3*b^9) + 9*(a^2 - b^2)*(d*x + c)
/(a^4 + 2*a^2*b^2 + b^4) + 6*(pi*floor((d*x + c)/pi + 1/2)*sgn((-a/b)^(1/3)
) + arctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*tan(d*x + c))/(-a/b)^(1/3)))*((2*a
^3 - a*b^2)*(-a*b^2)^(2/3) + (4*a^2*b^2 + b^4)*(-a*b^2)^(1/3))/(sqrt(3)*a^6
*b + 2*sqrt(3)*a^4*b^3 + sqrt(3)*a^2*b^5) - ((2*a^3 - a*b^2)*(-a*b^2)^(2/3)
```

```
 - (4*a^2*b^2 + b^4)*(-a*b^2)^(1/3))*log(tan(d*x + c)^2 + (-a/b)^(1/3)*tan(
d*x + c) + (-a/b)^(2/3))/(a^6*b + 2*a^4*b^3 + a^2*b^5) + 3*(2*a^2*b^2*tan(d
*x + c)^3 - a^3*b*tan(d*x + c)^2 - a*b^3*tan(d*x + c)^2 + a^2*b^2*tan(d*x +
 c) + b^4*tan(d*x + c) + 3*a^3*b + a*b^3)/((a^5 + 2*a^3*b^2 + a*b^4)*(b*tan
(d*x + c)^3 + a)))/d
```

3.380 $\int \frac{1}{1+\tan^3(x)}\,dx$

Optimal. Leaf size=37

$$\frac{x}{2}-\frac{1}{3}\log\left(\tan^2(x)-\tan(x)+1\right)+\frac{1}{6}\log(\tan(x)+1)-\frac{1}{2}\log(\cos(x))$$

```
[Out] x/2 - Log[Cos[x]]/2 + Log[1 + Tan[x]]/6 - Log[1 - Tan[x] + Tan[x]^2]/3
```

Rubi [A] time = 0.0612837, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3661, 2074, 635, 203, 260, 628}

$$\frac{x}{2}-\frac{1}{3}\log\left(\tan^2(x)-\tan(x)+1\right)+\frac{1}{6}\log(\tan(x)+1)-\frac{1}{2}\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tan[x]^3)^(-1),x]
```

```
[Out] x/2 - Log[Cos[x]]/2 + Log[1 + Tan[x]]/6 - Log[1 - Tan[x] + Tan[x]^2]/3
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{1+\tan^3(x)}\,dx = \text{Subst}\left(\int \frac{1}{(1+x^2)(1+x^3)}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{6(1+x)} + \frac{1+x}{2(1+x^2)} + \frac{1-2x}{3(1-x+x^2)}\right) dx, x, \tan(x)\right)$$
$$= \frac{1}{6}\log(1+\tan(x)) + \frac{1}{3}\text{Subst}\left(\int \frac{1-2x}{1-x+x^2}\,dx, x, \tan(x)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1+x}{1+x^2}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{6}\log(1+\tan(x)) - \frac{1}{3}\log\left(1-\tan(x)+\tan^2(x)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right) + \frac{1}{2}\text{Subst}\Big(\int$$
$$= \frac{x}{2} - \frac{1}{2}\log(\cos(x)) + \frac{1}{6}\log(1+\tan(x)) - \frac{1}{3}\log\left(1-\tan(x)+\tan^2(x)\right)$$

Mathematica [C] time = 0.0223794, size = 57, normalized size = 1.54

$$-\frac{1}{3}\log\left(\tan^2(x)-\tan(x)+1\right) + \left(\frac{1}{4}-\frac{i}{4}\right)\log(-\tan(x)+i) + \left(\frac{1}{4}+\frac{i}{4}\right)\log(\tan(x)+i) + \frac{1}{6}\log(\tan(x)+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tan[x]^3)^(-1),x]
```

```
[Out] (1/4 - I/4)*Log[I - Tan[x]] + (1/4 + I/4)*Log[I + Tan[x]] + Log[1 + Tan[x]]
/6 - Log[1 - Tan[x] + Tan[x]^2]/3
```

Maple [A] time = 0.026, size = 34, normalized size = 0.9

$$-\frac{\ln\left(1-\tan(x)+(\tan(x))^2\right)}{3} + \frac{\ln(1+\tan(x))}{6} + \frac{\ln\left(1+(\tan(x))^2\right)}{4} + \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+tan(x)^3),x)
```

```
[Out] -1/3*ln(1-tan(x)+tan(x)^2)+1/6*ln(1+tan(x))+1/4*ln(1+tan(x)^2)+1/2*x
```

Maxima [A] time = 1.50686, size = 45, normalized size = 1.22

$$\frac{1}{2}x - \frac{1}{3}\log\left(\tan(x)^2-\tan(x)+1\right) + \frac{1}{4}\log\left(\tan(x)^2+1\right) + \frac{1}{6}\log(\tan(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^3),x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/3*log(tan(x)^2 - tan(x) + 1) + 1/4*log(tan(x)^2 + 1) + 1/6*log(ta
n(x) + 1)
```

Fricas [A] time = 1.38414, size = 149, normalized size = 4.03

$$\frac{1}{2}x + \frac{1}{12}\log\left(\frac{\tan(x)^2 + 2\tan(x) + 1}{\tan(x)^2 + 1}\right) - \frac{1}{3}\log\left(\frac{\tan(x)^2 - \tan(x) + 1}{\tan(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^3),x, algorithm="fricas")
```

```
[Out] 1/2*x + 1/12*log((tan(x)^2 + 2*tan(x) + 1)/(tan(x)^2 + 1)) - 1/3*log((tan(x
)^2 - tan(x) + 1)/(tan(x)^2 + 1))
```

Sympy [A] time = 0.200639, size = 34, normalized size = 0.92

$$\frac{x}{2} + \frac{\log(\tan(x) + 1)}{6} + \frac{\log\left(\tan^2(x) + 1\right)}{4} - \frac{\log\left(\tan^2(x) - \tan(x) + 1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)**3),x)
```

```
[Out] x/2 + log(tan(x) + 1)/6 + log(tan(x)**2 + 1)/4 - log(tan(x)**2 - tan(x) + 1
)/3
```

Giac [A] time = 1.09872, size = 46, normalized size = 1.24

$$\frac{1}{2}x - \frac{1}{3}\log\left(\tan(x)^2 - \tan(x) + 1\right) + \frac{1}{4}\log\left(\tan(x)^2 + 1\right) + \frac{1}{6}\log(|\tan(x) + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^3),x, algorithm="giac")
```

```
[Out] 1/2*x - 1/3*log(tan(x)^2 - tan(x) + 1) + 1/4*log(tan(x)^2 + 1) + 1/6*log(ab
s(tan(x) + 1))
```

3.381 $\int \left(a+b\tan^4(c+dx)\right)^4 dx$

Optimal. Leaf size=216

$$\frac{b^2\left(6a^2+4ab+b^2\right)\tan^7(c+dx)}{7d}-\frac{b^2\left(6a^2+4ab+b^2\right)\tan^5(c+dx)}{5d}+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan^3(c+dx)}{3d}-\frac{b(2a+$$

```
[Out] (a + b)^4*x - (b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tan[c + d*x])/d + (b*(2*a
+ b)*(2*a^2 + 2*a*b + b^2)*Tan[c + d*x]^3)/(3*d) - (b^2*(6*a^2 + 4*a*b + b^
2)*Tan[c + d*x]^5)/(5*d) + (b^2*(6*a^2 + 4*a*b + b^2)*Tan[c + d*x]^7)/(7*d)
 - (b^3*(4*a + b)*Tan[c + d*x]^9)/(9*d) + (b^3*(4*a + b)*Tan[c + d*x]^11)/(
11*d) - (b^4*Tan[c + d*x]^13)/(13*d) + (b^4*Tan[c + d*x]^15)/(15*d)
```

Rubi [A] time = 0.129335, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3661, 1154, 203}

$$\frac{b^2\left(6a^2+4ab+b^2\right)\tan^7(c+dx)}{7d}-\frac{b^2\left(6a^2+4ab+b^2\right)\tan^5(c+dx)}{5d}+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan^3(c+dx)}{3d}-\frac{b(2a+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^4)^4,x]
```

```
[Out] (a + b)^4*x - (b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tan[c + d*x])/d + (b*(2*a
+ b)*(2*a^2 + 2*a*b + b^2)*Tan[c + d*x]^3)/(3*d) - (b^2*(6*a^2 + 4*a*b + b^
2)*Tan[c + d*x]^5)/(5*d) + (b^2*(6*a^2 + 4*a*b + b^2)*Tan[c + d*x]^7)/(7*d)
 - (b^3*(4*a + b)*Tan[c + d*x]^9)/(9*d) + (b^3*(4*a + b)*Tan[c + d*x]^11)/(
11*d) - (b^4*Tan[c + d*x]^13)/(13*d) + (b^4*Tan[c + d*x]^15)/(15*d)
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\tan^4(c+dx)\right)^4 dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^4\right)^4}{1+x^2} dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-b(2a+b)\left(2a^2+2ab+b^2\right)+b(2a+b)\left(2a^2+2ab+b^2\right)x^2-b^2\left(6a^2+4ab\right.\right.}{}$$
$$= -\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d}+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan^3(c+dx)}{3d}-$$
$$= (a+b)^4x-\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d}+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan^3(c}{3d}$$

Mathematica [A] time = 4.26395, size = 196, normalized size = 0.91

$$\frac{b\tan(c+dx)\left(6435b\left(6a^2+4ab+b^2\right)\tan^6(c+dx)-9009b\left(6a^2+4ab+b^2\right)\tan^4(c+dx)+15015\left(6a^2b+4a^3+4ab\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^4)^4,x]
```

```
[Out] ((a + b)^4*ArcTan[Tan[c + d*x]])/d + (b*Tan[c + d*x]*(-45045*(4*a^3 + 6*a^2
*b + 4*a*b^2 + b^3) + 15015*(4*a^3 + 6*a^2*b + 4*a*b^2 + b^3)*Tan[c + d*x]^
2 - 9009*b*(6*a^2 + 4*a*b + b^2)*Tan[c + d*x]^4 + 6435*b*(6*a^2 + 4*a*b + b
^2)*Tan[c + d*x]^6 - 5005*b^2*(4*a + b)*Tan[c + d*x]^8 + 4095*b^2*(4*a + b)
*Tan[c + d*x]^10 - 3465*b^3*Tan[c + d*x]^12 + 3003*b^3*Tan[c + d*x]^14))/(4
5045*d)
```

Maple [B] time = 0.005, size = 412, normalized size = 1.9

$$\frac{b^4\left(\tan\left(dx+c\right)\right)^7}{7d}-\frac{b^4\left(\tan\left(dx+c\right)\right)^9}{9d}+\frac{b^4\left(\tan\left(dx+c\right)\right)^{11}}{11d}-\frac{b^4\left(\tan\left(dx+c\right)\right)^{13}}{13d}+\frac{b^4\left(\tan\left(dx+c\right)\right)^{15}}{15d}-\frac{\left(\tan\left(dx+c\right.\right.}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^4)^4,x)
```

```
[Out] 1/7*b^4*tan(d*x+c)^7/d-1/9*b^4*tan(d*x+c)^9/d+1/11*b^4*tan(d*x+c)^11/d-1/13
*b^4*tan(d*x+c)^13/d+1/15*b^4*tan(d*x+c)^15/d-1/5/d*tan(d*x+c)^5*b^4+1/3/d*
tan(d*x+c)^3*b^4-1/d*b^4*tan(d*x+c)+1/d*arctan(tan(d*x+c))*a^4+1/d*arctan(t
an(d*x+c))*b^4-4/5/d*tan(d*x+c)^5*a*b^3+2/d*tan(d*x+c)^3*a^2*b^2+4/3/d*tan(
d*x+c)^3*a*b^3-4/d*tan(d*x+c)*a^3*b-6/d*a^2*b^2*tan(d*x+c)-4/d*a*b^3*tan(d*
x+c)+4/d*arctan(tan(d*x+c))*a^3*b+6/d*arctan(tan(d*x+c))*a^2*b^2+4/d*arctan
(tan(d*x+c))*a*b^3+4/3/d*tan(d*x+c)^3*a^3*b+4/11/d*tan(d*x+c)^11*a*b^3-4/9/
d*a*b^3*tan(d*x+c)^9+6/7/d*tan(d*x+c)^7*a^2*b^2+4/7/d*tan(d*x+c)^7*a*b^3-6/
5/d*tan(d*x+c)^5*a^2*b^2
```

Maxima [A] time = 1.51872, size = 358, normalized size = 1.66

$$a^4x+\frac{4\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^3b}{3\,d}+\frac{2\left(15\,\tan\left(dx+c\right)^7-21\,\tan\left(dx+c\right)^5+35\,\tan\left(dx+c\right)^3-\right.}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 4/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^3*b/d + 2/35*
(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x + 105*
c - 105*tan(d*x + c))*a^2*b^2/d + 4/3465*(315*tan(d*x + c)^11 - 385*tan(d*x
 + c)^9 + 495*tan(d*x + c)^7 - 693*tan(d*x + c)^5 + 1155*tan(d*x + c)^3 + 3
465*d*x + 3465*c - 3465*tan(d*x + c))*a*b^3/d + 1/45045*(3003*tan(d*x + c)^
15 - 3465*tan(d*x + c)^13 + 4095*tan(d*x + c)^11 - 5005*tan(d*x + c)^9 + 64
35*tan(d*x + c)^7 - 9009*tan(d*x + c)^5 + 15015*tan(d*x + c)^3 + 45045*d*x
+ 45045*c - 45045*tan(d*x + c))*b^4/d
```

Fricas [A] time = 1.54464, size = 564, normalized size = 2.61

$$\frac{3003\,b^4\tan(dx+c)^{15} - 3465\,b^4\tan(dx+c)^{13} + 4095\left(4\,ab^3+b^4\right)\tan(dx+c)^{11} - 5005\left(4\,ab^3+b^4\right)\tan(dx+c)^9 + 643}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^4,x, algorithm="fricas")
```

```
[Out] 1/45045*(3003*b^4*tan(d*x + c)^15 - 3465*b^4*tan(d*x + c)^13 + 4095*(4*a*b^
3 + b^4)*tan(d*x + c)^11 - 5005*(4*a*b^3 + b^4)*tan(d*x + c)^9 + 6435*(6*a^
2*b^2 + 4*a*b^3 + b^4)*tan(d*x + c)^7 - 9009*(6*a^2*b^2 + 4*a*b^3 + b^4)*ta
n(d*x + c)^5 + 15015*(4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*tan(d*x + c)^3 +
 45045*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x - 45045*(4*a^3*b + 6
*a^2*b^2 + 4*a*b^3 + b^4)*tan(d*x + c))/d
```

Sympy [A] time = 10.6628, size = 386, normalized size = 1.79

$$\begin{cases} a^4x + 4a^3bx + \frac{4a^3b\tan^3(c+dx)}{3d} - \frac{4a^3b\tan(c+dx)}{d} + 6a^2b^2x + \frac{6a^2b^2\tan^7(c+dx)}{7d} - \frac{6a^2b^2\tan^5(c+dx)}{5d} + \frac{2a^2b^2\tan^3(c+dx)}{d} - \frac{6a^2b^2\tan(c+dx)}{d} \\ x\left(a+b\tan^4(c)\right)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)**4*b)**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*x + 4*a**3*b*tan(c + d*x)**3/(3*d) - 4*a**3*b*
tan(c + d*x)/d + 6*a**2*b**2*x + 6*a**2*b**2*tan(c + d*x)**7/(7*d) - 6*a**2
*b**2*tan(c + d*x)**5/(5*d) + 2*a**2*b**2*tan(c + d*x)**3/d - 6*a**2*b**2*t
an(c + d*x)/d + 4*a*b**3*x + 4*a*b**3*tan(c + d*x)**11/(11*d) - 4*a*b**3*ta
n(c + d*x)**9/(9*d) + 4*a*b**3*tan(c + d*x)**7/(7*d) - 4*a*b**3*tan(c + d*x
)**5/(5*d) + 4*a*b**3*tan(c + d*x)**3/(3*d) - 4*a*b**3*tan(c + d*x)/d + b**
4*x + b**4*tan(c + d*x)**15/(15*d) - b**4*tan(c + d*x)**13/(13*d) + b**4*ta
n(c + d*x)**11/(11*d) - b**4*tan(c + d*x)**9/(9*d) + b**4*tan(c + d*x)**7/(
7*d) - b**4*tan(c + d*x)**5/(5*d) + b**4*tan(c + d*x)**3/(3*d) - b**4*tan(c
 + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c)**4)**4, True))
```

Giac [B] time = 156.397, size = 10450, normalized size = 48.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^4,x, algorithm="giac")
```

```
[Out] 1/45045*(45045*a^4*d*x*tan(d*x)^15*tan(c)^15 + 180180*a^3*b*d*x*tan(d*x)^15
*tan(c)^15 + 270270*a^2*b^2*d*x*tan(d*x)^15*tan(c)^15 + 180180*a*b^3*d*x*ta
n(d*x)^15*tan(c)^15 + 45045*b^4*d*x*tan(d*x)^15*tan(c)^15 - 675675*a^4*d*x*
tan(d*x)^14*tan(c)^14 - 2702700*a^3*b*d*x*tan(d*x)^14*tan(c)^14 - 4054050*a
^2*b^2*d*x*tan(d*x)^14*tan(c)^14 - 2702700*a*b^3*d*x*tan(d*x)^14*tan(c)^14
- 675675*b^4*d*x*tan(d*x)^14*tan(c)^14 + 180180*a^3*b*tan(d*x)^15*tan(c)^14
 + 270270*a^2*b^2*tan(d*x)^15*tan(c)^14 + 180180*a*b^3*tan(d*x)^15*tan(c)^1
4 + 45045*b^4*tan(d*x)^15*tan(c)^14 + 180180*a^3*b*tan(d*x)^14*tan(c)^15 +
270270*a^2*b^2*tan(d*x)^14*tan(c)^15 + 180180*a*b^3*tan(d*x)^14*tan(c)^15 +
 45045*b^4*tan(d*x)^14*tan(c)^15 + 4729725*a^4*d*x*tan(d*x)^13*tan(c)^13 +
18918900*a^3*b*d*x*tan(d*x)^13*tan(c)^13 + 28378350*a^2*b^2*d*x*tan(d*x)^13
*tan(c)^13 + 18918900*a*b^3*d*x*tan(d*x)^13*tan(c)^13 + 4729725*b^4*d*x*tan
(d*x)^13*tan(c)^13 - 60060*a^3*b*tan(d*x)^15*tan(c)^12 - 90090*a^2*b^2*tan(
d*x)^15*tan(c)^12 - 60060*a*b^3*tan(d*x)^15*tan(c)^12 - 15015*b^4*tan(d*x)^
15*tan(c)^12 - 2702700*a^3*b*tan(d*x)^14*tan(c)^13 - 4054050*a^2*b^2*tan(d*
x)^14*tan(c)^13 - 2702700*a*b^3*tan(d*x)^14*tan(c)^13 - 675675*b^4*tan(d*x)
^14*tan(c)^13 - 2702700*a^3*b*tan(d*x)^13*tan(c)^14 - 4054050*a^2*b^2*tan(d
*x)^13*tan(c)^14 - 2702700*a*b^3*tan(d*x)^13*tan(c)^14 - 675675*b^4*tan(d*x
)^13*tan(c)^14 - 60060*a^3*b*tan(d*x)^12*tan(c)^15 - 90090*a^2*b^2*tan(d*x)
^12*tan(c)^15 - 60060*a*b^3*tan(d*x)^12*tan(c)^15 - 15015*b^4*tan(d*x)^12*t
an(c)^15 - 20495475*a^4*d*x*tan(d*x)^12*tan(c)^12 - 81981900*a^3*b*d*x*tan(
d*x)^12*tan(c)^12 - 122972850*a^2*b^2*d*x*tan(d*x)^12*tan(c)^12 - 81981900*
a*b^3*d*x*tan(d*x)^12*tan(c)^12 - 20495475*b^4*d*x*tan(d*x)^12*tan(c)^12 +
54054*a^2*b^2*tan(d*x)^15*tan(c)^10 + 36036*a*b^3*tan(d*x)^15*tan(c)^10 + 9
009*b^4*tan(d*x)^15*tan(c)^10 + 720720*a^3*b*tan(d*x)^14*tan(c)^11 + 135135
0*a^2*b^2*tan(d*x)^14*tan(c)^11 + 900900*a*b^3*tan(d*x)^14*tan(c)^11 + 2252
25*b^4*tan(d*x)^14*tan(c)^11 + 18558540*a^3*b*tan(d*x)^13*tan(c)^12 + 28378
350*a^2*b^2*tan(d*x)^13*tan(c)^12 + 18918900*a*b^3*tan(d*x)^13*tan(c)^12 +
4729725*b^4*tan(d*x)^13*tan(c)^12 + 18558540*a^3*b*tan(d*x)^12*tan(c)^13 +
28378350*a^2*b^2*tan(d*x)^12*tan(c)^13 + 18918900*a*b^3*tan(d*x)^12*tan(c)^
13 + 4729725*b^4*tan(d*x)^12*tan(c)^13 + 720720*a^3*b*tan(d*x)^11*tan(c)^14
 + 1351350*a^2*b^2*tan(d*x)^11*tan(c)^14 + 900900*a*b^3*tan(d*x)^11*tan(c)^
14 + 225225*b^4*tan(d*x)^11*tan(c)^14 + 54054*a^2*b^2*tan(d*x)^10*tan(c)^15
 + 36036*a*b^3*tan(d*x)^10*tan(c)^15 + 9009*b^4*tan(d*x)^10*tan(c)^15 + 614
86425*a^4*d*x*tan(d*x)^11*tan(c)^11 + 245945700*a^3*b*d*x*tan(d*x)^11*tan(c
)^11 + 368918550*a^2*b^2*d*x*tan(d*x)^11*tan(c)^11 + 245945700*a*b^3*d*x*ta
n(d*x)^11*tan(c)^11 + 61486425*b^4*d*x*tan(d*x)^11*tan(c)^11 - 38610*a^2*b^
2*tan(d*x)^15*tan(c)^8 - 25740*a*b^3*tan(d*x)^15*tan(c)^8 - 6435*b^4*tan(d*
x)^15*tan(c)^8 - 810810*a^2*b^2*tan(d*x)^14*tan(c)^9 - 540540*a*b^3*tan(d*x
)^14*tan(c)^9 - 135135*b^4*tan(d*x)^14*tan(c)^9 - 3963960*a^3*b*tan(d*x)^13
*tan(c)^10 - 9459450*a^2*b^2*tan(d*x)^13*tan(c)^10 - 6306300*a*b^3*tan(d*x)
^13*tan(c)^10 - 1576575*b^4*tan(d*x)^13*tan(c)^10 - 77477400*a^3*b*tan(d*x)
^12*tan(c)^11 - 122972850*a^2*b^2*tan(d*x)^12*tan(c)^11 - 81981900*a*b^3*ta
n(d*x)^12*tan(c)^11 - 20495475*b^4*tan(d*x)^12*tan(c)^11 - 77477400*a^3*b*t
an(d*x)^11*tan(c)^12 - 122972850*a^2*b^2*tan(d*x)^11*tan(c)^12 - 81981900*a
*b^3*tan(d*x)^11*tan(c)^12 - 20495475*b^4*tan(d*x)^11*tan(c)^12 - 3963960*a
^3*b*tan(d*x)^10*tan(c)^13 - 9459450*a^2*b^2*tan(d*x)^10*tan(c)^13 - 630630
0*a*b^3*tan(d*x)^10*tan(c)^13 - 1576575*b^4*tan(d*x)^10*tan(c)^13 - 810810*
a^2*b^2*tan(d*x)^9*tan(c)^14 - 540540*a*b^3*tan(d*x)^9*tan(c)^14 - 135135*b
^4*tan(d*x)^9*tan(c)^14 - 38610*a^2*b^2*tan(d*x)^8*tan(c)^15 - 25740*a*b^3*
tan(d*x)^8*tan(c)^15 - 6435*b^4*tan(d*x)^8*tan(c)^15 - 135270135*a^4*d*x*ta
n(d*x)^10*tan(c)^10 - 541080540*a^3*b*d*x*tan(d*x)^10*tan(c)^10 - 811620810
*a^2*b^2*d*x*tan(d*x)^10*tan(c)^10 - 541080540*a*b^3*d*x*tan(d*x)^10*tan(c)
^10 - 135270135*b^4*d*x*tan(d*x)^10*tan(c)^10 + 20020*a*b^3*tan(d*x)^15*tan
(c)^6 + 5005*b^4*tan(d*x)^15*tan(c)^6 + 308880*a^2*b^2*tan(d*x)^14*tan(c)^7
```

+ 386100*a*b^3*tan(d*x)^14*tan(c)^7 + 96525*b^4*tan(d*x)^14*tan(c)^7 + 459
4590*a^2*b^2*tan(d*x)^13*tan(c)^8 + 3783780*a*b^3*tan(d*x)^13*tan(c)^8 + 94
5945*b^4*tan(d*x)^13*tan(c)^8 + 13213200*a^3*b*tan(d*x)^12*tan(c)^9 + 38468
430*a^2*b^2*tan(d*x)^12*tan(c)^9 + 27327300*a*b^3*tan(d*x)^12*tan(c)^9 + 68
31825*b^4*tan(d*x)^12*tan(c)^9 + 219999780*a^3*b*tan(d*x)^11*tan(c)^10 + 36
5134770*a^2*b^2*tan(d*x)^11*tan(c)^10 + 245945700*a*b^3*tan(d*x)^11*tan(c)^
10 + 61486425*b^4*tan(d*x)^11*tan(c)^10 + 219999780*a^3*b*tan(d*x)^10*tan(c
)^11 + 365134770*a^2*b^2*tan(d*x)^10*tan(c)^11 + 245945700*a*b^3*tan(d*x)^1
0*tan(c)^11 + 61486425*b^4*tan(d*x)^10*tan(c)^11 + 13213200*a^3*b*tan(d*x)^
9*tan(c)^12 + 38468430*a^2*b^2*tan(d*x)^9*tan(c)^12 + 27327300*a*b^3*tan(d*
x)^9*tan(c)^12 + 6831825*b^4*tan(d*x)^9*tan(c)^12 + 4594590*a^2*b^2*tan(d*x
)^8*tan(c)^13 + 3783780*a*b^3*tan(d*x)^8*tan(c)^13 + 945945*b^4*tan(d*x)^8*
tan(c)^13 + 308880*a^2*b^2*tan(d*x)^7*tan(c)^14 + 386100*a*b^3*tan(d*x)^7*t
an(c)^14 + 96525*b^4*tan(d*x)^7*tan(c)^14 + 20020*a*b^3*tan(d*x)^6*tan(c)^1
5 + 5005*b^4*tan(d*x)^6*tan(c)^15 + 225450225*a^4*d*x*tan(d*x)^9*tan(c)^9 +
901800900*a^3*b*d*x*tan(d*x)^9*tan(c)^9 + 1352701350*a^2*b^2*d*x*tan(d*x)^
9*tan(c)^9 + 901800900*a*b^3*d*x*tan(d*x)^9*tan(c)^9 + 225450225*b^4*d*x*ta
n(d*x)^9*tan(c)^9 - 16380*a*b^3*tan(d*x)^15*tan(c)^4 - 4095*b^4*tan(d*x)^15
*tan(c)^4 - 300300*a*b^3*tan(d*x)^14*tan(c)^5 - 75075*b^4*tan(d*x)^14*tan(c
)^5 - 1081080*a^2*b^2*tan(d*x)^13*tan(c)^6 - 2702700*a*b^3*tan(d*x)^13*tan(
c)^6 - 675675*b^4*tan(d*x)^13*tan(c)^6 - 14054040*a^2*b^2*tan(d*x)^12*tan(c
)^7 - 16396380*a*b^3*tan(d*x)^12*tan(c)^7 - 4099095*b^4*tan(d*x)^12*tan(c)^
7 - 29729700*a^3*b*tan(d*x)^11*tan(c)^8 - 99729630*a^2*b^2*tan(d*x)^11*tan(
c)^8 - 81981900*a*b^3*tan(d*x)^11*tan(c)^8 - 20495475*b^4*tan(d*x)^11*tan(c
)^8 - 449909460*a^3*b*tan(d*x)^10*tan(c)^9 - 777566790*a^2*b^2*tan(d*x)^10*
tan(c)^9 - 541080540*a*b^3*tan(d*x)^10*tan(c)^9 - 135270135*b^4*tan(d*x)^10
*tan(c)^9 - 449909460*a^3*b*tan(d*x)^9*tan(c)^10 - 777566790*a^2*b^2*tan(d*
x)^9*tan(c)^10 - 541080540*a*b^3*tan(d*x)^9*tan(c)^10 - 135270135*b^4*tan(d
*x)^9*tan(c)^10 - 29729700*a^3*b*tan(d*x)^8*tan(c)^11 - 99729630*a^2*b^2*ta
n(d*x)^8*tan(c)^11 - 81981900*a*b^3*tan(d*x)^8*tan(c)^11 - 20495475*b^4*tan
(d*x)^8*tan(c)^11 - 14054040*a^2*b^2*tan(d*x)^7*tan(c)^12 - 16396380*a*b^3*
tan(d*x)^7*tan(c)^12 - 4099095*b^4*tan(d*x)^7*tan(c)^12 - 1081080*a^2*b^2*t
an(d*x)^6*tan(c)^13 - 2702700*a*b^3*tan(d*x)^6*tan(c)^13 - 675675*b^4*tan(d
*x)^6*tan(c)^13 - 300300*a*b^3*tan(d*x)^5*tan(c)^14 - 75075*b^4*tan(d*x)^5*
tan(c)^14 - 16380*a*b^3*tan(d*x)^4*tan(c)^15 - 4095*b^4*tan(d*x)^4*tan(c)^1
5 - 289864575*a^4*d*x*tan(d*x)^8*tan(c)^8 - 1159458300*a^3*b*d*x*tan(d*x)^8
*tan(c)^8 - 1739187450*a^2*b^2*d*x*tan(d*x)^8*tan(c)^8 - 1159458300*a*b^3*d
*x*tan(d*x)^8*tan(c)^8 - 289864575*b^4*d*x*tan(d*x)^8*tan(c)^8 + 3465*b^4*t
an(d*x)^15*tan(c)^2 + 65520*a*b^3*tan(d*x)^14*tan(c)^3 + 61425*b^4*tan(d*x)
^14*tan(c)^3 + 1021020*a*b^3*tan(d*x)^13*tan(c)^4 + 525525*b^4*tan(d*x)^13*
tan(c)^4 + 2162160*a^2*b^2*tan(d*x)^12*tan(c)^5 + 7747740*a*b^3*tan(d*x)^12
*tan(c)^5 + 2927925*b^4*tan(d*x)^12*tan(c)^5 + 26486460*a^2*b^2*tan(d*x)^11
*tan(c)^6 + 39279240*a*b^3*tan(d*x)^11*tan(c)^6 + 12297285*b^4*tan(d*x)^11*
tan(c)^6 + 47567520*a^3*b*tan(d*x)^10*tan(c)^7 + 173513340*a^2*b^2*tan(d*x)
^10*tan(c)^7 + 162522360*a*b^3*tan(d*x)^10*tan(c)^7 + 45090045*b^4*tan(d*x)
^10*tan(c)^7 + 683783100*a^3*b*tan(d*x)^9*tan(c)^8 + 1214863650*a^2*b^2*tan
(d*x)^9*tan(c)^8 + 878017140*a*b^3*tan(d*x)^9*tan(c)^8 + 225450225*b^4*tan(
d*x)^9*tan(c)^8 + 683783100*a^3*b*tan(d*x)^8*tan(c)^9 + 1214863650*a^2*b^2*
tan(d*x)^8*tan(c)^9 + 878017140*a*b^3*tan(d*x)^8*tan(c)^9 + 225450225*b^4*t
an(d*x)^8*tan(c)^9 + 47567520*a^3*b*tan(d*x)^7*tan(c)^10 + 173513340*a^2*b^
2*tan(d*x)^7*tan(c)^10 + 162522360*a*b^3*tan(d*x)^7*tan(c)^10 + 45090045*b^
4*tan(d*x)^7*tan(c)^10 + 26486460*a^2*b^2*tan(d*x)^6*tan(c)^11 + 39279240*a
*b^3*tan(d*x)^6*tan(c)^11 + 12297285*b^4*tan(d*x)^6*tan(c)^11 + 2162160*a^2
*b^2*tan(d*x)^5*tan(c)^12 + 7747740*a*b^3*tan(d*x)^5*tan(c)^12 + 2927925*b^
4*tan(d*x)^5*tan(c)^12 + 1021020*a*b^3*tan(d*x)^4*tan(c)^13 + 525525*b^4*ta
n(d*x)^4*tan(c)^13 + 65520*a*b^3*tan(d*x)^3*tan(c)^14 + 61425*b^4*tan(d*x)^
3*tan(c)^14 + 3465*b^4*tan(d*x)^2*tan(c)^15 + 289864575*a^4*d*x*tan(d*x)^7*
tan(c)^7 + 1159458300*a^3*b*d*x*tan(d*x)^7*tan(c)^7 + 1739187450*a^2*b^2*d*
x*tan(d*x)^7*tan(c)^7 + 1159458300*a*b^3*d*x*tan(d*x)^7*tan(c)^7 + 28986457

5*b^4*d*x*tan(d*x)^7*tan(c)^7 - 3003*b^4*tan(d*x)^15 - 51975*b^4*tan(d*x)^1
4*tan(c) - 98280*a*b^3*tan(d*x)^13*tan(c)^2 - 429975*b^4*tan(d*x)^13*tan(c)
^2 - 1481480*a*b^3*tan(d*x)^12*tan(c)^3 - 2277275*b^4*tan(d*x)^12*tan(c)^3
- 2702700*a^2*b^2*tan(d*x)^11*tan(c)^4 - 10810800*a*b^3*tan(d*x)^11*tan(c)^
4 - 8783775*b^4*tan(d*x)^11*tan(c)^4 - 32540508*a^2*b^2*tan(d*x)^10*tan(c)^
5 - 52324272*a*b^3*tan(d*x)^10*tan(c)^5 - 27054027*b^4*tan(d*x)^10*tan(c)^5
- 55495440*a^3*b*tan(d*x)^9*tan(c)^6 - 208107900*a^2*b^2*tan(d*x)^9*tan(c)
^6 - 204804600*a*b^3*tan(d*x)^9*tan(c)^6 - 75150075*b^4*tan(d*x)^9*tan(c)^6
- 784864080*a^3*b*tan(d*x)^8*tan(c)^7 - 1408106700*a^2*b^2*tan(d*x)^8*tan(
c)^7 - 1034593560*a*b^3*tan(d*x)^8*tan(c)^7 - 289864575*b^4*tan(d*x)^8*tan(
c)^7 - 784864080*a^3*b*tan(d*x)^7*tan(c)^8 - 1408106700*a^2*b^2*tan(d*x)^7*
tan(c)^8 - 1034593560*a*b^3*tan(d*x)^7*tan(c)^8 - 289864575*b^4*tan(d*x)^7*
tan(c)^8 - 55495440*a^3*b*tan(d*x)^6*tan(c)^9 - 208107900*a^2*b^2*tan(d*x)^
6*tan(c)^9 - 204804600*a*b^3*tan(d*x)^6*tan(c)^9 - 75150075*b^4*tan(d*x)^6*
tan(c)^9 - 32540508*a^2*b^2*tan(d*x)^5*tan(c)^10 - 52324272*a*b^3*tan(d*x)^
5*tan(c)^10 - 27054027*b^4*tan(d*x)^5*tan(c)^10 - 2702700*a^2*b^2*tan(d*x)^
4*tan(c)^11 - 10810800*a*b^3*tan(d*x)^4*tan(c)^11 - 8783775*b^4*tan(d*x)^4*
tan(c)^11 - 1481480*a*b^3*tan(d*x)^3*tan(c)^12 - 2277275*b^4*tan(d*x)^3*tan
(c)^12 - 98280*a*b^3*tan(d*x)^2*tan(c)^13 - 429975*b^4*tan(d*x)^2*tan(c)^13
- 51975*b^4*tan(d*x)*tan(c)^14 - 3003*b^4*tan(c)^15 - 225450225*a^4*d*x*ta
n(d*x)^6*tan(c)^6 - 901800900*a^3*b*d*x*tan(d*x)^6*tan(c)^6 - 1352701350*a^
2*b^2*d*x*tan(d*x)^6*tan(c)^6 - 901800900*a*b^3*d*x*tan(d*x)^6*tan(c)^6 - 2
25450225*b^4*d*x*tan(d*x)^6*tan(c)^6 + 3465*b^4*tan(d*x)^13 + 65520*a*b^3*t
an(d*x)^12*tan(c) + 61425*b^4*tan(d*x)^12*tan(c) + 1021020*a*b^3*tan(d*x)^1
1*tan(c)^2 + 525525*b^4*tan(d*x)^11*tan(c)^2 + 2162160*a^2*b^2*tan(d*x)^10*
tan(c)^3 + 7747740*a*b^3*tan(d*x)^10*tan(c)^3 + 2927925*b^4*tan(d*x)^10*tan
(c)^3 + 26486460*a^2*b^2*tan(d*x)^9*tan(c)^4 + 39279240*a*b^3*tan(d*x)^9*ta
n(c)^4 + 12297285*b^4*tan(d*x)^9*tan(c)^4 + 47567520*a^3*b*tan(d*x)^8*tan(c
)^5 + 173513340*a^2*b^2*tan(d*x)^8*tan(c)^5 + 162522360*a*b^3*tan(d*x)^8*ta
n(c)^5 + 45090045*b^4*tan(d*x)^8*tan(c)^5 + 683783100*a^3*b*tan(d*x)^7*tan(
c)^6 + 1214863650*a^2*b^2*tan(d*x)^7*tan(c)^6 + 878017140*a*b^3*tan(d*x)^7*
tan(c)^6 + 225450225*b^4*tan(d*x)^7*tan(c)^6 + 683783100*a^3*b*tan(d*x)^6*t
an(c)^7 + 1214863650*a^2*b^2*tan(d*x)^6*tan(c)^7 + 878017140*a*b^3*tan(d*x)
^6*tan(c)^7 + 225450225*b^4*tan(d*x)^6*tan(c)^7 + 47567520*a^3*b*tan(d*x)^5
*tan(c)^8 + 173513340*a^2*b^2*tan(d*x)^5*tan(c)^8 + 162522360*a*b^3*tan(d*x
)^5*tan(c)^8 + 45090045*b^4*tan(d*x)^5*tan(c)^8 + 26486460*a^2*b^2*tan(d*x)
^4*tan(c)^9 + 39279240*a*b^3*tan(d*x)^4*tan(c)^9 + 12297285*b^4*tan(d*x)^4*
tan(c)^9 + 2162160*a^2*b^2*tan(d*x)^3*tan(c)^10 + 7747740*a*b^3*tan(d*x)^3*
tan(c)^10 + 2927925*b^4*tan(d*x)^3*tan(c)^10 + 1021020*a*b^3*tan(d*x)^2*tan
(c)^11 + 525525*b^4*tan(d*x)^2*tan(c)^11 + 65520*a*b^3*tan(d*x)*tan(c)^12 +
61425*b^4*tan(d*x)*tan(c)^12 + 3465*b^4*tan(c)^13 + 135270135*a^4*d*x*tan(
d*x)^5*tan(c)^5 + 541080540*a^3*b*d*x*tan(d*x)^5*tan(c)^5 + 811620810*a^2*b
^2*d*x*tan(d*x)^5*tan(c)^5 + 541080540*a*b^3*d*x*tan(d*x)^5*tan(c)^5 + 1352
70135*b^4*d*x*tan(d*x)^5*tan(c)^5 - 16380*a*b^3*tan(d*x)^11 - 4095*b^4*tan(
d*x)^11 - 300300*a*b^3*tan(d*x)^10*tan(c) - 75075*b^4*tan(d*x)^10*tan(c) -
1081080*a^2*b^2*tan(d*x)^9*tan(c)^2 - 2702700*a*b^3*tan(d*x)^9*tan(c)^2 - 6
75675*b^4*tan(d*x)^9*tan(c)^2 - 14054040*a^2*b^2*tan(d*x)^8*tan(c)^3 - 1639
6380*a*b^3*tan(d*x)^8*tan(c)^3 - 4099095*b^4*tan(d*x)^8*tan(c)^3 - 29729700
*a^3*b*tan(d*x)^7*tan(c)^4 - 99729630*a^2*b^2*tan(d*x)^7*tan(c)^4 - 8198190
0*a*b^3*tan(d*x)^7*tan(c)^4 - 20495475*b^4*tan(d*x)^7*tan(c)^4 - 449909460*
a^3*b*tan(d*x)^6*tan(c)^5 - 777566790*a^2*b^2*tan(d*x)^6*tan(c)^5 - 5410805
40*a*b^3*tan(d*x)^6*tan(c)^5 - 135270135*b^4*tan(d*x)^6*tan(c)^5 - 44990946
0*a^3*b*tan(d*x)^5*tan(c)^6 - 777566790*a^2*b^2*tan(d*x)^5*tan(c)^6 - 54108
0540*a*b^3*tan(d*x)^5*tan(c)^6 - 135270135*b^4*tan(d*x)^5*tan(c)^6 - 297297
00*a^3*b*tan(d*x)^4*tan(c)^7 - 99729630*a^2*b^2*tan(d*x)^4*tan(c)^7 - 81981
900*a*b^3*tan(d*x)^4*tan(c)^7 - 20495475*b^4*tan(d*x)^4*tan(c)^7 - 14054040
*a^2*b^2*tan(d*x)^3*tan(c)^8 - 16396380*a*b^3*tan(d*x)^3*tan(c)^8 - 4099095
*b^4*tan(d*x)^3*tan(c)^8 - 1081080*a^2*b^2*tan(d*x)^2*tan(c)^9 - 2702700*a*
b^3*tan(d*x)^2*tan(c)^9 - 675675*b^4*tan(d*x)^2*tan(c)^9 - 300300*a*b^3*tan

(d*x)*tan(c)^10 - 75075*b^4*tan(d*x)*tan(c)^10 - 16380*a*b^3*tan(c)^11 - 40
95*b^4*tan(c)^11 - 61486425*a^4*d*x*tan(d*x)^4*tan(c)^4 - 245945700*a^3*b*d
*x*tan(d*x)^4*tan(c)^4 - 368918550*a^2*b^2*d*x*tan(d*x)^4*tan(c)^4 - 245945
700*a*b^3*d*x*tan(d*x)^4*tan(c)^4 - 61486425*b^4*d*x*tan(d*x)^4*tan(c)^4 +
20020*a*b^3*tan(d*x)^9 + 5005*b^4*tan(d*x)^9 + 308880*a^2*b^2*tan(d*x)^8*ta
n(c) + 386100*a*b^3*tan(d*x)^8*tan(c) + 96525*b^4*tan(d*x)^8*tan(c) + 45945
90*a^2*b^2*tan(d*x)^7*tan(c)^2 + 3783780*a*b^3*tan(d*x)^7*tan(c)^2 + 945945
*b^4*tan(d*x)^7*tan(c)^2 + 13213200*a^3*b*tan(d*x)^6*tan(c)^3 + 38468430*a^
2*b^2*tan(d*x)^6*tan(c)^3 + 27327300*a*b^3*tan(d*x)^6*tan(c)^3 + 6831825*b^
4*tan(d*x)^6*tan(c)^3 + 219999780*a^3*b*tan(d*x)^5*tan(c)^4 + 365134770*a^2
*b^2*tan(d*x)^5*tan(c)^4 + 245945700*a*b^3*tan(d*x)^5*tan(c)^4 + 61486425*b
^4*tan(d*x)^5*tan(c)^4 + 219999780*a^3*b*tan(d*x)^4*tan(c)^5 + 365134770*a^
2*b^2*tan(d*x)^4*tan(c)^5 + 245945700*a*b^3*tan(d*x)^4*tan(c)^5 + 61486425*
b^4*tan(d*x)^4*tan(c)^5 + 13213200*a^3*b*tan(d*x)^3*tan(c)^6 + 38468430*a^2
*b^2*tan(d*x)^3*tan(c)^6 + 27327300*a*b^3*tan(d*x)^3*tan(c)^6 + 6831825*b^4
*tan(d*x)^3*tan(c)^6 + 4594590*a^2*b^2*tan(d*x)^2*tan(c)^7 + 3783780*a*b^3*
tan(d*x)^2*tan(c)^7 + 945945*b^4*tan(d*x)^2*tan(c)^7 + 308880*a^2*b^2*tan(d
*x)*tan(c)^8 + 386100*a*b^3*tan(d*x)*tan(c)^8 + 96525*b^4*tan(d*x)*tan(c)^8
 + 20020*a*b^3*tan(c)^9 + 5005*b^4*tan(c)^9 + 20495475*a^4*d*x*tan(d*x)^3*t
an(c)^3 + 81981900*a^3*b*d*x*tan(d*x)^3*tan(c)^3 + 122972850*a^2*b^2*d*x*ta
n(d*x)^3*tan(c)^3 + 81981900*a*b^3*d*x*tan(d*x)^3*tan(c)^3 + 20495475*b^4*d
*x*tan(d*x)^3*tan(c)^3 - 38610*a^2*b^2*tan(d*x)^7 - 25740*a*b^3*tan(d*x)^7
 - 6435*b^4*tan(d*x)^7 - 810810*a^2*b^2*tan(d*x)^6*tan(c) - 540540*a*b^3*tan
(d*x)^6*tan(c) - 135135*b^4*tan(d*x)^6*tan(c) - 3963960*a^3*b*tan(d*x)^5*ta
n(c)^2 - 9459450*a^2*b^2*tan(d*x)^5*tan(c)^2 - 6306300*a*b^3*tan(d*x)^5*tan
(c)^2 - 1576575*b^4*tan(d*x)^5*tan(c)^2 - 77477400*a^3*b*tan(d*x)^4*tan(c)^
3 - 122972850*a^2*b^2*tan(d*x)^4*tan(c)^3 - 81981900*a*b^3*tan(d*x)^4*tan(c
)^3 - 20495475*b^4*tan(d*x)^4*tan(c)^3 - 77477400*a^3*b*tan(d*x)^3*tan(c)^4
 - 122972850*a^2*b^2*tan(d*x)^3*tan(c)^4 - 81981900*a*b^3*tan(d*x)^3*tan(c)
^4 - 20495475*b^4*tan(d*x)^3*tan(c)^4 - 3963960*a^3*b*tan(d*x)^2*tan(c)^5 -
 9459450*a^2*b^2*tan(d*x)^2*tan(c)^5 - 6306300*a*b^3*tan(d*x)^2*tan(c)^5 -
1576575*b^4*tan(d*x)^2*tan(c)^5 - 810810*a^2*b^2*tan(d*x)*tan(c)^6 - 540540
*a*b^3*tan(d*x)*tan(c)^6 - 135135*b^4*tan(d*x)*tan(c)^6 - 38610*a^2*b^2*tan
(c)^7 - 25740*a*b^3*tan(c)^7 - 6435*b^4*tan(c)^7 - 4729725*a^4*d*x*tan(d*x)
^2*tan(c)^2 - 18918900*a^3*b*d*x*tan(d*x)^2*tan(c)^2 - 28378350*a^2*b^2*d*x
*tan(d*x)^2*tan(c)^2 - 18918900*a*b^3*d*x*tan(d*x)^2*tan(c)^2 - 4729725*b^4
*d*x*tan(d*x)^2*tan(c)^2 + 54054*a^2*b^2*tan(d*x)^5 + 36036*a*b^3*tan(d*x)^
5 + 9009*b^4*tan(d*x)^5 + 720720*a^3*b*tan(d*x)^4*tan(c) + 1351350*a^2*b^2*
tan(d*x)^4*tan(c) + 900900*a*b^3*tan(d*x)^4*tan(c) + 225225*b^4*tan(d*x)^4*
tan(c) + 18558540*a^3*b*tan(d*x)^3*tan(c)^2 + 28378350*a^2*b^2*tan(d*x)^3*t
an(c)^2 + 18918900*a*b^3*tan(d*x)^3*tan(c)^2 + 4729725*b^4*tan(d*x)^3*tan(c
)^2 + 18558540*a^3*b*tan(d*x)^2*tan(c)^3 + 28378350*a^2*b^2*tan(d*x)^2*tan(
c)^3 + 18918900*a*b^3*tan(d*x)^2*tan(c)^3 + 4729725*b^4*tan(d*x)^2*tan(c)^3
 + 720720*a^3*b*tan(d*x)*tan(c)^4 + 1351350*a^2*b^2*tan(d*x)*tan(c)^4 + 900
900*a*b^3*tan(d*x)*tan(c)^4 + 225225*b^4*tan(d*x)*tan(c)^4 + 54054*a^2*b^2*
tan(c)^5 + 36036*a*b^3*tan(c)^5 + 9009*b^4*tan(c)^5 + 675675*a^4*d*x*tan(d*
x)*tan(c) + 2702700*a^3*b*d*x*tan(d*x)*tan(c) + 4054050*a^2*b^2*d*x*tan(d*x
)*tan(c) + 2702700*a*b^3*d*x*tan(d*x)*tan(c) + 675675*b^4*d*x*tan(d*x)*tan(
c) - 60060*a^3*b*tan(d*x)^3 - 90090*a^2*b^2*tan(d*x)^3 - 60060*a*b^3*tan(d*
x)^3 - 15015*b^4*tan(d*x)^3 - 2702700*a^3*b*tan(d*x)^2*tan(c) - 4054050*a^2
*b^2*tan(d*x)^2*tan(c) - 2702700*a*b^3*tan(d*x)^2*tan(c) - 675675*b^4*tan(d
*x)^2*tan(c) - 2702700*a^3*b*tan(d*x)*tan(c)^2 - 4054050*a^2*b^2*tan(d*x)*t
an(c)^2 - 2702700*a*b^3*tan(d*x)*tan(c)^2 - 675675*b^4*tan(d*x)*tan(c)^2 -
60060*a^3*b*tan(c)^3 - 90090*a^2*b^2*tan(c)^3 - 60060*a*b^3*tan(c)^3 - 1501
5*b^4*tan(c)^3 - 45045*a^4*d*x - 180180*a^3*b*d*x - 270270*a^2*b^2*d*x - 18
0180*a*b^3*d*x - 45045*b^4*d*x + 180180*a^3*b*tan(d*x) + 270270*a^2*b^2*tan
(d*x) + 180180*a*b^3*tan(d*x) + 45045*b^4*tan(d*x) + 180180*a^3*b*tan(c) +
270270*a^2*b^2*tan(c) + 180180*a*b^3*tan(c) + 45045*b^4*tan(c))/(d*tan(d*x)
^15*tan(c)^15 - 15*d*tan(d*x)^14*tan(c)^14 + 105*d*tan(d*x)^13*tan(c)^13 -

```
455*d*tan(d*x)^12*tan(c)^12 + 1365*d*tan(d*x)^11*tan(c)^11 - 3003*d*tan(d*x
)^10*tan(c)^10 + 5005*d*tan(d*x)^9*tan(c)^9 - 6435*d*tan(d*x)^8*tan(c)^8 +
6435*d*tan(d*x)^7*tan(c)^7 - 5005*d*tan(d*x)^6*tan(c)^6 + 3003*d*tan(d*x)^5
*tan(c)^5 - 1365*d*tan(d*x)^4*tan(c)^4 + 455*d*tan(d*x)^3*tan(c)^3 - 105*d*
tan(d*x)^2*tan(c)^2 + 15*d*tan(d*x)*tan(c) - d)
```

3.382 $\int \left(a+b\tan^4(c+dx)\right)^3 dx$

Optimal. Leaf size=144

$$\frac{b\left(3a^2+3ab+b^2\right)\tan^3(c+dx)}{3d}-\frac{b\left(3a^2+3ab+b^2\right)\tan(c+dx)}{d}+\frac{b^2(3a+b)\tan^7(c+dx)}{7d}-\frac{b^2(3a+b)\tan^5(c+dx)}{5d}+$$

```
[Out] (a + b)^3*x - (b*(3*a^2 + 3*a*b + b^2)*Tan[c + d*x])/d + (b*(3*a^2 + 3*a*b
+ b^2)*Tan[c + d*x]^3)/(3*d) - (b^2*(3*a + b)*Tan[c + d*x]^5)/(5*d) + (b^2*
(3*a + b)*Tan[c + d*x]^7)/(7*d) - (b^3*Tan[c + d*x]^9)/(9*d) + (b^3*Tan[c +
 d*x]^11)/(11*d)
```

Rubi [A] time = 0.0824736, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3661, 1154, 203}

$$\frac{b\left(3a^2+3ab+b^2\right)\tan^3(c+dx)}{3d}-\frac{b\left(3a^2+3ab+b^2\right)\tan(c+dx)}{d}+\frac{b^2(3a+b)\tan^7(c+dx)}{7d}-\frac{b^2(3a+b)\tan^5(c+dx)}{5d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^4)^3,x]
```

```
[Out] (a + b)^3*x - (b*(3*a^2 + 3*a*b + b^2)*Tan[c + d*x])/d + (b*(3*a^2 + 3*a*b
+ b^2)*Tan[c + d*x]^3)/(3*d) - (b^2*(3*a + b)*Tan[c + d*x]^5)/(5*d) + (b^2*
(3*a + b)*Tan[c + d*x]^7)/(7*d) - (b^3*Tan[c + d*x]^9)/(9*d) + (b^3*Tan[c +
 d*x]^11)/(11*d)
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\tan^4(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^4\right)^3}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-b\left(3a^2+3ab+b^2\right)+b\left(3a^2+3ab+b^2\right)x^2-b^2(3a+b)x^4+b^2(3a+b)x^6-\right.\right.}{d}$$
$$= -\frac{b\left(3a^2+3ab+b^2\right)\tan(c+dx)}{d}+\frac{b\left(3a^2+3ab+b^2\right)\tan^3(c+dx)}{3d}-\frac{b^2(3a+b)\tan^5(c}{5d}$$
$$= (a+b)^3x-\frac{b\left(3a^2+3ab+b^2\right)\tan(c+dx)}{d}+\frac{b\left(3a^2+3ab+b^2\right)\tan^3(c+dx)}{3d}-\frac{b^2(3a}{}$$

Mathematica [A] time = 0.973732, size = 128, normalized size = 0.89

$$\frac{b\tan(c+dx)\left(1155\left(3a^2+3ab+b^2\right)\tan^2(c+dx)-3465\left(3a^2+3ab+b^2\right)+495b(3a+b)\tan^6(c+dx)-693b(3a+b\right.}{3465d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^4)^3,x]
```

```
[Out] ((a + b)^3*ArcTan[Tan[c + d*x]])/d + (b*Tan[c + d*x]*(-3465*(3*a^2 + 3*a*b
+ b^2) + 1155*(3*a^2 + 3*a*b + b^2)*Tan[c + d*x]^2 - 693*b*(3*a + b)*Tan[c
+ d*x]^4 + 495*b*(3*a + b)*Tan[c + d*x]^6 - 385*b^2*Tan[c + d*x]^8 + 315*b^
2*Tan[c + d*x]^10))/(3465*d)
```

Maple [A] time = 0.006, size = 252, normalized size = 1.8

$$\frac{b^3(\tan(dx+c))^{11}}{11\,d}-\frac{b^3(\tan(dx+c))^9}{9\,d}+\frac{3\,(\tan(dx+c))^7\,ab^2}{7\,d}+\frac{b^3(\tan(dx+c))^7}{7\,d}-\frac{3\,ab^2(\tan(dx+c))^5}{5\,d}-\frac{b^3(\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^4)^3,x)
```

```
[Out] 1/11*b^3*tan(d*x+c)^11/d-1/9*b^3*tan(d*x+c)^9/d+3/7/d*tan(d*x+c)^7*a*b^2+1/
7/d*b^3*tan(d*x+c)^7-3/5*a*b^2*tan(d*x+c)^5/d-1/5*b^3*tan(d*x+c)^5/d+1/d*ta
n(d*x+c)^3*a^2*b+a*b^2*tan(d*x+c)^3/d+1/3/d*tan(d*x+c)^3*b^3-3/d*tan(d*x+c)
*a^2*b-3*a*b^2*tan(d*x+c)/d-1/d*b^3*tan(d*x+c)+1/d*arctan(tan(d*x+c))*a^3+3
/d*arctan(tan(d*x+c))*a^2*b+3/d*arctan(tan(d*x+c))*a*b^2+1/d*arctan(tan(d*x
+c))*b^3
```

Maxima [A] time = 1.53418, size = 225, normalized size = 1.56

$$a^3x+\frac{\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)a^2b}{d}+\frac{\left(15\,\tan(dx+c)^7-21\,\tan(dx+c)^5+35\,\tan(dx+c)^3+10\right.}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^3,x, algorithm="maxima")
```

```
[Out] a^3*x + (tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^2*b/d + 1/35*(15*
tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x + 105*c -
105*tan(d*x + c))*a*b^2/d + 1/3465*(315*tan(d*x + c)^11 - 385*tan(d*x + c)^
9 + 495*tan(d*x + c)^7 - 693*tan(d*x + c)^5 + 1155*tan(d*x + c)^3 + 3465*d*
x + 3465*c - 3465*tan(d*x + c))*b^3/d
```

Fricas [A] time = 1.37365, size = 367, normalized size = 2.55

$$\frac{315\,b^3\tan\left(dx+c\right)^{11}-385\,b^3\tan\left(dx+c\right)^9+495\left(3\,ab^2+b^3\right)\tan\left(dx+c\right)^7-693\left(3\,ab^2+b^3\right)\tan\left(dx+c\right)^5+1155\left(3\,a^2\right.}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^3,x, algorithm="fricas")
```

```
[Out] 1/3465*(315*b^3*tan(d*x + c)^11 - 385*b^3*tan(d*x + c)^9 + 495*(3*a*b^2 + b
^3)*tan(d*x + c)^7 - 693*(3*a*b^2 + b^3)*tan(d*x + c)^5 + 1155*(3*a^2*b + 3
*a*b^2 + b^3)*tan(d*x + c)^3 + 3465*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 3
465*(3*a^2*b + 3*a*b^2 + b^3)*tan(d*x + c))/d
```

Sympy [A] time = 4.19558, size = 224, normalized size = 1.56

$$\begin{cases} a^3x+3a^2bx+\frac{a^2b\tan^3(c+dx)}{d}-\frac{3a^2b\tan(c+dx)}{d}+3ab^2x+\frac{3ab^2\tan^7(c+dx)}{7d}-\frac{3ab^2\tan^5(c+dx)}{5d}+\frac{ab^2\tan^3(c+dx)}{d}-\frac{3ab^2\tan(c+dx)}{d}+b^3x \\ x\left(a+b\tan^4(c)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)**4*b)**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*x + a**2*b*tan(c + d*x)**3/d - 3*a**2*b*tan(c
+ d*x)/d + 3*a*b**2*x + 3*a*b**2*tan(c + d*x)**7/(7*d) - 3*a*b**2*tan(c + d
*x)**5/(5*d) + a*b**2*tan(c + d*x)**3/d - 3*a*b**2*tan(c + d*x)/d + b**3*x
+ b**3*tan(c + d*x)**11/(11*d) - b**3*tan(c + d*x)**9/(9*d) + b**3*tan(c +
d*x)**7/(7*d) - b**3*tan(c + d*x)**5/(5*d) + b**3*tan(c + d*x)**3/(3*d) - b
**3*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c)**4)**3, True))
```

Giac [B] time = 34.4993, size = 4724, normalized size = 32.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^3,x, algorithm="giac")
```

```
[Out] 1/3465*(3465*a^3*d*x*tan(d*x)^11*tan(c)^11 + 10395*a^2*b*d*x*tan(d*x)^11*ta
n(c)^11 + 10395*a*b^2*d*x*tan(d*x)^11*tan(c)^11 + 3465*b^3*d*x*tan(d*x)^11*
tan(c)^11 - 38115*a^3*d*x*tan(d*x)^10*tan(c)^10 - 114345*a^2*b*d*x*tan(d*x)
^10*tan(c)^10 - 114345*a*b^2*d*x*tan(d*x)^10*tan(c)^10 - 38115*b^3*d*x*tan(
d*x)^10*tan(c)^10 + 10395*a^2*b*tan(d*x)^11*tan(c)^10 + 10395*a*b^2*tan(d*x
)^11*tan(c)^10 + 3465*b^3*tan(d*x)^11*tan(c)^10 + 10395*a^2*b*tan(d*x)^10*t
an(c)^11 + 10395*a*b^2*tan(d*x)^10*tan(c)^11 + 3465*b^3*tan(d*x)^10*tan(c)^
```

```
11 + 190575*a^3*d*x*tan(d*x)^9*tan(c)^9 + 571725*a^2*b*d*x*tan(d*x)^9*tan(c
)^9 + 571725*a*b^2*d*x*tan(d*x)^9*tan(c)^9 + 190575*b^3*d*x*tan(d*x)^9*tan(
c)^9 - 3465*a^2*b*tan(d*x)^11*tan(c)^8 - 3465*a*b^2*tan(d*x)^11*tan(c)^8 -
1155*b^3*tan(d*x)^11*tan(c)^8 - 114345*a^2*b*tan(d*x)^10*tan(c)^9 - 114345*
a*b^2*tan(d*x)^10*tan(c)^9 - 38115*b^3*tan(d*x)^10*tan(c)^9 - 114345*a^2*b*
tan(d*x)^9*tan(c)^10 - 114345*a*b^2*tan(d*x)^9*tan(c)^10 - 38115*b^3*tan(d*
x)^9*tan(c)^10 - 3465*a^2*b*tan(d*x)^8*tan(c)^11 - 3465*a*b^2*tan(d*x)^8*ta
n(c)^11 - 1155*b^3*tan(d*x)^8*tan(c)^11 - 571725*a^3*d*x*tan(d*x)^8*tan(c)^
8 - 1715175*a^2*b*d*x*tan(d*x)^8*tan(c)^8 - 1715175*a*b^2*d*x*tan(d*x)^8*ta
n(c)^8 - 571725*b^3*d*x*tan(d*x)^8*tan(c)^8 + 2079*a*b^2*tan(d*x)^11*tan(c)
^6 + 693*b^3*tan(d*x)^11*tan(c)^6 + 27720*a^2*b*tan(d*x)^10*tan(c)^7 + 3811
5*a*b^2*tan(d*x)^10*tan(c)^7 + 12705*b^3*tan(d*x)^10*tan(c)^7 + 550935*a^2*
b*tan(d*x)^9*tan(c)^8 + 571725*a*b^2*tan(d*x)^9*tan(c)^8 + 190575*b^3*tan(d
*x)^9*tan(c)^8 + 550935*a^2*b*tan(d*x)^8*tan(c)^9 + 571725*a*b^2*tan(d*x)^8
*tan(c)^9 + 190575*b^3*tan(d*x)^8*tan(c)^9 + 27720*a^2*b*tan(d*x)^7*tan(c)^
10 + 38115*a*b^2*tan(d*x)^7*tan(c)^10 + 12705*b^3*tan(d*x)^7*tan(c)^10 + 20
79*a*b^2*tan(d*x)^6*tan(c)^11 + 693*b^3*tan(d*x)^6*tan(c)^11 + 1143450*a^3*
d*x*tan(d*x)^7*tan(c)^7 + 3430350*a^2*b*d*x*tan(d*x)^7*tan(c)^7 + 3430350*a
*b^2*d*x*tan(d*x)^7*tan(c)^7 + 1143450*b^3*d*x*tan(d*x)^7*tan(c)^7 - 1485*a
*b^2*tan(d*x)^11*tan(c)^4 - 495*b^3*tan(d*x)^11*tan(c)^4 - 22869*a*b^2*tan(
d*x)^10*tan(c)^5 - 7623*b^3*tan(d*x)^10*tan(c)^5 - 97020*a^2*b*tan(d*x)^9*t
an(c)^6 - 190575*a*b^2*tan(d*x)^9*tan(c)^6 - 63525*b^3*tan(d*x)^9*tan(c)^6
 - 1538460*a^2*b*tan(d*x)^8*tan(c)^7 - 1715175*a*b^2*tan(d*x)^8*tan(c)^7 - 5
71725*b^3*tan(d*x)^8*tan(c)^7 - 1538460*a^2*b*tan(d*x)^7*tan(c)^8 - 1715175
*a*b^2*tan(d*x)^7*tan(c)^8 - 571725*b^3*tan(d*x)^7*tan(c)^8 - 97020*a^2*b*t
an(d*x)^6*tan(c)^9 - 190575*a*b^2*tan(d*x)^6*tan(c)^9 - 63525*b^3*tan(d*x)^
6*tan(c)^9 - 22869*a*b^2*tan(d*x)^5*tan(c)^10 - 7623*b^3*tan(d*x)^5*tan(c)^
10 - 1485*a*b^2*tan(d*x)^4*tan(c)^11 - 495*b^3*tan(d*x)^4*tan(c)^11 - 16008
30*a^3*d*x*tan(d*x)^6*tan(c)^6 - 4802490*a^2*b*d*x*tan(d*x)^6*tan(c)^6 - 48
02490*a*b^2*d*x*tan(d*x)^6*tan(c)^6 - 1600830*b^3*d*x*tan(d*x)^6*tan(c)^6 +
 385*b^3*tan(d*x)^11*tan(c)^2 + 5940*a*b^2*tan(d*x)^10*tan(c)^3 + 5445*b^3*
tan(d*x)^10*tan(c)^3 + 72765*a*b^2*tan(d*x)^9*tan(c)^4 + 38115*b^3*tan(d*x)
^9*tan(c)^4 + 194040*a^2*b*tan(d*x)^8*tan(c)^5 + 474705*a*b^2*tan(d*x)^8*ta
n(c)^5 + 190575*b^3*tan(d*x)^8*tan(c)^5 + 2765070*a^2*b*tan(d*x)^7*tan(c)^6
 + 3284820*a*b^2*tan(d*x)^7*tan(c)^6 + 1143450*b^3*tan(d*x)^7*tan(c)^6 + 27
65070*a^2*b*tan(d*x)^6*tan(c)^7 + 3284820*a*b^2*tan(d*x)^6*tan(c)^7 + 11434
50*b^3*tan(d*x)^6*tan(c)^7 + 194040*a^2*b*tan(d*x)^5*tan(c)^8 + 474705*a*b^
2*tan(d*x)^5*tan(c)^8 + 190575*b^3*tan(d*x)^5*tan(c)^8 + 72765*a*b^2*tan(d*
x)^4*tan(c)^9 + 38115*b^3*tan(d*x)^4*tan(c)^9 + 5940*a*b^2*tan(d*x)^3*tan(c
)^10 + 5445*b^3*tan(d*x)^3*tan(c)^10 + 385*b^3*tan(d*x)^2*tan(c)^11 + 16008
30*a^3*d*x*tan(d*x)^5*tan(c)^5 + 4802490*a^2*b*d*x*tan(d*x)^5*tan(c)^5 + 48
02490*a*b^2*d*x*tan(d*x)^5*tan(c)^5 + 1600830*b^3*d*x*tan(d*x)^5*tan(c)^5 -
 315*b^3*tan(d*x)^11 - 4235*b^3*tan(d*x)^10*tan(c) - 8910*a*b^2*tan(d*x)^9*
tan(c)^2 - 27225*b^3*tan(d*x)^9*tan(c)^2 - 103950*a*b^2*tan(d*x)^8*tan(c)^3
 - 114345*b^3*tan(d*x)^8*tan(c)^3 - 242550*a^2*b*tan(d*x)^7*tan(c)^4 - 6375
60*a*b^2*tan(d*x)^7*tan(c)^4 - 381150*b^3*tan(d*x)^7*tan(c)^4 - 3347190*a^2
*b*tan(d*x)^6*tan(c)^5 - 4074840*a*b^2*tan(d*x)^6*tan(c)^5 - 1600830*b^3*ta
n(d*x)^6*tan(c)^5 - 3347190*a^2*b*tan(d*x)^5*tan(c)^6 - 4074840*a*b^2*tan(d
*x)^5*tan(c)^6 - 1600830*b^3*tan(d*x)^5*tan(c)^6 - 242550*a^2*b*tan(d*x)^4*
tan(c)^7 - 637560*a*b^2*tan(d*x)^4*tan(c)^7 - 381150*b^3*tan(d*x)^4*tan(c)^
7 - 103950*a*b^2*tan(d*x)^3*tan(c)^8 - 114345*b^3*tan(d*x)^3*tan(c)^8 - 891
0*a*b^2*tan(d*x)^2*tan(c)^9 - 27225*b^3*tan(d*x)^2*tan(c)^9 - 4235*b^3*tan(
d*x)*tan(c)^10 - 315*b^3*tan(c)^11 - 1143450*a^3*d*x*tan(d*x)^4*tan(c)^4 -
3430350*a^2*b*d*x*tan(d*x)^4*tan(c)^4 - 3430350*a*b^2*d*x*tan(d*x)^4*tan(c)
^4 - 1143450*b^3*d*x*tan(d*x)^4*tan(c)^4 + 385*b^3*tan(d*x)^9 + 5940*a*b^2*
tan(d*x)^8*tan(c) + 5445*b^3*tan(d*x)^8*tan(c) + 72765*a*b^2*tan(d*x)^7*tan
(c)^2 + 38115*b^3*tan(d*x)^7*tan(c)^2 + 194040*a^2*b*tan(d*x)^6*tan(c)^3 +
474705*a*b^2*tan(d*x)^6*tan(c)^3 + 190575*b^3*tan(d*x)^6*tan(c)^3 + 2765070
*a^2*b*tan(d*x)^5*tan(c)^4 + 3284820*a*b^2*tan(d*x)^5*tan(c)^4 + 1143450*b^
```

3*tan(d*x)^5*tan(c)^4 + 2765070*a^2*b*tan(d*x)^4*tan(c)^5 + 3284820*a*b^2*t
an(d*x)^4*tan(c)^5 + 1143450*b^3*tan(d*x)^4*tan(c)^5 + 194040*a^2*b*tan(d*x
)^3*tan(c)^6 + 474705*a*b^2*tan(d*x)^3*tan(c)^6 + 190575*b^3*tan(d*x)^3*tan
(c)^6 + 72765*a*b^2*tan(d*x)^2*tan(c)^7 + 38115*b^3*tan(d*x)^2*tan(c)^7 + 5
940*a*b^2*tan(d*x)*tan(c)^8 + 5445*b^3*tan(d*x)*tan(c)^8 + 385*b^3*tan(c)^9
 + 571725*a^3*d*x*tan(d*x)^3*tan(c)^3 + 1715175*a^2*b*d*x*tan(d*x)^3*tan(c)
^3 + 1715175*a*b^2*d*x*tan(d*x)^3*tan(c)^3 + 571725*b^3*d*x*tan(d*x)^3*tan(
c)^3 - 1485*a*b^2*tan(d*x)^7 - 495*b^3*tan(d*x)^7 - 22869*a*b^2*tan(d*x)^6*
tan(c) - 7623*b^3*tan(d*x)^6*tan(c) - 97020*a^2*b*tan(d*x)^5*tan(c)^2 - 190
575*a*b^2*tan(d*x)^5*tan(c)^2 - 63525*b^3*tan(d*x)^5*tan(c)^2 - 1538460*a^2
*b*tan(d*x)^4*tan(c)^3 - 1715175*a*b^2*tan(d*x)^4*tan(c)^3 - 571725*b^3*tan
(d*x)^4*tan(c)^3 - 1538460*a^2*b*tan(d*x)^3*tan(c)^4 - 1715175*a*b^2*tan(d*
x)^3*tan(c)^4 - 571725*b^3*tan(d*x)^3*tan(c)^4 - 97020*a^2*b*tan(d*x)^2*tan
(c)^5 - 190575*a*b^2*tan(d*x)^2*tan(c)^5 - 63525*b^3*tan(d*x)^2*tan(c)^5 -
22869*a*b^2*tan(d*x)*tan(c)^6 - 7623*b^3*tan(d*x)*tan(c)^6 - 1485*a*b^2*tan
(c)^7 - 495*b^3*tan(c)^7 - 190575*a^3*d*x*tan(d*x)^2*tan(c)^2 - 571725*a^2*
b*d*x*tan(d*x)^2*tan(c)^2 - 571725*a*b^2*d*x*tan(d*x)^2*tan(c)^2 - 190575*b
^3*d*x*tan(d*x)^2*tan(c)^2 + 2079*a*b^2*tan(d*x)^5 + 693*b^3*tan(d*x)^5 + 2
7720*a^2*b*tan(d*x)^4*tan(c) + 38115*a*b^2*tan(d*x)^4*tan(c) + 12705*b^3*ta
n(d*x)^4*tan(c) + 550935*a^2*b*tan(d*x)^3*tan(c)^2 + 571725*a*b^2*tan(d*x)^
3*tan(c)^2 + 190575*b^3*tan(d*x)^3*tan(c)^2 + 550935*a^2*b*tan(d*x)^2*tan(c
)^3 + 571725*a*b^2*tan(d*x)^2*tan(c)^3 + 190575*b^3*tan(d*x)^2*tan(c)^3 + 2
7720*a^2*b*tan(d*x)*tan(c)^4 + 38115*a*b^2*tan(d*x)*tan(c)^4 + 12705*b^3*ta
n(d*x)*tan(c)^4 + 2079*a*b^2*tan(c)^5 + 693*b^3*tan(c)^5 + 38115*a^3*d*x*ta
n(d*x)*tan(c) + 114345*a^2*b*d*x*tan(d*x)*tan(c) + 114345*a*b^2*d*x*tan(d*x
)*tan(c) + 38115*b^3*d*x*tan(d*x)*tan(c) - 3465*a^2*b*tan(d*x)^3 - 3465*a*b
^2*tan(d*x)^3 - 1155*b^3*tan(d*x)^3 - 114345*a^2*b*tan(d*x)^2*tan(c) - 1143
45*a*b^2*tan(d*x)^2*tan(c) - 38115*b^3*tan(d*x)^2*tan(c) - 114345*a^2*b*tan
(d*x)*tan(c)^2 - 114345*a*b^2*tan(d*x)*tan(c)^2 - 38115*b^3*tan(d*x)*tan(c)
^2 - 3465*a^2*b*tan(c)^3 - 3465*a*b^2*tan(c)^3 - 1155*b^3*tan(c)^3 - 3465*a
^3*d*x - 10395*a^2*b*d*x - 10395*a*b^2*d*x - 3465*b^3*d*x + 10395*a^2*b*tan
(d*x) + 10395*a*b^2*tan(d*x) + 3465*b^3*tan(d*x) + 10395*a^2*b*tan(c) + 103
95*a*b^2*tan(c) + 3465*b^3*tan(c))/(d*tan(d*x)^11*tan(c)^11 - 11*d*tan(d*x)
^10*tan(c)^10 + 55*d*tan(d*x)^9*tan(c)^9 - 165*d*tan(d*x)^8*tan(c)^8 + 330*
d*tan(d*x)^7*tan(c)^7 - 462*d*tan(d*x)^6*tan(c)^6 + 462*d*tan(d*x)^5*tan(c)
^5 - 330*d*tan(d*x)^4*tan(c)^4 + 165*d*tan(d*x)^3*tan(c)^3 - 55*d*tan(d*x)^
2*tan(c)^2 + 11*d*tan(d*x)*tan(c) - d)

3.383 $\int \left(a+b\tan^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=82

$$\frac{b(2a+b)\tan^3(c+dx)}{3d}-\frac{b(2a+b)\tan(c+dx)}{d}+x(a+b)^2+\frac{b^2\tan^7(c+dx)}{7d}-\frac{b^2\tan^5(c+dx)}{5d}$$

```
[Out] (a + b)^2*x - (b*(2*a + b)*Tan[c + d*x])/d + (b*(2*a + b)*Tan[c + d*x]^3)/(
3*d) - (b^2*Tan[c + d*x]^5)/(5*d) + (b^2*Tan[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0547545, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3661, 1154, 203}

$$\frac{b(2a+b)\tan^3(c+dx)}{3d}-\frac{b(2a+b)\tan(c+dx)}{d}+x(a+b)^2+\frac{b^2\tan^7(c+dx)}{7d}-\frac{b^2\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^4)^2,x]
```

```
[Out] (a + b)^2*x - (b*(2*a + b)*Tan[c + d*x])/d + (b*(2*a + b)*Tan[c + d*x]^3)/(
3*d) - (b^2*Tan[c + d*x]^5)/(5*d) + (b^2*Tan[c + d*x]^7)/(7*d)
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\tan^4(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^4\right)^2}{1+x^2} dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-b(2a+b)+b(2a+b)x^2-b^2x^4+b^2x^6+\frac{(a+b)^2}{1+x^2}\right) dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{b(2a+b)\tan(c+dx)}{d}+\frac{b(2a+b)\tan^3(c+dx)}{3d}-\frac{b^2\tan^5(c+dx)}{5d}+\frac{b^2\tan^7(c+dx)}{7d}+ \ldots$$
$$= (a+b)^2x-\frac{b(2a+b)\tan(c+dx)}{d}+\frac{b(2a+b)\tan^3(c+dx)}{3d}-\frac{b^2\tan^5(c+dx)}{5d}+\frac{b^2\tan^7(c \ldots}{7d}$$

Mathematica [A] time = 0.526095, size = 75, normalized size = 0.91

$$\frac{105(a+b)^2\tan^{-1}(\tan(c+dx))+b\tan(c+dx)\left(35(2a+b)\tan^2(c+dx)-105(2a+b)+15b\tan^6(c+dx)-21b\tan^4(c+ \ldots\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^4)^2,x]
```

```
[Out] (105*(a + b)^2*ArcTan[Tan[c + d*x]] + b*Tan[c + d*x]*(-105*(2*a + b) + 35*(
2*a + b)*Tan[c + d*x]^2 - 21*b*Tan[c + d*x]^4 + 15*b*Tan[c + d*x]^6))/(105*
d)
```

Maple [A] time = 0.006, size = 134, normalized size = 1.6

$$\frac{b^2(\tan(dx+c))^7}{7d}-\frac{b^2(\tan(dx+c))^5}{5d}+\frac{2ab(\tan(dx+c))^3}{3d}+\frac{b^2(\tan(dx+c))^3}{3d}-2\frac{a\tan(dx+c)b}{d}-\frac{b^2\tan(dx+c)}{d} \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^4)^2,x)
```

```
[Out] 1/7*b^2*tan(d*x+c)^7/d-1/5*b^2*tan(d*x+c)^5/d+2/3*a*b*tan(d*x+c)^3/d+1/3*b^
2*tan(d*x+c)^3/d-2/d*tan(d*x+c)*a*b-b^2*tan(d*x+c)/d+1/d*arctan(tan(d*x+c))
*a^2+2/d*arctan(tan(d*x+c))*a*b+1/d*arctan(tan(d*x+c))*b^2
```

Maxima [A] time = 1.57343, size = 123, normalized size = 1.5

$$a^2x+\frac{2\left(\tan(dx+c)^3+3dx+3c-3\tan(dx+c)\right)ab}{3d}+\frac{\left(15\tan(dx+c)^7-21\tan(dx+c)^5+35\tan(dx+c)^3+105 \ldots\right.}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 2/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a*b/d + 1/105*(
15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x + 105*c
 - 105*tan(d*x + c))*b^2/d
```

Fricas [A] time = 1.43597, size = 208, normalized size = 2.54

$$\frac{15\,b^2\tan(dx+c)^7 - 21\,b^2\tan(dx+c)^5 + 35\left(2\,ab+b^2\right)\tan(dx+c)^3 + 105\left(a^2+2\,ab+b^2\right)dx - 105\left(2\,ab+b^2\right)\tan(dx+c)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*tan(d*x + c)^7 - 21*b^2*tan(d*x + c)^5 + 35*(2*a*b + b^2)*tan
(d*x + c)^3 + 105*(a^2 + 2*a*b + b^2)*d*x - 105*(2*a*b + b^2)*tan(d*x + c))
/d
```

Sympy [A] time = 1.35195, size = 116, normalized size = 1.41

$$\begin{cases} a^2x + 2abx + \frac{2ab\tan^3(c+dx)}{3d} - \frac{2ab\tan(c+dx)}{d} + b^2x + \frac{b^2\tan^7(c+dx)}{7d} - \frac{b^2\tan^5(c+dx)}{5d} + \frac{b^2\tan^3(c+dx)}{3d} - \frac{b^2\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a+b\tan^4(c)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)**4*b)**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x + 2*a*b*tan(c + d*x)**3/(3*d) - 2*a*b*tan(c + d
*x)/d + b**2*x + b**2*tan(c + d*x)**7/(7*d) - b**2*tan(c + d*x)**5/(5*d) +
b**2*tan(c + d*x)**3/(3*d) - b**2*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(
c)**4)**2, True))
```

Giac [B] time = 6.44607, size = 1594, normalized size = 19.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^2,x, algorithm="giac")
```

```
[Out] 1/105*(105*a^2*d*x*tan(d*x)^7*tan(c)^7 + 210*a*b*d*x*tan(d*x)^7*tan(c)^7 +
105*b^2*d*x*tan(d*x)^7*tan(c)^7 - 735*a^2*d*x*tan(d*x)^6*tan(c)^6 - 1470*a*
b*d*x*tan(d*x)^6*tan(c)^6 - 735*b^2*d*x*tan(d*x)^6*tan(c)^6 + 210*a*b*tan(d
*x)^7*tan(c)^6 + 105*b^2*tan(d*x)^7*tan(c)^6 + 210*a*b*tan(d*x)^6*tan(c)^7
+ 105*b^2*tan(d*x)^6*tan(c)^7 + 2205*a^2*d*x*tan(d*x)^5*tan(c)^5 + 4410*a*b
*d*x*tan(d*x)^5*tan(c)^5 + 2205*b^2*d*x*tan(d*x)^5*tan(c)^5 - 70*a*b*tan(d*
x)^7*tan(c)^4 - 35*b^2*tan(d*x)^7*tan(c)^4 - 1470*a*b*tan(d*x)^6*tan(c)^5 -
 735*b^2*tan(d*x)^6*tan(c)^5 - 1470*a*b*tan(d*x)^5*tan(c)^6 - 735*b^2*tan(d
*x)^5*tan(c)^6 - 70*a*b*tan(d*x)^4*tan(c)^7 - 35*b^2*tan(d*x)^4*tan(c)^7 -
3675*a^2*d*x*tan(d*x)^4*tan(c)^4 - 7350*a*b*d*x*tan(d*x)^4*tan(c)^4 - 3675*
b^2*d*x*tan(d*x)^4*tan(c)^4 + 21*b^2*tan(d*x)^7*tan(c)^2 + 280*a*b*tan(d*x)
^6*tan(c)^3 + 245*b^2*tan(d*x)^6*tan(c)^3 + 3990*a*b*tan(d*x)^5*tan(c)^4 +
2205*b^2*tan(d*x)^5*tan(c)^4 + 3990*a*b*tan(d*x)^4*tan(c)^5 + 2205*b^2*tan(
d*x)^4*tan(c)^5 + 280*a*b*tan(d*x)^3*tan(c)^6 + 245*b^2*tan(d*x)^3*tan(c)^6
 + 21*b^2*tan(d*x)^2*tan(c)^7 + 3675*a^2*d*x*tan(d*x)^3*tan(c)^3 + 7350*a*b
*d*x*tan(d*x)^3*tan(c)^3 + 3675*b^2*d*x*tan(d*x)^3*tan(c)^3 - 15*b^2*tan(d*
x)^7 - 147*b^2*tan(d*x)^6*tan(c) - 420*a*b*tan(d*x)^5*tan(c)^2 - 735*b^2*ta
```

```
n(d*x)^5*tan(c)^2 - 5460*a*b*tan(d*x)^4*tan(c)^3 - 3675*b^2*tan(d*x)^4*tan(
c)^3 - 5460*a*b*tan(d*x)^3*tan(c)^4 - 3675*b^2*tan(d*x)^3*tan(c)^4 - 420*a*
b*tan(d*x)^2*tan(c)^5 - 735*b^2*tan(d*x)^2*tan(c)^5 - 147*b^2*tan(d*x)*tan(
c)^6 - 15*b^2*tan(c)^7 - 2205*a^2*d*x*tan(d*x)^2*tan(c)^2 - 4410*a*b*d*x*ta
n(d*x)^2*tan(c)^2 - 2205*b^2*d*x*tan(d*x)^2*tan(c)^2 + 21*b^2*tan(d*x)^5 +
280*a*b*tan(d*x)^4*tan(c) + 245*b^2*tan(d*x)^4*tan(c) + 3990*a*b*tan(d*x)^3
*tan(c)^2 + 2205*b^2*tan(d*x)^3*tan(c)^2 + 3990*a*b*tan(d*x)^2*tan(c)^3 + 2
205*b^2*tan(d*x)^2*tan(c)^3 + 280*a*b*tan(d*x)*tan(c)^4 + 245*b^2*tan(d*x)*
tan(c)^4 + 21*b^2*tan(c)^5 + 735*a^2*d*x*tan(d*x)*tan(c) + 1470*a*b*d*x*tan
(d*x)*tan(c) + 735*b^2*d*x*tan(d*x)*tan(c) - 70*a*b*tan(d*x)^3 - 35*b^2*tan
(d*x)^3 - 1470*a*b*tan(d*x)^2*tan(c) - 735*b^2*tan(d*x)^2*tan(c) - 1470*a*b
*tan(d*x)*tan(c)^2 - 735*b^2*tan(d*x)*tan(c)^2 - 70*a*b*tan(c)^3 - 35*b^2*t
an(c)^3 - 105*a^2*d*x - 210*a*b*d*x - 105*b^2*d*x + 210*a*b*tan(d*x) + 105*
b^2*tan(d*x) + 210*a*b*tan(c) + 105*b^2*tan(c))/(d*tan(d*x)^7*tan(c)^7 - 7*
d*tan(d*x)^6*tan(c)^6 + 21*d*tan(d*x)^5*tan(c)^5 - 35*d*tan(d*x)^4*tan(c)^4
 + 35*d*tan(d*x)^3*tan(c)^3 - 21*d*tan(d*x)^2*tan(c)^2 + 7*d*tan(d*x)*tan(c
) - d)
```

3.384 $\int \left(a+b\tan^4(c+dx)\right)\,dx$

Optimal. Leaf size=35

$$ax+\frac{b\tan^3(c+dx)}{3d}-\frac{b\tan(c+dx)}{d}+bx$$

```
[Out] a*x + b*x - (b*Tan[c + d*x])/d + (b*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0251603, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3473, 8}

$$ax+\frac{b\tan^3(c+dx)}{3d}-\frac{b\tan(c+dx)}{d}+bx$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Tan[c + d*x]^4,x]
```

```
[Out] a*x + b*x - (b*Tan[c + d*x])/d + (b*Tan[c + d*x]^3)/(3*d)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tan^4(c+dx)\right)\,dx &= ax+b\int \tan^4(c+dx)\,dx\\ &= ax+\frac{b\tan^3(c+dx)}{3d}-b\int \tan^2(c+dx)\,dx\\ &= ax-\frac{b\tan(c+dx)}{d}+\frac{b\tan^3(c+dx)}{3d}+b\int 1\,dx\\ &= ax+bx-\frac{b\tan(c+dx)}{d}+\frac{b\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0263659, size = 44, normalized size = 1.26

$$ax+\frac{b\tan^3(c+dx)}{3d}+\frac{b\tan^{-1}(\tan(c+dx))}{d}-\frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Tan[c + d*x]^4,x]
```

```
[Out] a*x + (b*ArcTan[Tan[c + d*x]])/d - (b*Tan[c + d*x])/d + (b*Tan[c + d*x]^3)/
(3*d)
```

Maple [A] time = 0.006, size = 43, normalized size = 1.2

$$ax+\frac{b\left(\tan\left(dx+c\right)\right)^{3}}{3d}-\frac{b\tan\left(dx+c\right)}{d}+\frac{b\arctan\left(\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*tan(d*x+c)^4,x)
```

```
[Out] a*x+1/3*b*tan(d*x+c)^3/d-b*tan(d*x+c)/d+b/d*arctan(tan(d*x+c))
```

Maxima [A] time = 1.49651, size = 46, normalized size = 1.31

$$ax+\frac{\left(\tan\left(dx+c\right)^{3}+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)b}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+tan(d*x+c)^4*b,x, algorithm="maxima")
```

```
[Out] a*x + 1/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*b/d
```

Fricas [A] time = 1.39265, size = 82, normalized size = 2.34

$$\frac{b\tan\left(dx+c\right)^{3}+3\left(a+b\right)dx-3\,b\tan\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+tan(d*x+c)^4*b,x, algorithm="fricas")
```

```
[Out] 1/3*(b*tan(d*x + c)^3 + 3*(a + b)*d*x - 3*b*tan(d*x + c))/d
```

Sympy [A] time = 0.261537, size = 32, normalized size = 0.91

$$ax+b\left(\begin{cases}x+\frac{\tan^{3}\left(c+dx\right)}{3d}-\frac{\tan\left(c+dx\right)}{d} & \text{for}\ d\neq 0\\ x\tan^{4}\left(c\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+tan(d*x+c)**4*b,x)
```

```
[Out] a*x + b*Piecewise((x + tan(c + d*x)**3/(3*d) - tan(c + d*x)/d, Ne(d, 0)), (
x*tan(c)**4, True))
```

Giac [B] time = 2.25889, size = 797, normalized size = 22.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+tan(d*x+c)^4*b,x, algorithm="giac")
```

```
[Out] a*x + 1/12*(3*pi + 12*d*x*tan(d*x)^3*tan(c)^3 - 3*pi*sgn(2*tan(d*x)^2*tan(c
) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*tan(c)^3 - 3*pi
*tan(d*x)^3*tan(c)^3 + 6*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*
tan(d*x)^3*tan(c)^3 + 6*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*t
an(d*x)^3*tan(c)^3 - 36*d*x*tan(d*x)^2*tan(c)^2 + 9*pi*sgn(2*tan(d*x)^2*tan
(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 9*
pi*tan(d*x)^2*tan(c)^2 - 18*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)
))*tan(d*x)^2*tan(c)^2 - 18*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1
))*tan(d*x)^2*tan(c)^2 + 12*tan(d*x)^3*tan(c)^2 + 12*tan(d*x)^2*tan(c)^3 +
36*d*x*tan(d*x)*tan(c) - 9*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2
 - 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c) - 4*tan(d*x)^3 - 9*pi*tan(d*x)*ta
n(c) + 18*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c)
 + 18*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)*tan(c) - 3
6*tan(d*x)^2*tan(c) - 36*tan(d*x)*tan(c)^2 - 4*tan(c)^3 - 12*d*x + 3*pi*sgn
(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c)) - 6*arc
tan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c))) - 6*arctan((tan(d*x) + tan(c
))/(tan(d*x)*tan(c) - 1)) + 12*tan(d*x) + 12*tan(c))*b/(d*tan(d*x)^3*tan(c)
^3 - 3*d*tan(d*x)^2*tan(c)^2 + 3*d*tan(d*x)*tan(c) - d)
```

3.385 $\int \frac{1}{a+b\tan^4(c+dx)}\,dx$

Optimal. Leaf size=302

$$\frac{\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}d(a+b)}-\frac{\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}d(a+b)}-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,\mathrm{ta}\right.}{4\sqrt{2}a^{3/}}$$

```
[Out] x/(a + b) + ((Sqrt[a] - Sqrt[b])*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Tan[c
+ d*x])/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b)*d) - ((Sqrt[a] - Sqrt[b])*b^(1
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
(a + b)*d) - ((Sqrt[a] + Sqrt[b])*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/(4*Sqrt[2]*a^(3/4)*(a + b)*d)
+ ((Sqrt[a] + Sqrt[b])*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c
+ d*x] + Sqrt[b]*Tan[c + d*x]^2])/(4*Sqrt[2]*a^(3/4)*(a + b)*d)
```

Rubi [A] time = 0.321428, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {3661, 1171, 203, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}d(a+b)}-\frac{\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}d(a+b)}-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,\mathrm{ta}\right.}{4\sqrt{2}a^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^4)^(-1),x]
```

```
[Out] x/(a + b) + ((Sqrt[a] - Sqrt[b])*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Tan[c
+ d*x])/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b)*d) - ((Sqrt[a] - Sqrt[b])*b^(1
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
(a + b)*d) - ((Sqrt[a] + Sqrt[b])*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/(4*Sqrt[2]*a^(3/4)*(a + b)*d)
+ ((Sqrt[a] + Sqrt[b])*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c
+ d*x] + Sqrt[b]*Tan[c + d*x]^2])/(4*Sqrt[2]*a^(3/4)*(a + b)*d)
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{a+b\tan^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^4)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)(1+x^2)} + \frac{b-bx^2}{(a+b)(a+bx^4)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{(a+b)d} + \frac{\text{Subst}\left(\int \frac{b-bx^2}{a+bx^4}\,dx, x, \tan(c+dx)\right)}{(a+b)d}$$

$$= \frac{x}{a+b} - \frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx, x, \tan(c+dx)\right)}{2(a+b)d} + \frac{\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx, x,\right.}{2(a+b)d}$$

$$= \frac{x}{a+b} - \frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \tan(c+dx)\right)}{4(a+b)d} - \frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\right.}{4(a+b)d}$$

$$= \frac{x}{a+b} - \frac{\left(\sqrt{a}+\sqrt{b}\right)\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\tan(c+dx)+\sqrt{b}\tan^2(c+dx)\right)}{4\sqrt{2}a^{3/4}(a+b)d} + \frac{\left(\sqrt{a}+\sqrt{b}\right)\sqrt[4]{b}}{}$$

$$= \frac{x}{a+b} + \frac{\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(a+b)d} - \frac{\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(a+b)d}$$

Mathematica [A] time = 0.540501, size = 228, normalized size = 0.75

$$\frac{8a^{3/4}\tan^{-1}(\tan(c+dx)) + \sqrt{2}\sqrt[4]{b}\left(2\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right) - 2\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}+1\right) - (\sqrt{\right.}{8a^{3/4}d(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^4)^(-1),x]
```

```
[Out] (8*a^(3/4)*ArcTan[Tan[c + d*x]] + Sqrt[2]*b^(1/4)*(2*(Sqrt[a] - Sqrt[b])*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)] - 2*(Sqrt[a] - Sqrt[b])*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)] - (Sqrt[a] + Sqrt[b])*(Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2] -
 Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^
2])))/(8*a^(3/4)*(a + b)*d)
```

Maple [A] time = 0.031, size = 374, normalized size = 1.2

$$\frac{\arctan(\tan(dx+c))}{d(a+b)} + \frac{b\sqrt{2}}{8d(a+b)a}\sqrt[4]{\frac{a}{b}}\ln\left(\left((\tan(dx+c))^2 + \sqrt[4]{\frac{a}{b}}\tan(dx+c)\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left((\tan(dx+c))^2 - \sqrt[4]{\frac{a}{b}}\tan(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^4),x)
```

```
[Out] 1/d/(a+b)*arctan(tan(d*x+c))+1/8/d*b/(a+b)*(a/b)^(1/4)/a*2^(1/2)*ln((tan(d*
x+c)^2+(a/b)^(1/4)*tan(d*x+c)*2^(1/2)+(a/b)^(1/2))/(tan(d*x+c)^2-(a/b)^(1/4
```

```
)*tan(d*x+c)*2^(1/2)+(a/b)^(1/2)))+1/4/d*b/(a+b)*(a/b)^(1/4)/a*2^(1/2)*arct
an(2^(1/2)/(a/b)^(1/4)*tan(d*x+c)+1)-1/4/d*b/(a+b)*(a/b)^(1/4)/a*2^(1/2)*ar
ctan(-2^(1/2)/(a/b)^(1/4)*tan(d*x+c)+1)-1/8/d/(a+b)/(a/b)^(1/4)*2^(1/2)*ln(
(tan(d*x+c)^2-(a/b)^(1/4)*tan(d*x+c)*2^(1/2)+(a/b)^(1/2))/(tan(d*x+c)^2+(a/
b)^(1/4)*tan(d*x+c)*2^(1/2)+(a/b)^(1/2)))-1/4/d/(a+b)/(a/b)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a/b)^(1/4)*tan(d*x+c)+1)+1/4/d/(a+b)/(a/b)^(1/4)*2^(1/2)*arc
tan(-2^(1/2)/(a/b)^(1/4)*tan(d*x+c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.99136, size = 3252, normalized size = 10.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b),x, algorithm="fricas")
```

```
[Out] 1/8*((a + b)*sqrt(((a^3 + 2*a^2*b + a*b^2)*d^2*sqrt(-(a^2*b - 2*a*b^2 + b^3
)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)) + 2*b)/((a^3 + 2
*a^2*b + a*b^2)*d^2))*log((2*(a^3 - a*b^2)*d*sqrt(((a^3 + 2*a^2*b + a*b^2)*
d^2*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 +
 a^3*b^4)*d^4)) + 2*b)/((a^3 + 2*a^2*b + a*b^2)*d^2))*tan(d*x + c) + (a*b -
 b^2)*tan(d*x + c)^2 + a^2 - a*b + ((a^4 + 2*a^3*b + a^2*b^2)*d^2*tan(d*x +
 c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d^2)*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7
+ 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)))/(tan(d*x + c)^2 + 1)) -
 (a + b)*sqrt(((a^3 + 2*a^2*b + a*b^2)*d^2*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((
a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)) + 2*b)/((a^3 + 2*a^2
*b + a*b^2)*d^2))*log(-(2*(a^3 - a*b^2)*d*sqrt(((a^3 + 2*a^2*b + a*b^2)*d^2
*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^
3*b^4)*d^4)) + 2*b)/((a^3 + 2*a^2*b + a*b^2)*d^2))*tan(d*x + c) - (a*b - b^
2)*tan(d*x + c)^2 - a^2 + a*b - ((a^4 + 2*a^3*b + a^2*b^2)*d^2*tan(d*x + c)
^2 - (a^4 + 2*a^3*b + a^2*b^2)*d^2)*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4
*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)))/(tan(d*x + c)^2 + 1)) + (a
 + b)*sqrt(-((a^3 + 2*a^2*b + a*b^2)*d^2*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^
7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)) - 2*b)/((a^3 + 2*a^2*b
 + a*b^2)*d^2))*log(-(2*(a^3 - a*b^2)*d*sqrt(-((a^3 + 2*a^2*b + a*b^2)*d^2*
sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3
*b^4)*d^4)) - 2*b)/((a^3 + 2*a^2*b + a*b^2)*d^2))*tan(d*x + c) + (a*b - b^2
)*tan(d*x + c)^2 + a^2 - a*b - ((a^4 + 2*a^3*b + a^2*b^2)*d^2*tan(d*x + c)^
2 - (a^4 + 2*a^3*b + a^2*b^2)*d^2)*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4*
a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)))/(tan(d*x + c)^2 + 1)) - (a
+ b)*sqrt(-((a^3 + 2*a^2*b + a*b^2)*d^2*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7
 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)) - 2*b)/((a^3 + 2*a^2*b
+ a*b^2)*d^2))*log((2*(a^3 - a*b^2)*d*sqrt(-((a^3 + 2*a^2*b + a*b^2)*d^2*sq
rt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b
^4)*d^4)) - 2*b)/((a^3 + 2*a^2*b + a*b^2)*d^2))*tan(d*x + c) - (a*b - b^2)*
```

```
tan(d*x + c)^2 - a^2 + a*b + ((a^4 + 2*a^3*b + a^2*b^2)*d^2*tan(d*x + c)^2
- (a^4 + 2*a^3*b + a^2*b^2)*d^2)*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4*a^
6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)))/(tan(d*x + c)^2 + 1)) + 8*x)/
(a + b)
```

Sympy [A] time = 24.9238, size = 2759, normalized size = 9.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)**4*b),x)
```

```
[Out] Piecewise((zoo*x/tan(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x + 1/(d*tan
(c + d*x)) - 1/(3*d*tan(c + d*x)**3))/b, Eq(a, 0)), (-4*d*x*tan(c + d*x)**2
/(8*b*d*tan(c + d*x)**2 + 8*b*d) - 4*d*x/(8*b*d*tan(c + d*x)**2 + 8*b*d) +
log(tan(c + d*x) - 1)*tan(c + d*x)**2/(8*b*d*tan(c + d*x)**2 + 8*b*d) + log
(tan(c + d*x) - 1)/(8*b*d*tan(c + d*x)**2 + 8*b*d) - log(tan(c + d*x) + 1)*
tan(c + d*x)**2/(8*b*d*tan(c + d*x)**2 + 8*b*d) - log(tan(c + d*x) + 1)/(8*
b*d*tan(c + d*x)**2 + 8*b*d) - 2*tan(c + d*x)/(8*b*d*tan(c + d*x)**2 + 8*b*
d), Eq(a, -b)), (x/(a + b*tan(c)**4), Eq(d, 0)), (x/a, Eq(b, 0)), (-2*(-1)*
*(1/4)*a**(33/4)*b**4*(atan((-1)**(3/4)*tan(c + d*x)/(a**(1/4)*(1/b)**(1/4)
)) + pi*floor((c + d*x - pi/2)/pi)*sign((-1)**(3/4)/(a**(1/4)*(1/b)**(1/4))
))*(1/b)**(21/4)/(-4*I*a**(19/2)*b**2*d*(1/b)**(7/2) + 8*I*a**(17/2)*b**3*d
*(1/b)**(7/2) + 12*I*a**(15/2)*b**4*d*(1/b)**(7/2) - 12*a**9*d/b - 8*a**8*d
 + 4*a**7*b*d) - (-1)**(1/4)*a**(33/4)*b*(1/b)**(9/4)*log((-1)**(1/4)*a**(1
/4)*(1/b)**(1/4) + tan(c + d*x))/(-4*I*a**(19/2)*b**2*d*(1/b)**(7/2) + 8*I*
a**(17/2)*b**3*d*(1/b)**(7/2) + 12*I*a**(15/2)*b**4*d*(1/b)**(7/2) - 12*a**
9*d/b - 8*a**8*d + 4*a**7*b*d) + (-1)**(1/4)*a**(33/4)*(1/b)**(5/4)*log(-(-
1)**(1/4)*a**(1/4)*(1/b)**(1/4) + tan(c + d*x))/(-4*I*a**(19/2)*b**2*d*(1/b
)**(7/2) + 8*I*a**(17/2)*b**3*d*(1/b)**(7/2) + 12*I*a**(15/2)*b**4*d*(1/b)*
*(7/2) - 12*a**9*d/b - 8*a**8*d + 4*a**7*b*d) + 8*(-1)**(3/4)*a**(31/4)*b**
5*(atan((-1)**(3/4)*tan(c + d*x)/(a**(1/4)*(1/b)**(1/4))) + pi*floor((c + d
*x - pi/2)/pi)*sign((-1)**(3/4)/(a**(1/4)*(1/b)**(1/4))))*(1/b)**(23/4)/(-4
*I*a**(19/2)*b**2*d*(1/b)**(7/2) + 8*I*a**(17/2)*b**3*d*(1/b)**(7/2) + 12*I
*a**(15/2)*b**4*d*(1/b)**(7/2) - 12*a**9*d/b - 8*a**8*d + 4*a**7*b*d) + 2*(
-1)**(3/4)*a**(31/4)*b**2*(1/b)**(11/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/
4) + tan(c + d*x))/(-4*I*a**(19/2)*b**2*d*(1/b)**(7/2) + 8*I*a**(17/2)*b**3
*d*(1/b)**(7/2) + 12*I*a**(15/2)*b**4*d*(1/b)**(7/2) - 12*a**9*d/b - 8*a**8
*d + 4*a**7*b*d) - 2*(-1)**(3/4)*a**(31/4)*b*(1/b)**(7/4)*log(-(-1)**(1/4)*
a**(1/4)*(1/b)**(1/4) + tan(c + d*x))/(-4*I*a**(19/2)*b**2*d*(1/b)**(7/2) +
 8*I*a**(17/2)*b**3*d*(1/b)**(7/2) + 12*I*a**(15/2)*b**4*d*(1/b)**(7/2) - 1
2*a**9*d/b - 8*a**8*d + 4*a**7*b*d) + 12*(-1)**(1/4)*a**(29/4)*b**5*(atan((
-1)**(3/4)*tan(c + d*x)/(a**(1/4)*(1/b)**(1/4))) + pi*floor((c + d*x - pi/2
)/pi)*sign((-1)**(3/4)/(a**(1/4)*(1/b)**(1/4))))*(1/b)**(21/4)/(-4*I*a**(19
/2)*b**2*d*(1/b)**(7/2) + 8*I*a**(17/2)*b**3*d*(1/b)**(7/2) + 12*I*a**(15/2
)*b**4*d*(1/b)**(7/2) - 12*a**9*d/b - 8*a**8*d + 4*a**7*b*d) - 2*(-1)**(3/4
)*a**(27/4)*b**7*(atan((-1)**(3/4)*tan(c + d*x)/(a**(1/4)*(1/b)**(1/4))) +
pi*floor((c + d*x - pi/2)/pi)*sign((-1)**(3/4)/(a**(1/4)*(1/b)**(1/4))))*(1
/b)**(27/4)/(-4*I*a**(19/2)*b**2*d*(1/b)**(7/2) + 8*I*a**(17/2)*b**3*d*(1/b
)**(7/2) + 12*I*a**(15/2)*b**4*d*(1/b)**(7/2) - 12*a**9*d/b - 8*a**8*d + 4*
a**7*b*d) - 6*(-1)**(3/4)*a**(27/4)*b**6*(atan((-1)**(3/4)*tan(c + d*x)/(a*
*(1/4)*(1/b)**(1/4))) + pi*floor((c + d*x - pi/2)/pi)*sign((-1)**(3/4)/(a**
(1/4)*(1/b)**(1/4))))*(1/b)**(23/4)/(-4*I*a**(19/2)*b**2*d*(1/b)**(7/2) + 8
*I*a**(17/2)*b**3*d*(1/b)**(7/2) + 12*I*a**(15/2)*b**4*d*(1/b)**(7/2) - 12*
a**9*d/b - 8*a**8*d + 4*a**7*b*d) + 2*(-1)**(3/4)*a**(27/4)*b**3*(1/b)**(11
/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + tan(c + d*x))/(-4*I*a**(19/2)*b
```

```
**2*d*(1/b)**(7/2) + 8*I*a**(17/2)*b**3*d*(1/b)**(7/2) + 12*I*a**(15/2)*b**
4*d*(1/b)**(7/2) - 12*a**9*d/b - 8*a**8*d + 4*a**7*b*d) - 2*(-1)**(3/4)*a**
(27/4)*b**2*(1/b)**(7/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + tan(c + d
*x))/(-4*I*a**(19/2)*b**2*d*(1/b)**(7/2) + 8*I*a**(17/2)*b**3*d*(1/b)**(7/2
) + 12*I*a**(15/2)*b**4*d*(1/b)**(7/2) - 12*a**9*d/b - 8*a**8*d + 4*a**7*b*
d) - 2*(-1)**(1/4)*a**(25/4)*b**6*(atan((-1)**(3/4)*tan(c + d*x)/(a**(1/4)*
(1/b)**(1/4))) + pi*floor((c + d*x - pi/2)/pi)*sign((-1)**(3/4)/(a**(1/4)*(
1/b)**(1/4))))*(1/b)**(21/4)/(-4*I*a**(19/2)*b**2*d*(1/b)**(7/2) + 8*I*a**(
17/2)*b**3*d*(1/b)**(7/2) + 12*I*a**(15/2)*b**4*d*(1/b)**(7/2) - 12*a**9*d/
b - 8*a**8*d + 4*a**7*b*d) + (-1)**(1/4)*a**(25/4)*b**3*(1/b)**(9/4)*log((-
1)**(1/4)*a**(1/4)*(1/b)**(1/4) + tan(c + d*x))/(-4*I*a**(19/2)*b**2*d*(1/b
)**(7/2) + 8*I*a**(17/2)*b**3*d*(1/b)**(7/2) + 12*I*a**(15/2)*b**4*d*(1/b)*
*(7/2) - 12*a**9*d/b - 8*a**8*d + 4*a**7*b*d) - (-1)**(1/4)*a**(25/4)*b**2*
(1/b)**(5/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + tan(c + d*x))/(-4*I*a
**(19/2)*b**2*d*(1/b)**(7/2) + 8*I*a**(17/2)*b**3*d*(1/b)**(7/2) + 12*I*a**
(15/2)*b**4*d*(1/b)**(7/2) - 12*a**9*d/b - 8*a**8*d + 4*a**7*b*d) - 4*I*a**
(17/2)*b**2*d*x*(1/b)**(7/2)/(-4*I*a**(19/2)*b**2*d*(1/b)**(7/2) + 8*I*a**(
17/2)*b**3*d*(1/b)**(7/2) + 12*I*a**(15/2)*b**4*d*(1/b)**(7/2) - 12*a**9*d/
b - 8*a**8*d + 4*a**7*b*d) + 12*I*a**(15/2)*b**3*d*x*(1/b)**(7/2)/(-4*I*a**
(19/2)*b**2*d*(1/b)**(7/2) + 8*I*a**(17/2)*b**3*d*(1/b)**(7/2) + 12*I*a**(1
5/2)*b**4*d*(1/b)**(7/2) - 12*a**9*d/b - 8*a**8*d + 4*a**7*b*d) - 12*a**8*d
*x/(-4*I*a**(19/2)*b**3*d*(1/b)**(7/2) + 8*I*a**(17/2)*b**4*d*(1/b)**(7/2)
+ 12*I*a**(15/2)*b**5*d*(1/b)**(7/2) - 12*a**9*d - 8*a**8*b*d + 4*a**7*b**2
*d) + 4*a**7*d*x/(-4*I*a**(19/2)*b**2*d*(1/b)**(7/2) + 8*I*a**(17/2)*b**3*d
*(1/b)**(7/2) + 12*I*a**(15/2)*b**4*d*(1/b)**(7/2) - 12*a**9*d/b - 8*a**8*d
 + 4*a**7*b*d), True))
```

Giac [A] time = 2.27099, size = 478, normalized size = 1.58

$$\frac{\frac{2\left(\left(ab^3\right)^{\frac{1}{4}}b^2-\left(ab^3\right)^{\frac{3}{4}}\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\tan(dx+c)\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{\sqrt{2}a^2b^2+\sqrt{2}ab^3}+\frac{2\left(\left(ab^3\right)^{\frac{1}{4}}b^2-\left(ab^3\right)^{\frac{3}{4}}\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\tan(dx+c)\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{\sqrt{2}a^2b^2+\sqrt{2}ab^3}+\frac{\left((ab \cdots}{\cdots}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b),x, algorithm="giac")
```

```
[Out] 1/4*(2*((a*b^3)^(1/4)*b^2 - (a*b^3)^(3/4))*(pi*floor((d*x + c)/pi + 1/2) +
arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*tan(d*x + c))/(a/b)^(1/4)))/(sq
rt(2)*a^2*b^2 + sqrt(2)*a*b^3) + 2*((a*b^3)^(1/4)*b^2 - (a*b^3)^(3/4))*(pi*
floor((d*x + c)/pi + 1/2) + arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*ta
n(d*x + c))/(a/b)^(1/4)))/(sqrt(2)*a^2*b^2 + sqrt(2)*a*b^3) + ((a*b^3)^(1/4
)*b^2 + (a*b^3)^(3/4))*log(tan(d*x + c)^2 + sqrt(2)*(a/b)^(1/4)*tan(d*x + c
) + sqrt(a/b))/(sqrt(2)*a^2*b^2 + sqrt(2)*a*b^3) - ((a*b^3)^(1/4)*b^2 + (a*
b^3)^(3/4))*log(tan(d*x + c)^2 - sqrt(2)*(a/b)^(1/4)*tan(d*x + c) + sqrt(a/
b))/(sqrt(2)*a^2*b^2 + sqrt(2)*a*b^3) + 4*(d*x + c)/(a + b))/d
```

3.386 $\int \frac{1}{\left(a+b\tan^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=648

$$\frac{\sqrt[4]{b}\left(\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}d(a+b)}+\frac{\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}d(a+b)^2}-\frac{\sqrt[4]{b}\left(\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\tan(\right.}{\sqrt[4]{a}}}{8\sqrt{2}a^{7/4}d(a+b)}$$

```
[Out] x/(a + b)^2 + ((Sqrt[a] - Sqrt[b])*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Tan[
c + d*x])/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b)^2*d) + ((Sqrt[a] - 3*Sqrt[b]
)*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/(8*Sqrt[2]*a^
(7/4)*(a + b)*d) - ((Sqrt[a] - Sqrt[b])*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b)^2*d) - ((Sqrt[a] - 3*Sq
rt[b])*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/(8*Sqrt[
2]*a^(7/4)*(a + b)*d) - ((Sqrt[a] + Sqrt[b])*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/(4*Sqrt[2]*a^(3/4)*
(a + b)^2*d) - ((Sqrt[a] + 3*Sqrt[b])*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/(16*Sqrt[2]*a^(7/4)*(a + b
)*d) + ((Sqrt[a] + Sqrt[b])*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*T
an[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/(4*Sqrt[2]*a^(3/4)*(a + b)^2*d) + ((
Sqrt[a] + 3*Sqrt[b])*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c +
d*x] + Sqrt[b]*Tan[c + d*x]^2])/(16*Sqrt[2]*a^(7/4)*(a + b)*d) + (b*Tan[c +
 d*x]*(1 - Tan[c + d*x]^2))/(4*a*(a + b)*d*(a + b*Tan[c + d*x]^4))
```

Rubi [A] time = 0.661279, antiderivative size = 648, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3661, 1239, 203, 1179, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{b}\left(\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}d(a+b)}+\frac{\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}d(a+b)^2}-\frac{\sqrt[4]{b}\left(\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\tan(\right.}{\sqrt[4]{a}}}{8\sqrt{2}a^{7/4}d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^4)^(-2),x]
```

```
[Out] x/(a + b)^2 + ((Sqrt[a] - Sqrt[b])*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Tan[
c + d*x])/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b)^2*d) + ((Sqrt[a] - 3*Sqrt[b]
)*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/(8*Sqrt[2]*a^
(7/4)*(a + b)*d) - ((Sqrt[a] - Sqrt[b])*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b)^2*d) - ((Sqrt[a] - 3*Sq
rt[b])*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/(8*Sqrt[
2]*a^(7/4)*(a + b)*d) - ((Sqrt[a] + Sqrt[b])*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/(4*Sqrt[2]*a^(3/4)*
(a + b)^2*d) - ((Sqrt[a] + 3*Sqrt[b])*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/(16*Sqrt[2]*a^(7/4)*(a + b
)*d) + ((Sqrt[a] + Sqrt[b])*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*T
an[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/(4*Sqrt[2]*a^(3/4)*(a + b)^2*d) + ((
Sqrt[a] + 3*Sqrt[b])*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c +
d*x] + Sqrt[b]*Tan[c + d*x]^2])/(16*Sqrt[2]*a^(7/4)*(a + b)*d) + (b*Tan[c +
 d*x]*(1 - Tan[c + d*x]^2))/(4*a*(a + b)*d*(a + b*Tan[c + d*x]^4))
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
```

```
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 1239

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^4)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)^2(1+x^2)} + \frac{b-bx^2}{(a+b)(a+bx^4)^2} + \frac{b-bx^2}{(a+b)^2(a+bx^4)}\right) dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{(a+b)^2 d} + \frac{\text{Subst}\left(\int \frac{b-bx^2}{a+bx^4}\,dx, x, \tan(c+dx)\right)}{(a+b)^2 d} + \frac{\text{Subst}\left(\int \frac{b-bx^2}{(a+bx^4}\right.}{(a}$$

$$= \frac{x}{(a+b)^2} + \frac{b\tan(c+dx)\left(1-\tan^2(c+dx)\right)}{4a(a+b)d\left(a+b\tan^4(c+dx)\right)} - \frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx, x, \tan(c+d\right.}{2(a+b)^2 d}$$

$$= \frac{x}{(a+b)^2} + \frac{b\tan(c+dx)\left(1-\tan^2(c+dx)\right)}{4a(a+b)d\left(a+b\tan^4(c+dx)\right)} - \frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \tan(c\right.}{4(a+b)^2 d}$$

$$= \frac{x}{(a+b)^2} - \frac{\left(\sqrt{a}+\sqrt{b}\right)\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\tan(c+dx)+\sqrt{b}\tan^2(c+dx)\right)}{4\sqrt{2}a^{3/4}(a+b)^2 d} + \frac{\left(\sqrt{a}+\sqrt{\right.}{}$$

$$= \frac{x}{(a+b)^2} + \frac{\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(a+b)^2 d} - \frac{\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\tan(\right.}{\sqrt[4]{a}}}{2\sqrt{2}a^{3/4}(a+b)^2 d}$$

$$= \frac{x}{(a+b)^2} + \frac{\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(a+b)^2 d} + \frac{\left(\sqrt{a}-3\sqrt{b}\right)\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan\right.}{\sqrt[4]{a}}}{8\sqrt{2}a^{7/4}(a+b)d}$$

Mathematica [C] time = 6.27356, size = 598, normalized size = 0.92

$$-\frac{b\tan^3(c+dx)\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-\frac{b\tan^4(c+dx)}{a}\right)}{3a^2d(a+b)} - \frac{3\left(\frac{2\left(\frac{\sqrt{2}b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}} - \frac{\sqrt{2}b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{a}}\right)}{\sqrt{a}} + \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^4)^(-2),x]
```

```
[Out] ArcTan[Tan[c + d*x]]/((a + b)^2*d) + ((Sqrt[a] - Sqrt[b])*((Sqrt[2]*b^(1/4)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/a^(1/4) - (Sqrt[2]*b^(
```

```
1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/a^(1/4)))/(4*Sqrt[
a]*(a + b)^2*d) - ((Sqrt[a] + Sqrt[b])*((Sqrt[2]*b^(1/4)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/a^(1/4) - (Sqrt
[2]*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Ta
n[c + d*x]^2])/a^(1/4)))/(8*Sqrt[a]*(a + b)^2*d) - (3*((2*((Sqrt[2]*b^(3/4)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/a^(1/4) - (Sqrt[2]*b^(
3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/a^(1/4)))/Sqrt[a]
+ ((Sqrt[2]*b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c + d*x] + Sq
rt[b]*Tan[c + d*x]^2])/a^(1/4) - (Sqrt[2]*b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/a^(1/4))/Sqrt[a]))/(32
*a*(a + b)*d) - (b*Hypergeometric2F1[3/4, 2, 7/4, -((b*Tan[c + d*x]^4)/a)]*
Tan[c + d*x]^3)/(3*a^2*(a + b)*d) + (b*Tan[c + d*x])/(4*a*(a + b)*d*(a + b*
Tan[c + d*x]^4))
```

Maple [A] time = 0.037, size = 886, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^4)^2,x)
```

```
[Out] 1/d/(a+b)^2*arctan(tan(d*x+c))-1/4/d/(a+b)^2*b/(a+b*tan(d*x+c)^4)*tan(d*x+c
)^3-1/4/d/(a+b)^2*b^2/(a+b*tan(d*x+c)^4)/a*tan(d*x+c)^3+1/4/d/(a+b)^2*b/(a+
b*tan(d*x+c)^4)*tan(d*x+c)+1/4/d/(a+b)^2*b^2/(a+b*tan(d*x+c)^4)/a*tan(d*x+c
)+7/16/d/(a+b)^2*b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*tan(d*x
+c)+1)+3/16/d/(a+b)^2*b^2/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4
)*tan(d*x+c)+1)-7/16/d/(a+b)^2*b/a*(a/b)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(a/b
)^(1/4)*tan(d*x+c)+1)-3/16/d/(a+b)^2*b^2/a^2*(a/b)^(1/4)*2^(1/2)*arctan(-2^
(1/2)/(a/b)^(1/4)*tan(d*x+c)+1)+7/32/d/(a+b)^2*b/a*(a/b)^(1/4)*2^(1/2)*ln((
tan(d*x+c)^2+(a/b)^(1/4)*tan(d*x+c)*2^(1/2)+(a/b)^(1/2))/(tan(d*x+c)^2-(a/b
)^(1/4)*tan(d*x+c)*2^(1/2)+(a/b)^(1/2)))+3/32/d/(a+b)^2*b^2/a^2*(a/b)^(1/4)
*2^(1/2)*ln((tan(d*x+c)^2+(a/b)^(1/4)*tan(d*x+c)*2^(1/2)+(a/b)^(1/2))/(tan(
d*x+c)^2-(a/b)^(1/4)*tan(d*x+c)*2^(1/2)+(a/b)^(1/2)))-5/32/d/(a+b)^2/(a/b)^
(1/4)*2^(1/2)*ln((tan(d*x+c)^2-(a/b)^(1/4)*tan(d*x+c)*2^(1/2)+(a/b)^(1/2))/
(tan(d*x+c)^2+(a/b)^(1/4)*tan(d*x+c)*2^(1/2)+(a/b)^(1/2)))-1/32/d/(a+b)^2*b
/a/(a/b)^(1/4)*2^(1/2)*ln((tan(d*x+c)^2-(a/b)^(1/4)*tan(d*x+c)*2^(1/2)+(a/b
)^(1/2))/(tan(d*x+c)^2+(a/b)^(1/4)*tan(d*x+c)*2^(1/2)+(a/b)^(1/2)))-5/16/d/
(a+b)^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*tan(d*x+c)+1)-1/16/d
/(a+b)^2*b/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*tan(d*x+c)+1)+5
/16/d/(a+b)^2/(a/b)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(a/b)^(1/4)*tan(d*x+c)+1)
+1/16/d/(a+b)^2*b/a/(a/b)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(a/b)^(1/4)*tan(d*x
+c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.67707, size = 9750, normalized size = 15.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(32*a*b*d*x*tan(d*x + c)^4 + 32*a^2*d*x - 8*(a*b + b^2)*tan(d*x + c)^3
 + ((a^3*b + 2*a^2*b^2 + a*b^3)*d*tan(d*x + c)^4 + (a^4 + 2*a^3*b + a^2*b^2
)*d)*sqrt(((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2*sqrt(-(625
*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^4 + 2383*a^2*b^5 + 738*a*b
^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 56*a^12*b^3 + 70*a^11*b^4 +
56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d^4)) + 70*a^2*b + 44*a*b^2
 + 6*b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2))*log((625
*a^5 - 750*a^4*b - 1376*a^3*b^2 - 594*a^2*b^3 - 81*a*b^4 + (625*a^4*b - 750
*a^3*b^2 - 1376*a^2*b^3 - 594*a*b^4 - 81*b^5)*tan(d*x + c)^2 + 2*(2*(a^11 +
 5*a^10*b + 10*a^9*b^2 + 10*a^8*b^3 + 5*a^7*b^4 + a^6*b^5)*d^3*sqrt(-(625*a
^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6
 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56
*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d^4))*tan(d*x + c) + (125*a^7
 + 5*a^6*b - 442*a^5*b^2 - 490*a^4*b^3 - 195*a^3*b^4 - 27*a^2*b^5)*d*tan(d*
x + c))*sqrt(((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2*sqrt(-(
625*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^4 + 2383*a^2*b^5 + 738*
a*b^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 56*a^12*b^3 + 70*a^11*b^4
 + 56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d^4)) + 70*a^2*b + 44*a*
b^2 + 6*b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2)) + ((2
5*a^9 + 109*a^8*b + 186*a^7*b^2 + 154*a^6*b^3 + 61*a^5*b^4 + 9*a^4*b^5)*d^2
*tan(d*x + c)^2 - (25*a^9 + 109*a^8*b + 186*a^7*b^2 + 154*a^6*b^3 + 61*a^5*
b^4 + 9*a^4*b^5)*d^2)*sqrt(-(625*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*
a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^
2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*
b^8)*d^4)))/(tan(d*x + c)^2 + 1)) - ((a^3*b + 2*a^2*b^2 + a*b^3)*d*tan(d*x
+ c)^4 + (a^4 + 2*a^3*b + a^2*b^2)*d)*sqrt(((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*
a^4*b^3 + a^3*b^4)*d^2*sqrt(-(625*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748
*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b
^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7
*b^8)*d^4)) + 70*a^2*b + 44*a*b^2 + 6*b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*
a^4*b^3 + a^3*b^4)*d^2))*log((625*a^5 - 750*a^4*b - 1376*a^3*b^2 - 594*a^2*
b^3 - 81*a*b^4 + (625*a^4*b - 750*a^3*b^2 - 1376*a^2*b^3 - 594*a*b^4 - 81*b
^5)*tan(d*x + c)^2 - 2*(2*(a^11 + 5*a^10*b + 10*a^9*b^2 + 10*a^8*b^3 + 5*a^
7*b^4 + a^6*b^5)*d^3*sqrt(-(625*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a
^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2
 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b
^8)*d^4))*tan(d*x + c) + (125*a^7 + 5*a^6*b - 442*a^5*b^2 - 490*a^4*b^3 - 1
95*a^3*b^4 - 27*a^2*b^5)*d*tan(d*x + c))*sqrt(((a^7 + 4*a^6*b + 6*a^5*b^2 +
 4*a^4*b^3 + a^3*b^4)*d^2*sqrt(-(625*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2
748*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^1
3*b^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 +
a^7*b^8)*d^4)) + 70*a^2*b + 44*a*b^2 + 6*b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 +
 4*a^4*b^3 + a^3*b^4)*d^2)) + ((25*a^9 + 109*a^8*b + 186*a^7*b^2 + 154*a^6*
b^3 + 61*a^5*b^4 + 9*a^4*b^5)*d^2*tan(d*x + c)^2 - (25*a^9 + 109*a^8*b + 18
6*a^7*b^2 + 154*a^6*b^3 + 61*a^5*b^4 + 9*a^4*b^5)*d^2)*sqrt(-(625*a^6*b - 1
950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^
7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^
5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d^4)))/(tan(d*x + c)^2 + 1)) - ((a^3*
b + 2*a^2*b^2 + a*b^3)*d*tan(d*x + c)^4 + (a^4 + 2*a^3*b + a^2*b^2)*d)*sqrt
(-((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2*sqrt(-(625*a^6*b -
```

```
 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*
b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*
b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d^4)) - 70*a^2*b - 44*a*b^2 - 6*b^3
)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2))*log(-(625*a^5 -
750*a^4*b - 1376*a^3*b^2 - 594*a^2*b^3 - 81*a*b^4 + (625*a^4*b - 750*a^3*b^
2 - 1376*a^2*b^3 - 594*a*b^4 - 81*b^5)*tan(d*x + c)^2 + 2*(2*(a^11 + 5*a^10
*b + 10*a^9*b^2 + 10*a^8*b^3 + 5*a^7*b^4 + a^6*b^5)*d^3*sqrt(-(625*a^6*b -
1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b
^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b
^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d^4))*tan(d*x + c) - (125*a^7 + 5*a^
6*b - 442*a^5*b^2 - 490*a^4*b^3 - 195*a^3*b^4 - 27*a^2*b^5)*d*tan(d*x + c))
*sqrt(-((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2*sqrt(-(625*a^
6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6
+ 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*
a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d^4)) - 70*a^2*b - 44*a*b^2 -
6*b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2)) - ((25*a^9
+ 109*a^8*b + 186*a^7*b^2 + 154*a^6*b^3 + 61*a^5*b^4 + 9*a^4*b^5)*d^2*tan(d
*x + c)^2 - (25*a^9 + 109*a^8*b + 186*a^7*b^2 + 154*a^6*b^3 + 61*a^5*b^4 +
9*a^4*b^5)*d^2)*sqrt(-(625*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^
4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 56
*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d
^4)))/(tan(d*x + c)^2 + 1)) + ((a^3*b + 2*a^2*b^2 + a*b^3)*d*tan(d*x + c)^4
 + (a^4 + 2*a^3*b + a^2*b^2)*d)*sqrt(-((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b
^3 + a^3*b^4)*d^2*sqrt(-(625*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*
b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 +
56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)
*d^4)) - 70*a^2*b - 44*a*b^2 - 6*b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b
^3 + a^3*b^4)*d^2))*log(-(625*a^5 - 750*a^4*b - 1376*a^3*b^2 - 594*a^2*b^3
- 81*a*b^4 + (625*a^4*b - 750*a^3*b^2 - 1376*a^2*b^3 - 594*a*b^4 - 81*b^5)*
tan(d*x + c)^2 - 2*(2*(a^11 + 5*a^10*b + 10*a^9*b^2 + 10*a^8*b^3 + 5*a^7*b^
4 + a^6*b^5)*d^3*sqrt(-(625*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b
^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 5
6*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*
d^4))*tan(d*x + c) - (125*a^7 + 5*a^6*b - 442*a^5*b^2 - 490*a^4*b^3 - 195*a
^3*b^4 - 27*a^2*b^5)*d*tan(d*x + c))*sqrt(-((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*
a^4*b^3 + a^3*b^4)*d^2*sqrt(-(625*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748
*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b
^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7
*b^8)*d^4)) - 70*a^2*b - 44*a*b^2 - 6*b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*
a^4*b^3 + a^3*b^4)*d^2)) - ((25*a^9 + 109*a^8*b + 186*a^7*b^2 + 154*a^6*b^3
 + 61*a^5*b^4 + 9*a^4*b^5)*d^2*tan(d*x + c)^2 - (25*a^9 + 109*a^8*b + 186*a
^7*b^2 + 154*a^6*b^3 + 61*a^5*b^4 + 9*a^4*b^5)*d^2)*sqrt(-(625*a^6*b - 1950
*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^7)/
((a^15 + 8*a^14*b + 28*a^13*b^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^5 +
 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d^4)))/(tan(d*x + c)^2 + 1)) + 8*(a*b +
b^2)*tan(d*x + c))/((a^3*b + 2*a^2*b^2 + a*b^3)*d*tan(d*x + c)^4 + (a^4 + 2
*a^3*b + a^2*b^2)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)**4*b)**2,x)

[Out] Timed out
```

Giac [A] time = 2.39658, size = 698, normalized size = 1.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b)^2,x, algorithm="giac")
```

```
[Out] -1/16*(2*(pi*floor((d*x + c)/pi + 1/2) + arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^
(1/4) + 2*tan(d*x + c))/(a/b)^(1/4)))*((a*b^3)^(3/4)*(5*a + b) - (a*b^3)^(1
/4)*(7*a*b^2 + 3*b^3))/(sqrt(2)*a^4*b^2 + 2*sqrt(2)*a^3*b^3 + sqrt(2)*a^2*b
^4) + 2*(pi*floor((d*x + c)/pi + 1/2) + arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^
(1/4) - 2*tan(d*x + c))/(a/b)^(1/4)))*((a*b^3)^(3/4)*(5*a + b) - (a*b^3)^(1
/4)*(7*a*b^2 + 3*b^3))/(sqrt(2)*a^4*b^2 + 2*sqrt(2)*a^3*b^3 + sqrt(2)*a^2*b
^4) - ((a*b^3)^(3/4)*(5*a + b) + (a*b^3)^(1/4)*(7*a*b^2 + 3*b^3))*log(tan(d
*x + c)^2 + sqrt(2)*(a/b)^(1/4)*tan(d*x + c) + sqrt(a/b))/(sqrt(2)*a^4*b^2
+ 2*sqrt(2)*a^3*b^3 + sqrt(2)*a^2*b^4) + ((a*b^3)^(3/4)*(5*a + b) + (a*b^3)
^(1/4)*(7*a*b^2 + 3*b^3))*log(tan(d*x + c)^2 - sqrt(2)*(a/b)^(1/4)*tan(d*x
+ c) + sqrt(a/b))/(sqrt(2)*a^4*b^2 + 2*sqrt(2)*a^3*b^3 + sqrt(2)*a^2*b^4) -
 16*(d*x + c)/(a^2 + 2*a*b + b^2) + 4*(b*tan(d*x + c)^3 - b*tan(d*x + c))/(
(b*tan(d*x + c)^4 + a)*(a^2 + a*b)))/d
```

3.387 $\int \sqrt{a+b\tan^4(c+dx)}\,dx$

Optimal. Leaf size=650

$$-\frac{\sqrt[4]{b}(a+b)\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)\sqrt{\frac{a+b\tan^4(c+dx)}{\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\left(\sqrt{a}-\sqrt{b}\right)\sqrt{a+b\tan^4(c+dx)}}+\frac{\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\left(\sqrt{a}+\right.}{}$$

```
[Out] (Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]^4]])
/(2*d) + (Sqrt[b]*Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]^4])/(d*(Sqrt[a] + Sq
rt[b]*Tan[c + d*x]^2)) - (a^(1/4)*b^(1/4)*EllipticE[2*ArcTan[(b^(1/4)*Tan[c
 + d*x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[
c + d*x]^4)/(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)^2])/(d*Sqrt[a + b*Tan[c + d*
x]^4]) + ((Sqrt[a] - Sqrt[b])*b^(1/4)*EllipticF[2*ArcTan[(b^(1/4)*Tan[c + d
*x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c +
d*x]^4)/(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)^2])/(2*a^(1/4)*d*Sqrt[a + b*Tan[
c + d*x]^4]) - (b^(1/4)*(a + b)*EllipticF[2*ArcTan[(b^(1/4)*Tan[c + d*x])/a
^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c + d*x]^4
)/(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)^2])/(2*a^(1/4)*(Sqrt[a] - Sqrt[b])*d*S
qrt[a + b*Tan[c + d*x]^4]) + ((Sqrt[a] + Sqrt[b])*(a + b)*EllipticPi[-(Sqrt
[a] - Sqrt[b])^2/(4*Sqrt[a]*Sqrt[b]), 2*ArcTan[(b^(1/4)*Tan[c + d*x])/a^(1/
4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c + d*x]^4)/(S
qrt[a] + Sqrt[b]*Tan[c + d*x]^2)^2])/(4*a^(1/4)*(Sqrt[a] - Sqrt[b])*b^(1/4)
*d*Sqrt[a + b*Tan[c + d*x]^4])
```

Rubi [A] time = 0.559916, antiderivative size = 650, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3661, 1209, 1198, 220, 1196, 1217, 1707}

$$\frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a+b\tan^4(c+dx)}}\right)}{2d}+\frac{\sqrt{b}\tan(c+dx)\sqrt{a+b\tan^4(c+dx)}}{d\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)}-\frac{\sqrt[4]{b}(a+b)\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)\sqrt{\frac{a+b\tan}{\left(\sqrt{a}+\sqrt{b}\tan\right.}}}{2\sqrt[4]{a}d\left(\sqrt{a}-\sqrt{b}\right)\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[c + d*x]^4],x]
```

```
[Out] (Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]^4]])
/(2*d) + (Sqrt[b]*Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]^4])/(d*(Sqrt[a] + Sq
rt[b]*Tan[c + d*x]^2)) - (a^(1/4)*b^(1/4)*EllipticE[2*ArcTan[(b^(1/4)*Tan[c
 + d*x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[
c + d*x]^4)/(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)^2])/(d*Sqrt[a + b*Tan[c + d*
x]^4]) + ((Sqrt[a] - Sqrt[b])*b^(1/4)*EllipticF[2*ArcTan[(b^(1/4)*Tan[c + d
*x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c +
d*x]^4)/(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)^2])/(2*a^(1/4)*d*Sqrt[a + b*Tan[
c + d*x]^4]) - (b^(1/4)*(a + b)*EllipticF[2*ArcTan[(b^(1/4)*Tan[c + d*x])/a
^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c + d*x]^4
)/(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)^2])/(2*a^(1/4)*(Sqrt[a] - Sqrt[b])*d*S
qrt[a + b*Tan[c + d*x]^4]) + ((Sqrt[a] + Sqrt[b])*(a + b)*EllipticPi[-(Sqrt
[a] - Sqrt[b])^2/(4*Sqrt[a]*Sqrt[b]), 2*ArcTan[(b^(1/4)*Tan[c + d*x])/a^(1/
4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c + d*x]^4)/(S
qrt[a] + Sqrt[b]*Tan[c + d*x]^2)^2])/(4*a^(1/4)*(Sqrt[a] - Sqrt[b])*b^(1/4)
*d*Sqrt[a + b*Tan[c + d*x]^4])
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\tan^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^4}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{b-bx^2}{\sqrt{a+bx^4}}\,dx, x, \tan(c+dx)\right)}{d} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^4}}\,dx, x, \tan(c+dx)\right.}{d}$$

$$= -\frac{\left(\sqrt{a}\sqrt{b}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx, x, \tan(c+dx)\right)}{d} + \frac{\left(\left(\sqrt{a}-\sqrt{b}\right)\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x,\right.}{d}$$

$$= \frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a+b\tan^4(c+dx)}}\right)}{2d} + \frac{\sqrt{b}\tan(c+dx)\sqrt{a+b\tan^4(c+dx)}}{d\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)} - \frac{\sqrt[4]{a}\sqrt[4]{b}E\left(2\tan^{-}\right.}{}$$

Mathematica [C] time = 0.725444, size = 219, normalized size = 0.34

$$\frac{\sqrt{\frac{b\tan^4(c+dx)}{a}+1}\left(\left(\sqrt{a}-i\sqrt{b}\right)\left(\left(\sqrt{b}-i\sqrt{a}\right)\Pi\left(-\frac{i\sqrt{a}}{\sqrt{b}}; i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\right)\middle|-1\right)-\sqrt{b}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{}}{\sqrt{}}}\right.\right.\right.\right.}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+b\tan^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[c + d*x]^4],x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[c + d*x
]], -1] + (Sqrt[a] - I*Sqrt[b])*(-(Sqrt[b]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt
[b])/Sqrt[a]]*Tan[c + d*x]], -1]) + ((-I)*Sqrt[a] + Sqrt[b])*EllipticPi[((-
I)*Sqrt[a])/Sqrt[b], I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[c + d*x]], -1]
))*Sqrt[1 + (b*Tan[c + d*x]^4)/a])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*d*Sqrt[a + b*
Tan[c + d*x]^4])
```

Maple [C] time = 0.12, size = 531, normalized size = 0.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^4)^(1/2),x)
```

```
[Out] 1/d*(-b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)*
(1+I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)/(a+b*tan(d*x+c)^4)^(1/2)*EllipticF
(tan(d*x+c)*(I/a^(1/2)*b^(1/2))^(1/2),I)+I*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/
2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan
(d*x+c)^2)^(1/2)/(a+b*tan(d*x+c)^4)^(1/2)*EllipticF(tan(d*x+c)*(I/a^(1/2)*b
^(1/2))^(1/2),I)-I*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b
^(1/2)*tan(d*x+c)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)/(a+b*ta
n(d*x+c)^4)^(1/2)*EllipticE(tan(d*x+c)*(I/a^(1/2)*b^(1/2))^(1/2),I)+a/(I/a^
(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)*(1+I/a^(1/2)*
b^(1/2)*tan(d*x+c)^2)^(1/2)/(a+b*tan(d*x+c)^4)^(1/2)*EllipticPi(tan(d*x+c)*
(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a
```

```
^(1/2)*b^(1/2))^(1/2))+b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan
(d*x+c)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)/(a+b*tan(d*x+c)^4
)^(1/2)*EllipticPi(tan(d*x+c)*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2),(
-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \tan (dx + c)^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(d*x + c)^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \tan (dx + c)^4 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*tan(d*x + c)^4 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \tan^4 (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)**4*b)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(c + d*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \tan (dx + c)^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(d*x + c)^4 + a), x)
```

3.388 $\int \frac{1}{\sqrt{a+b\tan^4(c+dx)}}\,dx$

Optimal. Leaf size=348

$$-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)\sqrt{\frac{a+b\tan^4(c+dx)}{\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\left(\sqrt{a}-\sqrt{b}\right)\sqrt{a+b\tan^4(c+dx)}}+\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a+b\tan^4(c+dx)}}\right)}{2d\sqrt{a+b}}+\frac{(\sqrt{}}{}$$

```
[Out] ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]^4]]/(2*Sqrt[a + b
]*d) - (b^(1/4)*EllipticF[2*ArcTan[(b^(1/4)*Tan[c + d*x])/a^(1/4)], 1/2]*(S
qrt[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c + d*x]^4)/(Sqrt[a] + Sqr
t[b]*Tan[c + d*x]^2)^2])/(2*a^(1/4)*(Sqrt[a] - Sqrt[b])*d*Sqrt[a + b*Tan[c
+ d*x]^4]) + ((Sqrt[a] + Sqrt[b])*EllipticPi[-(Sqrt[a] - Sqrt[b])^2/(4*Sqrt
[a]*Sqrt[b]), 2*ArcTan[(b^(1/4)*Tan[c + d*x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqr
t[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[b]*Tan[c +
 d*x]^2)^2])/(4*a^(1/4)*(Sqrt[a] - Sqrt[b])*b^(1/4)*d*Sqrt[a + b*Tan[c + d*
x]^4])
```

Rubi [A] time = 0.224285, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3661, 1217, 220, 1707}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a+b\tan^4(c+dx)}}\right)}{2d\sqrt{a+b}}-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)\sqrt{\frac{a+b\tan^4(c+dx)}{\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}d\left(\sqrt{a}-\sqrt{b}\right)\sqrt{a+b\tan^4(c+dx)}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)(}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Tan[c + d*x]^4],x]
```


```
[Out] ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]^4]]/(2*Sqrt[a + b
]*d) - (b^(1/4)*EllipticF[2*ArcTan[(b^(1/4)*Tan[c + d*x])/a^(1/4)], 1/2]*(S
qrt[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c + d*x]^4)/(Sqrt[a] + Sqr
t[b]*Tan[c + d*x]^2)^2])/(2*a^(1/4)*(Sqrt[a] - Sqrt[b])*d*Sqrt[a + b*Tan[c
+ d*x]^4]) + ((Sqrt[a] + Sqrt[b])*EllipticPi[-(Sqrt[a] - Sqrt[b])^2/(4*Sqrt
[a]*Sqrt[b]), 2*ArcTan[(b^(1/4)*Tan[c + d*x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqr
t[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[b]*Tan[c +
 d*x]^2)^2])/(4*a^(1/4)*(Sqrt[a] - Sqrt[b])*b^(1/4)*d*Sqrt[a + b*Tan[c + d*
x]^4])
```


Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
```

```
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\tan^4(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^4}}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{(1+x^2)\sqrt{a+bx^4}}\,dx, x, \tan(c+dx)\right)}{\left(\sqrt{a}-\sqrt{b}\right)d} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \tan(c+dx)\right)}{\left(\sqrt{a}-\sqrt{b}\right)d}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a+b\tan^4(c+dx)}}\right)}{2\sqrt{a+b}d} - \frac{\sqrt[4]{b}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)\sqrt{\frac{a+b\tan^4(c+dx)}{\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)^2}}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)d\sqrt{a+b\tan^4(c+dx)}}$$

Mathematica [C] time = 0.360024, size = 106, normalized size = 0.3

$$-\frac{i\sqrt{\frac{b\tan^4(c+dx)}{a}+1}\,\Pi\left(-\frac{i\sqrt{a}}{\sqrt{b}};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\right)\middle|-1\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+b\tan^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Tan[c + d*x]^4],x]
```

```
[Out] ((-I)*EllipticPi[((-I)*Sqrt[a])/Sqrt[b], I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]
]*Tan[c + d*x]], -1]*Sqrt[1 + (b*Tan[c + d*x]^4)/a])/(Sqrt[(I*Sqrt[b])/Sqrt
[a]]*d*Sqrt[a + b*Tan[c + d*x]^4])
```

Maple [C] time = 0.069, size = 123, normalized size = 0.4

$$\frac{1}{d}\sqrt{1-i(\tan(dx+c))^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+i(\tan(dx+c))^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticPi}\left(\tan(dx+c)\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\sqrt{a}\frac{1}{\sqrt{b}},\sqrt{-i\sqrt{b}\frac{1}{\sqrt{a}}}\frac{1}{\sqrt{i\sqrt{b}\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^4)^(1/2),x)
```

```
[Out] 1/d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)/(a+b*tan(d*x+c)^4)^(1/2)*EllipticPi(ta
n(d*x+c)*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(
1/2)/(I/a^(1/2)*b^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\tan\left(dx+c\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*tan(d*x + c)^4 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\tan^4\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)**4*b)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*tan(c + d*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\tan\left(dx+c\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*tan(d*x + c)^4 + a), x)
```

3.389 $\int \tan^3(x)\sqrt{a+b\tan^4(x)}\,dx$

Optimal. Leaf size=103

$$-\frac{1}{4}\left(2-\tan^2(x)\right)\sqrt{a+b\tan^4(x)}+\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{4\sqrt{b}}+\frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)$$

```
[Out] ((a + 2*b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/(4*Sqrt[b]) +
(Sqrt[a + b]*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/
2 - ((2 - Tan[x]^2)*Sqrt[a + b*Tan[x]^4])/4
```

Rubi [A] time = 0.206797, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3670, 1252, 815, 844, 217, 206, 725}

$$-\frac{1}{4}\left(2-\tan^2(x)\right)\sqrt{a+b\tan^4(x)}+\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{4\sqrt{b}}+\frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3*Sqrt[a + b*Tan[x]^4],x]
```


```
[Out] ((a + 2*b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/(4*Sqrt[b]) +
(Sqrt[a + b]*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/
2 - ((2 - Tan[x]^2)*Sqrt[a + b*Tan[x]^4])/4
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$
\begin{aligned}
\int \tan^3(x)\sqrt{a+b\tan^4(x)}\,dx &= \text{Subst}\left(\int \frac{x^3\sqrt{a+bx^4}}{1+x^2}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x\sqrt{a+bx^2}}{1+x}\,dx, x, \tan^2(x)\right)\\
&= -\frac{1}{4}\left(2-\tan^2(x)\right)\sqrt{a+b\tan^4(x)}+\frac{\text{Subst}\left(\int \frac{-ab+b(a+2b)x}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)}{4b}\\
&= -\frac{1}{4}\left(2-\tan^2(x)\right)\sqrt{a+b\tan^4(x)}+\frac{1}{2}(-a-b)\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)\\
&= -\frac{1}{4}\left(2-\tan^2(x)\right)\sqrt{a+b\tan^4(x)}+\frac{1}{2}(a+b)\,\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)\\
&= \frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{4\sqrt{b}}+\frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)-\frac{1}{4}\left(2-\tan^2
\end{aligned}
$$

Mathematica [A] time = 3.67383, size = 145, normalized size = 1.41

$$
\frac{1}{4}\left(\frac{\frac{a^{3/2}\sqrt{\frac{b\tan^4(x)}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a}}\right)}{\sqrt{b}}+\left(\tan^2(x)-2\right)\left(a+b\tan^4(x)\right)}{\sqrt{a+b\tan^4(x)}}+2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)+2\sqrt{a+b}\tanh^{-1}\left(\frac{}{\sqrt{}}\right.\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3*Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]] + 2*Sqrt[a + b]
*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] + ((-2 + Tan[
x]^2)*(a + b*Tan[x]^4) + (a^(3/2)*ArcSinh[(Sqrt[b]*Tan[x]^2)/Sqrt[a]]*Sqrt[
1 + (b*Tan[x]^4)/a])/Sqrt[b])/Sqrt[a + b*Tan[x]^4])/4
```

Maple [B] time = 0.085, size = 181, normalized size = 1.8

$$\frac{(\tan(x))^2}{4}\sqrt{a+b(\tan(x))^4}+\frac{a}{4}\ln\left(\sqrt{b}(\tan(x))^2+\sqrt{a+b(\tan(x))^4}\right)\frac{1}{\sqrt{b}}-\frac{1}{2}\sqrt{b\left(1+(\tan(x))^2\right)^2-2b\left(1+(\tan(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(x)^4)^(1/2)*tan(x)^3,x)
```

```
[Out] 1/4*(a+b*tan(x)^4)^(1/2)*tan(x)^2+1/4*a/b^(1/2)*ln(b^(1/2)*tan(x)^2+(a+b*ta
n(x)^4)^(1/2))-1/2*(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2)+1/2*b^(1/2
)*ln((b*(1+tan(x)^2)-b)/b^(1/2)+(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/
2))+1/2*(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)
^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\tan(x)^4+a}\tan(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(x)^4 + a)*tan(x)^3, x)
```

Fricas [A] time = 2.67354, size = 1443, normalized size = 14.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x)^3,x, algorithm="fricas")
```

```
[Out] [1/8*((a + 2*b)*sqrt(b)*log(-2*b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*
tan(x)^2 - a) + 2*sqrt(a + b)*b*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^
2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan
(x)^4 + 2*tan(x)^2 + 1)) + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*b))/b, -1
/4*((a + 2*b)*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) -
 sqrt(a + b)*b*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(
x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^
2 + 1)) - sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*b))/b, 1/8*(4*sqrt(-a - b)*b
*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan
(x)^4 + a^2 + a*b)) + (a + 2*b)*sqrt(b)*log(-2*b*tan(x)^4 - 2*sqrt(b*tan(x)
^4 + a)*sqrt(b)*tan(x)^2 - a) + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*b))/
b, 1/4*(2*sqrt(-a - b)*b*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(
-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) - (a + 2*b)*sqrt(-b)*arctan(sqr
```

```
t(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + sqrt(b*tan(x)^4 + a)*(b*tan(x)^2
 - 2*b))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\tan^4(x)}\tan^3(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)**4)**(1/2)*tan(x)**3,x)
```

```
[Out] Integral(sqrt(a + b*tan(x)**4)*tan(x)**3, x)
```

Giac [A] time = 1.22797, size = 144, normalized size = 1.4

$$\frac{1}{4}\sqrt{b\tan(x)^4+a}\left(\tan(x)^2-2\right)-\frac{(a+b)\arctan\left(-\frac{\sqrt{b}\tan(x)^2-\sqrt{b\tan(x)^4+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}-\frac{\left(a\sqrt{b}+2b^{\frac{3}{2}}\right)\log\left(\left|-\sqrt{b}\tan(x)^2+\sqrt{b\tan(x)^4+a}\right|\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x)^3,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*tan(x)^4 + a)*(tan(x)^2 - 2) - (a + b)*arctan(-(sqrt(b)*tan(x)^2
 - sqrt(b*tan(x)^4 + a) + sqrt(b))/sqrt(-a - b))/sqrt(-a - b) - 1/4*(a*sqrt
(b) + 2*b^(3/2))*log(abs(-sqrt(b)*tan(x)^2 + sqrt(b*tan(x)^4 + a)))/b
```

3.390 $\int \tan(x)\sqrt{a+b\tan^4(x)}\,dx$

Optimal. Leaf size=90

$$\frac{1}{2}\sqrt{a+b\tan^4(x)}-\frac{1}{2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)-\frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)$$

```
[Out] -(Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/2 - (Sqrt[a + b
]*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/2 + Sqrt[a
+ b*Tan[x]^4]/2
```

Rubi [A] time = 0.117951, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3670, 1248, 735, 844, 217, 206, 725}

$$\frac{1}{2}\sqrt{a+b\tan^4(x)}-\frac{1}{2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)-\frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]*Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] -(Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/2 - (Sqrt[a + b
]*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/2 + Sqrt[a
+ b*Tan[x]^4]/2
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}
\int \tan(x)\sqrt{a+b\tan^4(x)}\,dx &= \text{Subst}\left(\int \frac{x\sqrt{a+bx^4}}{1+x^2}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1+x}\,dx, x, \tan^2(x)\right)\\
&= \frac{1}{2}\sqrt{a+b\tan^4(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{a-bx}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)\\
&= \frac{1}{2}\sqrt{a+b\tan^4(x)} - \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right) + \frac{1}{2}(a+b)\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{}}\right.\\
&= \frac{1}{2}\sqrt{a+b\tan^4(x)} + \frac{1}{2}(-a-b)\,\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) - \frac{1}{2}b\,\text{Subst}\left(\int\right.\\
&= -\frac{1}{2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) - \frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right) + \frac{1}{2}\sqrt{a+b\,\mathrm{t}}
\end{aligned}$$

Mathematica [A] time = 0.0405648, size = 86, normalized size = 0.96

$$\frac{1}{2}\left(\sqrt{a+b\tan^4(x)} - \sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) - \sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]*Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] (-(Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]]) - Sqrt[a + b]*
ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] + Sqrt[a + b*T
an[x]^4])/2
```

Maple [A] time = 0.048, size = 139, normalized size = 1.5

$$\frac{1}{2}\sqrt{b\left(1+(\tan(x))^2\right)^2-2b\left(1+(\tan(x))^2\right)+a+b}-\frac{1}{2}\sqrt{b}\ln\left(\left(b\left(1+(\tan(x))^2\right)-b\right)\frac{1}{\sqrt{b}}+\sqrt{b\left(1+(\tan(x))^2\right)^2-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(x)^4)^(1/2)*tan(x),x)
```

```
[Out] 1/2*(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2)-1/2*b^(1/2)*ln((b*(1+tan(
x)^2)-b)/b^(1/2)+(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2))-1/2*(a+b)^(
1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*(1+ta
n(x)^2)+a+b)^(1/2))/(1+tan(x)^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\tan(x)^4+a}\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(x)^4 + a)*tan(x), x)
```

Fricas [A] time = 2.48373, size = 1285, normalized size = 14.28

$$\left[\frac{1}{4}\sqrt{b}\log\left(-2b\tan(x)^4+2\sqrt{b\tan(x)^4+a}\sqrt{b}\tan(x)^2-a\right)+\frac{1}{4}\sqrt{a+b}\log\left(\frac{\left(ab+2b^2\right)\tan(x)^4-2ab\tan(x)^2+2}{\tan(x)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(b)*log(-2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 -
a) + 1/4*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 + 2*sqrt(
b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*t
an(x)^2 + 1)) + 1/2*sqrt(b*tan(x)^4 + a), 1/2*sqrt(-b)*arctan(sqrt(b*tan(x)
^4 + a)*sqrt(-b)/(b*tan(x)^2)) + 1/4*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^
4 - 2*a*b*tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) +
2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 1/2*sqrt(b*tan(x)^4 + a), -1/2*
sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*
b + b^2)*tan(x)^4 + a^2 + a*b)) + 1/4*sqrt(b)*log(-2*b*tan(x)^4 + 2*sqrt(b*
tan(x)^4 + a)*sqrt(b)*tan(x)^2 - a) + 1/2*sqrt(b*tan(x)^4 + a), -1/2*sqrt(-
a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^
2)*tan(x)^4 + a^2 + a*b)) + 1/2*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-
b)/(b*tan(x)^2)) + 1/2*sqrt(b*tan(x)^4 + a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\tan^4(x)}\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)**4)**(1/2)*tan(x),x)
```

```
[Out] Integral(sqrt(a + b*tan(x)**4)*tan(x), x)
```

Giac [A] time = 1.18093, size = 117, normalized size = 1.3

$$-\sqrt{-a-b}\arctan\left(-\frac{\sqrt{b}\tan(x)^2-\sqrt{b\tan(x)^4+a}+\sqrt{b}}{\sqrt{-a-b}}\right)+\frac{1}{2}\sqrt{b}\log\left(\left|-\sqrt{b}\tan(x)^2+\sqrt{b\tan(x)^4+a}\right|\right)+\frac{1}{2}\sqrt{b\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x),x, algorithm="giac")
```

```
[Out] -sqrt(-a - b)*arctan(-(sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a) + sqrt(b))/s
qrt(-a - b)) + 1/2*sqrt(b)*log(abs(-sqrt(b)*tan(x)^2 + sqrt(b*tan(x)^4 + a)
)) + 1/2*sqrt(b*tan(x)^4 + a)
```

3.391 $\int \cot(x)\sqrt{a+b\tan^4(x)}\,dx$

Optimal. Leaf size=102

$$\frac{1}{2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)+\frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)-\frac{1}{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)$$

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/2 + (Sqrt[a + b]
*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/2 - (Sqrt[a]
*ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]])/2
```

Rubi [A] time = 0.170897, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3670, 1252, 896, 266, 63, 208, 844, 217, 206, 725}

$$\frac{1}{2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)+\frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)-\frac{1}{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/2 + (Sqrt[a + b]
*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/2 - (Sqrt[a]
*ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]])/2
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 896

```
Int[((a_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))),
 x_Symbol] :> Dist[(c*d^2 + a*e^2)/(e*(e*f - d*g)), Int[(a + c*x^2)^(p - 1)
/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), Int[(Simp[c*d*f + a*e*g - c*(e
*f - d*g)*x, x]*(a + c*x^2)^(p - 1))/(f + g*x), x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[p] &
& GtQ[p, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \cot(x)\sqrt{a+b\tan^4(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+bx^4}}{x\left(1+x^2\right)}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x(1+x)}\,dx, x, \tan^2(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a-bx}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)\right) + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, \tan^2\right.$$
$$= \frac{1}{4}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tan^4(x)\right) + \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right) - \frac{1}{2}(a$$
$$= -\left(\frac{1}{2}(-a-b)\,\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)\right) + \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \right.}{2b}$$
$$= \frac{1}{2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) + \frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right) - \frac{1}{2}\sqrt{a}\tan$$

Mathematica [A] time = 0.069431, size = 98, normalized size = 0.96

$$\frac{1}{2}\left(\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) + \sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right) - \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]] + Sqrt[a + b]*Arc
Tanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] - Sqrt[a]*ArcTanh
[Sqrt[a + b*Tan[x]^4]/Sqrt[a]])/2
```

Maple [F] time = 0.16, size = 0, normalized size = 0.

$$\int \cot(x)\sqrt{a+b(\tan(x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+b*tan(x)^4)^(1/2),x)
```

```
[Out] int(cot(x)*(a+b*tan(x)^4)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan(x)^4 + a}\cot(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(x)^4 + a)*cot(x), x)
```

Fricas [A] time = 3.58301, size = 2809, normalized size = 27.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(b)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 + a
) + 1/4*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b
*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*ta
n(x)^2 + 1)) + 1/4*sqrt(a)*log((b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(a)
 + 2*a)/tan(x)^4), -1/2*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*ta
n(x)^2)) + 1/4*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2
*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4
 + 2*tan(x)^2 + 1)) + 1/4*sqrt(a)*log((b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*
sqrt(a) + 2*a)/tan(x)^4), 1/2*sqrt(-a)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)
/a) + 1/4*sqrt(b)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^
2 + a) + 1/4*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*s
qrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 +
 2*tan(x)^2 + 1)), 1/2*sqrt(-a)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) - 1
/2*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + 1/4*sqrt(a
 + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)
*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)),
1/2*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a - b)/(b*tan(x)^2 - a))
 + 1/4*sqrt(b)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 +
 a) + 1/4*sqrt(a)*log((b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(a) + 2*a)/t
an(x)^4), 1/2*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a - b)/(b*tan(
x)^2 - a)) - 1/2*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)
) + 1/4*sqrt(a)*log((b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(a) + 2*a)/tan
(x)^4), 1/2*sqrt(-a)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + 1/2*sqrt(-a
- b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a - b)/(b*tan(x)^2 - a)) + 1/4*sqrt(
b)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 + a), 1/2*sqr
t(-a)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + 1/2*sqrt(-a - b)*arctan(sqr
t(b*tan(x)^4 + a)*sqrt(-a - b)/(b*tan(x)^2 - a)) - 1/2*sqrt(-b)*arctan(sqrt
(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\tan^4(x)}\cot(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(x)**4)*cot(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan(x)^4 + a}\cot(x)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(x)^4 + a)*cot(x), x)
```

3.392 $\int \tan^2(x)\sqrt{a+b\tan^4(x)}\,dx$

Optimal. Leaf size=643

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+b\tan^4(x)}}+\frac{\sqrt[4]{b}(a+b)\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x}{\left(\sqrt{a}+\sqrt{b}\tan^2(\right.}}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{a}}$$

```
[Out] -(Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a + b*Tan[x]^4]])/2 + (Tan[x
]*Sqrt[a + b*Tan[x]^4])/3 - (Sqrt[b]*Tan[x]*Sqrt[a + b*Tan[x]^4])/(Sqrt[a]
+ Sqrt[b]*Tan[x]^2) + (a^(1/4)*b^(1/4)*EllipticE[2*ArcTan[(b^(1/4)*Tan[x])/
a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(a + b*Tan[x]^4)/(Sqrt[a]
+ Sqrt[b]*Tan[x]^2)^2])/Sqrt[a + b*Tan[x]^4] + (a^(3/4)*EllipticF[2*ArcTan[
(b^(1/4)*Tan[x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(a + b*Ta
n[x]^4)/(Sqrt[a] + Sqrt[b]*Tan[x]^2)^2])/(3*b^(1/4)*Sqrt[a + b*Tan[x]^4]) -
 ((Sqrt[a] - Sqrt[b])*b^(1/4)*EllipticF[2*ArcTan[(b^(1/4)*Tan[x])/a^(1/4)],
 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(a + b*Tan[x]^4)/(Sqrt[a] + Sqrt[b]
*Tan[x]^2)^2])/(2*a^(1/4)*Sqrt[a + b*Tan[x]^4]) + (b^(1/4)*(a + b)*Elliptic
F[2*ArcTan[(b^(1/4)*Tan[x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqr
t[(a + b*Tan[x]^4)/(Sqrt[a] + Sqrt[b]*Tan[x]^2)^2])/(2*a^(1/4)*(Sqrt[a] - S
qrt[b])*Sqrt[a + b*Tan[x]^4]) - ((Sqrt[a] + Sqrt[b])*(a + b)*EllipticPi[-(S
qrt[a] - Sqrt[b])^2/(4*Sqrt[a]*Sqrt[b]), 2*ArcTan[(b^(1/4)*Tan[x])/a^(1/4)]
, 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(a + b*Tan[x]^4)/(Sqrt[a] + Sqrt[b
]*Tan[x]^2)^2])/(4*a^(1/4)*(Sqrt[a] - Sqrt[b])*b^(1/4)*Sqrt[a + b*Tan[x]^4]
)
```

Rubi [A] time = 0.49573, antiderivative size = 643, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {3670, 1336, 195, 220, 1209, 1198, 1196, 1217, 1707}

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+b\tan^4(x)}}-\frac{1}{2}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a+b\tan^4(x)}}\right)+\frac{1}{3}\tan(x)\sqrt{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2*Sqrt[a + b*Tan[x]^4],x]
```


```
[Out] -(Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a + b*Tan[x]^4]])/2 + (Tan[x
]*Sqrt[a + b*Tan[x]^4])/3 - (Sqrt[b]*Tan[x]*Sqrt[a + b*Tan[x]^4])/(Sqrt[a]
+ Sqrt[b]*Tan[x]^2) + (a^(1/4)*b^(1/4)*EllipticE[2*ArcTan[(b^(1/4)*Tan[x])/
a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(a + b*Tan[x]^4)/(Sqrt[a]
+ Sqrt[b]*Tan[x]^2)^2])/Sqrt[a + b*Tan[x]^4] + (a^(3/4)*EllipticF[2*ArcTan[
(b^(1/4)*Tan[x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(a + b*Ta
n[x]^4)/(Sqrt[a] + Sqrt[b]*Tan[x]^2)^2])/(3*b^(1/4)*Sqrt[a + b*Tan[x]^4]) -
 ((Sqrt[a] - Sqrt[b])*b^(1/4)*EllipticF[2*ArcTan[(b^(1/4)*Tan[x])/a^(1/4)],
 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(a + b*Tan[x]^4)/(Sqrt[a] + Sqrt[b]
*Tan[x]^2)^2])/(2*a^(1/4)*Sqrt[a + b*Tan[x]^4]) + (b^(1/4)*(a + b)*Elliptic
F[2*ArcTan[(b^(1/4)*Tan[x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqr
t[(a + b*Tan[x]^4)/(Sqrt[a] + Sqrt[b]*Tan[x]^2)^2])/(2*a^(1/4)*(Sqrt[a] - S
qrt[b])*Sqrt[a + b*Tan[x]^4]) - ((Sqrt[a] + Sqrt[b])*(a + b)*EllipticPi[-(S
qrt[a] - Sqrt[b])^2/(4*Sqrt[a]*Sqrt[b]), 2*ArcTan[(b^(1/4)*Tan[x])/a^(1/4)]
, 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(a + b*Tan[x]^4)/(Sqrt[a] + Sqrt[b
]*Tan[x]^2)^2])/(4*a^(1/4)*(Sqrt[a] - Sqrt[b])*b^(1/4)*Sqrt[a + b*Tan[x]^4]
```

)

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 1336

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
```

```
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \tan^2(x)\sqrt{a+b\tan^4(x)}\,dx = \text{Subst}\left(\int \frac{x^2\sqrt{a+bx^4}}{1+x^2}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\sqrt{a+bx^4}-\frac{\sqrt{a+bx^4}}{1+x^2}\right)dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \sqrt{a+bx^4}\,dx, x, \tan(x)\right) - \text{Subst}\left(\int \frac{\sqrt{a+bx^4}}{1+x^2}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{3}\tan(x)\sqrt{a+b\tan^4(x)} + \frac{1}{3}(2a)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \tan(x)\right) - (a+b)\,\text{Subst}\left(\int \right.$$
$$= \frac{1}{3}\tan(x)\sqrt{a+b\tan^4(x)} + \frac{a^{3/4}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x}{\left(\sqrt{a}+\sqrt{b}\tan^2\right.}}}{3\sqrt[4]{b}\sqrt{a+b\tan^4(x)}}$$
$$= -\frac{1}{2}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a+b\tan^4(x)}}\right) + \frac{1}{3}\tan(x)\sqrt{a+b\tan^4(x)} - \frac{\sqrt{b}\tan(x)\sqrt{a+b\tan^4(x}}{\sqrt{a}+\sqrt{b}\tan^2(x)}$$

Mathematica [C] time = 16.8358, size = 550, normalized size = 0.86

$$\left(\frac{\tan(x)}{3}-\frac{1}{2}\sin(2x)\right)\sqrt{\frac{4a\cos(2x)+a\cos(4x)+3a-4b\cos(2x)+b\cos(4x)+3b}{4\cos(2x)+\cos(4x)+3}} + \frac{\left(3\sqrt{a}\sqrt{b}-2ia-3ib\right)\left(\tan^2(x)+1\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2*Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] Sqrt[(3*a + 3*b + 4*a*Cos[2*x] - 4*b*Cos[2*x] + a*Cos[4*x] + b*Cos[4*x])/(3
 + 4*Cos[2*x] + Cos[4*x])]*(-Sin[2*x]/2 + Tan[x]/3) + (3*a*Sqrt[(I*Sqrt[b])
/Sqrt[a]]*Tan[x] + 3*Sqrt[(I*Sqrt[b])/Sqrt[a]]*b*Tan[x]^5 + (3*I)*a*Ellipti
cPi[((-I)*Sqrt[a])/Sqrt[b], I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[x]], -1
]*Sqrt[1 + (b*Tan[x]^4)/a] + (3*I)*b*EllipticPi[((-I)*Sqrt[a])/Sqrt[b], I*A
rcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[x]], -1]*Sqrt[1 + (b*Tan[x]^4)/a] + (3
*I)*a*EllipticPi[((-I)*Sqrt[a])/Sqrt[b], I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]
]*Tan[x]], -1]*Tan[x]^2*Sqrt[1 + (b*Tan[x]^4)/a] + (3*I)*b*EllipticPi[((-I)
*Sqrt[a])/Sqrt[b], I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[x]], -1]*Tan[x]^
```

```
2*Sqrt[1 + (b*Tan[x]^4)/a] - 3*Sqrt[a]*Sqrt[b]*EllipticE[I*ArcSinh[Sqrt[(I*
Sqrt[b])/Sqrt[a]]*Tan[x]], -1]*(1 + Tan[x]^2)*Sqrt[1 + (b*Tan[x]^4)/a] + ((
-2*I)*a + 3*Sqrt[a]*Sqrt[b] - (3*I)*b)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])
/Sqrt[a]]*Tan[x]], -1]*(1 + Tan[x]^2)*Sqrt[1 + (b*Tan[x]^4)/a])/(3*Sqrt[(I*
Sqrt[b])/Sqrt[a]]*(1 + Tan[x]^2)*Sqrt[a + b*Tan[x]^4])
```

Maple [C] time = 0.056, size = 537, normalized size = 0.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(x)^4)^(1/2)*tan(x)^2,x)
```

```
[Out] 1/3*(a+b*tan(x)^4)^(1/2)*tan(x)+2/3*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2
)*b^(1/2)*tan(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)/(a+b*tan(x)^
4)^(1/2)*EllipticF(tan(x)*(I/a^(1/2)*b^(1/2))^(1/2),I)+b/(I/a^(1/2)*b^(1/2)
)^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(x)^2)
^(1/2)/(a+b*tan(x)^4)^(1/2)*EllipticF(tan(x)*(I/a^(1/2)*b^(1/2))^(1/2),I)-I
*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^2)^(
1/2)*(1+I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)/(a+b*tan(x)^4)^(1/2)*EllipticF(ta
n(x)*(I/a^(1/2)*b^(1/2))^(1/2),I)+I*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/
2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2
)/(a+b*tan(x)^4)^(1/2)*EllipticE(tan(x)*(I/a^(1/2)*b^(1/2))^(1/2),I)-a/(I/a
^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)*(1+I/a^(1/2)*b^(
1/2)*tan(x)^2)^(1/2)/(a+b*tan(x)^4)^(1/2)*EllipticPi(tan(x)*(I/a^(1/2)*b^(1
/2))^(1/2),I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))
^(1/2))-b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)*(1
+I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)/(a+b*tan(x)^4)^(1/2)*EllipticPi(tan(x)*(
I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^
(1/2)*b^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan(x)^4 + a}\tan(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(x)^4 + a)*tan(x)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \tan^4(x)} \tan^2(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)**4)**(1/2)*tan(x)**2,x)
```

```
[Out] Integral(sqrt(a + b*tan(x)**4)*tan(x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \tan(x)^4 + a} \tan(x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(x)^4 + a)*tan(x)^2, x)
```

3.393 $\int \tan^3(x)\left(a+b\tan^4(x)\right)^{3/2}\,dx$

Optimal. Leaf size=148

$$\frac{(3a^2+12ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{16\sqrt{b}}-\frac{1}{24}\left(4-3\tan^2(x)\right)\left(a+b\tan^4(x)\right)^{3/2}-\frac{1}{16}\left(8(a+b)-(3a+4b)\tan^2(x)\right)$$

```
[Out] ((3*a^2 + 12*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])
/(16*Sqrt[b]) + ((a + b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a
 + b*Tan[x]^4])])/2 - ((8*(a + b) - (3*a + 4*b)*Tan[x]^2)*Sqrt[a + b*Tan[x]
^4])/16 - ((4 - 3*Tan[x]^2)*(a + b*Tan[x]^4)^(3/2))/24
```

Rubi [A] time = 0.31002, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3670, 1252, 815, 844, 217, 206, 725}

$$\frac{(3a^2+12ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{16\sqrt{b}}-\frac{1}{24}\left(4-3\tan^2(x)\right)\left(a+b\tan^4(x)\right)^{3/2}-\frac{1}{16}\left(8(a+b)-(3a+4b)\tan^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3*(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] ((3*a^2 + 12*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])
/(16*Sqrt[b]) + ((a + b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a
 + b*Tan[x]^4])])/2 - ((8*(a + b) - (3*a + 4*b)*Tan[x]^2)*Sqrt[a + b*Tan[x]
^4])/16 - ((4 - 3*Tan[x]^2)*(a + b*Tan[x]^4)^(3/2))/24
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
```

```
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \tan^3(x)\left(a+b\tan^4(x)\right)^{3/2}dx = \text{Subst}\left(\int \frac{x^3\left(a+bx^4\right)^{3/2}}{1+x^2}dx, x, \tan(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{x\left(a+bx^2\right)^{3/2}}{1+x}dx, x, \tan^2(x)\right)$$

$$= -\frac{1}{24}\left(4-3\tan^2(x)\right)\left(a+b\tan^4(x)\right)^{3/2} + \frac{\text{Subst}\left(\int \frac{(-ab+b(3a+4b)x)\sqrt{a+bx^2}}{1+x}dx, x, \tan^2(x)\right)}{8b}$$

$$= -\frac{1}{16}\left(8(a+b)-(3a+4b)\tan^2(x)\right)\sqrt{a+b\tan^4(x)} - \frac{1}{24}\left(4-3\tan^2(x)\right)\left(a+b\tan^4(x\right.$$

$$= -\frac{1}{16}\left(8(a+b)-(3a+4b)\tan^2(x)\right)\sqrt{a+b\tan^4(x)} - \frac{1}{24}\left(4-3\tan^2(x)\right)\left(a+b\tan^4(x\right.$$

$$= -\frac{1}{16}\left(8(a+b)-(3a+4b)\tan^2(x)\right)\sqrt{a+b\tan^4(x)} - \frac{1}{24}\left(4-3\tan^2(x)\right)\left(a+b\tan^4(x\right.$$

$$= \frac{\left(3a^2+12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{16\sqrt{b}} + \frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x\right.$$

Mathematica [A] time = 6.04437, size = 189, normalized size = 1.28

$$\frac{1}{48}\left(\sqrt{a+b\tan^4(x)}\left(3(5a+4b)\tan^2(x)-8(4a+3b)+6b\tan^6(x)-8b\tan^4(x)\right)+24(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\tan}{\sqrt{a+b}\sqrt{a+}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3*(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] (24*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]] + 24*(
a + b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] +
 (3*Sqrt[a]*(3*a + 4*b)*ArcSinh[(Sqrt[b]*Tan[x]^2)/Sqrt[a]]*Sqrt[a + b*Tan[
x]^4])/(Sqrt[b]*Sqrt[1 + (b*Tan[x]^4)/a]) + Sqrt[a + b*Tan[x]^4]*(-8*(4*a +
 3*b) + 3*(5*a + 4*b)*Tan[x]^2 - 8*b*Tan[x]^4 + 6*b*Tan[x]^6))/48
```

Maple [B] time = 0.058, size = 374, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3*(a+b*tan(x)^4)^(3/2),x)
```

```
[Out] 1/8*b*tan(x)^6*(a+b*tan(x)^4)^(1/2)+5/16*a*tan(x)^2*(a+b*tan(x)^4)^(1/2)+3/
16*a^2*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)^4)^(1/2))/b^(1/2)-1/6*b*tan(x)^4*(a+
b*tan(x)^4)^(1/2)-2/3*a*(a+b*tan(x)^4)^(1/2)+1/4*b*tan(x)^2*(a+b*tan(x)^4)^
(1/2)+3/4*a*b^(1/2)*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)^4)^(1/2))-1/2*b*(a+b*ta
n(x)^4)^(1/2)+1/2*b^(3/2)*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)^4)^(1/2))+1/2/(a+
b)^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*(
1+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^2))*a^2+1/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+
tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2))/(1+t
an(x)^2))*a*b+1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b
*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan(x)^4+a\right)^{\frac{3}{2}}\tan(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3*(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(x)^4 + a)^(3/2)*tan(x)^3, x)
```

Fricas [A] time = 4.99827, size = 1937, normalized size = 13.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3*(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(3*a^2 + 12*a*b + 8*b^2)*sqrt(b)*log(-2*b*tan(x)^4 - 2*sqrt(b*tan(
x)^4 + a)*sqrt(b)*tan(x)^2 - a) + 24*(a*b + b^2)*sqrt(a + b)*log(((a*b + 2*
b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sq
rt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 2*(6*b^2*tan(x)^6 -
 8*b^2*tan(x)^4 + 3*(5*a*b + 4*b^2)*tan(x)^2 - 32*a*b - 24*b^2)*sqrt(b*tan(
x)^4 + a))/b, -1/48*(3*(3*a^2 + 12*a*b + 8*b^2)*sqrt(-b)*arctan(sqrt(b*tan(
x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) - 12*(a*b + b^2)*sqrt(a + b)*log(((a*b + 2
*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*s
qrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) - (6*b^2*tan(x)^6 -
8*b^2*tan(x)^4 + 3*(5*a*b + 4*b^2)*tan(x)^2 - 32*a*b - 24*b^2)*sqrt(b*tan(x
)^4 + a))/b, 1/96*(48*(a*b + b^2)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*
(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) + 3*(3*a^
2 + 12*a*b + 8*b^2)*sqrt(b)*log(-2*b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt
(b)*tan(x)^2 - a) + 2*(6*b^2*tan(x)^6 - 8*b^2*tan(x)^4 + 3*(5*a*b + 4*b^2)*
tan(x)^2 - 32*a*b - 24*b^2)*sqrt(b*tan(x)^4 + a))/b, 1/48*(24*(a*b + b^2)*s
qrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b
 + b^2)*tan(x)^4 + a^2 + a*b)) - 3*(3*a^2 + 12*a*b + 8*b^2)*sqrt(-b)*arctan
(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + (6*b^2*tan(x)^6 - 8*b^2*tan(
x)^4 + 3*(5*a*b + 4*b^2)*tan(x)^2 - 32*a*b - 24*b^2)*sqrt(b*tan(x)^4 + a))/
b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tan^4(x)\right)^{\frac{3}{2}} \tan^3(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3*(a+b*tan(x)**4)**(3/2),x)
```

```
[Out] Integral((a + b*tan(x)**4)**(3/2)*tan(x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan(x)^4 + a\right)^{\frac{3}{2}} \tan(x)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3*(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(x)^4 + a)^(3/2)*tan(x)^3, x)
```

3.394 $\int \tan(x)\left(a+b\tan^4(x)\right)^{3/2}\,dx$

Optimal. Leaf size=126

$$\frac{1}{6}\left(a+b\tan^4(x)\right)^{3/2}+\frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)}-\frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)-\frac{1}{4}\sqrt{b}($$

```
[Out] -(Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/4 -
 ((a + b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])
])/2 + ((2*(a + b) - b*Tan[x]^2)*Sqrt[a + b*Tan[x]^4])/4 + (a + b*Tan[x]^4)
^(3/2)/6
```

Rubi [A] time = 0.208331, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3670, 1248, 735, 815, 844, 217, 206, 725}

$$\frac{1}{6}\left(a+b\tan^4(x)\right)^{3/2}+\frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)}-\frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)-\frac{1}{4}\sqrt{b}($$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]*(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] -(Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/4 -
 ((a + b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])
])/2 + ((2*(a + b) - b*Tan[x]^2)*Sqrt[a + b*Tan[x]^4])/4 + (a + b*Tan[x]^4)
^(3/2)/6
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_^2)]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \tan(x)\left(a+b\tan^4(x)\right)^{3/2}dx = \text{Subst}\left(\int \frac{x\left(a+bx^4\right)^{3/2}}{1+x^2}dx, x, \tan(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{1+x}dx, x, \tan^2(x)\right)$$

$$= \frac{1}{6}\left(a+b\tan^4(x)\right)^{3/2} + \frac{1}{2}\text{Subst}\left(\int \frac{(a-bx)\sqrt{a+bx^2}}{1+x}dx, x, \tan^2(x)\right)$$

$$= \frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)} + \frac{1}{6}\left(a+b\tan^4(x)\right)^{3/2} + \frac{\text{Subst}\left(\int \frac{ab(2a+b)-b^2(3}{(1+x)\sqrt{a+}}\right.}{4}$$

$$= \frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)} + \frac{1}{6}\left(a+b\tan^4(x)\right)^{3/2} + \frac{1}{2}(a+b)^2\,\text{Subst}\left(\int \right.$$

$$= \frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)} + \frac{1}{6}\left(a+b\tan^4(x)\right)^{3/2} - \frac{1}{2}(a+b)^2\,\text{Subst}\left(\int \right.$$

$$= -\frac{1}{4}\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) - \frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right.$$

Mathematica [A] time = 4.39079, size = 166, normalized size = 1.32

$$\frac{1}{12}\left(\sqrt{a+b\tan^4(x)}\left(8a+2b\tan^4(x)-3b\tan^2(x)+6b\right)-6(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)-6\sqrt{b}(a+b)\,\mathrm{t}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]*(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] (-6*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]] - 6*(a
 + b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] +
Sqrt[a + b*Tan[x]^4]*(8*a + 6*b - 3*b*Tan[x]^2 + 2*b*Tan[x]^4) - (3*Sqrt[a]
*Sqrt[b]*ArcSinh[(Sqrt[b]*Tan[x]^2)/Sqrt[a]]*Sqrt[a + b*Tan[x]^4])/Sqrt[1 +
 (b*Tan[x]^4)/a])/12
```

Maple [B] time = 0.046, size = 313, normalized size = 2.5

$$\frac{b(\tan(x))^4}{6}\sqrt{a+b(\tan(x))^4}+\frac{2a}{3}\sqrt{a+b(\tan(x))^4}-\frac{b(\tan(x))^2}{4}\sqrt{a+b(\tan(x))^4}-\frac{3a}{4}\sqrt{b}\ln\left(\sqrt{b}(\tan(x))^2+\sqrt{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a+b*tan(x)^4)^(3/2),x)
```

```
[Out] 1/6*b*tan(x)^4*(a+b*tan(x)^4)^(1/2)+2/3*a*(a+b*tan(x)^4)^(1/2)-1/4*b*tan(x)
^2*(a+b*tan(x)^4)^(1/2)-3/4*a*b^(1/2)*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)^4)^(1
/2))+1/2*b*(a+b*tan(x)^4)^(1/2)-1/2*b^(3/2)*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)
^4)^(1/2))-1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1
+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^2))*a^2-1/(a+b)^(1/2)*l
n((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2
)+a+b)^(1/2))/(1+tan(x)^2))*a*b-1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2
)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^2)
)*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan(x)^4+a\right)^{\frac{3}{2}}\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(x)^4 + a)^(3/2)*tan(x), x)
```

Fricas [A] time = 4.23001, size = 1609, normalized size = 12.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a + 2*b)*sqrt(b)*log(-2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)
*tan(x)^2 - a) + 1/4*(a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(
x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(
tan(x)^4 + 2*tan(x)^2 + 1)) + 1/12*(2*b*tan(x)^4 - 3*b*tan(x)^2 + 8*a + 6*b
)*sqrt(b*tan(x)^4 + a), 1/4*(3*a + 2*b)*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a
)*sqrt(-b)/(b*tan(x)^2)) + 1/4*(a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 -
2*a*b*tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^
2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 1/12*(2*b*tan(x)^4 - 3*b*tan(x)^2 +
 8*a + 6*b)*sqrt(b*tan(x)^4 + a), -1/2*(a + b)*sqrt(-a - b)*arctan(sqrt(b*t
an(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*
b)) + 1/8*(3*a + 2*b)*sqrt(b)*log(-2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sq
rt(b)*tan(x)^2 - a) + 1/12*(2*b*tan(x)^4 - 3*b*tan(x)^2 + 8*a + 6*b)*sqrt(b
*tan(x)^4 + a), -1/2*(a + b)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*ta
n(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) + 1/4*(3*a + 2
*b)*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + 1/12*(2*b
*tan(x)^4 - 3*b*tan(x)^2 + 8*a + 6*b)*sqrt(b*tan(x)^4 + a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tan^4(x)\right)^{\frac{3}{2}} \tan(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+b*tan(x)**4)**(3/2),x)
```

```
[Out] Integral((a + b*tan(x)**4)**(3/2)*tan(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan(x)^4 + a\right)^{\frac{3}{2}} \tan(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(x)^4 + a)^(3/2)*tan(x), x)
```

3.395 $\int \cot(x) \left(a+b\tan^4(x)\right)^{3/2} dx$

Optimal. Leaf size=155

$$-\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)+\frac{1}{2}a\sqrt{a+b\tan^4(x)}-\frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)}+\frac{1}{4}\sqrt{b}(3a+2b)\tan$$

```
[Out] (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/4 +
((a + b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]
)/2 - (a^(3/2)*ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]])/2 + (a*Sqrt[a + b*Tan
[x]^4])/2 - ((2*(a + b) - b*Tan[x]^2)*Sqrt[a + b*Tan[x]^4])/4
```

Rubi [A] time = 0.266859, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {3670, 1252, 896, 266, 50, 63, 208, 815, 844, 217, 206, 725}

$$-\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)+\frac{1}{2}a\sqrt{a+b\tan^4(x)}-\frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)}+\frac{1}{4}\sqrt{b}(3a+2b)\tan$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*(a + b*Tan[x]^4)^(3/2),x]
```


```
[Out] (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/4 +
((a + b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]
)/2 - (a^(3/2)*ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]])/2 + (a*Sqrt[a + b*Tan
[x]^4])/2 - ((2*(a + b) - b*Tan[x]^2)*Sqrt[a + b*Tan[x]^4])/4
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 896

```
Int[((a_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))),
 x_Symbol] :> Dist[(c*d^2 + a*e^2)/(e*(e*f - d*g)), Int[(a + c*x^2)^(p - 1)
/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), Int[(Simp[c*d*f + a*e*g - c*(e
*f - d*g)*x, x]*(a + c*x^2)^(p - 1))/(f + g*x), x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[p] &
& GtQ[p, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
```

```
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \cot(x)\left(a+b\tan^4(x)\right)^{3/2}dx = \text{Subst}\left(\int \frac{\left(a+bx^4\right)^{3/2}}{x\left(1+x^2\right)}dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x(1+x)}dx, x, \tan^2(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a-bx)\sqrt{a+bx^2}}{1+x}dx, x, \tan^2(x)\right)\right)+\frac{1}{2}a\,\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x}dx, x, t$$
$$= -\frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)}+\frac{1}{4}a\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}dx, x, \tan^4(x)\right)-$$
$$= \frac{1}{2}a\sqrt{a+b\tan^4(x)}-\frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)}+\frac{1}{4}a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a}$$
$$= \frac{1}{2}a\sqrt{a+b\tan^4(x)}-\frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}d$$
$$= \frac{1}{4}\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)+\frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(}$$

Mathematica [A] time = 2.99893, size = 190, normalized size = 1.23

$$\frac{1}{4}\left(-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)+b\tan^2(x)\sqrt{a+b\tan^4(x)}-2b\sqrt{a+b\tan^4(x)}+2\sqrt{b}(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan^2}{\sqrt{a+b\tan}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] (2*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]] + 2*(a
+ b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] - 2
*a^(3/2)*ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]] - 2*b*Sqrt[a + b*Tan[x]^4] +
 b*Tan[x]^2*Sqrt[a + b*Tan[x]^4] + (Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[b]*Tan[x]
^2)/Sqrt[a]]*Sqrt[a + b*Tan[x]^4])/Sqrt[1 + (b*Tan[x]^4)/a])/4
```

Maple [F] time = 0.138, size = 0, normalized size = 0.

$$\int \cot(x)\left(a+b(\tan(x))^4\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+b*tan(x)^4)^(3/2),x)
```

```
[Out] int(cot(x)*(a+b*tan(x)^4)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(x\right)^{4}+a\right)^{\frac{3}{2}}\cot\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(x)^4 + a)^(3/2)*cot(x), x)
```

Fricas [A] time = 131.972, size = 3501, normalized size = 22.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a + 2*b)*sqrt(b)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*
tan(x)^2 + a) + 1/4*(a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x
)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(t
an(x)^4 + 2*tan(x)^2 + 1)) + 1/4*a^(3/2)*log((b*tan(x)^4 - 2*sqrt(b*tan(x)^
4 + a)*sqrt(a) + 2*a)/tan(x)^4) + 1/4*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*
b), -1/4*(3*a + 2*b)*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x
)^2)) + 1/4*(a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*
sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4
+ 2*tan(x)^2 + 1)) + 1/4*a^(3/2)*log((b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*s
qrt(a) + 2*a)/tan(x)^4) + 1/4*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*b), 1/2*
sqrt(-a)*a*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + 1/8*(3*a + 2*b)*sqrt(b
)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 + a) + 1/4*(a
+ b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4
 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 +
1)) + 1/4*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*b), 1/2*sqrt(-a)*a*arctan(sq
rt(b*tan(x)^4 + a)*sqrt(-a)/a) - 1/4*(3*a + 2*b)*sqrt(-b)*arctan(sqrt(b*tan
(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + 1/4*(a + b)^(3/2)*log(((a*b + 2*b^2)*ta
n(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a +
b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 1/4*sqrt(b*tan(x)^4 + a)*(
b*tan(x)^2 - 2*b), 1/2*(a + b)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*sqr
t(-a - b)/(b*tan(x)^2 - a)) + 1/8*(3*a + 2*b)*sqrt(b)*log(2*b*tan(x)^4 + 2*
sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 + a) + 1/4*a^(3/2)*log((b*tan(x)^4 -
2*sqrt(b*tan(x)^4 + a)*sqrt(a) + 2*a)/tan(x)^4) + 1/4*sqrt(b*tan(x)^4 + a)*
(b*tan(x)^2 - 2*b), 1/2*(a + b)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*sq
rt(-a - b)/(b*tan(x)^2 - a)) - 1/4*(3*a + 2*b)*sqrt(-b)*arctan(sqrt(b*tan(x
)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + 1/4*a^(3/2)*log((b*tan(x)^4 - 2*sqrt(b*ta
n(x)^4 + a)*sqrt(a) + 2*a)/tan(x)^4) + 1/4*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2
 - 2*b), 1/2*sqrt(-a)*a*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + 1/2*(a +
b)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a - b)/(b*tan(x)^2 - a))
+ 1/8*(3*a + 2*b)*sqrt(b)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)
*tan(x)^2 + a) + 1/4*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*b), 1/2*sqrt(-a)*
a*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + 1/2*(a + b)*sqrt(-a - b)*arctan
(sqrt(b*tan(x)^4 + a)*sqrt(-a - b)/(b*tan(x)^2 - a)) - 1/4*(3*a + 2*b)*sqrt
(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + 1/4*sqrt(b*tan(x)
```

```
^4 + a)*(b*tan(x)^2 - 2*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tan^4(x)\right)^{\frac{3}{2}} \cot(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)**4)**(3/2),x)
```

```
[Out] Integral((a + b*tan(x)**4)**(3/2)*cot(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan(x)^4 + a\right)^{\frac{3}{2}} \cot(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(x)^4 + a)^(3/2)*cot(x), x)
```

3.396 $\int \frac{\tan^3(x)}{\sqrt{a+b\tan^4(x)}}\,dx$

Optimal. Leaf size=74

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}$$

```
[Out] ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]]/(2*Sqrt[b]) + ArcTanh[(a -
 b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*Sqrt[a + b])
```

Rubi [A] time = 0.128865, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 1252, 844, 217, 206, 725}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3/Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]]/(2*Sqrt[b]) + ArcTanh[(a -
 b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*Sqrt[a + b])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(x)}{\sqrt{a+b\tan^4(x)}}\,dx &= \text{Subst}\left(\int \frac{x^3}{(1+x^2)\sqrt{a+bx^4}}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}
\end{aligned}$$

Mathematica [A] time = 0.0532017, size = 74, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3/Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]]/(2*Sqrt[b]) + ArcTanh[(a -
 b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*Sqrt[a + b])
```

Maple [A] time = 0.065, size = 91, normalized size = 1.2

$$\frac{1}{2}\ln\left(\sqrt{b}(\tan(x))^2+\sqrt{a+b(\tan(x))^4}\right)\frac{1}{\sqrt{b}}+\frac{1}{2}\ln\left(\frac{1}{1+(\tan(x))^2}\left(2a+2b-2b\left(1+(\tan(x))^2\right)+2\sqrt{a+b}\sqrt{b\left(1-\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a+b*tan(x)^4)^(1/2),x)
```

```
[Out] 1/2*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)^4)^(1/2))/b^(1/2)+1/2/(a+b)^(1/2)*ln((2
*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+
```

```
b)^(1/2))/(1+tan(x)^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)^3}{\sqrt{b\tan(x)^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)^3/sqrt(b*tan(x)^4 + a), x)
```

Fricas [A] time = 2.97415, size = 1251, normalized size = 16.91

$$\left[\frac{(a+b)\sqrt{b}\log\left(-2\,b\tan(x)^4 - 2\,\sqrt{b\tan(x)^4 + a}\sqrt{b}\tan(x)^2 - a\right) + \sqrt{a+b}b\log\left(\frac{(ab+2\,b^2)\tan(x)^4 - 2\,ab\tan(x)^2 - 2\,\sqrt{b\tan(x)^4 + a}(b}{\tan(x)^4 + 2\,\tan(x)^2 + 1}\right.}{4\,(ab+b^2)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a + b)*sqrt(b)*log(-2*b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*ta
n(x)^2 - a) + sqrt(a + b)*b*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 -
2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^
4 + 2*tan(x)^2 + 1)))/(a*b + b^2), -1/4*(2*(a + b)*sqrt(-b)*arctan(sqrt(b*t
an(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) - sqrt(a + b)*b*log(((a*b + 2*b^2)*tan(
x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b)
 + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)))/(a*b + b^2), 1/4*(2*sqrt(-a -
 b)*b*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2
)*tan(x)^4 + a^2 + a*b)) + (a + b)*sqrt(b)*log(-2*b*tan(x)^4 - 2*sqrt(b*tan
(x)^4 + a)*sqrt(b)*tan(x)^2 - a))/(a*b + b^2), 1/2*(sqrt(-a - b)*b*arctan(s
qrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a
^2 + a*b)) - (a + b)*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x
)^2)))/(a*b + b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(x)}{\sqrt{a + b\tan^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3/(a+b*tan(x)**4)**(1/2),x)
```

```
[Out] Integral(tan(x)**3/sqrt(a + b*tan(x)**4), x)
```

Giac [A] time = 1.16823, size = 101, normalized size = 1.36

$$-\frac{\arctan\left(-\frac{\sqrt{b}\tan(x)^2-\sqrt{b\tan(x)^4+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}-\frac{\log\left(\left|-\sqrt{b}\tan\left(x\right)^2+\sqrt{b\tan\left(x\right)^4+a}\right|\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -arctan(-(sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a) + sqrt(b))/sqrt(-a - b))/
sqrt(-a - b) - 1/2*log(abs(-sqrt(b)*tan(x)^2 + sqrt(b*tan(x)^4 + a)))/sqrt(
b)
```

3.397 $\int \frac{\tan(x)}{\sqrt{a+b\tan^4(x)}}\,dx$

Optimal. Leaf size=41

$$-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}$$

```
[Out] -ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*Sqrt[a + b
])
```

Rubi [A] time = 0.0679814, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3670, 1248, 725, 206}

$$-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] -ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*Sqrt[a + b
])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \text{Subst}\left(\int \frac{x}{(1+x^2)\sqrt{a+bx^4}}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}$$

Mathematica [A] time = 0.0151436, size = 41, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] -ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*Sqrt[a + b
])
```

Maple [A] time = 0.053, size = 65, normalized size = 1.6

$$-\frac{1}{2}\ln\left(\frac{1}{1+(\tan(x))^2}\left(2a+2b-2b\left(1+(\tan(x))^2\right)+2\sqrt{a+b}\sqrt{b\left(1+(\tan(x))^2\right)^2-2b\left(1+(\tan(x))^2\right)+a+b}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*tan(x)^4)^(1/2),x)
```

```
[Out] -1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)
^2-2*b*(1+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)}{\sqrt{b\tan(x)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)/sqrt(b*tan(x)^4 + a), x)
```

Fricas [A] time = 2.71372, size = 385, normalized size = 9.39

$$\left[\frac{\log\left(\frac{\left(ab+2\,b^2\right)\tan(x)^4-2\,ab\tan(x)^2+2\,\sqrt{b\tan(x)^4+a}\left(b\tan(x)^2-a\right)\sqrt{a+b}+2\,a^2+ab}{\tan(x)^4+2\,\tan(x)^2+1}\right)}{4\,\sqrt{a+b}},\ -\frac{\sqrt{-a-b}\arctan\left(\frac{\sqrt{b\tan(x)^4+a}\left(b\tan(x)^2-a\right)\sqrt{-a-b}}{\left(ab+b^2\right)\tan(x)^4+a^2+ab}\right)}{2\,(a+b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)*
(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1))/sq
rt(a + b), -1/2*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*s
qrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b))/(a + b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)}{\sqrt{a+b\tan^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)**4)**(1/2),x)
```

```
[Out] Integral(tan(x)/sqrt(a + b*tan(x)**4), x)
```

Giac [A] time = 1.12953, size = 62, normalized size = 1.51

$$\frac{\arctan\left(-\frac{\sqrt{b}\tan(x)^2-\sqrt{b\tan(x)^4+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(-(sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a) + sqrt(b))/sqrt(-a - b))/s
qrt(-a - b)
```

3.398 $\int \frac{\cot(x)}{\sqrt{a+b\tan^4(x)}}\,dx$

Optimal. Leaf size=70

$$\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

```
[Out] ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*Sqrt[a + b]
) - ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]]/(2*Sqrt[a])
```

Rubi [A] time = 0.158019, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3670, 1252, 961, 725, 206, 266, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*Sqrt[a + b]
) - ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]]/(2*Sqrt[a])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot(x)}{\sqrt{a+b\tan^4(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x\left(1+x^2\right)\sqrt{a+bx^4}}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{(-1-x)\sqrt{a+bx^2}}+\frac{1}{x\sqrt{a+bx^2}}\right)dx, x, \tan^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1-x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tan^4(x)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{-a+b\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)\\
&= \frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^4(x)}\right)}{2b}\\
&= \frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}
\end{aligned}
$$

Mathematica [A] time = 0.0484304, size = 70, normalized size = 1.

$$
\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*Sqrt[a + b]
) - ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]]/(2*Sqrt[a])
```

Maple [F] time = 0.148, size = 0, normalized size = 0.

$$\int \cot(x) \frac{1}{\sqrt{a + b(\tan(x))^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*tan(x)^4)^(1/2),x)
```

```
[Out] int(cot(x)/(a+b*tan(x)^4)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{\sqrt{b\tan(x)^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(x)/sqrt(b*tan(x)^4 + a), x)
```

Fricas [A] time = 3.1643, size = 1226, normalized size = 17.51

$$\left[\frac{\sqrt{a+b}a\log\left(\frac{(ab+2b^2)\tan(x)^4-2ab\tan(x)^2-2\sqrt{b\tan(x)^4+a}(b\tan(x)^2-a)\sqrt{a+b}+2a^2+ab}{\tan(x)^4+2\tan(x)^2+1}\right)+(a+b)\sqrt{a}\log\left(-\frac{b\tan(x)^4-2\sqrt{b\tan(x)^4+a}\sqrt{a}+}{\tan(x)^4}\right.}{4(a^2+ab)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*a*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b
*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*ta
n(x)^2 + 1)) + (a + b)*sqrt(a)*log(-(b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sq
rt(a) + 2*a)/tan(x)^4))/(a^2 + a*b), 1/4*(2*sqrt(-a)*(a + b)*arctan(sqrt(b*
tan(x)^4 + a)*sqrt(-a)/a) + sqrt(a + b)*a*log(((a*b + 2*b^2)*tan(x)^4 - 2*a
*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 +
 a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)))/(a^2 + a*b), 1/4*(2*a*sqrt(-a - b)*arct
an(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4
 + a^2 + a*b)) + (a + b)*sqrt(a)*log(-(b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*
sqrt(a) + 2*a)/tan(x)^4))/(a^2 + a*b), 1/2*(a*sqrt(-a - b)*arctan(sqrt(b*ta
n(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b
```

```
)) + sqrt(-a)*(a + b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a))/(a^2 + a*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{\sqrt{a + b\tan^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)**4)**(1/2),x)
```

```
[Out] Integral(cot(x)/sqrt(a + b*tan(x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{\sqrt{b\tan(x)^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(x)/sqrt(b*tan(x)^4 + a), x)
```

3.399 $\int \frac{\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\,dx$

Optimal. Leaf size=291

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{a+b\tan^4(x)}}-\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}+\right.}{}$$

```
[Out] -ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a + b*Tan[x]^4]]/(2*Sqrt[a + b]) + (a^(1/
4)*EllipticF[2*ArcTan[(b^(1/4)*Tan[x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Ta
n[x]^2)*Sqrt[(a + b*Tan[x]^4)/(Sqrt[a] + Sqrt[b]*Tan[x]^2)^2])/(2*(Sqrt[a]
- Sqrt[b])*b^(1/4)*Sqrt[a + b*Tan[x]^4]) - ((Sqrt[a] + Sqrt[b])*EllipticPi[
-(Sqrt[a] - Sqrt[b])^2/(4*Sqrt[a]*Sqrt[b]), 2*ArcTan[(b^(1/4)*Tan[x])/a^(1/
4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(a + b*Tan[x]^4)/(Sqrt[a] + Sqr
t[b]*Tan[x]^2)^2])/(4*a^(1/4)*(Sqrt[a] - Sqrt[b])*b^(1/4)*Sqrt[a + b*Tan[x]
^4])
```

Rubi [A] time = 0.240282, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3670, 1320, 220, 1707}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{a+b\tan^4(x)}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2/Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] -ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a + b*Tan[x]^4]]/(2*Sqrt[a + b]) + (a^(1/
4)*EllipticF[2*ArcTan[(b^(1/4)*Tan[x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Ta
n[x]^2)*Sqrt[(a + b*Tan[x]^4)/(Sqrt[a] + Sqrt[b]*Tan[x]^2)^2])/(2*(Sqrt[a]
- Sqrt[b])*b^(1/4)*Sqrt[a + b*Tan[x]^4]) - ((Sqrt[a] + Sqrt[b])*EllipticPi[
-(Sqrt[a] - Sqrt[b])^2/(4*Sqrt[a]*Sqrt[b]), 2*ArcTan[(b^(1/4)*Tan[x])/a^(1/
4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(a + b*Tan[x]^4)/(Sqrt[a] + Sqr
t[b]*Tan[x]^2)^2])/(4*a^(1/4)*(Sqrt[a] - Sqrt[b])*b^(1/4)*Sqrt[a + b*Tan[x]
^4])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 1320

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2]}, -Dist[(a*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a +
c*x^4], x], x] + Dist[(a*d*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
```

```
a*e^2, 0] && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)\sqrt{a+bx^4}}\,dx, x, \tan(x)\right)$$

$$= \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \tan(x)\right)}{\sqrt{a}-\sqrt{b}} - \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\left(1+x^2\right)\sqrt{a+bx^4}}\,dx, x, \tan(x)\right)}{\sqrt{a}-\sqrt{b}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}} + \frac{\sqrt[4]{a}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}}{2\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\sqrt{a+b\tan^4(x)}} - \frac{(\sqrt{a}\ldots}{\ldots}$$

Mathematica [C] time = 2.0611, size = 122, normalized size = 0.42

$$-\frac{i\sqrt{\frac{b\tan^4(x)}{a}+1}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\tan(x)\right),-1\right)-\Pi\left(-\frac{i\sqrt{a}}{\sqrt{b}};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\tan(x)\right)\middle|-1\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+b\tan^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2/Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] ((-I)*(EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[x]], -1] - Ellipti
cPi[((-I)*Sqrt[a])/Sqrt[b], I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[x]], -1
])*Sqrt[1 + (b*Tan[x]^4)/a])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*Sqrt[a + b*Tan[x]^4
])
```

Maple [C] time = 0.059, size = 179, normalized size = 0.6

$$\sqrt{1-i(\tan(x))^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+i(\tan(x))^2\sqrt{b}\frac{1}{\sqrt{a}}}\,\text{EllipticF}\left(\tan(x)\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{a+b(\tan(x))^4}} - \sqrt{1-i(\tan\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a+b*tan(x)^4)^(1/2),x)
```

```
[Out] 1/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)*(1+I/a^(1/
2)*b^(1/2)*tan(x)^2)^(1/2)/(a+b*tan(x)^4)^(1/2)*EllipticF(tan(x)*(I/a^(1/2)
*b^(1/2))^(1/2),I)-1/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^
2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)/(a+b*tan(x)^4)^(1/2)*Elliptic
Pi(tan(x)*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^
(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)^2}{\sqrt{b\tan(x)^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*tan(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)^2/sqrt(b*tan(x)^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tan(x)^2}{\sqrt{b\tan(x)^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*tan(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(tan(x)^2/sqrt(b*tan(x)^4 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(x)}{\sqrt{a + b\tan^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2/(a+b*tan(x)**4)**(1/2),x)
```

```
[Out] Integral(tan(x)**2/sqrt(a + b*tan(x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)^2}{\sqrt{b\tan(x)^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*tan(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(x)^2/sqrt(b*tan(x)^4 + a), x)
```

3.400 $\int \frac{\tan^3(x)}{\left(a+b\tan^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=71

$$\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}-\frac{1-\tan^2(x)}{2(a+b)\sqrt{a+b\tan^4(x)}}$$

```
[Out] ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*(a + b)^(3/
2)) - (1 - Tan[x]^2)/(2*(a + b)*Sqrt[a + b*Tan[x]^4])
```

Rubi [A] time = 0.162221, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 1252, 823, 12, 725, 206}

$$\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}-\frac{1-\tan^2(x)}{2(a+b)\sqrt{a+b\tan^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3/(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*(a + b)^(3/
2)) - (1 - Tan[x]^2)/(2*(a + b)*Sqrt[a + b*Tan[x]^4])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(x)}{\left(a+b\tan^4(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{x^3}{\left(1+x^2\right)\left(a+bx^4\right)^{3/2}}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1+x)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan^2(x)\right)\\
&= -\frac{1-\tan^2(x)}{2(a+b)\sqrt{a+b\tan^4(x)}} - \frac{\text{Subst}\left(\int \frac{ab}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)}{2ab(a+b)}\\
&= -\frac{1-\tan^2(x)}{2(a+b)\sqrt{a+b\tan^4(x)}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)}{2(a+b)}\\
&= -\frac{1-\tan^2(x)}{2(a+b)\sqrt{a+b\tan^4(x)}} + \frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)}\\
&= \frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}} - \frac{1-\tan^2(x)}{2(a+b)\sqrt{a+b\tan^4(x)}}
\end{aligned}$$

Mathematica [A] time = 0.321206, size = 67, normalized size = 0.94

$$\frac{1}{2}\left(\frac{\tan^2(x)-1}{(a+b)\sqrt{a+b\tan^4(x)}} + \frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3/(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] (ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(a + b)^(3/2)
 + (-1 + Tan[x]^2)/((a + b)*Sqrt[a + b*Tan[x]^4]))/2
```

Maple [B] time = 0.076, size = 267, normalized size = 3.8

$$\frac{(\tan(x))^2}{2a}\frac{1}{\sqrt{a+b(\tan(x))^4}}+\frac{1}{4a}\sqrt{b\left((\tan(x))^2+\frac{1}{b}\sqrt{-ab}\right)^2-2\sqrt{-ab}\left((\tan(x))^2+\frac{\sqrt{-ab}}{b}\right)}\left(\sqrt{-ab}-b\right)^{-1}\left((\tan(x))\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a+b*tan(x)^4)^(3/2),x)
```

```
[Out] 1/2*tan(x)^2/a/(a+b*tan(x)^4)^(1/2)+1/4/((-a*b)^(1/2)-b)/a/(tan(x)^2+(-a*b)
^(1/2)/b)*(b*(tan(x)^2+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(tan(x)^2+(-a*b)^(1
/2)/b))^(1/2)-1/2*b/((-a*b)^(1/2)+b)/((-a*b)^(1/2)-b)/(a+b)^(1/2)*ln((2*a+2
*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(
1/2))/(1+tan(x)^2))-1/4/((-a*b)^(1/2)+b)/a/(tan(x)^2-(-a*b)^(1/2)/b)*(b*(ta
n(x)^2-(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)^(1/2)/b))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)^3}{\left(b\tan(x)^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)^3/(b*tan(x)^4 + a)^(3/2), x)
```

Fricas [B] time = 3.27235, size = 721, normalized size = 10.15

$$\left[\frac{\left(b\tan(x)^4+a\right)\sqrt{a+b}\log\left(\frac{\left(ab+2b^2\right)\tan(x)^4-2ab\tan(x)^2-2\sqrt{b\tan(x)^4+a}\left(b\tan(x)^2-a\right)\sqrt{a+b}+2a^2+ab}{\tan(x)^4+2\tan(x)^2+1}\right)+2\sqrt{b\tan(x)^4+a}\left((a+b)\right.}{4\left(\left(a^2b+2ab^2+b^3\right)\tan(x)^4+a^3+2a^2b+ab^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*tan(x)^4 + a)*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(
x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(
tan(x)^4 + 2*tan(x)^2 + 1)) + 2*sqrt(b*tan(x)^4 + a)*((a + b)*tan(x)^2 - a
- b))/((a^2*b + 2*a*b^2 + b^3)*tan(x)^4 + a^3 + 2*a^2*b + a*b^2), 1/2*((b*t
an(x)^4 + a)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt
(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) + sqrt(b*tan(x)^4 + a)*((a + b
)*tan(x)^2 - a - b))/((a^2*b + 2*a*b^2 + b^3)*tan(x)^4 + a^3 + 2*a^2*b + a*
b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(x)}{\left(a+b\tan^4(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3/(a+b*tan(x)**4)**(3/2),x)
```

```
[Out] Integral(tan(x)**3/(a + b*tan(x)**4)**(3/2), x)
```

Giac [A] time = 1.31742, size = 139, normalized size = 1.96

$$\frac{\frac{(a+b)\tan(x)^2}{a^2+2\,ab+b^2}-\frac{a+b}{a^2+2\,ab+b^2}}{2\sqrt{b\tan(x)^4+a}}+\frac{\arctan\left(\frac{\sqrt{b}\tan(x)^2-\sqrt{b\tan(x)^4+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{(a+b)\sqrt{-a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((a + b)*tan(x)^2/(a^2 + 2*a*b + b^2) - (a + b)/(a^2 + 2*a*b + b^2))/sq
rt(b*tan(x)^4 + a) + arctan((sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a) + sqrt
(b))/sqrt(-a - b))/((a + b)*sqrt(-a - b))
```

3.401 $\int \frac{\tan(x)}{\left(a+b\tan^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}$$

```
[Out] -ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*(a + b)^(3
/2)) + (a + b*Tan[x]^2)/(2*a*(a + b)*Sqrt[a + b*Tan[x]^4])
```

Rubi [A] time = 0.114571, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3670, 1248, 741, 12, 725, 206}

$$\frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] -ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*(a + b)^(3
/2)) + (a + b*Tan[x]^2)/(2*a*(a + b)*Sqrt[a + b*Tan[x]^4])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(x)}{\left(a+b\tan^4(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{x}{\left(1+x^2\right)\left(a+bx^4\right)^{3/2}}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan^2(x)\right)\\
&= \frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}+\frac{\text{Subst}\left(\int \frac{a}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)}{2a(a+b)}\\
&= \frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}+\frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)}{2(a+b)}\\
&= \frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}-\frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)}\\
&= -\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}+\frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}
\end{aligned}$$

Mathematica [A] time = 0.27361, size = 73, normalized size = 0.99

$$\frac{1}{2}\left(\frac{a+b\tan^2(x)}{a(a+b)\sqrt{a+b\tan^4(x)}}-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] (-(ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(a + b)^(3/
2)) + (a + b*Tan[x]^2)/(a*(a + b)*Sqrt[a + b*Tan[x]^4]))/2
```

Maple [B] time = 0.056, size = 248, normalized size = 3.4

$$-\frac{1}{4a}\sqrt{b\left((\tan(x))^2+\frac{1}{b}\sqrt{-ab}\right)^2-2\sqrt{-ab}\left((\tan(x))^2+\frac{\sqrt{-ab}}{b}\right)}\left(\sqrt{-ab}-b\right)^{-1}\left((\tan(x))^2+\frac{1}{b}\sqrt{-ab}\right)^{-1}+\frac{b}{2}\ln\left(\frac{}{1+(\tan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*tan(x)^4)^(3/2),x)
```

```
[Out] -1/4/((-a*b)^(1/2)-b)/a/(tan(x)^2+(-a*b)^(1/2)/b)*(b*(tan(x)^2+(-a*b)^(1/2)
/b)^2-2*(-a*b)^(1/2)*(tan(x)^2+(-a*b)^(1/2)/b))^(1/2)+1/2*b/((-a*b)^(1/2)+b
)/((-a*b)^(1/2)-b)/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(
b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^2))+1/4/((-a*b)^(1/
2)+b)/a/(tan(x)^2-(-a*b)^(1/2)/b)*(b*(tan(x)^2-(-a*b)^(1/2)/b)^2+2*(-a*b)^(
1/2)*(tan(x)^2-(-a*b)^(1/2)/b))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)}{\left(b\tan(x)^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)/(b*tan(x)^4 + a)^(3/2), x)
```

Fricas [B] time = 3.2692, size = 771, normalized size = 10.42

$$\left[\frac{\left(ab\tan(x)^4+a^2\right)\sqrt{a+b}\log\left(\frac{(ab+2b^2)\tan(x)^4-2ab\tan(x)^2+2\sqrt{b\tan(x)^4+a}\left(b\tan(x)^2-a\right)\sqrt{a+b}+2a^2+ab}{\tan(x)^4+2\tan(x)^2+1}\right)+2\sqrt{b\tan(x)^4+a}\left((ab+\right.}{4\left(\left(a^3b+2a^2b^2+ab^3\right)\tan(x)^4+a^4+2a^3b+a^2b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b*tan(x)^4 + a^2)*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*
tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*
b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 2*sqrt(b*tan(x)^4 + a)*((a*b + b^2)*tan(x
)^2 + a^2 + a*b))/((a^3*b + 2*a^2*b^2 + a*b^3)*tan(x)^4 + a^4 + 2*a^3*b + a
^2*b^2), -1/2*((a*b*tan(x)^4 + a^2)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a
)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) - sqrt(
b*tan(x)^4 + a)*((a*b + b^2)*tan(x)^2 + a^2 + a*b))/((a^3*b + 2*a^2*b^2 + a
*b^3)*tan(x)^4 + a^4 + 2*a^3*b + a^2*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)}{\left(a+b\tan^4(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)**4)**(3/2),x)
```

```
[Out] Integral(tan(x)/(a + b*tan(x)**4)**(3/2), x)
```

Giac [A] time = 1.24129, size = 161, normalized size = 2.18

$$\frac{\frac{(ab+b^2)\tan(x)^2}{a^3+2\,a^2b+ab^2}+\frac{a^2+ab}{a^3+2\,a^2b+ab^2}}{2\,\sqrt{b\tan(x)^4+a}}-\frac{\arctan\left(\frac{\sqrt{b}\tan(x)^2-\sqrt{b\tan(x)^4+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{(a+b)\sqrt{-a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((a*b + b^2)*tan(x)^2/(a^3 + 2*a^2*b + a*b^2) + (a^2 + a*b)/(a^3 + 2*a^
2*b + a*b^2))/sqrt(b*tan(x)^4 + a) - arctan((sqrt(b)*tan(x)^2 - sqrt(b*tan(
x)^4 + a) + sqrt(b))/sqrt(-a - b))/((a + b)*sqrt(-a - b))
```

3.402 $\int \frac{\cot(x)}{\left(a+b\tan^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=121

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}+\frac{1}{2a\sqrt{a+b\tan^4(x)}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}$$

```
[Out] ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*(a + b)^(3/
2)) - ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]]/(2*a^(3/2)) + 1/(2*a*Sqrt[a + b
*Tan[x]^4]) - (a + b*Tan[x]^2)/(2*a*(a + b)*Sqrt[a + b*Tan[x]^4])
```

Rubi [A] time = 0.214166, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {3670, 1252, 961, 741, 12, 725, 206, 266, 51, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}+\frac{1}{2a\sqrt{a+b\tan^4(x)}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*(a + b)^(3/
2)) - ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]]/(2*a^(3/2)) + 1/(2*a*Sqrt[a + b
*Tan[x]^4]) - (a + b*Tan[x]^2)/(2*a*(a + b)*Sqrt[a + b*Tan[x]^4])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot(x)}{\left(a+b\tan^4(x)\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1}{x\left(1+x^2\right)\left(a+bx^4\right)^{3/2}}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan^2(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{(-1-x)\left(a+bx^2\right)^{3/2}}+\frac{1}{x\left(a+bx^2\right)^{3/2}}\right)dx, x, \tan^2(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1-x)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan^2(x)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(a+bx^2\right)^{3/2}}\,dx, x, \tan^2(\right.$$
$$= -\frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \tan^4(x)\right)+\frac{\text{Subst}\left(\int \frac{a}{(-1-x)\sqrt{a}\cdots}\right.}{2a(\cdots}$$
$$= \frac{1}{2a\sqrt{a+b\tan^4(x)}}-\frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tan^4(x)\right)}{4a}+\frac{\text{Subst}\cdots}{\cdots}$$
$$= \frac{1}{2a\sqrt{a+b\tan^4(x)}}-\frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^4(x)}\right)}{2ab}-\cdots$$
$$= \frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{1}{2a\sqrt{a+b\tan^4(x)}}-\frac{a+b\tan^2(\cdots}{2a(a+b)\sqrt{a+b\cdots}}$$

Mathematica [C] time = 0.560755, size = 108, normalized size = 0.89

$$\frac{1}{2}\left(\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{b\tan^4(x)}{a}+1\right)}{a\sqrt{a+b\tan^4(x)}}-\frac{a+b\tan^2(x)}{a(a+b)\sqrt{a+b\tan^4(x)}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] (ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(a + b)^(3/2)
 + Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Tan[x]^4)/a]/(a*Sqrt[a + b*Tan[x]
^4]) - (a + b*Tan[x]^2)/(a*(a + b)*Sqrt[a + b*Tan[x]^4]))/2
```

Maple [F] time = 0.125, size = 0, normalized size = 0.

$$\int \cot(x)\left(a+b(\tan(x))^4\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*tan(x)^4)^(3/2),x)
```

```
[Out] int(cot(x)/(a+b*tan(x)^4)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 4.7339, size = 2276, normalized size = 18.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a^2*b*tan(x)^4 + a^3)*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*
b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 +
a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + ((a^2*b + 2*a*b^2 + b^3)*tan(x)^4 + a^3
 + 2*a^2*b + a*b^2)*sqrt(a)*log(-(b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(
a) + 2*a)/tan(x)^4) + 2*sqrt(b*tan(x)^4 + a)*(a^2*b + a*b^2 - (a^2*b + a*b^
2)*tan(x)^2))/(a^5 + 2*a^4*b + a^3*b^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*tan(
x)^4), 1/4*(2*((a^2*b + 2*a*b^2 + b^3)*tan(x)^4 + a^3 + 2*a^2*b + a*b^2)*sq
rt(-a)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + (a^2*b*tan(x)^4 + a^3)*sqr
t(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 +
 a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)
) + 2*sqrt(b*tan(x)^4 + a)*(a^2*b + a*b^2 - (a^2*b + a*b^2)*tan(x)^2))/(a^5
 + 2*a^4*b + a^3*b^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*tan(x)^4), 1/4*(2*(a^2
*b*tan(x)^4 + a^3)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a
)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) + ((a^2*b + 2*a*b^2 + b^
3)*tan(x)^4 + a^3 + 2*a^2*b + a*b^2)*sqrt(a)*log(-(b*tan(x)^4 - 2*sqrt(b*ta
n(x)^4 + a)*sqrt(a) + 2*a)/tan(x)^4) + 2*sqrt(b*tan(x)^4 + a)*(a^2*b + a*b^
2 - (a^2*b + a*b^2)*tan(x)^2))/(a^5 + 2*a^4*b + a^3*b^2 + (a^4*b + 2*a^3*b^
2 + a^2*b^3)*tan(x)^4), 1/2*((a^2*b*tan(x)^4 + a^3)*sqrt(-a - b)*arctan(sqr
t(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2
 + a*b)) + ((a^2*b + 2*a*b^2 + b^3)*tan(x)^4 + a^3 + 2*a^2*b + a*b^2)*sqrt(
-a)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + sqrt(b*tan(x)^4 + a)*(a^2*b +
 a*b^2 - (a^2*b + a*b^2)*tan(x)^2))/(a^5 + 2*a^4*b + a^3*b^2 + (a^4*b + 2*a
^3*b^2 + a^2*b^3)*tan(x)^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{\left(a + b\tan^4(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)**4)**(3/2),x)
```

```
[Out] Integral(cot(x)/(a + b*tan(x)**4)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{\left(b\tan(x)^4 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(x)/(b*tan(x)^4 + a)^(3/2), x)
```

3.403 $\int \frac{\tan^3(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=109

$$-\frac{(b-2a)\tan^2(x)+3a}{6a(a+b)^2\sqrt{a+b\tan^4(x)}}-\frac{1-\tan^2(x)}{6(a+b)\left(a+b\tan^4(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}}$$

```
[Out] ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*(a + b)^(5/
2)) - (1 - Tan[x]^2)/(6*(a + b)*(a + b*Tan[x]^4)^(3/2)) - (3*a + (-2*a + b)
*Tan[x]^2)/(6*a*(a + b)^2*Sqrt[a + b*Tan[x]^4])
```

Rubi [A] time = 0.227757, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 1252, 823, 12, 725, 206}

$$-\frac{(b-2a)\tan^2(x)+3a}{6a(a+b)^2\sqrt{a+b\tan^4(x)}}-\frac{1-\tan^2(x)}{6(a+b)\left(a+b\tan^4(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3/(a + b*Tan[x]^4)^(5/2),x]
```

```
[Out] ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*(a + b)^(5/
2)) - (1 - Tan[x]^2)/(6*(a + b)*(a + b*Tan[x]^4)^(3/2)) - (3*a + (-2*a + b)
*Tan[x]^2)/(6*a*(a + b)^2*Sqrt[a + b*Tan[x]^4])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx &= \text{Subst}\left(\int \frac{x^3}{\left(1+x^2\right)\left(a+bx^4\right)^{5/2}}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1+x)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan^2(x)\right)\\
&= -\frac{1-\tan^2(x)}{6(a+b)\left(a+b\tan^4(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{ab-2abx}{(1+x)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan^2(x)\right)}{6ab(a+b)}\\
&= -\frac{1-\tan^2(x)}{6(a+b)\left(a+b\tan^4(x)\right)^{3/2}} - \frac{3a-(2a-b)\tan^2(x)}{6a(a+b)^2\sqrt{a+b\tan^4(x)}} + \frac{\text{Subst}\left(\int -\frac{3a^2b^2}{(1+x)\sqrt{a+bx^2}}\,dx, x, \text{ta}\right.}{6a^2b^2(a+b)^2}\\
&= -\frac{1-\tan^2(x)}{6(a+b)\left(a+b\tan^4(x)\right)^{3/2}} - \frac{3a-(2a-b)\tan^2(x)}{6a(a+b)^2\sqrt{a+b\tan^4(x)}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\,dx, x, \text{tan}\right.}{2(a+b)^2}\\
&= -\frac{1-\tan^2(x)}{6(a+b)\left(a+b\tan^4(x)\right)^{3/2}} - \frac{3a-(2a-b)\tan^2(x)}{6a(a+b)^2\sqrt{a+b\tan^4(x)}} + \frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\tan^2}{\sqrt{a+b\tan}}\right.}{2(a+b)^2}\\
&= \frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}} - \frac{1-\tan^2(x)}{6(a+b)\left(a+b\tan^4(x)\right)^{3/2}} - \frac{3a-(2a-b)\tan^2(x)}{6a(a+b)^2\sqrt{a+b\tan^4(x)}}
\end{aligned}$$

Mathematica [A] time = 0.7698, size = 104, normalized size = 0.95

$$\frac{1}{6}\left(\frac{3a^2\tan^2(x)+b(2a-b)\tan^6(x)-3ab\tan^4(x)-a(4a+b)}{a(a+b)^2\left(a+b\tan^4(x)\right)^{3/2}}+\frac{3\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3/(a + b*Tan[x]^4)^(5/2),x]
```

```
[Out] ((3*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/(a + b)^(
5/2) + (-(a*(4*a + b)) + 3*a^2*Tan[x]^2 - 3*a*b*Tan[x]^4 + (2*a - b)*b*Tan[
x]^6)/(a*(a + b)^2*(a + b*Tan[x]^4)^(3/2)))/6
```

Maple [B] time = 0.093, size = 654, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a+b*tan(x)^4)^(5/2),x)
```

```
[Out] 1/6*(a+b*tan(x)^4)^(1/2)*tan(x)^2*(2*b*tan(x)^4+3*a)/a^2/(b^2*tan(x)^8+2*ta
n(x)^4*a*b+a^2)-1/24/((-a*b)^(1/2)+b)/a/(-a*b)^(1/2)/(tan(x)^2-(-a*b)^(1/2)
/b)^2*(b*(tan(x)^2-(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)^(1/2)/
b))^(1/2)-1/24/((-a*b)^(1/2)+b)/a^2/(tan(x)^2-(-a*b)^(1/2)/b)*(b*(tan(x)^2-
(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)^(1/2)/b))^(1/2)+1/8*(2*(-
a*b)^(1/2)-b)/((-a*b)^(1/2)-b)^2/a^2/(tan(x)^2+(-a*b)^(1/2)/b)*(b*(tan(x)^2
+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(tan(x)^2+(-a*b)^(1/2)/b))^(1/2)+1/2*b^2/
((-a*b)^(1/2)+b)^2/((-a*b)^(1/2)-b)^2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)
^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^
2))-1/8*(2*(-a*b)^(1/2)+b)/((-a*b)^(1/2)+b)^2/a^2/(tan(x)^2-(-a*b)^(1/2)/b)
*(b*(tan(x)^2-(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)^(1/2)/b))^(
1/2)-1/24/((-a*b)^(1/2)-b)/a/(-a*b)^(1/2)/(tan(x)^2+(-a*b)^(1/2)/b)^2*(b*(t
an(x)^2+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(tan(x)^2+(-a*b)^(1/2)/b))^(1/2)+1
/24/((-a*b)^(1/2)-b)/a^2/(tan(x)^2+(-a*b)^(1/2)/b)*(b*(tan(x)^2+(-a*b)^(1/2
)/b)^2-2*(-a*b)^(1/2)*(tan(x)^2+(-a*b)^(1/2)/b))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)^3}{\left(b\tan(x)^4 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)^3/(b*tan(x)^4 + a)^(5/2), x)
```

Fricas [B] time = 4.04866, size = 1273, normalized size = 11.68

$$\left[\frac{3\left(ab^2\tan(x)^8 + 2a^2b\tan(x)^4 + a^3\right)\sqrt{a+b}\log\left(\frac{\left(ab+2b^2\right)\tan(x)^4 - 2ab\tan(x)^2 - 2\sqrt{b\tan(x)^4+a}\left(b\tan(x)^2-a\right)\sqrt{a+b} + 2a^2 + ab}{\tan(x)^4 + 2\tan(x)^2 + 1}\right) + 2\left(\left(2a^2\right.\right.}{12\left(\left(a^4b^2 + 3a^3b^3 + 3a^2b^4 + ab^5\right)\tan(x)^8 + a^6 + 3a^5b + 3a^4b^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*b^2*tan(x)^8 + 2*a^2*b*tan(x)^4 + a^3)*sqrt(a + b)*log(((a*b +
2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*
sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 2*((2*a^2*b + a*b
^2 - b^3)*tan(x)^6 - 3*(a^2*b + a*b^2)*tan(x)^4 - 4*a^3 - 5*a^2*b - a*b^2 +
 3*(a^3 + a^2*b)*tan(x)^2)*sqrt(b*tan(x)^4 + a))/((a^4*b^2 + 3*a^3*b^3 + 3*
a^2*b^4 + a*b^5)*tan(x)^8 + a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 + 2*(a^5*b
+ 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*tan(x)^4), 1/6*(3*(a*b^2*tan(x)^8 + 2*a^
2*b*tan(x)^4 + a^3)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 -
a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) + ((2*a^2*b + a*b^2 - b
^3)*tan(x)^6 - 3*(a^2*b + a*b^2)*tan(x)^4 - 4*a^3 - 5*a^2*b - a*b^2 + 3*(a^
3 + a^2*b)*tan(x)^2)*sqrt(b*tan(x)^4 + a))/((a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^
4 + a*b^5)*tan(x)^8 + a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 + 2*(a^5*b + 3*a^
4*b^2 + 3*a^3*b^3 + a^2*b^4)*tan(x)^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(x)}{\left(a + b\tan^4(x)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3/(a+b*tan(x)**4)**(5/2),x)
```

```
[Out] Integral(tan(x)**3/(a + b*tan(x)**4)**(5/2), x)
```

Giac [B] time = 1.249, size = 806, normalized size = 7.39

$$\frac{\left(\left(\frac{\left(2a^7b^2+11a^6b^3+24a^5b^4+25a^4b^5+10a^3b^6-3a^2b^7-4ab^8-b^9\right)\tan(x)^2}{a^9b+8a^8b^2+28a^7b^3+56a^6b^4+70a^5b^5+56a^4b^6+28a^3b^7+8a^2b^8+ab^9} - \frac{3\left(a^7b^2+6a^6b^3+15a^5b^4+20a^4b^5+15a^3b^6+6a^2b^7+ab^8\right)}{a^9b+8a^8b^2+28a^7b^3+56a^6b^4+70a^5b^5+56a^4b^6+28a^3b^7+8a^2b^8+ab^9}\right)\tan(x \cdots}{6\left(b\tan \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((((2*a^7*b^2 + 11*a^6*b^3 + 24*a^5*b^4 + 25*a^4*b^5 + 10*a^3*b^6 - 3*a
^2*b^7 - 4*a*b^8 - b^9)*tan(x)^2/(a^9*b + 8*a^8*b^2 + 28*a^7*b^3 + 56*a^6*b
^4 + 70*a^5*b^5 + 56*a^4*b^6 + 28*a^3*b^7 + 8*a^2*b^8 + a*b^9) - 3*(a^7*b^2
 + 6*a^6*b^3 + 15*a^5*b^4 + 20*a^4*b^5 + 15*a^3*b^6 + 6*a^2*b^7 + a*b^8)/(a
^9*b + 8*a^8*b^2 + 28*a^7*b^3 + 56*a^6*b^4 + 70*a^5*b^5 + 56*a^4*b^6 + 28*a
^3*b^7 + 8*a^2*b^8 + a*b^9))*tan(x)^2 + 3*(a^8*b + 6*a^7*b^2 + 15*a^6*b^3 +
 20*a^5*b^4 + 15*a^4*b^5 + 6*a^3*b^6 + a^2*b^7)/(a^9*b + 8*a^8*b^2 + 28*a^7
*b^3 + 56*a^6*b^4 + 70*a^5*b^5 + 56*a^4*b^6 + 28*a^3*b^7 + 8*a^2*b^8 + a*b^
9))*tan(x)^2 - (4*a^8*b + 25*a^7*b^2 + 66*a^6*b^3 + 95*a^5*b^4 + 80*a^4*b^5
 + 39*a^3*b^6 + 10*a^2*b^7 + a*b^8)/(a^9*b + 8*a^8*b^2 + 28*a^7*b^3 + 56*a^
6*b^4 + 70*a^5*b^5 + 56*a^4*b^6 + 28*a^3*b^7 + 8*a^2*b^8 + a*b^9))/(b*tan(x
)^4 + a)^(3/2) + arctan((sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a) + sqrt(b))
/sqrt(-a - b))/((a^2 + 2*a*b + b^2)*sqrt(-a - b))
```

3.404 $\int \frac{\tan(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=117

$$\frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}}+\frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}}$$

```
[Out] -ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*(a + b)^(5
/2)) + (a + b*Tan[x]^2)/(6*a*(a + b)*(a + b*Tan[x]^4)^(3/2)) + (3*a^2 + b*(
5*a + 2*b)*Tan[x]^2)/(6*a^2*(a + b)^2*Sqrt[a + b*Tan[x]^4])
```

Rubi [A] time = 0.18687, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3670, 1248, 741, 823, 12, 725, 206}

$$\frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}}+\frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(a + b*Tan[x]^4)^(5/2),x]
```

```
[Out] -ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*(a + b)^(5
/2)) + (a + b*Tan[x]^2)/(6*a*(a + b)*(a + b*Tan[x]^4)^(3/2)) + (3*a^2 + b*(
5*a + 2*b)*Tan[x]^2)/(6*a^2*(a + b)^2*Sqrt[a + b*Tan[x]^4])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{x}{\left(1+x^2\right)\left(a+bx^4\right)^{5/2}}\,dx, x, \tan(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan^2(x)\right)$$

$$= \frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3a-2b-2bx}{(1+x)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan^2(x)\right)}{6a(a+b)}$$

$$= \frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}} + \frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}} + \frac{\text{Subst}\left(\int \frac{3a^2b}{(1+x)\sqrt{a+bx^2}}\,dx, x, \text{ta}\ldots\right.}{6a^2b(a+b)^2}$$

$$= \frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}} + \frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\,dx, x, \text{ta}\ldots\right.}{2(a+b)^2}$$

$$= \frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}} + \frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}} - \frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\tan\ldots}{\sqrt{a+b\tan\ldots}}\right.}{2(a+b)^2}$$

$$= -\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}} + \frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}} + \frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}}$$

Mathematica [A] time = 0.765351, size = 113, normalized size = 0.97

$$\frac{1}{6}\left(\frac{3a^2b\tan^4(x)+a^2(4a+b)+b^2(5a+2b)\tan^6(x)+3ab(2a+b)\tan^2(x)}{a^2(a+b)^2\left(a+b\tan^4(x)\right)^{3/2}}-\frac{3\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(a + b*Tan[x]^4)^(5/2),x]
```

```
[Out] ((-3*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/(a + b)^
(5/2) + (a^2*(4*a + b) + 3*a*b*(2*a + b)*Tan[x]^2 + 3*a^2*b*Tan[x]^4 + b^2*
(5*a + 2*b)*Tan[x]^6)/(a^2*(a + b)^2*(a + b*Tan[x]^4)^(3/2)))/6
```

Maple [B] time = 0.065, size = 602, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*tan(x)^4)^(5/2),x)
```

```
[Out] 1/24/((-a*b)^(1/2)+b)/a/(-a*b)^(1/2)/(tan(x)^2-(-a*b)^(1/2)/b)^2*(b*(tan(x)
^2-(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)^(1/2)/b))^(1/2)+1/24/(
(-a*b)^(1/2)+b)/a^2/(tan(x)^2-(-a*b)^(1/2)/b)*(b*(tan(x)^2-(-a*b)^(1/2)/b)^
2+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)^(1/2)/b))^(1/2)-1/8*(2*(-a*b)^(1/2)-b)/((
-a*b)^(1/2)-b)^2/a^2/(tan(x)^2+(-a*b)^(1/2)/b)*(b*(tan(x)^2+(-a*b)^(1/2)/b)
^2-2*(-a*b)^(1/2)*(tan(x)^2+(-a*b)^(1/2)/b))^(1/2)-1/2*b^2/((-a*b)^(1/2)+b)
^2/((-a*b)^(1/2)-b)^2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2
)*(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^2))+1/8*(2*(-a*b
)^(1/2)+b)/((-a*b)^(1/2)+b)^2/a^2/(tan(x)^2-(-a*b)^(1/2)/b)*(b*(tan(x)^2-(-
a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)^(1/2)/b))^(1/2)+1/24/((-a*b
)^(1/2)-b)/a/(-a*b)^(1/2)/(tan(x)^2+(-a*b)^(1/2)/b)^2*(b*(tan(x)^2+(-a*b)^(
1/2)/b)^2-2*(-a*b)^(1/2)*(tan(x)^2+(-a*b)^(1/2)/b))^(1/2)-1/24/((-a*b)^(1/2
)-b)/a^2/(tan(x)^2+(-a*b)^(1/2)/b)*(b*(tan(x)^2+(-a*b)^(1/2)/b)^2-2*(-a*b)^
(1/2)*(tan(x)^2+(-a*b)^(1/2)/b))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)}{\left(b\tan(x)^4+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)/(b*tan(x)^4 + a)^(5/2), x)
```

Fricas [B] time = 4.12989, size = 1355, normalized size = 11.58

$$\left[\frac{3\left(a^2b^2\tan(x)^8+2a^3b\tan(x)^4+a^4\right)\sqrt{a+b}\log\left(\frac{(ab+2b^2)\tan(x)^4-2ab\tan(x)^2+2\sqrt{b\tan(x)^4+a}\left(b\tan(x)^2-a\right)\sqrt{a+b}+2a^2+ab}{\tan(x)^4+2\tan(x)^2+1}\right)+2((}{12\left(\left(a^5b^2+3a^4b^3+3a^3b^4+a^2b^5\right)\tan(x)^8+a^7+3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*b^2*tan(x)^8 + 2*a^3*b*tan(x)^4 + a^4)*sqrt(a + b)*log(((a*b
+ 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a
)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 2*((5*a^2*b^2 +
 7*a*b^3 + 2*b^4)*tan(x)^6 + 3*(a^3*b + a^2*b^2)*tan(x)^4 + 4*a^4 + 5*a^3*b
 + a^2*b^2 + 3*(2*a^3*b + 3*a^2*b^2 + a*b^3)*tan(x)^2)*sqrt(b*tan(x)^4 + a)
)/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*tan(x)^8 + a^7 + 3*a^6*b + 3
*a^5*b^2 + a^4*b^3 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*tan(x)^4),
 -1/6*(3*(a^2*b^2*tan(x)^8 + 2*a^3*b*tan(x)^4 + a^4)*sqrt(-a - b)*arctan(sq
rt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^
2 + a*b)) - ((5*a^2*b^2 + 7*a*b^3 + 2*b^4)*tan(x)^6 + 3*(a^3*b + a^2*b^2)*t
an(x)^4 + 4*a^4 + 5*a^3*b + a^2*b^2 + 3*(2*a^3*b + 3*a^2*b^2 + a*b^3)*tan(x
)^2)*sqrt(b*tan(x)^4 + a))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*tan
(x)^8 + a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*
b^3 + a^3*b^4)*tan(x)^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)}{\left(a+b\tan^4(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)**4)**(5/2),x)
```

```
[Out] Integral(tan(x)/(a + b*tan(x)**4)**(5/2), x)
```

Giac [B] time = 1.37004, size = 834, normalized size = 7.13

$$\frac{\left(\left(\left(\frac{(5a^7b^3+32a^6b^4+87a^5b^5+130a^4b^6+115a^3b^7+60a^2b^8+17ab^9+2b^{10})\tan(x)^2}{a^{10}b+8a^9b^2+28a^8b^3+56a^7b^4+70a^6b^5+56a^5b^6+28a^4b^7+8a^3b^8+a^2b^9}+\frac{3(a^8b^2+6a^7b^3+15a^6b^4+20a^5b^5+15a^4b^6+6a^3b^7+a^2b^8)}{a^{10}b+8a^9b^2+28a^8b^3+56a^7b^4+70a^6b^5+56a^5b^6+28a^4b^7+8a^3b^8+a^2b^9}\right.\right.\right.}{6(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((((5*a^7*b^3 + 32*a^6*b^4 + 87*a^5*b^5 + 130*a^4*b^6 + 115*a^3*b^7 + 6
0*a^2*b^8 + 17*a*b^9 + 2*b^10)*tan(x)^2/(a^10*b + 8*a^9*b^2 + 28*a^8*b^3 +
56*a^7*b^4 + 70*a^6*b^5 + 56*a^5*b^6 + 28*a^4*b^7 + 8*a^3*b^8 + a^2*b^9) +
3*(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 +
```

```
 a^2*b^8)/(a^10*b + 8*a^9*b^2 + 28*a^8*b^3 + 56*a^7*b^4 + 70*a^6*b^5 + 56*a
^5*b^6 + 28*a^4*b^7 + 8*a^3*b^8 + a^2*b^9))*tan(x)^2 + 3*(2*a^8*b^2 + 13*a^
7*b^3 + 36*a^6*b^4 + 55*a^5*b^5 + 50*a^4*b^6 + 27*a^3*b^7 + 8*a^2*b^8 + a*b
^9)/(a^10*b + 8*a^9*b^2 + 28*a^8*b^3 + 56*a^7*b^4 + 70*a^6*b^5 + 56*a^5*b^6
 + 28*a^4*b^7 + 8*a^3*b^8 + a^2*b^9))*tan(x)^2 + (4*a^9*b + 25*a^8*b^2 + 66
*a^7*b^3 + 95*a^6*b^4 + 80*a^5*b^5 + 39*a^4*b^6 + 10*a^3*b^7 + a^2*b^8)/(a^
10*b + 8*a^9*b^2 + 28*a^8*b^3 + 56*a^7*b^4 + 70*a^6*b^5 + 56*a^5*b^6 + 28*a
^4*b^7 + 8*a^3*b^8 + a^2*b^9))/(b*tan(x)^4 + a)^(3/2) - arctan((sqrt(b)*tan
(x)^2 - sqrt(b*tan(x)^4 + a) + sqrt(b))/sqrt(-a - b))/((a^2 + 2*a*b + b^2)*
sqrt(-a - b))
```

3.405 $\int \frac{\cot(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}}+\frac{1}{2a^2\sqrt{a+b\tan^4(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}+\frac{}{6a\left(a+b\right.}$$

```
[Out] ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*(a + b)^(5/
2)) - ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]]/(2*a^(5/2)) + 1/(6*a*(a + b*Tan
[x]^4)^(3/2)) - (a + b*Tan[x]^2)/(6*a*(a + b)*(a + b*Tan[x]^4)^(3/2)) + 1/(
2*a^2*Sqrt[a + b*Tan[x]^4]) - (3*a^2 + b*(5*a + 2*b)*Tan[x]^2)/(6*a^2*(a +
b)^2*Sqrt[a + b*Tan[x]^4])
```

Rubi [A] time = 0.302311, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {3670, 1252, 961, 741, 823, 12, 725, 206, 266, 51, 63, 208}

$$-\frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}}+\frac{1}{2a^2\sqrt{a+b\tan^4(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}+\frac{}{6a\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(a + b*Tan[x]^4)^(5/2),x]
```

```
[Out] ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*(a + b)^(5/
2)) - ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]]/(2*a^(5/2)) + 1/(6*a*(a + b*Tan
[x]^4)^(3/2)) - (a + b*Tan[x]^2)/(6*a*(a + b)*(a + b*Tan[x]^4)^(3/2)) + 1/(
2*a^2*Sqrt[a + b*Tan[x]^4]) - (3*a^2 + b*(5*a + 2*b)*Tan[x]^2)/(6*a^2*(a +
b)^2*Sqrt[a + b*Tan[x]^4])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
```

```
m, 0] || IGtQ[n, 0])
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{x\left(1+x^2\right)\left(a+bx^4\right)^{5/2}}\,dx, x, \tan(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan^2(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{(-1-x)\left(a+bx^2\right)^{5/2}}+\frac{1}{x\left(a+bx^2\right)^{5/2}}\right)dx, x, \tan^2(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1-x)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan^2(x)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(a+bx^2\right)^{5/2}}\,dx, x, \tan^2(\right.$$

$$= -\frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \tan^4(x)\right)-\frac{\text{Subst}\left(\int \frac{-3a}{(-1-x)}\right.}{6}$$

$$= \frac{1}{6a\left(a+b\tan^4(x)\right)^{3/2}}-\frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}-\frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}}+\frac{\text{Subst}}{}$$

$$= \frac{1}{6a\left(a+b\tan^4(x)\right)^{3/2}}-\frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}+\frac{1}{2a^2\sqrt{a+b\tan^4(x)}}-\frac{3a^2+b(5a+}{6a^2(a+b)^2\sqrt{a}}$$

$$= \frac{1}{6a\left(a+b\tan^4(x)\right)^{3/2}}-\frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}+\frac{1}{2a^2\sqrt{a+b\tan^4(x)}}-\frac{3a^2+b(5a+}{6a^2(a+b)^2\sqrt{a}}$$

$$= \frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2a^{5/2}}+\frac{1}{6a\left(a+b\tan^4(x)\right)^{3/2}}-\frac{a+b\,\text{t}}{6a(a+b)\left(a+\right.}$$

Mathematica [C] time = 1.40658, size = 149, normalized size = 0.81

$$\frac{1}{6}\left(\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{b\tan^4(x)}{a}+1\right)}{a\left(a+b\tan^4(x)\right)^{3/2}}-\frac{3a^2b\tan^4(x)+a^2(4a+b)+b^2(5a+2b)\tan^6(x)+3ab(2a+b)\,\text{t}}{a^2(a+b)^2\left(a+b\tan^4(x)\right)^{3/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a + b*Tan[x]^4)^(5/2),x]
```

```
[Out] ((3*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/(a + b)^(
5/2) + Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Tan[x]^4)/a]/(a*(a + b*Tan[x
```

```
]^4)^(3/2)) - (a^2*(4*a + b) + 3*a*b*(2*a + b)*Tan[x]^2 + 3*a^2*b*Tan[x]^4
+ b^2*(5*a + 2*b)*Tan[x]^6)/(a^2*(a + b)^2*(a + b*Tan[x]^4)^(3/2)))/6
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int \cot(x)\left(a+b(\tan(x))^4\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*tan(x)^4)^(5/2),x)

[Out] int(cot(x)/(a+b*tan(x)^4)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError
```

Fricas [B] time = 7.16085, size = 3970, normalized size = 21.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="fricas")

[Out] [1/12*(3*(a^3*b^2*tan(x)^8 + 2*a^4*b*tan(x)^4 + a^5)*sqrt(a + b)*log(((a*b
+ 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a
)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 3*((a^3*b^2 + 3
*a^2*b^3 + 3*a*b^4 + b^5)*tan(x)^8 + a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 +
2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*tan(x)^4)*sqrt(a)*log(-(b*tan(x)^
4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(a) + 2*a)/tan(x)^4) - 2*((5*a^3*b^2 + 7*a^2
*b^3 + 2*a*b^4)*tan(x)^6 - 7*a^4*b - 11*a^3*b^2 - 4*a^2*b^3 - 3*(2*a^3*b^2
+ 3*a^2*b^3 + a*b^4)*tan(x)^4 + 3*(2*a^4*b + 3*a^3*b^2 + a^2*b^3)*tan(x)^2)
*sqrt(b*tan(x)^4 + a))/((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*tan(x)^
8 + a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3
+ a^4*b^4)*tan(x)^4), 1/12*(6*((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*tan(x)
^8 + a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3
 + a*b^4)*tan(x)^4)*sqrt(-a)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + 3*(a
^3*b^2*tan(x)^8 + 2*a^4*b*tan(x)^4 + a^5)*sqrt(a + b)*log(((a*b + 2*b^2)*ta
n(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a +
b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) - 2*((5*a^3*b^2 + 7*a^2*b^3
+ 2*a*b^4)*tan(x)^6 - 7*a^4*b - 11*a^3*b^2 - 4*a^2*b^3 - 3*(2*a^3*b^2 + 3*a
^2*b^3 + a*b^4)*tan(x)^4 + 3*(2*a^4*b + 3*a^3*b^2 + a^2*b^3)*tan(x)^2)*sqrt
(b*tan(x)^4 + a))/((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*tan(x)^8 + a
^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4
```

```
*b^4)*tan(x)^4), 1/12*(6*(a^3*b^2*tan(x)^8 + 2*a^4*b*tan(x)^4 + a^5)*sqrt(-
a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^
2)*tan(x)^4 + a^2 + a*b)) + 3*((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*tan(x)
^8 + a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3
 + a*b^4)*tan(x)^4)*sqrt(a)*log(-(b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(
a) + 2*a)/tan(x)^4) - 2*((5*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*tan(x)^6 - 7*a^4
*b - 11*a^3*b^2 - 4*a^2*b^3 - 3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*tan(x)^4 +
3*(2*a^4*b + 3*a^3*b^2 + a^2*b^3)*tan(x)^2)*sqrt(b*tan(x)^4 + a))/((a^6*b^2
 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*tan(x)^8 + a^8 + 3*a^7*b + 3*a^6*b^2 +
a^5*b^3 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*tan(x)^4), 1/6*(3*(a^
3*b^2*tan(x)^8 + 2*a^4*b*tan(x)^4 + a^5)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^
4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) +
3*((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*tan(x)^8 + a^5 + 3*a^4*b + 3*a^3*b
^2 + a^2*b^3 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*tan(x)^4)*sqrt(-a)
*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) - ((5*a^3*b^2 + 7*a^2*b^3 + 2*a*b^
4)*tan(x)^6 - 7*a^4*b - 11*a^3*b^2 - 4*a^2*b^3 - 3*(2*a^3*b^2 + 3*a^2*b^3 +
 a*b^4)*tan(x)^4 + 3*(2*a^4*b + 3*a^3*b^2 + a^2*b^3)*tan(x)^2)*sqrt(b*tan(x
)^4 + a))/((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*tan(x)^8 + a^8 + 3*a
^7*b + 3*a^6*b^2 + a^5*b^3 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*ta
n(x)^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{\left(a + b\tan^4(x)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)**4)**(5/2),x)
```

```
[Out] Integral(cot(x)/(a + b*tan(x)**4)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{\left(b\tan(x)^4 + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(x)/(b*tan(x)^4 + a)^(5/2), x)
```

3.406 $\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right)^2 dx$

Optimal. Leaf size=212

$$\frac{\left(a^2-b^2\sqrt{-c^2}\right)\tan(e+fx)(d\tan(e+fx))^m\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)}{2f(m+1)}+\frac{\left(a^2+b^2\sqrt{-c^2}\right)\tan(e$$

```
[Out] ((a^2 - b^2*Sqrt[-c^2])*Hypergeometric2F1[1, 1 + m, 2 + m, -((c*Tan[e + f*x
])/Sqrt[-c^2])]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(2*f*(1 + m)) + ((a^2 + b^
2*Sqrt[-c^2])*Hypergeometric2F1[1, 1 + m, 2 + m, (c*Tan[e + f*x])/Sqrt[-c^2
]]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(2*f*(1 + m)) + (4*a*b*Hypergeometric2F
1[1, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[e + f*x]^2]*(c*Tan[e + f*x])^(3/2)*(d*T
an[e + f*x])^m)/(c*f*(3 + 2*m))
```

Rubi [A] time = 0.712759, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3670, 15, 1831, 364, 1286}

$$\frac{\left(a^2-b^2\sqrt{-c^2}\right)\tan(e+fx)(d\tan(e+fx))^m\,{}_2F_1\left(1,m+1;m+2;-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)}{2f(m+1)}+\frac{\left(a^2+b^2\sqrt{-c^2}\right)\tan(e+fx)(d\tan(e+fx}{2f(m+}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^m*(a + b*Sqrt[c*Tan[e + f*x]])^2,x]
```


```
[Out] ((a^2 - b^2*Sqrt[-c^2])*Hypergeometric2F1[1, 1 + m, 2 + m, -((c*Tan[e + f*x
])/Sqrt[-c^2])]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(2*f*(1 + m)) + ((a^2 + b^
2*Sqrt[-c^2])*Hypergeometric2F1[1, 1 + m, 2 + m, (c*Tan[e + f*x])/Sqrt[-c^2
]]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(2*f*(1 + m)) + (4*a*b*Hypergeometric2F
1[1, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[e + f*x]^2]*(c*Tan[e + f*x])^(3/2)*(d*T
an[e + f*x])^m)/(c*f*(3 + 2*m))
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1286

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (c_.)*(x_)^4), x_Sym
bol] :> With[{q = Rt[-(a*c), 2]}, -Dist[e/2 + (c*d)/(2*q), Int[(f*x)^m/(q -
 c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[(f*x)^m/(q + c*x^2), x], x]]
/; FreeQ[{a, c, d, e, f, m}, x]
```

Rubi steps

$$\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right)^2 dx = \frac{c\,\text{Subst}\left(\int \frac{\left(a+b\sqrt{x}\right)^2\left(\frac{dx}{c}\right)^m}{c^2+x^2}dx,x,c\tan(e+fx)\right)}{f}$$

$$= \frac{(2c)\,\text{Subst}\left(\int \frac{x\left(\frac{dx^2}{c}\right)^m(a+bx)^2}{c^2+x^4}dx,x,\sqrt{c\tan(e+fx)}\right)}{f}$$

$$= \frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int \frac{x^{1+2m}(a+bx)^2}{c^2+x^4}dx,x,\sqrt{\right.}}{f}$$

$$= \frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int\left(\frac{2abx^{2+2m}}{c^2+x^4}+\frac{x^{1+2m}(a^2}{c^2+}\right.\right.}{f}$$

$$= \frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int \frac{x^{1+2m}\left(a^2+b^2x^2\right)}{c^2+x^4}dx,x,\right.}{f}$$

$$= \frac{4ab\,{}_2F_1\left(1,\frac{1}{4}(3+2m);\frac{1}{4}(7+2m);-\tan^2(e+fx)\right)(c\tan(e+fx))^{3/2}(}{cf(3+2m)}$$

$$= \frac{\left(a^2-b^2\sqrt{-c^2}\right){}_2F_1\left(1,1+m;2+m;-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)\tan(e+fx)(d\tan(}{2f(1+m)}$$

Mathematica [A] time = 1.28323, size = 151, normalized size = 0.71

$$\frac{\tan(e+fx)(d\tan(e+fx))^m\left(\frac{a^2\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(e+fx)\right)}{m+1}+b\left(\frac{4a\sqrt{c\tan(e+fx)}\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2m+3),\frac{1}{4}(2m\right.}{2m+3}\right.\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^m*(a + b*Sqrt[c*Tan[e + f*x]])^2,x]
```

```
[Out] (Tan[e + f*x]*(d*Tan[e + f*x])^m*((a^2*Hypergeometric2F1[1, (1 + m)/2, (3 +
 m)/2, -Tan[e + f*x]^2])/(1 + m) + b*((b*c*Hypergeometric2F1[1, (2 + m)/2,
(4 + m)/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(2 + m) + (4*a*Hypergeometric2F1[
1, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[e + f*x]^2]*Sqrt[c*Tan[e + f*x]])/(3 + 2*
m))))/f
```

Maple [F] time = 0.257, size = 0, normalized size = 0.

$$\int \left(a+b\sqrt{c\tan\left(fx+e\right)}\right)^{2} \left(d\tan\left(fx+e\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*tan(f*x+e))^(1/2))^2*(d*tan(f*x+e))^m,x)
```

```
[Out] int((a+b*(c*tan(f*x+e))^(1/2))^2*(d*tan(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sqrt{c\tan\left(fx+e\right)}b+a\right)^{2} \left(d\tan\left(fx+e\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^(1/2))^2*(d*tan(f*x+e))^m,x, algorithm="maxim
a")
```

```
[Out] integrate((sqrt(c*tan(f*x + e))*b + a)^2*(d*tan(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(2\sqrt{c\tan\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{m}ab+\left(b^{2}c\tan\left(fx+e\right)+a^{2}\right)\left(d\tan\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^(1/2))^2*(d*tan(f*x+e))^m,x, algorithm="frica
s")
```

```
[Out] integral(2*sqrt(c*tan(f*x + e))*(d*tan(f*x + e))^m*a*b + (b^2*c*tan(f*x + e
) + a^2)*(d*tan(f*x + e))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\tan\left(e+fx\right)\right)^{m} \left(a+b\sqrt{c\tan\left(e+fx\right)}\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))**(1/2))**2*(d*tan(f*x+e))**m,x)
```

```
[Out] Integral((d*tan(e + f*x))**m*(a + b*sqrt(c*tan(e + f*x)))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sqrt{c\tan\left(fx+e\right)}b+a\right)^{2} \left(d\tan\left(fx+e\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^(1/2))^2*(d*tan(f*x+e))^m,x, algorithm="giac"
)
```

```
[Out] integrate((sqrt(c*tan(f*x + e))*b + a)^2*(d*tan(f*x + e))^m, x)
```

3.407 $\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right) dx$

Optimal. Leaf size=121

$$\frac{a\tan(e+fx)(d\tan(e+fx))^m\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(e+fx)\right)}{f(m+1)}+\frac{2b(c\tan(e+fx))^{3/2}(d\tan(e+fx))}{}$$

```
[Out] (a*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[e + f*x]^2]*Tan[e + f*x]
*(d*Tan[e + f*x])^m)/(f*(1 + m)) + (2*b*Hypergeometric2F1[1, (3 + 2*m)/4, (
7 + 2*m)/4, -Tan[e + f*x]^2]*(c*Tan[e + f*x])^(3/2)*(d*Tan[e + f*x])^m)/(c*
f*(3 + 2*m))
```

Rubi [A] time = 0.330969, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3670, 15, 1831, 364}

$$\frac{a\tan(e+fx)(d\tan(e+fx))^m\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(e+fx)\right)}{f(m+1)}+\frac{2b(c\tan(e+fx))^{3/2}(d\tan(e+fx))^m\,{}_2F_1\left(1,\frac{1}{4}(2m+3\right.}{cf(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^m*(a + b*Sqrt[c*Tan[e + f*x]]),x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[e + f*x]^2]*Tan[e + f*x]
*(d*Tan[e + f*x])^m)/(f*(1 + m)) + (2*b*Hypergeometric2F1[1, (3 + 2*m)/4, (
7 + 2*m)/4, -Tan[e + f*x]^2]*(c*Tan[e + f*x])^(3/2)*(d*Tan[e + f*x])^m)/(c*
f*(3 + 2*m))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right) dx = \frac{c\,\text{Subst}\left(\int \frac{\left(a+b\sqrt{x}\right)\left(\frac{dx}{c}\right)^m}{c^2+x^2}\,dx, x, c\tan(e+fx)\right)}{f}$$

$$= \frac{(2c)\,\text{Subst}\left(\int \frac{x\left(\frac{dx^2}{c}\right)^m (a+bx)}{c^2+x^4}\,dx, x, \sqrt{c\tan(e+fx)}\right)}{f}$$

$$= \frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int \frac{x^{1+2m}(a+bx)}{c^2+x^4}\,dx, x, \sqrt{c\right.}$$

$$= \frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int \left(\frac{ax^{1+2m}}{c^2+x^4}+\frac{bx^{2+2m}}{c^2+x^4}\right) dx\right.}{f}$$

$$= \frac{\left(2ac(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int \frac{x^{1+2m}}{c^2+x^4}\,dx, x, \sqrt{c\tan\right.}{f}$$

$$= \frac{a\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(e+fx)\right)\tan(e+fx)(d\tan(e+fx))^m}{f(1+m)} + \frac{2b}{}$$

Mathematica [C] time = 0.603302, size = 304, normalized size = 2.51

$$\frac{\tan(e+fx)(d\tan(e+fx))^m\left(\left(a-b\sqrt[4]{-c^2}\right)\text{Hypergeometric2F1}\left(1,2(m+1),2m+3,-\frac{\sqrt{c\tan(e+fx)}}{\sqrt[4]{-c^2}}\right)+\left(a+ib\sqrt[4]{-c^2}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^m*(a + b*Sqrt[c*Tan[e + f*x]]),x]
```

```
[Out] (((a - b*(-c^2)^(1/4))*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, -(Sqrt[c*Ta
n[e + f*x]]/(-c^2)^(1/4))] + (a + I*b*(-c^2)^(1/4))*Hypergeometric2F1[1, 2*
(1 + m), 3 + 2*m, ((-I)*Sqrt[c*Tan[e + f*x]])/(-c^2)^(1/4)] + a*Hypergeomet
ric2F1[1, 2*(1 + m), 3 + 2*m, (I*Sqrt[c*Tan[e + f*x]])/(-c^2)^(1/4)] - I*b*
(-c^2)^(1/4)*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, (I*Sqrt[c*Tan[e + f*x
]])/(-c^2)^(1/4)] + a*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, Sqrt[c*Tan[e
 + f*x]]/(-c^2)^(1/4)] + b*(-c^2)^(1/4)*Hypergeometric2F1[1, 2*(1 + m), 3 +
 2*m, Sqrt[c*Tan[e + f*x]]/(-c^2)^(1/4)])*Tan[e + f*x]*(d*Tan[e + f*x])^m)/
(4*f*(1 + m))
```

Maple [F] time = 0.198, size = 0, normalized size = 0.

$$\int \left(a+b\sqrt{c\tan(fx+e)}\right)\left(d\tan(fx+e)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*tan(f*x+e))^(1/2))*(d*tan(f*x+e))^m,x)
```

```
[Out] int((a+b*(c*tan(f*x+e))^(1/2))*(d*tan(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sqrt{c\tan\left(fx+e\right)}b+a\right)\left(d\tan\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^(1/2))*(d*tan(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] integrate((sqrt(c*tan(f*x + e))*b + a)*(d*tan(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{c\tan\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{m}b+\left(d\tan\left(fx+e\right)\right)^{m}a,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^(1/2))*(d*tan(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*tan(f*x + e))*(d*tan(f*x + e))^m*b + (d*tan(f*x + e))^m*a,
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\tan\left(e+fx\right)\right)^{m}\left(a+b\sqrt{c\tan\left(e+fx\right)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))**(1/2))*(d*tan(f*x+e))**m,x)
```

```
[Out] Integral((d*tan(e + f*x))**m*(a + b*sqrt(c*tan(e + f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sqrt{c\tan\left(fx+e\right)}b+a\right)\left(d\tan\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^(1/2))*(d*tan(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((sqrt(c*tan(f*x + e))*b + a)*(d*tan(f*x + e))^m, x)
```

3.408 $\int \frac{(d\tan(e+fx))^m}{a+b\sqrt{c\tan(e+fx)}}\,dx$

Optimal. Leaf size=460

$$-\frac{b\left(a^2-b^2\sqrt{-c^2}\right)(c\tan(e+fx))^{3/2}(d\tan(e+fx))^m\text{Hypergeometric2F1}\left(1,\frac{1}{2}(2m+3),\frac{1}{2}(2m+5),-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)}{cf(2m+3)\left(a^4+b^4c^2\right)}-b$$

```
[Out] (a*(a^2 - b^2*Sqrt[-c^2])*Hypergeometric2F1[1, 1 + m, 2 + m, -((c*Tan[e + f
*x])/Sqrt[-c^2])]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(2*(a^4 + b^4*c^2)*f*(1
+ m)) + (a*(a^2 + b^2*Sqrt[-c^2])*Hypergeometric2F1[1, 1 + m, 2 + m, (c*Tan
[e + f*x])/Sqrt[-c^2]]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(2*(a^4 + b^4*c^2)*
f*(1 + m)) + (b^4*c^2*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, -((b*Sqrt[c*
Tan[e + f*x]])/a)]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(a*(a^4 + b^4*c^2)*f*(1
 + m)) - (b*(a^2 - b^2*Sqrt[-c^2])*Hypergeometric2F1[1, (3 + 2*m)/2, (5 + 2
*m)/2, -((c*Tan[e + f*x])/Sqrt[-c^2])]*(c*Tan[e + f*x])^(3/2)*(d*Tan[e + f*
x])^m)/(c*(a^4 + b^4*c^2)*f*(3 + 2*m)) - (b*(a^2 + b^2*Sqrt[-c^2])*Hypergeo
metric2F1[1, (3 + 2*m)/2, (5 + 2*m)/2, (c*Tan[e + f*x])/Sqrt[-c^2]]*(c*Tan[
e + f*x])^(3/2)*(d*Tan[e + f*x])^m)/(c*(a^4 + b^4*c^2)*f*(3 + 2*m))
```

Rubi [A] time = 1.28252, antiderivative size = 460, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {3670, 15, 6725, 64, 1831, 1286, 364}

$$-\frac{b\left(a^2-b^2\sqrt{-c^2}\right)(c\tan(e+fx))^{3/2}(d\tan(e+fx))^m\,{}_2F_1\left(1,\frac{1}{2}(2m+3);\frac{1}{2}(2m+5);-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)}{cf(2m+3)\left(a^4+b^4c^2\right)}-\frac{b\left(a^2+b^2\sqrt{-c^2}\right)(c\,t}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^m/(a + b*Sqrt[c*Tan[e + f*x]]),x]
```


```
[Out] (a*(a^2 - b^2*Sqrt[-c^2])*Hypergeometric2F1[1, 1 + m, 2 + m, -((c*Tan[e + f
*x])/Sqrt[-c^2])]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(2*(a^4 + b^4*c^2)*f*(1
+ m)) + (a*(a^2 + b^2*Sqrt[-c^2])*Hypergeometric2F1[1, 1 + m, 2 + m, (c*Tan
[e + f*x])/Sqrt[-c^2]]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(2*(a^4 + b^4*c^2)*
f*(1 + m)) + (b^4*c^2*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, -((b*Sqrt[c*
Tan[e + f*x]])/a)]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(a*(a^4 + b^4*c^2)*f*(1
 + m)) - (b*(a^2 - b^2*Sqrt[-c^2])*Hypergeometric2F1[1, (3 + 2*m)/2, (5 + 2
*m)/2, -((c*Tan[e + f*x])/Sqrt[-c^2])]*(c*Tan[e + f*x])^(3/2)*(d*Tan[e + f*
x])^m)/(c*(a^4 + b^4*c^2)*f*(3 + 2*m)) - (b*(a^2 + b^2*Sqrt[-c^2])*Hypergeo
metric2F1[1, (3 + 2*m)/2, (5 + 2*m)/2, (c*Tan[e + f*x])/Sqrt[-c^2]]*(c*Tan[
e + f*x])^(3/2)*(d*Tan[e + f*x])^m)/(c*(a^4 + b^4*c^2)*f*(3 + 2*m))
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
```

```
&& !IntegerQ[m]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 1286

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (c_.)*(x_)^4), x_Sym
bol] :> With[{q = Rt[-(a*c), 2]}, -Dist[e/2 + (c*d)/(2*q), Int[(f*x)^m/(q -
 c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[(f*x)^m/(q + c*x^2), x], x]]
/; FreeQ[{a, c, d, e, f, m}, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^m}{a+b\sqrt{c\tan(e+fx)}}\,dx = \frac{c\,\text{Subst}\left(\int \frac{\left(\frac{dx}{c}\right)^m}{(a+b\sqrt{x})(c^2+x^2)}\,dx, x, c\tan(e+fx)\right)}{f}$$

$$= \frac{(2c)\,\text{Subst}\left(\int \frac{x\left(\frac{dx^2}{c}\right)^m}{(a+bx)(c^2+x^4)}\,dx, x, \sqrt{c\tan(e+fx)}\right)}{f}$$

$$= \frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int \frac{x^{1+2m}}{(a+bx)(c^2+x^4)}\,dx, x, \sqrt{c\tan(e+fx)}\right)}{f}$$

$$= \frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int \left(\frac{b^4x^{1+2m}}{(a^4+b^4c^2)(a+bx)} + \frac{x^{1+2m}(a^3-a^2bx+ab^2x^2-b^3x^3}{(a^4+b^4c^2)(c^2+x^4)}\right.\right.}{f}$$

$$= \frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int \frac{x^{1+2m}(a^3-a^2bx+ab^2x^2-b^3x^3)}{c^2+x^4}\,dx, x, \sqrt{c\tan(e}\right.}{(a^4+b^4c^2)f}$$

$$= \frac{b^4c^2\,{}_2F_1\left(1,2(1+m);3+2m;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)\tan(e+fx)(d\tan(e+fx))^m}{a(a^4+b^4c^2)f(1+m)} + \frac{\left(2c(c\tan(e\right.}{}$$

$$= \frac{b^4c^2\,{}_2F_1\left(1,2(1+m);3+2m;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)\tan(e+fx)(d\tan(e+fx))^m}{a(a^4+b^4c^2)f(1+m)} + \frac{\left(2c(c\tan(e\right.}{}$$

$$= \frac{b^4c^2\,{}_2F_1\left(1,2(1+m);3+2m;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)\tan(e+fx)(d\tan(e+fx))^m}{a(a^4+b^4c^2)f(1+m)} + \frac{\left(ac\left(b^2-\right.\right.}{}$$

$$= \frac{a\left(a^2-b^2\sqrt{-c^2}\right){}_2F_1\left(1,1+m;2+m;-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)\tan(e+fx)(d\tan(e+fx))^m}{2(a^4+b^4c^2)f(1+m)} + \frac{a\left(a^2\right.}{}$$

Mathematica [A] time = 6.27851, size = 385, normalized size = 0.84

$$\frac{2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\left(\frac{a^3(c\tan(e+fx))^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(e+fx)\right)}{2c^2(m+1)(a^4+b^4c^2)} + \frac{ab^2(c\tan(e+fx))^{m+2}\text{Hyperge}}{2c^2(m}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^m/(a + b*Sqrt[c*Tan[e + f*x]]),x]
```

```
[Out] (2*c*(d*Tan[e + f*x])^m*((a^3*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -T
an[e + f*x]^2]*(c*Tan[e + f*x])^(1 + m))/(2*c^2*(a^4 + b^4*c^2)*(1 + m)) +
(b^4*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, -((b*Sqrt[c*Tan[e + f*x]])/a)
]*(c*Tan[e + f*x])^(1 + m))/(2*a*(a^4 + b^4*c^2)*(1 + m)) + (a*b^2*Hypergeo
metric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[e + f*x]^2]*(c*Tan[e + f*x])^(2 + m
))/(2*c^2*(a^4 + b^4*c^2)*(2 + m)) - (a^2*b*Hypergeometric2F1[1, (3 + 2*m)/
4, (7 + 2*m)/4, -Tan[e + f*x]^2]*(c*Tan[e + f*x])^((3 + 2*m)/2))/(c^2*(a^4
+ b^4*c^2)*(3 + 2*m)) - (b^3*Hypergeometric2F1[1, (5 + 2*m)/4, (9 + 2*m)/4,
 -Tan[e + f*x]^2]*(c*Tan[e + f*x])^((5 + 2*m)/2))/(c^2*(a^4 + b^4*c^2)*(5 +
 2*m))))/(f*(c*Tan[e + f*x])^m)
```

Maple [F] time = 0.201, size = 0, normalized size = 0.

$$\int \left(d\tan\left(fx+e\right)\right)^{m} \left(a+b\sqrt{c\tan\left(fx+e\right)}\right)^{-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2)),x)
```

```
[Out] int((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\tan\left(fx+e\right)\right)^{m}}{\sqrt{c\tan\left(fx+e\right)}b+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2)),x, algorithm="maxima"
)
```

```
[Out] integrate((d*tan(f*x + e))^m/(sqrt(c*tan(f*x + e))*b + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c\tan\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{m}b-\left(d\tan\left(fx+e\right)\right)^{m}a}{b^{2}c\tan\left(fx+e\right)-a^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2)),x, algorithm="fricas"
)
```

```
[Out] integral((sqrt(c*tan(f*x + e))*(d*tan(f*x + e))^m*b - (d*tan(f*x + e))^m*a)
/(b^2*c*tan(f*x + e) - a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\tan\left(e+fx\right)\right)^{m}}{a+b\sqrt{c\tan\left(e+fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**m/(a+b*(c*tan(f*x+e))**(1/2)),x)
```

```
[Out] Integral((d*tan(e + f*x))**m/(a + b*sqrt(c*tan(e + f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\tan\left(fx+e\right)\right)^{m}}{\sqrt{c\tan\left(fx+e\right)}b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2)),x, algorithm="giac")
```

```
[Out] integrate((d*tan(f*x + e))^m/(sqrt(c*tan(f*x + e))*b + a), x)
```

3.409 $\int \frac{(d\tan(e+fx))^m}{\left(a+b\sqrt{c\tan(e+fx)}\right)^2}\,dx$

Optimal. Leaf size=617

$$-\frac{2ab\left(-2a^2b^2\sqrt{-c^2}+a^4-b^4c^2\right)(c\tan(e+fx))^{3/2}(d\tan(e+fx))^m\text{Hypergeometric2F1}\left(1,\frac{1}{2}(2m+3),\frac{1}{2}(2m+5),-\frac{c\tan}{\sqrt{}}\right.}{cf(2m+3)\left(a^4+b^4c^2\right)^2}$$

```
[Out] ((a^6 - 3*a^2*b^4*c^2 - 3*a^4*b^2*Sqrt[-c^2] - b^6*(-c^2)^(3/2))*Hypergeome
tric2F1[1, 1 + m, 2 + m, -((c*Tan[e + f*x])/Sqrt[-c^2])]*Tan[e + f*x]*(d*Ta
n[e + f*x])^m)/(2*(a^4 + b^4*c^2)^2*f*(1 + m)) + ((a^6 - 3*a^2*b^4*c^2 + 3*
a^4*b^2*Sqrt[-c^2] + b^6*(-c^2)^(3/2))*Hypergeometric2F1[1, 1 + m, 2 + m, (
c*Tan[e + f*x])/Sqrt[-c^2]]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(2*(a^4 + b^4*
c^2)^2*f*(1 + m)) + (4*a^2*b^4*c^2*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m,
 -((b*Sqrt[c*Tan[e + f*x]])/a)]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/((a^4 + b^
4*c^2)^2*f*(1 + m)) + (b^4*c^2*Hypergeometric2F1[2, 2*(1 + m), 3 + 2*m, -((
b*Sqrt[c*Tan[e + f*x]])/a)]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(a^2*(a^4 + b^
4*c^2)*f*(1 + m)) - (2*a*b*(a^4 - b^4*c^2 - 2*a^2*b^2*Sqrt[-c^2])*Hypergeom
etric2F1[1, (3 + 2*m)/2, (5 + 2*m)/2, -((c*Tan[e + f*x])/Sqrt[-c^2])]*(c*Ta
n[e + f*x])^(3/2)*(d*Tan[e + f*x])^m)/(c*(a^4 + b^4*c^2)^2*f*(3 + 2*m)) - (
2*a*b*(a^4 - b^4*c^2 + 2*a^2*b^2*Sqrt[-c^2])*Hypergeometric2F1[1, (3 + 2*m)
/2, (5 + 2*m)/2, (c*Tan[e + f*x])/Sqrt[-c^2]]*(c*Tan[e + f*x])^(3/2)*(d*Tan
[e + f*x])^m)/(c*(a^4 + b^4*c^2)^2*f*(3 + 2*m))
```

Rubi [A] time = 1.57946, antiderivative size = 617, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {3670, 15, 6725, 64, 1831, 1286, 364}

$$-\frac{2ab\left(-2a^2b^2\sqrt{-c^2}+a^4-b^4c^2\right)(c\tan(e+fx))^{3/2}(d\tan(e+fx))^m\,{}_2F_1\left(1,\frac{1}{2}(2m+3);\frac{1}{2}(2m+5);-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)}{cf(2m+3)\left(a^4+b^4c^2\right)^2}-\frac{2ab\left(2a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^m/(a + b*Sqrt[c*Tan[e + f*x]])^2,x]
```


```
[Out] ((a^6 - 3*a^2*b^4*c^2 - 3*a^4*b^2*Sqrt[-c^2] - b^6*(-c^2)^(3/2))*Hypergeome
tric2F1[1, 1 + m, 2 + m, -((c*Tan[e + f*x])/Sqrt[-c^2])]*Tan[e + f*x]*(d*Ta
n[e + f*x])^m)/(2*(a^4 + b^4*c^2)^2*f*(1 + m)) + ((a^6 - 3*a^2*b^4*c^2 + 3*
a^4*b^2*Sqrt[-c^2] + b^6*(-c^2)^(3/2))*Hypergeometric2F1[1, 1 + m, 2 + m, (
c*Tan[e + f*x])/Sqrt[-c^2]]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(2*(a^4 + b^4*
c^2)^2*f*(1 + m)) + (4*a^2*b^4*c^2*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m,
 -((b*Sqrt[c*Tan[e + f*x]])/a)]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/((a^4 + b^
4*c^2)^2*f*(1 + m)) + (b^4*c^2*Hypergeometric2F1[2, 2*(1 + m), 3 + 2*m, -((
b*Sqrt[c*Tan[e + f*x]])/a)]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(a^2*(a^4 + b^
4*c^2)*f*(1 + m)) - (2*a*b*(a^4 - b^4*c^2 - 2*a^2*b^2*Sqrt[-c^2])*Hypergeom
etric2F1[1, (3 + 2*m)/2, (5 + 2*m)/2, -((c*Tan[e + f*x])/Sqrt[-c^2])]*(c*Ta
n[e + f*x])^(3/2)*(d*Tan[e + f*x])^m)/(c*(a^4 + b^4*c^2)^2*f*(3 + 2*m)) - (
2*a*b*(a^4 - b^4*c^2 + 2*a^2*b^2*Sqrt[-c^2])*Hypergeometric2F1[1, (3 + 2*m)
/2, (5 + 2*m)/2, (c*Tan[e + f*x])/Sqrt[-c^2]]*(c*Tan[e + f*x])^(3/2)*(d*Tan
[e + f*x])^m)/(c*(a^4 + b^4*c^2)^2*f*(3 + 2*m))
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
```

```
x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 1286

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (c_.)*(x_)^4), x_Sym
bol] :> With[{q = Rt[-(a*c), 2]}, -Dist[e/2 + (c*d)/(2*q), Int[(f*x)^m/(q -
 c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[(f*x)^m/(q + c*x^2), x], x]]
/; FreeQ[{a, c, d, e, f, m}, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^m}{\left(a+b\sqrt{c\tan(e+fx)}\right)^2}\,dx = \frac{c\,\text{Subst}\left(\int \frac{\left(\frac{dx}{c}\right)^m}{\left(a+b\sqrt{x}\right)^2(c^2+x^2)}\,dx, x, c\tan(e+fx)\right)}{f}$$
$$= \frac{(2c)\,\text{Subst}\left(\int \frac{x\left(\frac{dx^2}{c}\right)^m}{(a+bx)^2(c^2+x^4)}\,dx, x, \sqrt{c\tan(e+fx)}\right)}{f}$$
$$= \frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int \frac{x^{1+2m}}{(a+bx)^2(c^2+x^4)}\,dx, x, \sqrt{c\tan(e+fx)}\right)}{f}$$
$$= \frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int\left(\frac{b^4x^{1+2m}}{(a^4+b^4c^2)(a+bx)^2} + \frac{4a^3b^4x^{1+2m}}{(a^4+b^4c^2)^2(a+bx)} + \frac{x^{1+2m}}{}\right.\right.}{f}$$
$$= \frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int \frac{x^{1+2m}\left(a^2\left(a^4-3b^4c^2\right)-2ab\left(a^4-b^4c^2\right)x+b^2\left(3a^4-b^4c^2\right)\right.}{c^2+x^4}\right.}{\left(a^4+b^4c^2\right)^2 f}$$
$$= \frac{4a^2b^4c^2\,{}_2F_1\left(1,2(1+m);3+2m;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)\tan(e+fx)(d\tan(e+fx))^m}{\left(a^4+b^4c^2\right)^2 f(1+m)} + \frac{b^4c^2\,{}_2F_1}{}$$
$$= \frac{4a^2b^4c^2\,{}_2F_1\left(1,2(1+m);3+2m;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)\tan(e+fx)(d\tan(e+fx))^m}{\left(a^4+b^4c^2\right)^2 f(1+m)} + \frac{b^4c^2\,{}_2F_1}{}$$
$$= \frac{4a^2b^4c^2\,{}_2F_1\left(1,2(1+m);3+2m;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)\tan(e+fx)(d\tan(e+fx))^m}{\left(a^4+b^4c^2\right)^2 f(1+m)} + \frac{b^4c^2\,{}_2F_1}{}$$
$$= \frac{\left(a^6-3a^2b^4c^2-3a^4b^2\sqrt{-c^2}-b^6\left(-c^2\right)^{3/2}\right){}_2F_1\left(1,1+m;2+m;-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)\tan(e+fx)}{2\left(a^4+b^4c^2\right)^2 f(1+m)}$$

Mathematica [A] time = 5.89128, size = 381, normalized size = 0.62

$$\frac{c(d\tan(e+fx))^m\left(-\frac{8a^3b^3(c\tan(e+fx))^{5/2}\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2m+5),\frac{1}{4}(2m+9),-\tan^2(e+fx)\right)}{c^2(2m+5)} + \frac{b^2\left(3a^4-b^4c^2\right)\tan^2(e+fx)\text{Hypergeometric2F1}}{m+2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^m/(a + b*Sqrt[c*Tan[e + f*x]])^2,x]
```

```
[Out] (c*(d*Tan[e + f*x])^m*((a^2*(a^4 - 3*b^4*c^2)*Hypergeometric2F1[1, (1 + m)/
2, (3 + m)/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(c*(1 + m)) + (4*a^2*b^4*c*Hyp
ergeometric2F1[1, 2*(1 + m), 3 + 2*m, -((b*Sqrt[c*Tan[e + f*x]])/a)]*Tan[e
+ f*x])/(1 + m) + (b^4*c*(a^4 + b^4*c^2)*Hypergeometric2F1[2, 2*(1 + m), 3
+ 2*m, -((b*Sqrt[c*Tan[e + f*x]])/a)]*Tan[e + f*x])/(a^2*(1 + m)) + (b^2*(3
*a^4 - b^4*c^2)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[e + f*x]^2]
*Tan[e + f*x]^2)/(2 + m) + (4*a*b*(-a^4 + b^4*c^2)*Hypergeometric2F1[1, (3
+ 2*m)/4, (7 + 2*m)/4, -Tan[e + f*x]^2]*(c*Tan[e + f*x])^(3/2))/(c^2*(3 + 2
*m)) - (8*a^3*b^3*Hypergeometric2F1[1, (5 + 2*m)/4, (9 + 2*m)/4, -Tan[e + f
```

```
*x]^2]*(c*Tan[e + f*x])^(5/2))/(c^2*(5 + 2*m))))/((a^4 + b^4*c^2)^2*f)
```

Maple [F] time = 0.286, size = 0, normalized size = 0.

$$\int \left(d\tan\left(fx+e\right)\right)^{m}\left(a+b\sqrt{c\tan\left(fx+e\right)}\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2))^2,x)
```

```
[Out] int((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2))^2,x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{2\sqrt{c\tan\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{m}ab-\left(b^{2}c\tan\left(fx+e\right)+a^{2}\right)\left(d\tan\left(fx+e\right)\right)^{m}}{b^{4}c^{2}\tan\left(fx+e\right)^{2}-2\,a^{2}b^{2}c\tan\left(fx+e\right)+a^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2))^2,x, algorithm="frica
s")
```

```
[Out] integral(-(2*sqrt(c*tan(f*x + e))*(d*tan(f*x + e))^m*a*b - (b^2*c*tan(f*x +
 e) + a^2)*(d*tan(f*x + e))^m)/(b^4*c^2*tan(f*x + e)^2 - 2*a^2*b^2*c*tan(f*
x + e) + a^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\tan\left(e+fx\right)\right)^{m}}{\left(a+b\sqrt{c\tan\left(e+fx\right)}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**m/(a+b*(c*tan(f*x+e))**(1/2))**2,x)
```

```
[Out] Integral((d*tan(e + f*x))**m/(a + b*sqrt(c*tan(e + f*x)))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\tan\left(fx+e\right)\right)^{m}}{\left(\sqrt{c\tan\left(fx+e\right)}b+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2))^2,x, algorithm="giac"
)
```

```
[Out] integrate((d*tan(f*x + e))^m/(sqrt(c*tan(f*x + e))*b + a)^2, x)
```

3.410 $\int (d\tan(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=74

$$\frac{\tan(e+fx)(d\tan(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \text{Hypergeometric2F1}\left(1,\frac{1}{2}(m+np+1),\frac{1}{2}(m+np+3),-\tan^2(e+\right.}{f(m+np+1)}$$

```
[Out] (Hypergeometric2F1[1, (1 + m + n*p)/2, (3 + m + n*p)/2, -Tan[e + f*x]^2]*Ta
n[e + f*x]*(d*Tan[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + m + n*p))
```

Rubi [A] time = 0.0993308, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3578, 20, 3476, 364}

$$\frac{\tan(e+fx)(d\tan(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p {}_2F_1\left(1,\frac{1}{2}(m+np+1);\frac{1}{2}(m+np+3);-\tan^2(e+fx)\right)}{f(m+np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + m + n*p)/2, (3 + m + n*p)/2, -Tan[e + f*x]^2]*Ta
n[e + f*x]*(d*Tan[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + m + n*p))
```

Rule 3578

```
Int[((c_.)*((d_.)*tan[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Tan[e + f*x
])^p)^FracPart[n])/(d*Tan[e + f*x])^(p*FracPart[n]), Int[(a + b*Tan[e + f*x
])^m*(d*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && !IntegerQ[n] && !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (d\tan(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{np}(d\tan(e+fx$$
$$= \left((c\tan(e+fx))^{-m-np}(d\tan(e+fx))^m\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+f$$
$$= \frac{\left(c(c\tan(e+fx))^{-m-np}(d\tan(e+fx))^m\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \frac{x^{m+}}{c^2+}\right.}{f}$$
$$= \frac{{}_2F_1\left(1,\frac{1}{2}(1+m+np);\frac{1}{2}(3+m+np);-\tan^2(e+fx)\right)\tan(e+fx)(d\tan(e}{f(1+m+np)}$$

Mathematica **[A]** time = 0.0807976, size = 76, normalized size = 1.03

$$\frac{\tan(e+fx)(d\tan(e+fx))^m\left(b(c\tan(e+fx))^n\right)^p\text{Hypergeometric2F1}\left(1,\frac{1}{2}(m+np+1),\frac{1}{2}(m+np+1)+1,-\tan^2(e+\right.}{f(m+np+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + m + n*p)/2, 1 + (1 + m + n*p)/2, -Tan[e + f*x]^2
]*Tan[e + f*x]*(d*Tan[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + m + n*p
))
```

Maple **[F]** time = 3.97, size = 0, normalized size = 0.

$$\int \left(d\tan\left(fx+e\right)\right)^m\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima **[F]** time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\left(d\tan\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*tan(f*x + e))^m, x)
```

Fricas **[F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\left(d\tan\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*(d*tan(f*x + e))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \left(c \tan \left(e + f x\right)\right)^{n}\right)^{p} \left(d \tan \left(e + f x\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*(d*tan(e + f*x))**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c \tan \left(f x + e\right)\right)^{n} b\right)^{p} \left(d \tan \left(f x + e\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*tan(f*x + e))^m, x)
```

3.411 $\int \tan^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=63

$$\frac{\tan^3(e+fx)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+3),\frac{1}{2}(np+5),-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+3)}$$

```
[Out] (Hypergeometric2F1[1, (3 + n*p)/2, (5 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]^3*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 + n*p))
```

Rubi [A] time = 0.093727, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3659, 16, 3476, 364}

$$\frac{\tan^3(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+3);\frac{1}{2}(np+5);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (3 + n*p)/2, (5 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]^3*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 + n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \tan^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \tan^2(e+fx)(c\tan(e+fx))^{np}\,d$$

$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int(c\tan(e+fx))^{2+np}\,dx}{c^2}$$

$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int\frac{x^{2+np}}{c^2+x^2}\,dx,x,c\tan(e+fx\right.}{cf}$$

$$= \frac{{}_2F_1\left(1,\frac{1}{2}(3+np);\frac{1}{2}(5+np);-\tan^2(e+fx)\right)\tan^3(e+fx)\left(b(c\tan(e+fx\right.}{f(3+np)}$$

Mathematica [A] time = 0.0763673, size = 65, normalized size = 1.03

$$\frac{\tan^3(e+fx)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+3),\frac{1}{2}(np+3)+1,-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (3 + n*p)/2, 1 + (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e
+ f*x]^3*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 + n*p))
```

Maple [F] time = 3.128, size = 0, normalized size = 0.

$$\int \left(\tan\left(fx+e\right)\right)^2\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(tan(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\tan\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\tan^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*tan(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^2, x)
```

3.412 $\int \left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=61

$$\frac{\tan(e+fx)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+1),\frac{1}{2}(np+3),-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rubi [A] time = 0.0493278, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{\tan(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{np}\,dx$$
$$= \frac{\left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \frac{x^{np}}{c^2+x^2}\,dx, x, c\tan(e+fx)\right)}{f}$$
$$= \frac{{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)}$$

Mathematica [A] time = 0.0381326, size = 59, normalized size = 0.97

$$\frac{\tan(e+fx)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+1),\frac{1}{2}(np+3),-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f + f*n*p)
```

Maple [F] time = 0.002, size = 0, normalized size = 0.

$$\int \left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p, x)
```

3.413 $\int \cot^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=63

$$-\frac{\cot(e+fx)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np-1),\frac{1}{2}(np+1),-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}$$

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[1, (-1 + n*p)/2, (1 + n*p)/2, -Tan[e + f*
x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - n*p)))
```

Rubi [A] time = 0.109797, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3659, 16, 3476, 364}

$$-\frac{\cot(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np-1);\frac{1}{2}(np+1);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[1, (-1 + n*p)/2, (1 + n*p)/2, -Tan[e + f*
x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - n*p)))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \cot^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \cot^2(e+fx)(c\tan(e+fx))^{np}\,d$$
$$= \left(c^2(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{-2+np}\,dx$$
$$= \frac{\left(c^3(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\mathrm{Subst}\left(\int \frac{x^{-2+np}}{c^2+x^2}\,dx,x,c\tan(e+\right.}{f}$$
$$= -\frac{\cot(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(-1+np);\frac{1}{2}(1+np);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx\right.}{f(1-np)}$$

Mathematica [A] time = 0.0492503, size = 61, normalized size = 0.97

$$\frac{\cot(e+fx)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np-1),\frac{1}{2}(np+1),-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[e + f*x]*Hypergeometric2F1[1, (-1 + n*p)/2, (1 + n*p)/2, -Tan[e + f*x]
^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(-1 + n*p))
```

Maple [F] time = 3.553, size = 0, normalized size = 0.

$$\int \left(\cot\left(fx+e\right)\right)^2\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cot(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \cot\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\cot^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*cot(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\cot\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^2, x)
```

3.414 $\int \cot^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=65

$$-\frac{\cot^3(e+fx)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np-3),\frac{1}{2}(np-1),-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(3-np)}$$

```
[Out] -((Cot[e + f*x]^3*Hypergeometric2F1[1, (-3 + n*p)/2, (-1 + n*p)/2, -Tan[e +
 f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 - n*p)))
```

Rubi [A] time = 0.113695, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3659, 16, 3476, 364}

$$-\frac{\cot^3(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np-3);\frac{1}{2}(np-1);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(3-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((Cot[e + f*x]^3*Hypergeometric2F1[1, (-3 + n*p)/2, (-1 + n*p)/2, -Tan[e +
 f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 - n*p)))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \cot^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \cot^4(e+fx)(c\tan(e+fx))^{np}\,dx$$
$$= \left(c^4(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{-4+np}\,dx$$
$$= \frac{\left(c^5(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \frac{x^{-4+np}}{c^2+x^2}\,dx, x, c\tan(e+fx\right.}{f}$$
$$= -\frac{\cot^3(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(-3+np);\frac{1}{2}(-1+np);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx\right.}{f(3-np)}$$

Mathematica [A] time = 0.0755062, size = 65, normalized size = 1.

$$\frac{\cot^3(e+fx)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np-3),\frac{1}{2}(np-3)+1,-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np-3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[e + f*x]^3*Hypergeometric2F1[1, (-3 + n*p)/2, 1 + (-3 + n*p)/2, -Tan[e
 + f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(-3 + n*p))
```

Maple [F] time = 3.549, size = 0, normalized size = 0.

$$\int \left(\cot(fx+e)\right)^4\left(b\left(c\tan(fx+e)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cot(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan(fx+e)\right)^n b\right)^p\cot(fx+e)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\cot\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^4, x)
```

3.415 $\int \cot^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=65

$$-\frac{\cot^5(e+fx)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np-5),\frac{1}{2}(np-3),-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(5-np)}$$

```
[Out] -((Cot[e + f*x]^5*Hypergeometric2F1[1, (-5 + n*p)/2, (-3 + n*p)/2, -Tan[e +
 f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(5 - n*p)))
```

Rubi [A] time = 0.111782, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3659, 16, 3476, 364}

$$-\frac{\cot^5(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np-5);\frac{1}{2}(np-3);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(5-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((Cot[e + f*x]^5*Hypergeometric2F1[1, (-5 + n*p)/2, (-3 + n*p)/2, -Tan[e +
 f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(5 - n*p)))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \cot^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \cot^6(e+fx)(c\tan(e+fx))^{np}\,d$$
$$= \left(c^6(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{-6+np}\,dx$$
$$= \frac{\left(c^7(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \frac{x^{-6+np}}{c^2+x^2}\,dx,x,c\tan(e+\right.}{f}$$
$$= -\frac{\cot^5(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(-5+np);\frac{1}{2}(-3+np);-\tan^2(e+fx)\right)\left(b(c\tan(e+\right.}{f(5-np)}$$

Mathematica [A] time = 0.0808054, size = 65, normalized size = 1.

$$\frac{\cot^5(e+fx)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np-5),\frac{1}{2}(np-5)+1,-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np-5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[e + f*x]^5*Hypergeometric2F1[1, (-5 + n*p)/2, 1 + (-5 + n*p)/2, -Tan[e
 + f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(-5 + n*p))
```

Maple [F] time = 3.42, size = 0, normalized size = 0.

$$\int \left(\cot\left(fx+e\right)\right)^6\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cot(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\cot\left(fx+e\right)^6,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \cot\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^6, x)
```

3.416 $\int \tan^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=63

$$\frac{\tan^4(e+fx)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+4),\frac{1}{2}(np+6),-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+4)}$$

```
[Out] (Hypergeometric2F1[1, (4 + n*p)/2, (6 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]^4*(b*(c*Tan[e + f*x])^n)^p)/(f*(4 + n*p))
```

Rubi [A] time = 0.0921519, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3659, 16, 3476, 364}

$$\frac{\tan^4(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+4);\frac{1}{2}(np+6);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (4 + n*p)/2, (6 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]^4*(b*(c*Tan[e + f*x])^n)^p)/(f*(4 + n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \tan^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \tan^3(e+fx)(c\tan(e+fx))^{np}\,dx$$
$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{3+np}\,dx}{c^3}$$
$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \frac{x^{3+np}}{c^2+x^2}\,dx, x, c\tan(e+fx)\right)}{c^2 f}$$
$$= \frac{{}_2F_1\left(1,\frac{1}{2}(4+np);\frac{1}{2}(6+np);-\tan^2(e+fx)\right)\tan^4(e+fx)\left(b(c\tan(e+fx))^n\right)}{f(4+np)}$$

Mathematica [A] time = 0.063495, size = 61, normalized size = 0.97

$$\frac{\tan^4(e+fx)\text{Hypergeometric2F1}\left(1,\frac{np}{2}+2,\frac{np}{2}+3,-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, 2 + (n*p)/2, 3 + (n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]^4*(b*(c*Tan[e + f*x])^n)^p)/(f*(4 + n*p))
```

Maple [F] time = 0.505, size = 0, normalized size = 0.

$$\int \left(\tan\left(fx+e\right)\right)^3\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(tan(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\tan\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\tan\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\tan^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*tan(e + f*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\tan\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^3, x)
```

3.417 $\int \tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=63

$$\frac{\tan^2(e+fx)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+2),\frac{1}{2}(np+4),-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+2)}$$

```
[Out] (Hypergeometric2F1[1, (2 + n*p)/2, (4 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]^2*(b*(c*Tan[e + f*x])^n)^p)/(f*(2 + n*p))
```

Rubi [A] time = 0.068729, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3659, 16, 3476, 364}

$$\frac{\tan^2(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+2);\frac{1}{2}(np+4);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (2 + n*p)/2, (4 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]^2*(b*(c*Tan[e + f*x])^n)^p)/(f*(2 + n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \tan(e+fx)(c\tan(e+fx))^{np}\,dx$$
$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{1+np}\,dx}{c}$$
$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \frac{x^{1+np}}{c^2+x^2}\,dx,x,c\tan(e+fx)\right)}{f}$$
$$= \frac{{}_2F_1\left(1,\frac{1}{2}(2+np);\frac{1}{2}(4+np);-\tan^2(e+fx)\right)\tan^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(2+np)}$$

Mathematica [A] time = 0.0574892, size = 61, normalized size = 0.97

$$\frac{\tan^2(e+fx)\text{Hypergeometric2F1}\left(1,\frac{np}{2}+1,\frac{np}{2}+2,-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + (n*p)/2, 2 + (n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]^2*(b*(c*Tan[e + f*x])^n)^p)/(f*(2 + n*p))
```

Maple [F] time = 5.706, size = 0, normalized size = 0.

$$\int \tan(fx+e)\left(b\left(c\tan(fx+e)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan(fx+e)\right)^n b\right)^p\tan(fx+e)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan(fx+e)\right)^n b\right)^p\tan(fx+e),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*tan(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\tan\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*tan(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e), x)
```

3.418 $\int \cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=50

$$\frac{\text{Hypergeometric2F1}\left(1,\frac{np}{2},\frac{np}{2}+1,-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp}$$

```
[Out] (Hypergeometric2F1[1, (n*p)/2, 1 + (n*p)/2, -Tan[e + f*x]^2]*(b*(c*Tan[e +
f*x])^n)^p)/(f*n*p)
```

Rubi [A] time = 0.0800734, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3659, 16, 3476, 364}

$$\frac{{}_2F_1\left(1,\frac{np}{2};\frac{np}{2}+1;-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (n*p)/2, 1 + (n*p)/2, -Tan[e + f*x]^2]*(b*(c*Tan[e +
f*x])^n)^p)/(f*n*p)
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \cot(e+fx)(c\tan(e+fx))^{np}\,dx$$
$$= \left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{-1+np}\,dx$$
$$= \frac{\left(c^2(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \frac{x^{-1+np}}{c^2+x^2}dx, x, c\tan(e+fx)\right)}{f}$$
$$= \frac{{}_2F_1\left(1,\frac{np}{2};1+\frac{np}{2};-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp}$$

Mathematica [A] time = 0.0158514, size = 50, normalized size = 1.

$$\frac{\text{Hypergeometric2F1}\left(1,\frac{np}{2},\frac{np}{2}+1,-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (n*p)/2, 1 + (n*p)/2, -Tan[e + f*x]^2]*(b*(c*Tan[e +
f*x])^n)^p)/(f*n*p)
```

Maple [F] time = 5.779, size = 0, normalized size = 0.

$$\int \cot(fx+e)\left(b\left(c\tan(fx+e)\right)^n\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan(fx+e)\right)^n b\right)^p \cot(fx+e)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan(fx+e)\right)^n b\right)^p \cot(fx+e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*cot(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*cot(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e), x)
```

3.419 $\int \cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=62

$$-\frac{\cot^2(e+fx)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np-2),\frac{np}{2},-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(2-np)}$$

```
[Out] -((Cot[e + f*x]^2*Hypergeometric2F1[1, (-2 + n*p)/2, (n*p)/2, -Tan[e + f*x]
^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(2 - n*p)))
```

Rubi [A] time = 0.108209, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3659, 16, 3476, 364}

$$-\frac{\cot^2(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np-2);\frac{np}{2};-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(2-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((Cot[e + f*x]^2*Hypergeometric2F1[1, (-2 + n*p)/2, (n*p)/2, -Tan[e + f*x]
^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(2 - n*p)))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx &= \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \cot^3(e+fx)(c\tan(e+fx))^{np}\,d\\ &= \left(c^3(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{-3+np}\,dx\\ &= \frac{\left(c^4(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \frac{x^{-3+np}}{c^2+x^2}\,dx,x,c\tan(e+\right.}{f}\\ &= -\frac{\cot^2(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(-2+np);\frac{np}{2};-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(2-np)}\end{aligned}$$

Mathematica [A] time = 0.0562243, size = 59, normalized size = 0.95

$$\frac{\cot^2(e+fx)\text{Hypergeometric2F1}\left(1,\frac{np}{2}-1,\frac{np}{2},-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[e + f*x]^2*Hypergeometric2F1[1, -1 + (n*p)/2, (n*p)/2, -Tan[e + f*x]^2
]*(b*(c*Tan[e + f*x])^n)^p)/(f*(-2 + n*p))
```

Maple [F] time = 3.477, size = 0, normalized size = 0.

$$\int \left(\cot\left(fx+e\right)\right)^3\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cot(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\cot\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\cot\left(fx+e\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\cot^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*cot(e + f*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\cot\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^3, x)
```

3.420 $\int (d\tan(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left((d\tan(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable[(d*Tan[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi [A] time = 0.0587434, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (d\tan(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*Tan[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Defer[Int][(d*Tan[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi steps

$$\int (d\tan(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p\,dx = \int (d\tan(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Mathematica [A] time = 3.52545, size = 0, normalized size = 0.

$$\int (d\tan(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Tan[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Integrate[(d*Tan[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Maple [A] time = 0.489, size = 0, normalized size = 0.

$$\int \left(d\tan\left(fx+e\right)\right)^m \left(a+b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b + a\right)^p \left(d\tan\left(fx+e\right)\right)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b + a\right)^p \left(d\tan\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**m*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b + a\right)^p \left(d\tan\left(fx+e\right)\right)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

3.421 $\int (d\cot(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=78

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p (d\cot(e+fx))^m \text{Hypergeometric2F1}\left(1, \frac{1}{2}(-m+2p+1), \frac{1}{2}(-m+2p+3), -\tan^2(e+\right.}{f(-m+2p+1)}$$

```
[Out] ((d*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + 2*p)/2, (3 - m + 2*p)/2,
-Tan[e + f*x]^2]*Tan[e + f*x]*(b*Tan[e + f*x]^2)^p)/(f*(1 - m + 2*p))
```

Rubi [A] time = 0.122044, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3658, 2604, 3476, 364}

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p (d\cot(e+fx))^m {}_2F_1\left(1, \frac{1}{2}(-m+2p+1); \frac{1}{2}(-m+2p+3); -\tan^2(e+fx)\right)}{f(-m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((d*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + 2*p)/2, (3 - m + 2*p)/2,
-Tan[e + f*x]^2]*Tan[e + f*x]*(b*Tan[e + f*x]^2)^p)/(f*(1 - m + 2*p))
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2604

```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m, Int[(b*Tan[e +
 f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (d\cot(e+fx))^m \left(b\tan^2(e+fx)\right)^p dx = \left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int (d\cot(e+fx))^m \tan^{2p}(e+fx)\,dx$$
$$= \left((d\cot(e+fx))^m \tan^{m-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int \tan^{-m+2p}(e+fx)\,dx$$
$$= \frac{\left((d\cot(e+fx))^m \tan^{m-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\text{Subst}\left(\int \frac{x^{-m+2p}}{1+x^2}dx, x, \text{ta}\right.}{f}$$
$$= \frac{(d\cot(e+fx))^m\, {}_2F_1\left(1,\frac{1}{2}(1-m+2p);\frac{1}{2}(3-m+2p);-\tan^2(e+fx)\right)\tan(e+}{f(1-m+2p)}$$

Mathematica [A] time = 0.136519, size = 70, normalized size = 0.9

$$-\frac{d\left(b\tan^2(e+fx)\right)^p (d\cot(e+fx))^{m-1}\text{Hypergeometric2F1}\left(1,-\frac{m}{2}+p+\frac{1}{2},-\frac{m}{2}+p+\frac{3}{2},-\tan^2(e+fx)\right)}{f(m-2p-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((d*(d*Cot[e + f*x])^(-1 + m)*Hypergeometric2F1[1, 1/2 - m/2 + p, 3/2 - m/
2 + p, -Tan[e + f*x]^2]*(b*Tan[e + f*x]^2)^p)/(f*(-1 + m - 2*p)))
```

Maple [F] time = 0.646, size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^m \left(b\left(\tan\left(fx+e\right)\right)^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*cot(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2\right)^p \left(d\cot\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*cot(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2\right)^p \left(d\cot\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2)^p*(d*cot(f*x + e))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan^2\left(e+fx\right)\right)^p\left(d\cot\left(e+fx\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**p*(d*cot(e + f*x))**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2\right)^p\left(d\cot\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*cot(f*x + e))^m, x)
```

3.422 $\int (d\cot(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=107

$$\frac{\tan(e+fx)(d\cot(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1-m}{2};1,-p;\frac{3-m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f(1-m)}$$

```
[Out] (AppellF1[(1 - m)/2, 1, -p, (3 - m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/a)]*(d*Cot[e + f*x])^m*Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1 - m)*
(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.198803, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3674, 3670, 511, 510}

$$\frac{\tan(e+fx)(d\cot(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1-m}{2};1,-p;\frac{3-m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```



```
[Out] (AppellF1[(1 - m)/2, 1, -p, (3 - m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/a)]*(d*Cot[e + f*x])^m*Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1 - m)*
(1 + (b*Tan[e + f*x]^2)/a)^p)
```


Rule 3674

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (
f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(d*Cot[e + f*x])^FracPart[m]*(Tan
[e + f*x]/d)^FracPart[m], Int[(a + b*(c*Tan[e + f*x])^n)^p/(Tan[e + f*x]/d)
^m, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !IntegerQ[m]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (d\cot(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx = \left((d\cot(e+fx))^m \left(\frac{\tan(e+fx)}{d}\right)^m\right)\int \left(\frac{\tan(e+fx)}{d}\right)^{-m} \left(a+b\tan^2(e-$$

$$= \frac{\left((d\cot(e+fx))^m \left(\frac{\tan(e+fx)}{d}\right)^m\right)\text{Subst}\left(\int \frac{\left(\frac{x}{d}\right)^{-m}\left(a+bx^2\right)^p}{1+x^2}dx, x, \tan(e+f\right.}{f}$$

$$= \frac{\left((d\cot(e+fx))^m \left(\frac{\tan(e+fx)}{d}\right)^m \left(a+b\tan^2(e+fx)\right)^p \left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-}\right.}{f}$$

$$= \frac{F_1\left(\frac{1-m}{2};1,-p;\frac{3-m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)(d\cot(e+fx))^m\tan(}{f(1-m)}$$

Mathematica [B] time = 2.33114, size = 265, normalized size = 2.48

$$-\frac{a(m-3)\cos^2(e+fx)\cot(e+fx)(d\cot(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p F_1\left(\frac{1-}{2}\right.}{f(m-1)\left(-2bpF_1\left(\frac{3-m}{2};1-p,1;\frac{5-m}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx)\right)+2aF_1\left(\frac{3-m}{2};-p,2;\frac{5-m}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cot[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((a*(-3 + m)*AppellF1[(1 - m)/2, -p, 1, (3 - m)/2, -((b*Tan[e + f*x]^2)/a)
, -Tan[e + f*x]^2]*Cos[e + f*x]^2*Cot[e + f*x]*(d*Cot[e + f*x])^m*(a + b*Ta
n[e + f*x]^2)^p)/(f*(-1 + m)*(-2*b*p*AppellF1[(3 - m)/2, 1 - p, 1, (5 - m)/
2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*a*AppellF1[(3 - m)/2, -p,
2, (5 - m)/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + a*(-3 + m)*Appell
F1[(1 - m)/2, -p, 1, (3 - m)/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*C
ot[e + f*x]^2)))
```

Maple [F] time = 0.522, size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^m \left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*cot(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p \left(d\cot\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*cot(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\left(d\cot\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*(d*cot(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\left(d\cot\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*cot(f*x + e))^m, x)
```

3.423 $\int (d\cot(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \,dx$

Optimal. Leaf size=80

$$\frac{\tan(e+fx)(d\cot(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \text{Hypergeometric2F1}\left(1,\frac{1}{2}(-m+np+1),\frac{1}{2}(-m+np+3),-\tan^2(e+fx)\right)}{f(-m+np+1)}$$

```
[Out] ((d*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + n*p)/2, (3 - m + n*p)/2,
-Tan[e + f*x]^2]*Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - m + n*p))
```

Rubi [A] time = 0.142618, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3659, 2604, 3476, 364}

$$\frac{\tan(e+fx)(d\cot(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p {}_2F_1\left(1,\frac{1}{2}(-m+np+1);\frac{1}{2}(-m+np+3);-\tan^2(e+fx)\right)}{f(-m+np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((d*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + n*p)/2, (3 - m + n*p)/2,
-Tan[e + f*x]^2]*Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - m + n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2604

```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m, Int[(b*Tan[e +
 f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (d\cot(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (d\cot(e+fx))^m(c\tan(e+fx))$$

$$= \left((d\cot(e+fx))^m(c\tan(e+fx))^{m-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx$$

$$= \frac{\left(c(d\cot(e+fx))^m(c\tan(e+fx))^{m-np}\left(b(c\tan(e+fx))^n\right)^p\right)\mathrm{Subst}\left(\int \frac{x^{-m+}}{c^2+x}\right.}{f}$$

$$= \frac{(d\cot(e+fx))^m\, {}_2F_1\left(1,\frac{1}{2}(1-m+np);\frac{1}{2}(3-m+np);-\tan^2(e+fx)\right)\tan}{f(1-m+np)}$$

Mathematica [A] time = 0.118189, size = 77, normalized size = 0.96

$$\frac{d(d\cot(e+fx))^{m-1}\left(b(c\tan(e+fx))^n\right)^p \text{Hypergeometric2F1}\left(1,\frac{1}{2}(-m+np+1),\frac{1}{2}(-m+np+3),-\tan^2(e+fx)\right)}{f(-m+np+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (d*(d*Cot[e + f*x])^(-1 + m)*Hypergeometric2F1[1, (1 - m + n*p)/2, (3 - m +
 n*p)/2, -Tan[e + f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - m + n*p))
```

Maple [F] time = 4.112, size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^m \left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*cot(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \left(d\cot\left(fx+e\right)\right)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*cot(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \left(d\cot\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*(d*cot(f*x + e))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\left(d\cot\left(e+fx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*(d*cot(e + f*x))**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\left(d\cot\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*cot(f*x + e))^m, x)
```

3.424 $\int (d\cot(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=56

$$\left(\frac{\tan(e+fx)}{d}\right)^m (d\cot(e+fx))^m \text{Unintegrable}\left(\left(\frac{\tan(e+fx)}{d}\right)^{-m}\left(a+b(c\tan(e+fx))^n\right)^p, x\right)$$

```
[Out] (d*Cot[e + f*x])^m*(Tan[e + f*x]/d)^m*Unintegrable[(a + b*(c*Tan[e + f*x])^
n)^p/(Tan[e + f*x]/d)^m, x]
```

Rubi [A] time = 0.138833, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (d\cot(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*Cot[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (d*Cot[e + f*x])^m*(Tan[e + f*x]/d)^m*Defer[Int][(a + b*(c*Tan[e + f*x])^n)
^p/(Tan[e + f*x]/d)^m, x]
```

Rubi steps

$$\int (d\cot(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p\,dx = \left((d\cot(e+fx))^m \left(\frac{\tan(e+fx)}{d}\right)^m\right)\int \left(\frac{\tan(e+fx)}{d}\right)^{-m}\left(a+b(c\tan(e$$

Mathematica [A] time = 9.02188, size = 0, normalized size = 0.

$$\int (d\cot(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Cot[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Integrate[(d*Cot[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Maple [A] time = 0.536, size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^m \left(a+b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*cot(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p \left(d\cot\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*cot(f*x + e))^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p \left(d\cot\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*(d*cot(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**m*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p \left(d\cot\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*cot(f*x + e))^m, x)
```

3.425 $\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=70

$$\frac{(4a-b)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4a-b)\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}$$

```
[Out] ((4*a - b)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*a - b)*Sec[c + d*x]*Tan[c + d
*x])/(8*d) + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.0513948, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3676, 385, 199, 206}

$$\frac{(4a-b)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4a-b)\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((4*a - b)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*a - b)*Sec[c + d*x]*Tan[c + d
*x])/(8*d) + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{a-(a-b)x^2}{(1-x^2)^3}dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{b\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{(4a-b)\,\text{Subst}\left(\int \frac{1}{(1-x^2)^2}dx, x, \sin(c+dx)\right)}{4d}$$
$$= \frac{(4a-b)\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{(4a-b)\,\text{Su}}{}$$
$$= \frac{(4a-b)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{(4a-b)\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c}{}$$

Mathematica [A] time = 0.0670364, size = 93, normalized size = 1.33

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)\sec(c+dx)}{2d} - \frac{b\tanh^{-1}(\sin(c+dx))}{8d} + \frac{b\tan(c+dx)\sec^3(c+dx)}{4d} - \frac{b\tan(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) - (b*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Sec[
c + d*x]*Tan[c + d*x])/(2*d) - (b*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b*Sec
[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Maple [A] time = 0.088, size = 116, normalized size = 1.7

$$\frac{b(\sin(dx+c))^3}{4d(\cos(dx+c))^4} + \frac{b(\sin(dx+c))^3}{8d(\cos(dx+c))^2} + \frac{\sin(dx+c)b}{8d} - \frac{b\ln(\sec(dx+c)+\tan(dx+c))}{8d} + \frac{a\sec(dx+c)\tan(dx+}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/4/d*b*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*b*sin(d*x+c)^3/cos(d*x+c)^2+1/8/d*s
in(d*x+c)*b-1/8/d*b*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a*sec(d*x+c)*tan(d*x+c)
+1/2/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.10627, size = 128, normalized size = 1.83

$$\frac{(4a-b)\log(\sin(dx+c)+1) - (4a-b)\log(\sin(dx+c)-1) - \frac{2\left((4a-b)\sin(dx+c)^3-(4a+b)\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/16*((4*a - b)*log(sin(d*x + c) + 1) - (4*a - b)*log(sin(d*x + c) - 1) - 2
*((4*a - b)*sin(d*x + c)^3 - (4*a + b)*sin(d*x + c))/(sin(d*x + c)^4 - 2*si
```

```
n(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.56155, size = 235, normalized size = 3.36

$$\frac{(4\,a-b)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-(4\,a-b)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left((4\,a-b)\cos\left(dx+c\right)^{2}+2\right.}{16\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/16*((4*a - b)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (4*a - b)*cos(d*x +
c)^4*log(-sin(d*x + c) + 1) + 2*((4*a - b)*cos(d*x + c)^2 + 2*b)*sin(d*x +
c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan^{2}\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*sec(c + d*x)**3, x)
```

Giac [A] time = 1.70409, size = 132, normalized size = 1.89

$$\frac{(4\,a-b)\log\left(|\sin\left(dx+c\right)+1|\right)-(4\,a-b)\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{2\left(4\,a\sin(dx+c)^{3}-b\sin(dx+c)^{3}-4\,a\sin(dx+c)-b\sin(dx+c)\right)}{\left(\sin(dx+c)^{2}-1\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/16*((4*a - b)*log(abs(sin(d*x + c) + 1)) - (4*a - b)*log(abs(sin(d*x + c)
 - 1)) - 2*(4*a*sin(d*x + c)^3 - b*sin(d*x + c)^3 - 4*a*sin(d*x + c) - b*si
n(d*x + c))/(sin(d*x + c)^2 - 1)^2)/d
```

3.426 $\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=42

$$\frac{(2a-b)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] ((2*a - b)*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]*Tan[c + d*x])/(2*
d)
```

Rubi [A] time = 0.0342149, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3676, 385, 206}

$$\frac{(2a-b)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((2*a - b)*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]*Tan[c + d*x])/(2*
d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{a-(a-b)x^2}{\left(1-x^2\right)^2}\,dx,x,\sin(c+dx)\right)}{d}\\ &= \frac{b\sec(c+dx)\tan(c+dx)}{2d}+\frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(c+dx)\right)}{2d}\\ &= \frac{(2a-b)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0220192, size = 48, normalized size = 1.14

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{b\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (b*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c +
d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.032, size = 75, normalized size = 1.8

$$\frac{b\,(\sin(dx+c))^3}{2\,d\,(\cos(dx+c))^2}+\frac{\sin(dx+c)\,b}{2\,d}-\frac{b\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}+\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/2/d*b*sin(d*x+c)^3/cos(d*x+c)^2+1/2/d*sin(d*x+c)*b-1/2/d*b*ln(sec(d*x+c)+
tan(d*x+c))+1/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.21137, size = 84, normalized size = 2.

$$\frac{(2\,a-b)\log(\sin(dx+c)+1)-(2\,a-b)\log(\sin(dx+c)-1)-\frac{2\,b\sin(dx+c)}{\sin(dx+c)^2-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*((2*a - b)*log(sin(d*x + c) + 1) - (2*a - b)*log(sin(d*x + c) - 1) - 2*
b*sin(d*x + c)/(sin(d*x + c)^2 - 1))/d
```

Fricas [B] time = 1.32347, size = 192, normalized size = 4.57

$$\frac{(2\,a-b)\cos(dx+c)^2\log(\sin(dx+c)+1)-(2\,a-b)\cos(dx+c)^2\log(-\sin(dx+c)+1)+2\,b\sin(dx+c)}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*((2*a - b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a - b)*cos(d*x + c
)^2*log(-sin(d*x + c) + 1) + 2*b*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tan^2\left(c + dx\right)\right) \sec\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*sec(c + d*x), x)
```

Giac [A] time = 1.66836, size = 86, normalized size = 2.05

$$\frac{(2a-b)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-(2a-b)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2b\sin(dx+c)}{\sin(dx+c)^2-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*((2*a - b)*log(abs(sin(d*x + c) + 1)) - (2*a - b)*log(abs(sin(d*x + c)
- 1)) - 2*b*sin(d*x + c)/(sin(d*x + c)^2 - 1))/d
```

3.427 $\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=28

$$\frac{(a-b)\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d + ((a - b)*Sin[c + d*x])/d
```

Rubi [A] time = 0.0327812, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3676, 388, 206}

$$\frac{(a-b)\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d + ((a - b)*Sin[c + d*x])/d
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{a-(a-b)x^2}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d}\\
&= \frac{(a-b)\sin(c+dx)}{d}+\frac{b\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d}\\
&= \frac{b\tanh^{-1}(\sin(c+dx))}{d}+\frac{(a-b)\sin(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.025269, size = 47, normalized size = 1.68

$$\frac{a\sin(c)\cos(dx)}{d}+\frac{a\cos(c)\sin(dx)}{d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
 - (b*Sin[c + d*x])/d
```

Maple [A] time = 0.037, size = 44, normalized size = 1.6

$$\frac{\sin(dx+c)\,a}{d}-\frac{\sin(dx+c)\,b}{d}+\frac{b\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*tan(d*x+c)^2),x)
```

```
[Out] a*sin(d*x+c)/d-1/d*sin(d*x+c)*b+1/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.08571, size = 62, normalized size = 2.21

$$\frac{b(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\sin(dx+c))+2\,a\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) + 2
*a*sin(d*x + c))/d
```

Fricas [A] time = 1.53931, size = 115, normalized size = 4.11

$$\frac{b\log(\sin(dx+c)+1)-b\log(-\sin(dx+c)+1)+2\,(a-b)\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*log(sin(d*x + c) + 1) - b*log(-sin(d*x + c) + 1) + 2*(a - b)*sin(d*x
 + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan^2(c+dx)\right)\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*cos(c + d*x), x)
```

Giac [A] time = 1.68142, size = 65, normalized size = 2.32

$$\frac{b(\log(|\sin(dx+c)+1|) - \log(|\sin(dx+c)-1|) - 2\sin(dx+c)) + 2a\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(b*(log(abs(sin(d*x + c) + 1)) - log(abs(sin(d*x + c) - 1)) - 2*sin(d*x
 + c)) + 2*a*sin(d*x + c))/d
```

3.428 $\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=32

$$\frac{a\sin(c+dx)}{d}-\frac{(a-b)\sin^3(c+dx)}{3d}$$

```
[Out] (a*Sin[c + d*x])/d - ((a - b)*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0345442, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3676}

$$\frac{a\sin(c+dx)}{d}-\frac{(a-b)\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*Sin[c + d*x])/d - ((a - b)*Sin[c + d*x]^3)/(3*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx&=\frac{\text{Subst}\left(\int\left(a-(a-b)x^2\right)\,dx,x,\sin(c+dx)\right)}{d}\\&=\frac{a\sin(c+dx)}{d}-\frac{(a-b)\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0168688, size = 44, normalized size = 1.38

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (b*Sin[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.077, size = 36, normalized size = 1.1

$$\frac{1}{d}\left(\frac{b\left(\sin(dx+c)\right)^3}{3}+\frac{a\left(2+\left(\cos(dx+c)\right)^2\right)\sin(dx+c)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*b*sin(d*x+c)^3+1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.05688, size = 39, normalized size = 1.22

$$-\frac{(a-b)\sin(dx+c)^3 - 3\,a\sin(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/3*((a - b)*sin(d*x + c)^3 - 3*a*sin(d*x + c))/d
```

Fricas [A] time = 1.42889, size = 74, normalized size = 2.31

$$\frac{\left((a-b)\cos(dx+c)^2 + 2\,a + b\right)\sin(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/3*((a - b)*cos(d*x + c)^2 + 2*a + b)*sin(d*x + c)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tan^2(c+dx)\right)\cos^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*cos(c + d*x)**3, x)
```

Giac [A] time = 1.55848, size = 49, normalized size = 1.53

$$-\frac{a\sin(dx+c)^3 - b\sin(dx+c)^3 - 3\,a\sin(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/3*(a*sin(d*x + c)^3 - b*sin(d*x + c)^3 - 3*a*sin(d*x + c))/d
```

3.429 $\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=54

$$\frac{(a-b)\sin^5(c+dx)}{5d}-\frac{(2a-b)\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}$$

```
[Out] (a*Sin[c + d*x])/d - ((2*a - b)*Sin[c + d*x]^3)/(3*d) + ((a - b)*Sin[c + d*
x]^5)/(5*d)
```

Rubi [A] time = 0.0496329, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3676, 373}

$$\frac{(a-b)\sin^5(c+dx)}{5d}-\frac{(2a-b)\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*Sin[c + d*x])/d - ((2*a - b)*Sin[c + d*x]^3)/(3*d) + ((a - b)*Sin[c + d*
x]^5)/(5*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)dx &= \frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a-(a-b)x^2\right)dx, x, \sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(a-(2a-b)x^2+(a-b)x^4\right)dx, x, \sin(c+dx)\right)}{d}\\
&= \frac{a\sin(c+dx)}{d}-\frac{(2a-b)\sin^3(c+dx)}{3d}+\frac{(a-b)\sin^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] time = 0.185842, size = 52, normalized size = 0.96

$$\frac{\sin(c+dx)(4(7a-2b)\cos(2(c+dx))+3(a-b)\cos(4(c+dx))+89a+11b)}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((89*a + 11*b + 4*(7*a - 2*b)*Cos[2*(c + d*x)] + 3*(a - b)*Cos[4*(c + d*x)]
)*Sin[c + d*x])/(120*d)
```

Maple [A] time = 0.082, size = 72, normalized size = 1.3

$$\frac{1}{d}\left(b\left(-\frac{\sin(dx+c)(\cos(dx+c))^4}{5}+\frac{\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{15}\right)+\frac{\sin(dx+c)\,a}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4\,(\cos(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))+1/5*
a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.01943, size = 63, normalized size = 1.17

$$\frac{3\,(a-b)\sin(dx+c)^5-5\,(2\,a-b)\sin(dx+c)^3+15\,a\sin(dx+c)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*(a - b)*sin(d*x + c)^5 - 5*(2*a - b)*sin(d*x + c)^3 + 15*a*sin(d*x
+ c))/d
```

Fricas [A] time = 1.48501, size = 117, normalized size = 2.17

$$\frac{\left(3\,(a-b)\cos(dx+c)^4+(4\,a+b)\cos(dx+c)^2+8\,a+2\,b\right)\sin(dx+c)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*(a - b)*cos(d*x + c)^4 + (4*a + b)*cos(d*x + c)^2 + 8*a + 2*b)*sin(
d*x + c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 37.2951, size = 2898, normalized size = 53.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2/15*(15*a*tan(1/2*d*x)^10*tan(1/2*c)^9 + 15*a*tan(1/2*d*x)^9*tan(1/2*c)^1
0 + 20*a*tan(1/2*d*x)^10*tan(1/2*c)^7 + 20*b*tan(1/2*d*x)^10*tan(1/2*c)^7 -
 75*a*tan(1/2*d*x)^9*tan(1/2*c)^8 + 60*b*tan(1/2*d*x)^9*tan(1/2*c)^8 - 75*a
*tan(1/2*d*x)^8*tan(1/2*c)^9 + 60*b*tan(1/2*d*x)^8*tan(1/2*c)^9 + 20*a*tan(
1/2*d*x)^7*tan(1/2*c)^10 + 20*b*tan(1/2*d*x)^7*tan(1/2*c)^10 + 58*a*tan(1/2
*d*x)^10*tan(1/2*c)^5 - 8*b*tan(1/2*d*x)^10*tan(1/2*c)^5 + 150*a*tan(1/2*d*
x)^9*tan(1/2*c)^6 - 180*b*tan(1/2*d*x)^9*tan(1/2*c)^6 + 700*a*tan(1/2*d*x)^
8*tan(1/2*c)^7 - 500*b*tan(1/2*d*x)^8*tan(1/2*c)^7 + 700*a*tan(1/2*d*x)^7*t
an(1/2*c)^8 - 500*b*tan(1/2*d*x)^7*tan(1/2*c)^8 + 150*a*tan(1/2*d*x)^6*tan(
1/2*c)^9 - 180*b*tan(1/2*d*x)^6*tan(1/2*c)^9 + 58*a*tan(1/2*d*x)^5*tan(1/2*
c)^10 - 8*b*tan(1/2*d*x)^5*tan(1/2*c)^10 + 20*a*tan(1/2*d*x)^10*tan(1/2*c)^
3 + 20*b*tan(1/2*d*x)^10*tan(1/2*c)^3 - 150*a*tan(1/2*d*x)^9*tan(1/2*c)^4 +
 180*b*tan(1/2*d*x)^9*tan(1/2*c)^4 - 610*a*tan(1/2*d*x)^8*tan(1/2*c)^5 + 10
40*b*tan(1/2*d*x)^8*tan(1/2*c)^5 - 2200*a*tan(1/2*d*x)^7*tan(1/2*c)^6 + 236
0*b*tan(1/2*d*x)^7*tan(1/2*c)^6 - 2200*a*tan(1/2*d*x)^6*tan(1/2*c)^7 + 2360
*b*tan(1/2*d*x)^6*tan(1/2*c)^7 - 610*a*tan(1/2*d*x)^5*tan(1/2*c)^8 + 1040*b
*tan(1/2*d*x)^5*tan(1/2*c)^8 - 150*a*tan(1/2*d*x)^4*tan(1/2*c)^9 + 180*b*ta
n(1/2*d*x)^4*tan(1/2*c)^9 + 20*a*tan(1/2*d*x)^3*tan(1/2*c)^10 + 20*b*tan(1/
2*d*x)^3*tan(1/2*c)^10 + 15*a*tan(1/2*d*x)^10*tan(1/2*c) + 75*a*tan(1/2*d*x
)^9*tan(1/2*c)^2 - 60*b*tan(1/2*d*x)^9*tan(1/2*c)^2 + 700*a*tan(1/2*d*x)^8*
tan(1/2*c)^3 - 500*b*tan(1/2*d*x)^8*tan(1/2*c)^3 + 2200*a*tan(1/2*d*x)^7*ta
n(1/2*c)^4 - 2360*b*tan(1/2*d*x)^7*tan(1/2*c)^4 + 5380*a*tan(1/2*d*x)^6*tan
(1/2*c)^5 - 5000*b*tan(1/2*d*x)^6*tan(1/2*c)^5 + 5380*a*tan(1/2*d*x)^5*tan(
1/2*c)^6 - 5000*b*tan(1/2*d*x)^5*tan(1/2*c)^6 + 2200*a*tan(1/2*d*x)^4*tan(1
/2*c)^7 - 2360*b*tan(1/2*d*x)^4*tan(1/2*c)^7 + 700*a*tan(1/2*d*x)^3*tan(1/2
*c)^8 - 500*b*tan(1/2*d*x)^3*tan(1/2*c)^8 + 75*a*tan(1/2*d*x)^2*tan(1/2*c)^
9 - 60*b*tan(1/2*d*x)^2*tan(1/2*c)^9 + 15*a*tan(1/2*d*x)*tan(1/2*c)^10 - 15
*a*tan(1/2*d*x)^9 - 75*a*tan(1/2*d*x)^8*tan(1/2*c) + 60*b*tan(1/2*d*x)^8*ta
n(1/2*c) - 700*a*tan(1/2*d*x)^7*tan(1/2*c)^2 + 500*b*tan(1/2*d*x)^7*tan(1/2
*c)^2 - 2200*a*tan(1/2*d*x)^6*tan(1/2*c)^3 + 2360*b*tan(1/2*d*x)^6*tan(1/2*
c)^3 - 5380*a*tan(1/2*d*x)^5*tan(1/2*c)^4 + 5000*b*tan(1/2*d*x)^5*tan(1/2*c
)^4 - 5380*a*tan(1/2*d*x)^4*tan(1/2*c)^5 + 5000*b*tan(1/2*d*x)^4*tan(1/2*c)
^5 - 2200*a*tan(1/2*d*x)^3*tan(1/2*c)^6 + 2360*b*tan(1/2*d*x)^3*tan(1/2*c)^
6 - 700*a*tan(1/2*d*x)^2*tan(1/2*c)^7 + 500*b*tan(1/2*d*x)^2*tan(1/2*c)^7 -
 75*a*tan(1/2*d*x)*tan(1/2*c)^8 + 60*b*tan(1/2*d*x)*tan(1/2*c)^8 - 15*a*tan
(1/2*c)^9 - 20*a*tan(1/2*d*x)^7 - 20*b*tan(1/2*d*x)^7 + 150*a*tan(1/2*d*x)^
6*tan(1/2*c) - 180*b*tan(1/2*d*x)^6*tan(1/2*c) + 610*a*tan(1/2*d*x)^5*tan(1
/2*c)^2 - 1040*b*tan(1/2*d*x)^5*tan(1/2*c)^2 + 2200*a*tan(1/2*d*x)^4*tan(1/
2*c)^3 - 2360*b*tan(1/2*d*x)^4*tan(1/2*c)^3 + 2200*a*tan(1/2*d*x)^3*tan(1/2
*c)^4 - 2360*b*tan(1/2*d*x)^3*tan(1/2*c)^4 + 610*a*tan(1/2*d*x)^2*tan(1/2*c
)^5 - 1040*b*tan(1/2*d*x)^2*tan(1/2*c)^5 + 150*a*tan(1/2*d*x)*tan(1/2*c)^6
 - 180*b*tan(1/2*d*x)*tan(1/2*c)^6 - 20*a*tan(1/2*c)^7 - 20*b*tan(1/2*c)^7 -
 58*a*tan(1/2*d*x)^5 + 8*b*tan(1/2*d*x)^5 - 150*a*tan(1/2*d*x)^4*tan(1/2*c)
 + 180*b*tan(1/2*d*x)^4*tan(1/2*c) - 700*a*tan(1/2*d*x)^3*tan(1/2*c)^2 + 50
0*b*tan(1/2*d*x)^3*tan(1/2*c)^2 - 700*a*tan(1/2*d*x)^2*tan(1/2*c)^3 + 500*b
*tan(1/2*d*x)^2*tan(1/2*c)^3 - 150*a*tan(1/2*d*x)*tan(1/2*c)^4 + 180*b*tan(
1/2*d*x)*tan(1/2*c)^4 - 58*a*tan(1/2*c)^5 + 8*b*tan(1/2*c)^5 - 20*a*tan(1/2
```

```
*d*x)^3 - 20*b*tan(1/2*d*x)^3 + 75*a*tan(1/2*d*x)^2*tan(1/2*c) - 60*b*tan(1
/2*d*x)^2*tan(1/2*c) + 75*a*tan(1/2*d*x)*tan(1/2*c)^2 - 60*b*tan(1/2*d*x)*t
an(1/2*c)^2 - 20*a*tan(1/2*c)^3 - 20*b*tan(1/2*c)^3 - 15*a*tan(1/2*d*x) - 1
5*a*tan(1/2*c))/(d*tan(1/2*d*x)^10*tan(1/2*c)^10 + 5*d*tan(1/2*d*x)^10*tan(
1/2*c)^8 + 5*d*tan(1/2*d*x)^8*tan(1/2*c)^10 + 10*d*tan(1/2*d*x)^10*tan(1/2*
c)^6 + 25*d*tan(1/2*d*x)^8*tan(1/2*c)^8 + 10*d*tan(1/2*d*x)^6*tan(1/2*c)^10
 + 10*d*tan(1/2*d*x)^10*tan(1/2*c)^4 + 50*d*tan(1/2*d*x)^8*tan(1/2*c)^6 + 5
0*d*tan(1/2*d*x)^6*tan(1/2*c)^8 + 10*d*tan(1/2*d*x)^4*tan(1/2*c)^10 + 5*d*t
an(1/2*d*x)^10*tan(1/2*c)^2 + 50*d*tan(1/2*d*x)^8*tan(1/2*c)^4 + 100*d*tan(
1/2*d*x)^6*tan(1/2*c)^6 + 50*d*tan(1/2*d*x)^4*tan(1/2*c)^8 + 5*d*tan(1/2*d*
x)^2*tan(1/2*c)^10 + d*tan(1/2*d*x)^10 + 25*d*tan(1/2*d*x)^8*tan(1/2*c)^2 +
 100*d*tan(1/2*d*x)^6*tan(1/2*c)^4 + 100*d*tan(1/2*d*x)^4*tan(1/2*c)^6 + 25
*d*tan(1/2*d*x)^2*tan(1/2*c)^8 + d*tan(1/2*c)^10 + 5*d*tan(1/2*d*x)^8 + 50*
d*tan(1/2*d*x)^6*tan(1/2*c)^2 + 100*d*tan(1/2*d*x)^4*tan(1/2*c)^4 + 50*d*ta
n(1/2*d*x)^2*tan(1/2*c)^6 + 5*d*tan(1/2*c)^8 + 10*d*tan(1/2*d*x)^6 + 50*d*t
an(1/2*d*x)^4*tan(1/2*c)^2 + 50*d*tan(1/2*d*x)^2*tan(1/2*c)^4 + 10*d*tan(1/
2*c)^6 + 10*d*tan(1/2*d*x)^4 + 25*d*tan(1/2*d*x)^2*tan(1/2*c)^2 + 10*d*tan(
1/2*c)^4 + 5*d*tan(1/2*d*x)^2 + 5*d*tan(1/2*c)^2 + d)
```

3.430 $\int \cos^7(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=76

$$-\frac{(a-b)\sin^7(c+dx)}{7d}+\frac{(3a-2b)\sin^5(c+dx)}{5d}-\frac{(3a-b)\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}$$

```
[Out] (a*Sin[c + d*x])/d - ((3*a - b)*Sin[c + d*x]^3)/(3*d) + ((3*a - 2*b)*Sin[c
+ d*x]^5)/(5*d) - ((a - b)*Sin[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0586209, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3676, 373}

$$-\frac{(a-b)\sin^7(c+dx)}{7d}+\frac{(3a-2b)\sin^5(c+dx)}{5d}-\frac{(3a-b)\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*Sin[c + d*x])/d - ((3*a - b)*Sin[c + d*x]^3)/(3*d) + ((3*a - 2*b)*Sin[c
+ d*x]^5)/(5*d) - ((a - b)*Sin[c + d*x]^7)/(7*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx&=\frac{\text{Subst}\left(\int \left(1-x^2\right)^2\left(a-(a-b)x^2\right)\,dx,x,\sin(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int \left(a-(3a-b)x^2+(3a-2b)x^4-(a-b)x^6\right)\,dx,x,\sin(c+dx)\right)}{d}\\&=\frac{a\sin(c+dx)}{d}-\frac{(3a-b)\sin^3(c+dx)}{3d}+\frac{(3a-2b)\sin^5(c+dx)}{5d}-\frac{(a-b)\sin^7(c+dx)}{7d}\end{aligned}$$

Mathematica [A] time = 0.28332, size = 75, normalized size = 0.99

$$\frac{\sin(c+dx)((897a-113b)\cos(2(c+dx))+6(27a-13b)\cos(4(c+dx))+15a\cos(6(c+dx))+2286a-15b\cos(6(c+dx)) \ldots}{3360d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((2286*a + 206*b + (897*a - 113*b)*Cos[2*(c + d*x)] + 6*(27*a - 13*b)*Cos[4
*(c + d*x)] + 15*a*Cos[6*(c + d*x)] - 15*b*Cos[6*(c + d*x)])*Sin[c + d*x])/
(3360*d)
```

Maple [A] time = 0.081, size = 92, normalized size = 1.2

$$\frac{1}{d}\left(b\left(-\frac{\sin(dx+c)(\cos(dx+c))^6}{7}+\frac{\sin(dx+c)}{35}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)\right)+\frac{\sin(dx+c)a}{7}\left(\frac{16}{5}+(\cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^
2)*sin(d*x+c))+1/7*a*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*
sin(d*x+c))
```

Maxima [A] time = 1.0054, size = 86, normalized size = 1.13

$$-\frac{15(a-b)\sin(dx+c)^7-21(3a-2b)\sin(dx+c)^5+35(3a-b)\sin(dx+c)^3-105a\sin(dx+c)}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/105*(15*(a - b)*sin(d*x + c)^7 - 21*(3*a - 2*b)*sin(d*x + c)^5 + 35*(3*a
 - b)*sin(d*x + c)^3 - 105*a*sin(d*x + c))/d
```

Fricas [A] time = 1.52069, size = 163, normalized size = 2.14

$$\frac{\left(15(a-b)\cos(dx+c)^6+3(6a+b)\cos(dx+c)^4+4(6a+b)\cos(dx+c)^2+48a+8b\right)\sin(dx+c)}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*(a - b)*cos(d*x + c)^6 + 3*(6*a + b)*cos(d*x + c)^4 + 4*(6*a + b)
*cos(d*x + c)^2 + 48*a + 8*b)*sin(d*x + c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.431 $\int \sec^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=68

$$\frac{(a+2b)\tan^5(c+dx)}{5d}+\frac{(2a+b)\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\tan^7(c+dx)}{7d}$$

```
[Out] (a*Tan[c + d*x])/d + ((2*a + b)*Tan[c + d*x]^3)/(3*d) + ((a + 2*b)*Tan[c +
d*x]^5)/(5*d) + (b*Tan[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0505921, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3675, 373}

$$\frac{(a+2b)\tan^5(c+dx)}{5d}+\frac{(2a+b)\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\tan^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*Tan[c + d*x])/d + ((2*a + b)*Tan[c + d*x]^3)/(3*d) + ((a + 2*b)*Tan[c +
d*x]^5)/(5*d) + (b*Tan[c + d*x]^7)/(7*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx&=\frac{\text{Subst}\left(\int\left(1+x^2\right)^2\left(a+bx^2\right)\,dx,x,\tan(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int\left(a+(2a+b)x^2+(a+2b)x^4+bx^6\right)\,dx,x,\tan(c+dx)\right)}{d}\\&=\frac{a\tan(c+dx)}{d}+\frac{(2a+b)\tan^3(c+dx)}{3d}+\frac{(a+2b)\tan^5(c+dx)}{5d}+\frac{b\tan^7(c+dx)}{7d}\end{aligned}$$

Mathematica [A] time = 0.247424, size = 75, normalized size = 1.1

$$\frac{\tan(c+dx)\left(21a\tan^4(c+dx)+70a\tan^2(c+dx)+105a+15b\sec^6(c+dx)-3b\sec^4(c+dx)-4b\sec^2(c+dx)-8b\right)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (Tan[c + d*x]*(105*a - 8*b - 4*b*Sec[c + d*x]^2 - 3*b*Sec[c + d*x]^4 + 15*b
*Sec[c + d*x]^6 + 70*a*Tan[c + d*x]^2 + 21*a*Tan[c + d*x]^4))/(105*d)
```

Maple [A] time = 0.087, size = 94, normalized size = 1.4

$$\frac{1}{d}\left(b\left(\frac{(\sin(dx+c))^3}{7(\cos(dx+c))^7}+\frac{4(\sin(dx+c))^3}{35(\cos(dx+c))^5}+\frac{8(\sin(dx+c))^3}{105(\cos(dx+c))^3}\right)-a\left(-\frac{8}{15}-\frac{(\sec(dx+c))^4}{5}-\frac{4(\sec(dx+c))^2}{15}\right)\mathrm{ta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/105*
sin(d*x+c)^3/cos(d*x+c)^3)-a*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan
(d*x+c))
```

Maxima [A] time = 1.11457, size = 76, normalized size = 1.12

$$\frac{15\,b\tan(dx+c)^7+21\,(a+2\,b)\tan(dx+c)^5+35\,(2\,a+b)\tan(dx+c)^3+105\,a\tan(dx+c)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*b*tan(d*x + c)^7 + 21*(a + 2*b)*tan(d*x + c)^5 + 35*(2*a + b)*tan
(d*x + c)^3 + 105*a*tan(d*x + c))/d
```

Fricas [A] time = 1.35736, size = 180, normalized size = 2.65

$$\frac{\left(8\,(7\,a-b)\cos(dx+c)^6+4\,(7\,a-b)\cos(dx+c)^4+3\,(7\,a-b)\cos(dx+c)^2+15\,b\right)\sin(dx+c)}{105\,d\cos(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(7*a - b)*cos(d*x + c)^6 + 4*(7*a - b)*cos(d*x + c)^4 + 3*(7*a - b
)*cos(d*x + c)^2 + 15*b)*sin(d*x + c)/(d*cos(d*x + c)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan^2(c+dx)\right)\sec^6(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*sec(c + d*x)**6, x)
```

Giac [A] time = 1.46791, size = 95, normalized size = 1.4

$$\frac{15\,b\tan\left(dx+c\right)^{7}+21\,a\tan\left(dx+c\right)^{5}+42\,b\tan\left(dx+c\right)^{5}+70\,a\tan\left(dx+c\right)^{3}+35\,b\tan\left(dx+c\right)^{3}+105\,a\tan\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/105*(15*b*tan(d*x + c)^7 + 21*a*tan(d*x + c)^5 + 42*b*tan(d*x + c)^5 + 70
*a*tan(d*x + c)^3 + 35*b*tan(d*x + c)^3 + 105*a*tan(d*x + c))/d
```

3.432 $\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=46

$$\frac{(a+b)\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\tan^5(c+dx)}{5d}$$

```
[Out] (a*Tan[c + d*x])/d + ((a + b)*Tan[c + d*x]^3)/(3*d) + (b*Tan[c + d*x]^5)/(5
*d)
```

Rubi [A] time = 0.0405622, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3675, 373}

$$\frac{(a+b)\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*Tan[c + d*x])/d + ((a + b)*Tan[c + d*x]^3)/(3*d) + (b*Tan[c + d*x]^5)/(5
*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \left(1+x^2\right)\left(a+bx^2\right)\,dx,x,\tan(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(a+(a+b)x^2+bx^4\right)\,dx,x,\tan(c+dx)\right)}{d}\\
&= \frac{a\tan(c+dx)}{d}+\frac{(a+b)\tan^3(c+dx)}{3d}+\frac{b\tan^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] time = 0.138067, size = 53, normalized size = 1.15

$$\frac{\tan(c+dx)\left(5a\tan^2(c+dx)+15a+3b\sec^4(c+dx)-b\sec^2(c+dx)-2b\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (Tan[c + d*x]*(15*a - 2*b - b*Sec[c + d*x]^2 + 3*b*Sec[c + d*x]^4 + 5*a*Tan
[c + d*x]^2))/(15*d)
```

Maple [A] time = 0.043, size = 66, normalized size = 1.4

$$\frac{1}{d}\left(b\left(\frac{(\sin(dx+c))^3}{5(\cos(dx+c))^5}+\frac{2(\sin(dx+c))^3}{15(\cos(dx+c))^3}\right)-a\left(-\frac{2}{3}-\frac{(\sec(dx+c))^2}{3}\right)\tan(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)-a*(-2
/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.00245, size = 53, normalized size = 1.15

$$\frac{3\,b\tan(dx+c)^5+5\,(a+b)\tan(dx+c)^3+15\,a\tan(dx+c)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*b*tan(d*x + c)^5 + 5*(a + b)*tan(d*x + c)^3 + 15*a*tan(d*x + c))/d
```

Fricas [A] time = 1.31229, size = 135, normalized size = 2.93

$$\frac{\left(2\,(5\,a-b)\cos(dx+c)^4+(5\,a-b)\cos(dx+c)^2+3\,b\right)\sin(dx+c)}{15\,d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(5*a - b)*cos(d*x + c)^4 + (5*a - b)*cos(d*x + c)^2 + 3*b)*sin(d*x
+ c)/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan^2(c+dx)\right)\sec^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*sec(c + d*x)**4, x)
```

Giac [A] time = 1.53058, size = 65, normalized size = 1.41

$$\frac{3\,b\tan\left(dx+c\right)^{5}+5\,a\tan\left(dx+c\right)^{3}+5\,b\tan\left(dx+c\right)^{3}+15\,a\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/15*(3*b*tan(d*x + c)^5 + 5*a*tan(d*x + c)^3 + 5*b*tan(d*x + c)^3 + 15*a*t
an(d*x + c))/d
```

3.433 $\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)dx$

Optimal. Leaf size=28

$$\frac{a\tan(c+dx)}{d}+\frac{b\tan^3(c+dx)}{3d}$$

```
[Out] (a*Tan[c + d*x])/d + (b*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0289415, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3675}

$$\frac{a\tan(c+dx)}{d}+\frac{b\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x]^2),x]

[Out] (a*Tan[c + d*x])/d + (b*Tan[c + d*x]^3)/(3*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)dx &= \frac{\text{Subst}\left(\int\left(a+bx^2\right)dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{a\tan(c+dx)}{d}+\frac{b\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0109921, size = 28, normalized size = 1.

$$\frac{a\tan(c+dx)}{d}+\frac{b\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x]^2),x]

[Out] (a*Tan[c + d*x])/d + (b*Tan[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.038, size = 33, normalized size = 1.2

$$\frac{1}{d}\left(\frac{b\left(\sin(dx+c)\right)^3}{3\left(\cos(dx+c)\right)^3}+a\tan(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*b*sin(d*x+c)^3/cos(d*x+c)^3+a*tan(d*x+c))
```

Maxima [A] time = 1.12524, size = 34, normalized size = 1.21

$$\frac{b\tan(dx+c)^3 + 3\,a\tan(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(b*tan(d*x + c)^3 + 3*a*tan(d*x + c))/d
```

Fricas [A] time = 1.48839, size = 92, normalized size = 3.29

$$\frac{\left((3\,a-b)\cos(dx+c)^2 + b\right)\sin(dx+c)}{3\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/3*((3*a - b)*cos(d*x + c)^2 + b)*sin(d*x + c)/(d*cos(d*x + c)^3)
```

Sympy [A] time = 1.76578, size = 36, normalized size = 1.29

$$\begin{cases} \frac{a\tan(c+dx)+\frac{b\tan^3(c+dx)}{3}}{d} & \text{for}\ d\neq 0 \\ x\left(a+b\tan^2(c)\right)\sec^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Piecewise(((a*tan(c + d*x) + b*tan(c + d*x)**3/3)/d, Ne(d, 0)), (x*(a + b*t
an(c)**2)*sec(c)**2, True))
```

Giac [A] time = 1.46188, size = 34, normalized size = 1.21

$$\frac{b\tan(dx+c)^3 + 3\,a\tan(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(b*tan(d*x + c)^3 + 3*a*tan(d*x + c))/d
```

3.434 $\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)dx$

Optimal. Leaf size=33

$$\frac{(a-b)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(a+b)$$

```
[Out] ((a + b)*x)/2 + ((a - b)*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0385714, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3675, 385, 203}

$$\frac{(a-b)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(a+b)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((a + b)*x)/2 + ((a - b)*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)dx &= \frac{\text{Subst}\left(\int \frac{a+bx^2}{\left(1+x^2\right)^2}\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{(a-b)\cos(c+dx)\sin(c+dx)}{2d}+\frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(c+dx)\right)}{2d}\\ &= \frac{1}{2}(a+b)x+\frac{(a-b)\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0564466, size = 32, normalized size = 0.97

$$\frac{2(a+b)(c+dx)+(a-b)\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (2*(a + b)*(c + d*x) + (a - b)*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.036, size = 54, normalized size = 1.6

$$\frac{1}{d}\left(b\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a*(1/2*cos(d*x+c)*sin(d*x
+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.63994, size = 53, normalized size = 1.61

$$\frac{(dx+c)(a+b)+\frac{(a-b)\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((d*x + c)*(a + b) + (a - b)*tan(d*x + c)/(tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.72183, size = 77, normalized size = 2.33

$$\frac{(a+b)dx+(a-b)\cos(dx+c)\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*((a + b)*d*x + (a - b)*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan^2(c+dx)\right)\cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*cos(c + d*x)**2, x)
```

Giac [B] time = 1.40934, size = 228, normalized size = 6.91

$$\frac{adx\tan(dx)^2\tan(c)^2 + bdx\tan(dx)^2\tan(c)^2 + adx\tan(dx)^2 + bdx\tan(dx)^2 + adx\tan(c)^2 + bdx\tan(c)^2 - a\tan(dx)}{2\left(d\tan(dx)^2\tan(c)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(a*d*x*tan(d*x)^2*tan(c)^2 + b*d*x*tan(d*x)^2*tan(c)^2 + a*d*x*tan(d*x)
^2 + b*d*x*tan(d*x)^2 + a*d*x*tan(c)^2 + b*d*x*tan(c)^2 - a*tan(d*x)^2*tan(
c) + b*tan(d*x)^2*tan(c) - a*tan(d*x)*tan(c)^2 + b*tan(d*x)*tan(c)^2 + a*d*
x + b*d*x + a*tan(d*x) - b*tan(d*x) + a*tan(c) - b*tan(c))/(d*tan(d*x)^2*ta
n(c)^2 + d*tan(d*x)^2 + d*tan(c)^2 + d)
```

3.435 $\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=61

$$\frac{(a-b)\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{(3a+b)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(3a+b)$$

```
[Out] ((3*a + b)*x)/8 + ((3*a + b)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + ((a - b)*Co
s[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.0464012, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3675, 385, 199, 203}

$$\frac{(a-b)\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{(3a+b)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(3a+b)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((3*a + b)*x)/8 + ((3*a + b)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + ((a - b)*Co
s[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{a+bx^2}{(1+x^2)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{(a-b)\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{(3a+b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)^2}\,dx, x, \tan(c+dx)\right)}{4d}$$

$$= \frac{(3a+b)\cos(c+dx)\sin(c+dx)}{8d} + \frac{(a-b)\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{(3a+b)\,\text{S}}{}$$

$$= \frac{1}{8}(3a+b)x + \frac{(3a+b)\cos(c+dx)\sin(c+dx)}{8d} + \frac{(a-b)\cos^3(c+dx)\sin(c+dx}{4d}$$

Mathematica [A] time = 0.12773, size = 46, normalized size = 0.75

$$\frac{(a-b)\sin(4(c+dx)) + 12a(c+dx) + 8a\sin(2(c+dx)) + 4bdx}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (4*b*d*x + 12*a*(c + d*x) + 8*a*Sin[2*(c + d*x)] + (a - b)*Sin[4*(c + d*x)]
)/(32*d)
```

Maple [A] time = 0.082, size = 81, normalized size = 1.3

$$\frac{1}{d}\left(b\left(-\frac{\sin(dx+c)(\cos(dx+c))^3}{4} + \frac{\cos(dx+c)\sin(dx+c)}{8} + \frac{dx}{8} + \frac{c}{8}\right) + a\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3 + \frac{3\cos(dx+}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*
c)+a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.6495, size = 93, normalized size = 1.52

$$\frac{(dx+c)(3a+b) + \frac{(3a+b)\tan(dx+c)^3 + (5a-b)\tan(dx+c)}{\tan(dx+c)^4 + 2\tan(dx+c)^2 + 1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*((d*x + c)*(3*a + b) + ((3*a + b)*tan(d*x + c)^3 + (5*a - b)*tan(d*x +
c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.68508, size = 122, normalized size = 2.

$$\frac{(3\,a+b)dx+\left(2\,(a-b)\cos\left(dx+c\right)^{3}+(3\,a+b)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*((3*a + b)*d*x + (2*(a - b)*cos(d*x + c)^3 + (3*a + b)*cos(d*x + c))*si
n(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan^{2}\left(c+dx\right)\right)\cos^{4}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*cos(c + d*x)**4, x)
```

Giac [B] time = 4.16552, size = 2714, normalized size = 44.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/64*(3*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*ta
n(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 24*a*d*x*tan
(d*x)^4*tan(c)^4 + 8*b*d*x*tan(d*x)^4*tan(c)^4 + 3*pi*b*sgn(-2*tan(d*x)^2*t
an(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 +
6*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)
*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^2 + 6*pi*b*sgn(2*tan(d
*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(
d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 + 6*b*arctan((tan(d*x) + tan(c))/(tan(
d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^4 - 6*b*arctan(-(tan(d*x) - tan(c))/(ta
n(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4 + 48*a*d*x*tan(d*x)^4*tan(c)^2 + 16
*b*d*x*tan(d*x)^4*tan(c)^2 + 6*pi*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*t
an(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^2 + 48*a*d*x*tan(d*x)^2*
tan(c)^4 + 16*b*d*x*tan(d*x)^2*tan(c)^4 + 6*pi*b*sgn(-2*tan(d*x)^2*tan(c) +
 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 + 3*pi*b*
sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)
^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 + 12*pi*b*sgn(2*tan(d*x)^2*tan(c)^2
- 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c)
)*tan(d*x)^2*tan(c)^2 + 12*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) -
1))*tan(d*x)^4*tan(c)^2 - 12*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c)
 + 1))*tan(d*x)^4*tan(c)^2 - 40*a*tan(d*x)^4*tan(c)^3 + 8*b*tan(d*x)^4*tan(
c)^3 + 3*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*t
an(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^4 + 12*b*arctan((tan(d*x)
+ tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^4 - 12*b*arctan(-(tan(d*
```

```
x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^4 - 40*a*tan(d*x)^3*t
an(c)^4 + 8*b*tan(d*x)^3*tan(c)^4 + 24*a*d*x*tan(d*x)^4 + 8*b*d*x*tan(d*x)^
4 + 3*pi*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*
tan(c))*tan(d*x)^4 + 96*a*d*x*tan(d*x)^2*tan(c)^2 + 32*b*d*x*tan(d*x)^2*tan
(c)^2 + 12*pi*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x)
 - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 24*a*d*x*tan(c)^4 + 8*b*d*x*tan(c)^4 + 3
*pi*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c
))*tan(c)^4 + 6*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c
) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2 + 6*b*arctan((t
an(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4 - 6*b*arctan(-(tan(d*x)
 - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4 - 24*a*tan(d*x)^4*tan(c) - 8*b
*tan(d*x)^4*tan(c) + 6*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^
2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 + 24*b*arc
tan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^2 - 24*b*a
rctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 + 48*
a*tan(d*x)^3*tan(c)^2 - 48*b*tan(d*x)^3*tan(c)^2 + 48*a*tan(d*x)^2*tan(c)^3
 - 48*b*tan(d*x)^2*tan(c)^3 + 6*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(
c) - 1))*tan(c)^4 - 6*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*
tan(c)^4 - 24*a*tan(d*x)*tan(c)^4 - 8*b*tan(d*x)*tan(c)^4 + 48*a*d*x*tan(d*
x)^2 + 16*b*d*x*tan(d*x)^2 + 6*pi*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*t
an(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2 + 48*a*d*x*tan(c)^2 + 16*b*d*x*
tan(c)^2 + 6*pi*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*
x) - 2*tan(c))*tan(c)^2 + 3*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(
d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c)) + 12*b*arctan(
(tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2 - 12*b*arctan(-(tan(d
*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2 + 24*a*tan(d*x)^3 + 8*b*tan
(d*x)^3 - 48*a*tan(d*x)^2*tan(c) + 48*b*tan(d*x)^2*tan(c) + 12*b*arctan((ta
n(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(c)^2 - 12*b*arctan(-(tan(d*x) -
 tan(c))/(tan(d*x)*tan(c) + 1))*tan(c)^2 - 48*a*tan(d*x)*tan(c)^2 + 48*b*ta
n(d*x)*tan(c)^2 + 24*a*tan(c)^3 + 8*b*tan(c)^3 + 24*a*d*x + 8*b*d*x + 3*pi*
b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c)) +
 6*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1)) - 6*b*arctan(-(tan(d
*x) - tan(c))/(tan(d*x)*tan(c) + 1)) + 40*a*tan(d*x) - 8*b*tan(d*x) + 40*a*
tan(c) - 8*b*tan(c))/(d*tan(d*x)^4*tan(c)^4 + 2*d*tan(d*x)^4*tan(c)^2 + 2*d
*tan(d*x)^2*tan(c)^4 + d*tan(d*x)^4 + 4*d*tan(d*x)^2*tan(c)^2 + d*tan(c)^4
+ 2*d*tan(d*x)^2 + 2*d*tan(c)^2 + d)
```

3.436 $\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=87

$$\frac{(a-b)\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{(5a+b)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(5a+b)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x(5a+b)$$

```
[Out] ((5*a + b)*x)/16 + ((5*a + b)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + ((5*a + b
)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + ((a - b)*Cos[c + d*x]^5*Sin[c + d*x
])/(6*d)
```

Rubi [A] time = 0.0551299, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3675, 385, 199, 203}

$$\frac{(a-b)\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{(5a+b)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(5a+b)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x(5a+b)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((5*a + b)*x)/16 + ((5*a + b)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + ((5*a + b
)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + ((a - b)*Cos[c + d*x]^5*Sin[c + d*x
])/(6*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{a+bx^2}{(1+x^2)^4}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{(a-b)\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{(5a+b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)^3}\,dx, x, \tan(c+dx)\right)}{6d}$$
$$= \frac{(5a+b)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{(a-b)\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{(5a+b)}{}$$
$$= \frac{(5a+b)\cos(c+dx)\sin(c+dx)}{16d} + \frac{(5a+b)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{(a-b)\,c}{}$$
$$= \frac{1}{16}(5a+b)x + \frac{(5a+b)\cos(c+dx)\sin(c+dx)}{16d} + \frac{(5a+b)\cos^3(c+dx)\sin(c+}{24d}$$

Mathematica [A] time = 0.21104, size = 74, normalized size = 0.85

$$\frac{3(15a+b)\sin(2(c+dx))+(9a-3b)\sin(4(c+dx))+a\sin(6(c+dx))+60ac+60adx-b\sin(6(c+dx))+12bdx}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (60*a*c + 60*a*d*x + 12*b*d*x + 3*(15*a + b)*Sin[2*(c + d*x)] + (9*a - 3*b)
*Sin[4*(c + d*x)] + a*Sin[6*(c + d*x)] - b*Sin[6*(c + d*x)])/(192*d)
```

Maple [A] time = 0.085, size = 102, normalized size = 1.2

$$\frac{1}{d}\left(b\left(-\frac{\sin(dx+c)(\cos(dx+c))^5}{6}+\frac{\sin(dx+c)}{24}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{dx}{16}+\frac{c}{16}\right)+a\left(\frac{\sin(dx+c)}{6}\left((\cos(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin
(d*x+c)+1/16*d*x+1/16*c)+a*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x
+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A] time = 1.70511, size = 131, normalized size = 1.51

$$\frac{3(dx+c)(5a+b)+\frac{3(5a+b)\tan(dx+c)^5+8(5a+b)\tan(dx+c)^3+3(11a-b)\tan(dx+c)}{\tan(dx+c)^6+3\tan(dx+c)^4+3\tan(dx+c)^2+1}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/48*(3*(d*x + c)*(5*a + b) + (3*(5*a + b)*tan(d*x + c)^5 + 8*(5*a + b)*tan
(d*x + c)^3 + 3*(11*a - b)*tan(d*x + c))/(tan(d*x + c)^6 + 3*tan(d*x + c)^4
 + 3*tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.67815, size = 167, normalized size = 1.92

$$\frac{3\,(5\,a+b)dx+\left(8\,(a-b)\cos\left(dx+c\right)^{5}+2\,(5\,a+b)\cos\left(dx+c\right)^{3}+3\,(5\,a+b)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/48*(3*(5*a + b)*d*x + (8*(a - b)*cos(d*x + c)^5 + 2*(5*a + b)*cos(d*x + c
)^3 + 3*(5*a + b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 5.93165, size = 5072, normalized size = 58.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/96*(3*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*ta
n(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*tan(c)^6 + 30*a*d*x*tan
(d*x)^6*tan(c)^6 + 6*b*d*x*tan(d*x)^6*tan(c)^6 + 3*pi*b*sgn(-2*tan(d*x)^2*t
an(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*tan(c)^6 +
9*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)
*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*tan(c)^4 + 9*pi*b*sgn(2*tan(d
*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(
d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^6 + 6*b*arctan((tan(d*x) + tan(c))/(tan(
d*x)*tan(c) - 1))*tan(d*x)^6*tan(c)^6 - 6*b*arctan(-(tan(d*x) - tan(c))/(ta
n(d*x)*tan(c) + 1))*tan(d*x)^6*tan(c)^6 + 90*a*d*x*tan(d*x)^6*tan(c)^4 + 18
*b*d*x*tan(d*x)^6*tan(c)^4 + 9*pi*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*t
an(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*tan(c)^4 + 90*a*d*x*tan(d*x)^4*
tan(c)^6 + 18*b*d*x*tan(d*x)^4*tan(c)^6 + 9*pi*b*sgn(-2*tan(d*x)^2*tan(c) +
 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^6 + 9*pi*b*
sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)
^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*tan(c)^2 + 27*pi*b*sgn(2*tan(d*x)^2*
tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) -
 2*tan(c))*tan(d*x)^4*tan(c)^4 + 18*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*
```

```
tan(c) - 1))*tan(d*x)^6*tan(c)^4 - 18*b*arctan(-(tan(d*x) - tan(c))/(tan(d*
x)*tan(c) + 1))*tan(d*x)^6*tan(c)^4 - 66*a*tan(d*x)^6*tan(c)^5 + 6*b*tan(d*
x)^6*tan(c)^5 + 9*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan
(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^6 + 18
*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^6 -
18*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^6
 - 66*a*tan(d*x)^5*tan(c)^6 + 6*b*tan(d*x)^5*tan(c)^6 + 90*a*d*x*tan(d*x)^6
*tan(c)^2 + 18*b*d*x*tan(d*x)^6*tan(c)^2 + 9*pi*b*sgn(-2*tan(d*x)^2*tan(c)
+ 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*tan(c)^2 + 270*a*
d*x*tan(d*x)^4*tan(c)^4 + 54*b*d*x*tan(d*x)^4*tan(c)^4 + 27*pi*b*sgn(-2*tan
(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*ta
n(c)^4 + 90*a*d*x*tan(d*x)^2*tan(c)^6 + 18*b*d*x*tan(d*x)^2*tan(c)^6 + 9*pi
*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*
tan(d*x)^2*tan(c)^6 + 3*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)
^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6 + 27*pi
*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan
(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^2 + 18*b*arctan((tan(d*x)
+ tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^6*tan(c)^2 - 18*b*arctan(-(tan(d*
x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^6*tan(c)^2 - 80*a*tan(d*x)^6*t
an(c)^3 - 16*b*tan(d*x)^6*tan(c)^3 + 27*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)
*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*ta
n(d*x)^2*tan(c)^4 + 54*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*
tan(d*x)^4*tan(c)^4 - 54*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1
))*tan(d*x)^4*tan(c)^4 + 90*a*tan(d*x)^5*tan(c)^4 - 78*b*tan(d*x)^5*tan(c)^
4 + 90*a*tan(d*x)^4*tan(c)^5 - 78*b*tan(d*x)^4*tan(c)^5 + 3*pi*b*sgn(2*tan(
d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan
(d*x) - 2*tan(c))*tan(c)^6 + 18*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(
c) - 1))*tan(d*x)^2*tan(c)^6 - 18*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*t
an(c) + 1))*tan(d*x)^2*tan(c)^6 - 80*a*tan(d*x)^3*tan(c)^6 - 16*b*tan(d*x)^
3*tan(c)^6 + 30*a*d*x*tan(d*x)^6 + 6*b*d*x*tan(d*x)^6 + 3*pi*b*sgn(-2*tan(d
*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6 + 27
0*a*d*x*tan(d*x)^4*tan(c)^2 + 54*b*d*x*tan(d*x)^4*tan(c)^2 + 27*pi*b*sgn(-2
*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^
4*tan(c)^2 + 270*a*d*x*tan(d*x)^2*tan(c)^4 + 54*b*d*x*tan(d*x)^2*tan(c)^4 +
 27*pi*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*ta
n(c))*tan(d*x)^2*tan(c)^4 + 30*a*d*x*tan(c)^6 + 6*b*d*x*tan(c)^6 + 3*pi*b*s
gn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(
c)^6 + 9*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*t
an(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 + 6*b*arctan((tan(d*x)
 + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^6 - 6*b*arctan(-(tan(d*x) - tan(
c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^6 - 30*a*tan(d*x)^6*tan(c) - 6*b*tan(d*
x)^6*tan(c) + 27*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(
c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 54*
b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^2 - 5
4*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^2
+ 90*a*tan(d*x)^5*tan(c)^2 + 18*b*tan(d*x)^5*tan(c)^2 - 240*a*tan(d*x)^4*ta
n(c)^3 + 144*b*tan(d*x)^4*tan(c)^3 + 9*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*
sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan
(c)^4 + 54*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*t
an(c)^4 - 54*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^
2*tan(c)^4 - 240*a*tan(d*x)^3*tan(c)^4 + 144*b*tan(d*x)^3*tan(c)^4 + 90*a*t
an(d*x)^2*tan(c)^5 + 18*b*tan(d*x)^2*tan(c)^5 + 6*b*arctan((tan(d*x) + tan(
c))/(tan(d*x)*tan(c) - 1))*tan(c)^6 - 6*b*arctan(-(tan(d*x) - tan(c))/(tan(
d*x)*tan(c) + 1))*tan(c)^6 - 30*a*tan(d*x)*tan(c)^6 - 6*b*tan(d*x)*tan(c)^6
 + 90*a*d*x*tan(d*x)^4 + 18*b*d*x*tan(d*x)^4 + 9*pi*b*sgn(-2*tan(d*x)^2*tan
(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 + 270*a*d*x*t
an(d*x)^2*tan(c)^2 + 54*b*d*x*tan(d*x)^2*tan(c)^2 + 27*pi*b*sgn(-2*tan(d*x)
^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^
2 + 90*a*d*x*tan(c)^4 + 18*b*d*x*tan(c)^4 + 9*pi*b*sgn(-2*tan(d*x)^2*tan(c)
```

```
 + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^4 + 9*pi*b*sgn(2*tan
(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*ta
n(d*x) - 2*tan(c))*tan(d*x)^2 + 18*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*t
an(c) - 1))*tan(d*x)^4 - 18*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c)
+ 1))*tan(d*x)^4 + 30*a*tan(d*x)^5 + 6*b*tan(d*x)^5 - 90*a*tan(d*x)^4*tan(c
) - 18*b*tan(d*x)^4*tan(c) + 9*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*t
an(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 +
54*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^2
- 54*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)
^2 + 240*a*tan(d*x)^3*tan(c)^2 - 144*b*tan(d*x)^3*tan(c)^2 + 240*a*tan(d*x)
^2*tan(c)^3 - 144*b*tan(d*x)^2*tan(c)^3 + 18*b*arctan((tan(d*x) + tan(c))/(
tan(d*x)*tan(c) - 1))*tan(c)^4 - 18*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)
*tan(c) + 1))*tan(c)^4 - 90*a*tan(d*x)*tan(c)^4 - 18*b*tan(d*x)*tan(c)^4 +
30*a*tan(c)^5 + 6*b*tan(c)^5 + 90*a*d*x*tan(d*x)^2 + 18*b*d*x*tan(d*x)^2 +
9*pi*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(
c))*tan(d*x)^2 + 90*a*d*x*tan(c)^2 + 18*b*d*x*tan(c)^2 + 9*pi*b*sgn(-2*tan(
d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 + 3*p
i*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*ta
n(c)^2 + 2*tan(d*x) - 2*tan(c)) + 18*b*arctan((tan(d*x) + tan(c))/(tan(d*x)
*tan(c) - 1))*tan(d*x)^2 - 18*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c
) + 1))*tan(d*x)^2 + 80*a*tan(d*x)^3 + 16*b*tan(d*x)^3 - 90*a*tan(d*x)^2*ta
n(c) + 78*b*tan(d*x)^2*tan(c) + 18*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*t
an(c) - 1))*tan(c)^2 - 18*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) +
1))*tan(c)^2 - 90*a*tan(d*x)*tan(c)^2 + 78*b*tan(d*x)*tan(c)^2 + 80*a*tan(c
)^3 + 16*b*tan(c)^3 + 30*a*d*x + 6*b*d*x + 3*pi*b*sgn(-2*tan(d*x)^2*tan(c)
+ 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c)) + 6*b*arctan((tan(d*x) + tan
(c))/(tan(d*x)*tan(c) - 1)) - 6*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan
(c) + 1)) + 66*a*tan(d*x) - 6*b*tan(d*x) + 66*a*tan(c) - 6*b*tan(c))/(d*tan
(d*x)^6*tan(c)^6 + 3*d*tan(d*x)^6*tan(c)^4 + 3*d*tan(d*x)^4*tan(c)^6 + 3*d*
tan(d*x)^6*tan(c)^2 + 9*d*tan(d*x)^4*tan(c)^4 + 3*d*tan(d*x)^2*tan(c)^6 + d
*tan(d*x)^6 + 9*d*tan(d*x)^4*tan(c)^2 + 9*d*tan(d*x)^2*tan(c)^4 + d*tan(c)^
6 + 3*d*tan(d*x)^4 + 9*d*tan(d*x)^2*tan(c)^2 + 3*d*tan(c)^4 + 3*d*tan(d*x)^
2 + 3*d*tan(c)^2 + d)
```

3.437 $\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=128

$$\frac{\left(8a^2-4ab+b^2\right)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{\left(8a^2-4ab+b^2\right)\tan(c+dx)\sec(c+dx)}{16d}+\frac{b(8a-3b)\tan(c+dx)\sec^3(c+dx)}{24d}\cdot$$

```
[Out] ((8*a^2 - 4*a*b + b^2)*ArcTanh[Sin[c + d*x]])/(16*d) + ((8*a^2 - 4*a*b + b^
2)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + ((8*a - 3*b)*b*Sec[c + d*x]^3*Tan[c
+ d*x])/(24*d) + (b*Sec[c + d*x]^5*(a - (a - b)*Sin[c + d*x]^2)*Tan[c + d*x
])/(6*d)
```

Rubi [A] time = 0.164569, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3676, 413, 385, 199, 206}

$$\frac{\left(8a^2-4ab+b^2\right)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{\left(8a^2-4ab+b^2\right)\tan(c+dx)\sec(c+dx)}{16d}+\frac{b(8a-3b)\tan(c+dx)\sec^3(c+dx)}{24d}\cdot$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 - 4*a*b + b^2)*ArcTanh[Sin[c + d*x]])/(16*d) + ((8*a^2 - 4*a*b + b^
2)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + ((8*a - 3*b)*b*Sec[c + d*x]^3*Tan[c
+ d*x])/(24*d) + (b*Sec[c + d*x]^5*(a - (a - b)*Sin[c + d*x]^2)*Tan[c + d*x
])/(6*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
```

```
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{\left(1-x^2\right)^4}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{b\sec^5(c+dx)\left(a-(a-b)\sin^2(c+dx)\right)\tan(c+dx)}{6d} - \frac{\text{Subst}\left(\int \frac{-a(6a-b)+3\cdots}{(1-\cdots}\right.}{\cdots}$$

$$= \frac{(8a-3b)b\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{b\sec^5(c+dx)\left(a-(a-b)\sin^2(c+\cdots\right.}{6d}$$

$$= \frac{\left(8a^2-4ab+b^2\right)\sec(c+dx)\tan(c+dx)}{16d} + \frac{(8a-3b)b\sec^3(c+dx)\tan(c\cdots}{24d}$$

$$= \frac{\left(8a^2-4ab+b^2\right)\tanh^{-1}(\sin(c+dx))}{16d} + \frac{\left(8a^2-4ab+b^2\right)\sec(c+dx)\tan(\cdots}{16d}$$

Mathematica [C] time = 10.8173, size = 875, normalized size = 6.84

$$\frac{\sin(c+dx)\left(380(a-b)^2\text{HypergeometricPFQ}\left(\left\{\frac{3}{2},2,2,2\right\},\left\{1,1,\frac{9}{2}\right\},\sin^2(c+dx)\right)\sqrt{\sin^2(c+dx)}\sin^{10}(c+dx)+12\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (Sin[c + d*x]*(65625*a^2*ArcTanh[Sqrt[Sin[c + d*x]^2]] - 36855*a^2*ArcTanh[
Sqrt[Sin[c + d*x]^2]]*Sin[c + d*x]^2 - 91875*a*(a - b)*ArcTanh[Sqrt[Sin[c +
 d*x]^2]]*Sin[c + d*x]^2 + 1680*a^2*ArcTanh[Sqrt[Sin[c + d*x]^2]]*Sin[c + d
*x]^4 + 54180*a*(a - b)*ArcTanh[Sqrt[Sin[c + d*x]^2]]*Sin[c + d*x]^4 + 3297
0*(a - b)^2*ArcTanh[Sqrt[Sin[c + d*x]^2]]*Sin[c + d*x]^4 - 1365*a*(a - b)*A
rcTanh[Sqrt[Sin[c + d*x]^2]]*Sin[c + d*x]^6 - 19845*(a - b)^2*ArcTanh[Sqrt[
Sin[c + d*x]^2]]*Sin[c + d*x]^6 + 525*(a - b)^2*ArcTanh[Sqrt[Sin[c + d*x]^2
]]*Sin[c + d*x]^8 - 65625*a^2*Sqrt[Sin[c + d*x]^2] - 23555*a*(a - b)*Sin[c
+ d*x]^4*Sqrt[Sin[c + d*x]^2] - 32970*(a - b)^2*Sin[c + d*x]^4*Sqrt[Sin[c +
 d*x]^2] + 8855*(a - b)^2*Sin[c + d*x]^6*Sqrt[Sin[c + d*x]^2] + 620*a^2*Hyp
ergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^6*
Sqrt[Sin[c + d*x]^2] + 160*a^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1,
1, 9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^6*Sqrt[Sin[c + d*x]^2] + 16*a^2*Hyper
geometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c
 + d*x]^6*Sqrt[Sin[c + d*x]^2] - 968*a*(a - b)*HypergeometricPFQ[{3/2, 2, 2
, 2}, {1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^8*Sqrt[Sin[c + d*x]^2] - 28
8*a*(a - b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, Sin[c + d*
x]^2]*Sin[c + d*x]^8*Sqrt[Sin[c + d*x]^2] - 32*a*(a - b)*HypergeometricPFQ[
```

```
{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^8*Sqr
t[Sin[c + d*x]^2] + 380*(a - b)^2*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1,
9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^10*Sqrt[Sin[c + d*x]^2] + 128*(a - b)^2*
HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c
+ d*x]^10*Sqrt[Sin[c + d*x]^2] + 16*(a - b)^2*HypergeometricPFQ[{3/2, 2, 2,
 2, 2, 2}, {1, 1, 1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^10*Sqrt[Sin[c +
d*x]^2] + 14980*a^2*(Sin[c + d*x]^2)^(3/2) + 91875*a*(a - b)*(Sin[c + d*x]^
2)^(3/2)))/(2520*d*(Sin[c + d*x]^2)^(5/2))
```

Maple [B] time = 0.066, size = 248, normalized size = 1.9

$$\frac{b^2 (\sin (dx+c))^5}{6 d (\cos (dx+c))^6}+\frac{b^2 (\sin (dx+c))^5}{24 d (\cos (dx+c))^4}-\frac{b^2 (\sin (dx+c))^5}{48 d (\cos (dx+c))^2}-\frac{b^2 (\sin (dx+c))^3}{48 d}-\frac{b^2 \sin (dx+c)}{16 d}+\frac{b^2 \ln (\sec (dx+c)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/6/d*b^2*sin(d*x+c)^5/cos(d*x+c)^6+1/24/d*b^2*sin(d*x+c)^5/cos(d*x+c)^4-1/
48/d*b^2*sin(d*x+c)^5/cos(d*x+c)^2-1/48/d*b^2*sin(d*x+c)^3-1/16/d*b^2*sin(d
*x+c)+1/16/d*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a*b*sin(d*x+c)^3/cos(d*x+c
)^4+1/4/d*a*b*sin(d*x+c)^3/cos(d*x+c)^2+1/4/d*a*b*sin(d*x+c)-1/4/d*a*b*ln(s
ec(d*x+c)+tan(d*x+c))+1/2/d*a^2*sec(d*x+c)*tan(d*x+c)+1/2/d*a^2*ln(sec(d*x+
c)+tan(d*x+c))
```

Maxima [A] time = 1.12479, size = 211, normalized size = 1.65

$$\frac{3 \left(8 a^2-4 a b+b^2\right) \log (\sin (dx+c)+1)-3 \left(8 a^2-4 a b+b^2\right) \log (\sin (dx+c)-1)-\frac{2 \left(3 \left(8 a^2-4 a b+b^2\right) \sin (dx+c)^5-8 \left(6 a^2-b^2\right) \sin}{\sin (dx+c)^6-3 \sin (dx+c)^4}}{96 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(8*a^2 - 4*a*b + b^2)*log(sin(d*x + c) + 1) - 3*(8*a^2 - 4*a*b + b^
2)*log(sin(d*x + c) - 1) - 2*(3*(8*a^2 - 4*a*b + b^2)*sin(d*x + c)^5 - 8*(6
*a^2 - b^2)*sin(d*x + c)^3 + 3*(8*a^2 + 4*a*b - b^2)*sin(d*x + c))/(sin(d*x
 + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1))/d
```

Fricas [A] time = 1.77664, size = 343, normalized size = 2.68

$$\frac{3 \left(8 a^2-4 a b+b^2\right) \cos (dx+c)^6 \log (\sin (dx+c)+1)-3 \left(8 a^2-4 a b+b^2\right) \cos (dx+c)^6 \log (-\sin (dx+c)+1)+2 (3 (8}{96 d \cos (dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/96*(3*(8*a^2 - 4*a*b + b^2)*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 3*(8*a
^2 - 4*a*b + b^2)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(3*(8*a^2 - 4*a
*b + b^2)*cos(d*x + c)^4 + 2*(12*a*b - 7*b^2)*cos(d*x + c)^2 + 8*b^2)*sin(d
```

```
*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\tan^2(c+dx)\right)^2\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*sec(c + d*x)**3, x)
```

Giac [A] time = 1.86341, size = 225, normalized size = 1.76

$$\frac{3\left(8a^2-4ab+b^2\right)\log\left(|\sin(dx+c)+1|\right)-3\left(8a^2-4ab+b^2\right)\log\left(|\sin(dx+c)-1|\right)-\frac{2\left(24a^2\sin(dx+c)^5-12ab\sin(dx+c)^5+\right.}{}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/96*(3*(8*a^2 - 4*a*b + b^2)*log(abs(sin(d*x + c) + 1)) - 3*(8*a^2 - 4*a*b
 + b^2)*log(abs(sin(d*x + c) - 1)) - 2*(24*a^2*sin(d*x + c)^5 - 12*a*b*sin(
d*x + c)^5 + 3*b^2*sin(d*x + c)^5 - 48*a^2*sin(d*x + c)^3 + 8*b^2*sin(d*x +
 c)^3 + 24*a^2*sin(d*x + c) + 12*a*b*sin(d*x + c) - 3*b^2*sin(d*x + c))/(si
n(d*x + c)^2 - 1)^3)/d
```

3.438 $\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=96

$$\frac{(8a^2-8ab+3b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3b(2a-b)\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)\left(a-(a-b)\sin\right.}{4d}$$

```
[Out] ((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (3*(2*a - b)*b*Sec[
c + d*x]*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*(a - (a - b)*Sin[c + d*x]^
2)*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.0859652, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3676, 413, 385, 206}

$$\frac{(8a^2-8ab+3b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3b(2a-b)\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)\left(a-(a-b)\sin\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Tan[c + d*x]^2)^2,x]
```


```
[Out] ((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (3*(2*a - b)*b*Sec[
c + d*x]*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*(a - (a - b)*Sin[c + d*x]^
2)*Tan[c + d*x])/(4*d)
```


Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{\left(1-x^2\right)^3}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{b\sec^3(c+dx)\left(a-(a-b)\sin^2(c+dx)\right)\tan(c+dx)}{4d} - \frac{\text{Subst}\left(\int \frac{-a(4a-b)+(4a}{(1-x}\right.}{}$$

$$= \frac{3(2a-b)b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)\left(a-(a-b)\sin^2(c+dx\right.}{4d}$$

$$= \frac{\left(8a^2-8ab+3b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{3(2a-b)b\sec(c+dx)\tan(c+dx}{8d}$$

Mathematica [C] time = 8.49057, size = 347, normalized size = 3.61

$$\frac{\csc^3(c+dx)\left(128\sin^6(c+dx)\left(\frac{1}{2}a^2(5\cos(2(c+dx))+9)\cos^2(c+dx)+b\sin^2(c+dx)\left(5a\cos(2(c+dx))+7a+5b\,\text{si}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (Csc[c + d*x]^3*(128*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, S
in[c + d*x]^2]*Sin[c + d*x]^6*(a + (-a + b)*Sin[c + d*x]^2)^2 + 128*Hyperge
ometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^6*((a^
2*Cos[c + d*x]^2*(9 + 5*Cos[2*(c + d*x)]))/2 + b*Sin[c + d*x]^2*(7*a + 5*a*
Cos[2*(c + d*x)] + 5*b*Sin[c + d*x]^2)) + 35*(-3375*a^2 + 3*a*(1969*a - 175
0*b)*Sin[c + d*x]^2 + (-3161*a^2 + 5108*a*b - 1947*b^2)*Sin[c + d*x]^4 + 48
5*(a - b)^2*Sin[c + d*x]^6 + (3*ArcTanh[Sqrt[Sin[c + d*x]^2]]*(1125*a^2 - 2
*a*(1172*a - 875*b)*Sin[c + d*x]^2 + (1674*a^2 - 2286*a*b + 649*b^2)*Sin[c
+ d*x]^4 + (-400*a^2 + 778*a*b - 378*b^2)*Sin[c + d*x]^6 + 9*(a - b)^2*Sin[
c + d*x]^8))/Sqrt[Sin[c + d*x]^2])))/(6720*d)
```

Maple [A] time = 0.04, size = 178, normalized size = 1.9

$$\frac{b^2(\sin(dx+c))^5}{4d(\cos(dx+c))^4} - \frac{b^2(\sin(dx+c))^5}{8d(\cos(dx+c))^2} - \frac{b^2(\sin(dx+c))^3}{8d} - \frac{3b^2\sin(dx+c)}{8d} + \frac{3b^2\ln(\sec(dx+c)+\tan(dx+c))}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/4/d*b^2*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*b^2*sin(d*x+c)^5/cos(d*x+c)^2-1/8
/d*b^2*sin(d*x+c)^3-3/8/d*b^2*sin(d*x+c)+3/8/d*b^2*ln(sec(d*x+c)+tan(d*x+c)
)+1/d*a*b*sin(d*x+c)^3/cos(d*x+c)^2+1/d*a*b*sin(d*x+c)-1/d*a*b*ln(sec(d*x+c
)+tan(d*x+c))+1/d*a^2*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.04013, size = 161, normalized size = 1.68

$$\frac{\left(8a^2-8ab+3b^2\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(8a^2-8ab+3b^2\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(\left(8ab-5b^2\right)\sin(dx+c)^3-\left(8ab-3b^2\right)\sin(dx+\right.}{\sin(dx+c)^4-2\sin(dx+c)^2+1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/16*((8*a^2 - 8*a*b + 3*b^2)*log(sin(d*x + c) + 1) - (8*a^2 - 8*a*b + 3*b^
2)*log(sin(d*x + c) - 1) - 2*((8*a*b - 5*b^2)*sin(d*x + c)^3 - (8*a*b - 3*b
^2)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.40442, size = 284, normalized size = 2.96

$$\frac{\left(8a^2-8ab+3b^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-\left(8a^2-8ab+3b^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(\left(8a\right.\right.}{16d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/16*((8*a^2 - 8*a*b + 3*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (8*a^2
 - 8*a*b + 3*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*((8*a*b - 5*b^2
)*cos(d*x + c)^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan^2\left(c+dx\right)\right)^2\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*sec(c + d*x), x)
```

Giac [A] time = 1.82696, size = 162, normalized size = 1.69

$$\frac{\left(8a^2-8ab+3b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-\left(8a^2-8ab+3b^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(8ab\sin(dx+c)^3-5b^2\sin(dx+c)^3-8ab\sin\right.}{\left(\sin(dx+c)^2-1\right)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*((8*a^2 - 8*a*b + 3*b^2)*log(abs(sin(d*x + c) + 1)) - (8*a^2 - 8*a*b +
 3*b^2)*log(abs(sin(d*x + c) - 1)) - 2*(8*a*b*sin(d*x + c)^3 - 5*b^2*sin(d*
x + c)^3 - 8*a*b*sin(d*x + c) + 3*b^2*sin(d*x + c))/(sin(d*x + c)^2 - 1)^2)
/d
```

3.439 $\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=62

$$\frac{(a-b)^2\sin(c+dx)}{d}+\frac{b(4a-3b)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] ((4*a - 3*b)*b*ArcTanh[Sin[c + d*x]])/(2*d) + ((a - b)^2*Sin[c + d*x])/d +
(b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0896454, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3676, 390, 385, 206}

$$\frac{(a-b)^2\sin(c+dx)}{d}+\frac{b(4a-3b)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((4*a - 3*b)*b*ArcTanh[Sin[c + d*x]])/(2*d) + ((a - b)^2*Sin[c + d*x])/d +
(b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{\left(1-x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left((a-b)^2 + \frac{(2a-b)b-2(a-b)bx^2}{\left(1-x^2\right)^2}\right)dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{(a-b)^2\sin(c+dx)}{d} + \frac{\text{Subst}\left(\int \frac{(2a-b)b-2(a-b)bx^2}{\left(1-x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{(a-b)^2\sin(c+dx)}{d} + \frac{b^2\sec(c+dx)\tan(c+dx)}{2d} + \frac{((4a-3b)b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\right.}{2d}$$

$$= \frac{(4a-3b)b\tanh^{-1}(\sin(c+dx))}{2d} + \frac{(a-b)^2\sin(c+dx)}{d} + \frac{b^2\sec(c+dx)\tan(c+}{2d}$$

Mathematica [A] time = 0.367144, size = 66, normalized size = 1.06

$$\frac{\tan(c+dx)\sec(c+dx)\left(a^2+(a-b)^2\cos(2(c+dx))-2ab+2b^2\right)+b(4a-3b)\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((4*a - 3*b)*b*ArcTanh[Sin[c + d*x]] + (a^2 - 2*a*b + 2*b^2 + (a - b)^2*Cos
[2*(c + d*x)])*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [B] time = 0.046, size = 125, normalized size = 2.

$$\frac{b^2(\sin(dx+c))^5}{2d(\cos(dx+c))^2} + \frac{b^2(\sin(dx+c))^3}{2d} + \frac{3b^2\sin(dx+c)}{2d} - \frac{3b^2\ln(\sec(dx+c)+\tan(dx+c))}{2d} - 2\frac{ab\sin(dx+c)}{d} + 2\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d*b^2*sin(d*x+c)^5/cos(d*x+c)^2+1/2/d*b^2*sin(d*x+c)^3+3/2/d*b^2*sin(d*
x+c)-3/2/d*b^2*ln(sec(d*x+c)+tan(d*x+c))-2/d*a*b*sin(d*x+c)+2/d*a*b*ln(sec(
d*x+c)+tan(d*x+c))+a^2*sin(d*x+c)/d
```

Maxima [A] time = 1.0981, size = 142, normalized size = 2.29

$$-\frac{b^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} + 3\log(\sin(dx+c)+1) - 3\log(\sin(dx+c)-1) - 4\sin(dx+c)\right) - 4ab(\log(\sin(dx+c)+1) - \log(s}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1) -
3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) - 4*a*b*(log(sin(d*x + c) + 1) -
log(sin(d*x + c) - 1) - 2*sin(d*x + c)) - 4*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.5416, size = 266, normalized size = 4.29

$$\frac{\left(4\,ab-3\,b^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(4\,ab-3\,b^2\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\left(a^2-2\,ab+\right.\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((4*a*b - 3*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (4*a*b - 3*b^2)
*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*(a^2 - 2*a*b + b^2)*cos(d*x +
 c)^2 + b^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan^2\left(c+dx\right)\right)^2\cos\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*cos(c + d*x), x)
```

Giac [A] time = 1.90617, size = 140, normalized size = 2.26

$$\frac{4\,a^2\sin\left(dx+c\right)-8\,ab\sin\left(dx+c\right)+4\,b^2\sin\left(dx+c\right)+\left(4\,ab-3\,b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-\left(4\,ab-3\,b^2\right)\log\left(\left|\sin\left(d\right.\right.\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(4*a^2*sin(d*x + c) - 8*a*b*sin(d*x + c) + 4*b^2*sin(d*x + c) + (4*a*b
- 3*b^2)*log(abs(sin(d*x + c) + 1)) - (4*a*b - 3*b^2)*log(abs(sin(d*x + c)
- 1)) - 2*b^2*sin(d*x + c)/(sin(d*x + c)^2 - 1))/d
```

3.440 $\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=56

$$\frac{(a^2-b^2)\sin(c+dx)}{d}-\frac{(a-b)^2\sin^3(c+dx)}{3d}+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (b^2*ArcTanh[Sin[c + d*x]])/d + ((a^2 - b^2)*Sin[c + d*x])/d - ((a - b)^2*S
in[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.064285, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3676, 390, 206}

$$\frac{(a^2-b^2)\sin(c+dx)}{d}-\frac{(a-b)^2\sin^3(c+dx)}{3d}+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (b^2*ArcTanh[Sin[c + d*x]])/d + ((a^2 - b^2)*Sin[c + d*x])/d - ((a - b)^2*S
in[c + d*x]^3)/(3*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2-b^2-(a-b)^2x^2+\frac{b^2}{1-x^2}\right)dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\left(a^2-b^2\right)\sin(c+dx)}{d} - \frac{(a-b)^2\sin^3(c+dx)}{3d} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin\right.}{d}$$
$$= \frac{b^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{\left(a^2-b^2\right)\sin(c+dx)}{d} - \frac{(a-b)^2\sin^3(c+dx)}{3d}$$

Mathematica [A] time = 0.40799, size = 71, normalized size = 1.27

$$\frac{\sin(c+dx)\left(\frac{3b^2\tanh^{-1}\left(\sqrt{\sin^2(c+dx)}\right)}{\sqrt{\sin^2(c+dx)}} - (a-b)\left((a-b)\sin^2(c+dx)-3(a+b)\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (Sin[c + d*x]*((3*b^2*ArcTanh[Sqrt[Sin[c + d*x]^2]])/Sqrt[Sin[c + d*x]^2] -
 (a - b)*(-3*(a + b) + (a - b)*Sin[c + d*x]^2)))/(3*d)
```

Maple [A] time = 0.049, size = 104, normalized size = 1.9

$$-\frac{b^2(\sin(dx+c))^3}{3d} - \frac{b^2\sin(dx+c)}{d} + \frac{b^2\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{2ab(\sin(dx+c))^3}{3d} + \frac{\sin(dx+c)(\cos(dx}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] -1/3/d*b^2*sin(d*x+c)^3-1/d*b^2*sin(d*x+c)+1/d*b^2*ln(sec(d*x+c)+tan(d*x+c)
)+2/3/d*a*b*sin(d*x+c)^3+1/3/d*sin(d*x+c)*cos(d*x+c)^2*a^2+2/3*a^2*sin(d*x+
c)/d
```

Maxima [A] time = 1.16113, size = 97, normalized size = 1.73

$$-\frac{2\left(a^2-2\,ab+b^2\right)\sin(dx+c)^3 - 3\,b^2\log(\sin(dx+c)+1) + 3\,b^2\log(\sin(dx+c)-1) - 6\left(a^2-b^2\right)\sin(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(a^2 - 2*a*b + b^2)*sin(d*x + c)^3 - 3*b^2*log(sin(d*x + c) + 1) +
3*b^2*log(sin(d*x + c) - 1) - 6*(a^2 - b^2)*sin(d*x + c))/d
```

Fricas [A] time = 1.47976, size = 197, normalized size = 3.52

$$\frac{3\,b^2\log\left(\sin\left(dx+c\right)+1\right)-3\,b^2\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(\left(a^2-2\,ab+b^2\right)\cos\left(dx+c\right)^2+2\,a^2+2\,ab-4\,b^2\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^2*log(sin(d*x + c) + 1) - 3*b^2*log(-sin(d*x + c) + 1) + 2*((a^2 -
 2*a*b + b^2)*cos(d*x + c)^2 + 2*a^2 + 2*a*b - 4*b^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.9412, size = 130, normalized size = 2.32

$$-\frac{2\,a^2\sin\left(dx+c\right)^3-4\,ab\sin\left(dx+c\right)^3+2\,b^2\sin\left(dx+c\right)^3-3\,b^2\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+3\,b^2\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-6\,a^2\sin\left(dx+c\right)+6\,b^2\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(2*a^2*sin(d*x + c)^3 - 4*a*b*sin(d*x + c)^3 + 2*b^2*sin(d*x + c)^3 -
3*b^2*log(abs(sin(d*x + c) + 1)) + 3*b^2*log(abs(sin(d*x + c) - 1)) - 6*a^2
*sin(d*x + c) + 6*b^2*sin(d*x + c))/d
```

3.441 $\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=57

$$\frac{a^2\sin(c+dx)}{d}+\frac{(a-b)^2\sin^5(c+dx)}{5d}-\frac{2a(a-b)\sin^3(c+dx)}{3d}$$

```
[Out] (a^2*Sin[c + d*x])/d - (2*a*(a - b)*Sin[c + d*x]^3)/(3*d) + ((a - b)^2*Sin[
c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.059917, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3676, 194}

$$\frac{a^2\sin(c+dx)}{d}+\frac{(a-b)^2\sin^5(c+dx)}{5d}-\frac{2a(a-b)\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Sin[c + d*x])/d - (2*a*(a - b)*Sin[c + d*x]^3)/(3*d) + ((a - b)^2*Sin[
c + d*x]^5)/(5*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx&=\frac{\text{Subst}\left(\int\left(a-(a-b)x^2\right)^2\,dx,x,\sin(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int\left(a^2-2a(a-b)x^2+(a-b)^2x^4\right)\,dx,x,\sin(c+dx)\right)}{d}\\&=\frac{a^2\sin(c+dx)}{d}-\frac{2a(a-b)\sin^3(c+dx)}{3d}+\frac{(a-b)^2\sin^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.158537, size = 52, normalized size = 0.91

$$\frac{15a^2\sin(c+dx)+3(a-b)^2\sin^5(c+dx)-10a(a-b)\sin^3(c+dx)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (15*a^2*Sin[c + d*x] - 10*a*(a - b)*Sin[c + d*x]^3 + 3*(a - b)^2*Sin[c + d*
x]^5)/(15*d)
```

Maple [A] time = 0.052, size = 89, normalized size = 1.6

$$\frac{1}{d}\left(\frac{b^2(\sin(dx+c))^5}{5}+2ab\left(-1/5\sin(dx+c)(\cos(dx+c))^4+1/15\left(2+(\cos(dx+c))^2\right)\sin(dx+c)\right)+\frac{a^2\sin(dx+c)}{5}\right($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(1/5*b^2*sin(d*x+c)^5+2*a*b*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d
*x+c)^2)*sin(d*x+c))+1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)
)
```

Maxima [A] time = 1.01503, size = 76, normalized size = 1.33

$$\frac{3\left(a^2-2ab+b^2\right)\sin(dx+c)^5-10\left(a^2-ab\right)\sin(dx+c)^3+15a^2\sin(dx+c)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*(a^2 - 2*a*b + b^2)*sin(d*x + c)^5 - 10*(a^2 - a*b)*sin(d*x + c)^3
+ 15*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.47696, size = 169, normalized size = 2.96

$$\frac{\left(3\left(a^2-2ab+b^2\right)\cos(dx+c)^4+2\left(2a^2+ab-3b^2\right)\cos(dx+c)^2+8a^2+4ab+3b^2\right)\sin(dx+c)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*(a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(2*a^2 + a*b - 3*b^2)*cos(d*
x + c)^2 + 8*a^2 + 4*a*b + 3*b^2)*sin(d*x + c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*tan(d*x+c)**2)**2,x)
```

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.442 $\int \cos^7(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=86

$$\frac{a^2\sin(c+dx)}{d}-\frac{(a-b)^2\sin^7(c+dx)}{7d}+\frac{(a-b)(3a-b)\sin^5(c+dx)}{5d}-\frac{a(3a-2b)\sin^3(c+dx)}{3d}$$

```
[Out] (a^2*Sin[c + d*x])/d - (a*(3*a - 2*b)*Sin[c + d*x]^3)/(3*d) + ((a - b)*(3*a
 - b)*Sin[c + d*x]^5)/(5*d) - ((a - b)^2*Sin[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0804554, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3676, 373}

$$\frac{a^2\sin(c+dx)}{d}-\frac{(a-b)^2\sin^7(c+dx)}{7d}+\frac{(a-b)(3a-b)\sin^5(c+dx)}{5d}-\frac{a(3a-2b)\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Sin[c + d*x])/d - (a*(3*a - 2*b)*Sin[c + d*x]^3)/(3*d) + ((a - b)*(3*a
 - b)*Sin[c + d*x]^5)/(5*d) - ((a - b)^2*Sin[c + d*x]^7)/(7*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a-(a-b)x^2\right)^2\,dx,x,\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int\left(a^2-a(3a-2b)x^2+\left(3a^2-4ab+b^2\right)x^4-(a-b)^2x^6\right)dx,x,\sin(c+\right.}{d}\\ &= \frac{a^2\sin(c+dx)}{d}-\frac{a(3a-2b)\sin^3(c+dx)}{3d}+\frac{(a-b)(3a-b)\sin^5(c+dx)}{5d}-\frac{(a-}{}\end{aligned}$$

Mathematica [A] time = 0.366442, size = 77, normalized size = 0.9

$$\frac{21\left(3a^2-4ab+b^2\right)\sin^5(c+dx)+105a^2\sin(c+dx)-15(a-b)^2\sin^7(c+dx)-35a(3a-2b)\sin^3(c+dx)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (105*a^2*Sin[c + d*x] - 35*a*(3*a - 2*b)*Sin[c + d*x]^3 + 21*(3*a^2 - 4*a*b
 + b^2)*Sin[c + d*x]^5 - 15*(a - b)^2*Sin[c + d*x]^7)/(105*d)
```

Maple [A] time = 0.056, size = 153, normalized size = 1.8

$$\frac{1}{d}\left(b^2\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^4}{7}-\frac{3\sin(dx+c)(\cos(dx+c))^4}{35}+\frac{\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{35}\right)+2ab\left(-1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*(-1/7*sin(d*x+c)^3*cos(d*x+c)^4-3/35*sin(d*x+c)*cos(d*x+c)^4+1/35*
(2+cos(d*x+c)^2)*sin(d*x+c))+2*a*b*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+
cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+1/7*a^2*(16/5+cos(d*x+c)^6+6/5*c
os(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.09244, size = 109, normalized size = 1.27

$$-\frac{15\left(a^2-2ab+b^2\right)\sin(dx+c)^7-21\left(3a^2-4ab+b^2\right)\sin(dx+c)^5+35\left(3a^2-2ab\right)\sin(dx+c)^3-105a^2\sin(dx\ldots}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/105*(15*(a^2 - 2*a*b + b^2)*sin(d*x + c)^7 - 21*(3*a^2 - 4*a*b + b^2)*si
n(d*x + c)^5 + 35*(3*a^2 - 2*a*b)*sin(d*x + c)^3 - 105*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.52151, size = 231, normalized size = 2.69

$$\frac{\left(15\left(a^2-2ab+b^2\right)\cos(dx+c)^6+6\left(3a^2+ab-4b^2\right)\cos(dx+c)^4+\left(24a^2+8ab+3b^2\right)\cos(dx+c)^2+48a^2+16\ldots}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*(a^2 - 2*a*b + b^2)*cos(d*x + c)^6 + 6*(3*a^2 + a*b - 4*b^2)*cos(
d*x + c)^4 + (24*a^2 + 8*a*b + 3*b^2)*cos(d*x + c)^2 + 48*a^2 + 16*a*b + 6*
b^2)*sin(d*x + c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.443 $\int \cos^9(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=114

$$\frac{\left(6a^2-6ab+b^2\right)\sin^5(c+dx)}{5d}+\frac{a^2\sin(c+dx)}{d}+\frac{(a-b)^2\sin^9(c+dx)}{9d}-\frac{2(a-b)(2a-b)\sin^7(c+dx)}{7d}-\frac{2a(2a-b)\sin}{3d}$$

```
[Out] (a^2*Sin[c + d*x])/d - (2*a*(2*a - b)*Sin[c + d*x]^3)/(3*d) + ((6*a^2 - 6*a
*b + b^2)*Sin[c + d*x]^5)/(5*d) - (2*(a - b)*(2*a - b)*Sin[c + d*x]^7)/(7*d
) + ((a - b)^2*Sin[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.0989362, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3676, 373}

$$\frac{\left(6a^2-6ab+b^2\right)\sin^5(c+dx)}{5d}+\frac{a^2\sin(c+dx)}{d}+\frac{(a-b)^2\sin^9(c+dx)}{9d}-\frac{2(a-b)(2a-b)\sin^7(c+dx)}{7d}-\frac{2a(2a-b)\sin}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^9*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Sin[c + d*x])/d - (2*a*(2*a - b)*Sin[c + d*x]^3)/(3*d) + ((6*a^2 - 6*a
*b + b^2)*Sin[c + d*x]^5)/(5*d) - (2*(a - b)*(2*a - b)*Sin[c + d*x]^7)/(7*d
) + ((a - b)^2*Sin[c + d*x]^9)/(9*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^9(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx&=\frac{\text{Subst}\left(\int\left(1-x^2\right)^2\left(a-(a-b)x^2\right)^2\,dx,x,\sin(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int\left(a^2-2a(2a-b)x^2+\left(6a^2-6ab+b^2\right)x^4-2\left(2a^2-3ab+b^2\right)x^6\right.\right.}{d}\\&=\frac{a^2\sin(c+dx)}{d}-\frac{2a(2a-b)\sin^3(c+dx)}{3d}+\frac{\left(6a^2-6ab+b^2\right)\sin^5(c+dx)}{5d}-\end{aligned}$$

Mathematica [A] time = 0.608363, size = 116, normalized size = 1.02

$$\frac{630\left(63a^2+14ab+3b^2\right)\sin(c+dx)+420\left(21a^2-b^2\right)\sin(3(c+dx))+252\left(9a^2-4ab-b^2\right)\sin(5(c+dx))+35(a-b}{80640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^9*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (630*(63*a^2 + 14*a*b + 3*b^2)*Sin[c + d*x] + 420*(21*a^2 - b^2)*Sin[3*(c +
 d*x)] + 252*(9*a^2 - 4*a*b - b^2)*Sin[5*(c + d*x)] + 45*(a - b)*(9*a - b)*
Sin[7*(c + d*x)] + 35*(a - b)^2*Sin[9*(c + d*x)])/(80640*d)
```

Maple [A] time = 0.093, size = 183, normalized size = 1.6

$$\frac{1}{d}\left(b^2\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^6}{9}-\frac{\sin(dx+c)(\cos(dx+c))^6}{21}+\frac{\sin(dx+c)}{105}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c)}{3}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^9*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105
*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+2*a*b*(-1/9*sin(d*x+c)*cos
(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*
x+c))+1/9*a^2*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/3
5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.12922, size = 140, normalized size = 1.23

$$\frac{35\left(a^2-2\,ab+b^2\right)\sin(dx+c)^9-90\left(2\,a^2-3\,ab+b^2\right)\sin(dx+c)^7+63\left(6\,a^2-6\,ab+b^2\right)\sin(dx+c)^5-210\left(2\,a^2-ab\right.}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/315*(35*(a^2 - 2*a*b + b^2)*sin(d*x + c)^9 - 90*(2*a^2 - 3*a*b + b^2)*sin
(d*x + c)^7 + 63*(6*a^2 - 6*a*b + b^2)*sin(d*x + c)^5 - 210*(2*a^2 - a*b)*s
in(d*x + c)^3 + 315*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.51343, size = 290, normalized size = 2.54

$$\frac{\left(35\left(a^2-2\,ab+b^2\right)\cos(dx+c)^8+10\left(4\,a^2+ab-5\,b^2\right)\cos(dx+c)^6+3\left(16\,a^2+4\,ab+b^2\right)\cos(dx+c)^4+4\left(16\,a^2+4\right.\right.}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/315*(35*(a^2 - 2*a*b + b^2)*cos(d*x + c)^8 + 10*(4*a^2 + a*b - 5*b^2)*cos
(d*x + c)^6 + 3*(16*a^2 + 4*a*b + b^2)*cos(d*x + c)^4 + 4*(16*a^2 + 4*a*b +
 b^2)*cos(d*x + c)^2 + 128*a^2 + 32*a*b + 8*b^2)*sin(d*x + c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**9*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.444 $\int \sec^6(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=96

$$\frac{(a^2+4ab+b^2)\tan^5(c+dx)}{5d}+\frac{a^2\tan(c+dx)}{d}+\frac{2b(a+b)\tan^7(c+dx)}{7d}+\frac{2a(a+b)\tan^3(c+dx)}{3d}+\frac{b^2\tan^9(c+dx)}{9d}$$

```
[Out] (a^2*Tan[c + d*x])/d + (2*a*(a + b)*Tan[c + d*x]^3)/(3*d) + ((a^2 + 4*a*b +
 b^2)*Tan[c + d*x]^5)/(5*d) + (2*b*(a + b)*Tan[c + d*x]^7)/(7*d) + (b^2*Tan
[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.0844828, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 373}

$$\frac{(a^2+4ab+b^2)\tan^5(c+dx)}{5d}+\frac{a^2\tan(c+dx)}{d}+\frac{2b(a+b)\tan^7(c+dx)}{7d}+\frac{2a(a+b)\tan^3(c+dx)}{3d}+\frac{b^2\tan^9(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tan[c + d*x])/d + (2*a*(a + b)*Tan[c + d*x]^3)/(3*d) + ((a^2 + 4*a*b +
 b^2)*Tan[c + d*x]^5)/(5*d) + (2*b*(a + b)*Tan[c + d*x]^7)/(7*d) + (b^2*Tan
[c + d*x]^9)/(9*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int (1+x^2)^2\left(a+bx^2\right)^2\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a^2+2a(a+b)x^2+\left(a^2+4ab+b^2\right)x^4+2b(a+b)x^6+b^2x^8\right)dx,x,\right.}{d}\\ &= \frac{a^2\tan(c+dx)}{d}+\frac{2a(a+b)\tan^3(c+dx)}{3d}+\frac{(a^2+4ab+b^2)\tan^5(c+dx)}{5d}+\frac{2b}{}\end{aligned}$$

Mathematica [A] time = 0.354463, size = 106, normalized size = 1.1

$$\frac{\tan(c+dx)\left(3\left(21a^2-6ab+b^2\right)\sec^4(c+dx)+4\left(21a^2-6ab+b^2\right)\sec^2(c+dx)+8\left(21a^2-6ab+b^2\right)+10b(9a-5b)\sec\right.}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((8*(21*a^2 - 6*a*b + b^2) + 4*(21*a^2 - 6*a*b + b^2)*Sec[c + d*x]^2 + 3*(2
1*a^2 - 6*a*b + b^2)*Sec[c + d*x]^4 + 10*(9*a - 5*b)*b*Sec[c + d*x]^6 + 35*
b^2*Sec[c + d*x]^8)*Tan[c + d*x])/(315*d)
```

Maple [A] time = 0.062, size = 157, normalized size = 1.6

$$\frac{1}{d}\left(b^2\left(\frac{(\sin(dx+c))^5}{9(\cos(dx+c))^9}+\frac{4(\sin(dx+c))^5}{63(\cos(dx+c))^7}+\frac{8(\sin(dx+c))^5}{315(\cos(dx+c))^5}\right)+2ab\left(1/7\frac{(\sin(dx+c))^3}{(\cos(dx+c))^7}+\frac{4(\sin(dx+c))^3}{35(\cos(dx+c))^5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*(1/9*sin(d*x+c)^5/cos(d*x+c)^9+4/63*sin(d*x+c)^5/cos(d*x+c)^7+8/31
5*sin(d*x+c)^5/cos(d*x+c)^5)+2*a*b*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(
d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)-a^2*(-8/15-1/5*sec(d
*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.11271, size = 115, normalized size = 1.2

$$\frac{35\,b^2\tan(dx+c)^9+90\left(ab+b^2\right)\tan(dx+c)^7+63\left(a^2+4\,ab+b^2\right)\tan(dx+c)^5+210\left(a^2+ab\right)\tan(dx+c)^3+31}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/315*(35*b^2*tan(d*x + c)^9 + 90*(a*b + b^2)*tan(d*x + c)^7 + 63*(a^2 + 4*
a*b + b^2)*tan(d*x + c)^5 + 210*(a^2 + a*b)*tan(d*x + c)^3 + 315*a^2*tan(d*
x + c))/d
```

Fricas [A] time = 1.42991, size = 284, normalized size = 2.96

$$\frac{\left(8\left(21\,a^2-6\,ab+b^2\right)\cos(dx+c)^8+4\left(21\,a^2-6\,ab+b^2\right)\cos(dx+c)^6+3\left(21\,a^2-6\,ab+b^2\right)\cos(dx+c)^4+10\left(9\,ab\right.\right.}{315\,d\cos(dx+c)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/315*(8*(21*a^2 - 6*a*b + b^2)*cos(d*x + c)^8 + 4*(21*a^2 - 6*a*b + b^2)*c
os(d*x + c)^6 + 3*(21*a^2 - 6*a*b + b^2)*cos(d*x + c)^4 + 10*(9*a*b - 5*b^2
)*cos(d*x + c)^2 + 35*b^2)*sin(d*x + c)/(d*cos(d*x + c)^9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tan^2\left(c + dx\right)\right)^2 \sec^6\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*sec(c + d*x)**6, x)
```

Giac [A] time = 2.41169, size = 159, normalized size = 1.66

$$\frac{35\,b^2\tan\left(dx+c\right)^9 + 90\,ab\tan\left(dx+c\right)^7 + 90\,b^2\tan\left(dx+c\right)^7 + 63\,a^2\tan\left(dx+c\right)^5 + 252\,ab\tan\left(dx+c\right)^5 + 63\,b^2\tan\left(d\right.}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/315*(35*b^2*tan(d*x + c)^9 + 90*a*b*tan(d*x + c)^7 + 90*b^2*tan(d*x + c)^
7 + 63*a^2*tan(d*x + c)^5 + 252*a*b*tan(d*x + c)^5 + 63*b^2*tan(d*x + c)^5
+ 210*a^2*tan(d*x + c)^3 + 210*a*b*tan(d*x + c)^3 + 315*a^2*tan(d*x + c))/d
```

3.445 $\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=74

$$\frac{a^2\tan(c+dx)}{d}+\frac{b(2a+b)\tan^5(c+dx)}{5d}+\frac{a(a+2b)\tan^3(c+dx)}{3d}+\frac{b^2\tan^7(c+dx)}{7d}$$

```
[Out] (a^2*Tan[c + d*x])/d + (a*(a + 2*b)*Tan[c + d*x]^3)/(3*d) + (b*(2*a + b)*Ta
n[c + d*x]^5)/(5*d) + (b^2*Tan[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0693627, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 373}

$$\frac{a^2\tan(c+dx)}{d}+\frac{b(2a+b)\tan^5(c+dx)}{5d}+\frac{a(a+2b)\tan^3(c+dx)}{3d}+\frac{b^2\tan^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tan[c + d*x])/d + (a*(a + 2*b)*Tan[c + d*x]^3)/(3*d) + (b*(2*a + b)*Ta
n[c + d*x]^5)/(5*d) + (b^2*Tan[c + d*x]^7)/(7*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx&=\frac{\text{Subst}\left(\int\left(1+x^2\right)\left(a+bx^2\right)^2\,dx,x,\tan(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int\left(a^2+a(a+2b)x^2+b(2a+b)x^4+b^2x^6\right)dx,x,\tan(c+dx)\right)}{d}\\&=\frac{a^2\tan(c+dx)}{d}+\frac{a(a+2b)\tan^3(c+dx)}{3d}+\frac{b(2a+b)\tan^5(c+dx)}{5d}+\frac{b^2\tan^7(c+dx)}{7d}\end{aligned}$$

Mathematica [A] time = 0.514383, size = 83, normalized size = 1.12

$$\frac{\tan(c+dx)\left(\left(35a^2-14ab+3b^2\right)\sec^2(c+dx)+70a^2+6b(7a-4b)\sec^4(c+dx)-28ab+15b^2\sec^6(c+dx)+6b^2\right)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((70*a^2 - 28*a*b + 6*b^2 + (35*a^2 - 14*a*b + 3*b^2)*Sec[c + d*x]^2 + 6*(7
*a - 4*b)*b*Sec[c + d*x]^4 + 15*b^2*Sec[c + d*x]^6)*Tan[c + d*x])/(105*d)
```

Maple [A] time = 0.059, size = 111, normalized size = 1.5

$$\frac{1}{d}\left(b^2\left(\frac{(\sin(dx+c))^5}{7(\cos(dx+c))^7}+\frac{2(\sin(dx+c))^5}{35(\cos(dx+c))^5}\right)+2ab\left(1/5\frac{(\sin(dx+c))^3}{(\cos(dx+c))^5}+2/15\frac{(\sin(dx+c))^3}{(\cos(dx+c))^3}\right)-a^2\left(-\frac{2}{3}-\frac{(\sec(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*(1/7*sin(d*x+c)^5/cos(d*x+c)^7+2/35*sin(d*x+c)^5/cos(d*x+c)^5)+2*a
*b*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)-a^2*(-2/3
-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.13133, size = 89, normalized size = 1.2

$$\frac{15\,b^2\tan(dx+c)^7+21\left(2\,ab+b^2\right)\tan(dx+c)^5+35\left(a^2+2\,ab\right)\tan(dx+c)^3+105\,a^2\tan(dx+c)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/105*(15*b^2*tan(d*x + c)^7 + 21*(2*a*b + b^2)*tan(d*x + c)^5 + 35*(a^2 +
2*a*b)*tan(d*x + c)^3 + 105*a^2*tan(d*x + c))/d
```

Fricas [A] time = 1.51317, size = 231, normalized size = 3.12

$$\frac{\left(2\left(35\,a^2-14\,ab+3\,b^2\right)\cos(dx+c)^6+\left(35\,a^2-14\,ab+3\,b^2\right)\cos(dx+c)^4+6\left(7\,ab-4\,b^2\right)\cos(dx+c)^2+15\,b^2\right)\sin(dx}{105\,d\cos(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/105*(2*(35*a^2 - 14*a*b + 3*b^2)*cos(d*x + c)^6 + (35*a^2 - 14*a*b + 3*b^
2)*cos(d*x + c)^4 + 6*(7*a*b - 4*b^2)*cos(d*x + c)^2 + 15*b^2)*sin(d*x + c)
/(d*cos(d*x + c)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan^2(c+dx)\right)^2\sec^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*sec(c + d*x)**4, x)
```

Giac [A] time = 2.26377, size = 108, normalized size = 1.46

$$\frac{15\,b^2\tan\left(dx+c\right)^7+42\,ab\tan\left(dx+c\right)^5+21\,b^2\tan\left(dx+c\right)^5+35\,a^2\tan\left(dx+c\right)^3+70\,ab\tan\left(dx+c\right)^3+105\,a^2\tan}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^2*tan(d*x + c)^7 + 42*a*b*tan(d*x + c)^5 + 21*b^2*tan(d*x + c)^
5 + 35*a^2*tan(d*x + c)^3 + 70*a*b*tan(d*x + c)^3 + 105*a^2*tan(d*x + c))/d
```

3.446 $\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=49

$$\frac{a^2\tan(c+dx)}{d}+\frac{2ab\tan^3(c+dx)}{3d}+\frac{b^2\tan^5(c+dx)}{5d}$$

```
[Out] (a^2*Tan[c + d*x])/d + (2*a*b*Tan[c + d*x]^3)/(3*d) + (b^2*Tan[c + d*x]^5)/
(5*d)
```

Rubi [A] time = 0.0535059, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 194}

$$\frac{a^2\tan(c+dx)}{d}+\frac{2ab\tan^3(c+dx)}{3d}+\frac{b^2\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tan[c + d*x])/d + (2*a*b*Tan[c + d*x]^3)/(3*d) + (b^2*Tan[c + d*x]^5)/
(5*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx&=\frac{\operatorname{Subst}\left(\int \left(a+bx^2\right)^2\,dx,x,\tan(c+dx)\right)}{d}\\&=\frac{\operatorname{Subst}\left(\int \left(a^2+2abx^2+b^2x^4\right)\,dx,x,\tan(c+dx)\right)}{d}\\&=\frac{a^2\tan(c+dx)}{d}+\frac{2ab\tan^3(c+dx)}{3d}+\frac{b^2\tan^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.140486, size = 49, normalized size = 1.

$$\frac{a^2\tan(c+dx)}{d}+\frac{2ab\tan^3(c+dx)}{3d}+\frac{b^2\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tan[c + d*x])/d + (2*a*b*Tan[c + d*x]^3)/(3*d) + (b^2*Tan[c + d*x]^5)/
(5*d)
```

Maple [A] time = 0.053, size = 57, normalized size = 1.2

$$\frac{1}{d}\left(\frac{b^{2} (\sin (dx+c))^{5}}{5 (\cos (dx+c))^{5}}+\frac{2 ab (\sin (dx+c))^{3}}{3 (\cos (dx+c))^{3}}+a^{2} \tan (dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(1/5*b^2*sin(d*x+c)^5/cos(d*x+c)^5+2/3*a*b*sin(d*x+c)^3/cos(d*x+c)^3+a^
2*tan(d*x+c))
```

Maxima [A] time = 1.02884, size = 57, normalized size = 1.16

$$\frac{3 b^{2} \tan (dx+c)^{5}+10 ab \tan (dx+c)^{3}+15 a^{2} \tan (dx+c)}{15 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*b^2*tan(d*x + c)^5 + 10*a*b*tan(d*x + c)^3 + 15*a^2*tan(d*x + c))/d
```

Fricas [A] time = 1.44614, size = 167, normalized size = 3.41

$$\frac{\left(\left(15 a^{2}-10 ab+3 b^{2}\right) \cos (dx+c)^{4}+2 \left(5 ab-3 b^{2}\right) \cos (dx+c)^{2}+3 b^{2}\right) \sin (dx+c)}{15 d \cos (dx+c)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*((15*a^2 - 10*a*b + 3*b^2)*cos(d*x + c)^4 + 2*(5*a*b - 3*b^2)*cos(d*x
+ c)^2 + 3*b^2)*sin(d*x + c)/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b \tan^{2} (c+dx)\right)^{2} \sec^{2} (c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*sec(c + d*x)**2, x)
```

Giac [A] time = 1.77604, size = 57, normalized size = 1.16

$$\frac{3 b^2 \tan(dx + c)^5 + 10 ab \tan(dx + c)^3 + 15 a^2 \tan(dx + c)}{15 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*b^2*tan(d*x + c)^5 + 10*a*b*tan(d*x + c)^3 + 15*a^2*tan(d*x + c))/d
```

3.447 $\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=55

$$\frac{(a-b)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(a+3b)(a-b)+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] ((a - b)*(a + 3*b)*x)/2 + ((a - b)^2*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (b^
2*Tan[c + d*x])/d
```

Rubi [A] time = 0.0772035, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3675, 390, 385, 203}

$$\frac{(a-b)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(a+3b)(a-b)+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((a - b)*(a + 3*b)*x)/2 + ((a - b)^2*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (b^
2*Tan[c + d*x])/d
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{\left(1+x^2\right)^2} dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(b^2 + \frac{a^2-b^2+2(a-b)bx^2}{\left(1+x^2\right)^2}\right) dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{b^2\tan(c+dx)}{d} + \frac{\text{Subst}\left(\int \frac{a^2-b^2+2(a-b)bx^2}{\left(1+x^2\right)^2} dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{(a-b)^2\cos(c+dx)\sin(c+dx)}{2d} + \frac{b^2\tan(c+dx)}{d} + \frac{((a-b)(a+3b))\,\text{Subst}\left(\right.}{2}$$
$$= \frac{1}{2}(a-b)(a+3b)x + \frac{(a-b)^2\cos(c+dx)\sin(c+dx)}{2d} + \frac{b^2\tan(c+dx)}{d}$$

Mathematica [A] time = 0.394028, size = 55, normalized size = 1.

$$\frac{2\left(a^2+2ab-3b^2\right)(c+dx)+(a-b)^2\sin(2(c+dx))+4b^2\tan(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (2*(a^2 + 2*a*b - 3*b^2)*(c + d*x) + (a - b)^2*Sin[2*(c + d*x)] + 4*b^2*Tan
[c + d*x])/(4*d)
```

Maple [B] time = 0.043, size = 111, normalized size = 2.

$$\frac{1}{d}\left(b^2\left(\frac{(\sin(dx+c))^5}{\cos(dx+c)} + \left((\sin(dx+c))^3 + \frac{3\sin(dx+c)}{2}\right)\cos(dx+c) - \frac{3dx}{2} - \frac{3c}{2}\right) + 2ab\left(-1/2\cos(dx+c)\sin(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-
3/2*d*x-3/2*c)+2*a*b*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^2*(1/2*co
s(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.67371, size = 89, normalized size = 1.62

$$\frac{2b^2\tan(dx+c) + \left(a^2+2ab-3b^2\right)(dx+c) + \frac{\left(a^2-2ab+b^2\right)\tan(dx+c)}{\tan(dx+c)^2+1}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^2*tan(d*x + c) + (a^2 + 2*a*b - 3*b^2)*(d*x + c) + (a^2 - 2*a*b +
b^2)*tan(d*x + c)/(tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.41154, size = 166, normalized size = 3.02

$$\frac{\left(a^2 + 2\,ab - 3\,b^2\right)dx\cos\left(dx + c\right) + \left(\left(a^2 - 2\,ab + b^2\right)\cos\left(dx + c\right)^2 + 2\,b^2\right)\sin\left(dx + c\right)}{2\,d\cos\left(dx + c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 + 2*a*b - 3*b^2)*d*x*cos(d*x + c) + ((a^2 - 2*a*b + b^2)*cos(d*x
+ c)^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tan^2\left(c + dx\right)\right)^2\cos^2\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*cos(c + d*x)**2, x)
```

Giac [B] time = 1.95463, size = 802, normalized size = 14.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*d*x*tan(d*x)^3*tan(c)^3 + 2*a*b*d*x*tan(d*x)^3*tan(c)^3 - 3*b^2*d*
x*tan(d*x)^3*tan(c)^3 + a^2*d*x*tan(d*x)^3*tan(c) + 2*a*b*d*x*tan(d*x)^3*ta
n(c) - 3*b^2*d*x*tan(d*x)^3*tan(c) - a^2*d*x*tan(d*x)^2*tan(c)^2 - 2*a*b*d*
x*tan(d*x)^2*tan(c)^2 + 3*b^2*d*x*tan(d*x)^2*tan(c)^2 + a^2*d*x*tan(d*x)*ta
n(c)^3 + 2*a*b*d*x*tan(d*x)*tan(c)^3 - 3*b^2*d*x*tan(d*x)*tan(c)^3 - a^2*ta
n(d*x)^3*tan(c)^2 + 2*a*b*tan(d*x)^3*tan(c)^2 - 3*b^2*tan(d*x)^3*tan(c)^2 -
 a^2*tan(d*x)^2*tan(c)^3 + 2*a*b*tan(d*x)^2*tan(c)^3 - 3*b^2*tan(d*x)^2*tan
(c)^3 - a^2*d*x*tan(d*x)^2 - 2*a*b*d*x*tan(d*x)^2 + 3*b^2*d*x*tan(d*x)^2 +
a^2*d*x*tan(d*x)*tan(c) + 2*a*b*d*x*tan(d*x)*tan(c) - 3*b^2*d*x*tan(d*x)*ta
n(c) - a^2*d*x*tan(c)^2 - 2*a*b*d*x*tan(c)^2 + 3*b^2*d*x*tan(c)^2 - 2*b^2*t
an(d*x)^3 + 2*a^2*tan(d*x)^2*tan(c) - 4*a*b*tan(d*x)^2*tan(c) + 2*a^2*tan(d
*x)*tan(c)^2 - 4*a*b*tan(d*x)*tan(c)^2 - 2*b^2*tan(c)^3 - a^2*d*x - 2*a*b*d
*x + 3*b^2*d*x - a^2*tan(d*x) + 2*a*b*tan(d*x) - 3*b^2*tan(d*x) - a^2*tan(c
) + 2*a*b*tan(c) - 3*b^2*tan(c))/(d*tan(d*x)^3*tan(c)^3 + d*tan(d*x)^3*tan(
c) - d*tan(d*x)^2*tan(c)^2 + d*tan(d*x)*tan(c)^3 - d*tan(d*x)^2 + d*tan(d*x
)*tan(c) - d*tan(c)^2 - d)
```

3.448 $\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=87

$$\frac{3\left(a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(3a^2+2ab+3b^2\right)+\frac{(a-b)\sin(c+dx)\cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)}{4d}$$

```
[Out] ((3*a^2 + 2*a*b + 3*b^2)*x)/8 + (3*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(
8*d) + ((a - b)*Cos[c + d*x]^3*Sin[c + d*x]*(a + b*Tan[c + d*x]^2))/(4*d)
```

Rubi [A] time = 0.0849135, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3675, 413, 385, 203}

$$\frac{3\left(a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(3a^2+2ab+3b^2\right)+\frac{(a-b)\sin(c+dx)\cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x]^2)^2,x]
```


```
[Out] ((3*a^2 + 2*a*b + 3*b^2)*x)/8 + (3*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(
8*d) + ((a - b)*Cos[c + d*x]^3*Sin[c + d*x]*(a + b*Tan[c + d*x]^2))/(4*d)
```


Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{\left(1+x^2\right)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{(a-b)\cos^3(c+dx)\sin(c+dx)\left(a+b\tan^2(c+dx)\right)}{4d} + \frac{\text{Subst}\left(\int \frac{a(3a+b)+b(}{(1+x^2}\right.}{}$$

$$= \frac{3\left(a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{8d} + \frac{(a-b)\cos^3(c+dx)\sin(c+dx)\left(a+\right.}{4d}$$

$$= \frac{1}{8}\left(3a^2+2ab+3b^2\right)x + \frac{3\left(a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{8d} + \frac{(a-b)\cos^3(}{}$$

Mathematica [A] time = 0.323917, size = 65, normalized size = 0.75

$$\frac{4\left(3a^2+2ab+3b^2\right)(c+dx)+8\left(a^2-b^2\right)\sin(2(c+dx))+(a-b)^2\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (4*(3*a^2 + 2*a*b + 3*b^2)*(c + d*x) + 8*(a^2 - b^2)*Sin[2*(c + d*x)] + (a
- b)^2*Sin[4*(c + d*x)])/(32*d)
```

Maple [A] time = 0.044, size = 122, normalized size = 1.4

$$\frac{1}{d}\left(b^2\left(-\frac{\cos(dx+c)}{4}\left((\sin(dx+c))^3+\frac{3\sin(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(-1/4\sin(dx+c)(\cos(dx+c))^3+1/8\,\text{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+2*a*
b*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)+a^
2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.58422, size = 131, normalized size = 1.51

$$\frac{\left(3a^2+2ab+3b^2\right)(dx+c)+\frac{\left(3a^2+2ab-5b^2\right)\tan(dx+c)^3+\left(5a^2-2ab-3b^2\right)\tan(dx+c)}{\tan(dx+c)^4+2\tan(dx+c)^2+1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2 + 2*a*b + 3*b^2)*(d*x + c) + ((3*a^2 + 2*a*b - 5*b^2)*tan(d*x +
 c)^3 + (5*a^2 - 2*a*b - 3*b^2)*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x +
```

```
c)^2 + 1))/d
```

Fricas [A] time = 1.38311, size = 176, normalized size = 2.02

$$\frac{\left(3\,a^2+2\,ab+3\,b^2\right)dx+\left(2\left(a^2-2\,ab+b^2\right)\cos\left(dx+c\right)^3+\left(3\,a^2+2\,ab-5\,b^2\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((3*a^2 + 2*a*b + 3*b^2)*d*x + (2*(a^2 - 2*a*b + b^2)*cos(d*x + c)^3 +
(3*a^2 + 2*a*b - 5*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 47.8567, size = 5287, normalized size = 60.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/32*(3*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*
tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 - 5*pi*b^2*s
gn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^
2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 12*a^2*d*x*tan(d*x)^4*tan(
c)^4 + 8*a*b*d*x*tan(d*x)^4*tan(c)^4 + 12*b^2*d*x*tan(d*x)^4*tan(c)^4 + 3*p
i*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c
))*tan(d*x)^4*tan(c)^4 - 5*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan
(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 6*pi*a*b*sgn(2*tan(d*x
)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*
x) - 2*tan(c))*tan(d*x)^4*tan(c)^2 - 10*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 -
2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*
tan(d*x)^4*tan(c)^2 + 6*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*
x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c
)^4 - 10*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2
*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 + 6*a*b*arc
tan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^4 - 10*b^2
*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^4 - 6*
a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4
+ 10*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(
c)^4 + 24*a^2*d*x*tan(d*x)^4*tan(c)^2 + 16*a*b*d*x*tan(d*x)^4*tan(c)^2 + 24
```

```
*b^2*d*x*tan(d*x)^4*tan(c)^2 + 6*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*
x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^2 - 10*pi*b^2*sgn(-2
*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^
4*tan(c)^2 + 24*a^2*d*x*tan(d*x)^2*tan(c)^4 + 16*a*b*d*x*tan(d*x)^2*tan(c)^
4 + 24*b^2*d*x*tan(d*x)^2*tan(c)^4 + 6*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*
tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 - 10*pi*b^2*
sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan
(d*x)^2*tan(c)^4 + 3*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^
2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 - 5*pi*b
^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan
(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 + 12*pi*a*b*sgn(2*tan(d*x)^2*tan(
c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*t
an(c))*tan(d*x)^2*tan(c)^2 - 20*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-
2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)
^2*tan(c)^2 + 12*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(
d*x)^4*tan(c)^2 - 20*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*
tan(d*x)^4*tan(c)^2 - 12*a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) +
 1))*tan(d*x)^4*tan(c)^2 + 20*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan
(c) + 1))*tan(d*x)^4*tan(c)^2 - 20*a^2*tan(d*x)^4*tan(c)^3 + 8*a*b*tan(d*x)
^4*tan(c)^3 + 12*b^2*tan(d*x)^4*tan(c)^3 + 3*pi*a*b*sgn(2*tan(d*x)^2*tan(c)
^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan
(c))*tan(c)^4 - 5*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*t
an(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^4 + 12*a*b*arct
an((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^4 - 20*b^2*
arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^4 - 12*
a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^4
 + 20*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(
c)^4 - 20*a^2*tan(d*x)^3*tan(c)^4 + 8*a*b*tan(d*x)^3*tan(c)^4 + 12*b^2*tan(
d*x)^3*tan(c)^4 + 12*a^2*d*x*tan(d*x)^4 + 8*a*b*d*x*tan(d*x)^4 + 12*b^2*d*x
*tan(d*x)^4 + 3*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*t
an(d*x) - 2*tan(c))*tan(d*x)^4 - 5*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(
d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 + 48*a^2*d*x*tan(d*x)^2*t
an(c)^2 + 32*a*b*d*x*tan(d*x)^2*tan(c)^2 + 48*b^2*d*x*tan(d*x)^2*tan(c)^2 +
 12*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*
tan(c))*tan(d*x)^2*tan(c)^2 - 20*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*
x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 12*a^2*d*x*tan(c
)^4 + 8*a*b*d*x*tan(c)^4 + 12*b^2*d*x*tan(c)^4 + 3*pi*a*b*sgn(-2*tan(d*x)^2
*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^4 - 5*pi*b^2*
sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan
(c)^4 + 6*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) +
2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2 - 10*pi*b^2*sgn(2*t
an(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*
tan(d*x) - 2*tan(c))*tan(d*x)^2 + 6*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x
)*tan(c) - 1))*tan(d*x)^4 - 10*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan
(c) - 1))*tan(d*x)^4 - 6*a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) +
 1))*tan(d*x)^4 + 10*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))
*tan(d*x)^4 - 12*a^2*tan(d*x)^4*tan(c) - 8*a*b*tan(d*x)^4*tan(c) + 20*b^2*t
an(d*x)^4*tan(c) + 6*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^
2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 - 10*pi*b^
2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(
c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 + 24*a*b*arctan((tan(d*x) + tan(c))/
(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^2 - 40*b^2*arctan((tan(d*x) + tan(
c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^2 - 24*a*b*arctan(-(tan(d*x) -
 tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 + 40*b^2*arctan(-(tan(d
*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 + 24*a^2*tan(d*x)^
3*tan(c)^2 - 48*a*b*tan(d*x)^3*tan(c)^2 + 24*b^2*tan(d*x)^3*tan(c)^2 + 24*a
^2*tan(d*x)^2*tan(c)^3 - 48*a*b*tan(d*x)^2*tan(c)^3 + 24*b^2*tan(d*x)^2*tan
(c)^3 + 6*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(c)^4 -
10*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(c)^4 - 6*a*b*a
```

```
rctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(c)^4 + 10*b^2*arctan(
-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(c)^4 - 12*a^2*tan(d*x)*tan(
c)^4 - 8*a*b*tan(d*x)*tan(c)^4 + 20*b^2*tan(d*x)*tan(c)^4 + 24*a^2*d*x*tan(
d*x)^2 + 16*a*b*d*x*tan(d*x)^2 + 24*b^2*d*x*tan(d*x)^2 + 6*pi*a*b*sgn(-2*ta
n(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2 -
 10*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*
tan(c))*tan(d*x)^2 + 24*a^2*d*x*tan(c)^2 + 16*a*b*d*x*tan(c)^2 + 24*b^2*d*x
*tan(c)^2 + 6*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan
(d*x) - 2*tan(c))*tan(c)^2 - 10*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x
)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 + 3*pi*a*b*sgn(2*tan(d*x)^2*ta
n(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2
*tan(c)) - 5*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c)
 + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c)) + 12*a*b*arctan((tan(d*x) +
 tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2 - 20*b^2*arctan((tan(d*x) + tan(
c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2 - 12*a*b*arctan(-(tan(d*x) - tan(c))/
(tan(d*x)*tan(c) + 1))*tan(d*x)^2 + 20*b^2*arctan(-(tan(d*x) - tan(c))/(tan
(d*x)*tan(c) + 1))*tan(d*x)^2 + 12*a^2*tan(d*x)^3 + 8*a*b*tan(d*x)^3 - 20*b
^2*tan(d*x)^3 - 24*a^2*tan(d*x)^2*tan(c) + 48*a*b*tan(d*x)^2*tan(c) - 24*b^
2*tan(d*x)^2*tan(c) + 12*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) -
1))*tan(c)^2 - 20*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan
(c)^2 - 12*a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(c)^2
+ 20*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(c)^2 - 24*a
^2*tan(d*x)*tan(c)^2 + 48*a*b*tan(d*x)*tan(c)^2 - 24*b^2*tan(d*x)*tan(c)^2
+ 12*a^2*tan(c)^3 + 8*a*b*tan(c)^3 - 20*b^2*tan(c)^3 + 12*a^2*d*x + 8*a*b*d
*x + 12*b^2*d*x + 3*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 +
 2*tan(d*x) - 2*tan(c)) - 5*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*ta
n(c)^2 + 2*tan(d*x) - 2*tan(c)) + 6*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x
)*tan(c) - 1)) - 10*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1)) -
 6*a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1)) + 10*b^2*arctan(-
(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1)) + 20*a^2*tan(d*x) - 8*a*b*tan(d*
x) - 12*b^2*tan(d*x) + 20*a^2*tan(c) - 8*a*b*tan(c) - 12*b^2*tan(c))/(d*tan
(d*x)^4*tan(c)^4 + 2*d*tan(d*x)^4*tan(c)^2 + 2*d*tan(d*x)^2*tan(c)^4 + d*ta
n(d*x)^4 + 4*d*tan(d*x)^2*tan(c)^2 + d*tan(c)^4 + 2*d*tan(d*x)^2 + 2*d*tan(
c)^2 + d)
```

3.449 $\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=122

$$\frac{\left(5a^2+2ab+b^2\right)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x\left(5a^2+2ab+b^2\right)+\frac{(a-b)(5a+3b)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(a-b)}{}$$

```
[Out] ((5*a^2 + 2*a*b + b^2)*x)/16 + ((5*a^2 + 2*a*b + b^2)*Cos[c + d*x]*Sin[c +
d*x])/(16*d) + ((a - b)*(5*a + 3*b)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (
(a - b)*Cos[c + d*x]^5*Sin[c + d*x]*(a + b*Tan[c + d*x]^2))/(6*d)
```

Rubi [A] time = 0.131196, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3675, 413, 385, 199, 203}

$$\frac{\left(5a^2+2ab+b^2\right)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x\left(5a^2+2ab+b^2\right)+\frac{(a-b)(5a+3b)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(a-b)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((5*a^2 + 2*a*b + b^2)*x)/16 + ((5*a^2 + 2*a*b + b^2)*Cos[c + d*x]*Sin[c +
d*x])/(16*d) + ((a - b)*(5*a + 3*b)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (
(a - b)*Cos[c + d*x]^5*Sin[c + d*x]*(a + b*Tan[c + d*x]^2))/(6*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
```

```
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{\left(1+x^2\right)^4}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{(a-b)\cos^5(c+dx)\sin(c+dx)\left(a+b\tan^2(c+dx)\right)}{6d} + \frac{\text{Subst}\left(\int \frac{a(5a+b)+3b(a+}{\left(1+x^2\right)^3}\right.}{6}$$

$$= \frac{(a-b)(5a+3b)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{(a-b)\cos^5(c+dx)\sin(c+dx)\left(a\right.}{6d}$$

$$= \frac{\left(5a^2+2ab+b^2\right)\cos(c+dx)\sin(c+dx)}{16d} + \frac{(a-b)(5a+3b)\cos^3(c+dx)\sin(}{24d}$$

$$= \frac{1}{16}\left(5a^2+2ab+b^2\right)x + \frac{\left(5a^2+2ab+b^2\right)\cos(c+dx)\sin(c+dx)}{16d} + \frac{(a-b)(5}{}$$

Mathematica [C] time = 0.358395, size = 87, normalized size = 0.71

$$\frac{12(b+(1-2i)a)(b+(1+2i)a)(c+dx)+(a-b)^2\sin(6(c+dx))+3(3a+b)(a-b)\sin(4(c+dx))+3(5a-b)(3a+b)\sin(}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (12*((1 - 2*I)*a + b)*((1 + 2*I)*a + b)*(c + d*x) + 3*(5*a - b)*(3*a + b)*S
in[2*(c + d*x)] + 3*(a - b)*(3*a + b)*Sin[4*(c + d*x)] + (a - b)^2*Sin[6*(c
 + d*x)])/(192*d)
```

Maple [A] time = 0.053, size = 166, normalized size = 1.4

$$\frac{1}{d}\left(b^2\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^3}{6}-\frac{\sin(dx+c)(\cos(dx+c))^3}{8}+\frac{\cos(dx+c)\sin(dx+c)}{16}+\frac{dx}{16}+\frac{c}{16}\right)+2ab\left(-1/6\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*sin(d*x+c)*cos(d*x+c)^3+1/16*c
os(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+2*a*b*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1
/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)+a^2*(1/6*(cos
```

```
(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A] time = 1.55486, size = 177, normalized size = 1.45

$$\frac{3\left(5a^2+2ab+b^2\right)(dx+c)+\frac{3\left(5a^2+2ab+b^2\right)\tan(dx+c)^5+8\left(5a^2+2ab-b^2\right)\tan(dx+c)^3+3\left(11a^2-2ab-b^2\right)\tan(dx+c)}{\tan(dx+c)^6+3\tan(dx+c)^4+3\tan(dx+c)^2+1}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(5*a^2 + 2*a*b + b^2)*(d*x + c) + (3*(5*a^2 + 2*a*b + b^2)*tan(d*x
+ c)^5 + 8*(5*a^2 + 2*a*b - b^2)*tan(d*x + c)^3 + 3*(11*a^2 - 2*a*b - b^2)*
tan(d*x + c))/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.41763, size = 235, normalized size = 1.93

$$\frac{3\left(5a^2+2ab+b^2\right)dx+\left(8\left(a^2-2ab+b^2\right)\cos(dx+c)^5+2\left(5a^2+2ab-7b^2\right)\cos(dx+c)^3+3\left(5a^2+2ab+b^2\right)\mathrm{co}\right.}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(5*a^2 + 2*a*b + b^2)*d*x + (8*(a^2 - 2*a*b + b^2)*cos(d*x + c)^5 +
 2*(5*a^2 + 2*a*b - 7*b^2)*cos(d*x + c)^3 + 3*(5*a^2 + 2*a*b + b^2)*cos(d*x
 + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 61.6446, size = 6057, normalized size = 49.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/48*(3*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*
tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*tan(c)^6 + 15*a^2*d*x
*tan(d*x)^6*tan(c)^6 + 6*a*b*d*x*tan(d*x)^6*tan(c)^6 + 3*b^2*d*x*tan(d*x)^6
```

*tan(c)^6 + 3*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*tan(c)^6 + 9*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*tan(c)^4 + 9*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^6 + 6*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^6*tan(c)^6 - 6*a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^6*tan(c)^6 + 45*a^2*d*x*tan(d*x)^6*tan(c)^4 + 18*a*b*d*x*tan(d*x)^6*tan(c)^4 + 9*b^2*d*x*tan(d*x)^6*tan(c)^4 + 9*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*tan(c)^4 + 45*a^2*d*x*tan(d*x)^4*tan(c)^6 + 18*a*b*d*x*tan(d*x)^4*tan(c)^6 + 9*b^2*d*x*tan(d*x)^4*tan(c)^6 + 9*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^6 + 9*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*tan(c)^2 + 27*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 18*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^6*tan(c)^4 - 18*a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^6*tan(c)^4 - 33*a^2*tan(d*x)^6*tan(c)^5 + 6*a*b*tan(d*x)^6*tan(c)^5 + 3*b^2*tan(d*x)^6*tan(c)^5 + 9*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^6 + 18*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^6 - 18*a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^6 - 33*a^2*tan(d*x)^5*tan(c)^6 + 6*a*b*tan(d*x)^5*tan(c)^6 + 3*b^2*tan(d*x)^5*tan(c)^6 + 45*a^2*d*x*tan(d*x)^6*tan(c)^2 + 18*a*b*d*x*tan(d*x)^6*tan(c)^2 + 9*b^2*d*x*tan(d*x)^6*tan(c)^2 + 9*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*tan(c)^2 + 135*a^2*d*x*tan(d*x)^4*tan(c)^4 + 54*a*b*d*x*tan(d*x)^4*tan(c)^4 + 27*b^2*d*x*tan(d*x)^4*tan(c)^4 + 27*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 45*a^2*d*x*tan(d*x)^2*tan(c)^6 + 18*a*b*d*x*tan(d*x)^2*tan(c)^6 + 9*b^2*d*x*tan(d*x)^2*tan(c)^6 + 9*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^6 + 3*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6 + 27*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^2 + 18*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^6*tan(c)^2 - 18*a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^6*tan(c)^2 - 40*a^2*tan(d*x)^6*tan(c)^3 - 16*a*b*tan(d*x)^6*tan(c)^3 + 8*b^2*tan(d*x)^6*tan(c)^3 + 27*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 + 54*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^4 - 54*a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4 + 45*a^2*tan(d*x)^5*tan(c)^4 - 78*a*b*tan(d*x)^5*tan(c)^4 + 9*b^2*tan(d*x)^5*tan(c)^4 + 45*a^2*tan(d*x)^4*tan(c)^5 - 78*a*b*tan(d*x)^4*tan(c)^5 + 9*b^2*tan(d*x)^4*tan(c)^5 + 3*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^6 + 18*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^6 - 18*a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^6 - 40*a^2*tan(d*x)^3*tan(c)^6 - 16*a*b*tan(d*x)^3*tan(c)^6 + 8*b^2*tan(d*x)^3*tan(c)^6 + 15*a^2*d*x*tan(d*x)^6 + 6*a*b*d*x*tan(d*x)^6 + 3*b^2*d*x*tan(d*x)^6 + 3*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6 + 135*a^2*d*x*tan(d*x)^4*tan(c)^2 + 54*a*b*d*x*tan(d*x)^4*tan(c)^2 + 27*b^2*d*x*tan(d*x)^4*tan(c)^2 + 27*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^2 + 135*a^2*d*x*tan(d*x)^2*tan(c)^4 + 54*a*b*d*x*tan(d*x)^2*tan(c)^4 + 27*b^2*d*x*tan(d*x)^2*tan(c)^4 + 27*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 + 15*a^2*d*x*tan(c)^6 + 6*a*b*d*x*tan(c)^6 + 3*b^2*d*x*tan(c)^6 + 3*pi*a*b*sg

n(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c
)^6 + 9*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*
tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 + 6*a*b*arctan((tan(d
*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^6 - 6*a*b*arctan(-(tan(d*x) -
tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^6 - 15*a^2*tan(d*x)^6*tan(c) - 6*a
*b*tan(d*x)^6*tan(c) - 3*b^2*tan(d*x)^6*tan(c) + 27*pi*a*b*sgn(2*tan(d*x)^2
*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x)
- 2*tan(c))*tan(d*x)^2*tan(c)^2 + 54*a*b*arctan((tan(d*x) + tan(c))/(tan(d*
x)*tan(c) - 1))*tan(d*x)^4*tan(c)^2 - 54*a*b*arctan(-(tan(d*x) - tan(c))/(t
an(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^2 + 45*a^2*tan(d*x)^5*tan(c)^2 + 18*
a*b*tan(d*x)^5*tan(c)^2 - 39*b^2*tan(d*x)^5*tan(c)^2 - 120*a^2*tan(d*x)^4*t
an(c)^3 + 144*a*b*tan(d*x)^4*tan(c)^3 - 72*b^2*tan(d*x)^4*tan(c)^3 + 9*pi*a
*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan
(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^4 + 54*a*b*arctan((tan(d*x) + tan(c))
/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^4 - 54*a*b*arctan(-(tan(d*x) - ta
n(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^4 - 120*a^2*tan(d*x)^3*tan(c
)^4 + 144*a*b*tan(d*x)^3*tan(c)^4 - 72*b^2*tan(d*x)^3*tan(c)^4 + 45*a^2*tan
(d*x)^2*tan(c)^5 + 18*a*b*tan(d*x)^2*tan(c)^5 - 39*b^2*tan(d*x)^2*tan(c)^5
+ 6*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(c)^6 - 6*a*b*
arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(c)^6 - 15*a^2*tan(d*
x)*tan(c)^6 - 6*a*b*tan(d*x)*tan(c)^6 - 3*b^2*tan(d*x)*tan(c)^6 + 45*a^2*d*
x*tan(d*x)^4 + 18*a*b*d*x*tan(d*x)^4 + 9*b^2*d*x*tan(d*x)^4 + 9*pi*a*b*sgn(
-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x
)^4 + 135*a^2*d*x*tan(d*x)^2*tan(c)^2 + 54*a*b*d*x*tan(d*x)^2*tan(c)^2 + 27
*b^2*d*x*tan(d*x)^2*tan(c)^2 + 27*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d
*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 45*a^2*d*x*tan(
c)^4 + 18*a*b*d*x*tan(c)^4 + 9*b^2*d*x*tan(c)^4 + 9*pi*a*b*sgn(-2*tan(d*x)^
2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^4 + 9*pi*a*b
*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c
)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2 + 18*a*b*arctan((tan(d*x) + tan(c))
/(tan(d*x)*tan(c) - 1))*tan(d*x)^4 - 18*a*b*arctan(-(tan(d*x) - tan(c))/(ta
n(d*x)*tan(c) + 1))*tan(d*x)^4 + 15*a^2*tan(d*x)^5 + 6*a*b*tan(d*x)^5 + 3*b
^2*tan(d*x)^5 - 45*a^2*tan(d*x)^4*tan(c) - 18*a*b*tan(d*x)^4*tan(c) + 39*b^
2*tan(d*x)^4*tan(c) + 9*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*
x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 + 54*a*
b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^2 - 5
4*a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^
2 + 120*a^2*tan(d*x)^3*tan(c)^2 - 144*a*b*tan(d*x)^3*tan(c)^2 + 72*b^2*tan(
d*x)^3*tan(c)^2 + 120*a^2*tan(d*x)^2*tan(c)^3 - 144*a*b*tan(d*x)^2*tan(c)^3
+ 72*b^2*tan(d*x)^2*tan(c)^3 + 18*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x)
*tan(c) - 1))*tan(c)^4 - 18*a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c
) + 1))*tan(c)^4 - 45*a^2*tan(d*x)*tan(c)^4 - 18*a*b*tan(d*x)*tan(c)^4 + 39
*b^2*tan(d*x)*tan(c)^4 + 15*a^2*tan(c)^5 + 6*a*b*tan(c)^5 + 3*b^2*tan(c)^5
+ 45*a^2*d*x*tan(d*x)^2 + 18*a*b*d*x*tan(d*x)^2 + 9*b^2*d*x*tan(d*x)^2 + 9*
pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(
c))*tan(d*x)^2 + 45*a^2*d*x*tan(c)^2 + 18*a*b*d*x*tan(c)^2 + 9*b^2*d*x*tan(
c)^2 + 9*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x)
- 2*tan(c))*tan(c)^2 + 3*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(
d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c)) + 18*a*b*arcta
n((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2 - 18*a*b*arctan(-(t
an(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2 + 40*a^2*tan(d*x)^3 + 1
6*a*b*tan(d*x)^3 - 8*b^2*tan(d*x)^3 - 45*a^2*tan(d*x)^2*tan(c) + 78*a*b*tan
(d*x)^2*tan(c) - 9*b^2*tan(d*x)^2*tan(c) + 18*a*b*arctan((tan(d*x) + tan(c)
)/(tan(d*x)*tan(c) - 1))*tan(c)^2 - 18*a*b*arctan(-(tan(d*x) - tan(c))/(tan
(d*x)*tan(c) + 1))*tan(c)^2 - 45*a^2*tan(d*x)*tan(c)^2 + 78*a*b*tan(d*x)*ta
n(c)^2 - 9*b^2*tan(d*x)*tan(c)^2 + 40*a^2*tan(c)^3 + 16*a*b*tan(c)^3 - 8*b^
2*tan(c)^3 + 15*a^2*d*x + 6*a*b*d*x + 3*b^2*d*x + 3*pi*a*b*sgn(-2*tan(d*x)^
2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c)) + 6*a*b*arctan((tan
(d*x) + tan(c))/(tan(d*x)*tan(c) - 1)) - 6*a*b*arctan(-(tan(d*x) - tan(c))/

```
(tan(d*x)*tan(c) + 1)) + 33*a^2*tan(d*x) - 6*a*b*tan(d*x) - 3*b^2*tan(d*x)
+ 33*a^2*tan(c) - 6*a*b*tan(c) - 3*b^2*tan(c))/(d*tan(d*x)^6*tan(c)^6 + 3*d
*tan(d*x)^6*tan(c)^4 + 3*d*tan(d*x)^4*tan(c)^6 + 3*d*tan(d*x)^6*tan(c)^2 +
9*d*tan(d*x)^4*tan(c)^4 + 3*d*tan(d*x)^2*tan(c)^6 + d*tan(d*x)^6 + 9*d*tan(
d*x)^4*tan(c)^2 + 9*d*tan(d*x)^2*tan(c)^4 + d*tan(c)^6 + 3*d*tan(d*x)^4 + 9
*d*tan(d*x)^2*tan(c)^2 + 3*d*tan(c)^4 + 3*d*tan(d*x)^2 + 3*d*tan(c)^2 + d)
```

3.450 $\int \frac{\sec^5(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=90

$$\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^2d}-\frac{(2a-3b)\tanh^{-1}(\sin(c+dx))}{2b^2d}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

```
[Out] -((2*a - 3*b)*ArcTanh[Sin[c + d*x]])/(2*b^2*d) + ((a - b)^(3/2)*ArcTanh[(Sq
rt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^2*d) + (Sec[c + d*x]*Tan[c + d
*x])/(2*b*d)
```

Rubi [A] time = 0.14345, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3676, 414, 522, 206, 208}

$$\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^2d}-\frac{(2a-3b)\tanh^{-1}(\sin(c+dx))}{2b^2d}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] -((2*a - 3*b)*ArcTanh[Sin[c + d*x]])/(2*b^2*d) + ((a - b)^(3/2)*ArcTanh[(Sq
rt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^2*d) + (Sec[c + d*x]*Tan[c + d
*x])/(2*b*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a-(a-b)x^2)}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\sec(c+dx)\tan(c+dx)}{2bd} + \frac{\text{Subst}\left(\int \frac{-a+2b+(-a+b)x^2}{(1-x^2)(a+(-a+b)x^2)}\,dx, x, \sin(c+dx)\right)}{2bd}$$
$$= \frac{\sec(c+dx)\tan(c+dx)}{2bd} - \frac{(2a-3b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{2b^2d} + \frac{(a-b)^2\,\text{Subst}\left(\int \frac{}{a+}\right.}{}$$
$$= -\frac{(2a-3b)\tanh^{-1}(\sin(c+dx))}{2b^2d} + \frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd}$$

Mathematica [B] time = 1.32839, size = 207, normalized size = 2.3

$$\frac{-\frac{2(a-b)^{3/2}\log\left(\sqrt{a}-\sqrt{a-b}\sin(c+dx)\right)}{\sqrt{a}} + \frac{2(a-b)^{3/2}\log\left(\sqrt{a-b}\sin(c+dx)+\sqrt{a}\right)}{\sqrt{a}} + 2(2a-3b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2(3b-}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (2*(2*a - 3*b)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*(-2*a + 3*b)*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (2*(a - b)^(3/2)*Log[Sqrt[a] - Sqr
t[a - b]*Sin[c + d*x]])/Sqrt[a] + (2*(a - b)^(3/2)*Log[Sqrt[a] + Sqrt[a - b
]*Sin[c + d*x]])/Sqrt[a] + b/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - b/(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(4*b^2*d)
```

Maple [B] time = 0.087, size = 224, normalized size = 2.5

$$\frac{a^2}{db^2}\text{Artanh}\left((a-b)\sin(dx+c)\frac{1}{\sqrt{a(a-b)}}\right)\frac{1}{\sqrt{a(a-b)}} - 2\frac{a}{db\sqrt{a(a-b)}}\text{Artanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right) + \frac{1}{d}\text{Artanh}\left((a-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d/b^2/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))*a^2-2/d/b
/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))*a+1/d/(a*(a-b))^
(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))-1/4/d/b/(sin(d*x+c)+1)-1/2/
d/b^2*ln(sin(d*x+c)+1)*a+3/4/d/b*ln(sin(d*x+c)+1)-1/4/d/b/(sin(d*x+c)-1)+1/
2/d/b^2*ln(sin(d*x+c)-1)*a-3/4/d/b*ln(sin(d*x+c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.03724, size = 724, normalized size = 8.04

$$\left[-\frac{2\,(a-b)\sqrt{\frac{a-b}{a}}\cos\left(dx+c\right)^2\log\left(-\frac{(a-b)\cos(dx+c)^2+2\,a\sqrt{\frac{a-b}{a}}\sin(dx+c)-2\,a+b}{(a-b)\cos(dx+c)^2+b}\right)+(2\,a-3\,b)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right.}{4\,b^2d\cos\left(dx+c\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a - b)*sqrt((a - b)/a)*cos(d*x + c)^2*log(-((a - b)*cos(d*x + c)^
2 + 2*a*sqrt((a - b)/a)*sin(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 + b
)) + (2*a - 3*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a - 3*b)*cos(d*x
 + c)^2*log(-sin(d*x + c) + 1) - 2*b*sin(d*x + c))/(b^2*d*cos(d*x + c)^2),
-1/4*(4*(a - b)*sqrt(-(a - b)/a)*arctan(sqrt(-(a - b)/a)*sin(d*x + c))*cos(
d*x + c)^2 + (2*a - 3*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a - 3*b)
*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*b*sin(d*x + c))/(b^2*d*cos(d*x +
 c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5\left(c+dx\right)}{a+b\tan^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*tan(c + d*x)**2), x)
```

Giac [A] time = 1.75967, size = 177, normalized size = 1.97

$$-\frac{\frac{(2\,a-3\,b)\log(|\sin(dx+c)+1|)}{b^2}-\frac{(2\,a-3\,b)\log(|\sin(dx+c)-1|)}{b^2}-\frac{4\left(a^2-2\,ab+b^2\right)\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,b^2}+\frac{2\,\sin(dx+c)}{\left(\sin(dx+c)^2-1\right)b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/4*((2*a - 3*b)*log(abs(sin(d*x + c) + 1))/b^2 - (2*a - 3*b)*log(abs(sin(
d*x + c) - 1))/b^2 - 4*(a^2 - 2*a*b + b^2)*arctan(-(a*sin(d*x + c) - b*sin(
d*x + c))/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*b^2) + 2*sin(d*x + c)/((sin(d
*x + c)^2 - 1)*b))/d
```

3.451 $\int \frac{\sec^3(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{\tanh^{-1}(\sin(c+dx))}{bd}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}bd}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(b*d) - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Sin[c + d*x
])/Sqrt[a]])/(Sqrt[a]*b*d)
```

Rubi [A] time = 0.0811648, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3676, 391, 206, 208}

$$\frac{\tanh^{-1}(\sin(c+dx))}{bd}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(b*d) - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Sin[c + d*x
])/Sqrt[a]])/(Sqrt[a]*b*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-(a-b)x^2)}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{bd} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \sin(c+dx)\right)}{bd}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}bd}$$

Mathematica [A] time = 0.103528, size = 53, normalized size = 0.9

$$\frac{\tanh^{-1}(\sin(c+dx)) - \frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]] - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sq
rt[a]])/Sqrt[a])/(b*d)
```

Maple [B] time = 0.077, size = 111, normalized size = 1.9

$$-\frac{a}{db}\text{Artanh}\left((a-b)\sin(dx+c)\frac{1}{\sqrt{a(a-b)}}\right)\frac{1}{\sqrt{a(a-b)}} + \frac{1}{d}\text{Artanh}\left((a-b)\sin(dx+c)\frac{1}{\sqrt{a(a-b)}}\right)\frac{1}{\sqrt{a(a-b)}} + \frac{\ln(\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*tan(d*x+c)^2),x)
```

```
[Out] -1/d/b/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))*a+1/d/(a*(
a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))+1/2/d/b*ln(sin(d*x+c)
+1)-1/2/d/b*ln(sin(d*x+c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69346, size = 413, normalized size = 7.

$$\left[\frac{\sqrt{\frac{a-b}{a}}\log\left(-\frac{(a-b)\cos(dx+c)^2+2a\sqrt{\frac{a-b}{a}}\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(-\sin\left(dx+c\right)+1\right)}{2\,bd}, \frac{2\sqrt{-\frac{a-b}{a}}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt((a - b)/a)*log(-((a - b)*cos(d*x + c)^2 + 2*a*sqrt((a - b)/a)*si
n(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 + b)) + log(sin(d*x + c) + 1)
 - log(-sin(d*x + c) + 1))/(b*d), 1/2*(2*sqrt(-(a - b)/a)*arctan(sqrt(-(a -
 b)/a)*sin(d*x + c)) + log(sin(d*x + c) + 1) - log(-sin(d*x + c) + 1))/(b*d
)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3\left(c+dx\right)}{a+b\tan^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*tan(c + d*x)**2), x)
```

Giac [A] time = 2.22181, size = 119, normalized size = 2.02

$$-\frac{\frac{2\,(a-b)\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}b}-\frac{\log(|\sin(dx+c)+1|)}{b}+\frac{\log(|\sin(dx+c)-1|)}{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*(a - b)*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))
/(sqrt(-a^2 + a*b)*b) - log(abs(sin(d*x + c) + 1))/b + log(abs(sin(d*x + c)
 - 1))/b)/d
```

3.452 $\int \frac{\sec(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d\sqrt{a-b}}$$

```
[Out] ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b]*d)
```

Rubi [A] time = 0.0461854, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3676, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b]*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{a+b\tan^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a-(a-b)x^2}\,dx, x, \sin(c+dx)\right)}{d}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a-b}d}\end{aligned}$$

Mathematica [A] time = 0.0472236, size = 40, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b]*d)
```

Maple [A] time = 0.056, size = 36, normalized size = 0.9

$$\frac{1}{d}\mathrm{Artanh}\left((a-b)\sin(dx+c)\frac{1}{\sqrt{a(a-b)}}\right)\frac{1}{\sqrt{a(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32869, size = 279, normalized size = 6.98

$$\left[\frac{\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)}{2\sqrt{a^2-ab}d}, -\frac{\sqrt{-a^2+ab}\arctan\left(\frac{\sqrt{-a^2+ab}\sin(dx+c)}{a}\right)}{(a^2-ab)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*b)*sin(d*x + c) - 2*a +
b)/((a - b)*cos(d*x + c)^2 + b))/(sqrt(a^2 - a*b)*d), -sqrt(-a^2 + a*b)*arc
tan(sqrt(-a^2 + a*b)*sin(d*x + c)/a)/((a^2 - a*b)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*tan(c + d*x)**2), x)
```

Giac [A] time = 1.72895, size = 63, normalized size = 1.58

$$-\frac{\arctan\left(\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -arctan((a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*
b)*d)
```

3.453 $\int \frac{\cos(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=60

$$\frac{\sin(c+dx)}{d(a-b)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)^{3/2}}$$

```
[Out] -((b*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(3/2)*d)
) + Sin[c + d*x]/((a - b)*d)
```

Rubi [A] time = 0.0797293, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3676, 388, 208}

$$\frac{\sin(c+dx)}{d(a-b)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(3/2)*d)
) + Sin[c + d*x]/((a - b)*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{a-(a-b)x^2}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\sin(c+dx)}{(a-b)d} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \sin(c+dx)\right)}{(a-b)d}$$
$$= -\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{3/2}d} + \frac{\sin(c+dx)}{(a-b)d}$$

Mathematica [A] time = 0.0937933, size = 60, normalized size = 1.

$$\frac{\sin(c+dx)}{d(a-b)} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(3/2)*d)
) + Sin[c + d*x]/((a - b)*d)
```

Maple [A] time = 0.063, size = 61, normalized size = 1.

$$\frac{1}{d}\left(\frac{\sin(dx+c)}{a-b} - \frac{b}{a-b}\text{Artanh}\left((a-b)\sin(dx+c)\frac{1}{\sqrt{a(a-b)}}\right)\frac{1}{\sqrt{a(a-b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*(1/(a-b)*sin(d*x+c)-1/(a-b)*b/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/
(a*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53467, size = 414, normalized size = 6.9

$$\left[-\frac{\sqrt{a^2-ab}b\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)-2\left(a^2-ab\right)\sin(dx+c)}{2\left(a^3-2a^2b+ab^2\right)d}, \frac{\sqrt{-a^2+ab}b\arctan\left(\frac{\sqrt{-a^2+ab}\sin(dx+c)}{a}\right)}{\left(a^3-2a^2b+ab^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a^2 - a*b)*b*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*b)*s
in(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 + b)) - 2*(a^2 - a*b)*sin(d*
x + c))/((a^3 - 2*a^2*b + a*b^2)*d), (sqrt(-a^2 + a*b)*b*arctan(sqrt(-a^2 +
 a*b)*sin(d*x + c)/a) + (a^2 - a*b)*sin(d*x + c))/((a^3 - 2*a^2*b + a*b^2)*
d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (c+dx)}{a+b\tan ^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*tan(c + d*x)**2), x)
```

Giac [A] time = 1.70302, size = 99, normalized size = 1.65

$$-\frac{\frac{b\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}(a-b)}-\frac{\sin(dx+c)}{a-b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(b*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/(sqrt(-a^2
+ a*b)*(a - b)) - sin(d*x + c)/(a - b))/d
```

3.454 $\int \frac{\cos^3(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=88

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)^{5/2}}-\frac{\sin^3(c+dx)}{3d(a-b)}+\frac{(a-2b)\sin(c+dx)}{d(a-b)^2}$$

```
[Out] (b^2*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(5/2)*d)
 + ((a - 2*b)*Sin[c + d*x])/((a - b)^2*d) - Sin[c + d*x]^3/(3*(a - b)*d)
```

Rubi [A] time = 0.122464, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3676, 390, 208}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)^{5/2}}-\frac{\sin^3(c+dx)}{3d(a-b)}+\frac{(a-2b)\sin(c+dx)}{d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (b^2*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(5/2)*d)
 + ((a - 2*b)*Sin[c + d*x])/((a - b)^2*d) - Sin[c + d*x]^3/(3*(a - b)*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{a-(a-b)x^2}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a-2b}{(a-b)^2} - \frac{x^2}{a-b} + \frac{b^2}{(a-b)^2(a-(a-b)x^2)}\right) dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{(a-2b)\sin(c+dx)}{(a-b)^2 d} - \frac{\sin^3(c+dx)}{3(a-b)d} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a-(a-b)x^2}\,dx, x, \sin(c+dx)\right)}{(a-b)^2 d}$$
$$= \frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{5/2} d} + \frac{(a-2b)\sin(c+dx)}{(a-b)^2 d} - \frac{\sin^3(c+dx)}{3(a-b)d}$$

Mathematica [A] time = 0.491502, size = 115, normalized size = 1.31

$$\frac{\frac{6b^2\left(\log\left(\sqrt{a-b}\sin(c+dx)+\sqrt{a}\right)-\log\left(\sqrt{a}-\sqrt{a-b}\sin(c+dx)\right)\right)}{\sqrt{a}(a-b)^{5/2}} + \frac{3(3a-7b)\sin(c+dx)}{(a-b)^2} + \frac{\sin(3(c+dx))}{a-b}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((6*b^2*(-Log[Sqrt[a] - Sqrt[a - b]*Sin[c + d*x]] + Log[Sqrt[a] + Sqrt[a -
b]*Sin[c + d*x]]))/(Sqrt[a]*(a - b)^(5/2)) + (3*(3*a - 7*b)*Sin[c + d*x])/(
a - b)^2 + Sin[3*(c + d*x)]/(a - b))/(12*d)
```

Maple [A] time = 0.064, size = 98, normalized size = 1.1

$$\frac{1}{d}\left(-\frac{1}{(a-b)^2}\left(\frac{(\sin(dx+c))^3 a}{3} - \frac{b(\sin(dx+c))^3}{3} - \sin(dx+c)\,a + 2\sin(dx+c)\,b\right) + \frac{b^2}{(a-b)^2}\text{Artanh}\left((a-b)\sin(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*(-1/(a-b)^2*(1/3*sin(d*x+c)^3*a-1/3*b*sin(d*x+c)^3-sin(d*x+c)*a+2*sin(d
*x+c)*b)+b^2/(a-b)^2/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/
2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57248, size = 617, normalized size = 7.01

$$\left[\frac{3\,\sqrt{a^2-ab}b^2\log\left(-\frac{(a-b)\cos(dx+c)^2-2\,\sqrt{a^2-ab}\sin(dx+c)-2\,a+b}{(a-b)\cos(dx+c)^2+b}\right)+2\left(2\,a^3-7\,a^2b+5\,ab^2+\left(a^3-2\,a^2b+ab^2\right)\cos\left(dx+c\right)^2\right)\sin\left(\right.}{6\left(a^4-3\,a^3b+3\,a^2b^2-ab^3\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a^2 - a*b)*b^2*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*b
)*sin(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 + b)) + 2*(2*a^3 - 7*a^2*
b + 5*a*b^2 + (a^3 - 2*a^2*b + a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/((a^4 -
 3*a^3*b + 3*a^2*b^2 - a*b^3)*d), -1/3*(3*sqrt(-a^2 + a*b)*b^2*arctan(sqrt(
-a^2 + a*b)*sin(d*x + c)/a) - (2*a^3 - 7*a^2*b + 5*a*b^2 + (a^3 - 2*a^2*b +
 a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*
d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.65282, size = 217, normalized size = 2.47

$$\frac{\frac{3\,b^2\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\left(a^2-2\,ab+b^2\right)\sqrt{-a^2+ab}}-\frac{a^2\sin(dx+c)^3-2\,ab\sin(dx+c)^3+b^2\sin(dx+c)^3-3\,a^2\sin(dx+c)+9\,ab\sin(dx+c)-6\,b^2\sin(dx+c)}{a^3-3\,a^2b+3\,ab^2-b^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*b^2*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/((a^
2 - 2*a*b + b^2)*sqrt(-a^2 + a*b)) - (a^2*sin(d*x + c)^3 - 2*a*b*sin(d*x +
c)^3 + b^2*sin(d*x + c)^3 - 3*a^2*sin(d*x + c) + 9*a*b*sin(d*x + c) - 6*b^2
*sin(d*x + c))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/d
```

3.455 $\int \frac{\cos^5(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=126

$$\frac{(a^2-3ab+3b^2)\sin(c+dx)}{d(a-b)^3}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)^{7/2}}+\frac{\sin^5(c+dx)}{5d(a-b)}-\frac{(2a-3b)\sin^3(c+dx)}{3d(a-b)^2}$$

```
[Out] -((b^3*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(7/2)*
d)) + ((a^2 - 3*a*b + 3*b^2)*Sin[c + d*x])/((a - b)^3*d) - ((2*a - 3*b)*Sin
[c + d*x]^3)/(3*(a - b)^2*d) + Sin[c + d*x]^5/(5*(a - b)*d)
```

Rubi [A] time = 0.147802, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3676, 390, 208}

$$\frac{(a^2-3ab+3b^2)\sin(c+dx)}{d(a-b)^3}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)^{7/2}}+\frac{\sin^5(c+dx)}{5d(a-b)}-\frac{(2a-3b)\sin^3(c+dx)}{3d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] -((b^3*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(7/2)*
d)) + ((a^2 - 3*a*b + 3*b^2)*Sin[c + d*x])/((a - b)^3*d) - ((2*a - 3*b)*Sin
[c + d*x]^3)/(3*(a - b)^2*d) + Sin[c + d*x]^5/(5*(a - b)*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{a-(a-b)x^2}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2-3ab+3b^2}{(a-b)^3} - \frac{(2a-3b)x^2}{(a-b)^2} + \frac{x^4}{a-b} - \frac{b^3}{(a-b)^3(a-(a-b)x^2)}\right)dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{(a^2-3ab+3b^2)\sin(c+dx)}{(a-b)^3 d} - \frac{(2a-3b)\sin^3(c+dx)}{3(a-b)^2 d} + \frac{\sin^5(c+dx)}{5(a-b)d} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{a-(a-b)x^2}\right.}{(a-b}$$
$$= -\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{7/2}d} + \frac{(a^2-3ab+3b^2)\sin(c+dx)}{(a-b)^3 d} - \frac{(2a-3b)\sin^3(c+dx)}{3(a-b)^2 d} + \frac{\sin^5(c}{5(a-}$$

Mathematica [A] time = 1.71354, size = 148, normalized size = 1.17

$$\frac{\frac{30(5a^2-16ab+19b^2)\sin(c+dx)}{(a-b)^3} + \frac{120b^3\left(\log\left(\sqrt{a}-\sqrt{a-b}\sin(c+dx)\right)-\log\left(\sqrt{a-b}\sin(c+dx)+\sqrt{a}\right)\right)}{\sqrt{a}(a-b)^{7/2}} + \frac{5(5a-9b)\sin(3(c+dx))}{(a-b)^2} + \frac{3\sin(5(c+dx))}{a-b}}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((120*b^3*(Log[Sqrt[a] - Sqrt[a - b]*Sin[c + d*x]] - Log[Sqrt[a] + Sqrt[a -
 b]*Sin[c + d*x]]))/(Sqrt[a]*(a - b)^(7/2)) + (30*(5*a^2 - 16*a*b + 19*b^2)
*Sin[c + d*x])/(a - b)^3 + (5*(5*a - 9*b)*Sin[3*(c + d*x)])/(a - b)^2 + (3*
Sin[5*(c + d*x)])/(a - b))/(240*d)
```

Maple [A] time = 0.071, size = 165, normalized size = 1.3

$$\frac{1}{d}\left(\frac{1}{(a-b)^3}\left(\frac{a^2(\sin(dx+c))^5}{5} - \frac{2(\sin(dx+c))^5 ab}{5} + \frac{b^2(\sin(dx+c))^5}{5} - \frac{2(\sin(dx+c))^3 a^2}{3} + \frac{5ab(\sin(dx+c))^3}{3} - (\text{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*(1/(a-b)^3*(1/5*a^2*sin(d*x+c)^5-2/5*sin(d*x+c)^5*a*b+1/5*b^2*sin(d*x+c
)^5-2/3*sin(d*x+c)^3*a^2+5/3*a*b*sin(d*x+c)^3-sin(d*x+c)^3*b^2+a^2*sin(d*x+
c)-3*sin(d*x+c)*a*b+3*b^2*sin(d*x+c))-b^3/(a-b)^3/(a*(a-b))^(1/2)*arctanh((
a-b)*sin(d*x+c)/(a*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76536, size = 892, normalized size = 7.08

$$\left[-\frac{15\sqrt{a^2-ab}b^3\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)-2\left(3\left(a^4-3a^3b+3a^2b^2-ab^3\right)\cos(dx+c)^4+8a^4-34\right.}{30\left(a^5-4a^4b+6a^3b^2-4a^2b^3+a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*sqrt(a^2 - a*b)*b^3*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 -
a*b)*sin(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 + b)) - 2*(3*(a^4 - 3*
a^3*b + 3*a^2*b^2 - a*b^3)*cos(d*x + c)^4 + 8*a^4 - 34*a^3*b + 59*a^2*b^2 -
 33*a*b^3 + (4*a^4 - 17*a^3*b + 22*a^2*b^2 - 9*a*b^3)*cos(d*x + c)^2)*sin(d
*x + c))/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d), 1/15*(15*sqrt
(-a^2 + a*b)*b^3*arctan(sqrt(-a^2 + a*b)*sin(d*x + c)/a) + (3*(a^4 - 3*a^3*
b + 3*a^2*b^2 - a*b^3)*cos(d*x + c)^4 + 8*a^4 - 34*a^3*b + 59*a^2*b^2 - 33*
a*b^3 + (4*a^4 - 17*a^3*b + 22*a^2*b^2 - 9*a*b^3)*cos(d*x + c)^2)*sin(d*x +
 c))/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.76499, size = 431, normalized size = 3.42

$$-\frac{\frac{15b^3\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\left(a^3-3a^2b+3ab^2-b^3\right)\sqrt{-a^2+ab}}-\frac{3a^4\sin(dx+c)^5-12a^3b\sin(dx+c)^5+18a^2b^2\sin(dx+c)^5-12ab^3\sin(dx+c)^5+3b^4\sin(dx+c)^5-10a^4\sin(dx+c)^3+45}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*b^3*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/(
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(-a^2 + a*b)) - (3*a^4*sin(d*x + c)^5 -
 12*a^3*b*sin(d*x + c)^5 + 18*a^2*b^2*sin(d*x + c)^5 - 12*a*b^3*sin(d*x + c
)^5 + 3*b^4*sin(d*x + c)^5 - 10*a^4*sin(d*x + c)^3 + 45*a^3*b*sin(d*x + c)^
3 - 75*a^2*b^2*sin(d*x + c)^3 + 55*a*b^3*sin(d*x + c)^3 - 15*b^4*sin(d*x +
c)^3 + 15*a^4*sin(d*x + c) - 75*a^3*b*sin(d*x + c) + 150*a^2*b^2*sin(d*x +
c) - 135*a*b^3*sin(d*x + c) + 45*b^4*sin(d*x + c))/(a^5 - 5*a^4*b + 10*a^3*
b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5))/d
```

3.456 $\int \frac{\sec^8(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=108

$$\frac{(a^2-3ab+3b^2)\tan(c+dx)}{b^3d}-\frac{(a-3b)\tan^3(c+dx)}{3b^2d}-\frac{(a-b)^3\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}d}+\frac{\tan^5(c+dx)}{5bd}$$

```
[Out] -(((a - b)^3*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(7/2)*d)) +
 ((a^2 - 3*a*b + 3*b^2)*Tan[c + d*x])/(b^3*d) - ((a - 3*b)*Tan[c + d*x]^3)/
(3*b^2*d) + Tan[c + d*x]^5/(5*b*d)
```

Rubi [A] time = 0.109796, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3675, 390, 205}

$$\frac{(a^2-3ab+3b^2)\tan(c+dx)}{b^3d}-\frac{(a-3b)\tan^3(c+dx)}{3b^2d}-\frac{(a-b)^3\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}d}+\frac{\tan^5(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] -(((a - b)^3*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(7/2)*d)) +
 ((a^2 - 3*a*b + 3*b^2)*Tan[c + d*x])/(b^3*d) - ((a - 3*b)*Tan[c + d*x]^3)/
(3*b^2*d) + Tan[c + d*x]^5/(5*b*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^8(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2-3ab+3b^2}{b^3} - \frac{(a-3b)x^2}{b^2} + \frac{x^4}{b} + \frac{-a^3+3a^2b-3ab^2+b^3}{b^3(a+bx^2)}\right) dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{(a^2-3ab+3b^2)\tan(c+dx)}{b^3d} - \frac{(a-3b)\tan^3(c+dx)}{3b^2d} + \frac{\tan^5(c+dx)}{5bd} - \frac{(a-b)^3\,\text{Subst}\left(\int\right.}{}$$
$$= -\frac{(a-b)^3\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}d} + \frac{(a^2-3ab+3b^2)\tan(c+dx)}{b^3d} - \frac{(a-3b)\tan^3(c+dx)}{3b^2d} + \frac{\text{t}}{}$$

Mathematica [A] time = 0.922971, size = 103, normalized size = 0.95

$$\frac{\sqrt{b}\tan(c+dx)\left(15a^2-b(5a-9b)\sec^2(c+dx)-40ab+3b^2\sec^4(c+dx)+33b^2\right)-\frac{15(a-b)^3\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}}}{15b^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((-15*(a - b)^3*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/Sqrt[a] + Sqrt[b]*(
15*a^2 - 40*a*b + 33*b^2 - (5*a - 9*b)*b*Sec[c + d*x]^2 + 3*b^2*Sec[c + d*x
]^4)*Tan[c + d*x])/(15*b^(7/2)*d)
```

Maple [B] time = 0.073, size = 206, normalized size = 1.9

$$\frac{(\tan(dx+c))^5}{5bd} - \frac{(\tan(dx+c))^3 a}{3db^2} + \frac{(\tan(dx+c))^3}{bd} + \frac{a^2\tan(dx+c)}{db^3} - 3\frac{a\tan(dx+c)}{db^2} + 3\frac{\tan(dx+c)}{bd} - \frac{a^3}{db^3}\text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/5*tan(d*x+c)^5/b/d-1/3/d/b^2*tan(d*x+c)^3*a+tan(d*x+c)^3/b/d+1/d/b^3*a^2*
tan(d*x+c)-3/d/b^2*tan(d*x+c)*a+3*tan(d*x+c)/b/d-1/d/b^3/(a*b)^(1/2)*arctan
(b*tan(d*x+c)/(a*b)^(1/2))*a^3+3/d/b^2/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b
)^(1/2))*a^2-3/d/b/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))*a+1/d/(a*b)
^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.78256, size = 992, normalized size = 9.19

$$\left[\frac{15\left(a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}\right)\sqrt{-ab}\cos\left(dx+c\right)^{5}\log\left(\frac{\left(a^{2}+6\,ab+b^{2}\right)\cos(dx+c)^{4}-2\left(3\,ab+b^{2}\right)\cos(dx+c)^{2}+4\left((a+b)\cos(dx+c)^{3}-b\cos(dx+c)\right)\sqrt{-ab}}{\left(a^{2}-2\,ab+b^{2}\right)\cos(dx+c)^{4}+2\left(ab-b^{2}\right)\cos(dx+c)^{2}+b^{2}}\right.}{60\,ab^{4}d\cos\left(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(-a*b)*cos(d*x + c)^5*log(((a
^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 + 4*((a +
 b)*cos(d*x + c)^3 - b*cos(d*x + c))*sqrt(-a*b)*sin(d*x + c) + b^2)/((a^2 -
 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 + b^2)) + 4*((1
5*a^3*b - 40*a^2*b^2 + 33*a*b^3)*cos(d*x + c)^4 + 3*a*b^3 - (5*a^2*b^2 - 9*
a*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a*b^4*d*cos(d*x + c)^5), 1/30*(15*(a^
3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x + c)^2 -
 b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c)))*cos(d*x + c)^5 + 2*((15*a^3*
b - 40*a^2*b^2 + 33*a*b^3)*cos(d*x + c)^4 + 3*a*b^3 - (5*a^2*b^2 - 9*a*b^3)
*cos(d*x + c)^2)*sin(d*x + c))/(a*b^4*d*cos(d*x + c)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.63043, size = 204, normalized size = 1.89

$$-\frac{\frac{15\left(a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)}{\sqrt{ab}b^{3}}-\frac{3\,b^{4}\tan(dx+c)^{5}-5\,ab^{3}\tan(dx+c)^{3}+15\,b^{4}\tan(dx+c)^{3}+15\,a^{2}b^{2}\tan(dx+c)-45\,ab^{3}\tan(dx}{b^{5}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(pi*floor((d*x + c)/pi + 1/2)*sgn
(b) + arctan(b*tan(d*x + c)/sqrt(a*b)))/(sqrt(a*b)*b^3) - (3*b^4*tan(d*x +
c)^5 - 5*a*b^3*tan(d*x + c)^3 + 15*b^4*tan(d*x + c)^3 + 15*a^2*b^2*tan(d*x
+ c) - 45*a*b^3*tan(d*x + c) + 45*b^4*tan(d*x + c))/b^5)/d
```

3.457 $\int \frac{\sec^6(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}d}-\frac{(a-2b)\tan(c+dx)}{b^2d}+\frac{\tan^3(c+dx)}{3bd}$$

```
[Out] ((a - b)^2*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)*d) - ((
a - 2*b)*Tan[c + d*x])/(b^2*d) + Tan[c + d*x]^3/(3*b*d)
```

Rubi [A] time = 0.091478, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3675, 390, 205}

$$\frac{(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}d}-\frac{(a-2b)\tan(c+dx)}{b^2d}+\frac{\tan^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((a - b)^2*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)*d) - ((
a - 2*b)*Tan[c + d*x])/(b^2*d) + Tan[c + d*x]^3/(3*b*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a-2b}{b^2}+\frac{x^2}{b}+\frac{a^2-2ab+b^2}{b^2(a+bx^2)}\right)dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{(a-2b)\tan(c+dx)}{b^2d}+\frac{\tan^3(c+dx)}{3bd}+\frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{b^2d}$$
$$= \frac{(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}d}-\frac{(a-2b)\tan(c+dx)}{b^2d}+\frac{\tan^3(c+dx)}{3bd}$$

Mathematica [A] time = 0.330773, size = 74, normalized size = 0.96

$$\frac{\frac{3(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}}+\sqrt{b}\tan(c+dx)\left(-3a+b\sec^2(c+dx)+5b\right)}{3b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((3*(a - b)^2*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/Sqrt[a] + Sqrt[b]*(-3
*a + 5*b + b*Sec[c + d*x]^2)*Tan[c + d*x])/(3*b^(5/2)*d)
```

Maple [A] time = 0.07, size = 127, normalized size = 1.7

$$\frac{(\tan(dx+c))^3}{3\,bd}-\frac{a\tan(dx+c)}{db^2}+2\,\frac{\tan(dx+c)}{bd}+\frac{a^2}{db^2}\arctan\left(b\tan(dx+c)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-2\,\frac{a}{bd\sqrt{ab}}\arctan\left(\frac{b\tan(dx}{\sqrt{ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/3*tan(d*x+c)^3/b/d-1/d/b^2*tan(d*x+c)*a+2*tan(d*x+c)/b/d+1/d/b^2/(a*b)^(1
/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))*a^2-2/d/b/(a*b)^(1/2)*arctan(b*tan(d*x
+c)/(a*b)^(1/2))*a+1/d/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53481, size = 807, normalized size = 10.48

$$\left[-\frac{3\left(a^2-2\,ab+b^2\right)\sqrt{-ab}\cos\left(dx+c\right)^3\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos(dx+c)^4-2\left(3\,ab+b^2\right)\cos(dx+c)^2+4\left((a+b)\cos(dx+c)^3-b\cos(dx+c)\right)\sqrt{-ab}\sin(d}{\left(a^2-2\,ab+b^2\right)\cos(dx+c)^4+2\left(ab-b^2\right)\cos(dx+c)^2+b^2}\right.}{12\,ab^3d\cos\left(dx+c\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(a^2 - 2*a*b + b^2)*sqrt(-a*b)*cos(d*x + c)^3*log(((a^2 + 6*a*b +
 b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 + 4*((a + b)*cos(d*x
+ c)^3 - b*cos(d*x + c))*sqrt(-a*b)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2
)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 + b^2)) - 4*(a*b^2 - (3*a^2
*b - 5*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(a*b^3*d*cos(d*x + c)^3), -1/6*
(3*(a^2 - 2*a*b + b^2)*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x + c)^2 - b)*sq
rt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c)))*cos(d*x + c)^3 - 2*(a*b^2 - (3*a^2
*b - 5*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(a*b^3*d*cos(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^6\left(c+dx\right)}{a+b\tan^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)**6/(a + b*tan(c + d*x)**2), x)
```

Giac [A] time = 1.6116, size = 130, normalized size = 1.69

$$\frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)\left(a^2-2\,ab+b^2\right)}{\sqrt{ab}b^2}+\frac{b^2\tan(dx+c)^3-3\,ab\tan(dx+c)+6\,b^2\tan(dx+c)}{b^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*
b)))*(a^2 - 2*a*b + b^2)/(sqrt(a*b)*b^2) + (b^2*tan(d*x + c)^3 - 3*a*b*tan(
d*x + c) + 6*b^2*tan(d*x + c))/b^3)/d
```

3.458 $\int \frac{\sec^4(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=52

$$\frac{\tan(c+dx)}{bd}-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}d}$$

```
[Out] -(((a - b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d)) + T
an[c + d*x]/(b*d)
```

Rubi [A] time = 0.0673449, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3675, 388, 205}

$$\frac{\tan(c+dx)}{bd}-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] -(((a - b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d)) + T
an[c + d*x]/(b*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{a+bx^2}\,dx,x,\tan(c+dx)\right)}{d}$$
$$= \frac{\tan(c+dx)}{bd} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\tan(c+dx)\right)}{bd}$$
$$= -\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}d} + \frac{\tan(c+dx)}{bd}$$

Mathematica [A] time = 0.143639, size = 52, normalized size = 1.

$$\frac{\tan(c+dx)}{bd} - \frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] -(((a - b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d)) + T
an[c + d*x]/(b*d)
```

Maple [A] time = 0.063, size = 66, normalized size = 1.3

$$\frac{\tan(dx+c)}{bd} - \frac{a}{bd}\arctan\left(b\tan(dx+c)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{1}{d}\arctan\left(b\tan(dx+c)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*tan(d*x+c)^2),x)
```

```
[Out] tan(d*x+c)/b/d-1/d/b/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))*a+1/d/(a*
b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.62674, size = 644, normalized size = 12.38

$$\left[\frac{\sqrt{-ab}(a-b)\cos(dx+c)\log\left(\frac{(a^2+6ab+b^2)\cos(dx+c)^4-2(3ab+b^2)\cos(dx+c)^2+4((a+b)\cos(dx+c)^3-b\cos(dx+c))\sqrt{-ab}\sin(dx+c)+b^2}{(a^2-2ab+b^2)\cos(dx+c)^4+2(ab-b^2)\cos(dx+c)^2+b^2}\right)+4a}{4ab^2d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-a*b)*(a - b)*cos(d*x + c)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)
^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x
 + c))*sqrt(-a*b)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 +
 2*(a*b - b^2)*cos(d*x + c)^2 + b^2)) + 4*a*b*sin(d*x + c))/(a*b^2*d*cos(d*
x + c)), 1/2*(sqrt(a*b)*(a - b)*arctan(1/2*((a + b)*cos(d*x + c)^2 - b)*sqr
t(a*b)/(a*b*cos(d*x + c)*sin(d*x + c)))*cos(d*x + c) + 2*a*b*sin(d*x + c))/
(a*b^2*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*tan(c + d*x)**2), x)
```

Giac [A] time = 1.65045, size = 84, normalized size = 1.62

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)(a-b)}{\sqrt{ab}\,b}-\frac{\tan(dx+c)}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b)))*
(a - b)/(sqrt(a*b)*b) - tan(d*x + c)/b)/d
```

3.459 $\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

```
[Out] ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rubi [A] time = 0.0531762, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}\end{aligned}$$

Mathematica [A] time = 0.05198, size = 32, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Maple [A] time = 0.064, size = 24, normalized size = 0.8

$$\frac{1}{d}\arctan\left(b\tan\left(dx+c\right)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.49803, size = 487, normalized size = 15.22

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos(dx+c)^4-2\left(3\,ab+b^2\right)\cos(dx+c)^2+4\left(\left(a+b\right)\cos(dx+c)^3-b\cos(dx+c)\right)\sqrt{-ab}\sin(dx+c)+b^2}{\left(a^2-2\,ab+b^2\right)\cos(dx+c)^4+2\left(ab-b^2\right)\cos(dx+c)^2+b^2}\right)}{4\,abd},-\frac{\sqrt{ab}\arctan\left(\frac{\left(\left(a+b\right)\cos(d\cdots}{2\,ab\cos(dx+\cdots}\right.}{2\,abd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a*b)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*
cos(d*x + c)^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x + c))*sqrt(-a*b)*sin
(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*
x + c)^2 + b^2))/(a*b*d), -1/2*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x + c)^2
 - b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c)))/(a*b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2\left(c+dx\right)}{a+b\tan^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*tan(c + d*x)**2), x)
```

Giac [A] time = 1.7099, size = 54, normalized size = 1.69

$$\frac{\pi \left\lfloor \frac{dx+c}{\pi} + \frac{1}{2} \right\rfloor \mathrm{sgn}\,(b) + \arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b)))/(s
qrt(a*b)*d)
```

3.460 $\int \frac{\cos^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=83

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)^2}+\frac{\sin(c+dx)\cos(c+dx)}{2d(a-b)}+\frac{x(a-3b)}{2(a-b)^2}$$

```
[Out] ((a - 3*b)*x)/(2*(a - b)^2) + (b^(3/2)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a
]])/(Sqrt[a]*(a - b)^2*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*(a - b)*d)
```

Rubi [A] time = 0.103657, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3675, 414, 522, 203, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)^2}+\frac{\sin(c+dx)\cos(c+dx)}{2d(a-b)}+\frac{x(a-3b)}{2(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((a - 3*b)*x)/(2*(a - b)^2) + (b^(3/2)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a
]])/(Sqrt[a]*(a - b)^2*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*(a - b)*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+bx^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d} - \frac{\text{Subst}\left(\int \frac{-a+2b-bx^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(c+dx)\right)}{2(a-b)d}$$

$$= \frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d} + \frac{(a-3b)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{2(a-b)^2d} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\right.}{(a-}$$

$$= \frac{(a-3b)x}{2(a-b)^2} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^2d} + \frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d}$$

Mathematica [A] time = 0.18469, size = 78, normalized size = 0.94

$$\frac{4b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right) + \sqrt{a}(2(a-3b)(c+dx)+(a-b)\sin(2(c+dx)))}{4\sqrt{a}d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (4*b^(3/2)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a]*(2*(a - 3*b)*(c
 + d*x) + (a - b)*Sin[2*(c + d*x)]))/(4*Sqrt[a]*(a - b)^2*d)
```

Maple [A] time = 0.069, size = 137, normalized size = 1.7

$$\frac{b^2}{d(a-b)^2}\arctan\left(b\tan(dx+c)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{a\tan(dx+c)}{2d(a-b)^2\left((\tan(dx+c))^2+1\right)} - \frac{b\tan(dx+c)}{2d(a-b)^2\left((\tan(dx+c))^2+1\right)} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*b^2/(a-b)^2/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))+1/2/d/(a-b)^2*
tan(d*x+c)/(tan(d*x+c)^2+1)*a-1/2/d/(a-b)^2*tan(d*x+c)/(tan(d*x+c)^2+1)*b+1
/2/d/(a-b)^2*arctan(tan(d*x+c))*a-3/2/d/(a-b)^2*arctan(tan(d*x+c))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68791, size = 690, normalized size = 8.31

$$\left[\frac{2\,(a-3\,b)dx+2\,(a-b)\cos\left(dx+c\right)\sin\left(dx+c\right)+b\sqrt{-\frac{b}{a}}\log\left(\frac{\left(a^{2}+6\,ab+b^{2}\right)\cos(dx+c)^{4}-2\left(3\,ab+b^{2}\right)\cos(dx+c)^{2}-4\left(\left(a^{2}+ab\right)\cos(dx+c)^{3}-a\right.}{\left(a^{2}-2\,ab+b^{2}\right)\cos(dx+c)^{4}+2\left(ab-b^{2}\right)\cos(dx+c)^{2}}\right.}{4\left(a^{2}-2\,ab+b^{2}\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a - 3*b)*d*x + 2*(a - b)*cos(d*x + c)*sin(d*x + c) + b*sqrt(-b/a)*
log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 -
4*((a^2 + a*b)*cos(d*x + c)^3 - a*b*cos(d*x + c))*sqrt(-b/a)*sin(d*x + c) +
 b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 +
b^2)))/((a^2 - 2*a*b + b^2)*d), 1/2*((a - 3*b)*d*x + (a - b)*cos(d*x + c)*s
in(d*x + c) - b*sqrt(b/a)*arctan(1/2*((a + b)*cos(d*x + c)^2 - b)*sqrt(b/a)
/(b*cos(d*x + c)*sin(d*x + c))))/((a^2 - 2*a*b + b^2)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.65799, size = 149, normalized size = 1.8

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)b^{2}}{\left(a^{2}-2\,ab+b^{2}\right)\sqrt{ab}}+\frac{(dx+c)(a-3\,b)}{a^{2}-2\,ab+b^{2}}+\frac{\tan(dx+c)}{\left(\tan(dx+c)^{2}+1\right)(a-b)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(2*(pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*
b)))*b^2/((a^2 - 2*a*b + b^2)*sqrt(a*b)) + (d*x + c)*(a - 3*b)/(a^2 - 2*a*b
 + b^2) + tan(d*x + c)/((tan(d*x + c)^2 + 1)*(a - b)))/d
```

3.461 $\int \frac{\cos^4(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=129

$$\frac{x(3a^2-10ab+15b^2)}{8(a-b)^3}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)^3}+\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a-b)}+\frac{(3a-7b)\sin(c+dx)\cos(c+dx)}{8d(a-b)^2}$$

```
[Out] ((3*a^2 - 10*a*b + 15*b^2)*x)/(8*(a - b)^3) - (b^(5/2)*ArcTan[(Sqrt[b]*Tan[
c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^3*d) + ((3*a - 7*b)*Cos[c + d*x]*Sin[c
 + d*x])/(8*(a - b)^2*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*(a - b)*d)
```

Rubi [A] time = 0.166607, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3675, 414, 527, 522, 203, 205}

$$\frac{x(3a^2-10ab+15b^2)}{8(a-b)^3}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)^3}+\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a-b)}+\frac{(3a-7b)\sin(c+dx)\cos(c+dx)}{8d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((3*a^2 - 10*a*b + 15*b^2)*x)/(8*(a - b)^3) - (b^(5/2)*ArcTan[(Sqrt[b]*Tan[
c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^3*d) + ((3*a - 7*b)*Cos[c + d*x]*Sin[c
 + d*x])/(8*(a - b)^2*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*(a - b)*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^3\left(a+bx^2\right)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d} - \frac{\text{Subst}\left(\int \frac{-3a+4b-3bx^2}{\left(1+x^2\right)^2\left(a+bx^2\right)}\,dx, x, \tan(c+dx)\right)}{4(a-b)d}$$

$$= \frac{(3a-7b)\cos(c+dx)\sin(c+dx)}{8(a-b)^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d} + \frac{\text{Subst}\left(\int \frac{3a^2-7ab+8b^2+(3a-7b)bx^2}{\left(1+x^2\right)\left(a+bx^2\right)}\right.}{8(a-b)^2d}$$

$$= \frac{(3a-7b)\cos(c+dx)\sin(c+dx)}{8(a-b)^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(c\right.}{(a-b)^3d}$$

$$= \frac{\left(3a^2-10ab+15b^2\right)x}{8(a-b)^3} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^3d} + \frac{(3a-7b)\cos(c+dx)\sin(c+dx)}{8(a-b)^2d} + \frac{\cos^3(}{}$$

Mathematica [A] time = 0.427649, size = 113, normalized size = 0.88

$$\frac{\sqrt{a}\left(4\left(3a^2-10ab+15b^2\right)(c+dx)+8\left(a^2-3ab+2b^2\right)\sin(2(c+dx))+(a-b)^2\sin(4(c+dx))\right)-32b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+}{\sqrt{a}}\right.}{32\sqrt{a}d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (-32*b^(5/2)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a]*(4*(3*a^2 - 1
0*a*b + 15*b^2)*(c + d*x) + 8*(a^2 - 3*a*b + 2*b^2)*Sin[2*(c + d*x)] + (a -
 b)^2*Sin[4*(c + d*x)]))/(32*Sqrt[a]*(a - b)^3*d)
```

Maple [B] time = 0.077, size = 303, normalized size = 2.4

$$-\frac{b^3}{d(a-b)^3}\arctan\left(b\tan(dx+c)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{3(\tan(dx+c))^3a^2}{8d(a-b)^3\left((\tan(dx+c))^2+1\right)^2} - \frac{5(\tan(dx+c))^3ab}{4d(a-b)^3\left((\tan(dx+c))^2+1\right)^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*tan(d*x+c)^2),x)
```

```
[Out] -1/d/(a-b)^3*b^3/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))+3/8/d/(a-b)^3
/(tan(d*x+c)^2+1)^2*tan(d*x+c)^3*a^2-5/4/d/(a-b)^3/(tan(d*x+c)^2+1)^2*tan(d
*x+c)^3*a*b+7/8/d/(a-b)^3/(tan(d*x+c)^2+1)^2*tan(d*x+c)^3*b^2-7/4/d/(a-b)^3
/(tan(d*x+c)^2+1)^2*tan(d*x+c)*a*b+9/8/d/(a-b)^3/(tan(d*x+c)^2+1)^2*tan(d*x
+c)*b^2+5/8/d/(a-b)^3/(tan(d*x+c)^2+1)^2*tan(d*x+c)*a^2+15/8/d/(a-b)^3*arct
an(tan(d*x+c))*b^2+3/8/d/(a-b)^3*arctan(tan(d*x+c))*a^2-5/4/d/(a-b)^3*arcta
n(tan(d*x+c))*a*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74992, size = 929, normalized size = 7.2

$$\left[-\frac{2\,b^2\sqrt{-\frac{b}{a}}\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos(dx+c)^4-2\left(3\,ab+b^2\right)\cos(dx+c)^2-4\left(\left(a^2+ab\right)\cos(dx+c)^3-ab\cos(dx+c)\right)\sqrt{-\frac{b}{a}}\sin(dx+c)+b^2}{\left(a^2-2\,ab+b^2\right)\cos(dx+c)^4+2\left(ab-b^2\right)\cos(dx+c)^2+b^2}\right)-\left(3\,a^2-10\,ab+\right.}{8\left(a^3-3\,a^2b+3\,ab^2-b^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/8*(2*b^2*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b
+ b^2)*cos(d*x + c)^2 - 4*((a^2 + a*b)*cos(d*x + c)^3 - a*b*cos(d*x + c))*s
qrt(-b/a)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b
- b^2)*cos(d*x + c)^2 + b^2)) - (3*a^2 - 10*a*b + 15*b^2)*d*x - (2*(a^2 - 2
*a*b + b^2)*cos(d*x + c)^3 + (3*a^2 - 10*a*b + 7*b^2)*cos(d*x + c))*sin(d*x
 + c))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d), 1/8*(4*b^2*sqrt(b/a)*arctan(1/2
*((a + b)*cos(d*x + c)^2 - b)*sqrt(b/a)/(b*cos(d*x + c)*sin(d*x + c))) + (3
*a^2 - 10*a*b + 15*b^2)*d*x + (2*(a^2 - 2*a*b + b^2)*cos(d*x + c)^3 + (3*a^
2 - 10*a*b + 7*b^2)*cos(d*x + c))*sin(d*x + c))/((a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*tan(d*x+c)**2),x)
```

[Out] Timed out

Giac [A] time = 1.65097, size = 247, normalized size = 1.91

$$-\frac{\frac{8\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)b^3}{(a^3-3a^2b+3ab^2-b^3)\sqrt{ab}}-\frac{(3a^2-10ab+15b^2)(dx+c)}{a^3-3a^2b+3ab^2-b^3}-\frac{3a\tan(dx+c)^3-7b\tan(dx+c)^3+5a\tan(dx+c)-9b\tan(dx+c)}{(a^2-2ab+b^2)(\tan(dx+c)^2+1)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/8*(8*(pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a
*b)))*b^3/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a*b)) - (3*a^2 - 10*a*b + 1
5*b^2)*(d*x + c)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (3*a*tan(d*x + c)^3 - 7*
b*tan(d*x + c)^3 + 5*a*tan(d*x + c) - 9*b*tan(d*x + c))/((a^2 - 2*a*b + b^2
)*(tan(d*x + c)^2 + 1)^2))/d
```

3.462 $\int \frac{\sec^7(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=167

$$\frac{(4a+b)(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d}+\frac{(2a-b)(a-b)\sin(c+dx)}{2ab^2d\left(a-(a-b)\sin^2(c+dx)\right)}-\frac{(4a-5b)\tanh^{-1}(\sin(c+dx))}{2b^3d}+\frac{\tan}{2bd\left(a\right.}$$

```
[Out] -((4*a - 5*b)*ArcTanh[Sin[c + d*x]])/(2*b^3*d) + ((a - b)^(3/2)*(4*a + b)*A
rcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^3*d) + ((a - b)*(2
*a - b)*Sin[c + d*x])/(2*a*b^2*d*(a - (a - b)*Sin[c + d*x]^2)) + (Sec[c + d
*x]*Tan[c + d*x])/(2*b*d*(a - (a - b)*Sin[c + d*x]^2))
```

Rubi [A] time = 0.267281, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3676, 414, 527, 522, 206, 208}

$$\frac{(4a+b)(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d}+\frac{(2a-b)(a-b)\sin(c+dx)}{2ab^2d\left(a-(a-b)\sin^2(c+dx)\right)}-\frac{(4a-5b)\tanh^{-1}(\sin(c+dx))}{2b^3d}+\frac{\tan}{2bd\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + b*Tan[c + d*x]^2)^2,x]
```


```
[Out] -((4*a - 5*b)*ArcTanh[Sin[c + d*x]])/(2*b^3*d) + ((a - b)^(3/2)*(4*a + b)*A
rcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^3*d) + ((a - b)*(2
*a - b)*Sin[c + d*x])/(2*a*b^2*d*(a - (a - b)*Sin[c + d*x]^2)) + (Sec[c + d
*x]*Tan[c + d*x])/(2*b*d*(a - (a - b)*Sin[c + d*x]^2))
```


Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a-(a-b)x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\sec(c+dx)\tan(c+dx)}{2bd\left(a-(a-b)\sin^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-a+2b-3(a-b)x^2}{\left(1-x^2\right)\left(a+(-a+b)x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{2bd}$$

$$= \frac{(a-b)(2a-b)\sin(c+dx)}{2ab^2d\left(a-(a-b)\sin^2(c+dx)\right)} + \frac{\sec(c+dx)\tan(c+dx)}{2bd\left(a-(a-b)\sin^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{2\left(2a^2-2ab-b^2\right)+\cdots}{\left(1-x^2\right)\left(a+(\cdots\right.}\right.}{\cdots}$$

$$= \frac{(a-b)(2a-b)\sin(c+dx)}{2ab^2d\left(a-(a-b)\sin^2(c+dx)\right)} + \frac{\sec(c+dx)\tan(c+dx)}{2bd\left(a-(a-b)\sin^2(c+dx)\right)} - \frac{(4a-5b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\cdots\right.}{2b^3d\cdots}$$

$$= -\frac{(4a-5b)\tanh^{-1}(\sin(c+dx))}{2b^3d} + \frac{(a-b)^{3/2}(4a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d} + \frac{(a-b)(2a\cdots}{2ab^2d\left(a-(\cdots\right.}$$

Mathematica [A] time = 4.10419, size = 254, normalized size = 1.52

$$-\frac{(4a+b)(a-b)^{3/2}\log\left(\sqrt{a}-\sqrt{a-b}\sin(c+dx)\right)}{a^{3/2}} + \frac{(4a+b)(a-b)^{3/2}\log\left(\sqrt{a-b}\sin(c+dx)+\sqrt{a}\right)}{a^{3/2}} + \frac{4b(a-b)^2\sin(c+dx)}{a((a-b)\cos(2(c+dx))+a+b)} + 2(4a-5b)\log\left(\cos\left(\frac{1}{2}(c+\cdots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (2*(4*a - 5*b)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*(-4*a + 5*b)*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - ((a - b)^(3/2)*(4*a + b)*Log[Sqrt[
a] - Sqrt[a - b]*Sin[c + d*x]])/a^(3/2) + ((a - b)^(3/2)*(4*a + b)*Log[Sqrt
[a] + Sqrt[a - b]*Sin[c + d*x]])/a^(3/2) + b/(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])^2 - b/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*(a - b)^2*b*Sin[
```

```
c + d*x])/(a*(a + b + (a - b)*Cos[2*(c + d*x)])))/(4*b^3*d)
```

Maple [B] time = 0.109, size = 389, normalized size = 2.3

$$-\frac{\sin (dx+c)\, a}{2\, db^2 \left(a (\sin (dx+c))^2 - b (\sin (dx+c))^2 - a\right)} + \frac{\sin (dx+c)}{db \left(a (\sin (dx+c))^2 - b (\sin (dx+c))^2 - a\right)} - \frac{\sin (}{2\, da \left(a (\sin (dx+c))^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d/b^2*a*sin(d*x+c)/(a*sin(d*x+c)^2-b*sin(d*x+c)^2-a)+1/d/b*sin(d*x+c)/
(a*sin(d*x+c)^2-b*sin(d*x+c)^2-a)-1/2/d/a*sin(d*x+c)/(a*sin(d*x+c)^2-b*sin(
d*x+c)^2-a)+2/d/b^3/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2
))*a^2-7/2/d/b^2/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))*
a+1/d/b/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))+1/2/d/a/(
a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))-1/4/d/b^2/(sin(d*x
+c)+1)-1/d/b^3*ln(sin(d*x+c)+1)*a+5/4/d/b^2*ln(sin(d*x+c)+1)-1/4/d/b^2/(sin
(d*x+c)-1)+1/d/b^3*ln(sin(d*x+c)-1)*a-5/4/d/b^2*ln(sin(d*x+c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.42098, size = 1447, normalized size = 8.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(((4*a^3 - 7*a^2*b + 2*a*b^2 + b^3)*cos(d*x + c)^4 + (4*a^2*b - 3*a*b
^2 - b^3)*cos(d*x + c)^2)*sqrt((a - b)/a)*log(-((a - b)*cos(d*x + c)^2 + 2*
a*sqrt((a - b)/a)*sin(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 + b)) + (
(4*a^3 - 9*a^2*b + 5*a*b^2)*cos(d*x + c)^4 + (4*a^2*b - 5*a*b^2)*cos(d*x +
c)^2)*log(sin(d*x + c) + 1) - ((4*a^3 - 9*a^2*b + 5*a*b^2)*cos(d*x + c)^4 +
 (4*a^2*b - 5*a*b^2)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a*b^2 + (2
*a^2*b - 3*a*b^2 + b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a*b^4*d*cos(d*x + c)
^2 + (a^2*b^3 - a*b^4)*d*cos(d*x + c)^4), -1/4*(2*((4*a^3 - 7*a^2*b + 2*a*b
^2 + b^3)*cos(d*x + c)^4 + (4*a^2*b - 3*a*b^2 - b^3)*cos(d*x + c)^2)*sqrt(-
(a - b)/a)*arctan(sqrt(-(a - b)/a)*sin(d*x + c)) + ((4*a^3 - 9*a^2*b + 5*a*
b^2)*cos(d*x + c)^4 + (4*a^2*b - 5*a*b^2)*cos(d*x + c)^2)*log(sin(d*x + c)
+ 1) - ((4*a^3 - 9*a^2*b + 5*a*b^2)*cos(d*x + c)^4 + (4*a^2*b - 5*a*b^2)*co
s(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a*b^2 + (2*a^2*b - 3*a*b^2 + b^3)
*cos(d*x + c)^2)*sin(d*x + c))/(a*b^4*d*cos(d*x + c)^2 + (a^2*b^3 - a*b^4)*
```

```
d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.8084, size = 331, normalized size = 1.98

$$-\frac{\frac{(4\,a-5\,b)\log(|\sin(dx+c)+1|)}{b^3}-\frac{(4\,a-5\,b)\log(|\sin(dx+c)-1|)}{b^3}-\frac{2\left(4\,a^3-7\,a^2b+2\,ab^2+b^3\right)\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}ab^3}+\frac{2\left(2\,a^2\sin(dx+c)^3-3\,ab\sin(dx+c\right.}{\left(a\sin(dx+c)^4-b\,s\right.}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/4*((4*a - 5*b)*log(abs(sin(d*x + c) + 1))/b^3 - (4*a - 5*b)*log(abs(sin(
d*x + c) - 1))/b^3 - 2*(4*a^3 - 7*a^2*b + 2*a*b^2 + b^3)*arctan(-(a*sin(d*x
 + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*a*b^3) + 2*(2*a
^2*sin(d*x + c)^3 - 3*a*b*sin(d*x + c)^3 + b^2*sin(d*x + c)^3 - 2*a^2*sin(d
*x + c) + 2*a*b*sin(d*x + c) - b^2*sin(d*x + c))/((a*sin(d*x + c)^4 - b*sin
(d*x + c)^4 - 2*a*sin(d*x + c)^2 + b*sin(d*x + c)^2 + a)*a*b^2))/d
```

3.463 $\int \frac{\sec^5(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=109

$$-\frac{\sqrt{a-b}(2a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d}-\frac{(a-b)\sin(c+dx)}{2abd\left(a-(a-b)\sin^2(c+dx)\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(b^2*d) - (Sqrt[a - b]*(2*a + b)*ArcTanh[(Sqrt[a - b]
*Sin[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^2*d) - ((a - b)*Sin[c + d*x])/(2*a*b*
d*(a - (a - b)*Sin[c + d*x]^2))
```

Rubi [A] time = 0.140827, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3676, 414, 522, 206, 208}

$$-\frac{\sqrt{a-b}(2a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d}-\frac{(a-b)\sin(c+dx)}{2abd\left(a-(a-b)\sin^2(c+dx)\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(b^2*d) - (Sqrt[a - b]*(2*a + b)*ArcTanh[(Sqrt[a - b]
*Sin[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^2*d) - ((a - b)*Sin[c + d*x])/(2*a*b*
d*(a - (a - b)*Sin[c + d*x]^2))
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a-(a-b)x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{(a-b)\sin(c+dx)}{2abd\left(a-(a-b)\sin^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-a-b+(-a+b)x^2}{(1-x^2)\left(a+(-a+b)x^2\right)}\,dx, x, \sin(c+dx)\right)}{2abd}$$

$$= -\frac{(a-b)\sin(c+dx)}{2abd\left(a-(a-b)\sin^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{b^2d} - \frac{((a-b)(2a+b))\,\text{Su}}{}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{\sqrt{a-b}(2a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d} - \frac{(a-b)\sin(c+dx)}{2abd\left(a-(a-b)\sin^2(c+\right.}$$

Mathematica [A] time = 0.806989, size = 191, normalized size = 1.75

$$\frac{\frac{\left(-2a^2+ab+b^2\right)\log\left(\sqrt{a-b}\sin(c+dx)+\sqrt{a}\right)}{a^{3/2}\sqrt{a-b}} + \frac{\sqrt{a-b}(2a+b)\log\left(\sqrt{a}-\sqrt{a-b}\sin(c+dx)\right)}{a^{3/2}} + \frac{4b(b-a)\sin(c+dx)}{a((a-b)\cos(2(c+dx))+a+b)} - 4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}\right.\right.}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (-4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]] + (Sqrt[a - b]*(2*a + b)*Log[Sqrt[a] - Sqrt[a - b]*Sin[c + d
*x]])/a^(3/2) + ((-2*a^2 + a*b + b^2)*Log[Sqrt[a] + Sqrt[a - b]*Sin[c + d*x
]])/(a^(3/2)*Sqrt[a - b]) + (4*b*(-a + b)*Sin[c + d*x])/(a*(a + b + (a - b)
*Cos[2*(c + d*x)])))/(4*b^2*d)
```

Maple [B] time = 0.099, size = 236, normalized size = 2.2

$$\frac{\sin(dx+c)}{2db\left(a(\sin(dx+c))^2-b(\sin(dx+c))^2-a\right)} - \frac{a}{db^2}\text{Artanh}\left((a-b)\sin(dx+c)\frac{1}{\sqrt{a(a-b)}}\right)\frac{1}{\sqrt{a(a-b)}} + \frac{1}{2db}\text{Artanh}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d/b*sin(d*x+c)/(a*sin(d*x+c)^2-b*sin(d*x+c)^2-a)-1/d/b^2/(a*(a-b))^(1/2
)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))*a+1/2/d/b/(a*(a-b))^(1/2)*arcta
nh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))-1/2/d/a*sin(d*x+c)/(a*sin(d*x+c)^2-b*s
```

```
in(d*x+c)^2-a)+1/2/d/a/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(
1/2))+1/2/d/b^2*ln(sin(d*x+c)+1)-1/2/d/b^2*ln(sin(d*x+c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.0195, size = 929, normalized size = 8.52

$$\left[\frac{\left(\left(2a^2-ab-b^2\right)\cos\left(dx+c\right)^2+2ab+b^2\right)\sqrt{\frac{a-b}{a}}\log\left(-\frac{(a-b)\cos(dx+c)^2+2a\sqrt{\frac{a-b}{a}}\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)+2\left(\left(a^2-ab\right)\cos\left(dx+c\right.\right.}{4\left(ab^3d+\left(a^2b^2-ab^3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a^2 - a*b - b^2)*cos(d*x + c)^2 + 2*a*b + b^2)*sqrt((a - b)/a)*lo
g(-((a - b)*cos(d*x + c)^2 + 2*a*sqrt((a - b)/a)*sin(d*x + c) - 2*a + b)/((
a - b)*cos(d*x + c)^2 + b)) + 2*((a^2 - a*b)*cos(d*x + c)^2 + a*b)*log(sin(
d*x + c) + 1) - 2*((a^2 - a*b)*cos(d*x + c)^2 + a*b)*log(-sin(d*x + c) + 1)
 - 2*(a*b - b^2)*sin(d*x + c))/(a*b^3*d + (a^2*b^2 - a*b^3)*d*cos(d*x + c)^
2), 1/2*(((2*a^2 - a*b - b^2)*cos(d*x + c)^2 + 2*a*b + b^2)*sqrt(-(a - b)/a
)*arctan(sqrt(-(a - b)/a)*sin(d*x + c)) + ((a^2 - a*b)*cos(d*x + c)^2 + a*b
)*log(sin(d*x + c) + 1) - ((a^2 - a*b)*cos(d*x + c)^2 + a*b)*log(-sin(d*x +
 c) + 1) - (a*b - b^2)*sin(d*x + c))/(a*b^3*d + (a^2*b^2 - a*b^3)*d*cos(d*x
 + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.81859, size = 207, normalized size = 1.9

$$\frac{\frac{\log(|\sin(dx+c)+1|)}{b^2}-\frac{\log(|\sin(dx+c)-1|)}{b^2}-\frac{\left(2a^2-ab-b^2\right)\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}ab^2}+\frac{a\sin(dx+c)-b\sin(dx+c)}{\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)ab}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(sin(d*x + c) + 1))/b^2 - log(abs(sin(d*x + c) - 1))/b^2 - (2*a
^2 - a*b - b^2)*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))
/(sqrt(-a^2 + a*b)*a*b^2) + (a*sin(d*x + c) - b*sin(d*x + c))/((a*sin(d*x +
 c)^2 - b*sin(d*x + c)^2 - a)*a*b))/d
```

3.464 $\int \frac{\sec^3(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=79

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d\sqrt{a-b}}+\frac{\sin(c+dx)}{2ad\left(a-(a-b)\sin^2(c+dx)\right)}$$

```
[Out] ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a - b]*d) + Sin
[c + d*x]/(2*a*d*(a - (a - b)*Sin[c + d*x]^2))
```

Rubi [A] time = 0.0795542, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3676, 199, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d\sqrt{a-b}}+\frac{\sin(c+dx)}{2ad\left(a-(a-b)\sin^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a - b]*d) + Sin
[c + d*x]/(2*a*d*(a - (a - b)*Sin[c + d*x]^2))
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a-(a-b)x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\sin(c+dx)}{2ad\left(a-(a-b)\sin^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \sin(c+dx)\right)}{2ad}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a-b}d} + \frac{\sin(c+dx)}{2ad\left(a-(a-b)\sin^2(c+dx)\right)}$$

Mathematica [A] time = 0.237823, size = 75, normalized size = 0.95

$$\frac{\frac{\sqrt{a}\sin(c+dx)}{(b-a)\sin^2(c+dx)+a} + \frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]]/Sqrt[a - b] + (Sqrt[a]*Sin[c +
 d*x])/(a + (-a + b)*Sin[c + d*x]^2))/(2*a^(3/2)*d)
```

Maple [A] time = 0.09, size = 80, normalized size = 1.

$$\frac{1}{d}\left(-\frac{\sin(dx+c)}{2a\left(a(\sin(dx+c))^2-b(\sin(dx+c))^2-a\right)}+\frac{1}{2a}\text{Artanh}\left((a-b)\sin(dx+c)\frac{1}{\sqrt{a(a-b)}}\right)\frac{1}{\sqrt{a(a-b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(-1/2*sin(d*x+c)/a/(a*sin(d*x+c)^2-b*sin(d*x+c)^2-a)+1/2/a/(a*(a-b))^(1
/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54069, size = 598, normalized size = 7.57

$$\left[\frac{\left((a-b)\cos\left(dx+c\right)^2+b\right)\sqrt{a^2-ab}\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)+2\left(a^2-ab\right)\sin\left(dx+c\right)}{4\left(\left(a^4-2a^3b+a^2b^2\right)d\cos\left(dx+c\right)^2+\left(a^3b-a^2b^2\right)d\right)},-\frac{\left((a-b)\cos\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((a - b)*cos(d*x + c)^2 + b)*sqrt(a^2 - a*b)*log(-((a - b)*cos(d*x +
c)^2 - 2*sqrt(a^2 - a*b)*sin(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 +
b)) + 2*(a^2 - a*b)*sin(d*x + c))/((a^4 - 2*a^3*b + a^2*b^2)*d*cos(d*x + c)
^2 + (a^3*b - a^2*b^2)*d), -1/2*(((a - b)*cos(d*x + c)^2 + b)*sqrt(-a^2 + a
*b)*arctan(sqrt(-a^2 + a*b)*sin(d*x + c)/a) - (a^2 - a*b)*sin(d*x + c))/((a
^4 - 2*a^3*b + a^2*b^2)*d*cos(d*x + c)^2 + (a^3*b - a^2*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^3\left(c+dx\right)}{\left(a+b\tan^2\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*tan(c + d*x)**2)**2, x)
```

Giac [A] time = 1.73869, size = 123, normalized size = 1.56

$$\frac{\frac{\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}a}-\frac{\sin(dx+c)}{\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/(sqrt(-a^2
 + a*b)*a) - sin(d*x + c)/((a*sin(d*x + c)^2 - b*sin(d*x + c)^2 - a)*a))/d
```

3.465 $\int \frac{\sec(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=94

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{3/2}}-\frac{b\sin(c+dx)}{2ad(a-b)\left(a-(a-b)\sin^2(c+dx)\right)}$$

```
[Out] ((2*a - b)*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^
(3/2)*d) - (b*Sin[c + d*x])/(2*a*(a - b)*d*(a - (a - b)*Sin[c + d*x]^2))
```

Rubi [A] time = 0.0830864, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3676, 385, 208}

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{3/2}}-\frac{b\sin(c+dx)}{2ad(a-b)\left(a-(a-b)\sin^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((2*a - b)*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^
(3/2)*d) - (b*Sin[c + d*x])/(2*a*(a - b)*d*(a - (a - b)*Sin[c + d*x]^2))
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a-(a-b)x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{b\sin(c+dx)}{2a(a-b)d\left(a-(a-b)\sin^2(c+dx)\right)} + \frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \sin(c+dx)\right)}{2a(a-b)d}$$

$$= \frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{3/2}d} - \frac{b\sin(c+dx)}{2a(a-b)d\left(a-(a-b)\sin^2(c+dx)\right)}$$

Mathematica [A] time = 0.237171, size = 92, normalized size = 0.98

$$\frac{\frac{\sqrt{a}b\sin(c+dx)}{(a-b)\left((a-b)\sin^2(c+dx)-a\right)} + \frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{(a-b)^{3/2}}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (((2*a - b)*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(a - b)^(3/2) + (S
qrt[a]*b*Sin[c + d*x])/((a - b)*(-a + (a - b)*Sin[c + d*x]^2)))/(2*a^(3/2)*
d)
```

Maple [A] time = 0.074, size = 102, normalized size = 1.1

$$\frac{1}{d}\left(\frac{\sin(dx+c)\,b}{2\,a\,(a-b)\left(a(\sin(dx+c))^2-b(\sin(dx+c))^2-a\right)} + \frac{2\,a-b}{2\,a\,(a-b)}\text{Artanh}\left((a-b)\sin(dx+c)\frac{1}{\sqrt{a\,(a-b)}}\right)\frac{1}{\sqrt{a\,(a-b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(1/2*b/a/(a-b)*sin(d*x+c)/(a*sin(d*x+c)^2-b*sin(d*x+c)^2-a)+1/2*(2*a-b)
/a/(a-b)/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62324, size = 738, normalized size = 7.85

$$\left[\frac{\left(\left(2\,a^{2}-3\,ab+b^{2}\right)\cos\left(dx+c\right)^{2}+2\,ab-b^{2}\right)\sqrt{a^{2}-ab}\log\left(-\frac{(a-b)\cos(dx+c)^{2}-2\,\sqrt{a^{2}-ab}\sin(dx+c)-2\,a+b}{(a-b)\cos(dx+c)^{2}+b}\right)-2\left(a^{2}b-ab^{2}\right)\sin\left(dx\right.}{4\left(\left(a^{5}-3\,a^{4}b+3\,a^{3}b^{2}-a^{2}b^{3}\right)d\cos\left(dx+c\right)^{2}+\left(a^{4}b-2\,a^{3}b^{2}+a^{2}b^{3}\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a^2 - 3*a*b + b^2)*cos(d*x + c)^2 + 2*a*b - b^2)*sqrt(a^2 - a*b)*
log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*b)*sin(d*x + c) - 2*a + b)/((
a - b)*cos(d*x + c)^2 + b)) - 2*(a^2*b - a*b^2)*sin(d*x + c))/((a^5 - 3*a^4
*b + 3*a^3*b^2 - a^2*b^3)*d*cos(d*x + c)^2 + (a^4*b - 2*a^3*b^2 + a^2*b^3)*
d), -1/2*(((2*a^2 - 3*a*b + b^2)*cos(d*x + c)^2 + 2*a*b - b^2)*sqrt(-a^2 +
a*b)*arctan(sqrt(-a^2 + a*b)*sin(d*x + c)/a) + (a^2*b - a*b^2)*sin(d*x + c)
)/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cos(d*x + c)^2 + (a^4*b - 2*a^3*
b^2 + a^2*b^3)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(c+dx\right)}{\left(a+b\tan^{2}\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*tan(c + d*x)**2)**2, x)
```

Giac [A] time = 1.78525, size = 151, normalized size = 1.61

$$-\frac{\frac{(2\,a-b)\arctan\left(\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^{2}+ab}}\right)}{\left(a^{2}-ab\right)\sqrt{-a^{2}+ab}}-\frac{b\sin(dx+c)}{\left(a\sin(dx+c)^{2}-b\sin(dx+c)^{2}-a\right)\left(a^{2}-ab\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((2*a - b)*arctan((a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/
((a^2 - a*b)*sqrt(-a^2 + a*b)) - b*sin(d*x + c)/((a*sin(d*x + c)^2 - b*sin(
d*x + c)^2 - a)*(a^2 - a*b)))/d
```

3.466 $\int \frac{\cos(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2} dx$

Optimal. Leaf size=114

$$-\frac{b(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{5/2}}+\frac{b^2\sin(c+dx)}{2ad(a-b)^2\left(a-(a-b)\sin^2(c+dx)\right)}+\frac{\sin(c+dx)}{d(a-b)^2}$$

```
[Out] -((4*a - b)*b*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a -
b)^(5/2)*d) + Sin[c + d*x]/((a - b)^2*d) + (b^2*Sin[c + d*x])/(2*a*(a - b)^
2*d*(a - (a - b)*Sin[c + d*x]^2))
```

Rubi [A] time = 0.181111, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3676, 390, 385, 208}

$$-\frac{b(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{5/2}}+\frac{b^2\sin(c+dx)}{2ad(a-b)^2\left(a-(a-b)\sin^2(c+dx)\right)}+\frac{\sin(c+dx)}{d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] -((4*a - b)*b*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a -
b)^(5/2)*d) + Sin[c + d*x]/((a - b)^2*d) + (b^2*Sin[c + d*x])/(2*a*(a - b)^
2*d*(a - (a - b)*Sin[c + d*x]^2))
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a-(a-b)x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a-b)^2} - \frac{(2a-b)b-2(a-b)bx^2}{(a-b)^2\left(a+(-a+b)x^2\right)^2}\right)dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\sin(c+dx)}{(a-b)^2d} - \frac{\text{Subst}\left(\int \frac{(2a-b)b-2(a-b)bx^2}{\left(a+(-a+b)x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{(a-b)^2d}$$
$$= \frac{\sin(c+dx)}{(a-b)^2d} + \frac{b^2\sin(c+dx)}{2a(a-b)^2d\left(a-(a-b)\sin^2(c+dx)\right)} - \frac{((4a-b)b)\,\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x\right.}{2a(a-b)^2d}$$
$$= -\frac{(4a-b)b\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{5/2}d} + \frac{\sin(c+dx)}{(a-b)^2d} + \frac{b^2\sin(c+dx)}{2a(a-b)^2d\left(a-(a-b)\sin^2(c+dx)\right)}$$

Mathematica [A] time = 0.52163, size = 119, normalized size = 1.04

$$\frac{-\frac{\sqrt{a}\sin(c+dx)\left(a^2+a(a-b)\cos(2(c+dx))+ab+b^2\right)}{(a-b)^2\left((a-b)\sin^2(c+dx)-a\right)} - \frac{b(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{(a-b)^{5/2}}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (-(((4*a - b)*b*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(a - b)^(5/2))
 - (Sqrt[a]*(a^2 + a*b + b^2 + a*(a - b)*Cos[2*(c + d*x)])*Sin[c + d*x])/((
a - b)^2*(-a + (a - b)*Sin[c + d*x]^2)))/(2*a^(3/2)*d)
```

Maple [A] time = 0.085, size = 118, normalized size = 1.

$$\frac{1}{d}\left(\frac{\sin(dx+c)}{a^2-2ab+b^2} + \frac{b}{(a-b)^2}\left(-\frac{\sin(dx+c)\,b}{2a\left(a(\sin(dx+c))^2-b(\sin(dx+c))^2-a\right)} - \frac{4a-b}{2a}\text{Artanh}\left((a-b)\sin(dx+c)\frac{1}{\sqrt{a(a-}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(1/(a^2-2*a*b+b^2)*sin(d*x+c)+b/(a-b)^2*(-1/2*b/a*sin(d*x+c)/(a*sin(d*x
+c)^2-b*sin(d*x+c)^2-a)-1/2*(4*a-b)/a/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x
+c)/(a*(a-b))^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.8103, size = 973, normalized size = 8.54

$$\left[-\frac{\left(4\,ab^2-b^3+\left(4\,a^2b-5\,ab^2+b^3\right)\cos\left(dx+c\right)^2\right)\sqrt{a^2-ab}\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\sin(dx+c)-2\,a+b}{(a-b)\cos(dx+c)^2+b}\right)-2\left(2\,a^3b-a^2b\right.}{4\left(\left(a^6-4\,a^5b+6\,a^4b^2-4\,a^3b^3+a^2b^4\right)d\cos\left(dx+c\right)^2+\left(a^5b-3\,a^4b^2+3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((4*a*b^2 - b^3 + (4*a^2*b - 5*a*b^2 + b^3)*cos(d*x + c)^2)*sqrt(a^2
- a*b)*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*b)*sin(d*x + c) - 2*a
+ b)/((a - b)*cos(d*x + c)^2 + b)) - 2*(2*a^3*b - a^2*b^2 - a*b^3 + 2*(a^4
- 2*a^3*b + a^2*b^2)*cos(d*x + c)^2)*sin(d*x + c))/((a^6 - 4*a^5*b + 6*a^4*
b^2 - 4*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^2 + (a^5*b - 3*a^4*b^2 + 3*a^3*b^
3 - a^2*b^4)*d), 1/2*((4*a*b^2 - b^3 + (4*a^2*b - 5*a*b^2 + b^3)*cos(d*x +
c)^2)*sqrt(-a^2 + a*b)*arctan(sqrt(-a^2 + a*b)*sin(d*x + c)/a) + (2*a^3*b -
 a^2*b^2 - a*b^3 + 2*(a^4 - 2*a^3*b + a^2*b^2)*cos(d*x + c)^2)*sin(d*x + c)
)/((a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^2 + (a^
5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.72388, size = 205, normalized size = 1.8

$$-\frac{\frac{b^2\sin(dx+c)}{\left(a^3-2\,a^2b+ab^2\right)\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)}+\frac{\left(4\,ab-b^2\right)\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\left(a^3-2\,a^2b+ab^2\right)\sqrt{-a^2+ab}}-\frac{2\,\sin(dx+c)}{a^2-2\,ab+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*sin(d*x + c)/((a^3 - 2*a^2*b + a*b^2)*(a*sin(d*x + c)^2 - b*sin(d
*x + c)^2 - a)) + (4*a*b - b^2)*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/s
qrt(-a^2 + a*b))/((a^3 - 2*a^2*b + a*b^2)*sqrt(-a^2 + a*b)) - 2*sin(d*x + c
)/(a^2 - 2*a*b + b^2))/d
```

3.467 $\int \frac{\cos^3(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=143

$$\frac{b^2(6a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{7/2}}-\frac{b^3\sin(c+dx)}{2ad(a-b)^3\left(a-(a-b)\sin^2(c+dx)\right)}-\frac{\sin^3(c+dx)}{3d(a-b)^2}+\frac{(a-3b)\sin(c+dx)}{d(a-b)^3}$$

```
[Out] ((6*a - b)*b^2*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a -
 b)^(7/2)*d) + ((a - 3*b)*Sin[c + d*x])/((a - b)^3*d) - Sin[c + d*x]^3/(3*(
a - b)^2*d) - (b^3*Sin[c + d*x])/(2*a*(a - b)^3*d*(a - (a - b)*Sin[c + d*x]
^2))
```

Rubi [A] time = 0.212886, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3676, 390, 385, 208}

$$\frac{b^2(6a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{7/2}}-\frac{b^3\sin(c+dx)}{2ad(a-b)^3\left(a-(a-b)\sin^2(c+dx)\right)}-\frac{\sin^3(c+dx)}{3d(a-b)^2}+\frac{(a-3b)\sin(c+dx)}{d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((6*a - b)*b^2*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a -
 b)^(7/2)*d) + ((a - 3*b)*Sin[c + d*x])/((a - b)^3*d) - Sin[c + d*x]^3/(3*(
a - b)^2*d) - (b^3*Sin[c + d*x])/(2*a*(a - b)^3*d*(a - (a - b)*Sin[c + d*x]
^2))
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a-(a-b)x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a-3b}{(a-b)^3}-\frac{x^2}{(a-b)^2}+\frac{(3a-b)b^2-3(a-b)b^2x^2}{(a-b)^3\left(a+(-a+b)x^2\right)^2}\right)dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{(a-3b)\sin(c+dx)}{(a-b)^3d}-\frac{\sin^3(c+dx)}{3(a-b)^2d}+\frac{\operatorname{Subst}\left(\int \frac{(3a-b)b^2-3(a-b)b^2x^2}{\left(a+(-a+b)x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{(a-b)^3d}$$

$$= \frac{(a-3b)\sin(c+dx)}{(a-b)^3d}-\frac{\sin^3(c+dx)}{3(a-b)^2d}-\frac{b^3\sin(c+dx)}{2a(a-b)^3d\left(a-(a-b)\sin^2(c+dx)\right)}+\frac{\left((6a-b)\right.}{}$$

$$= \frac{(6a-b)b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{7/2}d}+\frac{(a-3b)\sin(c+dx)}{(a-b)^3d}-\frac{\sin^3(c+dx)}{3(a-b)^2d}-\frac{b}{2a(a-b)^3d\left(\right.}$$

Mathematica [A] time = 1.5229, size = 147, normalized size = 1.03

$$\frac{\frac{3b^2(b-6a)\left(\log\left(\sqrt{a}-\sqrt{a-b}\sin(c+dx)\right)-\log\left(\sqrt{a-b}\sin(c+dx)+\sqrt{a}\right)\right)}{a^{3/2}(a-b)^{7/2}}+\frac{3\sin(c+dx)\left(-\frac{4b^3}{a((a-b)\cos(2(c+dx))+a+b)}+3a-11b\right)}{(a-b)^3}+\frac{\sin(3(c+dx))}{(a-b)^2}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((3*b^2*(-6*a + b)*(Log[Sqrt[a] - Sqrt[a - b]*Sin[c + d*x]] - Log[Sqrt[a] +
 Sqrt[a - b]*Sin[c + d*x]]))/(a^(3/2)*(a - b)^(7/2)) + (3*(3*a - 11*b - (4*
b^3)/(a*(a + b + (a - b)*Cos[2*(c + d*x)])))*Sin[c + d*x])/(a - b)^3 + Sin[
3*(c + d*x)]/(a - b)^2)/(12*d)
```

Maple [A] time = 0.092, size = 164, normalized size = 1.2

$$\frac{1}{d}\left(-\frac{1}{\left(a^2-2\,ab+b^2\right)(a-b)}\left(\frac{(\sin(dx+c))^3 a}{3}-\frac{b(\sin(dx+c))^3}{3}-\sin(dx+c)\,a+3\,\sin(dx+c)\,b\right)-\frac{b^2}{(a-b)^3}\left(-\frac{}{2\,a\,(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(-1/(a^2-2*a*b+b^2)/(a-b)*(1/3*sin(d*x+c)^3*a-1/3*b*sin(d*x+c)^3-sin(d*
x+c)*a+3*sin(d*x+c)*b)-b^2/(a-b)^3*(-1/2*b/a*sin(d*x+c)/(a*sin(d*x+c)^2-b*s
in(d*x+c)^2-a)-1/2*(6*a-b)/a/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a
-b))^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.08064, size = 1307, normalized size = 9.14

$$\left[\frac{3\left(6\,ab^3 - b^4 + \left(6\,a^2b^2 - 7\,ab^3 + b^4\right)\cos\left(dx + c\right)^2\right)\sqrt{a^2 - ab}\log\left(-\frac{(a-b)\cos(dx+c)^2 - 2\sqrt{a^2-ab}\sin(dx+c) - 2\,a+b}{(a-b)\cos(dx+c)^2+b}\right) + 2\left(4\,a^4b - 20\,a^3\right.}{12\left(\left(a^7 - 5\,a^6b + 10\,a^5b^2 - 10\,a^4b^3 + 5\,a^3b^4 - a^2b^5\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(6*a*b^3 - b^4 + (6*a^2*b^2 - 7*a*b^3 + b^4)*cos(d*x + c)^2)*sqrt(
a^2 - a*b)*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*b)*sin(d*x + c) -
2*a + b)/((a - b)*cos(d*x + c)^2 + b)) + 2*(4*a^4*b - 20*a^3*b^2 + 13*a^2*b
^3 + 3*a*b^4 + 2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*cos(d*x + c)^4 + 2*(
2*a^5 - 11*a^4*b + 16*a^3*b^2 - 7*a^2*b^3)*cos(d*x + c)^2)*sin(d*x + c))/((
a^7 - 5*a^6*b + 10*a^5*b^2 - 10*a^4*b^3 + 5*a^3*b^4 - a^2*b^5)*d*cos(d*x +
c)^2 + (a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d), -1/6*(3*(6
*a*b^3 - b^4 + (6*a^2*b^2 - 7*a*b^3 + b^4)*cos(d*x + c)^2)*sqrt(-a^2 + a*b)
*arctan(sqrt(-a^2 + a*b)*sin(d*x + c)/a) - (4*a^4*b - 20*a^3*b^2 + 13*a^2*b
^3 + 3*a*b^4 + 2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*cos(d*x + c)^4 + 2*(
2*a^5 - 11*a^4*b + 16*a^3*b^2 - 7*a^2*b^3)*cos(d*x + c)^2)*sin(d*x + c))/((
a^7 - 5*a^6*b + 10*a^5*b^2 - 10*a^4*b^3 + 5*a^3*b^4 - a^2*b^5)*d*cos(d*x +
c)^2 + (a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.75014, size = 444, normalized size = 3.1

$$\frac{\frac{3\,b^3\sin(dx+c)}{\left(a^4-3\,a^3b+3\,a^2b^2-ab^3\right)\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)} + \frac{3\left(6\,ab^2-b^3\right)\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\left(a^4-3\,a^3b+3\,a^2b^2-ab^3\right)\sqrt{-a^2+ab}} - \frac{2\left(a^4\sin(dx+c)^3-4\,a^3b\sin(dx+c)^3+6\,a^2b^2\sin(dx+c)\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*b^3*sin(d*x + c)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*(a*sin(d*x + c
)^2 - b*sin(d*x + c)^2 - a)) + 3*(6*a*b^2 - b^3)*arctan(-(a*sin(d*x + c) -
b*sin(d*x + c))/sqrt(-a^2 + a*b))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*sqrt
(-a^2 + a*b)) - 2*(a^4*sin(d*x + c)^3 - 4*a^3*b*sin(d*x + c)^3 + 6*a^2*b^2*
sin(d*x + c)^3 - 4*a*b^3*sin(d*x + c)^3 + b^4*sin(d*x + c)^3 - 3*a^4*sin(d*
x + c) + 18*a^3*b*sin(d*x + c) - 36*a^2*b^2*sin(d*x + c) + 30*a*b^3*sin(d*x
 + c) - 9*b^4*sin(d*x + c))/(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a
^2*b^4 - 6*a*b^5 + b^6))/d
```

3.468 $\int \frac{\sec^8(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=127

$$\frac{(5a+b)(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}d}-\frac{(a-b)^3\tan(c+dx)}{2ab^3d\left(a+b\tan^2(c+dx)\right)}-\frac{(2a-3b)\tan(c+dx)}{b^3d}+\frac{\tan^3(c+dx)}{3b^2d}$$

```
[Out] ((a - b)^2*(5*a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(
7/2)*d) - ((2*a - 3*b)*Tan[c + d*x])/(b^3*d) + Tan[c + d*x]^3/(3*b^2*d) - (
(a - b)^3*Tan[c + d*x])/(2*a*b^3*d*(a + b*Tan[c + d*x]^2))
```

Rubi [A] time = 0.141559, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3675, 390, 385, 205}

$$\frac{(5a+b)(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}d}-\frac{(a-b)^3\tan(c+dx)}{2ab^3d\left(a+b\tan^2(c+dx)\right)}-\frac{(2a-3b)\tan(c+dx)}{b^3d}+\frac{\tan^3(c+dx)}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((a - b)^2*(5*a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(
7/2)*d) - ((2*a - 3*b)*Tan[c + d*x])/(b^3*d) + Tan[c + d*x]^3/(3*b^2*d) - (
(a - b)^3*Tan[c + d*x])/(2*a*b^3*d*(a + b*Tan[c + d*x]^2))
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^8(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{\left(a+bx^2\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{2a-3b}{b^3}+\frac{x^2}{b^2}+\frac{(a-b)^2(2a+b)+3(a-b)^2bx^2}{b^3\left(a+bx^2\right)^2}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{(2a-3b)\tan(c+dx)}{b^3d}+\frac{\tan^3(c+dx)}{3b^2d}+\frac{\text{Subst}\left(\int \frac{(a-b)^2(2a+b)+3(a-b)^2bx^2}{\left(a+bx^2\right)^2}\,dx, x, \tan(c+\right.}{b^3d}$$

$$= -\frac{(2a-3b)\tan(c+dx)}{b^3d}+\frac{\tan^3(c+dx)}{3b^2d}-\frac{(a-b)^3\tan(c+dx)}{2ab^3d\left(a+b\tan^2(c+dx)\right)}+\frac{\left((a-b)^2(5a+\right.}{}$$

$$= \frac{(a-b)^2(5a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}d}-\frac{(2a-3b)\tan(c+dx)}{b^3d}+\frac{\tan^3(c+dx)}{3b^2d}-\frac{(a-}{2ab^3d\,(}$$

Mathematica [A] time = 0.732435, size = 135, normalized size = 1.06

$$\frac{\frac{3(a-b)^2(5a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}+4\sqrt{b}(4b-3a)\tan(c+dx)+\frac{3\sqrt{b}(b-a)^3\sin(2(c+dx))}{a((a-b)\cos(2(c+dx))+a+b)}+2b^{3/2}\tan(c+dx)\sec^2(c+dx)}{6b^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((3*(a - b)^2*(5*a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/a^(3/2) + (
3*Sqrt[b]*(-a + b)^3*Sin[2*(c + d*x)])/(a*(a + b + (a - b)*Cos[2*(c + d*x)]
)) + 4*Sqrt[b]*(-3*a + 4*b)*Tan[c + d*x] + 2*b^(3/2)*Sec[c + d*x]^2*Tan[c +
 d*x])/(6*b^(7/2)*d)
```

Maple [B] time = 0.093, size = 275, normalized size = 2.2

$$\frac{(\tan(dx+c))^3}{3b^2d}-2\frac{a\tan(dx+c)}{db^3}+3\frac{\tan(dx+c)}{b^2d}-\frac{a^2\tan(dx+c)}{2db^3\left(a+b(\tan(dx+c))^2\right)}+\frac{3a\tan(dx+c)}{2b^2d\left(a+b(\tan(dx+c))^2\right)}-\frac{}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/3*tan(d*x+c)^3/b^2/d-2/d/b^3*a*tan(d*x+c)+3*tan(d*x+c)/b^2/d-1/2/d/b^3*a^
2*tan(d*x+c)/(a+b*tan(d*x+c)^2)+3/2/d/b^2*a*tan(d*x+c)/(a+b*tan(d*x+c)^2)-3
/2/d/b*tan(d*x+c)/(a+b*tan(d*x+c)^2)+1/2*tan(d*x+c)/a/d/(a+b*tan(d*x+c)^2)+
5/2/d/b^3*a^2/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))-9/2/d/b^2*a/(a*b
)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))+3/2/d/b/(a*b)^(1/2)*arctan(b*tan(d
```

```
*x+c)/(a*b)^(1/2))+1/2/d/a/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.85338, size = 1349, normalized size = 10.62

$$\left[-\frac{3\left(\left(5\,a^4-14\,a^3b+12\,a^2b^2-2\,ab^3-b^4\right)\cos\left(dx+c\right)^5+\left(5\,a^3b-9\,a^2b^2+3\,ab^3+b^4\right)\cos\left(dx+c\right)^3\right)\sqrt{-ab}\log\left(\frac{\left(a^2+6\,ab+b\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((5*a^4 - 14*a^3*b + 12*a^2*b^2 - 2*a*b^3 - b^4)*cos(d*x + c)^5 +
 (5*a^3*b - 9*a^2*b^2 + 3*a*b^3 + b^4)*cos(d*x + c)^3)*sqrt(-a*b)*log(((a^2
 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 + 4*((a + b
)*cos(d*x + c)^3 - b*cos(d*x + c))*sqrt(-a*b)*sin(d*x + c) + b^2)/((a^2 - 2
*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 + b^2)) - 4*(2*a^
2*b^3 - (15*a^4*b - 37*a^3*b^2 + 25*a^2*b^3 - 3*a*b^4)*cos(d*x + c)^4 - 2*(
5*a^3*b^2 - 7*a^2*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^2*b^5*d*cos(d*x + c
)^3 + (a^3*b^4 - a^2*b^5)*d*cos(d*x + c)^5), -1/12*(3*((5*a^4 - 14*a^3*b +
12*a^2*b^2 - 2*a*b^3 - b^4)*cos(d*x + c)^5 + (5*a^3*b - 9*a^2*b^2 + 3*a*b^3
 + b^4)*cos(d*x + c)^3)*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x + c)^2 - b)*s
qrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c))) - 2*(2*a^2*b^3 - (15*a^4*b - 37*a
^3*b^2 + 25*a^2*b^3 - 3*a*b^4)*cos(d*x + c)^4 - 2*(5*a^3*b^2 - 7*a^2*b^3)*c
os(d*x + c)^2)*sin(d*x + c))/(a^2*b^5*d*cos(d*x + c)^3 + (a^3*b^4 - a^2*b^5
)*d*cos(d*x + c)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.70501, size = 243, normalized size = 1.91

$$\frac{\frac{3\left(5a^3-9a^2b+3ab^2+b^3\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)}{\sqrt{ab}ab^3}-\frac{3\left(a^3\tan(dx+c)-3a^2b\tan(dx+c)+3ab^2\tan(dx+c)-b^3\tan(dx+c)\right)}{\left(b\tan(dx+c)^2+a\right)ab^3}+\frac{2\left(b^4\tan(dx+c\right.}{}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*(pi*floor((d*x + c)/pi + 1/2)*sgn(
b) + arctan(b*tan(d*x + c)/sqrt(a*b)))/(sqrt(a*b)*a*b^3) - 3*(a^3*tan(d*x +
 c) - 3*a^2*b*tan(d*x + c) + 3*a*b^2*tan(d*x + c) - b^3*tan(d*x + c))/((b*t
an(d*x + c)^2 + a)*a*b^3) + 2*(b^4*tan(d*x + c)^3 - 6*a*b^3*tan(d*x + c) +
9*b^4*tan(d*x + c))/b^6)/d
```

3.469 $\int \frac{\sec^6(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=104

$$-\frac{\left(3a^2-2ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d}+\frac{(a-b)^2\tan(c+dx)}{2ab^2d\left(a+b\tan^2(c+dx)\right)}+\frac{\tan(c+dx)}{b^2d}$$

```
[Out] -((3*a^2 - 2*a*b - b^2)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(3/2)*
b^(5/2)*d) + Tan[c + d*x]/(b^2*d) + ((a - b)^2*Tan[c + d*x])/(2*a*b^2*d*(a
+ b*Tan[c + d*x]^2))
```

Rubi [A] time = 0.135615, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3675, 390, 385, 205}

$$-\frac{\left(3a^2-2ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d}+\frac{(a-b)^2\tan(c+dx)}{2ab^2d\left(a+b\tan^2(c+dx)\right)}+\frac{\tan(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] -((3*a^2 - 2*a*b - b^2)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(3/2)*
b^(5/2)*d) + Tan[c + d*x]/(b^2*d) + ((a - b)^2*Tan[c + d*x])/(2*a*b^2*d*(a
+ b*Tan[c + d*x]^2))
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(a+bx^2)^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{b^2} - \frac{a^2-b^2+2(a-b)bx^2}{b^2(a+bx^2)^2}\right) dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\tan(c+dx)}{b^2d} - \frac{\text{Subst}\left(\int \frac{a^2-b^2+2(a-b)bx^2}{(a+bx^2)^2}\,dx, x, \tan(c+dx)\right)}{b^2d}$$
$$= \frac{\tan(c+dx)}{b^2d} + \frac{(a-b)^2\tan(c+dx)}{2ab^2d\left(a+b\tan^2(c+dx)\right)} - \frac{((a-b)(3a+b))\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan\right.}{2ab^2d}$$
$$= -\frac{(a-b)(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d} + \frac{\tan(c+dx)}{b^2d} + \frac{(a-b)^2\tan(c+dx)}{2ab^2d\left(a+b\tan^2(c+dx)\right)}$$

Mathematica [A] time = 0.620238, size = 104, normalized size = 1.

$$\frac{-\frac{(3a+b)(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\sqrt{b}(a-b)^2\sin(2(c+dx))}{a((a-b)\cos(2(c+dx))+a+b)} + 2\sqrt{b}\tan(c+dx)}{2b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (-(((a - b)*(3*a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/a^(3/2)) + ((
a - b)^2*Sqrt[b]*Sin[2*(c + d*x)])/(a*(a + b + (a - b)*Cos[2*(c + d*x)])) +
 2*Sqrt[b]*Tan[c + d*x])/(2*b^(5/2)*d)
```

Maple [A] time = 0.085, size = 181, normalized size = 1.7

$$\frac{\tan(dx+c)}{b^2d} + \frac{a\tan(dx+c)}{2b^2d\left(a+b(\tan(dx+c))^2\right)} - \frac{\tan(dx+c)}{db\left(a+b(\tan(dx+c))^2\right)} + \frac{\tan(dx+c)}{2ad\left(a+b(\tan(dx+c))^2\right)} - \frac{3a}{2b^2d}\text{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] tan(d*x+c)/b^2/d+1/2/d/b^2*a*tan(d*x+c)/(a+b*tan(d*x+c)^2)-1/d/b*tan(d*x+c)
/(a+b*tan(d*x+c)^2)+1/2*tan(d*x+c)/a/d/(a+b*tan(d*x+c)^2)-3/2/d/b^2*a/(a*b)
^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))+1/d/b/(a*b)^(1/2)*arctan(b*tan(d*x+
c)/(a*b)^(1/2))+1/2/d/a/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.72883, size = 1089, normalized size = 10.47

$$\left[\frac{\left(\left(3\,a^{3}-5\,a^{2}b+ab^{2}+b^{3}\right)\cos\left(dx+c\right)^{3}+\left(3\,a^{2}b-2\,ab^{2}-b^{3}\right)\cos\left(dx+c\right)\right)\sqrt{-ab}\log\left(\frac{\left(a^{2}+6\,ab+b^{2}\right)\cos(dx+c)^{4}-2\left(3\,ab+b^{2}\right)\cos(dx}{\left(a^{2}-2\,ab+b^{2}\right)\cos(}\right.}{8\left(a^{2}b^{4}d\cos\left(dx+c\right)+\left(a^{3}b^{3}-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(((3*a^3 - 5*a^2*b + a*b^2 + b^3)*cos(d*x + c)^3 + (3*a^2*b - 2*a*b^2
- b^3)*cos(d*x + c))*sqrt(-a*b)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2
*(3*a*b + b^2)*cos(d*x + c)^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x + c))
*sqrt(-a*b)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*
b - b^2)*cos(d*x + c)^2 + b^2)) + 4*(2*a^2*b^2 + (3*a^3*b - 4*a^2*b^2 + a*b
^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^2*b^4*d*cos(d*x + c) + (a^3*b^3 - a^2*
b^4)*d*cos(d*x + c)^3), 1/4*(((3*a^3 - 5*a^2*b + a*b^2 + b^3)*cos(d*x + c)^
3 + (3*a^2*b - 2*a*b^2 - b^3)*cos(d*x + c))*sqrt(a*b)*arctan(1/2*((a + b)*c
os(d*x + c)^2 - b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c))) + 2*(2*a^2*b^
2 + (3*a^3*b - 4*a^2*b^2 + a*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^2*b^4*d*
cos(d*x + c) + (a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.73124, size = 173, normalized size = 1.66

$$\frac{\frac{2\tan(dx+c)}{b^{2}}-\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)\left(3\,a^{2}-2\,ab-b^{2}\right)}{\sqrt{ab}ab^{2}}+\frac{a^{2}\tan(dx+c)-2\,ab\tan(dx+c)+b^{2}\tan(dx+c)}{\left(b\tan(dx+c)^{2}+a\right)ab^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*tan(d*x + c)/b^2 - (pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*t
an(d*x + c)/sqrt(a*b)))*(3*a^2 - 2*a*b - b^2)/(sqrt(a*b)*a*b^2) + (a^2*tan(
d*x + c) - 2*a*b*tan(d*x + c) + b^2*tan(d*x + c))/((b*tan(d*x + c)^2 + a)*a
*b^2))/d
```

3.470 $\int \frac{\sec^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=77

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d}-\frac{(a-b)\tan(c+dx)}{2abd\left(a+b\tan^2(c+dx)\right)}$$

```
[Out] ((a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(3/2)*d) - ((
a - b)*Tan[c + d*x])/(2*a*b*d*(a + b*Tan[c + d*x]^2))
```

Rubi [A] time = 0.0759769, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3675, 385, 205}

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d}-\frac{(a-b)\tan(c+dx)}{2abd\left(a+b\tan^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(3/2)*d) - ((
a - b)*Tan[c + d*x])/(2*a*b*d*(a + b*Tan[c + d*x]^2))
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\left(a+bx^2\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{(a-b)\tan(c+dx)}{2abd\left(a+b\tan^2(c+dx)\right)} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{2abd}$$

$$= \frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d} - \frac{(a-b)\tan(c+dx)}{2abd\left(a+b\tan^2(c+dx)\right)}$$

Mathematica [A] time = 0.290043, size = 83, normalized size = 1.08

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right) + \frac{\sqrt{a}\sqrt{b}(b-a)\sin(2(c+dx))}{(a-b)\cos(2(c+dx))+a+b}}{2a^{3/2}b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]] + (Sqrt[a]*Sqrt[b]*(-a + b)
*Sin[2*(c + d*x)])/(a + b + (a - b)*Cos[2*(c + d*x)]))/(2*a^(3/2)*b^(3/2)*d
)
```

Maple [A] time = 0.088, size = 112, normalized size = 1.5

$$-\frac{\tan(dx+c)}{2\,db\left(a+b(\tan(dx+c))^2\right)} + \frac{\tan(dx+c)}{2\,ad\left(a+b(\tan(dx+c))^2\right)} + \frac{1}{2\,db}\arctan\left(b\tan(dx+c)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{1}{2\,ad}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d/b*tan(d*x+c)/(a+b*tan(d*x+c)^2)+1/2*tan(d*x+c)/a/d/(a+b*tan(d*x+c)^2
)+1/2/d/b/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))+1/2/d/a/(a*b)^(1/2)*
arctan(b*tan(d*x+c)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.73774, size = 844, normalized size = 10.96

$$\left[-\frac{4\left(a^{2}b-ab^{2}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}+ab+b^{2}\right)\sqrt{-ab}\log\left(\frac{\left(a^{2}+6\,ab+b^{2}\right)\cos(dx+c)^{4}-2\left(3\,ab+b^{2}\right)\cos(}{\left(a^{2}-2\,ab+b^{2}\right)\cos}\right.}{8\left(a^{2}b^{3}d+\left(a^{3}b^{2}-a^{2}b^{3}\right)d\cos\left(dx+c\right)^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(a^2*b - a*b^2)*cos(d*x + c)*sin(d*x + c) + ((a^2 - b^2)*cos(d*x +
 c)^2 + a*b + b^2)*sqrt(-a*b)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(
3*a*b + b^2)*cos(d*x + c)^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x + c))*s
qrt(-a*b)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b
- b^2)*cos(d*x + c)^2 + b^2)))/(a^2*b^3*d + (a^3*b^2 - a^2*b^3)*d*cos(d*x +
 c)^2), -1/4*(2*(a^2*b - a*b^2)*cos(d*x + c)*sin(d*x + c) + ((a^2 - b^2)*co
s(d*x + c)^2 + a*b + b^2)*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x + c)^2 - b)
*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c))))/(a^2*b^3*d + (a^3*b^2 - a^2*b^
3)*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{4}\left(c+dx\right)}{\left(a+b\tan^{2}\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*tan(c + d*x)**2)**2, x)
```

Giac [A] time = 1.70943, size = 124, normalized size = 1.61

$$\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)(a+b)}{\sqrt{ab}ab}-\frac{a\tan(dx+c)-b\tan(dx+c)}{\left(b\tan(dx+c)^{2}+a\right)ab}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b)
))*(a + b)/(sqrt(a*b)*a*b) - (a*tan(d*x + c) - b*tan(d*x + c))/((b*tan(d*x
+ c)^2 + a)*a*b))/d
```

3.471 $\int \frac{\sec^2(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=66

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d}+\frac{\tan(c+dx)}{2ad\left(a+b\tan^2(c+dx)\right)}$$

```
[Out] ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]*d) + Tan[c + d*x]
/(2*a*d*(a + b*Tan[c + d*x]^2))
```

Rubi [A] time = 0.0619239, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3675, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d}+\frac{\tan(c+dx)}{2ad\left(a+b\tan^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]*d) + Tan[c + d*x]
/(2*a*d*(a + b*Tan[c + d*x]^2))
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\tan(c+dx)}{2ad\left(a+b\tan^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{2ad}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d} + \frac{\tan(c+dx)}{2ad\left(a+b\tan^2(c+dx)\right)}$$

Mathematica [A] time = 0.266365, size = 63, normalized size = 0.95

$$\frac{\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{\sqrt{a}\tan(c+dx)}{a+b\tan^2(c+dx)}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/Sqrt[b] + (Sqrt[a]*Tan[c + d*x])/(a
 + b*Tan[c + d*x]^2))/(2*a^(3/2)*d)
```

Maple [A] time = 0.083, size = 57, normalized size = 0.9

$$\frac{\tan(dx+c)}{2\,ad\left(a+b\left(\tan(dx+c)\right)^2\right)} + \frac{1}{2\,ad}\arctan\left(b\tan(dx+c)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/2*tan(d*x+c)/a/d/(a+b*tan(d*x+c)^2)+1/2/d/a/(a*b)^(1/2)*arctan(b*tan(d*x+
c)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.69403, size = 771, normalized size = 11.68

$$\left[\frac{4\,ab\cos(dx+c)\sin(dx+c)-\left((a-b)\cos(dx+c)^2+b\right)\sqrt{-ab}\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos(dx+c)^4-2\left(3\,ab+b^2\right)\cos(dx+c)^2+4\left((a+b)\cos(\right.}{\left(a^2-2\,ab+b^2\right)\cos(dx+c)^4+2\left(ab-b^2\right)c}\right.}{8\left(a^2b^2d+\left(a^3b-a^2b^2\right)d\cos(dx+c)^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*a*b*cos(d*x + c)*sin(d*x + c) - ((a - b)*cos(d*x + c)^2 + b)*sqrt(-
a*b)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)
^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x + c))*sqrt(-a*b)*sin(d*x + c) +
b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 + b
^2)))/(a^2*b^2*d + (a^3*b - a^2*b^2)*d*cos(d*x + c)^2), 1/4*(2*a*b*cos(d*x
+ c)*sin(d*x + c) - ((a - b)*cos(d*x + c)^2 + b)*sqrt(a*b)*arctan(1/2*((a +
 b)*cos(d*x + c)^2 - b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c))))/(a^2*b^
2*d + (a^3*b - a^2*b^2)*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*tan(c + d*x)**2)**2, x)
```

Giac [A] time = 1.73682, size = 95, normalized size = 1.44

$$\frac{\frac{\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}a}+\frac{\tan(dx+c)}{\left(b\tan(dx+c)^2+a\right)a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b)
))/(sqrt(a*b)*a) + tan(d*x + c)/((b*tan(d*x + c)^2 + a)*a))/d
```

3.472 $\int \frac{\cos^2(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=148

$$\frac{b^{3/2}(5a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^3}+\frac{b(a+b)\tan(c+dx)}{2ad(a-b)^2\left(a+b\tan^2(c+dx)\right)}+\frac{\sin(c+dx)\cos(c+dx)}{2d(a-b)\left(a+b\tan^2(c+dx)\right)}+\frac{x(a-5b)}{2(a-b)^3}$$

```
[Out] ((a - 5*b)*x)/(2*(a - b)^3) + ((5*a - b)*b^(3/2)*ArcTan[(Sqrt[b]*Tan[c + d*
x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^3*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*(a -
 b)*d*(a + b*Tan[c + d*x]^2)) + (b*(a + b)*Tan[c + d*x])/(2*a*(a - b)^2*d*(
a + b*Tan[c + d*x]^2))
```

Rubi [A] time = 0.186098, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3675, 414, 527, 522, 203, 205}

$$\frac{b^{3/2}(5a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^3}+\frac{b(a+b)\tan(c+dx)}{2ad(a-b)^2\left(a+b\tan^2(c+dx)\right)}+\frac{\sin(c+dx)\cos(c+dx)}{2d(a-b)\left(a+b\tan^2(c+dx)\right)}+\frac{x(a-5b)}{2(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((a - 5*b)*x)/(2*(a - b)^3) + ((5*a - b)*b^(3/2)*ArcTan[(Sqrt[b]*Tan[c + d*
x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^3*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*(a -
 b)*d*(a + b*Tan[c + d*x]^2)) + (b*(a + b)*Tan[c + d*x])/(2*a*(a - b)^2*d*(
a + b*Tan[c + d*x]^2))
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
```

```
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\left(a+bx^2\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d\left(a+b\tan^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-a+2b-3bx^2}{\left(1+x^2\right)\left(a+bx^2\right)^2}\,dx, x, \tan(c+dx)\right)}{2(a-b)d}$$

$$= \frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d\left(a+b\tan^2(c+dx)\right)} + \frac{b(a+b)\tan(c+dx)}{2a(a-b)^2d\left(a+b\tan^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-2\left(a^2-4ab\right.}{(1+x}\right.}{}$$

$$= \frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d\left(a+b\tan^2(c+dx)\right)} + \frac{b(a+b)\tan(c+dx)}{2a(a-b)^2d\left(a+b\tan^2(c+dx)\right)} + \frac{(a-5b)\,\text{Subst}\left(\int\right.}{2(}$$

$$= \frac{(a-5b)x}{2(a-b)^3} + \frac{(5a-b)b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^3d} + \frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d\left(a+b\tan^2(c+dx)\right)} + \frac{b}{2a(a-}$$

Mathematica [A] time = 1.14763, size = 116, normalized size = 0.78

$$\frac{-\frac{2b^{3/2}(b-5a)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{2b^2(a-b)\sin(2(c+dx))}{a((a-b)\cos(2(c+dx))+a+b)} + 2(a-5b)(c+dx) + (a-b)\sin(2(c+dx))}{4d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (2*(a - 5*b)*(c + d*x) - (2*b^(3/2)*(-5*a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x]
)/Sqrt[a]])/a^(3/2) + (a - b)*Sin[2*(c + d*x)] + (2*(a - b)*b^2*Sin[2*(c +
d*x)])/(a*(a + b + (a - b)*Cos[2*(c + d*x)])))/(4*(a - b)^3*d)
```

Maple [A] time = 0.084, size = 248, normalized size = 1.7

$$\frac{b^2\tan(dx+c)}{2d(a-b)^3\left(a+b(\tan(dx+c))^2\right)}-\frac{b^3\tan(dx+c)}{2d(a-b)^3a\left(a+b(\tan(dx+c))^2\right)}+\frac{5b^2}{2d(a-b)^3}\arctan\left(b\tan(dx+c)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d*b^2/(a-b)^3*tan(d*x+c)/(a+b*tan(d*x+c)^2)-1/2/d*b^3/(a-b)^3/a*tan(d*x
+c)/(a+b*tan(d*x+c)^2)+5/2/d*b^2/(a-b)^3/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a
*b)^(1/2))-1/2/d*b^3/(a-b)^3/a/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))
+1/2/d/(a-b)^3*tan(d*x+c)/(tan(d*x+c)^2+1)*a-1/2/d/(a-b)^3*tan(d*x+c)/(tan(
d*x+c)^2+1)*b+1/2/d/(a-b)^3*arctan(tan(d*x+c))*a-5/2/d/(a-b)^3*arctan(tan(d
*x+c))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.95393, size = 1370, normalized size = 9.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^3 - 6*a^2*b + 5*a*b^2)*d*x*cos(d*x + c)^2 + 4*(a^2*b - 5*a*b^2)*
d*x + (5*a*b^2 - b^3 + (5*a^2*b - 6*a*b^2 + b^3)*cos(d*x + c)^2)*sqrt(-b/a)
*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 -
 4*((a^2 + a*b)*cos(d*x + c)^3 - a*b*cos(d*x + c))*sqrt(-b/a)*sin(d*x + c)
+ b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 +
 b^2)) + 4*((a^3 - 2*a^2*b + a*b^2)*cos(d*x + c)^3 + (a^2*b - b^3)*cos(d*x
+ c))*sin(d*x + c))/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cos(
d*x + c)^2 + (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d), 1/4*(2*(a^3 - 6*a^
2*b + 5*a*b^2)*d*x*cos(d*x + c)^2 + 2*(a^2*b - 5*a*b^2)*d*x - (5*a*b^2 - b^
3 + (5*a^2*b - 6*a*b^2 + b^3)*cos(d*x + c)^2)*sqrt(b/a)*arctan(1/2*((a + b)
*cos(d*x + c)^2 - b)*sqrt(b/a)/(b*cos(d*x + c)*sin(d*x + c))) + 2*((a^3 - 2
*a^2*b + a*b^2)*cos(d*x + c)^3 + (a^2*b - b^3)*cos(d*x + c))*sin(d*x + c))/
((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cos(d*x + c)^2 + (a^4*b
- 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 147.814, size = 1083, normalized size = 7.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((a^5*b - 12*a^4*b^2 + 22*a^3*b^3 - 12*a^2*b^4 + a*b^5 - a*b*abs(-a^4
+ 3*a^3*b - 3*a^2*b^2 + a*b^3) - b^2*abs(-a^4 + 3*a^3*b - 3*a^2*b^2 + a*b^3
))*(pi*floor((d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(d*x + c)/sqrt((a^
4 - a^3*b - a^2*b^2 + a*b^3 + sqrt((a^4 - a^3*b - a^2*b^2 + a*b^3)^2 - 4*(a
^4 - 2*a^3*b + a^2*b^2)*(a^3*b - 2*a^2*b^2 + a*b^3)))/(a^3*b - 2*a^2*b^2 +
a*b^3))))/(a^4*abs(-a^4 + 3*a^3*b - 3*a^2*b^2 + a*b^3) - a^3*b*abs(-a^4 + 3
*a^3*b - 3*a^2*b^2 + a*b^3) - a^2*b^2*abs(-a^4 + 3*a^3*b - 3*a^2*b^2 + a*b^
3) + a*b^3*abs(-a^4 + 3*a^3*b - 3*a^2*b^2 + a*b^3) + (a^4 - 3*a^3*b + 3*a^2
*b^2 - a*b^3)^2) + (sqrt(a*b)*(a + b)*abs(-a^4 + 3*a^3*b - 3*a^2*b^2 + a*b^
3)*abs(b) + (a^5 - 12*a^4*b + 22*a^3*b^2 - 12*a^2*b^3 + a*b^4)*sqrt(a*b)*ab
s(b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(d*x + c)/sqrt(
(a^4 - a^3*b - a^2*b^2 + a*b^3 - sqrt((a^4 - a^3*b - a^2*b^2 + a*b^3)^2 - 4
*(a^4 - 2*a^3*b + a^2*b^2)*(a^3*b - 2*a^2*b^2 + a*b^3)))/(a^3*b - 2*a^2*b^2
 + a*b^3))))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)^2*b - (a^4*b - a^3*b^2 -
a^2*b^3 + a*b^4)*abs(-a^4 + 3*a^3*b - 3*a^2*b^2 + a*b^3)) - (a*b*tan(d*x +
c)^3 + b^2*tan(d*x + c)^3 + a^2*tan(d*x + c) + b^2*tan(d*x + c))/((b*tan(d*
x + c)^4 + a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)*(a^3 - 2*a^2*b + a*b^2)
))/d
```

3.473 $\int \frac{\cos^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=212

$$-\frac{b^{5/2}(7a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^4}+\frac{x\left(3a^2-14ab+35b^2\right)}{8(a-b)^4}+\frac{b(a-4b)(3a+b)\tan(c+dx)}{8ad(a-b)^3\left(a+b\tan^2(c+dx)\right)}+\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a-b)\left(a+b\tan^2(c+dx)\right)}$$

```
[Out] ((3*a^2 - 14*a*b + 35*b^2)*x)/(8*(a - b)^4) - ((7*a - b)*b^(5/2)*ArcTan[(Sq
rt[b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^4*d) + (3*(a - 3*b)*Cos[c
+ d*x]*Sin[c + d*x])/(8*(a - b)^2*d*(a + b*Tan[c + d*x]^2)) + (Cos[c + d*x]
^3*Sin[c + d*x])/(4*(a - b)*d*(a + b*Tan[c + d*x]^2)) + ((a - 4*b)*b*(3*a +
 b)*Tan[c + d*x])/(8*a*(a - b)^3*d*(a + b*Tan[c + d*x]^2))
```

Rubi [A] time = 0.301012, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3675, 414, 527, 522, 203, 205}

$$-\frac{b^{5/2}(7a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^4}+\frac{x\left(3a^2-14ab+35b^2\right)}{8(a-b)^4}+\frac{b(a-4b)(3a+b)\tan(c+dx)}{8ad(a-b)^3\left(a+b\tan^2(c+dx)\right)}+\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a-b)\left(a+b\tan^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((3*a^2 - 14*a*b + 35*b^2)*x)/(8*(a - b)^4) - ((7*a - b)*b^(5/2)*ArcTan[(Sq
rt[b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^4*d) + (3*(a - 3*b)*Cos[c
+ d*x]*Sin[c + d*x])/(8*(a - b)^2*d*(a + b*Tan[c + d*x]^2)) + (Cos[c + d*x]
^3*Sin[c + d*x])/(4*(a - b)*d*(a + b*Tan[c + d*x]^2)) + ((a - 4*b)*b*(3*a +
 b)*Tan[c + d*x])/(8*a*(a - b)^3*d*(a + b*Tan[c + d*x]^2))
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^3\left(a+bx^2\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d\left(a+b\tan^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-3a+4b-5bx^2}{\left(1+x^2\right)^2\left(a+bx^2\right)^2}\,dx, x, \tan(c+dx)\right)}{4(a-b)d}$$

$$= \frac{3(a-3b)\cos(c+dx)\sin(c+dx)}{8(a-b)^2d\left(a+b\tan^2(c+dx)\right)} + \frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d\left(a+b\tan^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{3a^2-5ab+}{(1+x}\right.}{}$$

$$= \frac{3(a-3b)\cos(c+dx)\sin(c+dx)}{8(a-b)^2d\left(a+b\tan^2(c+dx)\right)} + \frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d\left(a+b\tan^2(c+dx)\right)} + \frac{(a-4b)b(3a+b}{8a(a-b)^3d\left(a+b\right.}$$

$$= \frac{3(a-3b)\cos(c+dx)\sin(c+dx)}{8(a-b)^2d\left(a+b\tan^2(c+dx)\right)} + \frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d\left(a+b\tan^2(c+dx)\right)} + \frac{(a-4b)b(3a+b}{8a(a-b)^3d\left(a+b\right.}$$

$$= \frac{\left(3a^2-14ab+35b^2\right)x}{8(a-b)^4} - \frac{(7a-b)b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^4d} + \frac{3(a-3b)\cos(c+dx)\sin(c}{8(a-b)^2d\left(a+b\tan^2(c+\right.}$$

Mathematica [A] time = 2.0214, size = 148, normalized size = 0.7

$$\frac{4\left(3a^2-14ab+35b^2\right)(c+dx) + \frac{16b^{5/2}(b-7a)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} - \frac{16b^3(a-b)\sin(2(c+dx))}{a((a-b)\cos(2(c+dx))+a+b)} + 8(a-3b)(a-b)\sin(2(c+dx)) +}{32d(a-b)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (4*(3*a^2 - 14*a*b + 35*b^2)*(c + d*x) + (16*b^(5/2)*(-7*a + b)*ArcTan[(Sqr
t[b]*Tan[c + d*x])/Sqrt[a]])/a^(3/2) + 8*(a - 3*b)*(a - b)*Sin[2*(c + d*x)]
 - (16*(a - b)*b^3*Sin[2*(c + d*x)])/(a*(a + b + (a - b)*Cos[2*(c + d*x)]))
 + (a - b)^2*Sin[4*(c + d*x)])/(32*(a - b)^4*d)
```

Maple [B] time = 0.088, size = 413, normalized size = 2.

$$-\frac{b^3\tan(dx+c)}{2d(a-b)^4\left(a+b(\tan(dx+c))^2\right)}+\frac{b^4\tan(dx+c)}{2d(a-b)^4a\left(a+b(\tan(dx+c))^2\right)}-\frac{7b^3}{2d(a-b)^4}\arctan\left(b\tan(dx+c)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d*b^3/(a-b)^4*tan(d*x+c)/(a+b*tan(d*x+c)^2)+1/2/d*b^4/(a-b)^4/a*tan(d*
x+c)/(a+b*tan(d*x+c)^2)-7/2/d*b^3/(a-b)^4/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(
a*b)^(1/2))+1/2/d*b^4/(a-b)^4/a/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2)
)+3/8/d/(a-b)^4/(tan(d*x+c)^2+1)^2*tan(d*x+c)^3*a^2-7/4/d/(a-b)^4/(tan(d*x+
c)^2+1)^2*tan(d*x+c)^3*a*b+11/8/d/(a-b)^4/(tan(d*x+c)^2+1)^2*tan(d*x+c)^3*b
^2-9/4/d/(a-b)^4/(tan(d*x+c)^2+1)^2*tan(d*x+c)*a*b+13/8/d/(a-b)^4/(tan(d*x+
c)^2+1)^2*tan(d*x+c)*b^2+5/8/d/(a-b)^4/(tan(d*x+c)^2+1)^2*tan(d*x+c)*a^2+35
/8/d/(a-b)^4*arctan(tan(d*x+c))*b^2+3/8/d/(a-b)^4*arctan(tan(d*x+c))*a^2-7/
4/d/(a-b)^4*arctan(tan(d*x+c))*a*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.14692, size = 1786, normalized size = 8.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*((3*a^4 - 17*a^3*b + 49*a^2*b^2 - 35*a*b^3)*d*x*cos(d*x + c)^2 + (3*a^
3*b - 14*a^2*b^2 + 35*a*b^3)*d*x - (7*a*b^3 - b^4 + (7*a^2*b^2 - 8*a*b^3 +
b^4)*cos(d*x + c)^2)*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2
*(3*a*b + b^2)*cos(d*x + c)^2 - 4*((a^2 + a*b)*cos(d*x + c)^3 - a*b*cos(d*x
 + c))*sqrt(-b/a)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 +
 2*(a*b - b^2)*cos(d*x + c)^2 + b^2)) + (2*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b
^3)*cos(d*x + c)^5 + 3*(a^4 - 5*a^3*b + 7*a^2*b^2 - 3*a*b^3)*cos(d*x + c)^3
 + (3*a^3*b - 14*a^2*b^2 + 7*a*b^3 + 4*b^4)*cos(d*x + c))*sin(d*x + c))/((a
^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d*cos(d*x + c)^
```

```
2 + (a^5*b - 4*a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d), 1/8*((3*a^4 - 1
7*a^3*b + 49*a^2*b^2 - 35*a*b^3)*d*x*cos(d*x + c)^2 + (3*a^3*b - 14*a^2*b^2
 + 35*a*b^3)*d*x + 2*(7*a*b^3 - b^4 + (7*a^2*b^2 - 8*a*b^3 + b^4)*cos(d*x +
 c)^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(d*x + c)^2 - b)*sqrt(b/a)/(b*cos(d
*x + c)*sin(d*x + c))) + (2*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*cos(d*x + c
)^5 + 3*(a^4 - 5*a^3*b + 7*a^2*b^2 - 3*a*b^3)*cos(d*x + c)^3 + (3*a^3*b - 1
4*a^2*b^2 + 7*a*b^3 + 4*b^4)*cos(d*x + c))*sin(d*x + c))/((a^6 - 5*a^5*b +
10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d*cos(d*x + c)^2 + (a^5*b - 4*
a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.73081, size = 363, normalized size = 1.71

$$-\frac{\frac{4\,b^3\tan(dx+c)}{\left(a^4-3\,a^3b+3\,a^2b^2-ab^3\right)\left(b\tan(dx+c)^2+a\right)}-\frac{\left(3\,a^2-14\,ab+35\,b^2\right)(dx+c)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}+\frac{4\left(7\,ab^3-b^4\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)}{\left(a^5-4\,a^4b+6\,a^3b^2-4\,a^2b^3+ab^4\right)\sqrt{ab}}-\frac{3\,a\tan(dx+c)^3-11}{\left(a^3-3\right.}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^3*tan(d*x + c)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*(b*tan(d*x +
c)^2 + a)) - (3*a^2 - 14*a*b + 35*b^2)*(d*x + c)/(a^4 - 4*a^3*b + 6*a^2*b^2
 - 4*a*b^3 + b^4) + 4*(7*a*b^3 - b^4)*(pi*floor((d*x + c)/pi + 1/2)*sgn(b)
+ arctan(b*tan(d*x + c)/sqrt(a*b)))/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3
 + a*b^4)*sqrt(a*b)) - (3*a*tan(d*x + c)^3 - 11*b*tan(d*x + c)^3 + 5*a*tan(
d*x + c) - 13*b*tan(d*x + c))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(tan(d*x + c
)^2 + 1)^2))/d
```

3.474 $\int (d\sec(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=95

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p (d\sec(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(m+2p+1)}\text{Hypergeometric2F1}\left(\frac{1}{2}(2p+1),\frac{1}{2}(m+2p+1),\frac{1}{2}(\right.}{f(2p+1)}$$

```
[Out] ((Cos[e + f*x]^2)^((1 + m + 2*p)/2)*Hypergeometric2F1[(1 + 2*p)/2, (1 + m +
 2*p)/2, (3 + 2*p)/2, Sin[e + f*x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*x]*(b*Ta
n[e + f*x]^2)^p)/(f*(1 + 2*p))
```

Rubi [A] time = 0.0981185, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3658, 2617}

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p (d\sec(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(m+2p+1)} {}_2F_1\left(\frac{1}{2}(2p+1),\frac{1}{2}(m+2p+1);\frac{1}{2}(2p+3);\sin^2(e+f\right.}{f(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + m + 2*p)/2)*Hypergeometric2F1[(1 + 2*p)/2, (1 + m +
 2*p)/2, (3 + 2*p)/2, Sin[e + f*x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*x]*(b*Ta
n[e + f*x]^2)^p)/(f*(1 + 2*p))
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int (d\sec(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx &= \left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int (d\sec(e+fx))^m \tan^{2p}(e+fx)\, dx\\ &= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+m+2p)} {}_2F_1\left(\frac{1}{2}(1+2p),\frac{1}{2}(1+m+2p);\frac{1}{2}(3+2p);\sin^2(e+fx)\right)(}{f(1+2p)}\end{aligned}$$

Mathematica [A] time = 0.18626, size = 81, normalized size = 0.85

$$\frac{\cot(e+fx)\left(-\tan^2(e+fx)\right)^{\frac{1}{2}-p}\left(b\tan^2(e+fx)\right)^p(d\sec(e+fx))^m\text{Hypergeometric2F1}\left(\frac{m}{2},\frac{1}{2}-p,\frac{m+2}{2},\sec^2(e+fx)\right)}{fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Cot[e + f*x]*Hypergeometric2F1[m/2, 1/2 - p, (2 + m)/2, Sec[e + f*x]^2]*(d
*Sec[e + f*x])^m*(-Tan[e + f*x]^2)^(1/2 - p)*(b*Tan[e + f*x]^2)^p)/(f*m)
```

Maple [F] time = 0.808, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^m\left(b\left(\tan\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*sec(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2\right)^p\left(d\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*sec(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2\right)^p\left(d\sec\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2)^p*(d*sec(f*x + e))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan^2\left(e+fx\right)\right)^p\left(d\sec\left(e+fx\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**p*(d*sec(e + f*x))**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2\right)^p \left(d\sec\left(fx+e\right)\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*sec(f*x + e))^m, x)
```

3.475 $\int (d\sec(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=108

$$\frac{\tan(e+fx)\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1-\frac{m}{2},-p;\frac{3}{2};-\tan^2(e+\ldots\right.}{f}$$

```
[Out] (AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*
(d*Sec[e + f*x])^m*Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(Sec[e + f*x]^
2)^(m/2)*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.0936662, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3679, 430, 429}

$$\frac{\tan(e+fx)\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1-\frac{m}{2},-p;\frac{3}{2};-\tan^2(e+\ldots\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*
(d*Sec[e + f*x])^m*Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(Sec[e + f*x]^
2)^(m/2)*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 3679

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(f
f*(d*Sec[e + f*x])^m)/(f*(Sec[e + f*x]^2)^(m/2)), Subst[Int[(1 + ff^2*x^2)^
(m/2 - 1)*(a + b*ff^2*x^2)^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (d\sec(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\left((d\sec(e+fx))^m \sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int \left(1+x^2\right)^{-1+\frac{m}{2}}\left(a+bx^2\right)^p\,dx,\right.}{f}$$

$$= \frac{\left((d\sec(e+fx))^m \sec^2(e+fx)^{-m/2}\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-}\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};1-\frac{m}{2},-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)(d\sec(e+fx))^m\sec^2(e+\right.}{f}$$

Mathematica [B] time = 16.1627, size = 2033, normalized size = 18.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
a)]*(d*Sec[e + f*x])^m*(Sec[e + f*x]^2)^(-1 + m/2)*Tan[e + f*x]*(a + b*Tan[
e + f*x]^2)^(2*p))/(f*(3*a*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2,
 -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + a*(-2 + m)*AppellF1[3/2, 2 - m/2, -p
, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2)*((6*a*b*p
*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*
(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(-1 + p))/(3*a
*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]
+ (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f
*x]^2)/a)] + a*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) + (3*a*AppellF1[1/2, 1 - m/2, -p, 3
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*(Sec[e + f*x]^2)^(m/2)*(a + b
*Tan[e + f*x]^2)^p)/(3*a*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -
((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e
+ f*x]^2, -((b*Tan[e + f*x]^2)/a)] + a*(-2 + m)*AppellF1[3/2, 2 - m/2, -p,
5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) + (6*a*(-1
+ m/2)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/a)]*(Sec[e + f*x]^2)^(-1 + m/2)*Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p)/
(3*a*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
a)] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
 + f*x]^2)/a)] + a*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) + (3*a*(Sec[e + f*x]^2)^(-1 + m
/2)*Tan[e + f*x]*((2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*(1 - m/2
)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]
*Sec[e + f*x]^2*Tan[e + f*x])/3)*(a + b*Tan[e + f*x]^2)^p)/(3*a*AppellF1[1/
2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + (2*b*p*App
ellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] +
 a*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f
*x]^2)/a)])*Tan[e + f*x]^2) - (3*a*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/a)]*(Sec[e + f*x]^2)^(-1 + m/2)*Tan[e + f*x]*
(a + b*Tan[e + f*x]^2)^p*(2*(2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + a*(-2 + m)*AppellF1[3/2, 2 - m/2, -p
, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Sec[e + f*x]^2*Tan[e + f*
x] + 3*a*((2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*T
an[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*(1 - m/2)*Appell
F1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e +
```

```
f*x]^2*Tan[e + f*x])/3) + Tan[e + f*x]^2*(2*b*p*((-6*b*(1 - p)*AppellF1[5/
2, 1 - m/2, 2 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f
*x]^2*Tan[e + f*x])/(5*a) - (6*(1 - m/2)*AppellF1[5/2, 2 - m/2, 1 - p, 7/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/5)
+ a*(-2 + m)*((6*b*p*AppellF1[5/2, 2 - m/2, 1 - p, 7/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/(5*a) - (6*(2 - m/2)*Ap
pellF1[5/2, 3 - m/2, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec
[e + f*x]^2*Tan[e + f*x])/5))))/(3*a*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p,
5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + a*(-2 + m)*AppellF1[3/2, 2
 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2)
^2))
```

Maple [F] time = 0.923, size = 0, normalized size = 0.

$$\int \left(d \sec \left(fx+e\right)\right)^{m} \left(a+b \left(\tan \left(fx+e\right)\right)^{2}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan \left(fx+e\right)^{2}+a\right)^{p} \left(d \sec \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \tan \left(fx+e\right)^{2}+a\right)^{p} \left(d \sec \left(fx+e\right)\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p \left(d\sec\left(fx+e\right)\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

3.476 $\int (d\sec(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx$

Optimal. Leaf size=97

$$\frac{\tan(e+fx)(d\sec(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(m+np+1)} \left(b(c\tan(e+fx))^n\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2}(np+1), \frac{1}{2}(m+np+\ldots\right.}{f(np+1)}$$

```
[Out] ((Cos[e + f*x]^2)^((1 + m + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (1 + m +
 n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*x]*(b*(c
*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rubi [A] time = 0.107514, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {3659, 2617}

$$\frac{\tan(e+fx)(d\sec(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(m+np+1)} \left(b(c\tan(e+fx))^n\right)^p {}_2F_1\left(\frac{1}{2}(np+1), \frac{1}{2}(m+np+1); \frac{1}{2}(np+3); \sin^2\ldots\right.}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + m + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (1 + m +
 n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*x]*(b*(c
*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int (d\sec(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx &= \left((c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p\right) \int (d\sec(e+fx))^m (c\tan(e+f\ldots \\ &= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+m+np)} {}_2F_1\left(\frac{1}{2}(1+np), \frac{1}{2}(1+m+np); \frac{1}{2}(3+np); \sin^2(e+\ldots\right.}{f(1+np)}\end{aligned}$$

Mathematica [A] time = 0.156979, size = 89, normalized size = 0.92

$$\frac{\cot(e+fx)(d\sec(e+fx))^m\left(-\tan^2(e+fx)\right)^{\frac{1}{2}(1-np)}\left(b(c\tan(e+fx))^n\right)^p\text{Hypergeometric2F1}\left(\frac{m}{2},\frac{1}{2}(1-np),\frac{m+2}{2},\sec^2(\right.}{fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[e + f*x]*Hypergeometric2F1[m/2, (1 - n*p)/2, (2 + m)/2, Sec[e + f*x]^2
]*(d*Sec[e + f*x])^m*(-Tan[e + f*x]^2)^((1 - n*p)/2)*(b*(c*Tan[e + f*x])^n)
^p)/(f*m)
```

Maple [F] time = 0.267, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^m\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*sec(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\left(d\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*sec(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\left(d\sec\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*(d*sec(f*x + e))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^n\right)^p\left(d\sec\left(e+fx\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*(d*sec(e + f*x))**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c \tan \left(fx+e\right)\right)^{n} b\right)^{p} \left(d \sec \left(fx+e\right)\right)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*sec(f*x + e))^m, x)
```

3.477 $\int \sec^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=99

$$\frac{\tan^5(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+5)}+\frac{2\tan^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+3)}+\frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

```
[Out] (Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p)) + (2*Tan[e + f*x]^3*(
b*(c*Tan[e + f*x])^n)^p)/(f*(3 + n*p)) + (Tan[e + f*x]^5*(b*(c*Tan[e + f*x]
)^n)^p)/(f*(5 + n*p))
```

Rubi [A] time = 0.134914, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3659, 2607, 270}

$$\frac{\tan^5(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+5)}+\frac{2\tan^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+3)}+\frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p)) + (2*Tan[e + f*x]^3*(
b*(c*Tan[e + f*x])^n)^p)/(f*(3 + n*p)) + (Tan[e + f*x]^5*(b*(c*Tan[e + f*x]
)^n)^p)/(f*(5 + n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sec^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sec^6(e+fx)(c\tan(e+fx))^{np}\,d$$
$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int (cx)^{np}\left(1+x^2\right)^2\,dx,x,\text{ta}\right.}{f}$$
$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \left((cx)^{np}+\frac{2(cx)^{2+np}}{c^2}+\frac{(cx)^{\cdot}}{c}\right.\right.}{f}$$
$$= \frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)}+\frac{2\tan^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(3+np)}+\cdot$$

Mathematica [A] time = 2.14253, size = 122, normalized size = 1.23

$$\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p\left(\tan^2(e+fx)\sec^4(e+fx)\left(2(np+3)\cos(2(e+fx))+\cos(4(e+fx))+n^2p^2+6np\right.\right.}{f(np+1)(np+3)(np+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p*((8 + 6*n*p + n^2*p^2 + 2*(3 + n*p)*
Cos[2*(e + f*x)] + Cos[4*(e + f*x)])*Sec[e + f*x]^4*Tan[e + f*x]^2 + 8*(-Ta
n[e + f*x]^2)^((1 - n*p)/2)))/(f*(1 + n*p)*(3 + n*p)*(5 + n*p))
```

Maple [F] time = 0.532, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^6\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sec(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A] time = 1.25765, size = 143, normalized size = 1.44

$$\frac{\frac{b^p(c^n)^p\left(\tan\left(fx+e\right)^n\right)^p\tan\left(fx+e\right)^5}{np+5}+\frac{2\,b^p(c^n)^p\left(\tan\left(fx+e\right)^n\right)^p\tan\left(fx+e\right)^3}{np+3}+\frac{b^p(c^n)^p\left(\tan\left(fx+e\right)^n\right)^p\tan\left(fx+e\right)}{np+1}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] (b^p*(c^n)^p*(tan(f*x + e)^n)^p*tan(f*x + e)^5/(n*p + 5) + 2*b^p*(c^n)^p*(t
an(f*x + e)^n)^p*tan(f*x + e)^3/(n*p + 3) + b^p*(c^n)^p*(tan(f*x + e)^n)^p*
tan(f*x + e)/(n*p + 1))/f
```

Fricas [A] time = 1.49455, size = 266, normalized size = 2.69

$$\frac{\left(n^2p^2+8\,\cos\left(fx+e\right)^4+4\left(np+1\right)\cos\left(fx+e\right)^2+4\,np+3\right)e^{\left(np\log\left(\frac{c\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)+p\log\left(b\right)\right)}\sin\left(fx+e\right)}{\left(fn^3p^3+9\,fn^2p^2+23\,fnp+15\,f\right)\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] (n^2*p^2 + 8*cos(f*x + e)^4 + 4*(n*p + 1)*cos(f*x + e)^2 + 4*n*p + 3)*e^(n*
p*log(c*sin(f*x + e)/cos(f*x + e)) + p*log(b))*sin(f*x + e)/((f*n^3*p^3 + 9
*f*n^2*p^2 + 23*f*n*p + 15*f)*cos(f*x + e)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.478 $\int \sec^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=65

$$\frac{\tan^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+3)}+\frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

```
[Out] (Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p)) + (Tan[e + f*x]^3*(b*
(c*Tan[e + f*x])^n)^p)/(f*(3 + n*p))
```

Rubi [A] time = 0.111186, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3659, 2607, 14}

$$\frac{\tan^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+3)}+\frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```


```
[Out] (Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p)) + (Tan[e + f*x]^3*(b*
(c*Tan[e + f*x])^n)^p)/(f*(3 + n*p))
```


Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sec^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sec^4(e+fx)(c\tan(e+fx))^{np}\,dx$$

$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int (cx)^{np}\left(1+x^2\right)dx,x,\tan(e$$

$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \left((cx)^{np}+\frac{(cx)^{2+np}}{c^2}\right)dx,x,\text{ta}$$

$$= \frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)} + \frac{\tan^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(3+np)}$$

Mathematica [A] time = 2.09433, size = 87, normalized size = 1.34

$$\frac{\cot(e+fx)\left(2\left(-\tan^2(e+fx)\right)^{\frac{1}{2}(1-np)}+\tan^2(e+fx)\left((np+1)\sec^2(e+fx)+2\right)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)(np+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p*((2 + (1 + n*p)*Sec[e + f*x]^2)*Tan[
e + f*x]^2 + 2*(-Tan[e + f*x]^2)^((1 - n*p)/2)))/(f*(1 + n*p)*(3 + n*p))
```

Maple [F] time = 0.514, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^4\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sec(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A] time = 1.11682, size = 96, normalized size = 1.48

$$\frac{\frac{b^p(c^n)^p\left(\tan\left(fx+e\right)^n\right)^p\tan\left(fx+e\right)^3}{np+3}+\frac{b^p(c^n)^p\left(\tan\left(fx+e\right)^n\right)^p\tan\left(fx+e\right)}{np+1}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] (b^p*(c^n)^p*(tan(f*x + e)^n)^p*tan(f*x + e)^3/(n*p + 3) + b^p*(c^n)^p*(tan
(f*x + e)^n)^p*tan(f*x + e)/(n*p + 1))/f
```

Fricas [A] time = 1.41611, size = 189, normalized size = 2.91

$$\frac{\left(np + 2\cos\left(fx + e\right)^{2} + 1\right)e^{\left(np\log\left(\frac{c\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)+p\log\left(b\right)\right)}\sin\left(fx + e\right)}{\left(fn^{2}p^{2} + 4\,fnp + 3\,f\right)\cos\left(fx + e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] (n*p + 2*cos(f*x + e)^2 + 1)*e^(n*p*log(c*sin(f*x + e)/cos(f*x + e)) + p*lo
g(b))*sin(f*x + e)/((f*n^2*p^2 + 4*f*n*p + 3*f)*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.479 $\int \sec^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=31

$$\frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

```
[Out] (Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rubi [A] time = 0.0925313, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3659, 2607, 32}

$$\frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx &= \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sec^2(e+fx)(c\tan(e+fx))^{np}\,dx\\ &= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int (cx)^{np}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)}\end{aligned}$$

Mathematica [A] time = 0.0228926, size = 31, normalized size = 1.

$$\frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Maple [C] time = 2.749, size = 25070, normalized size = 808.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 1.03106, size = 47, normalized size = 1.52

$$\frac{b^p(c^n)^p\left(\tan\left(fx+e\right)^n\right)^p\tan\left(fx+e\right)}{\left(np+1\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] b^p*(c^n)^p*(tan(f*x + e)^n)^p*tan(f*x + e)/((n*p + 1)*f)
```

Fricas [A] time = 1.42176, size = 126, normalized size = 4.06

$$\frac{e^{\left(np\log\left(\frac{c\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)+p\log(b)\right)}\sin\left(fx+e\right)}{\left(fnp+f\right)\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] e^(n*p*log(c*sin(f*x + e)/cos(f*x + e)) + p*log(b))*sin(f*x + e)/((f*n*p +
f)*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\left(c\tan\left(e+fx\right)\right)^n\right)^p\sec^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*sec(e + f*x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.480 $\int \left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=61

$$\frac{\tan(e+fx)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+1),\frac{1}{2}(np+3),-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rubi [A] time = 0.042406, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{\tan(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{np}\,dx$$
$$= \frac{\left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \frac{x^{np}}{c^2+x^2}\,dx, x, c\tan(e+fx)\right)}{f}$$
$$= \frac{{}_2F_1\left(1, \frac{1}{2}(1+np); \frac{1}{2}(3+np); -\tan^2(e+fx)\right)\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)}$$

Mathematica [A] time = 0.0052363, size = 59, normalized size = 0.97

$$\frac{\tan(e+fx)\text{Hypergeometric2F1}\left(1, \frac{1}{2}(np+1), \frac{1}{2}(np+3), -\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f + f*n*p)
```

Maple [F] time = 0.003, size = 0, normalized size = 0.

$$\int \left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p, x)
```

3.481 $\int \cos^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=61

$$\frac{\tan(e+fx)\text{Hypergeometric2F1}\left(2,\frac{1}{2}(np+1),\frac{1}{2}(np+3),-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

```
[Out] (Hypergeometric2F1[2, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rubi [A] time = 0.103805, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3659, 2607, 364}

$$\frac{\tan(e+fx)\,{}_2F_1\left(2,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[2, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \cos^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \cos^2(e+fx)(c\tan(e+fx))^{np}\,d$$
$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \frac{(cx)^{np}}{(1+x^2)^2}dx, x, \tan(e+fx\right.}{f}$$
$$= \frac{{}_2F_1\left(2,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\left(b(c\tan(e+fx))\right.}{f(1+np)}$$

Mathematica [C] time = 5.5656, size = 1060, normalized size = 17.38

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (2*(AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2] - 4*AppellF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + 4*AppellF1[(1 + n*p)/2, n*p, 3, (3 + n*p)/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Cos[e + f*x]^2*Tan[(e + f*x)/2]*(b*
(c*Tan[e + f*x])^n)^p)/(f*((AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*AppellF1[(1 + n*p)/2, n*p, 2, (3 +
 n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*AppellF1[(1 + n*p)/2,
 n*p, 3, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*
x)/2]^2 + n*p*(AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] - 4*AppellF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*AppellF1[(1 + n*p)/2, n*p, 3, (3 +
 n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Sec[e
 + f*x] + (2*(1 + n*p)*(-AppellF1[(3 + n*p)/2, n*p, 2, (5 + n*p)/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 8*AppellF1[(3 + n*p)/2, n*p, 3, (5 + n*
p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 12*AppellF1[(3 + n*p)/2, n
*p, 4, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*p*AppellF1
[(3 + n*p)/2, 1 + n*p, 1, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] - 4*n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + 4*n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 3, (5 + n*
p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e +
 f*x)/2]^2)/(3 + n*p) - 2*n*p*(AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*AppellF1[(1 + n*p)/2, n*p, 2, (
3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*AppellF1[(1 + n*p)
/2, n*p, 3, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[e +
f*x]*Tan[(e + f*x)/2]^2))
```

Maple [F] time = 5.872, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cos(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n} b\right)^{p} \cos\left(fx+e\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^{n} b\right)^{p} \cos\left(fx+e\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n} b\right)^{p} \cos\left(fx+e\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^2, x)
```

3.482 $\int \sec^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=93

$$\frac{\tan(e+fx)\sec^3(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+4)}\text{Hypergeometric2F1}\left(\frac{1}{2}(np+1),\frac{1}{2}(np+4),\frac{1}{2}(np+3),\sin^2(e+fx)\right)\left(b(\right.}{f(np+1)}$$

```
[Out] ((Cos[e + f*x]^2)^((4 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (4 + n*p)/2,
 (3 + n*p)/2, Sin[e + f*x]^2]*Sec[e + f*x]^3*Tan[e + f*x]*(b*(c*Tan[e + f*x
])^n)^p)/(f*(1 + n*p))
```

Rubi [A] time = 0.0968147, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3659, 2617}

$$\frac{\tan(e+fx)\sec^3(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+4)}\,{}_2F_1\left(\frac{1}{2}(np+1),\frac{1}{2}(np+4);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((4 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (4 + n*p)/2,
 (3 + n*p)/2, Sin[e + f*x]^2]*Sec[e + f*x]^3*Tan[e + f*x]*(b*(c*Tan[e + f*x
])^n)^p)/(f*(1 + n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx &= \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sec^3(e+fx)(c\tan(e+fx))^{np}\,d\\ &= \frac{\cos^2(e+fx)^{\frac{1}{2}(4+np)}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(4+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\sec^3}{f(1+np)}\end{aligned}$$

Mathematica [A] time = 0.111356, size = 81, normalized size = 0.87

$$\frac{\csc(e+fx)\sec^2(e+fx)\left(-\tan^2(e+fx)\right)^{\frac{1}{2}(1-np)}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}(1-np),\frac{5}{2},\sec^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Csc[e + f*x]*Hypergeometric2F1[3/2, (1 - n*p)/2, 5/2, Sec[e + f*x]^2]*Sec[
e + f*x]^2*(-Tan[e + f*x]^2)^((1 - n*p)/2)*(b*(c*Tan[e + f*x])^n)^p)/(3*f)
```

Maple [F] time = 0.537, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^3 \left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sec(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \sec\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sec(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \sec\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*sec(f*x + e)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^n\right)^p \sec^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*sec(e + f*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n} b\right)^{p} \sec\left(fx+e\right)^{3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sec(f*x + e)^3, x)
```

3.483 $\int \sec(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=91

$$\frac{\tan(e+fx)\sec(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+2)}\text{Hypergeometric2F1}\left(\frac{1}{2}(np+1),\frac{1}{2}(np+2),\frac{1}{2}(np+3),\sin^2(e+fx)\right)\left(b(c\tan\right.}{f(np+1)}$$

```
[Out] ((Cos[e + f*x]^2)^((2 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (2 + n*p)/2,
 (3 + n*p)/2, Sin[e + f*x]^2]*Sec[e + f*x]*Tan[e + f*x]*(b*(c*Tan[e + f*x])
^n)^p)/(f*(1 + n*p))
```

Rubi [A] time = 0.0566201, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3659, 2617}

$$\frac{\tan(e+fx)\sec(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+2)}\,{}_2F_1\left(\frac{1}{2}(np+1),\frac{1}{2}(np+2);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((2 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (2 + n*p)/2,
 (3 + n*p)/2, Sin[e + f*x]^2]*Sec[e + f*x]*Tan[e + f*x]*(b*(c*Tan[e + f*x])
^n)^p)/(f*(1 + n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rubi steps

$$\int \sec(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sec(e+fx)(c\tan(e+fx))^{np}\,dx$$

$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(2+np)}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(2+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\sec(e+\right.}{f(1+np)}$$

Mathematica [A] time = 0.0769891, size = 70, normalized size = 0.77

$$\frac{\csc(e+fx)\left(-\tan^2(e+fx)\right)^{\frac{1}{2}(1-np)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{3}{2},\sec^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Csc[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, 3/2, Sec[e + f*x]^2]*(-Ta
n[e + f*x]^2)^((1 - n*p)/2)*(b*(c*Tan[e + f*x])^n)^p)/f
```

Maple [F] time = 3.32, size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)\left(b\left(c\tan\left(fx+e\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sec(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*sec(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*sec(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n} b\right)^{p} \sec\left(fx+e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sec(f*x + e), x)
```

3.484 $\int \cos(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=79

$$\frac{\sin(e+fx)\cos^2(e+fx)^{\frac{np}{2}}\,\text{Hypergeometric2F1}\left(\frac{np}{2},\frac{1}{2}(np+1),\frac{1}{2}(np+3),\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

```
[Out] ((Cos[e + f*x]^2)^((n*p)/2)*Hypergeometric2F1[(n*p)/2, (1 + n*p)/2, (3 + n*
p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rubi [A] time = 0.0755335, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3659, 2617}

$$\frac{\sin(e+fx)\cos^2(e+fx)^{\frac{np}{2}}\,{}_2F_1\left(\frac{np}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((n*p)/2)*Hypergeometric2F1[(n*p)/2, (1 + n*p)/2, (3 + n*
p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rubi steps

$$\int \cos(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \cos(e+fx)(c\tan(e+fx))^{np}\,dx$$
$$= \frac{\cos^2(e+fx)^{\frac{np}{2}}\,{}_2F_1\left(\frac{np}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\sin(e+fx)\left(b(c\text{t}\right.}{f(1+np)}$$

Mathematica [C] time = 3.50856, size = 482, normalized size = 6.1

$$\frac{}{2f(np+1)\left((np+3)F_1\left(\frac{1}{2}(np+1);np,1;\frac{1}{2}(np+3);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\left(\tan^2\left(\frac{1}{2}(e+fx)\right)\left(F_1\left(\right.\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((3 + n*p)*(AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] - 2*AppellF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[2*(e + f*x)]*(b*(c*Tan[e + f*x])^n)^
p)/(2*f*(1 + n*p)*((3 + n*p)*AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*((3 + n*p)*AppellF1[(1 + n*p)/2,
n*p, 2, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (AppellF1[(
3 + n*p)/2, n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] -
 4*AppellF1[(3 + n*p)/2, n*p, 3, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] - n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 1, (5 + n*p)/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 2, (5
 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))
```

Maple [F] time = 7.773, size = 0, normalized size = 0.

$$\int \cos \left(fx+e\right) \left(b \left(c \tan \left(fx+e\right)\right)^{n}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cos(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c \tan \left(fx+e\right)\right)^{n} b\right)^{p} \cos \left(fx+e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\left(c \tan \left(fx+e\right)\right)^{n} b\right)^{p} \cos \left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*cos(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\cos\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*cos(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e), x)
```

3.485 $\int \cos^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=82

$$\frac{\sin(e+fx)\cos^2(e+fx)^{\frac{np}{2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(np-2),\frac{1}{2}(np+1),\frac{1}{2}(np+3),\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

```
[Out] ((Cos[e + f*x]^2)^((n*p)/2)*Hypergeometric2F1[(-2 + n*p)/2, (1 + n*p)/2, (3
 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n
*p))
```

Rubi [A] time = 0.0918643, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3659, 2617}

$$\frac{\sin(e+fx)\cos^2(e+fx)^{\frac{np}{2}}\,{}_2F_1\left(\frac{1}{2}(np-2),\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((n*p)/2)*Hypergeometric2F1[(-2 + n*p)/2, (1 + n*p)/2, (3
 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n
*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rubi steps

$$\int \cos^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \cos^3(e+fx)(c\tan(e+fx))^{np}\,dx$$
$$=\frac{\cos^2(e+fx)^{\frac{np}{2}}\,{}_2F_1\left(\frac{1}{2}(-2+np),\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\sin(e+fx}{f(1+np)}$$

Mathematica [C] time = 6.56998, size = 1552, normalized size = 18.93

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((6 + 2*n*p)*(AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] - 6*AppellF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 12*AppellF1[(1 + n*p)/2, n*p, 3, (3 +
 n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 8*AppellF1[(1 + n*p)/2,
 n*p, 4, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Cos[(e + f*
x)/2]^3*Cos[e + f*x]^3*Sin[(e + f*x)/2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n
*p)*(-AppellF1[(3 + n*p)/2, n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] + 12*AppellF1[(3 + n*p)/2, n*p, 3, (5 + n*p)/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] - 36*AppellF1[(3 + n*p)/2, n*p, 4, (5 + n*p)/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 32*AppellF1[(3 + n*p)/2, n*p,
5, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*p*AppellF1[(3
+ n*p)/2, 1 + n*p, 1, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 - 6*n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + 12*n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 3, (5 + n*p)/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 8*n*p*AppellF1[(3 + n*p)/2, 1
 + n*p, 4, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + n*p
)*AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2]*Cos[(e + f*x)/2]^2 - 18*AppellF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 - 6*n*p*Appe
llF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*Cos[(e + f*x)/2]^2 + 36*AppellF1[(1 + n*p)/2, n*p, 3, (3 + n*p)/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 12*n*p*AppellF1[
(1 + n*p)/2, n*p, 3, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*
Cos[(e + f*x)/2]^2 - 8*(3 + n*p)*AppellF1[(1 + n*p)/2, n*p, 4, (3 + n*p)/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + AppellF1[(3
+ n*p)/2, n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos
[e + f*x] - 12*AppellF1[(3 + n*p)/2, n*p, 3, (5 + n*p)/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x] + 36*AppellF1[(3 + n*p)/2, n*p, 4, (5
+ n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x] - 32*Appell
F1[(3 + n*p)/2, n*p, 5, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*Cos[e + f*x] - n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 1, (5 + n*p)/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x] + 6*n*p*AppellF1[(3 + n*p)/
2, 1 + n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e
+ f*x] - 12*n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 3, (5 + n*p)/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x] + 8*n*p*AppellF1[(3 + n*p)/2, 1 +
 n*p, 4, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]
))
```

Maple [F] time = 6.417, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^{3}\left(b\left(c\tan\left(fx+e\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cos(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \cos\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \cos\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \cos\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^3, x)
```

3.486 $\int (d\sec(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left((d\sec(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p, x\right)$$

[Out] Unintegrable[(d*Sec[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]

Rubi [A] time = 0.0564878, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (d\sec(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p dx$$

Verification is Not applicable to the result.

[In] Int[(d*Sec[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]

[Out] Defer[Int][(d*Sec[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]

Rubi steps

$$\int (d\sec(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p dx = \int (d\sec(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p dx$$

Mathematica [A] time = 2.68608, size = 0, normalized size = 0.

$$\int (d\sec(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p dx$$

Verification is Not applicable to the result.

[In] Integrate[(d*Sec[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]

[Out] Integrate[(d*Sec[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]

Maple [A] time = 2.257, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^m \left(a+b\left(c\tan\left(fx+e\right)\right)^n\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)

[Out] int((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b+a\right)^{p}\left(d\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*sec(f*x + e))^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^{n}b+a\right)^{p}\left(d\sec\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*(d*sec(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b+a\right)^{p}\left(d\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*sec(f*x + e))^m, x)
```

3.487 $\int \sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable[Sec[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi [A] time = 0.0532297, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Defer[Int][Sec[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Mathematica [A] time = 3.62258, size = 0, normalized size = 0.

$$\int \sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Integrate[Sec[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Maple [A] time = 0.381, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^3\left(a+b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b+a\right)^{p}\sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^{n}b+a\right)^{p}\sec\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b+a\right)^{p}\sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^3, x)
```

3.488 $\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable[Sec[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi [A] time = 0.0275891, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Defer[Int][Sec[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Mathematica [A] time = 1.74632, size = 0, normalized size = 0.

$$\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Integrate[Sec[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Maple [A] time = 0.305, size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)\left(a+b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b+a\right)^{p}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^{n}b+a\right)^{p}\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((a + b*(c*tan(e + f*x))**n)**p*sec(e + f*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b+a\right)^{p}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e), x)
```

3.489 $\int \cos(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\cos(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable[Cos[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi [A] time = 0.0414141, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \cos(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Defer[Int][Cos[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \cos(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \cos(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Mathematica [A] time = 3.2515, size = 0, normalized size = 0.

$$\int \cos(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Integrate[Cos[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Maple [A] time = 0.339, size = 0, normalized size = 0.

$$\int \cos\left(fx+e\right)\left(a+b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n} b + a\right)^{p} \cos\left(fx+e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^{n} b + a\right)^{p} \cos\left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n} b + a\right)^{p} \cos\left(fx+e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e), x)
```

3.490 $\int \cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

[Out] Unintegrable[Cos[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p, x]

Rubi [A] time = 0.053067, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

[In] Int[Cos[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p,x]

[Out] Defer[Int][Cos[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p, x]

Rubi steps

$$\int \cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Mathematica [A] time = 10.5382, size = 0, normalized size = 0.

$$\int \cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Cos[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p,x]

[Out] Integrate[Cos[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p, x]

Maple [A] time = 0.75, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^3\left(a+b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)

[Out] int(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c \tan \left(fx + e \right) \right)^n b + a \right)^p \cos \left(fx + e \right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c \tan \left(fx + e \right) \right)^n b + a \right)^p \cos \left(fx + e \right)^3, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c \tan \left(fx + e \right) \right)^n b + a \right)^p \cos \left(fx + e \right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e)^3, x)
```

3.491 $\int \sec^6(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=244

$$\frac{\tan^5(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{n},-p,\frac{n+5}{n},-\frac{b(c\tan(e+fx))^n}{a}\right)}{5f}+\frac{2\tan}{}$$

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e + f*x])^n)/a)]*Tan
[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p)/(f*(1 + (b*(c*Tan[e + f*x])^n)/a)^p
) + (2*Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*(c*Tan[e + f*x])^n)/a)]*T
an[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p)/(3*f*(1 + (b*(c*Tan[e + f*x])^n
)/a)^p) + (Hypergeometric2F1[5/n, -p, (5 + n)/n, -((b*(c*Tan[e + f*x])^n)/a
)]*Tan[e + f*x]^5*(a + b*(c*Tan[e + f*x])^n)^p)/(5*f*(1 + (b*(c*Tan[e + f*x
])^n)/a)^p)
```

Rubi [A] time = 0.190906, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3675, 1893, 246, 245, 365, 364}

$$\frac{\tan^5(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{n},-p;\frac{n+5}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)}{5f}+\frac{2\tan^3(e+fx)\left(a+b(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + b*(c*Tan[e + f*x])^n)^p,x]
```


```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e + f*x])^n)/a)]*Tan
[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p)/(f*(1 + (b*(c*Tan[e + f*x])^n)/a)^p
) + (2*Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*(c*Tan[e + f*x])^n)/a)]*T
an[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p)/(3*f*(1 + (b*(c*Tan[e + f*x])^n
)/a)^p) + (Hypergeometric2F1[5/n, -p, (5 + n)/n, -((b*(c*Tan[e + f*x])^n)/a
)]*Tan[e + f*x]^5*(a + b*(c*Tan[e + f*x])^n)^p)/(5*f*(1 + (b*(c*Tan[e + f*x
])^n)/a)^p)
```


Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
```

```
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \sec^6(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\frac{\text{Subst}\left(\int \left(c^2+x^2\right)^2(a+bx^n)^p\,dx,x,c\tan(e+fx)\right)}{c^5 f}$$

$$=\frac{\text{Subst}\left(\int \left(c^4(a+bx^n)^p+2c^2x^2(a+bx^n)^p+x^4(a+bx^n)^p\right)dx,x,c\tan(e+\right.}{c^5 f}$$

$$=\frac{\text{Subst}\left(\int x^4(a+bx^n)^p\,dx,x,c\tan(e+fx)\right)}{c^5 f}+\frac{2\,\text{Subst}\left(\int x^2(a+bx^n)^p\,dx\right.}{c^3 f}$$

$$=\frac{\left(\left(a+b(c\tan(e+fx))^n\right)^p\left(1+\frac{b(c\tan(e+fx))^n}{a}\right)^{-p}\right)\text{Subst}\left(\int x^4\left(1+\frac{bx^n}{a}\right)^p dx\right.}{c^5 f}$$

$$=\frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)\tan(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(1+\right.}{f}$$

Mathematica [A] time = 2.19323, size = 165, normalized size = 0.68

$$\frac{\tan(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}\left(3\tan^4(e+fx)\text{Hypergeometric2F1}\left(\frac{5}{n},-p,\frac{n+5}{n},-\frac{b(c\tan(e+fx)}{a}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Tan[e + f*x]*(15*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e +
 f*x])^n)/a)] + 10*Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*(c*Tan[e + f*
x])^n)/a)]*Tan[e + f*x]^2 + 3*Hypergeometric2F1[5/n, -p, (5 + n)/n, -((b*(c
*Tan[e + f*x])^n)/a)]*Tan[e + f*x]^4)*(a + b*(c*Tan[e + f*x])^n)^p)/(15*f*(
1 + (b*(c*Tan[e + f*x])^n)/a)^p)
```

Maple [F] time = 0.428, size = 0, normalized size = 0.

$$\int \left(\sec \left(fx+e\right)\right)^{6} \left(a+b\left(c\tan \left(fx+e\right)\right)^{n}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sec(f*x+e)^6*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\left(c\tan \left(fx+e\right)\right)^{n} b+a\right)^{p} \sec \left(fx+e\right)^{6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan \left(fx+e\right)\right)^{n} b+a\right)^{p} \sec \left(fx+e\right)^{6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^6, x)
```

3.492 $\int \sec^4(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=160

$$\frac{\tan^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{n},-p,\frac{n+3}{n},-\frac{b(c\tan(e+fx))^n}{a}\right)}{3f}+\frac{\tan(e}{}$$

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e + f*x])^n)/a)]*Tan
[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p)/(f*(1 + (b*(c*Tan[e + f*x])^n)/a)^p
) + (Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*(c*Tan[e + f*x])^n)/a)]*Tan
[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p)/(3*f*(1 + (b*(c*Tan[e + f*x])^n)/
a)^p)
```

Rubi [A] time = 0.130309, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3675, 1893, 246, 245, 365, 364}

$$\frac{\tan^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{n},-p;\frac{n+3}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)}{3f}+\frac{\tan(e+fx)\left(a+b(c\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e + f*x])^n)/a)]*Tan
[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p)/(f*(1 + (b*(c*Tan[e + f*x])^n)/a)^p
) + (Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*(c*Tan[e + f*x])^n)/a)]*Tan
[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p)/(3*f*(1 + (b*(c*Tan[e + f*x])^n)/
a)^p)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \sec^4(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx &= \frac{\text{Subst}\left(\int\left(c^2+x^2\right)(a+bx^n)^p\,dx,x,c\tan(e+fx)\right)}{c^3f}\\
&= \frac{\text{Subst}\left(\int\left(c^2(a+bx^n)^p+x^2(a+bx^n)^p\right)dx,x,c\tan(e+fx)\right)}{c^3f}\\
&= \frac{\text{Subst}\left(\int x^2(a+bx^n)^p\,dx,x,c\tan(e+fx)\right)}{c^3f}+\frac{\text{Subst}\left(\int(a+bx^n)^p\,dx,x,c\right.}{cf}\\
&= \frac{\left(\left(a+b(c\tan(e+fx))^n\right)^p\left(1+\frac{b(c\tan(e+fx))^n}{a}\right)^{-p}\right)\text{Subst}\left(\int x^2\left(1+\frac{bx^n}{a}\right)^p dx\right.}{c^3f}\\
&= \frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)\tan(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(1+\right.}{f}
\end{aligned}$$

Mathematica [A] time = 1.61931, size = 122, normalized size = 0.76

$$\frac{\tan(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}\left(\tan^2(e+fx)\text{Hypergeometric2F1}\left(\frac{3}{n},-p,\frac{n+3}{n},-\frac{b(c\tan(e+fx))^n}{a}\right.\right.}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Tan[e + f*x]*(3*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e +
f*x])^n)/a)] + Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*(c*Tan[e + f*x])^
n)/a)]*Tan[e + f*x]^2)*(a + b*(c*Tan[e + f*x])^n)^p)/(3*f*(1 + (b*(c*Tan[e
+ f*x])^n)/a)^p)
```

Maple [F] time = 0.378, size = 0, normalized size = 0.

$$\int\left(\sec\left(fx+e\right)\right)^4\left(a+b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sec(f*x+e)^4*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b+a\right)^{p}\sec\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^{n}b+a\right)^{p}\sec\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b+a\right)^{p}\sec\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^4, x)
```

3.493 $\int \sec^2(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=75

$$\frac{\tan(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{n},-p,\frac{1}{n}+1,-\frac{b(c\tan(e+fx))^n}{a}\right)}{f}$$

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e + f*x])^n)/a)]*Tan
[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p)/(f*(1 + (b*(c*Tan[e + f*x])^n)/a)^p
)
```

Rubi [A] time = 0.0769877, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3675, 246, 245}

$$\frac{\tan(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e + f*x])^n)/a)]*Tan
[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p)/(f*(1 + (b*(c*Tan[e + f*x])^n)/a)^p
)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx = \frac{\text{Subst}\left(\int (a+bx^n)^p\,dx, x, c\tan(e+fx)\right)}{cf}$$
$$= \frac{\left((a+b(c\tan(e+fx))^n)^p\left(1+\frac{b(c\tan(e+fx))^n}{a}\right)^{-p}\right)\text{Subst}\left(\int\left(1+\frac{bx^n}{a}\right)^p dx\right.}{cf}$$
$$= \frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)\tan(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\right.}{f}$$

Mathematica [A] time = 0.109588, size = 75, normalized size = 1.

$$\frac{\tan(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{n},-p,\frac{1}{n}+1,-\frac{b(c\tan(e+fx))^n}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e + f*x])^n)/a)]*Tan
[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p)/(f*(1 + (b*(c*Tan[e + f*x])^n)/a)^p
)
```

Maple [F] time = 0.333, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^2\left(a+b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sec(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p \sec\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p \sec\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b+a\right)^{p}\sec\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^2, x)
```

3.494 $\int \left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi [A] time = 0.0144434, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Defer[Int][(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Mathematica [A] time = 0.768884, size = 0, normalized size = 0.

$$\int \left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Integrate[(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Maple [A] time = 0.297, size = 0, normalized size = 0.

$$\int \left(a+b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c \tan \left(fx + e\right)\right)^{n} b + a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c \tan \left(fx + e\right)\right)^{n} b + a\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \left(c \tan \left(e + fx\right)\right)^{n}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((a + b*(c*tan(e + f*x))**n)**p, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c \tan \left(fx + e\right)\right)^{n} b + a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p, x)
```

3.495 $\int \cos^2(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\cos^2(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable[Cos[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi [A] time = 0.0533736, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \cos^2(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Defer[Int][Cos[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx = \int \cos^2(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Mathematica [A] time = 8.13873, size = 0, normalized size = 0.

$$\int \cos^2(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Integrate[Cos[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Maple [A] time = 0.501, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(a+b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b+a\right)^{p}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^{n}b+a\right)^{p}\cos\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b+a\right)^{p}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e)^2, x)
```

3.496 $\int (d\csc(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=98

$$\frac{\tan(e+fx)\cos^2(e+fx)^{p+\frac{1}{2}}\left(b\tan^2(e+fx)\right)^p(d\csc(e+fx))^m\text{Hypergeometric2F1}\left(\frac{1}{2}(2p+1),\frac{1}{2}(-m+2p+1),\frac{1}{2}(-\right.}{f(-m+2p+1)}$$

```
[Out] ((Cos[e + f*x]^2)^(1/2 + p)*(d*Csc[e + f*x])^m*Hypergeometric2F1[(1 + 2*p)/
2, (1 - m + 2*p)/2, (3 - m + 2*p)/2, Sin[e + f*x]^2]*Tan[e + f*x]*(b*Tan[e
+ f*x]^2)^p)/(f*(1 - m + 2*p))
```

Rubi [A] time = 0.189325, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3658, 2618, 2602, 2577}

$$\frac{\tan(e+fx)\cos^2(e+fx)^{p+\frac{1}{2}}\left(b\tan^2(e+fx)\right)^p(d\csc(e+fx))^m\,{}_2F_1\left(\frac{1}{2}(2p+1),\frac{1}{2}(-m+2p+1);\frac{1}{2}(-m+2p+3);\sin^2(\right.}{f(-m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^(1/2 + p)*(d*Csc[e + f*x])^m*Hypergeometric2F1[(1 + 2*p)/
2, (1 - m + 2*p)/2, (3 - m + 2*p)/2, Sin[e + f*x]^2]*Tan[e + f*x]*(b*Tan[e
+ f*x]^2)^p)/(f*(1 - m + 2*p))
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2618

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
```

```
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (d\csc(e+fx))^m \left(b\tan^2(e+fx)\right)^p dx = \left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int (d\csc(e+fx))^m \tan^{2p}(e+fx)\,dx$$
$$= \left((d\csc(e+fx))^m \left(\frac{\sin(e+fx)}{d}\right)^m \tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int \left(\frac{\sin(e$$
$$= \frac{\left(\cos^{2p}(e+fx)(d\csc(e+fx))^{1+m}\sin(e+fx)\left(\frac{\sin(e+fx)}{d}\right)^{m-2p}\left(b\tan^2(e+fx)\right)^p\right.}{d}$$
$$= \frac{\cos^2(e+fx)^{\frac{1}{2}+p}(d\csc(e+fx))^{1+m}\,{}_2F_1\left(\frac{1}{2}(1+2p),\frac{1}{2}(1-m+2p);\frac{1}{2}(3-m+2\right.}{df(1-m+2p)}$$

Mathematica [C] time = 1.92321, size = 299, normalized size = 3.05

$$-\frac{d(m-2p-3)\left(b\tan^2(e+fx)\right)^p}{f(m-2p-1)\left(2\tan^2\left(\frac{1}{2}(e+fx)\right)\left(-(m-1)F_1\left(-\frac{m}{2}+p+\frac{3}{2};2p,2-m;-\frac{m}{2}+p+\frac{5}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Csc[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((d*(-3 + m - 2*p)*AppellF1[1/2 - m/2 + p, 2*p, 1 - m, 3/2 - m/2 + p, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(d*Csc[e + f*x])^(-1 + m)*(b*Tan[e + f
*x]^2)^p)/(f*(-1 + m - 2*p)*((-3 + m - 2*p)*AppellF1[1/2 - m/2 + p, 2*p, 1
- m, 3/2 - m/2 + p, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*(-((-1 + m
)*AppellF1[3/2 - m/2 + p, 2*p, 2 - m, 5/2 - m/2 + p, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]) - 2*p*AppellF1[3/2 - m/2 + p, 1 + 2*p, 1 - m, 5/2 - m/2
 + p, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))
```

Maple [F] time = 0.924, size = 0, normalized size = 0.

$$\int \left(d\csc\left(fx+e\right)\right)^m \left(b\left(\tan\left(fx+e\right)\right)^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*csc(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2\right)^p \left(d\csc\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*csc(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^{2}\right)^{p}\left(d\csc\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2)^p*(d*csc(f*x + e))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan^{2}\left(e+fx\right)\right)^{p}\left(d\csc\left(e+fx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**p*(d*csc(e + f*x))**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(fx+e\right)^{2}\right)^{p}\left(d\csc\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*csc(f*x + e))^m, x)
```

3.497 $\int (d\csc(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx$

Optimal. Leaf size=127

$$\frac{\tan(e+fx)\sec^2(e+fx)^{-m/2}(d\csc(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1-m}{2};1-\frac{m}{2},-p;\frac{3-m}{2};-\tan^2(e+\right.}{f(1-m)}$$

```
[Out] (AppellF1[(1 - m)/2, 1 - m/2, -p, (3 - m)/2, -Tan[e + f*x]^2, -((b*Tan[e +
f*x]^2)/a)]*(d*Csc[e + f*x])^m*Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1
 - m)*(Sec[e + f*x]^2)^(m/2)*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.182889, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3681, 3667, 511, 510}

$$\frac{\tan(e+fx)\sec^2(e+fx)^{-m/2}(d\csc(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1-m}{2};1-\frac{m}{2},-p;\frac{3-m}{2};-\tan^2(e+\right.}{f(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[(1 - m)/2, 1 - m/2, -p, (3 - m)/2, -Tan[e + f*x]^2, -((b*Tan[e +
f*x]^2)/a)]*(d*Csc[e + f*x])^m*Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1
 - m)*(Sec[e + f*x]^2)^(m/2)*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 3681

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (
f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(d*Csc[e + f*x])^FracPart[m]*(Sin
[e + f*x]/d)^FracPart[m], Int[(a + b*(c*Tan[e + f*x])^n)^p/(Sin[e + f*x]/d)
^m, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !IntegerQ[m]
```

Rule 3667

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(f
f*(d*Sin[e + f*x])^m*(Sec[e + f*x]^2)^(m/2))/(f*Tan[e + f*x]^m), Subst[Int[
((ff*x)^m*(a + b*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]
/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (d\csc(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx = \left((d\csc(e+fx))^m \left(\frac{\sin(e+fx)}{d}\right)^m\right)\int \left(\frac{\sin(e+fx)}{d}\right)^{-m} \left(a+b\tan^2(e+\right.$$

$$= \frac{\left((d\csc(e+fx))^m \sec^2(e+fx)^{-m/2}\tan^m(e+fx)\right)\text{Subst}\left(\int x^{-m}\left(1+x^2\right.\right.}{f}$$

$$= \frac{\left((d\csc(e+fx))^m \sec^2(e+fx)^{-m/2}\tan^m(e+fx)\left(a+b\tan^2(e+fx)\right)^p\right.}{}$$

$$= \frac{F_1\left(\frac{1-m}{2};1-\frac{m}{2},-p;\frac{3-m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)(d\csc(e+fx))^m}{f(1-}$$

Mathematica [B] time = 3.50409, size = 292, normalized size = 2.3

$$-\frac{a(m-3)\cos^2(e+fx)\cot(e+fx)(d\csc(e+fx))^m\left(a+b\tan^2(e+f\right.}{f(m-1)\left(-2bpF_1\left(\frac{3}{2}-\frac{m}{2};1-\frac{m}{2},1-p;\frac{5}{2}-\frac{m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)-a(m-2)F_1\left(\frac{3}{2}-\frac{m}{2};2-\frac{m}{2},-p;\frac{5}{2}-\frac{m}{2};-\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Csc[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((a*(-3 + m)*AppellF1[1/2 - m/2, 1 - m/2, -p, 3/2 - m/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/a)]*Cos[e + f*x]^2*Cot[e + f*x]*(d*Csc[e + f*x])^m*(a
+ b*Tan[e + f*x]^2)^p)/(f*(-1 + m)*(-2*b*p*AppellF1[3/2 - m/2, 1 - m/2, 1 -
 p, 5/2 - m/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - a*(-2 + m)*Appel
lF1[3/2 - m/2, 2 - m/2, -p, 5/2 - m/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/a)] + a*(-3 + m)*AppellF1[1/2 - m/2, 1 - m/2, -p, 3/2 - m/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/a)]*Cot[e + f*x]^2)))
```

Maple [F] time = 0.773, size = 0, normalized size = 0.

$$\int \left(d\csc\left(fx+e\right)\right)^m \left(a+b\left(\tan\left(fx+e\right)\right)^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*csc(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p \left(d\csc\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*csc(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\left(d\csc\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*(d*csc(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(fx+e\right)^2+a\right)^p\left(d\csc\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*csc(f*x + e))^m, x)
```

3.498 $\int (d\csc(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx$

Optimal. Leaf size=104

$$\frac{\tan(e+fx)(d\csc(e+fx))^m\cos^2(e+fx)^{\frac{1}{2}(np+1)}\left(b(c\tan(e+fx))^n\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2}(np+1),\frac{1}{2}(-m+np+\right.}{f(-m+np+1)}$$

```
[Out] ((Cos[e + f*x]^2)^((1 + n*p)/2)*(d*Csc[e + f*x])^m*Hypergeometric2F1[(1 + n
*p)/2, (1 - m + n*p)/2, (3 - m + n*p)/2, Sin[e + f*x]^2]*Tan[e + f*x]*(b*(c
*Tan[e + f*x])^n)^p)/(f*(1 - m + n*p))
```

Rubi [A] time = 0.213576, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3659, 2618, 2602, 2577}

$$\frac{\tan(e+fx)(d\csc(e+fx))^m\cos^2(e+fx)^{\frac{1}{2}(np+1)}\left(b(c\tan(e+fx))^n\right)^p {}_2F_1\left(\frac{1}{2}(np+1),\frac{1}{2}(-m+np+1);\frac{1}{2}(-m+np+3)\right.}{f(-m+np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n*p)/2)*(d*Csc[e + f*x])^m*Hypergeometric2F1[(1 + n
*p)/2, (1 - m + n*p)/2, (3 - m + n*p)/2, Sin[e + f*x]^2]*Tan[e + f*x]*(b*(c
*Tan[e + f*x])^n)^p)/(f*(1 - m + n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2618

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
```

```
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (d\csc(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (d\csc(e+fx))^m(c\tan(e+fx))$$

$$= \left((d\csc(e+fx))^m\left(\frac{\sin(e+fx)}{d}\right)^m (c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)$$

$$= \frac{\left(\cos^{np}(e+fx)(d\csc(e+fx))^{1+m}\sin(e+fx)\left(\frac{\sin(e+fx)}{d}\right)^{m-np}\left(b(c\tan(e+f\right.\right.}{d}$$

$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}(d\csc(e+fx))^{1+m}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(1-m+np);\frac{1}{2}(3-\right.}{df(1-m+}$$

Mathematica [C] time = 2.07088, size = 319, normalized size = 3.07

$$-\frac{d(m-np-3)(d\csc(e+}{f(m-np-1)\left((m-np-3)F_1\left(\frac{1}{2}(-m+np+1);np,1-m;\frac{1}{2}(-m+np+3);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Csc[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((d*(-3 + m - n*p)*AppellF1[(1 - m + n*p)/2, n*p, 1 - m, (3 - m + n*p)/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(d*Csc[e + f*x])^(-1 + m)*(b*(c*Ta
n[e + f*x])^n)^p)/(f*(-1 + m - n*p)*((-3 + m - n*p)*AppellF1[(1 - m + n*p)/
2, n*p, 1 - m, (3 - m + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] -
2*((-1 + m)*AppellF1[(3 - m + n*p)/2, n*p, 2 - m, (5 - m + n*p)/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*p*AppellF1[(3 - m + n*p)/2, 1 + n*p, 1
 - m, (5 - m + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f
*x)/2]^2)))
```

Maple [F] time = 0.292, size = 0, normalized size = 0.

$$\int \left(d\csc\left(fx+e\right)\right)^m\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*csc(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\left(d\csc\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*csc(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\left(d\csc\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*(d*csc(f*x + e))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\left(d\csc\left(e+fx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*(d*csc(e + f*x))**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\left(d\csc\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*csc(f*x + e))^m, x)
```

3.499 $\int (d\csc(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p \, dx$

Optimal. Leaf size=56

$$\left(\frac{\sin(e+fx)}{d}\right)^m (d\csc(e+fx))^m \text{Unintegrable}\left(\left(\frac{\sin(e+fx)}{d}\right)^{-m} \left(a+b(c\tan(e+fx))^n\right)^p, x\right)$$

```
[Out] (d*Csc[e + f*x])^m*(Sin[e + f*x]/d)^m*Unintegrable[(a + b*(c*Tan[e + f*x])^
n)^p/(Sin[e + f*x]/d)^m, x]
```

Rubi [A] time = 0.131539, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (d\csc(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*Csc[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (d*Csc[e + f*x])^m*(Sin[e + f*x]/d)^m*Defer[Int][(a + b*(c*Tan[e + f*x])^n)
^p/(Sin[e + f*x]/d)^m, x]
```

Rubi steps

$$\int (d\csc(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p \, dx = \left((d\csc(e+fx))^m \left(\frac{\sin(e+fx)}{d}\right)^m\right) \int \left(\frac{\sin(e+fx)}{d}\right)^{-m} \left(a+b(c\tan(e+\right.$$

Mathematica [A] time = 2.68349, size = 0, normalized size = 0.

$$\int (d\csc(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Csc[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Integrate[(d*Csc[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Maple [A] time = 2.397, size = 0, normalized size = 0.

$$\int \left(d\csc\left(fx+e\right)\right)^m \left(a+b\left(c\tan\left(fx+e\right)\right)^n\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*csc(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c \tan \left(fx + e \right) \right)^n b + a \right)^p \left(d \csc \left(fx + e \right) \right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*csc(f*x + e))^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\left(c \tan \left(fx + e \right) \right)^n b + a \right)^p \left(d \csc \left(fx + e \right) \right)^m, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*(d*csc(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**m*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c \tan \left(fx + e \right) \right)^n b + a \right)^p \left(d \csc \left(fx + e \right) \right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*csc(f*x + e))^m, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
              print("both result and optimal complex");
           fi;
           #both result and optimal complex
           if leaf_count_result<=2*leaf_count_optimal then
              return "A";
           else
              return "B";
           end if
        else #result contains complex but optimal is not
           if debug then
              print("result contains complex but optimal is not");
           fi;
           return "C";
        end if
     else # result do not contain complex
          # this assumes optimal do not as well
          if debug then
              print("result do not contain complex, this assumes optimal do not as well");
          fi;
          if leaf_count_result<=2*leaf_count_optimal then
             if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
             fi;
             return "A";
          else
             if debug then
                print("leaf_count_result>2*leaf_count_optimal");
             fi;
             return "B";
          end if
     end if
  else #ExpnType(result) > ExpnType(optimal)
     if debug then
        print("ExpnType(result) > ExpnType(optimal)");
     fi;
     return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```